CRESTCENTER FOR RESEARCH IN ECONOMICS AND STATISTICS

WORKING PAPER SERIES

The Trilemma for Low Interest Rate Macroeconomics

Jean-Baptiste Michau

n°19/September 2022

CREST
Center for Research in Economics and Statistics
UMR 9194

5 Avenue Henry Le Chatelier
TSA 96642
91764 Palaiseau Cedex
FRANCE

Phone: +33 (0)1 70 26 67 00
Email: info@crest.science
https://crest.science/

The Trilemma for Low Interest Rate Macroeconomics

Jean-Baptiste Michau*

September 2022

Abstract

Three desirable goals of macroeconomic policy are: full employment, low inflation, and a low debt level with no Ponzi scheme. This paper shows that, when the natural real interest rate is persistently depressed, at most two of these three goals can be simultaneously achieved. Depending of the parameters of the economy, each of the three possibilities can be the preferred option, resulting in a non-trivial policy trilemma.

Keywords: Liquidity trap, Ponzi scheme, Secular stagnation

JEL Classification: E12, E31, E63, H63

1 Introduction

Over the past decades, the natural real interest rate has progressively and persistently declined across the industrialized world to such an extent that Japan, the Eurozone, and even the United States have spent long stretches of time with a binding zero lower bound on the nominal interest rate. It took massive fiscal stimuli following the Covid pandemic and large contractionary supply shocks to raise inflation above target. However, population aging, declining productivity growth, high inequality, and persistently low demand for investment suggest that the natural real interest rate is likely to remain depressed for the foreseeable future. This paper argues that this entails a major challenge to macroeconomic policy.

The traditional response to this challenge has been to advocate for a rise in the inflation target such as to prevent the zero lower bound from binding (Krugman, 1998). However, central banks are reluctant to raise their target above 2%. An alternative response to persistently depressed demand consists in implementing a fiscal expansion

*Ecole Polytechnique, France; jean-baptiste.michau@polytechnique.edu.

financed by an increase in the debt-to-GDP ratio, unbacked by future tax increases, which can be sustainable provided that the real interest rate is below the growth rate of the economy (Blanchard, 2019; Krugman, 2021). This paper formalizes this idea within a secular stagnation framework, while also highlighting the risk this strategy entails for price stability.

Secular stagnation is characterized by low inflation, determined by binding downward nominal wage rigidities, and a binding zero lower bound. This induces the real interest rate to be above its natural counterpart, resulting in underemployment. This suggests a policy dilemma: either raise the inflation target sufficiently to depart from the zero lower bound or keep the economy depressed. But, under secular stagnation, the natural real interest rate is so low that a Ponzi scheme of public debt is likely to be sustainable. Government transfers to households financed by rolling over debt, rather than by raising future taxes, generate a wealth effect that stimulate aggregate demand, which can restore full employment. However, a Ponzi scheme is inherently unstable and can collapse, either because households coordinate on running away from it following a sunspot shock or because a positive shock to the natural real interest rate brings stagnation to an end. Such a run restores the fiscal theory of the price level, which results in an upward jump in the price level that shrinks the Ponzi scheme to zero. While the Ponzi scheme can restore full employment without raising the inflation target, it entails a small probability of a sudden debasement of the currency.[1]

The government must therefore choose between a depressed economy, a higher inflation target, or a Ponzi scheme. If either the welfare cost of changes to the price level or the likelihood of collapse of the Ponzi scheme is sufficiently low, then the Ponzi debt scheme is optimal... until it collapses. Otherwise, the optimal policy consists in permanently higher inflation and full employment, unless the welfare cost of inflation is so high that a persistently depressed economy is preferable.

My analysis assumes a preference for wealth. This makes it possible to have secular stagnation (Michau, 2018) and rational bubbles (Michau, Ono and Schlegl, 2022) within a representative household model of the economy. However, the policy trilemma does not rely on this specific microfoundation. Indeed, any model of secular stagnation must have both a depressed natural real interest rate and a steady state with a finite elasticity of consumption with respect to the real interest rate, which are the ingredients needed for a rational bubble. In particular, the trilemma can be derived within the Eggertsson, Mehrotra and Robbins (2019) OLG model of secular stagna-

[1] A Ponzi scheme can also crowd out capital. However, for this to be an adverse effect, the marginal product of capital must be larger than the growth rate of the economy, which is itself larger than the real interest rate. This requires a friction such as imperfect competition or liquidity constraints. My analysis abstracts from capital and therefore abstracts from these effects.

tion.[2]

Related literature. In a highly influential AEA Presidential Lecture, Blanchard (2019) has argued that public debt sustainability need not be a concern in a low interest rate environment. Building on this insight, Krugman (2021) has argued that, under secular stagnation, public debt is an attractive alternative to higher inflation to achieve full employment. Mankiw (2022) has warned about the possibility that, even when a Ponzi scheme can be sustainable, a run on public debt can occur and is likely to be painful. This paper formalizes these insights.

Billi, Gali and Nakov (2022) have characterized the optimal trade-off between excessive inflation and insufficient economic activity within a New Keynesian economy with a persistently depressed natural real interest rate, but without bubbles. When a Ponzi scheme is sustainable, Kocherlakota (2022*b*) and Miao and Su (2021) have shown that fiscal policy can be essential to stabilize economic activity. My analysis emphasizes that such fiscal policy can reduce the inflation rate that is necessary to achieve full employment, but it entails the risk of a price level jump when the Ponzi scheme collapses.

Bassetto and Cui (2018) have shown that the fiscal theory of the price level does not uniquely pin down the price level when the interest rate is below the growth rate of the economy, since a Ponzi scheme may or may not arise. Brunnermeier, Merkel and Sannikov (2022) have argued that the steady state with a Ponzi scheme can be made the unique equilibrium provided that the government makes an off-the-equilibrium commitment to run primary surpluses forever if the Ponzi scheme collapses (which forces the real interest rate to be positive thereby ruling out alternative equilibrium possibilities). My analysis implicitly assumes that the government is not able to make such a strong commitment and therefore cannot prevent the possibility of a run on Ponzi debt.

Mian, Straub and Sufi (2022) have characterized the maximum budget deficit that can be sustained forever when the natural real interest rate is depressed. With a binding zero lower bound, this budget deficit, and the corresponding debt level, may not be sufficient for the economy to produce at full capacity. By contrast, my analysis determines the magnitude of the Ponzi scheme that is required for the economy to produce at full capacity.[3]

Michau (2022*b*) has shown that, under secular stagnation, helicopter drops of money

[2] The Ono (1994, 2001) model of secular stagnation assumes a constant marginal utility of wealth (or of real money balances), which annihilates the wealth effect from the Ponzi scheme. This results in a dilemma: higher inflation or underemployment.

[3] Barro (2020), Mehrotra and Sergeyev (2021), Reis (2021), Cochrane (2021), Kocherlakota (2022*a*) have also carefully investigated the sustainability of public debt in low interest rate environments, but in real economies without the possibility of depressed demand.

can be stimulative and need not be inflationary. Relying on similar microfoundations, the present analysis adds two features to have a non-trivial policy trilemma: price instability has a negative impact on welfare and the Ponzi scheme can collapse through a stochastic jump in the price level.

Section 2 presents the setup of the economy, section 3 defines the equilibrium, and section 4 characterizes the steady state equilibria. The policy trilemma is investigated in section 5. Section 6 discusses the nature of the shock inducing the price level jump. Possible ways to break through the trilemma are discussed in section 7. The paper ends with a conclusion.

2 Economy

The economy consists of identical firms, identical households, and a government. The only friction is a downward nominal wage rigidity.

Time is continuous. There is a unit mass of of infinitely lived households. Population within each household grows at rate n. At time t, the total population of the economy is equal to $N_t = e^{nt}$.

2.1 Government

Nominal indebtedness at time t amounts to B_t. Real lump-sum taxes per capita are set equal to τ_t. Public indebtedness therefore evolves according to

$$\dot{B}_t = i_t B_t - \tau_t P_t N_t, \tag{1}$$

where i_t denotes the nominal interest rate and P_t the aggregate price level at t.

Real indebtedness per capita is given by $b_t = B_t/(P_t N_t)$. Real primary surpluses per capita at time t amount to τ_t. I denote by Φ_t the expected present value of these real primary surpluses from time t onward. In the absence of Ponzi scheme, we would have $b_t = \Phi_t$. It is therefore natural to define the magnitude of the government Ponzi debt scheme as

$$\Delta_t = b_t - \Phi_t. \tag{2}$$

Whether a Ponzi is sustainable will be determined endogenously in equilibrium.

Throughout my analysis, I allow for the possibility that, if a Ponzi scheme exists, it can collapse with probability εdt at time t. This corresponds to a situation where, following a sunspot shock, households suddenly run away from the Ponzi scheme, which induces an upward jump in the price level that must increase by Δ_t/Φ_t.[4] The

[4]Indeed, as the price level increases from P_t to $P_t(1 + \Delta_t/\Phi_t)$, public indebtedness falls from

evolution of the price level is therefore given by the following stochastic process

$$dP_t = \pi_t P_t dt + \frac{\Delta_t}{\Phi_t} P_t dJ_t, \tag{3}$$

where π_t denotes the inflation rate at time t in the absence of price level jump, while dJ_t denotes the Poisson jump, which is equal to 1 with probability εdt and to 0 with probability $1 - \varepsilon dt$. Throughout my analysis, I assume that lump-sum taxes are set such that the present value of surpluses Φ_t is strictly positive; otherwise, an arbitrarily large price level jump would not be sufficient to eliminate the Ponzi scheme.

Using Itô's lemma with jumps, we can compute $d(1/P_t)$ and, hence, $d(B_t/(P_t N_t))$, which gives

$$db_t = [(i_t - \pi_t - n)b_t - \tau_t]\, dt - \Delta_t dJ_t. \tag{4}$$

When the shock occurs, public debt falls from b_t to $b_t - \Delta_t = \Phi_t$. The derivation is provided in appendix A.

