

CREST - GENES

Cours doctoraux 2024 – 2025

INFORMATION THEORY AND STATISTICAL PHYSICS PERSPECTIVES ON STOCHASTIC PROCESSES

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SCHEDULE	Monday	17 March 2024	From : 9:00 to 13:00	Room 2036
	Thursday	20 March 2024	From : 9:00 to 13:00	Room 2036

Summary :

Analysis of extremal behavior of stochastic processes is a key ingredient in a wide variety of applications, including probability, statistical physics, theoretical computer science, and learning theory. One way of capturing this quantitatively is via the expected supremum of the process. The fundamental result here is the majorizing measure theorem of Fernique and Talagrand. It relates the boundedness of random processes indexed by elements of a metric space to complexity measures arising from certain multiscale combinatorial structures, such as packing and covering trees.

The first part of the course will focus on subgaussian processes and will present an information-theoretic perspective on the majorizing measure theorem building on the ideas first outlined in a little-noticed preprint of Andreas Maurer. The main idea here is that the boundedness of random processes can be phrased in terms of the existence of efficient variable-length codes for the elements of the indexing metric space. The topics covered will include information-theoretic prerequisites (variable-length codes on compact metric spaces, Kraft-MacMillan inequality) and a self-contained derivation of upper bounds on expected suprema of subgaussian processes using several coding-theoretic constructions. A lower bound for Gaussian processes making use of Sudakov minoration and a recursive code construction will be presented as well.

The second part of the course will focus on Gaussian processes with a finite index set. Here, we will study expected values of certain smoothed, or “soft,” versions of the maximum. This circle of problems can be phrased using ideas and concepts from statistical physics of disordered systems. The topics covered will include prerequisites from statistical physics (the Gibbs variational principle for the equilibrium free energy and replica-symmetric representations of Gibbs averages) and from probability theory (Sudakov minoration). Then we will explain how these techniques lead to upper and lower bounds on expected soft maxima of a Gaussian process parametrized by the inverse temperature $\beta > 0$. In the zero-temperature limit $\beta \rightarrow \infty$, these bounds reduce to the usual Gaussian maximal inequalities. If time permits, we will provide an illustration in the context of the Random Energy Model, one of the simplest models of physical systems with random disorder.

Format, pre-requisites and audience

Prerequisites include basic familiarity with probability, stochastic processes, and concentration of measure. Background on information theory and statistical physics will not be assumed.

References

- Andreas Maurer, "Majorizing codes and measures," available at <http://www.andreas-maurer.eu/mmnotes.pdf>.
- Yifeng Chu and Maxim Raginsky, "Majorizing measures, codes, and information," in preparation (2025); conference version available at <https://arxiv.org/abs/2305.02960>.
- Yifeng Chu and Maxim Raginsky, "Talagrand meets Talagrand: upper and lower bounds on expected soft maxima of Gaussian processes with finite index sets" (2025); available at <https://arxiv.org/abs/2502.06709>.
- Michel Talagrand, *Upper and Lower Bounds for Stochastic Processes*, Springer, 2014.
- Michel Talagrand, *Mean Field Models for Spin Glasses, Volume I: Basic Examples*, Springer, 2011.