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Digital Business Models and Quality Investment*

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Abstract

In this paper, we study the competition between two horizontally differentiated digital platforms. Each platform can adopt either a *user-funded* or an *ad-funded* business model. The platforms also invest in service quality to attract users. We find that which business model is more conducive to quality investment depends on the relative value of the platforms' services to the marginal multi-homing user compared to the value of the marginal user to advertisers. From a consumer welfare perspective, in equilibrium, platforms tend to adopt the user-funded business model too often and insufficiently differentiate their business models. Our results also show that regulatory interventions in digital markets aiming at mitigating market power of ad-tech firms or improving privacy protection can induce shifts in business models, with potentially ambiguous effects on consumer welfare.

JEL Codes: L13, L41, L42.

Keywords: Platform, two-sided markets, business model, quality investment.

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1 Introduction

The last two decades have witnessed the emergence of a vibrant platform economy, bringing significant benefits to society and the economy due to sharp cost reductions (Goldfarb and Tucker, 2019) and innovation in products and services (Aguiar and Waldfogel, 2018). Incremental innovation by digital platforms to reduce service costs or improve quality is also occurring rapidly, particularly through the extensive use of user data and artificial intelligence algorithms. Still, the dominance of a few large platforms has raised competition and regulatory concerns about whether competition is working properly in these markets.

Digital platforms have adopted a variety of business models. Some platforms generate revenue by selling products or charging subscriptions for access to their digital services. Others offer their products or services for free to consumers and generate revenue by charging other sides of their market, typically advertisers. The choice of business model can affect competition in platform markets in several ways. In this paper we examine how platforms' choice of business model can affect their incentives to invest in quality-improving innovation, and how these investments, in turn, influence their business model decisions.

We study the competition between two horizontally differentiated platforms offering services (e.g., content) to consumers. Platforms can choose to be either *user-funded* or *ad-funded*. A user-funded platform is funded solely by the sale of a device or by charging membership fees to users. Examples of such platforms include Apple (which relies mainly on the sale of devices such as the iPhone) and Netflix (which relies primarily on subscription fees). Platforms can also choose to be (purely) ad-funded, that is, supported only by the sale of advertising space to advertisers. In this case, users access the platform for free, but ads can be a nuisance to them. Examples of ad-funded platforms include YouTube and Facebook.

Platforms can also invest to improve the quality of service for users. Consumers are heterogeneous in their preferences for the two platforms and decide which platform(s) to join. They can single-home (join only one platform) or multi-home (join both platforms), and we focus on equilibria with partial multi-homing. Consumers dislike ads, which represent a nuisance to them. As Anderson, Foros and Kind (2018), we assume that consumers do not observe the ad levels on the platforms before making their homing decision, but form rational expectations about these levels. We also assume that a second exposure to an ad is less valuable to the advertisers than the first exposure.

We begin by analyzing the platforms' pricing and quality investment decisions, taking their business models as given. We consider the three possible business model configurations that can arise: both platforms are user-funded, both are ad-funded, or one is user-funded while the other is ad-funded.

We find that quality investment is higher for a user-funded platform than for an ad-funded platform when the intrinsic value of the service to the marginal consumer, who is a multi-homer, exceeds the value that advertisers derive from exposing the marginal consumer to ads (and *vice versa*).

When a platform displays ads, we can consider the nuisance of advertising as the equivalent of a price to users. We can then compare the quality-adjusted "prices" in the different business model configurations. We find that this comparison depends only on the *return to ads*, that is, the ratio between the value of a marginal eyeball for advertisers and the nuisance of ads (Jullien and Bouvard, 2023), and the discount of a second impression for advertisers. When the return to ads is less than 1 (the "return" of a standard device or membership fee), a user-funded platform charges lower quality-adjusted prices. Conversely, if the return to ads is sufficiently higher than 1, the quality-adjusted price for the ad-funded platform is always lower.

In terms of total consumer surplus, when the return to ads is less than 1, consumers are better off when both platforms are user-funded. When the return to ads is greater than 1, consumers are better off when at least one platform is ad-funded. When the value of multi-homers in the advertising market is highly discounted, consumers prefer differentiated business models because it leads to higher quality investments.

In a second step, we endogenize the business model decisions of the platforms. In equilibrium, when the return to advertising is less than 1, both platforms choose to be user-funded, as it is more efficient to monetize the service through users than through advertisers. This configuration maximizes consumer surplus. However, as the return to ads increases and becomes slightly greater than 1, both platforms still tend to adopt the user-funded model, even though consumers would prefer at least one of them to be ad-funded. In other words, platforms choose the user-funded business model too often. This is because, as users have passive beliefs about the level of advertising, ad-funded platforms choose too high levels of advertising, which reduces the value of advertising as a revenue stream. When the return to ads is larger, at least one platform is ad-funded in equilibrium. In this case, however, we find that platforms do not sufficiently differentiate their business models in terms of consumer welfare.

Finally, we discuss the robustness of our results to two changes in assumptions. First, we allow platforms to adopt a *mixed* business model, where they can charge both users and advertisers. We show that under a simple condition, a platform operating under the mixed model does not charge users, in which case the mixed business model is equivalent to the ad-funded model. Second, we consider the case where users cannot multi-home and thus only single-home. We find that all three equilibrium business model configurations still exist. However, whenever the asymmetric business model configuration arises in equilibrium, it is not the preferred configuration for consumers, who would prefer that both platforms be ad-funded.

From a policy perspective, our findings suggest that platforms may adopt the user-funded business model too often and fail to sufficiently differentiate their business models. We also point out that regulatory interventions in digital markets – whether aimed at curbing the market power of ad-tech firms or strengthening privacy protection – can induce shifts in business models, with potentially ambiguous effects on consumer welfare.

Our paper is closely related to the strand of literature that investigates the business model decisions of multi-sided platforms. Casadesus-Masanell and Zhu (2010) study the best response of a high-quality incumbent platform when facing a low-quality, *ad-funded* entrant. They analyze strategic reactions in terms of prices or ad levels, but also consider the possibility for the incumbent to change its business model. They show that if the incumbent's quality advantage is not too large, it is optimal for the incumbent to adopt either a pure *user-funded* or *ad-funded* business model, rather than a mixed business model. Casadesus-Masanell and Hervas-Drane (2015) study the competition between two platforms that derive revenue from both subscriptions and the sale of consumer data in a secondary market, which consumers dislike. They show that the platforms have an incentive to focus on a single revenue stream and to differentiate their business models.

In Casadesus-Masanell and Zhu (2010) and Casadesus-Masanell and Hervas-Drane (2015), users are heterogeneous in their valuation for the service and can only single-home. By contrast, Calvano and Polo (2020) study the competition between two media platforms that can generate revenue from subscriptions and advertising, in a setting where users and advertisers are homogeneous and can multi-home. They show that even in the absence of agent heterogeneity, platforms can be differentiated in equilibrium. This is because the user-funded and ad-funded business models are strategic substitutes: if one media platform chooses to be more "user-funded" (by raising subscription fees and lowering ad levels), the rival has an incentive to be more "ad-funded" (by increasing ad levels).[1]

Our contribution to this line of literature is to endogenize the quality of service provided to users in a setting with multi-homing and to compare the business model configurations that emerge in equilibrium with those that would be preferred by consumers. In particular, we show that the relationship between the return to ads and consumer surplus is non-monotonic: a higher return to ads may induce a shift in business models, which may have ambiguous effects on consumer welfare.

Our paper also contributes to the strand of literature that investigates how platforms' innovation incentives are influenced by their business model.[2] In particular, Etro (2021) analyzes the competition between two platforms based on a device.[3] Each platform sells

[1] See also Kind, Nilssen and Sørgard (2009) and Weeds (2013).

[2] Other relevant, though less related, literature has examined the impact of the acquisition of startups by dominant platforms (see, e.g., Rasmusen (1988), Cunningham, Ederer and Ma (2021), Dijk, Moraga-González and Motchenkova (2024)) and the impact of mergers on innovation (see, e.g., Mermelstein et al. (2020), Federico, Langus and Valletti (2017), Denicolò and Polo (2018), Bourreau, Jullien and Lefouili (2024)).

[3] See also Casadesus-Masanell and Llanes (2015), who study the incentives to invest of a monopoly

a device with an app store that hosts services from independent app developers. One platform is "device-funded" (i.e., derives most of its revenue from the sale of the device, a situation we refer to as "user-funded" in our setup), while the other is "ad-funded" (i.e., derives revenue from advertising). Users choose between the user-funded or ad-funded platforms, i.e., single-home. Etro then analyzes the impact of the platforms' business models on their incentives to invest in platform quality and the development of new in-house apps. In particular, assuming that platform quality is a function of ad revenue per device, he shows that an ad-funded platform under-invests in quality (i.e., chooses too high an ad revenue) relative to a user-funded platform.

We complement his analysis by endogenizing the platforms' business model decisions in a setting where consumers can multi-home. We also model quality investment in a more direct way and find that quality investment may be higher on an ad-funded platform than on a user-funded platform if advertising revenues are sufficiently high.

The rest of the paper is organized as follows. In Section 2, we introduce the model. In Section 3, we take the platforms' business models as given and analyze their pricing and quality investment decisions in each possible business model configuration. In Section 4, we endogenize the business model decisions of platforms. In Section 5, we discuss the robustness of our results to changes in assumptions. First, we extend the baseline model to allow for a mixed business model in which platforms can charge both users and advertisers, and investigate whether platforms end up relying on a single revenue source in equilibrium. Second, we assume that users must single-home, which introduces more direct competitive interactions between the platforms. Finally, in Section 6 we discuss the policy implications of our results, and in Section 7 we conclude.

2 The model

We consider two platforms, 1 and 2, which offer services (e.g., content) to users at a constant marginal cost normalized to zero. The platforms are horizontally differentiated on an *Hotelling* line of length 1, with platform 1 located at $x_1 = 0$ and platform 2 at $x_2 = 1$. A mass 1 of users is uniformly distributed along the line. A user located at x incurs the transportation cost $|x - x_i|$ if she consumes the content of platform $i = 1, 2$. Users can single-home (join only one platform) or multi-home (join both platforms).[4]

The two platforms are also vertically differentiated. When consuming platform i's service, a consumer receives the utility $v + q_i$, where $v > 0$ is an intrinsic benefit and q_i is the quality of service for users. In order to provide a quality of service q_i, platform i must incur the investment cost $c(q_i) = q_i^2/2$. If the consumer joins both platforms, the

software or hardware platform to invest in quality, considering two possible business models, proprietary and open source. In particular, the authors show that investments in platform quality can be higher with the open source model if an open source platform attracts more users.

[4]In Section 5.2 we consider the case where users cannot multi-home and there is only single-homing.

total intrinsic benefit from the two platforms is $(1+\theta)v$, where $\theta \in [0,1]$. A lower value of θ means greater duplication in the intrinsic benefits offered by the platforms.[5]

A platform $i \in \{1,2\}$ can be either *user-funded* or *ad-funded*.[6] If it is user-funded, the platform charges users a device or membership fee $f_i \geq 0$ but does not display ads. For a user located at x, the net utility of joining platform i is then $u_i = v + q_i - f_i - |x - x_i|$. If it is ad-funded, the platform provides free access to users, but displays ads that users dislike. If users expect a level of advertising a_i^e on platform i, they face the disutility γa_i^e, with $\gamma > 0$. For a user located at x, the net utility of joining platform i is then $u_i = v + q_i - \gamma a_i^e - |x - x_i|$.

An ad-funded platform charges advertisers a price p_i to run an ad campaign. An ad campaign exposes all users of the platform to the ad. There is a unit mass of homogeneous advertisers who wish to run at most 1 ad campaign on each platform. Showing an ad once to a user generates a value of $\sigma > 0$ for an advertiser. Showing the ad twice to the same user generates a total value of $\sigma + \beta\sigma$, where $\beta \in [0,1]$ captures the discount applied to the second impression.[7]. The quantity of advertisers who eventually end up running an ad campaign on platform i is denoted by a_i.

To compare the ad-funded and user-funded business models, it will be useful to use the *return to ads* $r \equiv \sigma/\gamma$, as in Jullien and Bouvard (2023). When $r = 1$, ads provide the same return to the platforms as a user fee. The return to ads is higher than a user fee when $r > 1$, and lower when $r < 1$.

Timing. The timing is as follows:

- In Stage 1, platforms simultaneously choose their business model (user-funded or ad-funded). These choices are observable to all.
- In Stage 2, platforms choose their investment in quality q_i.
- In Stage 3, platforms observe each other's investment. The user-funded platforms

[5]For example, consider the case of music streaming platforms. The preference for one platform over another on the Hotelling line could be linked to the way each platform organizes and offers playlists to users. For instance, some users may prefer curated playlists, like on Apple Music, while others may prefer algorithmic playlists, like on Spotify. Then, θ would capture the overlap of music content between the two platforms, which tends to be relatively high in the case of music streaming. Finally, music streaming platforms may invest to improve the sound quality of their music content; e.g., Apple Music typically offers a higher sound quality (lossless ALAC at no extra cost) than Spotify, which does not offer lossless music and remains limited to compressed formats.

[6]As Etro (2021), we take the ad-funded and user-funded business models as given. In Section 5.1 we allow platforms to adopt a *mixed* business model, where they can charge both users and advertisers, and study whether the "pure" user-funded and ad-funded models can emerge endogenously. See also Casadesus-Masanell and Hervas-Drane (2015) and Calvano and Polo (2020) for models where business models emerge endogenously.

[7]The assumption that an ad is less effective if the user has already been informed is standard in the advertising literature when there is overlapping viewership. See, e.g., Ambrus and Reisinger (2006) and Anderson and Jullien (2015).

set their user fee f_i. The ad-funded platforms set their ad-campaign price p_i. Advertisers observe the ad prices and decide on which platform(s) to run an ad campaign.

- In Stage 4, users observe the business models of the platforms and the user fees, if any. They do not observe the level of advertising on the platforms, but form rational expectations about advertisers' participation on each platform. Users then decide which platform(s) to join.

With this timing, we assume that platforms can commit to a business model in Stage 1. In practice, platforms incur non-negligible costs in establishing a revenue stream, making the choice of a business model less flexible than pricing decisions.[8] Following Anderson, Foros and Kind (2018), we further assume that users do not observe ad levels and have passive beliefs about them. This means that they do not respond instantaneously to a change in ad levels, which seems reasonable in the context of digital services where platforms use sophisticated advertising techniques.[9] This also means that, in our setting, users are more reactive to prices (that they observe) than to ads.

We look for the subgame perfect Nash equilibria (SPNE) of this game.

Depending on platforms' business model decisions, either both platforms are user-funded, both are ad-funded, or one is ad-funded while the other is user-funded. In the rest of the paper, we denote by $X_{b_i b_j}$ the equilibrium value of X for platform i at Stage 2 of the game (where X can represent prices, quality investments, profits, etc.) when platform i has adopted the business model b_i and the rival platform $j \neq i$ has adopted the business model b_j, with $b_i, b_j \in \{A, U\}$.

Assumption 1 guarantees that for each business model configuration, in equilibrium the market is covered and there is *partial* multi-homing (i.e., some - but not all - users multi-home).