Finally, monetary policy follows a Taylor rule, unless the zero lower bound is binding, which implies

$$i_t = \max\{r^n + \pi^* + \phi[\pi_t - \pi^*], 0\}, \tag{5}$$

where π^* is the inflation target, r^n is the natural real interest rate to be subsequently defined, and ϕ determines the responsiveness of the nominal interest rate to inflation.

2.2 Firms

For simplicity, I assume that labor is the only factor of production. The representative firm employs L_t units of labor per capita to produce output Y_t using a constant returns to scale production function

$$Y_t = N_t L_t. \tag{6}$$

Employment therefore amounts to $N_t L_t$. The real wage w_t is equal to the marginal product of labor, which gives

$$w_t = 1. \tag{7}$$

2.3 Households

The representative household discounts the future at rate ρ, where $\rho > n$. It inelastically supplies one unit of labor per capita, resulting in aggregate labor supply being equal to N_t. The household drives utility $u(c_t)$ from consuming c_t per capita at time t, where $u'(\cdot) > 0$, $u''(\cdot) < 0$, and $\lim_{c \to 0} u'(c) = \infty$.

$B_t/(N_t P_t) = \Delta_t + \Phi_t$ to $B_t/(N_t P_t(1 + \Delta_t/\Phi_t)) = \Phi_t$.

The household also derives utility from holding wealth, which is equal to a_t per capita. However, government debt b_t must eventually be repaid through taxes; unless the government is running a Ponzi scheme. The expected present value of taxes is therefore equal to $b_t - \Delta_t$, where Δ_t denotes the magnitude of the government's Ponzi scheme. The household perceives its *net wealth* to be equal to $a_t - b_t + \Delta_t$ at time t, and derives utility $\gamma(a_t - b_t + \Delta_t)$ from holding it. This specification of net wealth implies that households are Ricardian. A lump-sum transfer eventually repaid through a lump-sum tax temporarily raises both a_t and b_t by the same amount, while leaving $a_t - b_t + \Delta_t$ unchanged. Thus, the marginal utility of wealth $\gamma'(a_t - b_t + \Delta_t)$ is unaffected by the transfer, consistently with the Ricardian equivalence proposition. The preference for wealth satisfies $\gamma'(\cdot) > 0$, $\gamma''(\cdot) < 0$, $\gamma'(0) < \infty$, $\lim_{k\to\infty} \gamma'(k) = 0$, and $\int_0^\infty \gamma'(e^{\lambda t})dt < \infty$ for any $\lambda > 0$.[5]

Finally, the household gets disutility $\psi c(dP_t/P_t)$ from changes to the price level, where ψ determines the strength of this disutility, while the function $c(\cdot)$ is given by

$$\begin{aligned} c\left(\frac{dP_t}{P_t}\right) &= \begin{cases} \frac{1}{dt}\left|\frac{dP_t}{P_t}\right| & \text{if } dJ_t = 0 \\ \frac{1}{dt}C\left(\left|\frac{dP_t}{P_t}\right|\right) & \text{if } dJ_t = 1 \end{cases}, \\ &= \begin{cases} |\pi_t| & \text{if } dJ_t = 0 \\ \frac{1}{dt}C\left(\frac{\Delta_t}{\Phi_t}\right) & \text{if } dJ_t = 1 \end{cases}, \end{aligned} \tag{8}$$

where the function $C(\cdot)$ satisfies $C(0) = 0$, $C'(0) \geq 1$, and $C''(\cdot) \geq 0$.[6] This specification implies that, in the absence of a price level jump, the household gets linear disutility from inflation. In addition, a price level jump entails a discrete cost, which is increasing and weakly convex in the magnitude of the jump. This utility cost of inflation has no impact on the behavior of the representative household, since inflation is beyond its control, but it will be relevant for our subsequent welfare analysis. The adverse effect of inflation can be interpreted as the mental cost of optimizing purchases when the price level changes, together with an additional convex cost from the financial disruption and the loss of monetary credibility entailed by a sudden debasement of the currency.

[5]This last technical condition, which makes it possible to rule out explosive Ponzi schemes, is very mild. It must be satisfied for the CRRA specification $\gamma(k) = [(k - \underline{k})^{(1-\sigma)} - 1]/(1-\sigma)$ with reference wealth level $\underline{k} < 0$.

[6]If we impose $C'(0) = 1$, then the two parts of (8) can be nested into the single expression $c\left(\frac{dP_t}{P_t}\right) = \frac{C(|dP_t/P_t|)}{dt}$. Indeed, by (3), if $dJ_t = 1$, we have $|dP_t/P_t| = \Delta_t/\Phi_t$; while, if $dJ_t = 0$, we have $\frac{C(|dP_t/P_t|)}{dt} = \frac{C(|\pi_t|dt)}{dt} = \frac{C(0)+|\pi_t|dtC'(0)}{dt} = |\pi_t|$.

The household's expected intertemporal utility is given by

$$\mathbb{E}_0\left[\int_0^\infty e^{-(\rho-n)t}\left[u(c_t)+\gamma(a_t-b_t+\Delta_t)-\psi c\left(\frac{dP_t}{P_t}\right)\right]\,dt\right]. \tag{9}$$

Let r_t denote the real return that is risk-free in *real* terms (whereas $i_t-\pi_t$ is the real return that is risk-free in *nominal* terms, but risky in real terms due to the inflation risk). The portfolio of the representative household h is composed of two assets: government bonds b_t^h and bonds that are risk-free in real terms d_t^h. Thus, at any point in time $a_t=b_t^h+d_t^h$. The risk-free bonds yield a return r_t-n per capita. Government bonds yield $i_t-\pi_t-n$ and their value drops by Δ_t/b_t when the price level jumps, which occurs with probability εdt at time t. The household receives labor income w_tL_t, pays lump-sum taxes τ_t, and consumes c_t per capita.[7] Hence, household wealth per capita follows

$$\begin{aligned} da_t &= \left[(r_t-n)d_t^h+(i_t-\pi_t-n)b_t^h+w_tL_t-\tau_t-c_t\right]dt-b_t^h\frac{\Delta_t}{b_t}dJ_t, \\ &= \left[(r_t-n)a_t+w_tL_t-\tau_t-c_t\right]dt+b_t^h\left[(i_t-\pi_t-r_t)dt-\frac{\Delta_t}{b_t}dJ_t\right]. \end{aligned} \tag{10}$$

Finally, the household is subject to a no-borrowing constraint

$$a_t\geq 0. \tag{11}$$

In equilibrium, this constraint is never binding since households are identical and the supply of assets is always positive.

The representative household maximizes its expected utility (9) subject to its flow of funds constraint (10) with initial wealth a_0 and to the no-borrowing constraint (11).

Before the jump in the price level has occurred, the intertemporal allocation of consumption satisfies the Euler equation

$$\frac{\dot{c}_t}{c_t}=\left(\frac{u'(c_t)}{-c_tu''(c_t)}\right)\left[r_t-\rho+\frac{\gamma'(a_t-b_t+\Delta_t)}{u'(c_t)}+\varepsilon\left(\frac{u'(\bar{c}_t)}{u'(c_t)}-1\right)\right], \tag{12}$$

where $\bar{c}_t$ denotes consumption at time t immediately after the price level jump. Once the price level jump has occurred, consumption follows a similar Euler equation with both Δ_t and ε equal to zero. The optimal portfolio allocation between risky government debt b_t^h and risk-free bonds d_t^h results in

$$r_t=i_t-\pi_t-\varepsilon\frac{\Delta_t}{b_t}\frac{u'(\bar{c}_t)}{u'(c_t)}. \tag{13}$$

[7] If there is less than full employment, labor demand L_t is below 1 and labor income is equal to w_tL_t.

The real return on government bonds $i_t - \pi_t$ is above the risk-free real interest rate r_t due to the price level risk. Finally, the optimizing behavior of the household implies that following transversality condition must be satisfied

$$\lim_{t\to\infty} \mathbb{E}_0 \left[e^{-(\rho-n)t} u'(c_t) a_t\right] = 0. \tag{14}$$

The Euler equation (12), the risk premium relationship (13), and the transversality condition (14) are sufficient conditions to characterize a solution to the household's problem. This is formally established in appendix B.

The role of the preference for wealth can be seen from the Euler equation (12): in addition to raising the propensity to save, it ensures that in steady state, i.e. when $\dot{c}_t = 0$, consumption is a decreasing function of the real interest rate. This is essential to allow for the possibility of a sustainable Ponzi scheme or of secular stagnation.

2.4 Downward nominal wage rigidity

Workers never accept a rate of nominal wage growth that falls below a reference rate of inflation π^R. But, the profit maximizing behavior of firms implies that, under our linear production function, the real wage must always be equal to 1, as given by (7). Hence, the price level P_t must be equal to the nominal wage rate. So, the downward nominal wage rigidity prevents inflation from ever falling below π^R. This results in two possibilities: if inflation is above π^R, the downward nominal wage rigidity is not binding, ensuring full employment with $L_t = 1$; conversely, if there is less than full employment with $L_t < 1$, the downward nominal wage rigidity must be binding, resulting in inflation equal to π^R. We must therefore have[8]

$$\pi_t \geq \pi^R \text{ and } L_t \leq 1 \text{ with complementary slackness.} \tag{15}$$

Throughout my analysis, I assume that the inflation target π^* from the Taylor rule (5) is greater or equal to the reference rate of inflation π^R.

