ESTIMATION OF FUNCTIONALS OF HIGH-DIMENSIONAL PARAMETERS: BIAS REDUCTION AND CONCENTRATION

Vladimir Koltchinskii

Georgia Institute of Technology

SCHEDULE	Monday	13th May 2024 27th May 2024	From 13:00 to 16:15	TBA
	Thursday	16th May 2024 23rd May 2024	From 13:00 to 16:15	TBA

Aims and objectives

The main focus of this course will be on a circle of problems related to estimation of real valued functionals of parameters of high-dimensional statistical models. In such problems, it is of interest to estimate one-dimensional features of a high-dimensional parameter that are often represented by nonlinear functionals of certain degree of smoothness defined on the parameter space. The functionals of interest could be estimated with faster convergence rates than the whole parameter (sometimes, even with parametric rates). The examples include, for instance, such problems as estimation of linear functionals of principal components (that are nonlinear functionals of unknown covariance) in high-dimensional PCA. The goal is to discuss several mathematical methods that provide a way to develop estimators of functionals of high-dimensional parameters with optimal error rates in classes of functionals of some Hölder smoothness. Moreover, when the degree of smoothness of the functional is above certain threshold, the estimators in question have parametric $\sqrt{n}$ error rate and are asymptotically efficient, whereas the error rates become slower than $\sqrt{n}$ when the degree of smoothness is below the threshold.

The following topics will be covered (at least, to some extent):

• preliminaries in high-dimensional probability and analysis (concentration inequalities, comparison inequalities, Hölder smoothness of operator functions, etc);

• non-asymptotic bounds and concentration inequalities for sample covariance in high-dimensional and dimension-free frameworks;

• some approaches to concentration inequalities for smooth functionals of statistical estimators;

• higher order bias reduction methods in functional estimation;

- methods based on Taylor expansion and estimation of polynomials with reduced bias;
- iterative bias reduction and bootstrap chains;
- linear aggregation of plug-in estimators with different sample sizes and jackknife estimators;

• minimax lower bounds in functional estimation (applications of van Trees inequality, Nemirovski's construction of least favorable functionals, etc);

• Examples:

  – high-dimensional and infinite dimensional Gaussian models: functionals of mean and of covariance; – log-concave models, in particular, log-concave location families;

  – high-dimensional exponential families;

  – nonparametric models, functionals of unknown density;

  – linear functionals of spectral projections of matrix parameters.

Pre-requisites

T.B.A.

Some related literature

R. Adamczak and P. Wolffe, Concentration inequalities for non-Lipschitz functions with bounded derivatives of higher orders, P*robability Theory and Related Fields*, 2015, 162 (3-4), 531–586.

J. Jiao and Y. Han, Bias correction with bootstrap, jackknife and Taylor series, *IEEE Transactions on Information Theory*, 2020, 66, 4392–4418.

V. Koltchinskii and K. Lounici, Concentration Inequalities and Moment Bounds for Sample Covariance Operators, *Bernoulli,* 2017, 23, 1, 110-133.

V. Koltchinskii, M. Löffler and R. Nickl, Efficient Estimation of Linear Functionals of Principal Components, *Annals of Statistics*, 2020, 48, 1, 464– 490.

V. Koltchinskii, Asymptotically Efficient Estimation of Smooth Functionals of Covariance Operators, *J. European Mathematical Society*, 2021, 23, 3, 765–845.

V. Koltchinskii and M. Zhilova, Estimation of Smooth Functionals in Normal Models: Bias Reduction and Asymptotic Efficiency, *Annals of Statistics*, 2021, 49, 5, 2577-2610.

V. Koltchinskii, Estimation of smooth functionals in high-dimensional models: bootstrap chains and Gaussian approximation, *Annals of Statistics*, 2022, 50(4), 2386–2415.

V. Koltchinskii, Estimation of smooth functionals of covariance operators: jackknife bias reduction and bounds in terms of effective rank, 2022, arXiv:2205.10280.

V. Koltchinskii and M. Wahl, Functional estimation in log-concave location families, In: *High Dimensional Probability*, 2023, Springer (to appear). arXiv:2108.00263.

Nemirovski, *Topics in Non-Parametric Statistics*, in: M. Emery, A. Nemirovski, D. Voiculescu, Lectures on Probability Theory and Statistics, Ecole d'ete de Probabilities de Saint-Flour, XXVIII - 1998, Editor: P. Bernard, Lecture Notes in Mathematics,1738, Springer, 2000.

R. Vershynin, *High-Dimensional Probability. An Introduction with Applications in Data Science*, Cambridge University Press, 2018.