Assumption 1 ***(Market coverage and partial multi-homing)***

(a) $1/2 < \theta v \leq 1$

(b) $\sigma_{min} \equiv \frac{1}{\beta} \cdot \left(\frac{1}{2} + \gamma - \theta v\right) \leq \sigma \leq \sigma_{max} \equiv 1 + \gamma - \theta v$

[8] For example, it is costly for a platform to set up an advertising solution. As an order of magnitude, the CMA study on online platforms and digital advertising (2019) indicates that it takes 3-6 months to 18 months for a publisher to switch ad server (see par 199, `https://assets.publishing.service.gov.uk/media/5dfa172240f0b6217b108351/Appendix_H2.pdf`). Similarly, there are costs associated with providing a payment solution to users.

[9] If users observe ad levels, platforms need to take into account the effect of ad levels on demand. When facing a user-funded platform, an ad-funded platform will choose the same ad level as a monopoly platform. If it faces an ad-funded platform, multi-homing reduces its attractiveness to advertisers. Ad-funded platforms may then have an incentive to increase their ad levels to reduce multi-homing. See Anderson and Jullien (2015) and Ambrus, Calvano and Reisinger (2016).

Assumption 1*(a)* implies that in the symmetric configuration, where both platforms are user-funded (UU), the market is covered and there is partial multi-homing. Assumption 1*(b)*, combined with the previous one, implies that the market is covered and we have partial multi-homing in the other two configurations (UA and AA).[10]

3 Pricing and investment decisions

In this section, we take the platforms' business models as given. For each business model configuration, we examine the platforms' pricing and investment decisions, as well as the participation decisions of users and advertisers.

3.1 Both platforms are user-funded (UU)

In the last stage, users can single-home or multi-home. A user located at x receives the utility $U_{12} = (1+\theta)v+q_1+q_2-f_1-f_2-x-(1-x)$ by joining both platforms, whereas joining only platform 1 yields $U_1 = v + q_1 - f_1 - x$ and joining only platform 2 yields $U_2 = v + q_2 - f_2 - (1-x)$. The consumer indifferent between joining only platform 1 and joining both platforms is thus located at $\hat{x}_2^{UU} \equiv 1 - (\theta v + q_2 - f_2)$, while the consumer indifferent between joining only platform 2 and joining both platforms is located at $\hat{x}_1^{UU} \equiv \theta v + q_1 - f_1$.

Let SH_i denote the single-homing demand for platform i and MH_{12} denote the demand from multi-homers. The single-homing and multi-homing demands are as follows:

$$SH_1 = \hat{x}_2^{UU} = 1 - (\theta v + q_2 - f_2) \tag{1}$$

$$MH_{12} = \hat{x}_1^{UU} - \hat{x}_2^{UU} = 2\theta v - 1 + q_1 + q_2 - f_1 - f_2 \tag{2}$$

$$SH_2 = 1 - \hat{x}_1^{UU} = 1 - (\theta v + q_1 - f_1). \tag{3}$$

The total demand of platform i is $SH_i + MH_{12} = \theta v + q_i - f_i$, and it depends only on its quality and price. However, the composition of its demand between single-homers (SH_i) and multi-homers (M_{12}) depends on the quality and price of the rival platform $j \neq i$.

User fees. In stage 3, platforms set their fees, anticipating users' homing decisions and taking as given quality investments. Platform i's profit is given by:

$$\Pi_i(q_i, f_i) = f_i \cdot (\theta v + q_i - f_i) - c(q_i). \tag{4}$$

[10]For more details, see Appendix A.1.

Solving the first-order condition yields the optimal price:

$$f_i(q_i) = \frac{\theta v + q_i}{2}. \tag{5}$$

When setting its fee, the platform considers its effect on the marginal user, who is a multi-homer. As the user-funded platform cannot price discriminate between single-homers and multi-homers, the optimal fee depends on the valuation of the service by a multi-homer, $\theta v + q_i$.

Quality investment. In stage 2, platforms choose their level of investment, anticipating the effect of their investment on user demand and their ability to charge higher fees. Platform i's profit is given by:

$$\Pi_i(q_i, f_i(q_i)) = f_i(q_i)\left(\theta v + q_i - f_i(q_i)\right) - \frac{q_i^2}{2}. \tag{6}$$

The first-order condition is

$$\frac{\partial \Pi_i}{\partial q_i} = f_i(q_i) - c'(q_i) = \frac{\theta v + q_i}{2} - q_i = 0,$$

which gives the platform's optimal level of investment $q_i = \theta v$.

When choosing its level of investment, the platform considers the effect of a quality increase on the marginal consumer, a multi-homer.

Lemma 1 *When the two platforms are user-funded, in equilibrium, the platforms invest $q_{UU} = \theta v$ in quality and charge the fee $f_{UU} = \theta v$. Platforms' profits are $\Pi_{UU} = \frac{(\theta v)^2}{2}$.*

The parameter θ captures the duplication of intrinsic benefits across platforms. As it decreases, platforms can extract less value from users, reducing their incentive to invest in quality.

One can easily verify that under Assumption 1, there is partial multi-homing in equilibrium. Also, notice that the equilibrium profit Π_{UU} is strictly positive, meaning that in this symmetric configuration, it is always sustainable to be user-funded.[11]

Finally, in equilibrium consumer surplus -defined as the sum of all of users' net utility- is as follows:

$$\begin{aligned} CS_{UU} = & \underbrace{\int_0^{1-\theta v} (v - x)\, dx}_{\text{CS from platform 1's single-homers}} + \underbrace{\int_{1-\theta v}^{\theta v} [(1+\theta)v - 1]\, dx}_{\text{CS from multi-homers}} + \underbrace{\int_{\theta v}^{1} [v - (1-x)]\, dx}_{\text{CS from platform 2's single-homers}} \\ & = (\theta v)^2 + v(1-\theta). \end{aligned}$$

[11] Assumption 1 requires that $\theta v > 1/2$, so we necessarily have $\theta v > 0$, hence $\Pi_{UU} > 0$.

In this configuration, consumer surplus depends on the baseline utility that consumers receive from joining a platform and the overlap in stand-alone benefits. Under Assumption 1, CS_{UU} increases with θ.

3.2 One platform is user-funded, the other is ad-funded (UA)

Now, consider the asymmetric configuration in which one platform is user-funded, whereas the other is ad-funded. Without loss of generality, we assume that platform 1 is user-funded and platform 2 is ad-funded.

The utility of joining both platforms is $U_{12} = (1+\theta)v + q_1 + q_2 - f_1 - \gamma a_2^e - x - (1-x)$, whereas joining only platform 1 yields $U_1 = v + q_1 - f_1 - x$ and joining only platform 2 yields $U_2 = v + q_2 - \gamma a_2^e - (1-x)$. The single-homing and multi-homing demands are then as follows:

$$SH_1 = \hat{x}_2^{AU} \equiv 1 - (\theta v + q_2 - \gamma a_2^e) \tag{7}$$

$$MH_{12} = \hat{x}_1^{UA} - \hat{x}_2^{AU} = 2\theta v - 1 + q_1 + q_2 - f_1 - \gamma a_2^e \tag{8}$$

$$SH_2 = 1 - \hat{x}_1^{UA} \equiv 1 - (\theta v + q_1 - f_1). \tag{9}$$

The total demand of platform 1 is $\theta v + q_1 - f_1$ and the total demand of platform 2 is $\theta v + q_2 - \gamma a_2^e$. As above, the total demand for a given platform depends only on its quality and "price", i.e., the fee for a the user-funded platform and the nuisance of advertising for an ad-funded platform. In contrast, the composition of demand between single-homers and multi-homers depends on the quality and "price" of the rival platform.

Advertising price and advertisers' participation decision. In this asymmetric configuration, only platform 2 offers advertisers the possibility to run an ad campaign. Therefore, each impression is unique and valued at σ by advertisers, regardless of whether users single-home or multi-home. So, advertisers decide to run an ad campaign on platform 2 if

$$\sigma(\theta v + q_2 - \gamma a_2^e) - p_2 \geq 0. \tag{10}$$

Lemma 2 *There exists a unique equilibrium where the ad-funded platform sets the advertising price*

$$p_2 = \sigma(\theta v + q_2 - \gamma),$$

each advertiser runs an ad-campaign on platform 2, and users rationally expect $a_2^e = 1$.

Proof. Platform 2 can raise its ad price until the advertisers' participation constraint (10) binds, so $p_2 = \sigma(\theta v + q_2 - \gamma a_2^e)$. At this price, all advertisers place an ad on the platform, leading to an ad intensity of 1, which users rationally expect, so $a_2^e = 1$, giving the result. ■

The ad-funded platform holds a viewership monopoly over its users, whether they single-home or multi-home, enabling it to capture all of the advertisers' surplus. It can set its ad price at the advertisers' valuation and still induce them to run an ad campaign. As users do not observe advertisers' participation before deciding which platform(s) to join, an *ex-post* increase or decrease in the ad level will not affect user demand. Hence, the ad-funded platform charges the maximum ad price consistent with advertiser participation.[12] Users anticipate this and form their expectations about the level of advertising accordingly.

User fee. Because it monetizes all users in the same way (i.e., it does not price discriminate between single-homers and multi-homers), the user-funded platform is unaffected by the other platform's business model. Its pricing decision is affected by the total demand, but not by the composition of demand, while the rival platform's decisions affect the composition of demand, but not the total demand. As a result, the optimal fee is the same as in the UU configuration:

$$f_1(q_1) = \frac{\theta v + q_1}{2}.$$

Quality investment. In stage 2, platforms decide how much to invest in quality. First, consider the user-funded platform. As explained above, its decision is unaffected by the other platform's business model, hence its optimal investment is the same as in the UU configuration. We thus have

$$q_1 = q_{UU} = \theta v \equiv q_{UA}, \ f_1 = f_{UU} = \theta v \equiv f_{UA}$$
$$\text{and}$$
$$\Pi_1 = \Pi_{UU} = \tfrac{(\theta v)^2}{2} \equiv \Pi_{UA}.$$

Consider now the ad-funded platform. It maximizes its profit, $p_2 - c(q_2) = \sigma(\theta v + q_2 - \gamma) - \frac{q_2^2}{2}$, with respect to the investment level q_2. The first-order condition yields

$$q_2 = \sigma. \tag{11}$$

The ability of an ad-funded platform to charge a high price for ad space depends on user participation. What matters is the value to advertisers of the marginal user, who is a multi-homer. As the rival platform is user-funded and does not display ads, the value of a multi-homer to advertisers is the same as a single-homer and is equal to σ. We thus obtain the following result:

[12] Note that a per-impression price would yield the same results, with a platform ad price of σ per impression in equilibrium. Considering a per-click price would also lead to the same results. Assume that users have the same click-through rate μ, and let us denote the expected profit from the advertised product by ω. In this case, the platform would charge ω per click, with an expected profit per impression of $\sigma \equiv \mu\omega$.

Lemma 3 *When one platform is user-funded and the other is ad-funded, in equilibrium, the user-funded platform's investment, fee and profit are* $q_{UA} = \theta v = q_{UU}$, $f_{UA} = \theta v = f_{UU}$, *and* $\Pi_{UA} = \frac{(\theta v)^2}{2} = \Pi_{UU}$, *whereas the ad-funded platform's investment, price for ad space and profit are* $q_{AU} = \sigma$, $p_{AU} = \sigma(\theta v + \sigma - \gamma)$, *and* $\Pi_{AU} = \sigma(\theta v + \sigma - \gamma) - \frac{\sigma^2}{2}$.

Assumption 1 guarantees that we have partial multi-homing in equilibrium. Note that the equilibrium profits of the two platforms decrease when θ becomes lower, as the competition for users intensifies.

It is also interesting to note that a user-funded platform makes the same strictly positive profit whether it competes with a user-funded or an ad-funded platform. It follows that:

Remark 1 *A user-funded platform is profitable, regardless of the business model of the competing platform.*

For this reason, we do not need to make assumptions about the profitability of each business model in each configuration. When we examine the business model decisions, platforms will always have the possibility to make positive profits by being user-funded.

As in the UU configuration, the user-funded platform extracts all of the additional surplus generated by its investment. The ad-funded platform, however, does not charge the users directly. It generates an additional surplus σ from its investment from advertisers and "charges" users with an ad nuisance of γ. Consumer surplus is then as follows:

$$CS_{UA} = \underbrace{\int_0^{1-(\theta v+\sigma-\gamma)} (v-x)\ dx}_{\text{platform 1's single-homers}} + \underbrace{\int_{1-(\theta v+\sigma-\gamma)}^{\theta v} [(1+\theta)v + \sigma - \gamma - 1]\ dx}_{\text{multi-homers}} + \underbrace{\int_{\theta v}^{1} [(v+\sigma-\gamma) - (1-x)]\ dx}_{\text{platform 2's single-homers}},$$

that is,

$$CS_{UA} = (\theta v)^2 + v(1-\theta) + \frac{1}{2}(\sigma - \gamma)(2\theta v + \sigma - \gamma).$$

Under Assumption 1, CS_{UA} increases with the stand-alone utility v and with θ. It also increases in the value of an impression σ, and decreases in the ad-nuisance parameter γ.

3.3 Both platforms are ad-funded (AA)

Finally, we consider the symmetric configuration AA in which both platforms are ad-funded. The platforms are free to users, but they offer ad space to advertisers, exposing users to nuisance from advertising.

Users' homing decision. Users know that both platforms display ads, but they do not observe the levels of advertising before deciding which platform(s) to join. So, they base their homing decision on their expectations a_i^e about the level of advertising on each platform $i \in \{1,2\}$.

The utility of joining both platforms is $U_{12} = (1+\theta)v + q_1 + q_2 - \gamma a_1^e - \gamma a_2^e - x - (1-x)$, whereas joining only platform 1 yields $U_1 = v + q_1 - \gamma a_1^e - x$ and joining only platform 2 yields $U_2 = v + q_2 - \gamma a_2^e - (1-x)$. Hence, the single-homing and multi-homing demands are as follows:

$$\begin{aligned} SH_1 &= \hat{x}_2^{AA} \equiv 1 - (\theta v + q_2 - \gamma a_2^e) \\ MH_{12} &= \hat{x}_1^{AA} - x_2^{AA} = 2\theta v - 1 + q_1 + q_2 - \gamma(a_1^e + a_2^e) \\ SH_2 &= 1 - \hat{x}_1^{AA} \equiv 1 - (\theta v + q_1 - \gamma a_1^e). \end{aligned}$$

The total demand of platform i is $SH_i + MH_{12} = \theta v + q_i - \gamma a_i^e$. Once again, the total demand for platform i depends on its quality and "price" (i.e., ad nuisance for users), whereas the composition of its demand between single-homers and multi-homers depends on the strategy of the rival platform $j \neq i$.

Advertising price and advertisers' participation decision. In this configuration, both platforms offer ad space and some users multi-home. If an advertiser runs an ad campaign on both platforms, multi-homing users will be exposed to the ad twice. As the second impression is discounted by β, the value of showing an ad twice to a multi-homer is $\sigma(1+\beta)$.