2.5 Market clearing

For the economy to be in equilibrium, markets must clear. Goods market clearing requires aggregate demand $N_t c_t$ to be equal to aggregate supply $Y_t = N_t L_t$, which

[8] Michau (2018) offers a slightly more flexible specification, whereby under-employment induces workers to accept rate of nominal wage growth below π^R. However, empirically, the Phillips curve is very flat at low rates of inflation (Forbes, Gagnon and Collins, 2021), suggesting that downward nominal wage flexibility is very limited.

gives

$$c_t = L_t. \tag{16}$$

Financial market clearing requires households' demand for government bonds b_t^h and for risk-free bonds d_t^h to be equal to their respective supply, equal to b_t and 0. As $a_t = b_t^h + d_t^h$, this implies

$$a_t = b_t. \tag{17}$$

Hence, net household wealth $a_t - b_t + \Delta_t$ must always be equal to Δ_t. Finally, the labor market clearing condition is replaced by the downward nominal wage rigidity (15).

3 Equilibrium

Given the structure of the economy, and the preference for wealth of the representative household, the stochastic discount factor must be given by $\Lambda_t = e^{-\int_0^t \left(\rho - n - \frac{\gamma'(\Delta_u)}{u'(c_u)}\right) du} u'(c_t)$. This allows us to provide a precise definition of the present value of primary surpluses Φ_t, from which the magnitude of the Ponzi debt scheme $\Delta_t = b_t - \Phi_t$ can be deduced.

Let the expected present value of real primary surpluses Φ_t be defined by

$$d\Phi_t = \left[(r_t - n)\Phi_t - \tau_t\right] dt, \tag{18}$$

together with the boundary condition $\lim_{T\to\infty} \mathbb{E}_t \left[\Lambda_T \Phi_T\right] = 0$. As the primary surpluses are not directly affected by the price level jump, they are discounted using the risk-free real interest rate, equal to $r_t - n$ per capita. As required, this definition implies

$$\Phi_t = \mathbb{E}_t \left[\int_t^\infty \frac{\Lambda_s}{\Lambda_t} \tau_s ds \right]. \tag{19}$$

This is formally established in appendix C. The evolution of public debt b_t and of the present value of surpluses Φ_t, respectively given by (4) and (18), together with expression for the risk premium (13), imply that the Ponzi scheme $\Delta_t = b_t - \Phi_t$ follows

$$d\Delta_t = \left[r_t - n + \varepsilon \frac{u'(\bar{c}_t)}{u'(c_t)} \right] \Delta_t dt - \Delta_t dJ_t, \tag{20}$$

which implies that

$$\Delta_t = \lim_{T\to\infty} \mathbb{E}_t \left[\frac{\Lambda_T}{\Lambda_t} \Delta_T \right]. \tag{21}$$

This is also shown in appendix C. Hence, a Ponzi scheme is only valuable if households expect it to be valuable in the future.

Throughout my analysis, I do not impose the government's no-Ponzi condition

$\Delta_t \leq 0$. Instead, the limit to public indebtedness is endogenously determined by households' willingness to lend to their government, which is itself determined by their transversality condition (14). Note that, in the absence of a preference for wealth, this transversality condition (14), together with the asset market clearing equation (17) and the expression for the stochastic discount factor, implies that $\Delta_t = 0$. But, with a preference for wealth, $\Delta_t > 0$ is possible.[9]

The no-borrowing constraint (11) prevents households from running Ponzi schemes. Hence, by Walras' law, the government's no-Ponzi condition must either be binding $\Delta_t = 0$ or violated $\Delta_t > 0$, but cannot be slack. I henceforth consider that $\Delta_t \geq 0$.[10]

Recall that Φ_t was assumed to be strictly positive; otherwise, by (3), an arbitrarily large price level jump could not eliminate the Ponzi scheme. The supply of asset $b_t = \Phi_t + \Delta_t$ must therefore always be strictly positive. This implies that the household's no-borrowing constraint (11) cannot be binding in equilibrium. Also, in equilibrium, the household's transversality condition (14) can be written as $\lim_{t\to\infty} e^{-(\rho-n+\varepsilon)t} u'(c_t)\Delta_t = 0$ conditional on the absence of a price level jump. This is shown in appendix D.

An equilibrium of the economy *before* the price level jump $(c_t, \Delta_t, i_t, \pi_t, r_t)_{t=0}^{\infty}$ is fully characterized by the Euler equation (12) with asset market clearing (17):

$$\frac{\dot{c}_t}{c_t} = \left(\frac{u'(c_t)}{-c_t u''(c_t)}\right)\left[r_t - \rho + \frac{\gamma'(\Delta_t)}{u'(c_t)} + \varepsilon\left(\frac{u'(\bar{c}_t)}{u'(c_t)} - 1\right)\right]; \tag{22}$$

the downward nominal wage rigidity (15) with goods market clearing (16):

$$\pi_t \geq \pi^R \text{ and } c_t \leq 1 \text{ with complementary slackness;} \tag{23}$$

the risk-premium equation (13) with $b_t = \Phi_t + \Delta_t$:

$$r_t = i_t - \pi_t - \varepsilon \frac{\Delta_t}{\Phi_t + \Delta_t} \frac{u'(\bar{c}_t)}{u'(c_t)}; \tag{24}$$

the Taylor rule (5):

$$i_t = \max\{r^n + \pi^* + \phi[\pi_t - \pi^*], 0\}; \tag{25}$$

[9] As $\lim_{T\to\infty} \mathbb{E}_t[\Lambda_T \Phi_T] = 0$, we have $\Lambda_t \Delta_t = \lim_{T\to\infty} \mathbb{E}_t[\Lambda_T \Delta_T] = \lim_{T\to\infty} \mathbb{E}_t[\Lambda_T(b_T - \Phi_T)] = \lim_{T\to\infty} \mathbb{E}_t[\Lambda_T b_T]$. Using the stochastic discount factor and the asset market clearing condition (17) yields $\Delta_t \Lambda_t = \lim_{T\to\infty} \mathbb{E}_t\left[e^{-\int_0^T \left(\rho - n - \frac{\gamma'(\Delta_u)}{u'(c_u)}\right) du} u'(c_T) a_T\right]$. Hence, without the preference for wealth, the households' transversality condition (14) implies $\Delta_t = 0$. But, as we shall see, with a preference for wealth, we can have $\Delta_t > 0$.

[10] Formally, since $\lim_{T\to\infty} \mathbb{E}_t[\Lambda_T \Phi_T] = 0$, we have $\Lambda_t \Delta_t = \lim_{T\to\infty} \mathbb{E}_t[\Lambda_T \Delta_T] = \lim_{T\to\infty} \mathbb{E}_t[\Lambda_T b_T]$. Hence, the no-borrowing constraint $a_t \geq 0$ together with the asset market clearing condition $a_t = b_t$ implies $\Delta_t \geq 0$.

the dynamics of the Ponzi scheme (20) with $dJ_t = 0$:

$$\dot{\Delta}_t = \left[r_t - n + \varepsilon \frac{u'(\bar{c}_t)}{u'(c_t)} \right] \Delta_t; \tag{26}$$

and the transversality condition conditional on the absence of a price level jump:

$$\lim_{t \to \infty} e^{-(\rho - n + \varepsilon)t} u'(c_t) \Delta_t = 0. \tag{27}$$

The equilibrium *after* the price level jump is also characterized by (22)-(27), but with $\Delta_t = 0$ and $\varepsilon = 0$. This determines the consumption level immediately after the price level jump $\bar{c}_t$, which affects the economy before the jump through (22), (24), and (26).

4 Steady state equilibria

Let us now characterize the steady state equilibria of the economy (c, π, r, i, Δ) before the occurrence of the price level jump. From the downward nominal wage rigidity (23), we must either have full employment with $c = 1$ or low inflation with $\pi = \pi^R$. From the dynamics of the Ponzi scheme (26), we must either have no Ponzi scheme with $\Delta = 0$ or a Ponzi scheme of constant magnitude with $r = n - \varepsilon u'(\bar{c})/u'(c)$.[11] This gives the following four steady state equilibrium possibilities:

- A *neoclassical steady state* with full employment $c = 1$ and no Ponzi scheme $\Delta = 0$;
- A *secular stagnation steady state* with low inflation $\pi = \pi^R$, no Ponzi scheme $\Delta = 0$, and under-employment $c < 1$;
- A *Ponzi steady state* with full employment $c = 1$, interest rate $r = n - \varepsilon u'(\bar{c})/u'(c)$, and a Ponzi scheme $\Delta > 0$;
- A *Ponzi-stagnation steady state* with low inflation $\pi = \pi^R$, interest rate $r = n - \varepsilon u'(\bar{c})/u'(c)$, under-employment $c < 1$, and a Ponzi scheme $\Delta > 0$.

Once the price level jump has occurred, only the first two steady state survive.

I now characterize each of these steady state equilibria.

4.1 Neoclassical steady state

A neoclassical steady state $(c^n, \pi^n, r^n, i^n, \Delta^n)$ is characterized by full employment $c^n = 1$ and no Ponzi scheme $\Delta^n = 0$. Recall from (3) that, in the absence of Ponzi scheme,

[11] Note that an explosive Ponzi scheme, with $\lim_{t \to \infty} \Delta_t = \infty$, cannot be an equilibrium outcome. This is shown in appendix E.

the price level cannot jump. We can therefore consider that, once in the neoclassical steady state, the economy remains there. Hence, $\bar{c} = c^n = 1$. From the consumption Euler equation (22), the real interest rate is therefore given by

$$r^n = \rho - \frac{\gamma'(0)}{u'(1)}. \tag{28}$$

This is the *natural real interest rate*, which enters the Taylor rule (25). A persistent lack of demand corresponds to a low natural real interest rate r^n. In this framework, this results from a strong marginal utility of wealth $\gamma'(0)$. This can be seen as a proxy for other factors depressing aggregate demand, such as population aging, which would not change the nature of the underlying policy trilemma.