Consider an advertiser running an ad campaign only on platform $i \in \{1,2\}$. This generates a gross value of $\sigma SH_i + \sigma MH_{12}$ for the advertiser, as the multi-homers are only reached once. The *incremental value* of running the ad campaign on the other platform $j \neq i$ as well is then $\sigma SH_j + \beta\sigma MH_{12}$. This allows the advertiser to reach platform j's single-homers for the first time and the multi-homers for the second time. This affects the platforms' pricing strategy, as shown by the following Lemma.

Lemma 4 *There exists a unique equilibrium in which each platform sets an ad-campaign price equal to*

$$p_i = \sigma SH_i + \beta\sigma MH_{12} \qquad (\text{for } i \in \{1,2\})$$

such that all advertisers participate, users rationally expect $a_i^e = 1$, $SH_i = 1-(\theta v+q_j-\gamma)$ *and* $MH_{12} = (\theta v + q_i - \gamma) + (\theta v + q_j - \gamma) - 1$.

Proof. The proof follows Anderson, Foros and Kind (2018)'s Proposition 1 and their *incremental pricing principle*. It is an equilibrium as no platform has an incentive to deviate. If a platform increases its price, it loses all advertiser demand. If the platform lowers its price, it will not be able to attract additional advertisers as they are all already

running an ad campaign. It is unique because in any other candidate equilibrium, the platforms have a profitable deviation which consists in pricing at this *incremental value* while still getting all advertisers to run an ad campaign. Hence, both platforms price at the *incremental value*, all advertisers advertise on both platforms, and users rationally expect $a_i^e = 1$ for $i \in \{1, 2\}$. ■

In this symmetric configuration, the discount parameter β captures the intensity of competition in the advertising market. As it decreases, the second impression becomes more discounted, leading to a lower *incremental value* and more intense competition in the advertising market. Multi-homers are less valuable to platforms than single-homers, as they do not enjoy a monopoly position on their viewership. Hence, in this symmetric configuration with two ad-funded platforms, the composition of demand matters for firms' profits.

Quality investment. In stage 2, platform i's profits are given by:

$$\Pi_i^{AA} = p_i - c(q_i) = \sigma SH_i(q_j) + \beta\sigma MH_{12}(q_i, q_j) - c(q_i). \qquad \text{(for } i \in \{1, 2\}\text{)}$$

Ad-funded platforms simultaneously choose their investment levels to maximize their profits. The first-order condition yields:

$$\beta\sigma \frac{\partial MH_{12}}{\partial q_i} = c'(q_i). \qquad \text{(for } i \in \{1, 2\}\text{)}$$

As $\partial MH_{12}/\partial q_i = 1$, we obtain $q_i = \beta\sigma$.

As in the other configurations, platforms consider the value of the marginal user they can attract by increasing their investment in quality. This user is a multi-homer, hence in equilibrium, platforms' investment is equal to $\beta\sigma$, which is lower than the investment made by an ad-funded platform in the asymmetric configuration UA.

Lemma 5 *When both platforms are ad-funded, in equilibrium, the platforms invest* $q_{AA} = \beta\sigma$ *in quality and charge* $p_{AA} = \sigma SH_{AA} + \beta\sigma MH_{AA}$ *for ad space, with* $SH_{AA} = 1 - (\theta v + \beta\sigma - \gamma)$ *and* $MH_{AA} = 2(\theta v + \beta\sigma - \gamma) - 1$. *Platforms' profits are* $\Pi_{AA} = \sigma(\theta v + \beta\sigma - \gamma) - \sigma(1 - \beta)\left[2(\theta v + \beta\sigma - \gamma) - 1\right] - \frac{(\beta\sigma)^2}{2}$.

Assumption 1 ensures that there is partial multi-homing in equilibrium.

Platform profits are decreasing in θ for $\beta < 1/2$ and increasing for $\beta > 1/2$.[13] The non-monotonicity of profits reflects two opposing effects. On the one hand, as θ increases (i.e., there is less duplication of intrinsic benefits for users), the total demand of each platform $(\theta v + \beta\sigma - \gamma)$ increases, which can be monetized with advertisers. On the other

[13] Denoting by $D_{AA} = \theta v + \beta\sigma - \gamma$ the total demand of each platform in equilibrium, we have $\frac{\partial \Pi_{AA}}{\partial \theta} = \sigma \frac{\partial D_{AA}}{\partial \theta} - \sigma(1 - \beta)\frac{\partial MH_{AA}}{\partial \theta} = \sigma \cdot v - \sigma(1 - \beta) \cdot (2v) > 0$ if and only if $\beta > 1/2$.

hand, multi-homing (MH_{AA}) also increases. Since multi-homers are less valuable than single-homers, this reduces the profit of the platforms. The first effect dominates the second when the discount applied to a second impression for advertisers is sufficiently low, that is, when β is sufficiently high.

Similarly, platform profits are decreasing in β for $\beta < \frac{1}{3} - \frac{2\theta v - 2\gamma - 1}{3\sigma}$, and increasing otherwise. Intuitively, while a higher β increases the value of multi-homers (taking the demand structure as given), it also increases the number of multi-homers that are less valuable.

Finally, in equilibrium consumer surplus is as follows:

$$CS_{AA} = \underbrace{\int_0^{1-(\theta v+\beta\sigma-\gamma)} (v+\beta\sigma-\gamma-x)\ dx}_{\text{platform 1's single-homers}} + \underbrace{\int_{1-(\theta v+\beta\sigma-\gamma)}^{\theta v+\beta\sigma-\gamma} [(1+\theta)v+2(\beta\sigma-\gamma)-1]\ dx}_{\text{multi-homers}} + \underbrace{\int_{\theta v+\beta\sigma-\gamma}^{1} [v+\beta\sigma-\gamma-(1-x)]\ dx}_{\text{platform 2's single-homers}},$$

that is,

$$CS_{AA} = (\theta v)^2 + v(1-\theta) + (\beta\sigma-\gamma)(2\theta v+\beta\sigma-\gamma).$$

Consumer surplus increases with the baseline utility v and the value of an impression σ, and decreases with the ad-nuisance parameter γ.

3.4 Business model configurations, quality of service and consumer surplus

Finally, we compare quality investments, quality-adjusted prices, and consumer surplus in the different business model configurations. The following table summarizes the equilibrium outcomes for each configuration:

	Fee	**Ad-nuisance**	**Quality**
User-funded platform (UU)	$f_{UU} = \theta v$	None	$q_{UU} = \theta v$
User-funded platform (UA)	$f_{UA} = \theta v$	None	$q_{UA} = \theta v$
Ad-funded platform (AU)	None	γ	$q_{AU} = \sigma$
Ad-funded platform (AA)	None	γ	$q_{AA} = \beta\sigma$

Table 1: Equilibrium outcomes in each business model configuration

Quality investments. A user-funded platform makes the same investment in quality regardless of the business model chosen by its competitor. By contrast, an ad-funded platform invests more when its rival is user-funded than when it is ad-funded, because

the value of an ad that reaches a multi-homer is not discounted when the rival does not display ads.

Proposition 1 *Quality investments in the different business model configurations compare as follows:*

- *If $\sigma < \theta v$, $q_{UU} = q_{UA} > q_{AU} \geq q_{AA}$. Regardless of the rival's business model, a user-funded platform invests in quality more than an ad-funded platform.*
- *If $\theta v \leq \sigma < \theta v/\beta$, $q_{AU} \geq q_{UU} = q_{UA} > q_{AA}$. When facing a user-funded rival, an ad-funded platform invests more than a user-funded platform; by contrast, an ad-funded platform invests less than a user-funded platform when facing an ad-funded rival.*
- *If $\sigma \geq \theta v/\beta$, $q_{AU} \geq q_{AA} \geq q_{UA} = q_{UU}$. Regardless of the rival's business model, an ad-funded platform invests more than a user-funded platform.*

Quality-adjusted prices. As previously discussed, user-funded platforms are unaffected by the other platform's business model, so users derive the same utility from patronizing a user-funded platform in any configuration.

Furthermore, we have seen that ad-funded platforms expose their users to the same nuisance regardless of their rival's business model– the ad-nuisance cost will always be γ to the users. However, ad-funded platforms invest more if the rival platform is user-funded.

We can consider the nuisance of advertising as the equivalent of a price to users. We can then compare the quality-adjusted "prices" in the different business model configurations, which is equal to either the fee or the ad-nuisance incurred by the user of a given platform minus the additional quality the platform offers its users. Let us define $\hat{p}_{b_i b_j}$ as firm i's quality-adjusted price when it has adopted the business model b_i and its rival has adopted the business model b_j, with $b_i, b_j \in \{U, A\}$. For a user-funded platform, the quality-adjusted price is always 0, i.e., $\hat{p}_{UU} = \hat{p}_{UA} = 0$. For an ad-funded platform, the quality-adjusted price is $\hat{p}_{AU} = \gamma - \sigma$ if it competes with a user-funded platform, and $\hat{p}_{AA} = \gamma - \beta\sigma$ if it competes with an ad-funded platform.

Note that $\hat{p}_{AU} < 0$ if and only if $r > 1$, where $r = \sigma/\gamma$ is the return to ads. Similarly, $\hat{p}_{AA} < 0$ if and only if $r > 1/\beta$. We then have the following result:

Proposition 2 *Quality-adjusted prices in the different business model configurations compare as follows:*

- *If $r < 1$, $\hat{p}_{AA} \geq \hat{p}_{AU} > \hat{p}_{UA} = \hat{p}_{UU}$. The quality-adjusted price for the user-funded platform is always lower.*

- *If* $1 \leq r < 1/\beta$, $\hat{p}_{AA} > \hat{p}_{UA} = \hat{p}_{UU} \geq \hat{p}_{AU}$. *When the rival platform is user-funded, the quality-adjusted price is lower for an ad-funded platform than for a user-funded one; conversely, when the rival platform is ad-funded, the quality-adjusted price is lower for an a user-funded platform than for ad-funded one.*

- *If* $r \geq 1/\beta$, $\hat{p}_{UA} = \hat{p}_{UU} > \hat{p}_{AA} \geq \hat{p}_{AU}$. *The quality-adjusted price for the ad-funded platform is always lower.*

Consumer surplus. We now compare the consumer surplus in the different configurations.

Proposition 3 *Consumer surpluses in the different business model configurations compare as follows:*

- *If* $r < 1$, $CS_{UU} > CS_{UA} > CS_{AA}$.

- *If* $1 \leq r < 1/\beta$, $CS_{UA} > CS_{UU} > CS_{AA}$.

- *If* $r \geq 1/\beta$, $CS_{AA} > CS_{UU}$ *and* $CS_{UA} > CS_{UU}$*; furthermore, there exists a threshold* $\tilde{\beta} \in (1/r, 1)$ *such that* $CS_{UA} > CS_{AA}$ *if and only if* $\beta < \tilde{\beta}$.

Proof. See Appendix A.2. ■

Which configuration the consumers prefer depends on the return to ads r and the discount for a second impression β. If advertising is a less efficient source of revenue than charging users (i.e., $r < 1$), the consumers' preferred configuration is for both platforms to be user-funded. Otherwise, if $r > 1$, consumers prefer that one platform be ad-funded if $\beta < \tilde{\beta}$ and that both be ad-funded if $\beta > \tilde{\beta}$ (this follows from the proposition).

When both platforms are user-funded, consumers receive only the intrinsic benefit from joining a platform (which is discounted when second-homing), as the surplus from quality investment is fully extracted through the fee. When faced with a user-funded platform, an ad-funded platform provides a higher surplus to its users than the intrinsic benefit whenever the value of an ad to advertisers is larger than the disutility of an ad to consumers, that is, when $r \geq 1$. Conversely, if $r < 1$, the additional quality provided by the ad-funded platform's investment does not compensate for the ad nuisance it imposes on users, resulting in a lower consumer surplus than when both platforms are user-funded.

When both platforms are ad-funded, their incentives to invest in quality are weakened by the competition in the advertising market, which is not at play in an asymmetric configuration and reduces the value of the marginal consumer. Consumer surplus is then higher in the configuration in which both platforms are ad-funded than in the configuration with two user-funded platforms if and only if $r \geq 1/\beta$.

Finally, when a second impression on users is highly discounted ($\beta \leq \tilde{\beta}$), competition on the advertising market in a symmetric configuration is so fierce that it reduces the

investment incentives for ad-funded platforms: in this case, users prefer an asymmetric market configuration (i.e., UA over AA). By contrast, if the discount is not as strong ($\beta > \tilde{\beta}$), the asymmetric setting is preferred by users. Intuitively, if β is close to 1, as the value of an impression goes up for advertisers, ad-funded platforms become more and more attractive from the point of view of users, ultimately leading them to prefer that all of them adopt the same business model.

Figure 1 provides an illustration of Proposition 3 by showing the preferred configuration for consumers as a function of β on the horizontal axis and r on the vertical axis for some values of the parameters.

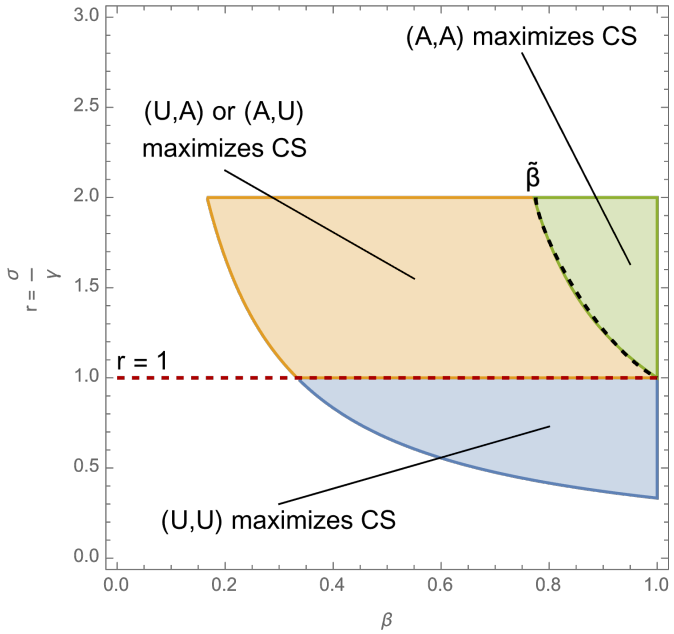

Figure 1: Consumer surplus maximizing configurations ($v = 1$, $\theta = 0.7$ and $\gamma = 0.3$).

Note: The white area is excluded by Assumption 1.

4 Business model decisions

In this section, we examine the business model decisions of platforms in stage 1, that is, whether to be user-funded or ad-funded. Note that a user-funded platform is indifferent to the other platform's business model and always earns the same profit, $(\theta v)^2/2$. By contrast, the profit of an ad-funded platform is affected by the rival platform's choice of business model.

Proposition 4 *Platform profits in the different business model configurations compare as follows:*

- *If* $r < \hat{r} \equiv 1 + \frac{\theta v}{\gamma}\left(\sqrt{1 + \left(1 - \frac{\gamma}{\theta v}\right)^2} - 1\right)$, $\Pi_{UU} = \Pi_{UA} > \Pi_{AU} \geq \Pi_{AA}$.

- *Otherwise, if* $r \geq \hat{r}$, $\Pi_{AU} \geq \Pi_{UA} = \Pi_{UU}$ *and* $\Pi_{AU} \geq \Pi_{AA}$.

Proof. See Appendix A.3. ■

As a result, it is a dominant strategy for a platform to be user-funded whenever $r < \hat{r}$, where $r = \sigma/\gamma$ represents the return to ads.

Notice that the threshold $\hat{r}$ defined in Proposition 4 is strictly higher than 1.[14] So, as expected, if the return to ads is less than 1, platforms make higher profits if they are user-funded, irrespective of their rival's business model. However, as $\hat{r} > 1$, platforms may still prefer to be user-funded when the return is (slightly) higher than 1. The return to ads must be sufficiently higher than 1 for the platforms to make higher profits with the ad-funded business model. The reason is that users in our setting have passive beliefs about advertising. Therefore, platforms do not take their effect on demand into account when setting ad prices, leading to too high ad levels in equilibrium. This diminishes the value of advertising as a revenue stream.

The proposition also shows that it is always more profitable for an ad-funded platform to be the only platform offering ad space ($\Pi_{AU} \geq \Pi_{AA}$). This is because, when both platforms are ad-funded, competition in the advertising market drives the platforms' profits down.

We have shown in Section 3 that being user-funded gives the same profit regardless of the other platform's business model ($\Pi_{UU} = \Pi_{UA}$). We also know that it is always more profitable to be an ad-funded platform when the other platform is user-funded rather than when it is ad-funded ($\Pi_{AU} \geq \Pi_{AA}$). From this we can derive the following proposition, which characterizes the equilibrium of the game.

Proposition 5 *In equilibrium, the business model configurations are as follows:*

(1) When $r < \hat{r}$*, there is a unique symmetric equilibrium in which both platforms are user-funded.*

(2) When $r > \hat{r}$*:*

- *(i) Whenever* $\Pi_{UU} \geq \Pi_{AA}$*, two mirror equilibria exist in which one platform chooses to be user-funded while the rival chooses to be ad-funded;*
- *(ii) Whenever* $\Pi_{UU} \leq \Pi_{AA}$*, there exists an equilibrium in which both platforms choose to be ad-funded.*

Proof. Point (1) derives from the fact that being user-funded is a dominant strategy if $r < \hat{r}$. Point $(2i)$ derives from $\Pi_{AA} \leq \Pi_{UA} = \Pi_{UU} < \Pi_{AU}$. Finally, point $(2ii)$ derives from $\Pi_{UA} = \Pi_{UU} \leq \Pi_{AA} \leq \Pi_{AU}$ and $\Pi_{UU} < \Pi_{AU}$. ■

[14] Note also that $\hat{r}$ depends on θ and that it increases with θ if $\theta v - \gamma > 0$.

Note that when $\Pi_{UU} = \Pi_{AU}$, both the asymmetric equilibrium AU and the symmetric equilibrium UU exist.[15]

Figure 2 illustrates platforms' business model decisions in equilibrium in the plane (β, r), where β is the discount for a second ad impression and $r = \sigma/\gamma$ is the return to ads, for the same values of the parameters v, θ and γ as in Figure 1.

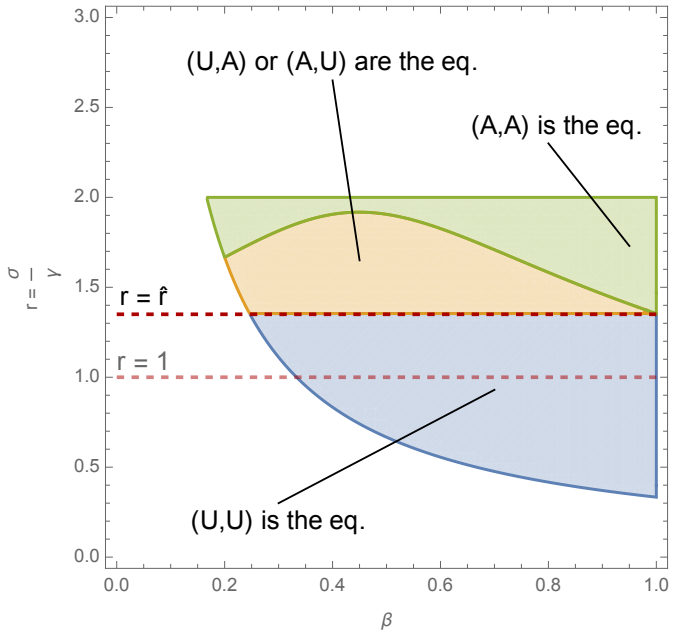

Figure 2: Equilibrium business model configurations
($v = 1$, $\theta = 0.7$ and $\gamma = 0.3$).

Note: The white area is excluded by Assumption 1.

Both platforms choose to be user-funded whenever the return to ads is sufficiently low ($r < \hat{r}$). For higher values of the return to ads ($r > \hat{r}$), choosing the ad-funded business model becomes a more profitable option. As seen in the figure, a symmetric equilibrium with two ad-funded platforms (AA) is then more likely to arise when β is either low or high. This is because platform profits in configuration AA are convex in β, as discussed after Lemma 5. When β is low and increases, the value of multi-homers increases but the number of (less valuable) multi-homers also increases, and the latter effect dominates the former, making the AA configuration less profitable. As β gets higher, the increased value of multi-homers becomes more important and the AA configuration becomes more attractive to platforms.

Finally, we compare the equilibrium business model configurations with those that maximize consumer surplus.[16] Figure 3 shows the shaded areas in the (β, r) space where

[15] In the degenerate case, where the four profits are equal, the three configurations can arise in equilibrium. We characterize the conditions under which the asymmetric equilibria arise in Appendix A.4.

[16] Note that we focus on consumer surplus, as it is the main criterion employed by competition authorities and regulators. Extending the comparison to total welfare should be done with caution, because the set of active firms varies across the different business model configurations: for instance, advertisers are

the equilibrium and CS-maximizing business model configurations coincide, for the same parameters as in Figures 1 and 2.

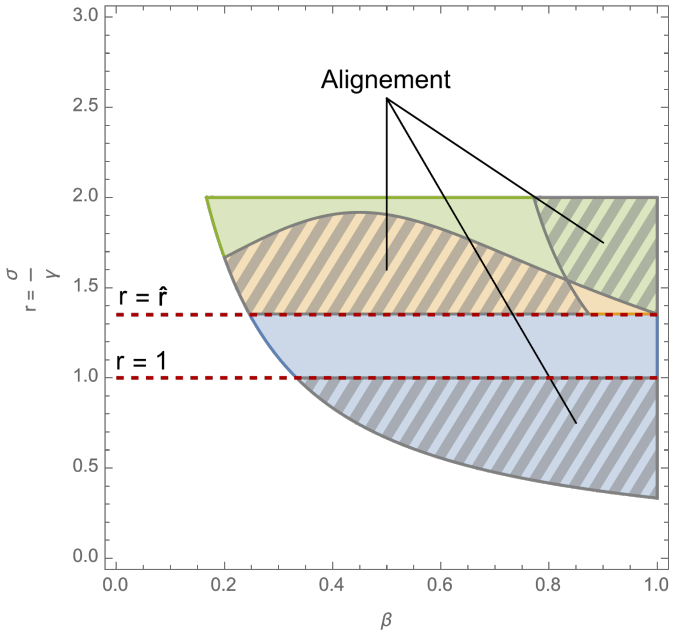

Figure 3: Equilibrium vs. CS-maximizing configurations ($v = 1$, $\theta = 0.7$ and $\gamma = 0.3$).

Note: The white area is excluded by Assumption 1.

Figure 3 shows two main distortions. First, platforms adopt the user-funded business model too often. Specifically, when $r \in (1, \hat{r})$, both platforms are user-funded in equilibrium, whereas consumers would prefer at least one of them to be ad-funded. As discussed above, because users hold passive beliefs about advertising, ad-funded platforms set ad levels too high, which undermines advertising as a revenue source relative to user funding. By contrast, consumers pay a lower quality-adjusted price from an ad-funded platform (in the asymmetric configuration AU) than from a user-funded platform whenever $r > 1$ (see Proposition 2).

Second, platforms also tend to adopt the ad-funded business model too often. In the green, unshaded area of the figure, both platforms are ad-funded in equilibrium, whereas consumers would prefer an asymmetric configuration with one ad-funded and one user-funded platform. The intuition is that competition in the advertising market becomes so intense when both platforms are ad-funded that it harms their incentives to invest. In this case, users prefer the asymmetric AU configuration, where quality investment is higher.

inactive in configuration UU.

5 Robustness and extensions

5.1 Mixed business model

In the baseline model, we assumed that a platform derives revenue from only one side, either users or advertisers. In practice, some platforms – such as traditional newspapers or video streaming services – operate under a mixed business model, generating revenue from both sides. Therefore, in this section, we extend the baseline model to allow for a mixed business model in which platforms can charge both users and advertisers. The question is whether, in this case, platforms can end up relying on a single revenue source in equilibrium.

We assume that in Stage 1, each platform commits to a business model among U, A, and M, where M stands for *mixed*. In the mixed business model, a platform i sets a user fee $f_i \geq 0$ and an advertising price $p_i \geq 0$. The total cost for a consumer of joining platform i as a single-homer or multi-homer is thus $f_i + \gamma a_i^e$, where a_i^e is the expected ad level on platform i.

There are three configurations in which at least one platform adopts the mixed business model: MU, MA and MM. We can show that in each of these configurations, a platform employing the mixed business model sets the same equilibrium user fee, regardless of the business model chosen by its competitor (all the proofs for this section are provided in Appendix A.5). As in the baseline model, this is due to fact that the platform does not price discriminate between single-homers and multi-homers when setting its fee. Hence, its pricing decision is affected by its total demand, not by the composition of demand.

Lemma 6 *In the configurations MU, MA and MM, in equilibrium a platform employing the mixed business model charges the user fee $f_M^* = \theta v - \gamma$ if $\theta v > \gamma$ and $f_M^* = 0$ otherwise.*

Under the ad-funded business model, a platform does not internalize the nuisance from advertising when choosing its ad price, as consumers have passive beliefs about advertising levels. The same is true under the mixed business model. However, when setting the user fee, the platform takes into account the disutility of advertising. Specifically, given that the ad level will always be equal to 1 in equilibrium, firm i chooses its user fee f_i to maximize $(f_i + \rho)(\theta v + q_i - f_i - \gamma) - c(q_i)$, where ρ represents the advertising revenue from the marginal user, i.e., $\rho = \sigma$ when facing a user-funded platform and $\rho = \beta\sigma$ when facing an ad-funded platform.

If $\theta v \leq \gamma$, a platform operating under the mixed business model charges a user fee equal to 0 in equilibrium, regardless of the business model of its competitor. In this case, the mixed business model is equivalent to the ad-funded business model and the analysis of the baseline model applies.

By contrast, if $\theta v > \gamma$, the firm charges a strictly positive user fee in equilibrium, regardless of the business model of its rival. The following proposition characterizes the

equilibrium in business models in this case.

Proposition 6 *Assume that platforms can adopt the U, A or M business model and that $\theta v > \gamma$. Then, in equilibrium:*

1. *If $r < 1$, the two platforms adopt the user-funded business model (U).*
2. *If $r > 1$, the two platforms adopt the mixed business model (M) if and only if*

$$(\sigma - \gamma)(2\theta v + \sigma - \gamma) > (1 - \beta)\sigma(4\theta v + (1 + 3\beta)\sigma - 4\gamma - 2).$$

Otherwise, one platform adopts the mixed business model and the other one is user-funded.

Therefore, in equilibrium, the ad-funded business model does not emerge. This is intuitive: the mixed business model always weakly dominates the ad-funded business model as a platform can charge a zero fee to users.[17]

Consumer surplus

Because in equilibrium, we have $CS_{AU} = CS_{MU}$, $CS_{AM} = CS_{MM}$ and $CS_{AA} = CS_{MM}$, we can focus on the business model configurations that can emerge in equilibrium, i.e., UU, MU and MM. Consumers would not obtain a larger surplus with a business model configuration involving A, i.e., AU, AM or AA. Furthermore, as $CS_{MU} = CS_{AU}$ and $CS_{MM} = CS_{AA}$, Proposition 3 applies simply by replacing A by M:

Proposition 7 *Assume that $\theta v > \gamma$. Consumer surpluses in the business model configurations UU, MU and MM compare as follows:*

- *If $r < 1$, $CS_{UU} > CS_{MU} > CS_{MM}$.*
- *If $1 \leq r < 1/\beta$, $CS_{MU} > CS_{UU} > CS_{MM}$.*
- *If $r \geq 1/\beta$, $CS_{MM} > CS_{UU}$ and $CS_{MU} > CS_{UU}$; furthermore, $CS_{MU} > CS_{MM}$ if and only if $\beta < \tilde{\beta}$.*

Note that the threshold $\tilde{\beta}$ in Proposition 7 is the same as in Proposition 3.

The configuration UU emerges in equilibrium if and only if $r \leq 1$, and it is the one preferred by consumers. This is because in the mixed business model, platforms internalize the nuisance from ads, leading to an efficient adoption of the user-funded business model.

[17] In contrast, the platform cannot replicate the user-funded model with the mixed business model. Since consumers do not observe the advertising price, the platform cannot commit to a zero level of advertising.

However, when $r > 1$, the equilibrium outcome is not necessarily the one preferred by consumers.

As an illustration, Figure 4 shows the shaded areas where the equilibrium outcome coincides with the CS-maximizing configurations when platforms can adopt the mixed business model, for the same parameters as in the other figures. Since $\theta v > \gamma$, the mixed business model emerges in equilibrium. As in the baseline model, we observe that the platforms do not sufficiently differentiate their business models from a consumer welfare perspective: in the green unshaded area, both platforms adopt the mixed business model, while consumers would prefer one of them to be user-funded.

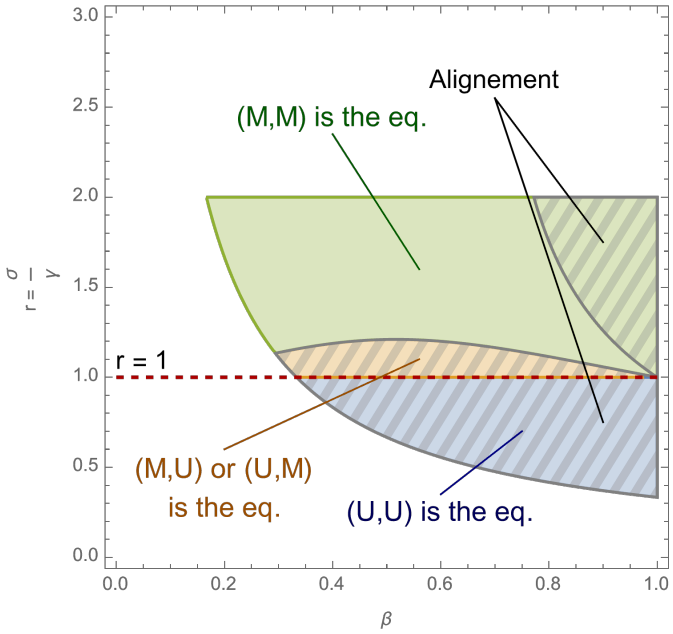

Figure 4: Equilibrium vs. CS-maximizing configurations ($v = 1$, $\theta = 0.7$ and $\gamma = 0.3$).