In the absence of price level risk, the risk premium in (24) is trivially equal to zero, which gives $i^n = r^n + \pi^n$. The Taylor rule (25) therefore entails $r^n = \max\{r^n + (\phi - 1)[\pi^n - \pi^*], -\pi^n\}$. Hence, we must either have $\pi^n = \pi^*$ or $\pi^n = -r^n$; and both possibilities require

$$\pi^* \geq -r^n. \tag{29}$$

This shows that, for the neoclassical steady state to exist with a depressed natural real interest rate r^n, the inflation target π^* must be sufficiently high to overcome the zero lower bound on the nominal interest rate. This is an important ingredient of the policy trilemma.

Finally, the downward nominal wage rigidity (23) requires $\pi^n \geq \pi^R$. But, as either $\pi^n = \pi^*$ or $\pi^n = -r^n$, while $\pi^* \geq -r^n$, we must have $\pi^n \geq -r^n$. And, as we are about to see, $-r^n > \pi^R$ is a necessary condition for the secular stagnation steady state to exist and, hence, for the trilemma to arise. It follows that $\pi^n > \pi^R$, implying that the downward nominal wage rigidity is slack.

4.2 Secular stagnation steady state

A secular stagnation steady state $(c^{ss}, \pi^{ss}, r^{ss}, i^{ss}, \Delta^{ss})$ is characterized by low inflation $\pi^{ss} = \pi^R$, no Ponzi scheme $\Delta^{ss} = 0$, and underemployment $c^{ss} < 1$. Again, in the absence of Ponzi scheme, the price level cannot jump and we can consider that, once in the secular stagnation steady state, the economy remains there. This implies that $\bar{c} = c^{ss}$ and $r^{ss} = i^{ss} - \pi^{ss}$. The Euler equation (22) in steady state, given by $1/u'(c^{ss}) = (\rho - r^{ss})/\gamma'(0)$, implies that c^{ss} is a decreasing function of r^{ss}. Hence, to have underemployment with $c^{ss} < 1 = c^n$, the stagnation real interest rate r^{ss} must be above the natural real interest rate r^n.

The Taylor rule $r^{ss} = \max\{r^n + (\phi - 1)[\pi^R - \pi^*], -\pi^R\}$ with $r^{ss} > r^n$ and $\pi^R \leq \pi^*$ implies that $r^{ss} = -\pi^R$ and, hence, $i^{ss} = 0$. Thus, for the secular stagnation steady state to exist, and for the trilemma to arise, aggregate demand must be so depressed that $r^n < -\pi^R$. I henceforth assume that this condition is satisfied.

Finally, by the Euler equation (22), output is demand determined with

$$\frac{1}{u'(c^{ss})} = \frac{\rho + \pi^R}{\gamma'(0)}, \tag{30}$$

where I consider that $\pi^R > -\rho$. Note that a relaxation of the downward nominal wage rigidity, through a reduction in π^R, raises the real interest rate $-\pi^R$, which further depresses the economy. This is the paradox of flexibility, which shows that the fundamental cause of stagnation is not the downward nominal wage rigidity, but the existence of money which prevents the nominal, and hence real, interest rate from being sufficiently low. Underemployment is a general equilibrium phenomenon: the interest rate is excessively high in the financial market, which depresses the demand for goods and, hence, firms' demand for labor. The downward nominal wage rigidity is only necessary to put a break on the deflationary spiral, which would otherwise be so strong as to prevent the existence of the secular stagnation steady state.

4.3 Ponzi steady state

A Ponzi steady state $(c^p, \pi^p, r^p, i^p, \Delta^p)$ is characterized by full employment $c^p = 1$ and $r^p = n - \varepsilon\frac{u'(\bar{c})}{u'(1)}$. Hence, by the Euler equation (22), the magnitude of the Ponzi scheme Δ^p is given by

$$\gamma'\left(\Delta^p\right) = (\rho - n + \varepsilon)u'(1). \tag{31}$$

The existence of this Ponzi steady state requires $\Delta^p > 0$ or, equivalently, $\gamma'(\Delta^p) < \gamma'(0)$. Hence, by (28) and (31), we must have $r^n < n - \varepsilon$. The larger the likelihood ε of a collapse of the Ponzi scheme, the more stringent the existence condition for this Ponzi steady state. This insight was originally obtained by Weil (1987) in his seminal analysis of stochastic bubbles.

From the risk-premium equation (24), together with $r^p = n - \varepsilon\frac{u'(\bar{c})}{u'(1)}$, we have $i^p - \pi^p = n - \varepsilon\frac{\Phi}{\Phi+\Delta^p}\frac{u'(\bar{c})}{u'(1)}$, where the present value of real primary surpluses Φ is assumed to be constant in the Ponzi steady state. The Taylor rule (25) can be written as $i^p - \pi^p = \max\{r^n + (\phi - 1)[\pi^p - \pi^*], -\pi^p\}$. Hence, from these two equations, we must either have $\pi^p = \varepsilon\frac{\Phi}{\Phi+\Delta^p}\frac{u'(\bar{c})}{u'(1)} - n$ or $\pi^p = \pi^* + \frac{1}{\phi-1}\left[n - r^n - \varepsilon\frac{\Phi}{\Phi+\Delta^p}\frac{u'(\bar{c})}{u'(1)}\right]$; and both possibilities require

$$\pi^* \geq -r^n - \frac{\phi}{\phi - 1}\left[(n - \varepsilon - r^n) + \varepsilon\left(1 - \frac{\Phi}{\Phi + \Delta^p}\frac{u'(\bar{c})}{u'(1)}\right)\right]. \tag{32}$$

If the collapse of the Ponzi scheme does not entail an output risk, i.e. if $\bar{c} = 1$, then the lower bound for the inflation target (32) is lower than in the neoclassical steady state (29). This is due to the fact that the Ponzi scheme Δ^p generates a wealth effect, which stimulates aggregate demand. Hence, the corresponding real interest rate $r^p = n - \varepsilon$ is higher than in the neoclassical steady state r^n, which relaxes the zero lower bound constraint. This is the essence of the policy trilemma: offsetting a depressed level of aggregate demand either requires high inflation or a Ponzi scheme.

Even with an output risk, i.e. with $\bar{c} < 1$, which depresses demand and reduces the real interest rate $r^p = n - \varepsilon \frac{u'(\bar{c})}{u'(1)}$, the threshold for the inflation target is likely to be lower in the Ponzi steady state (32) than in the neoclassical steady state (29). This is always the case when either ε or Φ is sufficiently close to zero.

Finally, to have full employment, the downward nominal wage rigidity must be non-binding. If $\varepsilon \frac{\Phi}{\Phi+\Delta^p} \frac{u'(\bar{c})}{u'(1)} - n \geq \pi^R$, then the constraint is always trivially satisfied. Otherwise, the threshold (32) for the inflation target must be raised to $\pi^* \geq \pi^R - \frac{1}{\phi-1}\left[n - r^n - \varepsilon \frac{\Phi}{\Phi+\Delta^p} \frac{u'(\bar{c})}{u'(1)}\right]$ (which is a mild condition since $\pi^* \geq \pi^R$) such as to have $\pi^p = \pi^* + \frac{1}{\phi-1}\left[n - r^n - \varepsilon \frac{\Phi}{\Phi+\Delta^p} \frac{u'(\bar{c})}{u'(1)}\right] \geq \pi^R$ as the unique possibility for inflation within the Ponzi steady state.

4.4 Ponzi-stagnation steady state

A Ponzi-stagnation steady state only exists under stringent conditions. It is derived in appendix F.

5 Policy options

Our first three steady state possibilities capture the essence of the policy trilemma. The neoclassical steady state has full employment and no Ponzi scheme, but fairly high inflation equal to at least $-r^n$. The secular stagnation steady state has low inflation equal to π^R and no Ponzi scheme, but under-employment with $c^{ss} < 1$. The Ponzi steady state has full employment and typically fairly low inflation, but with a Ponzi scheme $\Delta^p > 0$ that can collapse at any moment. For simplicity, I henceforth consider that $\pi^R \geq 0$, which implies that the downward nominal wage rigidity prevents the occurrence of deflation. Hence, from a pure welfare perspective, inflation cannot be excessively low.

However, the trilemma does not always arise. First, to have a policy trade-off, the secular stagnation steady state must exist, which requires aggregate demand to be sufficiently depressed to have $r^n < -\pi^R$. Otherwise, the neoclassical steady state can combine full employment, no Ponzi scheme, and inflation as low as π^R. Second, when $r^n \geq n - \varepsilon$, a Ponzi scheme is not sustainable. In that case, the policy options amount to a dilemma: either full employment or low inflation.

I now focus on steady state equilibria and assume that the government must choose the one the maximizes welfare. The government implements the value of Δ and sets the lowest inflation target π^* consistent with the welfare maximizing steady state. Throughout this analysis, I focus on the best case scenario where households spontaneously coordinate on the best equilibrium consistent with the policy chosen by the government.

Note that, once the Ponzi scheme has collapsed, the equilibrium (given by (22)-(27) with $\Delta_t = 0$ and $\varepsilon = 0$) implies that the economy must either jump to the neoclassical or to the secular stagnation steady state, without any transitional dynamics. If we denote by (c, π, Δ) the steady state equilibrium before the price level jump and by $(\bar{c}, \bar{\pi})$ the equilibrium afterwards, the expected welfare of the representative household (9) can be written as

$$\frac{1}{\rho - n}\left[\frac{\rho - n}{\rho - n + \varepsilon}\left(u(c) + \gamma(\Delta) - \psi\pi - \psi\varepsilon C\left(\frac{\Delta}{\Phi}\right)\right) + \frac{\varepsilon}{\rho - n + \varepsilon}\left(u(\bar{c}) + \gamma(0) - \psi\bar{\pi}\right)\right]. \quad (33)$$

This is derived in appendix G. At each point in time, a price level jump occurs with probability εdt and momentarily raises the cost of inflation to $\psi C(\Delta_t/\Phi_t)/dt$. This inflation risk is the welfare cost of running a Ponzi scheme of public debt. Note that, if households never run away from the Ponzi scheme, i.e. if $\varepsilon = 0$, then the Ponzi steady state is always superior to the neoclassical steady state, thanks to the welfare gain from higher wealth and to lower inflation equal to $\max\{-n, \pi^R\}$ instead of $-r^n$.

5.1 The dilemma

Before investigating the trilemma, let us characterize the dilemma facing the government in the absence of Ponzi scheme, either because a Ponzi scheme is not sustainable as $r^n \geq n - \varepsilon$ or because it has already collapsed. Hence, the economy must either be in the neoclassical or in the secular stagnation steady state. As the government sets the lowest inflation target π^* consistent with the desired steady state, inflation in the neoclassical steady state must be equal to $-r^n$. The welfare of the representative household is equal to $[u(1) + \gamma(0) + \psi r^n]/(\rho - n)$ in the neoclassical steady state and to $[u(c^{ss}) + \gamma(0) - \psi\pi^R]/(\rho - n)$ under secular stagnation. It follows that full employment is chosen if and only if the welfare cost of higher inflation $\psi(-r^n - \pi^R) > 0$ is lower than welfare cost of under-employment $u(1) - u(c^{ss}) > 0$. We therefore have

$$(\bar{c}, \bar{\pi}) = \begin{cases} (1, -r^n) & \text{if } \psi \leq \frac{u(1) - u(c^{ss})}{-r^n - \pi^R} \\ (c^{ss}, \pi^R) & \text{if } \psi > \frac{u(1) - u(c^{ss})}{-r^n - \pi^R} \end{cases}. \quad (34)$$

Note that, in the presence of a Ponzi scheme, $\bar{c}$ has an impact on welfare, as reflected by the risk-premium (24). I am assuming that the government chooses $\bar{c}$ once the Ponzi scheme has collapsed and cannot commit *ex-ante* to a different value of $\bar{c}$.

5.2 The trilemma

When $r^n < \min\{-\pi^R, n - \varepsilon\}$, the secular stagnation and the Ponzi steady state both exist, resulting in a policy trilemma. In the Ponzi steady state, when the inflation target is set as low as possible, we have $\pi^p = \max\{\varepsilon \frac{\Phi}{\Phi+\Delta^p} \frac{u'(\bar{c})}{u'(1)} - n, \pi^R\}$. Thus, from the welfare function (33), the Ponzi steady state is the preferred option if and only if

$$u(1) + \gamma(\Delta^p) - \psi \max\left\{\varepsilon \frac{\Phi}{\Phi + \Delta^p} \frac{u'(\bar{c})}{u'(1)} - n, \pi^R\right\} - \psi \varepsilon C\left(\frac{\Delta^p}{\Phi}\right) \geq u(\bar{c}) + \gamma(0) - \psi \bar{\pi}, \quad (35)$$

where $\bar{c}$ and $\bar{\pi}$ are given by (34) and Δ^p by (31).

To gain further insights about how the trilemma is affected by the welfare cost of price level changes ψ and the likelihood of collapse of the Ponzi scheme ε, I now rely on a calibration of the model to perform illustrative simulations.

5.2.1 Calibration

I assume constant relative risk aversion for consumption

$$u(c) = \frac{c^{1-\theta} - 1}{1 - \theta}, \quad (36)$$

and, following Kumhof, Rancière and Winant (2015) and Michau (2022*b*), constant relative risk aversion for wealth, relative to a reference wealth level $\underline{a} < 0$,

$$\gamma(a) = k \frac{(a - \underline{a})^{1-\sigma} - 1}{1 - \sigma}. \quad (37)$$

The convex welfare cost of a price level jump is given by

$$C(x) = \alpha \frac{(x + 1)^{\beta} - 1}{\beta}, \quad (38)$$

where $\alpha \geq 1$ determines the cost of a price level jump relative to the cost of a continuous increase in the price level[12] and $\beta \geq 1$ determines the convexity of this cost.

[12] An infinitesimally small price level jump $|dP_t/P_t|$ entails a welfare cost $\frac{C(|dP_t/P_t|)}{dt} = \frac{C(0)+|dP_t/P_t|C'(0)}{dt} = C'(0)|\pi_t| = \alpha|\pi_t|$. By (8), a continuous increase in price of the same magnitude entails a welfare cost $|\pi_t|$.

I assume a 5% discount rate and constant population. Wages cannot fall in nominal terms, i.e. $\pi^R = 0$. This implies that inflation is nil under secular stagnation, consistently with the experience of Japan over the past 25 years.[13]

Eggertsson, Mehrotra and Robbins (2019) and Rachel and Summers (2019) have estimated the U.S. natural real interest rate to be equal to -2.2% and 0.4%, respectively. However, throughout my analysis, the natural real interest rate r^n has been defined by (28) as the real interest rate consistent with full employment *in the absence of Ponzi scheme*. But, Eggertsson, Mehrotra and Robbins (2019) and Rachel and Summers (2019) found that the rise in public indebtedness in the U.S. over the past four decades has raised the natural real interest rate by about 2%. Hence, their estimation implies that the U.S. natural real interest rate as defined by (28) is between -4.2% and -1.6%. The natural real interest rate is probably even lower in the Eurozone and in Japan.[14] I therefore set the coefficient of relative risk aversion for consumption θ such that the natural real interest rate r^n is equal to -3%, which gives $\theta = 4.46$. Note that with $r^n = -3\%$, in the absence of Ponzi scheme, a 2% inflation target is inconsistent with the economy being at full employment.

I set the scale parameter k of the preference for wealth such that, under secular stagnation, the output gap amounts to 10% of GDP, i.e. $c^{ss} = 0.9c^n$ with c^n normalized to 1, which gives $k = 0.18$. According to Hausman and Wieland (2014), the output gap in Japan was about 10% in 2013, before the monetary and fiscal expansion of Abenomics, while Hall (2017) reported a 15% output gap for the U.S. in 2015. The reference wealth level $\underline{a}$, which gives the theoretical upper bound to household indebtedness, is set equal to two years of output at full employment, which gives $\underline{a} = -2$. The present value of surpluses is set equal to 100% of GDP at full employment, which implies $\Phi = 1$. Public debt in excess of that threshold must correspond to a Ponzi scheme. The maximal magnitude of a Ponzi scheme, reached when $\varepsilon = 0$, is also set equal to 100% of GDP, implying that public debt could potentially rise to 200% of GDP, but no higher, which gives $\sigma = 1.16$. This calibration is summarized in Table 1.

5.2.2 Simulations

The critical parameters for the policy trilemma are the the welfare cost of changes to the price level ψ and the likelihood of collapse of the Ponzi scheme ε. The parameters α and β of the cost of a price level jump (38) are also important. I therefore characterize the optimal steady state as a function of ψ and ε for given values of α and β.

I first consider that possibility that $\alpha = 4$ and $\beta = 1$. This implies that an increase in

[13]With $n + \pi^R = 0$, the Ponzi-stagnation steady state derived in appendix F does not exist.

[14]Holston, Laubach and Williams (2017) estimated the natural real interest rate in the Eurozone to be about 0.7% below the U.S..

Parameter	Calibrated value	Moment
Discount rate	$\rho = 5\%$	$\cdot$
Population growth	$n = 0\%$	$\cdot$
Reference rate of inflation for wage bargaining	$\pi^R = 0\%$	$\cdot$
CRRA for consumption	$\theta = 4.46$	$r^n = -3\%$
CRRA for wealth (relative to reference level)	$\sigma = 1.16$	$\Delta^p = 1$ when $\varepsilon = 0$
Scale parameter of preference for wealth	$k = 0.18$	$c^{ss} = (1 - 0.1)c^n$
Reference wealth level	$\underline{a} = -2$	$\underline{a} = -2c^n$
Present value of primary surpluses	$\Phi = 1$	$\Phi = c^n$

Table 1: Calibration of the model

the price level is 4 times more costly when it is due to the collapse of a Ponzi scheme than when it results from a high inflation target (see footnote 12). Intuitively, a 1% chance per year of having a 50% increase in the price level, which raises expected inflation by 0.5%, is as costly as a $0.005\alpha = 2\%$ increase in the inflation target. The idea is that sudden and uncontrolled fiscal inflation generates financial disruption and a loss of monetary policy credibility that makes it markedly more costly than a smooth increase in the price level induced by monetary policy. Similarly, the sovereign debt literature typically assumes a sizeable deadweight cost of default (Aguiar and Amador, 2021).

The optimal steady state, as a function of ψ and ε, is displayed in Figure 1. Recall that, when $\varepsilon = 0$, the Ponzi scheme is of maximal magnitude (calibrated to be equal to 100% of GDP) and never collapses; while, when $\varepsilon = n - r^n = 3\%$, the Ponzi scheme is of zero magnitude. Figure 1 shows that when either ε or ψ is close to zero, a Ponzi scheme is either so unlikely to collapse or the cost of a collapse is so low that it is the preferred option, thanks to the positive impact of the wealth effect on welfare. The vertical dashed line corresponds to the threshold from the dilemma, given by (34). After the collapse of the Ponzi scheme, the economy must in the neoclassical steady state to the left of this line and in secular stagnation to the right.[15]

As ε increases, a Ponzi scheme is more likely to collapse and to induce an upward jump in the price level, making it less attractive. At some point, a Ponzi scheme is no longer desirable. If the cost of inflation ψ is below the threshold from the dilemma, given by (34), then the neoclassical steady state is optimal even though inflation is permanently raised to $-r^n = 3\%$; while, if the cost of inflation is above the threshold, then a permanently depressed economy with consumption equal to $c^{ss} = 0.9 < c^n = 1$ is preferable.