Note: The white area is excluded by Assumption 1.

5.2 Single-homing

In our baseline model, we assume that users can multi-home. This assumption is consistent with real-life digital markets (Calvano and Polo, 2021). Under multi-homing, each platform operates as a local monopolist, and strategic interactions are mostly driven by the interaction in the advertising market: when both platforms are ad-funded, the advertisers' willingness to pay depends on the number of multi-homers because the value of a second impression is discounted. This assumption is standard in the literature (see, e.g., Anderson and Jullien (2015)).

In this section, we check the robustness of our results by introducing more direct competitive interactions, assuming that users cannot multi-home, while advertisers can still use both platforms. Under single-homing (henceforth SH), a consumer joins the

platform i that offers the highest surplus. In what follows, we present the implications of these changes on our results. The detailed analysis is presented in the Appendix A.6.

Quality investment in each business model configuration

Lemma 7 *With single-homing, equilibrium qualities are as follows:*

- *In the UU configuration,* $q_{UU}^{sh} = \frac{1}{3}$ *for both platforms;*
- *In the AA configuration,* $q_{AA}^{sh} = \frac{\sigma}{2}$ *for both platforms;*
- *In the UA configuration, the user-funded platform sets a quality* $q_{UA}^{sh} = \frac{4(1+\gamma)\sigma}{12}$*, and the ad-funded platform a quality* $q_{AU}^{sh} = \frac{\sigma}{4}$*.*

Proof. See Appendix A.6. ■

With single-homing, competition between the two platforms on the Hotelling segment drives the choice of quality. Note first that the optimal quality set by a user-funded platform now decreases in its rival's quality, because the demand, hence the optimal fee this platform charges decreases in her rival's quality. In contrast, under multi-homing, demand for a user-funded platform was independent on the rival's quality: the user-funded platform behaved as a monopolist and considered the effect of a quality increase on the marginal consumer, a multi-homer, thus setting a quality independent of the other platform's business model. The quality set by an ad-funded platform is still independent on the rival's quality, because the ad-funded platform's revenue per user is constant (σ) and the platform thus only accounts for the effect of its own quality on demand.

Finally, it is worth noting that a platform competing against a user-funded rival always sets a lower quality under single-homing than under multi-homing. Specifically, under Assumption 1, we have $q_{UU}^{sh} = \frac{1}{3} \leq \theta v = q_{UU}$, and $q_{AU}^{sh} = \frac{\sigma}{4} < \sigma = q_{AU}$. This occurs because, under single-homing, quality has a strategic effect that is absent in the multi-homing case: a reduction in the quality of a platform facing a user-funded rival increases the optimal fee of the rival under single-homing, whereas it has no impact on the rival's fee under multi-homing.

In Appendix A.6 we present the consumer surplus in the different business model configurations. Intuitively, consumer surplus is higher in the AA configuration than in the UU configuration whenever the disutility from advertising is low (namely, when $\gamma \leq \frac{3\sigma+4}{6}$). In the asymmetric equilibrium, consumer surplus is higher than in the UU configuration whenever the disutility from advertising is not too high, and higher than in the AA configuration whenever γ is high enough.

Choice of business model Comparing the profits in Appendix A.6, we obtain the equilibrium choice of business models. Figure 5 illustrates these results.

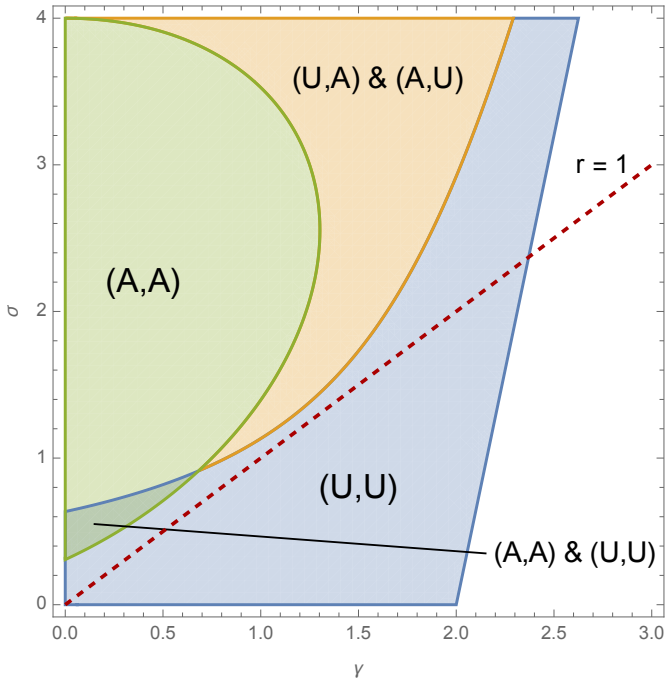

Figure 5: Equilibrium business model configuration (single-homing)

Note: The white area is excluded by Assumption 2.

As in the multi-homing baseline model, and despite more competitive interactions due to single-homing, three equilibrium configurations exist. Intuitively, in equilibrium, both platforms choose the user-funded model (blue area) when consumers dislike advertising (γ is high), or when the value of advertising for the advertiser is low (σ is low). Note that these two symmetric equilibria co-exist when γ is very low and σ is positive but low. When by contrast γ is low and σ is intermediate, both platforms choose the ad-funded model (green area). Interestingly, the asymmetric business model arises in equilibrium when σ is high enough for intermediate values of γ (yellow area). However, Figure 8 in Appendix A.6 shows that whenever the asymmetric business model configuration arises in equilibrium, it is not the preferred configuration for consumers, who would then always prefer that both platforms are ad-funded.

6 Discussion: policy implications

Regulatory or competition policy decisions aimed at improving consumer surplus typically take the firms' business models as given. However, our model shows that such policy decisions, by influencing the market parameters, can also influence the platform's business model decisions and thus indirectly affect consumer surplus.

For instance, decisions aiming at limiting market power in the ad-tech sector are likely to impact the value an ad generates for advertisers: within our framework, such measures

would correspond to an increase in σ.[18] Similarly, privacy regulations, such as the General Data Protection Regulation (GDPR) enacted in the European Union in 2018, can affect the efficiency of advertising (Wang, Jiang and Yang, 2024). Specifically, by making it harder to track whether consumers have already seen an ad, these regulations can reduce the value of a second impression relative to a first, thereby decreasing the parameter β in our model.

As shown in previous sections, consumer surplus may vary non-monotonically with respect to these parameters. Consider for instance a policy that leads to an increase in σ or equivalently an increase in the return to ads r.[19] In any given business model configuration, an increase in σ improves consumer welfare (showing this is straightforward from the expressions of consumer surplus in Appendix A.2). However, an increase in σ also affects the profitability of the ad-funded business model, and thus influences the business model decisions of platforms and indirectly consumer surplus. Figure 6 illustrates the effect of an increase in σ on the equilibrium outcome and consumer surplus. We assume that $\beta < \tilde{\beta}$: in this case, we know from Proposition 3 that the configuration with two ad-funded platforms is never the consumer surplus maximizing configuration. Let us denote by $\sigma_{UU\to UA}$ and $\sigma_{UA\to AA}$ the threshold values of σ such that the platforms switch their business model decisions in equilibrium (see Appendix A.4). Clearly, consumer surplus increases in σ when the equilibrium business model switches from UU to the asymmetric configuration UA, and increases thereafter until σ reaches the threshold $\sigma_{UA\to AA}$ (zones (I) and (II) in Figure 6). However, the switch (III) to the AA business model configuration may lead to a decrease in consumer surplus, especially if β is low. Note that such non-monotonicity also appears in the single-homing case.

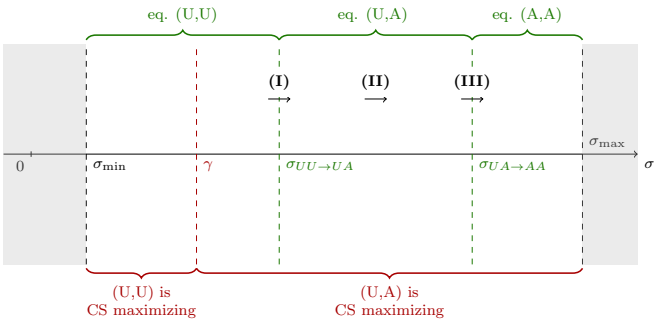

Figure 6: Impact of an increase in σ on equilibrium outcome and consumer surplus.

[18] Recent decisions by the UK CMA (ÄúOnline platforms and digital advertising: A market study.Äù https://www.gov.uk/cma-cases[...]) and the French Competition Authority (see decision 21-D-11 of August 7, 2021, available at: https://www.autoritedelaconcurrence.fr[...]) stress the importance of protecting advertisers and publishers from the excessive market power of Big Tech. See also D'Annunzio, Russo and Shekhar (2024) and Darlas (2024) for a discussion of the efficiency of various policies in limiting the market power in this industry.

[19] The same type of effect can be observed for a policy leading to a decrease in β.

7 Conclusion

In this paper, we consider two competing platforms that choose their business models and invest in improving the quality of service for users. We aim to understand how a platform's business model shapes its incentives to invest in quality and how its competitor's business model might influence those incentives. We also analyze the business model decisions of the platforms.

We find that quality investment is higher for a user-funded platform than for an ad-funded platform when the value of the service to the marginal multi-homing consumer is higher than the value of the marginal consumer to advertisers.

We also analyze which business model configurations emerge in equilibrium. Unsurprisingly, when the *return to ads* is very low, both platforms adopt the user-funded business model, and when it is very high, both platforms prefer to be ad-funded. But asymmetric business model configurations can also arise in equilibrium. Indeed, if the platforms differentiate their business models, there is no competition in the advertising market. Thus, the ad-funded platform enjoys a monopoly position over advertisers for its entire user base. This results in higher profitability for the ad-funded business model compared to a symmetric situation where both platforms would operate under the same business model. As a result, there may be more investment from platforms in an asymmetric configuration due to a greater ability to monetize. This is a driving force for the emergence of asymmetric equilibria, which can occur despite the fact that the platforms are identical *ex ante*.

We show that our results are robust to the following changes in the baseline model. First, we show that when platforms can charge both users and advertisers (i.e., adopt a "mixed" business model), they may still rely on a single revenue source in equilibrium and choose to be ad-funded only. Second, we show that our results also hold qualitatively when users must single-home.

From a consumer welfare perspective, we find that platforms tend to adopt the user-funded business model too often, but also fail to differentiate their business models sufficiently. Finally, we show that public policies must account for the potential adjustment of platform business models in response to regulation. In particular, we emphasize a non-monotonic relationship between the return to ads and consumer surplus: an increase in the return to ads may prompt a shift in business models with ambiguous effects on consumer welfare.

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A Appendices

A.1 Assumption 1 and parameter space

We want to ensure that in each configuration, in equilibrium, the market is covered and we have partial multi-homing. This requires that in equilibrium, we have $0 \leq \hat{x_2} \leq \hat{x_1} \leq 1$:

- In the UU configuration, to ensure that $\hat{x}_1^{UU}$ and $\hat{x}_2^{UU}$ are within $[0,1]$, we must assume that $0 \leq \theta v \leq 1$. Furthermore, there is partial multi-homing in equilibrium if $\hat{x}_1^{UU} - \hat{x}_2^{UU} > 0$, which is satisfied whenever $\theta v > 1/2$. Hence we assume that $\frac{1}{2} < \theta v \leq 1$.
- In the asymmetric UA configuration, to ensure that $\hat{x}_1^{UA}$ and $\hat{x}_2^{AU}$ are within $[0,1]$ in equilibrium, we must assume that $0 \leq \theta v \leq 1$ and $\gamma - \theta v \leq \sigma \leq 1 + \gamma - \theta v$. Furthermore, there is partial multi-homing in equilibrium if $\hat{x}_1^{UA} - \hat{x}_2^{AU} > 0$, which is satisfied whenever $\sigma > 1 + \gamma - 2\theta v$. Hence we assume that $0 \leq \theta v \leq 1$ and that $1 + \gamma - 2\theta v < \sigma \leq 1 + \gamma - \theta v$.
- In the AA configuration, to ensure that $\hat{x}_1^{AA}$ and $\hat{x}_2^{AA}$ are within $[0,1]$ in equilibrium, we must assume that $\frac{\gamma - \theta v}{\beta} \leq \sigma \leq \frac{1+\gamma-\theta v}{\beta}$. Furthermore, there is partial multi-homing in equilibrium if $\hat{x}_1^{AA} - \hat{x}_2^{AA} > 0$, which is satisfied whenever $\sigma > \frac{\frac{1}{2}+\gamma-\theta v}{\beta}$. Hence we assume that $\frac{1}{\beta} \cdot (\frac{1}{2} + \gamma - \theta v) \leq \sigma \leq \frac{1}{\beta} \cdot (1 + \gamma - \theta v)$.

As we need to make all the assumptions above, we end up assuming that $\theta v \in [\frac{1}{2}, 1]$ and $\frac{1}{\beta} \cdot \left(\frac{1}{2} + \gamma - \theta v\right) \leq \sigma \leq 1 + \gamma - \theta v$ as in Assumption 1. From the set of assumptions made in Section 2 and Assumption 1, we can then define the parameter space as follows:

$$\mathcal{P} = \begin{cases} \theta v \in [\frac{1}{2}, 1] \\ \beta \in [0,1] \\ \gamma \geq 0 \\ \max\left(0, \frac{1}{\beta} \cdot \left(\frac{1}{2} + \gamma - \theta v\right)\right) \leq \sigma \leq 1 + \gamma - \theta v \end{cases} \subset \mathbb{R}^4.$$

A.2 Proof of Proposition 3

The consumer surplus in configurations DD, DA and AA are

$$\begin{aligned} CS_{UU} &= (\theta v)^2 + v(1-\theta) \\ CS_{UA} &= CS_{UU} + \frac{1}{2}(\sigma - \gamma)(2\theta v + \sigma - \gamma) \\ CS_{AA} &= CS_{UU} + (\beta\sigma - \gamma)(2\theta v + \beta\sigma - \gamma), \end{aligned}$$

respectively. Note that under Assumption 1, we have $2\theta v+\sigma-\gamma>0$ and $2\theta v+\beta\sigma-\gamma>0$. Therefore, if $\sigma<\gamma$ (i.e., $r<1$), we have $CS_{UU}>CS_{UA}>CS_{AA}$. If $\gamma<\sigma<\gamma/\beta$ (i.e., $1<r<1/\beta$), we have $CS_{UA}>CS_{UU}>CS_{AA}$.