What is the solution to the trilemma when the cost of the price level jump is convex

[15]This implies that, before the price level jump, the Ponzi steady state yields slightly higher welfare to the left of that line, where $\bar{c} = 1$, than to the right, where $\bar{c} = c^{ss}$.

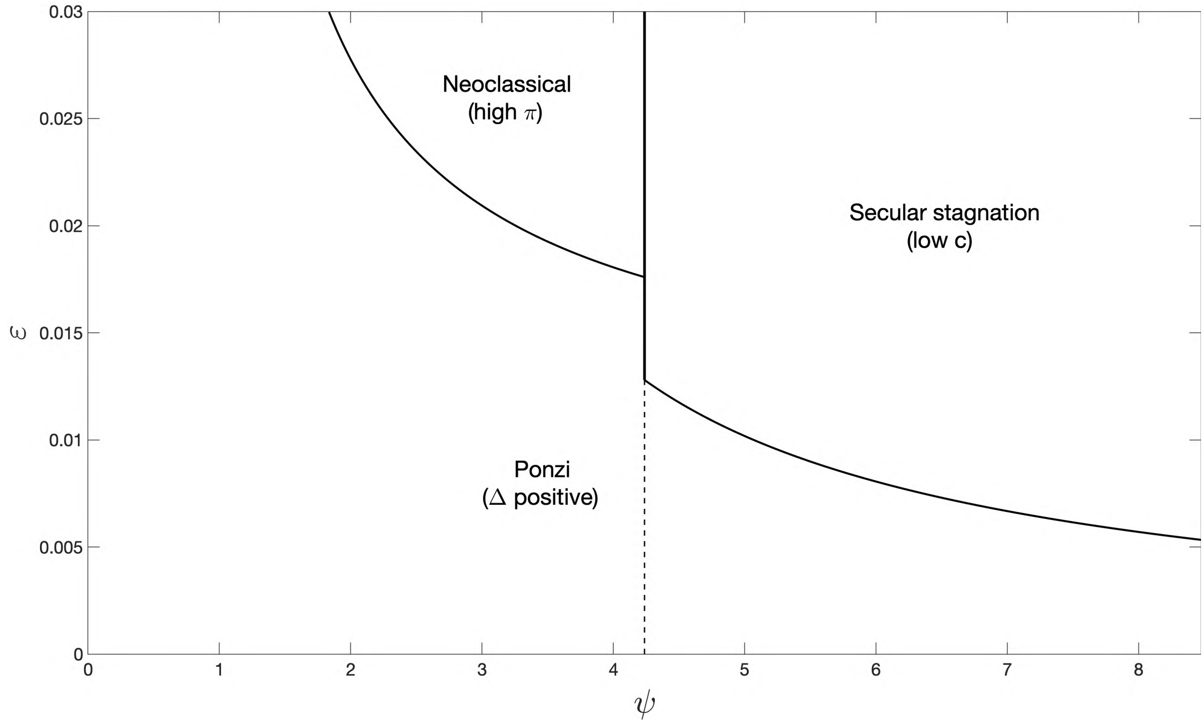

Figure 1: Trilemma for $\alpha = 4$ and $\beta = 1$

in its magnitude? To emphasize the effect of convexity, I set $\alpha = 1$ and $\beta = 8$. With $\alpha = 1$, there is no discontinuity between a continuous rise in the price level and an infinitesimal jump. With $\beta = 8$, a 100% increase in the price level is about 10 times more costly than a 50% increase. The corresponding trilemma is displayed in Figure 2. A small likelihood of collapse ε entails a large magnitude of the Ponzi scheme and, hence, a sizeable welfare cost when the price level jump does occur. The neoclassical steady state with permanently higher inflation becomes optimal for intermediate values of ε. The secular stagnation steady state remains optimal for a high welfare cost of inflation ψ, unless ε is so close to zero that a Ponzi scheme is unlikely to collapse.

The values of $\alpha = 1$ and $\beta = 8$ were chosen for illustrative purposes, to emphasize the difference with the case where $\alpha = 4$ and $\beta = 1$. If we set $\beta = 4$, implying that a 100% increase in the price level is 3.7 times more costly than a 50% increase, and keep $\alpha = 4$, we obtain a situation where the Ponzi steady state is much less desirable, as shown in Figure A1 from appendix H.

Another situation of interest arises when $\alpha = 1$ and $\beta = 1$, implying that changes to the price level are equally costly whether they occur through jumps or through continuous changes.[16] In that case, under our calibration, the neoclassical steady state is never preferred to the Ponzi steady state: the welfare benefit from the Ponzi scheme $\gamma(\Delta^p)$ outweighs the expected cost of a higher price level $\psi\varepsilon\Delta^p/\Phi$. This can be seen in

[16]Formally, when $\alpha = 1$ and $\beta = 1$, we have $C(x) = x$ and, hence, $c(dP_t/P_t) = |dP_t/P_t|/dt$ regardless of whether a shock occurs.

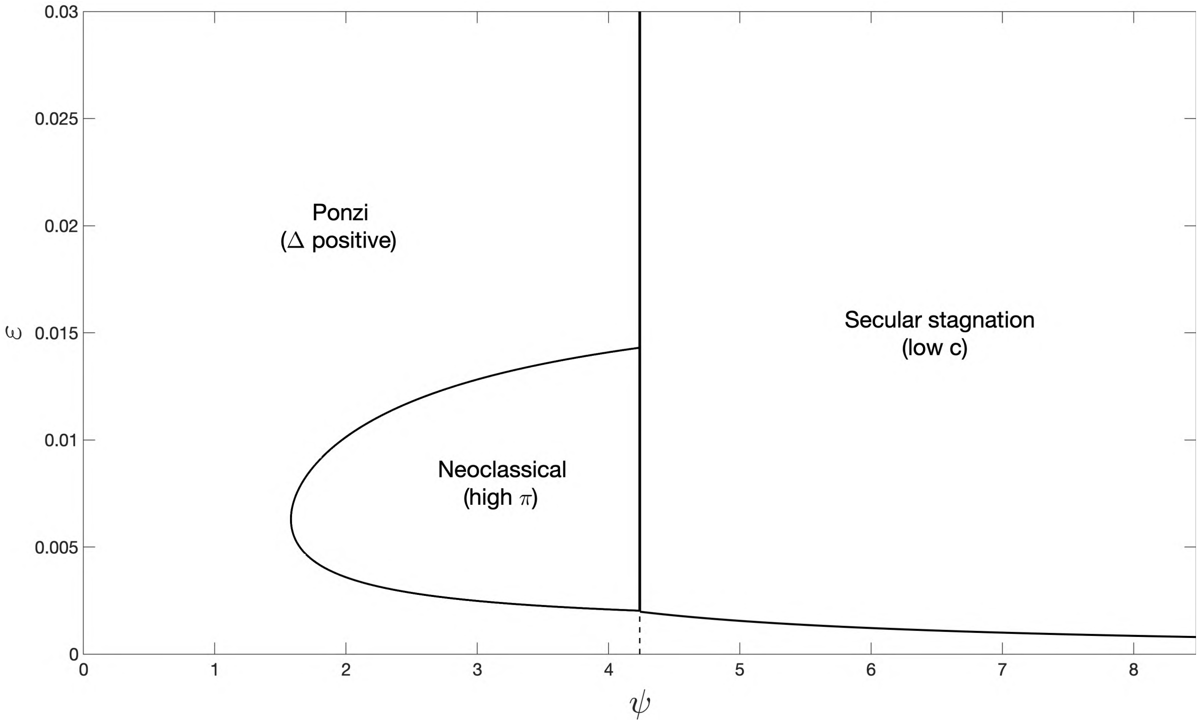

Figure 2: Trilemma for $\alpha = 1$ and $\beta = 8$

Figure A2 from appendix H.

In principle, welfare can be higher if the government is not constrained to choose among the three steady states. However, this potentially requires a time varying inflation target π_t^*. In particular, if the government wants the economy to operate at full capacity with $c_t = 1$, then choosing $\Delta_0 \in (0, \Delta^p)$ implies convergence to the neoclassical steady state with a rising inflation target.[17] But, if the inflation target is time-invariant and inflation is on target, then the Ponzi scheme $\Delta_0 \in (0, \Delta^p)$ fails to reduce inflation.

6 Persistent lack of demand

So far, we have assumed a *permanent* lack of demand, with a permanently depressed natural real interest rate. But the analysis remains unchanged for a sufficiently *persistent* lack of demand. In particular, we can assume that ε is the likelihood of a decline in the marginal utility of wealth, which raises aggregate demand sufficiently to eliminate the secular stagnation and Ponzi steady states. Hence, the upward jump in the price level can be driven by a fundamental shock, rather than by a sunspot shock. Appendix I shows that the analysis remains unchanged, with the equilibrium before the shock still given by (22)-(27), but with $\bar{c}_t = 1$.

[17] Convergence to the neoclassical steady state follows from (22) and (26) with $c_t = \bar{c}_t = 1$. Rising inflation follows from (24) with $i_t = 0$.

The steady states also remain unchanged, except for the secular stagnation steady state that is now characterized by a higher output level, i.e. by less underemployment, since the prospect of an economic recovery raises aggregate demand. The existence condition for the secular stagnation steady state remains unchanged, and given by $r^n < -\pi^R$. The existence of a Ponzi scheme still requires $\varepsilon < n - r^n$, with r^n given by (28), which now corresponds to a very persistent depression in aggregate demand.