Finally, if $\sigma>\gamma/\beta$ (i.e., $r>1/\beta$), we have $CS_{AA}>CS_{UU}$ and $CS_{UA}>CS_{UU}$. It remains to compare CS_{AA} and CS_{UA}. Let $\Delta CS\equiv CS_{UA}-CS_{AA}=\frac{1}{2}(\sigma-\gamma)(2\theta v+\sigma-\gamma)-(\beta\sigma-\gamma)(2\theta v+\beta\sigma-\gamma)$. We have:

$$\frac{\partial\Delta CS}{\partial\beta}=-2\sigma(\theta v+\beta\sigma-\gamma)<0,$$

where the inequality follows from Assumption 1. Therefore, ΔCS decreases with β. Furthermore, under Assumption 1, we have:

$$\Delta CS|_{\beta=\gamma/\sigma}=\frac{1}{2}(\sigma-\gamma)(2v\theta+\sigma-\gamma)>0 \text{ and } \Delta CS|_{\beta=1}=-\frac{1}{2}(\sigma-\gamma)(2v\theta+\sigma-\gamma)<0,$$

using the fact that in this case, $\gamma<\beta\sigma<\sigma$. Therefore, there exists a threshold value of β, which we denote by $\tilde{\beta}$, such that $\Delta CS>0$ if $\beta<\tilde{\beta}$ and $\Delta CS<0$ if $\beta>\tilde{\beta}$. Note that we have $\tilde{\beta}>\gamma/\sigma=1/r$.

A.3 Proof of Proposition 4

(a) First, we compare Π_{UU} to Π_{AU} (i.e., the profit of the ad-funded platform in configuration UA). We have:

$$\Pi_{AU}-\Pi_{UU}=\sigma(\theta v+\sigma-\gamma)-\sigma^2/2-(\theta v)^2/2=\frac{\sigma^2}{2}+\sigma(\theta v-\gamma)-\frac{(\theta v)^2}{2}.$$

Thus, $\Pi_{AU}>\Pi_{UU}$ if and only if

$$\sigma>\gamma-\theta v+\sqrt{(\theta v)^2+(\theta v-\gamma)^2},$$

or

$$r=\frac{\sigma}{\gamma}>1+\frac{\theta v}{\gamma}\left(\sqrt{1+\left(1-\frac{\gamma}{\theta v}\right)^2}-1\right)\equiv\hat{r}.$$

(b) Second, we prove that $\Pi_{AU}\geq\Pi_{AA}$. Note that if $\beta=1$, we have $\Pi_{AU}=\Pi_{AA}$. Now, assume that $\beta<1$. Note that

$$\Pi_{AU}=\sigma(\theta v+\sigma-\gamma)-\frac{\sigma^2}{2}\geq\sigma(\underbrace{\theta v+\beta\sigma-\gamma}_{=SH_{AA}+MH_{AA}})-\frac{(\beta\sigma)^2}{2},$$

as $\sigma = \underset{q}{\text{argmax}}\ \sigma(v+q-\gamma) - \frac{q^2}{2}$. Moreover:

$$\Pi_{AA} = \sigma SH_{AA} + \beta\sigma MH_{AA} - \frac{(\beta\sigma)^2}{2} \leq \sigma\left(SH_{AA} + MH_{AA}\right) - \frac{(\beta\sigma)^2}{2},$$

which proves that $\Pi_{AU} \geq \Pi_{AA}$.

(c) Summing up, if $r < \hat{r}$, we have $\Pi_{UU} = \Pi_{UA} > \Pi_{AU} \geq \Pi_{AA}$. If $r \geq \hat{r}$, we have $\Pi_{AU} \geq \Pi_{UA} = \Pi_{UU}$ and $\Pi_{AU} \geq \Pi_{AA}$.

A.4 Characterization of equilibria with asymmetric business models

We show that there are parameter values in our parameter space where platforms adopt different business models in equilibrium. Note that there is no asymmetric equilibrium when $\beta = 1$. In this case, ad-funded platforms are indifferent to the other platform's business model, leading only to symmetric equilibria. Therefore, for the rest of this proof, we assume that $\beta < 1$.

An asymmetric equilibrium arises if and only if platforms' best responses are (1) to be ad-funded when the other platform is user-funded ($\Pi_{AU} > \Pi_{UU}$), and (2) to be user-funded when the other platform is ad-funded ($\Pi_{UA} > \Pi_{AA}$).

Consider a vector $(v, \gamma, \beta, \sigma)$ that belongs to our parameter space, with $\beta < 1$. First, we can easily show that:

$$\frac{d\Pi_{UU}}{d\sigma} = \frac{d\Pi_{UA}}{d\sigma} = 0, \quad \frac{d\Pi_{AU}}{d\sigma} > 0, \quad \frac{d\Pi_{AA}}{d\sigma} > 0, \text{ and } \Pi_{AU} > \Pi_{AA}. \tag{12}$$

The strict monotonicity of both ad-funded profits, combined with the fact that user-funded profits are constant in σ, implies that there exists at most one solution to $\Pi_{AU} = \Pi_{UU}$ and to $\Pi_{AA} = \Pi_{UA}$. To simplify our notations, let us define:

$$f : \left| \begin{array}{rcl} [\sigma_{\min}, \sigma_{\max}] & \longrightarrow & \mathbb{R} \\ \sigma & \longmapsto & \Pi_{AU}(\sigma) - \frac{(\theta v)^2}{2} \end{array} \right.$$

and

$$g : \left| \begin{array}{rcl} [\sigma_{\min}, \sigma_{\max}] & \longrightarrow & \mathbb{R} \\ \sigma & \longmapsto & \Pi_{AA}(\sigma) - \frac{(\theta v)^2}{2} \end{array} \right.$$

where $\frac{(\theta v)^2}{2} = \Pi_{UU} = \Pi_{UA}$. From above, we have $f'(.) > 0$, $g'(.) > 0$ and $f > g$.

So, for any (v, γ, β) in our parameter space, there exists a pair of asymmetric equilibria, (U, A) and (A, U), if and only if there exists $\tilde{\sigma} \in [\sigma_{\min}, \sigma_{\max}]$ such that $f(\tilde{\sigma}) > 0 > g(\tilde{\sigma})$. A necessary and sufficient condition for this is $f(\sigma_{\max}) > 0$ and $g(\sigma_{\min}) < 0$.

Let us first analyze the case where $f(\sigma_{\max}) > 0$, meaning when there exists a σ in our parameter space such that being ad-funded is a best response when the other platform is user-funded.

We have $\sigma_{\max} = 1 - (\theta v - \gamma)$, and at this value of σ the equilibrium demand for an ad-funded platform in an asymmetric configuration, $\theta v + \sigma - \gamma$, equals 1. Thus, we have $\Pi_{AU}(\sigma_{\max}) = \sigma_{\max} \cdot 1 - \frac{\sigma_{\max}^2}{2}$ and $f(\sigma_{\max}) = \sigma_{\max} - \frac{\sigma_{\max}^2}{2} - \frac{(\theta v)^2}{2}$. It follows that:

$$f(\sigma_{\max}) > 0 \;\Leftrightarrow\; \sigma_{\max} \in \left(1 - \sqrt{1-(\theta v)^2}, 1 + \sqrt{1-(\theta v)^2}\right) \Leftrightarrow\; \gamma \in \left(\underline{\gamma}, \bar{\gamma}\right),$$

with $\underline{\gamma} \equiv \theta v - \sqrt{1-(\theta v)^2}$ and $\bar{\gamma} \equiv \theta v + \sqrt{1-(\theta v)^2}$.

Now, let us analyze the case where $g(\sigma_{\min}) < 0$. Verifying this condition implies that there exist a σ in our parameter space such that being user-funded can be a best response to an ad-funded platform. We have

$$\sigma_{\min} = \max\left(0, \frac{1}{\beta}\left(\frac{1}{2} - (\theta v - \gamma)\right)\right).$$

Suppose first that $\gamma \leq \theta v - \frac{1}{2}$. Then, we have $\sigma_{\min} = 0$ and $\Pi_{AA} = 0 < \Pi_{UA}$ and $g(\sigma_{\min}) < 0$.

If, instead, $\gamma > \theta v - \frac{1}{2}$, then $\sigma_{\min} = \frac{1}{\beta}\left(\frac{1}{2} - (\theta v - \gamma)\right)$. We have thus $\Pi_{AA}(\sigma_{\min}) = \frac{\sigma_{\min}}{2} - \frac{(\beta \cdot \sigma_{\min})^2}{2}$ and $g(\sigma_{\min}) = \frac{\sigma_{\min}}{2} - \frac{(\beta \cdot \sigma_{\min})^2}{2} - \frac{(\theta v)^2}{2}$. We distinguish two cases:

- If $\beta > \frac{1}{2(\theta v)}$, then one can easily verify that for all γ, $g(\sigma_{\min}) < 0$.
- Otherwise:

$$g(\sigma_{\min}) < 0 \;\Leftrightarrow\; \sigma_{\min} \notin \left(\frac{1 - \sqrt{1-(2\beta \cdot \theta v)^2}}{2}, \frac{1 + \sqrt{1-(2\beta \cdot \theta v)^2}}{2}\right)$$
$$\Leftrightarrow\; \gamma \notin \left(\underline{\gamma}', \bar{\gamma}'\right),$$

where

$$\underline{\gamma}' \equiv \theta v - \frac{1}{2} \cdot \left(\frac{1}{\beta} - 1 - \sqrt{\frac{1}{\beta^2} - (2(\theta v))^2}\right) > \theta v - \frac{1}{2}$$
$$\bar{\gamma}' \equiv \theta v - \frac{1}{2} \cdot \left(\frac{1}{\beta} - 1 + \sqrt{\frac{1}{\beta^2} - (2(\theta v))^2}\right)$$

To sum up, we have shown that, in our parameter space, there exists an interval $I \neq \emptyset$ and $I \subset [\sigma_{\min}, \sigma_{\max}]$ such that for any $(v, \gamma, \beta, \sigma)$ with $\sigma \in I$, an asymmetric equilibrium exists if and only if:

- either $\beta > \frac{1}{2\theta v}$ and $\gamma \in (\underline{\gamma}, \bar{\gamma})$
- or $\beta \leq \frac{1}{2\theta v}$ and $\gamma \in (\underline{\gamma}, \underline{\gamma}') \;\cup\; (\bar{\gamma}', \bar{\gamma})$

To be more precise, in both cases, an asymmetric equilibrium exists for intermediate values of σ. Let us denote by σ_{UU} the value of an impression such that $\Pi_{AU}(\sigma_{UU\to UA}) = \Pi_{UU}(\sigma_{UU\to UA})$, i.e. $f(\sigma_{UU\to UA}) = 0$, and $\sigma_{UA\to AA}$ such that $\Pi_{AA}(\sigma_{UA\to AA}) = \Pi_{UA}(\sigma_{UA\to AA})$, i.e. $g(\sigma_{UA\to AA}) = 0$. We know from (12) above that $\sigma_{UU\to UA} < \sigma_{UA\to AA}$ and by definition the conditions for an asymmetric equilibrium to arise would be verified for $\sigma \in [\sigma_{UU\to UA}, \sigma_{UA\to AA}]$. Denoting $\underline{\sigma} \equiv \max(\sigma_{UU\to UA}, \sigma_{\min})$ and $\bar{\sigma} \equiv \min(\sigma_{UA\to AA}, \sigma_{\max})$, we have shown that asymmetric equilibria exist for $\sigma \in [\underline{\sigma}, \bar{\sigma}]$.

Summary Consider any $(v, \beta, \gamma, \sigma)$ in our parameter space. We can characterize the equilibria of our game as follows.

- if $\beta \geq \frac{1}{2\theta v}$ and,
  - $\gamma \notin [\underline{\gamma}, \bar{\gamma}]$, the unique equilibrium is (U,U)
  - $\gamma \in [\underline{\gamma}, \bar{\gamma}]$ and,
    - $\sigma < \underline{\sigma}$, the unique equilibrium is (U,U)
    - $\sigma \in [\underline{\sigma}, \bar{\sigma}]$, the equilibria are (U,A) and (A,U)
    - $\sigma > \bar{\sigma}$, the unique equilibrium is (A,A)
- if $\beta < \frac{1}{2\theta v}$ and,
  - $\gamma \notin [\underline{\gamma}, \bar{\gamma}]$, the unique equilibrium is (U,U)
  - $\gamma \in [\underline{\gamma}, \underline{\gamma}'] \cup [\bar{\gamma}', \bar{\gamma}]$ and,
    - $\sigma < \underline{\sigma}$, the unique equilibrium is (U,U)
    - $\sigma \in [\underline{\sigma}, \bar{\sigma}]$, the equilibria are (U,A) and (A,U)
    - $\sigma > \bar{\sigma}$, the unique equilibrium is (A,A)
  - $\gamma \in (\underline{\gamma}', \bar{\gamma}')$, the unique equilibrium is (A,A)

A.5 Mixed business model

We assume that in Stage 1, platforms choose a business model among U, A, and M, where M stands for *mixed*. In the mixed business model M, a firm i earns revenue from both users and advertisers. Thus, it sets a user fee $f_i \geq 0$ and an advertising price $p_i \geq 0$.

We derive below the equilibrium for the three business model configurations that involve M: MU, MA and MM.

A.5.1 Configuration MU.

Suppose that firm 1 has adopted business model M and firm 2 has adopted business model U. A consumer located at x receives the net utility $U_1 = v + q_1 - f_1 - \gamma a_1^e - x$ from

single-homing on platform 1 and $U_2 = v+q_2-f_2-(1-x)$ from single-homing on platform 2. Joining both platforms gives the net utility $U_{12} = (1+\theta)v + q_1 + q_2 - f_1 - \gamma a_1^e - f_2 - 1$.

The consumer indifferent between joining platform 1 only and joining both platforms is thus located at $\hat{x}_2^M = 1-(\theta v + q_2 - f_2)$, while the consumer indifferent between joining platform 2 only and joining both platforms is located at $\hat{x}_1^M = \theta v + q_1 - f_1 - \gamma a_1^e$.

Prices. In Stage 3, firms simultaneously set their prices. Firm 2 sets its user fee f_2 to maximize its profit, which is equal to $f_2(\theta v + q_2 - f_2) - c(q_2)$, and the profit-maximizing fee is then $f_2(q_2) = (\theta v + q_2)/2$.

Platform 1 offers advertisers the possibility to run an ad campaign. Since each impression is unique and valued by advertisers at σ, regardless of whether users single-home or multi-home, platform 1 can charge the ad price $p_1 = \sigma(\theta v + q_1 - f_1 - \gamma)$ so all advertisers place an ad and $a_1^e = 1$. Then, firm 1 chooses its user fee f_1 to maximize $(f_1+\sigma)(\theta v + q_1 - f_1 - \gamma) - c(q_1)$, and we obtain $f_1(q_1) = \max\{0, (\theta v + q_1 - \gamma - \sigma)/2\}$.

Quality levels. In Stage 2, firms choose their quality investment. Platform 2 chooses its investment q_2 to maximize its profit, $f_2(q_2)(\theta v + q_2 - f_2(q_2)) - c(q_2)$, and we obtain $q_2 = \theta v$, i.e., the level of investment of a user-funded platform.