7 Breaking through the trilemma

Is there a way to break through the policy trilemma? First, the country can switch to electronic currency, i.e. abolish cash, such as to remove the zero lower bound on the nominal interest rate. A negative nominal interest rate can then be implemented by taxing bank deposits (which, in practice, would make such a reform politically difficult to implement). Secular stagnation would then never be optimal, while inflation could be set equal to π^R. The Ponzi steady state would be preferred to the neoclassical steady state if and only if $\gamma(\Delta^p) - \psi\varepsilon C\left(\Delta^p/\Phi\right) \geq \gamma(0)$.[18]

Another way to circumvent the zero lower bound is through tax policy. As shown in Michau (2018), a negative nominal interest rate can be replicated through either a wealth tax or an increasing rate of consumption tax, together with offsetting adjustments to the taxation of labor and investment. However, in practice, wealth is neither easily observable nor very liquid, while the rate of consumption tax can hardly keep increasing for a prolonged period of time.

Finally, the government can try to overcome the trilemma by raising the natural real interest rate. For instance, Rachel and Summers (2019) have advocated for redistribution from old to young and for enhanced social insurance. However, this can only work if households do not undo these policies through private transfers. Moreover, the scope for such policies seems limited in Europe and Japan, where aggregate demand is particularly weak despite an already extensive welfare state. Also, with heterogeneous households, a one-off redistribution of wealth from wealthy-thrifty households to poor-spendthrift ones can only temporarily boost aggregate demand, until wealth inequality regains its original level (Mian, Straub and Sufi, 2021; Illing, Ono and Schlegl, 2018). Government spending can boost aggregate demand but, if households do not value the goods and services that are publicly produced, it is preferable to have a negative output gap such as to minimize the disutility from supplying labor.

[18]Note that if the planner, unlike households, does not value Ponzi wealth, then a Ponzi scheme would never be desirable.

8 Conclusion

This paper has shown that, when aggregate demand is permanently or persistently depressed, we cannot simultaneously have full employment, low inflation, and a low debt level that is backed by future fiscal surpluses.

While I have assumed that inflation should ideally be as low as possible, alternatively the central bank can be strongly committed to its inflation target π^* with $\pi^* \in (\pi^R, -r^n)$, which rules out the neoclassical steady state. In that case, the government must choose between secular stagnation, where the liquidity trap makes monetary policy unable to raise inflation from π^R to π^*, and a Ponzi scheme, where monetary policy cannot prevent a price level jump that temporarily raises inflation much above π^*. In both cases, monetary policy is powerless. The inflation targeting framework is fundamentally challenged by a persistently low real interest rate, at least if the inflation target remains below $-r^n$.

My analysis has assumed that households spontaneously coordinate on the best equilibrium that is feasible given government policy. However, the experience of Japan over the past 25 years suggests that, once the economy is stuck into secular stagnation, it can be extremely difficult to raise inflation sufficiently to have full employment. With inflation persistence, moving from the secular stagnation to the neoclassical steady state requires massive fiscal stimulus (Michau, 2022*a*). This suggests that the post-Covid episode of high inflation is an ideal time to decide which solution to the trilemma to aim for.

References

Aguiar, M. and Amador, M. (2021), *The economics of sovereign debt and default*, Princeton University Press.

Barro, R. J. (2020), r minus g*, working paper, Harvard.

Bassetto, M. and Cui, W. (2018), 'The fiscal theory of the price level in a world of low interest rates', *Journal of Economic Dynamics and Control* **89**, 5–22.

Billi, R., Gali, J. and Nakov, A. (2022), Optimal monetary policy with $r^* < 0$, working paper, Sveriges Riksbank, CREI, and ECB.

Blanchard, O. (2019), 'Public debt and low interest rates', *American Economic Review* **109**(4), 1197–1229.

Brunnermeier, M. K., Merkel, S. and Sannikov, Y. (2022), The fiscal theory of the price level with a bubble, working paper, Princeton, Exeter, and Stanford.

Cochrane, J. H. (2021), $r < g$, working paper, Stanford.

Eggertsson, G. B., Mehrotra, N. R. and Robbins, J. A. (2019), 'A model of secular stagnation: Theory and quantitative evaluation', *American Economic Journal: Macroeconomics* **11**(1), 1–48.

Forbes, K. J., Gagnon, J. and Collins, C. G. (2021), Low inflation bends the phillips curve around the world: Extended results, working paper, MIT and Peterson Institute for International Economics.

Hall, R. E. (2017), 'The anatomy of stagnation in a modern economy', *Economica* **84**(333), 1–15.

Hausman, J. K. and Wieland, J. F. (2014), 'Abenomics: Preliminary analysis and outlook', *Brooking Papers on Economic Activity* **1**, 1–63.

Holston, K., Laubach, T. and Williams, J. C. (2017), 'Measuring the natural rate of interest: International trends and determinants', *Journal of International Economics* **108**, S59–S75.

Illing, G., Ono, Y. and Schlegl, M. (2018), 'Credit booms, debt overhang and secular stagnation', *European Economic Review* **108**, 78–104.

Kocherlakota, N. R. (2022*a*), Infinite debt rollover in stochastic economies, working paper, Rochester.

Kocherlakota, N. R. (2022*b*), 'Stabilization with fiscal policy', *Journal of Monetary Economics* .

Krugman, P. R. (1998), 'It's baaack: Japan's slump and the return of the liquidity trap', *Brookings Papers on Economic Activity* **1998**(2), 137–187.

Krugman, P. R. (2021), Credible irresponsibility revisited, working paper, City University of New York.

Kumhof, M., Rancière, R. and Winant, P. (2015), 'Inequality, leverage, and crises', *American Economic Review* **105**(3), 1217–1245.

Mankiw, N. G. (2022), Government debt and capital accumulation in an era of low interest rates, working paper, Harvard.

Mehrotra, N. R. and Sergeyev, D. (2021), 'Debt sustainability in a low interest rate world', *Journal of Monetary Economics* **124**, S1–S18.

Mian, A., Straub, L. and Sufi, A. (2021), 'Indebted demand', *The Quarterly Journal of Economics* **136**(4), 2243–2307.

Mian, A., Straub, L. and Sufi, A. (2022), A goldilocks theory of fiscal deficits, working paper, Princeton, Harvard, and Chicago Booth.

Miao, J. and Su, D. (2021), Fiscal and monetary policy interactions in a model with low interest rates, working paper, Boston University.

Michau, J.-B. (2018), 'Secular stagnation: Theory and remedies', *Journal of Economic Theory* **176**, 552–618.

Michau, J.-B. (2022*a*), Fiscal policy under secular stagnation: An optimal pump priming strategy, working paper, Ecole Polytechnique.

Michau, J.-B. (2022*b*), Helicopter drops of money under secular stagnation: From ponzi to pigou, working paper, Ecole Polytechnique.

Michau, J.-B., Ono, Y. and Schlegl, M. (2022), Wealth preference and rational bubbles, working paper, Ecole Polytechnique, Osaka University, and Sophia University.

Ono, Y. (1994), *Money, interest, and stagnation: Dynamic theory and Keynes's economics*, Oxford University Press.

Ono, Y. (2001), 'A reinterpretation of chapter 17 of keynes's general theory: Effective demand shortage under dynamic optimization', *International Economic Review* **42**(1), 207–236.

Rachel, L. and Summers, L. H. (2019), 'On secular stagnation in the industrialized world', *Brooking Papers on Economic Activity* **50**(1), 1–54.

Reis, R. (2021), The constraint on public debt when $r < g$ but $g < m$, working paper, LSE.

Weil, P. (1987), 'Confidence and the real value of money in an overlapping generations economy', *Quarterly Journal of Economics* **102**(1), 1–22.

A Government's flow of funds

By Itô's lemma with jumps, if X_t follows a continuous time stochastic process with jumps given by $dX_t = \mu(X_t)dt + \gamma(X_t)dJ_t$, then for any differentiable function $f(\cdot)$ we have

$$df(X_t) = \frac{\partial f(X_t)}{\partial X_t}\mu(X_t)dt + [f(X_t + \gamma(X_t)) - f(X_t)]\, dJ_t.$$

Applying this lemma to $dP_t = \pi_t P_t dt + (\Delta_t/\Phi_t)P_t dJ_t$ yields

$$\begin{aligned} d\left(\frac{1}{P_t}\right) &= \frac{-1}{P_t^2}\pi_t P_t dt + \left(\frac{1}{P_t + \frac{\Delta_t}{\Phi_t}P_t} - \frac{1}{P_t}\right) dJ_t, \\ &= \frac{-\pi_t}{P_t}dt - \frac{\Delta_t}{b_t}\frac{1}{P_t}dJ_t, \end{aligned} \tag{A1}$$

where $b_t = \Phi_t + \Delta_t$.

We therefore have

$$\begin{aligned} d\left(\frac{B_t}{P_t N_t}\right) &= \frac{dB_t}{P_t N_t} + \frac{B_t}{N_t}d\left(\frac{1}{P_t}\right) + \frac{B_t}{P_t}d\left(\frac{1}{N_t}\right), \\ &= \frac{i_t B_t - \tau_t P_t N_t}{P_t N_t}dt + \frac{B_t}{N_t}\left(\frac{-\pi_t}{P_t}dt - \frac{\Delta_t}{b_t}\frac{1}{P_t}dJ_t\right) - \frac{B_t}{P_t}\frac{nN_t}{N_t^2}dt, \\ &= [(i_t - \pi_t - n)b_t - \tau_t]\, dt - \Delta_t dJ_t, \end{aligned}$$

where the second line was obtained using equations (1), (A1), and the fact that $N_t = e^{nt}$.