Platform 1 chooses its quality investment q_1 to maximize its profit, $(f_1(q_1)+\sigma)(\theta v + q_1 - f_1(q_1) - \gamma) - c(q_1)$. If the user fee is interior, we obtain $q_1 = \theta v + \sigma - \gamma$ and $f_1 = \theta v - \gamma$. We have $f_1 > 0$ if and only if $\theta v > \gamma$. If the user fee is equal to 0, platform 1 chooses q_1 to maximize $\sigma(\theta v + q_1 - \gamma) - c(q_1)$, which is the profit of an ad-funded platform in the AU configuration. We then obtain $q_1 = \sigma$. At this candidate equilibrium, we have $f_1(q_1) = 0$ if and only if $\theta v \leq \gamma$.

Equilibrium prices and profits. In equilibrium, we have $f_1^* = \theta v - \gamma$ if $\theta v > \gamma$ and $f_1^* = 0$ otherwise. We always have $f_2^* = \theta v$. Therefore, if $\theta v \leq \gamma$, firm 1 charges no user fee and the equilibrium outcome is the same as for AU. If $\theta v > \gamma$, we have

$$\Pi_1^* = \Pi_{MU} = (\theta v + \sigma - \gamma)^2/2 \quad \text{and} \quad \Pi_2^* = \Pi_{UM} = (\theta v)^2/2 = \Pi_{UU}.$$

We have $\Pi_{MU} > \Pi_{UU}$ iff $\sigma > \gamma$, i.e., if the return to ads $r > 1$. Furthermore, we have $\Pi_{MU} - \Pi_{AU} = \frac{1}{2}(\theta v - \gamma)^2 > 0$. So platform 1 always prefers business model M over A when facing a user-funded rival.

We now derive Assumption 1 and consumer surplus when $\theta v > \gamma$.

Assumption 1. Assumption 1 holds if $0 < \hat{x}_2^M < \hat{x}_1^M < 1$, that is, $0 < 1 - \theta v < \theta v + \sigma - \gamma < 1$, which holds if $(1+\gamma-\sigma)/2 < \theta v < \min\{1, 1+\gamma-\sigma\}$ and corresponds to the condition for configuration AU.

Consumer surplus. Consumer surplus is given by:

$$CS_{MU} = \underbrace{\int_0^{1-\theta v} (v+\sigma-\gamma-x)\ dx}_{\text{platform 1's single-homers}} + \underbrace{\int_{1-\theta v}^{\theta v+\sigma-\gamma} [(1+\theta)v+\sigma-\gamma-1]\ dx}_{\text{multi-homers}} + \underbrace{\int_{\theta v+\sigma-\gamma}^{1} [v-(1-x)]\ dx}_{\text{platform 2's single-homers}}$$
$$= CS_{UU} + \frac{1}{2}(\sigma-\gamma)(2\theta v+\sigma-\gamma) = CS_{UA}.$$

Since $2\theta v+\sigma-\gamma > 0$ from Assumption 1, we have $CS_{MU} > CS_{UU}$ iff $\sigma > \gamma$.

A.5.2 Configuration MA.

Now, assume that firm 1 has adopted business model M and firm 2 business model A.

Prices. In Stage 3, firms set their prices simultaneously. For a given user fee f_1 set by firm 1, the advertising prices are the same as in the baseline model: we have $p_i = \sigma SH_i + \beta\sigma MH_{12}$ and $a_i = 1 = a_i^e$, for $i = 1, 2$, with:

$$SH_1 = 1-(\theta v+q_2-\gamma)$$
$$SH_2 = 1-(\theta v+q_1-f_1-\gamma)$$
$$MH_{12} = (\theta v+q_2-\gamma)+(\theta v+q_1-f_1-\gamma)-1.$$

Platform 1 then chooses its user fee f_1 to maximize its profit, which can be written as:

$$(f_1+\beta\sigma)(\theta v+q_1-f_1-\gamma)+(1-\beta)\sigma\left[1-(\theta v+q_2-\gamma)\right]-c(q_1).$$

The profit-maximizing fee is $f_1(q_1) = \max\{0, (\theta v+q_1-\beta\sigma-\gamma)/2\}$.

Quality levels. In Stage 2, firms choose their quality investment. Platform 1 chooses its quality investment q_1 to maximize its profit, $(f_1(q_1)+\beta\sigma)(\theta v+q_1-f_1(q_1)-\gamma)-c(q_1)$, where we have removed the part of the profit function that does not depend on q_1. If the user fee is interior, we have $q_1 = \theta v+\beta\sigma-\gamma$ and $f_1 = \theta v-\gamma$. We have $f_1 > 0$ if and only if $\theta v > \gamma$. If the user fee is equal to 0, platform 1 chooses q_1 to maximize $\beta\sigma(\theta v+q_1-\gamma)-c(q_1)$, which corresponds to the problem of an ad-funded platform in the AA configuration. We then obtain $q_1 = \beta\sigma$. At this candidate equilibrium, we have $f_1(q_1) = 0$ if and only if $\theta v \leq \gamma$.

As for firm 2, if we ignore the terms in its profit function that do not depend on q_2, it chooses q_2 to maximize $\beta\sigma(\theta v+q_2-\gamma)-q_2^2/2$, which is the problem of an ad-funded firm in the AA configuration. We then obtain $q_2 = \beta\sigma$.

Equilibrium prices and profits. In equilibrium, we have $f_1^* = \theta v-\gamma$ if $\theta v > \gamma$ and $f_1^* = 0$ otherwise. If $\theta v \leq \gamma$, firm 1 does not charge a user fee and the equilibrium outcome

is the same as for AA. If $\theta v > \gamma$, we have

$$\Pi_1^* = \Pi_{MA} = \frac{(\theta v + \beta\sigma - \gamma)^2}{2} + (1-\beta)\sigma\left[1 - (\theta v + \beta\sigma - \gamma)\right]$$
$$\Pi_2^* = \Pi_{AM} = \beta\sigma(\theta v + \beta\sigma - \gamma) + (1-\beta)\sigma\left[1 - (\theta v + \beta\sigma - \gamma)\right] - \frac{(\beta\sigma)^2}{2} = \Pi_{AA}.$$

We have $\Pi_{MA} > \Pi_{UU}$ if $\beta\sigma > \gamma$ (sufficient condition). Furthermore, we have $\Pi_{MA} - \Pi_{AA} = (\theta v - \gamma)^2/2 > 0$, so $\Pi_{MA} > \Pi_{AA}$.

We now derive Assumption 1 and consumer surplus when $\theta v > \gamma$.

Assumption 1. Assumption 1 holds if $0 < \hat{x}_2^M < \hat{x}_1^M < 1$, that is, if $0 < 1 - (\theta v + \beta\sigma - \gamma) < \theta v + \beta\sigma - \gamma < 1$, which is the same condition as for configuration AA.

Consumer surplus. Consumer surplus is given by:

$$CS_{MA} = \underbrace{\int_0^{1-(\theta v+\beta\sigma-\gamma)} (v + \beta\sigma - \gamma - x)\ dx}_{\text{platform 1's single-homers}} + \underbrace{\int_{1-(\theta v+\beta\sigma-\gamma)}^{\theta v+\beta\sigma-\gamma} [(1+\theta)v + 2(\beta\sigma-\gamma) - 1]\ dx}_{\text{multi-homers}}$$
$$+ \underbrace{\int_{\theta v+\beta\sigma-\gamma}^{1} [v + \beta\sigma - \gamma - (1-x)]\ dx}_{\text{platform 2's single-homers}},$$

that is, $CS_{MA} = CS_{UU} + (\beta\sigma - \gamma)(2\theta v + \beta\sigma - \gamma) = CS_{AA}$. So, $CS_{MA} > CS_{UU}$ iff $\beta\sigma > \gamma$.

A.5.3 Configuration MM.

Finally, we consider the case where both platforms have adopted the mixed business model M. A consumer located at x obtains the net utility $U_1 = v + q_1 - f_1 - \gamma a_1^e - x$ from single-homing on platform 1 and $U_2 = v + q_2 - f_2 - \gamma a_2^e - (1-x)$ from single-homing on platform 2. Joining both platforms gives the net utility $U_{12} = (1+\theta)v + q_1 + q_2 - f_1 - f_2 - \gamma(a_1^e + \gamma a_2^e) - 1$.

The consumer indifferent between joining platform 1 only and joining both platforms is thus located at $\hat{x}_2^M = 1 - (\theta v + q_2 - f_2 - \gamma a_2^e)$, while the consumer indifferent between joining platform 2 only and joining both platforms is located at $\hat{x}_1^M = \theta v + q_1 - f_1 - \gamma a_1^e$.

Prices. In Stage 3, firms simultaneously set their prices. Taking the user fees as given, the advertising prices are the same as in the baseline model: we have $p_i = \sigma SH_i + \beta\sigma MH$

and $a_i = 1 = a_i^e$, for $i = 1, 2$, with:

$$\begin{aligned} SH_1 &= 1 - (\theta v + q_2 - f_2 - \gamma) \\ SH_2 &= 1 - (\theta v + q_1 - f_1 - \gamma) \\ MH &= (\theta v + q_2 - f_2 - \gamma) + (\theta v + q_1 - f_1 - \gamma) - 1. \end{aligned}$$

Each platform i chooses its user fee f_i to maximize its profit, which can be written as:

$$(f_i + \beta\sigma)(\theta v + q_i - f_i - \gamma) + (1 - \beta)\sigma\left[1 - (\theta v + q_j - f_j - \gamma)\right] - c(q_i).$$

The profit-maximizing fee is $f_i(q_i) = \max\{0, (\theta v + q_i - \beta\sigma - \gamma)/2\}$.

Quality levels. At Stage 2, each platform i chooses its quality investment q_i to maximize

$$(f_i(q_i) + \beta\sigma)(\theta v + q_i - f_i(q_i) - \gamma) + (1 - \beta)\sigma\left[1 - (\theta v + q_j - f_j(q_j) - \gamma)\right] - (q_i)^2/2.$$

The analysis is similar to the previous case (MA). If $\theta v > \gamma$, we have $q_1 = q_2 = \theta v + \beta\sigma - \gamma$ and $f_1 = f_2 = \theta v - \gamma$. If $\theta v \leq \gamma$, we have $q_1 = q_2 = \beta\sigma$ and $f_1 = f_2 = 0$.

Equilibrium prices and profits. In equilibrium, we have $f_1^* = f_2^* = \theta v - \gamma$ if $\theta v > \gamma$ and $f_1^* = f_2^* = 0$ otherwise. If $\theta v \leq \gamma$, firms do not charge a user fee and the equilibrium outcome is the same as for AA. If $\theta v > \gamma$, we have

$$\Pi_1^* = \Pi_2^* = \Pi_{MM} = \frac{(\theta v + \beta\sigma - \gamma)^2}{2} + (1 - \beta)\sigma\left[1 - (\theta v + \beta\sigma - \gamma)\right] = \Pi_{MA}.$$

Since $\Pi_{MA} > \Pi_{AA}$, then $\Pi_{MM} > \Pi_{AM} = \Pi_{AA}$.

We now derive Assumption 1 and consumer surplus when $\theta v > \gamma$.

Assumption 1. Assumption 1 holds if $0 < \hat{x}_2^M < \hat{x}_1^M < 1$, that is, if $0 < 1 - (\theta v + \beta\sigma - \gamma) < \theta v + \beta\sigma - \gamma < 1$, which is the same condition as for configuration AA.

Consumer surplus. Consumer surplus is given by:

$$CS_{MM} = \underbrace{\int_0^{1-(\theta v+\beta\sigma-\gamma)} (v + \beta\sigma - \gamma - x)\ dx}_{\text{platform 1's single-homers}} + \underbrace{\int_{1-(\theta v+\beta\sigma-\gamma)}^{\theta v+\beta\sigma-\gamma} [(1+\theta)v + 2(\beta\sigma - \gamma) - 1]\ dx}_{\text{multi-homers}} + \underbrace{\int_{\theta v+\beta\sigma-\gamma}^{1} [v + \beta\sigma - \gamma - (1 - x)]\ dx}_{\text{platform 2's single-homers}},$$

that is, $CS_{MM} = CS_{UU} + (\beta\sigma - \gamma)(2\theta v + \beta\sigma - \gamma) = CS_{AA}$.

A.5.4 Existence of the mixed business model

With the previous analysis, we have shown that if $\theta v \leq \gamma$, a firm that adopts the mixed business model charges a user fee equal to 0 in equilibrium, regardless of the business model of its rival. In this case, the mixed business model is equivalent to the ad-funded business model. The analysis of the baseline model then applies.

By contrast, if $\theta v > \gamma$, the firm charges a strictly positive user fee $f^* = \theta v - \gamma$ in equilibrium, regardless of the business model of its rival.

A.5.5 Proof of Proposition 6

First, we show that no platform adopts business model A in equilibrium. Indeed, from the analysis above, we have: (i) $\Pi_{AM} < \Pi_{MM}$, so M is preferred to A when facing a rival with M; (ii) $\Pi_{AA} < \Pi_{MA}$, so M is preferred to A when facing a rival with A; (iii) $\Pi_{AU} < \Pi_{MU}$, so M is preferred to A when facing a rival with U.

Therefore, there are only three possible equilibria, UU, MU and MM, and everything is as if the platforms were choosing between U and M.

So, (U, U) is an equilibrium if and only if $\Pi_{UU} > \Pi_{MU}$, that is, if $\sigma < \gamma$ or $r < 1$. Similarly, (M, M) is an equilibrium if and only if $\Pi_{MM} > \Pi_{UM} = \Pi_{UU}$, that is, iff:

$$-(\sigma - \gamma)(2\theta v + \sigma - \gamma) + (1 - \beta)\sigma(4\theta v + (1 + 3\beta)\sigma - 4\gamma - 2) < 0. \tag{13}$$

Note that under Assumption 1, we have $2\theta v + \sigma - \gamma > 0$. Furthermore, we also have $4\theta v + (1 + 3\beta)\sigma - 4\gamma - 2 > 2(2\theta v + 2\beta\sigma - 2\gamma - 1) > 0$ under Assumption 1. Thus, if $\sigma < \gamma$, condition (13) does not hold. If $\sigma > \gamma$, it holds if the condition in the proposition holds.

Finally, (M, U) is an equilibrium if $\Pi_{MU} > \Pi_{UU}$, i.e., iff $\sigma > \gamma$ or $r > 1$, and $\Pi_{UM} > \Pi_{MM}$, i.e., condition (13) does not hold.

A.6 Single-homing

A.6.1 Demand

Suppose first that both platforms are user-funded. The consumer indifferent between joining platform 1 or 2 is located at $\tilde{x}^{UU} = \frac{1}{2} + \frac{q_1 - q_2}{2} - \frac{f_1 - f_2}{2}$, hence the demands are $D_1 = \tilde{x}^{UU}$ and $D_2 = 1 - \tilde{x}^{UU}$.

Suppose now that at least one platform is ad-funded. As users single-home, there is no discount for multiple impressions. Advertisers thus decide to run an ad campaign on platform i facing user demand D_i if the price p_i of the campaign satisfies $\sigma D_i - p_i \geq 0$. As in the baseline model with multi-homing (henceforth MH), there exists a unique

equilibrium in which an ad-funded platform sets the advertising price $p_i = \sigma D_i$, each advertiser runs an ad-campaign on both platforms, and users rationally expect $a_i^e = 1$. If both platforms are ad-funded, consumers face the same disutility from advertising on each platform: the demand for platform i is thus $D_i = \frac{1}{2} + \frac{q_i - q_j}{2}$. By contrast, if the two platforms choose asymmetric business models (say, platform 1 is user funded and platform 2 is ad-funded), the indifferent consumer is now located at $\tilde{x}^{UA} = \frac{1+(q_1-q_2)-(f_1-\gamma)}{2}$, and the demands are $D_1 = \tilde{x}^{UA}$ and $D_2 = 1 - \tilde{x}^{UA}$.

To ensure smooth profit functions and full market coverage, we make the following assumption:

Assumption 2 *Either (i)* $\gamma \leq 2$ *and* $0 \leq \sigma < 4$*, or (ii)* $2 \leq \gamma < \frac{21}{8}$ *and* $\frac{32(-2+\gamma)}{5} < \sigma < 4$.

In what follows we derive the equilibrium outcomes for each of the three business model configurations.

A.6.2 UU

Fees. Platform i's profit is given by:

$$\Pi_i = f_i(\frac{1}{2} + \frac{q_i - q_j}{2} - \frac{f_i - f_j}{2}) - c(q_i)$$

In Stage 2, the two platforms simultaneously set their price so as to maximize their profit. The equilibrium prices are:

$$f_i = 1 + \frac{q_i - q_j}{3}$$

Each platform's price increases in its own quality and decreases in its rival's quality.

Investment decision. In Stage 1, the first-order condition is now

$$\begin{aligned} 0 = \frac{\partial \Pi_i}{\partial q_i} &= \frac{\partial \Pi_i}{\partial q_i} + \frac{\partial \Pi_i}{\partial f_j}\frac{df_j(q_j, q_i)}{dq_i} - c'(q_i) \\ &= \frac{f_i(q_i, q_j)}{2} - \frac{1}{3}\frac{f_i(q_i, q_j)}{2} - q_i, \end{aligned}$$

which gives the platform's best response investment $q_i = \frac{3+q_i-q_j}{9}$. In equilibrium, each platform sets the optimal level of investment $q_i = \frac{1}{3}$.

In the single-homing configuration, when both platforms are user-funded, qualities are strategic substitutes. In equilibrium both platforms set a quality $q_{UU}^{sh} = \frac{1}{3}$.

Note that the marginal cost of the optimal quality is equal to two thirds of the fee instead of the full value of the fee under multi-homing (see equation 3.1). This results from two effects: first, with competition, demand for platform i is less elastic to q_i than the monopoly demand addressed by the platform under multi-homing (direct effect); second,

due to strategic interactions between the platforms, an increase in q_i now leads to a decrease in f_j, which in turn reduces platform i's demand (indirect effect).

Equilibrium outcomes As a result, equilibrium outcomes are as follows:

$$q_1 = q_2 = q_{UU}^{sh} = \frac{1}{3}$$
$$f_1 = f_2 = f_{UU}^{sh} = 1$$
$$\Pi_1 = \Pi_2 = \Pi_{UU}^{sh} = \frac{4}{9}.$$

Consumer surplus. In equilibrium each platform serves half of the users. Transport costs are thus $\frac{1}{4}$ and consumer surplus C_{UU} is

$$CS_{UU}^{sh} = v + \frac{1}{3} - 1 - \frac{1}{4} = v - \frac{11}{12}$$

A.6.3 AA

Investment decision. Platform i's profit is :

$$\Pi_i = \sigma \left(\frac{1}{2} + \frac{q_i - q_j}{2} \right) - c(q_i).$$

Each platform thus sets the following quality:

$$q_i = \frac{\sigma}{2}$$

In the AA configuration, platforms invest more in quality with single-homing than with multi-homing whenever $\beta \leq \frac{1}{2}$. This results from a trade-off: on one hand, under single-homing competition dissipates platforms' profits and demand is less elastic to quality than when users can multi-home ($\frac{1+q_i-q_j}{2}$ under SH *vs.* $\theta v + q_i - \gamma$ under MH), which drives quality down. On the other hand, when β is low the advertising price may be lower under multi-homing than under single-homing since ads are less valuable for multi-homers. This inefficiency reduces platforms' incentives to invest in quality. Interestingly, in the asymmetric market configuration, where the discount does not apply to multi-homers, the quality of the ad-funded platform q_{AU}^{sh} facing a user-funded platform is always lower under single-homing than under multi-homing (as $\frac{\sigma}{4} < \sigma$).

Equilibrium outcomes.

$$p_1 = p_2 = \sigma$$
$$q_1 = q_2 = q_{AA}^{sh} = \frac{\sigma}{2}$$
$$\Pi_1 = \Pi_2 = \Pi_{AA}^{sh} = \frac{\sigma}{2}\left(1 - \frac{\sigma}{4}\right)$$

Note that profits are concave in σ. For low values of σ, profits increase in σ because marginal revenue from the increased value of advertisers are higher than marginal cost from increased competition through innovation. For higher values of σ, the competition for user demand becomes so fierce that an increase in the value of advertisers leads to a decrease in profits.

Consumer surplus. Again in equilibrium $D_1 = D_2 = \frac{1}{2} = D_{AA}$ hence transports costs equal $\frac{1}{4}$ and consumer surplus is

$$CS_{AA}^{sh} = v + \frac{\sigma}{2} - \gamma - 1/4$$

A.6.4 UA

Suppose that platform 1 is user-funded while platform 2 is ad-funded (with a price $p = \sigma$).

Demands are:

$$D_1 = \frac{1 + (q_1 - q_2) - (f_1 - \gamma)}{2}$$
$$D_2 = \frac{1 + (q_2 - q_1) - (\gamma - f_1)}{2}$$

Prices Platform 1 maximizes its profit $\pi_1 = f_1 \frac{1+(q_1-q_2)-(f_1-\gamma)}{2} - \frac{q_1^2}{2}$ by setting $f_1 = \frac{1+q_1-q_2+\gamma}{2}$.

Quality investments Platform 1 maximizes its profit $\pi_1 = \frac{(1+q_1-q_2+\gamma)^2}{8} - \frac{q_1^2}{2}$ by setting $q_1 = \frac{1-q_2+\gamma}{3}$.

Platform 2 maximizes its profit $\pi_2 = \sigma\frac{1+(q_2-q_1)-(\gamma-f_1)}{2} - \frac{q_2^2}{2} = \sigma\frac{3+q_2-q_1-\gamma}{4} - \frac{q_2^2}{2}$ by setting $q_2 = \frac{\sigma}{4}$. Note that the ad-funded platform's optimal quality is independent on the rival's quality as the price received for each unit sold is constant. The user-funded platform responds by setting a quality $\frac{1}{3} + \frac{\gamma - q_{AU}^{sh}}{3} = \frac{1}{3} + \frac{\gamma - \sigma/4}{3}$. Note that the sign of $\gamma - \frac{\sigma}{4}$ reflects the balance between the "price" paid by the user (γ is the consumer's disutility from advertising) and the quality of the ad-funded platform ($\sigma/4$): when γ is large ($\gamma \geq \sigma/4$), the user-funded platform's quality is higher when it faces an ad-funded platform than

when it faces a user-funded rival. In that case, the quality of the user-funded platform can even be higher under single homing than under multi-homing.

Equilibrium outcomes.

$$
\begin{aligned}
p_2 &= \sigma \\
f_1 &= \frac{1+q_1-q_2+\gamma}{2} = \frac{4(1+\gamma)-\sigma}{6} \\
q_1 &= q_{UA}^{sh} = \frac{4(1+\gamma)-\sigma}{12} \\
q_2 &= q_{AU}^{sh} = \frac{\sigma}{4} \\
D_1 &= \frac{4(1+\gamma)-\sigma}{12} \\
D_2 &= 1 - D_1 \\
\Pi_1 &= \Pi_{UA}^{sh} = \frac{(4(1+\gamma)-\sigma)^2}{96} \\
\Pi_2 &= \Pi_{AU}^{sh} = \frac{\sigma(64-32\gamma+5\sigma)}{96}
\end{aligned}
$$

Consumer surplus $CS_1 = \int_0^{D_1} v + q_1 - f_1 - x dx = \frac{(4(1+\gamma)-\sigma)(8v-4(1+\gamma)+\sigma)}{96}$
$CS_2 = \int_{D_1}^1 v + q_2 - \gamma - (1-x)dx = \frac{(8-24v+20\gamma-5\sigma)(-8+4\gamma-\sigma)}{288}$
Hence the total CS: $CS_{UA}^{sh} = \frac{-56+144v+16\gamma^2+28\sigma+\sigma^2-8\gamma(14+\sigma)}{12^2}$

A.6.5 Quality-adjusted prices and consumer surplus

Under single-homing, the quality-adjusted prices of user-funded platforms are now positive irrespective of the business model of the rival, and no longer zero as in the case of multi-homing. Indeed, for a user-funded platform, demand is less elastic to quality under single homing compared to multi-homing (see Appendix A.6), resulting in lower quality levels relative to prices. The quality-adjusted price of a user-funded platform is lower if it faces an ad-funded platform than if it faces a user-funded rival iff σ is high enough (namely, $4(\gamma-1) \leq \sigma$).

For an ad-funded platform, the competition from another ad-funded platform is fiercer and leads to lower quality adjusted prices: we always have $\hat{p}_{AA} < \hat{p}_{AU}$. The quality-adjusted prices of an ad-funded platform facing a user-funded rival is also higher under single-homing than under multi-homing ($\hat{p}_{AU}^{sh} \equiv \gamma - \frac{\sigma}{4} \geq \hat{p}_{AU} = \gamma - \sigma$). However, the quality-adjusted price of an ad-funded platform facing an ad-funded rival can be higher under SH than under MH iff $\beta \geq \frac{1}{2}$.

We have presented above the consumer surplus in the different business model configurations. Intuitively, consumer surplus is higher in the AA configuration than in the UU configuration whenever the disutility from advertising is low (namely, when $\gamma \leq \frac{3\sigma+4}{6}$).

In the asymmetric equilibrium, consumer surplus is higher than in the UU configuration whenever the disutility from advertising is not too high ($\gamma \leq \frac{14+\sigma}{4} - \sqrt{\frac{15}{2}}$), and higher than in the AA configuration whenever $\gamma \geq \frac{\sigma-4}{4} + \frac{3\sqrt{1+\sigma}}{2}$.

Comparing consumer surplus in the different business model configurations, we obtain the configuration that would be preferred by consumers. Figure 7 illustrates this result.

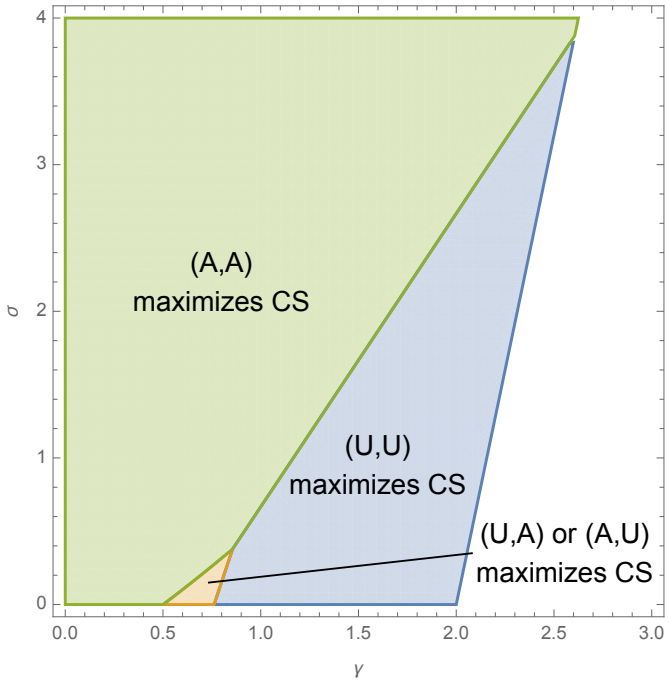

Figure 7: Consumer surplus maximizing configuration (single-homing)

Note: The white area is excluded by Assumption 2.

A.6.6 Business model decisions

Consider now the platform's business model decisions in stage 1.

Best response to rival choosing the user-funded business model Straightforward comparison of profits π^{sh}_{UU} and π^{sh}_{AU} reveals that the best response to the rival choosing the user-funded business model is to opt for the ad-funded business model whenever σ is sufficiently high: $\frac{8}{15}(-12+\sqrt{174}) < \sigma < 4$ and $0 < \gamma < \frac{-128+192\sigma+15\sigma^2}{96\sigma}$

Best response to rival choosing ad-funded business model Similarly, straightforward comparison of profits π^{sh}_{UA} and π^{sh}_{AA} reveals that the best response to the rival choosing the ad-funded business model is to opt for the user-funded business model whenever σ is low or γ is sufficiently high:

$$\sigma \leq \frac{4}{13}$$
$$\text{or } \frac{4}{13} \leq \sigma \leq 4 \text{ and } \frac{\sigma - 4}{4} + \frac{\sqrt{3}\sqrt{4\sigma - \sigma^2}}{2} \leq \gamma$$

As a result, within the parameter space delimited by Assumption 2, three types of equilibria can co-exit: both platforms choose to be ad-funded, both choose to be user-funded, or the two choose different business models. Figure 5 illustrates this result.

Finally, we compare the equilibrium business model configuration with the one that would maximize consumer surplus. Figure 8 below illustrates the areas, in which the business models that endogenously arise in equilibrium coincides with those that maximize consumer surplus.

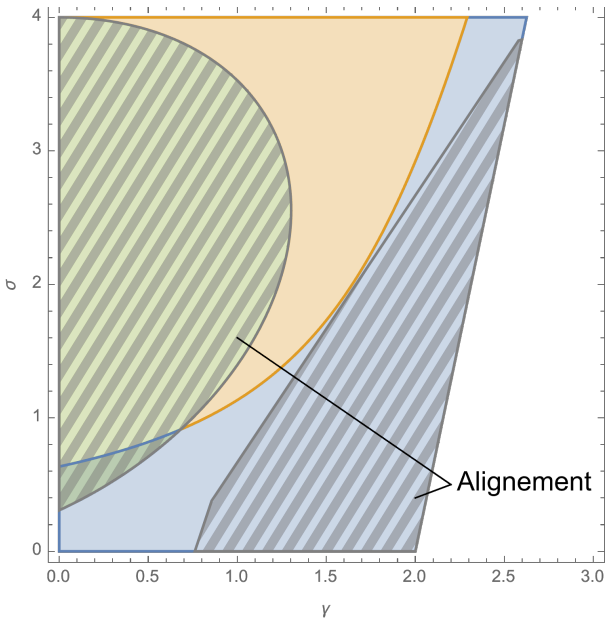

Figure 8: Equilibrium vs. CS-maximizing configurations (single-homing)

Note: The white area is excluded by Assumption 2.

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