B Solving the household's problem

To provide sufficient conditions that characterize a solution to the household's problem, let us introduce slightly more specific notations than in the text. The only source of uncertainty is the time T when the price level jump occurs, which follows an exponential distribution with parameter ε. A state-contingent allocation chosen by a household can therefore be denoted by $(\tilde{c}_t, \tilde{a}_t, \tilde{b}_t^h, (\bar{c}_s(t), \bar{a}_s(t))_{s=t}^{\infty})_{t=0}^{\infty}$, where $\tilde{c}_t$, $\tilde{a}_t$, and $\tilde{b}_t^h$ denote consumption, wealth, and government bond holdings at time t conditional on the absence of a jump, while $\bar{c}_t(T)$ and $\bar{a}_t(T)$ denote consumption and wealth at time t conditional on the price level having jumped at time T.[19]

[19] After time T, the economy is risk-free and the household no longer needs to make a portfolio decision.

Using these notations, the Euler equation before the jump in the price level is

$$\frac{\dot{\tilde{c}}_t}{\tilde{c}_t} = \left(\frac{u'(\tilde{c}_t)}{-\tilde{c}_t u''(\tilde{c}_t)}\right)\left[r_t - \rho + \frac{\gamma'(\tilde{a}_t - b_t + \Delta_t)}{u'(\tilde{c}_t)} + \varepsilon\left(\frac{u'(\bar{c}_t(t))}{u'(\tilde{c}_t)} - 1\right)\right], \tag{A2}$$

where $\bar{c}_t(t)$ is the consumption level immediately after a price level jump occurring at time t. Once the price level jump has occurred at time T, the Euler equation simplifies to

$$\frac{\dot{\bar{c}}_t(T)}{\bar{c}_t(T)} = \left(\frac{u'(\bar{c}_t(T))}{-\bar{c}_t(T)u''(\bar{c}_t(T))}\right)\left[r_t - \rho + \frac{\gamma'(\bar{a}_t(T) - b_t)}{u'(\bar{c}_t(T))}\right]. \tag{A3}$$

The expression for the risk premium (13) can be expressed as

$$r_t = i_t - \pi_t - \varepsilon \frac{\Delta_t}{b_t}\frac{u'(\bar{c}_t(t))}{u'(\tilde{c}_t)}. \tag{A4}$$

As T is exponentially distributed, we have

$$\begin{aligned}
\mathbb{E}_0\left[e^{-(\rho-n)t}u'(c_t)a_t\right] &= \int_0^\infty \varepsilon e^{-\varepsilon T}\left[e^{-(\rho-n)t}u'(c_t)a_t\right]dT,\\
&= \int_0^t \varepsilon e^{-\varepsilon T}\left[e^{-(\rho-n)t}u'(\bar{c}_t(T))\bar{a}_t(T)\right]dT\\
&\qquad + \int_t^\infty \varepsilon e^{-\varepsilon T}\left[e^{-(\rho-n)t}u'(\tilde{c}_t)\tilde{a}_t\right]dT,\\
&= \int_0^t \varepsilon e^{-\varepsilon T}\left[e^{-(\rho-n)t}u'(\bar{c}_t(T))\bar{a}_t(T)\right]dT,\\
&\qquad + e^{-(\rho-n+\varepsilon)t}u'(\tilde{c}_t)\tilde{a}_t.
\end{aligned} \tag{A5}$$

Hence, the transversality condition (14) can be written as

$$\lim_{t\to\infty}\left[e^{-(\rho-n+\varepsilon)t}u'(\tilde{c}_t)\tilde{a}_t + \int_0^t \varepsilon e^{-\varepsilon T}\left[e^{-(\rho-n)t}u'(\bar{c}_t(T))\bar{a}_t(T)\right]dT\right] = 0. \tag{A6}$$

I shall now prove that (A2), (A3), (A4), and (A6) are sufficient to characterize a solution to the household's problem.[20] Let $(\tilde{c}_t^*, \tilde{a}_t^*, \tilde{b}_t^{h*}, (\bar{c}_s^*(t), \bar{a}_s^*(t))_{s=t}^\infty)_{t=0}^\infty$ denote the allocation that satisfies these four equations. Throughout the proof, I am assuming that this allocation satisfies the no-borrowing constraint (11), which is therefore non-binding.

The objective of the household is to maximize the following objective, where the cost of changes to the price level is omitted since it is exogenous to the household's

[20]Note that the Euler equations (A2) and (A3) as well as the equation for the risk-premium (A4) can be derived from the Hamilton-Jacobi-Bellman equation for the household's problem.

behavior,

$$
\begin{aligned}
\mathbb{E}_0 & \left[\int_0^\infty e^{-(\rho-n)t}\left[u(c_t)+\gamma(a_t-b_t+\Delta_t)\right]dt\right] \\
&= \int_0^\infty \varepsilon e^{-\varepsilon T}\left[\int_0^T e^{-(\rho-n)t}\left[u(\tilde{c}_t)+\gamma(\tilde{a}_t-b_t+\Delta_t)\right]dt\right. \\
&\qquad \left. + \int_T^\infty e^{-(\rho-n)t}\left[u(\bar{c}_t(T))+\gamma(\bar{a}_t(T)-b_t)\right]dt\right]dT, \\
&= \int_0^\infty \left[\int_t^\infty \varepsilon e^{-\varepsilon T}dT\right] e^{-(\rho-n)t}\left[u(\tilde{c}_t)+\gamma(\tilde{a}_t-b_t+\Delta_t)\right]dt \\
&\qquad + \int_0^\infty \varepsilon e^{-\varepsilon T}\left[\int_T^\infty e^{-(\rho-n)t}\left[u(\bar{c}_t(T))+\gamma(\bar{a}_t(T)-b_t)\right]dt\right]dT, \\
&= \int_0^\infty e^{-(\rho-n+\varepsilon)t}\left[u(\tilde{c}_t)+\gamma(\tilde{a}_t-b_t+\Delta_t)\right]dt \\
&\qquad + \int_0^\infty \varepsilon e^{-\varepsilon T}\left[\int_T^\infty e^{-(\rho-n)t}\left[u(\bar{c}_t(T))+\gamma(\bar{a}_t(T)-b_t)\right]dt\right]dT.
\end{aligned}
$$

Let D be defined by

$$
D=\mathbb{E}_0\left[\int_0^\infty e^{-(\rho-n)t}\left[u(c_t^*)+\gamma(a_t^*-b_t+\Delta_t)-u(c_t)-\gamma(a_t-b_t+\Delta_t)\right]dt\right]. \tag{A7}
$$

I shall now show that, under equations (A2), (A3), (A6), and (A4), we must have $D \geq 0$.

To establish this result, the following lemma will be used.

Lemma A1 *For any differentiable and concave functions $u(\cdot)$ and $\gamma(\cdot)$, we have*

$$
u(c_t^*)+\gamma(a_t^*)-u(c_t)-\gamma(a_t) \geq (\dot{a}_t-\dot{a}_t^*)\,u'(c_t^*)-(a_t-a_t^*)\left[r_t u'(c_t^*)+\gamma'(a_t^*)\right]-i_t(b_t-b_t^*)u'(c_t^*),
$$

where c_t is given by $c_t=r_t a_t+i_t b_t+x_t-\dot{a}_t$ for some scalars r_t, i_t, and x_t at time t.

Proof. Let c_t be defined by

$$
c_t=r_t a_t+i_t b_t+x_t-\frac{a_{t+\delta t}-a_t}{\delta t},
$$

with $\delta>0$. We have

$$
\begin{aligned}
u(c_t)+\gamma(a_t) &= u\left(r_t a_t+i_t b_t+x_t-\frac{a_{t+\delta t}-a_t}{\delta t}\right)+\gamma(a_t), \\
&= u\left(\left(\frac{1}{\delta t}+r_t\right)a_t+i_t b_t+x_t-\frac{a_{t+\delta t}}{\delta t}\right)+\gamma(a_t).
\end{aligned}
$$

For δ sufficiently small, $1/(\delta t)+r_t$ must be positive. Hence, the above expression is concave in a_t, $-a_{t+\delta t}$, and $i_t b_t$. It follows that

$$\begin{aligned}
u(c_t)+\gamma(a_t) &\leq u(c_t^*)+\gamma(a_t^*)+(a_t-a_t^*)\left[\left(\frac{1}{\delta t}+r_t\right)u'(c_t^*)+\gamma'(a_t^*)\right] \\
&\qquad -(a_{t+\delta t}-a_{t+\delta t}^*)\frac{u'(c_t^*)}{\delta t}+i_t(b_t-b_t^*)u'(c_t^*), \\
&\leq u(c_t^*)+\gamma(a_t^*)+(a_t-a_t^*)\left[r_t u'(c_t^*)+\gamma'(a_t^*)\right] \\
&\qquad -\left(\frac{a_{t+\delta t}-a_t}{\delta t}-\frac{a_{t+\delta t}^*-a_t^*}{\delta t}\right)u'(c_t^*)+i_t(b_t-b_t^*)u'(c_t^*),
\end{aligned}$$

where $c_t^* = r_t a_t^* + i_t b_t^* + x_t - \frac{a_{t+\delta t}^*-a_t^*}{\delta t}$. Taking the limit as δ tends to zero gives the desired result. ■

For any $t \neq T$, by the household's flow of funds constraint (10), we have

$$\dot{a}_t = (r_t-n)a_t + w_t L_t - \tau_t - c_t + b_t^h(i_t-\pi_t-r_t),$$

with $r_t = i_t - \pi_t$ for $t \geq T$. Hence, from Lemma A1, for any $t \neq T$ we have

$$\begin{aligned}
&u(c_t^*)+\gamma(a_t^*-b_t+\Delta_t)-u(c_t)-\gamma(a_t-b_t+\Delta_t) \geq (\dot{a}_t-\dot{a}_t^*)\,u'(c_t^*) \\
&\qquad -(a_t-a_t^*)\left[(r_t-n)\,u'(c_t^*)+\gamma'(a_t^*-b_t+\Delta_t)\right]-(i_t-\pi_t-r_t)(b_t^h-b_t^{h*})u'(c_t^*).
\end{aligned}$$

Substituting this inequality into the expression for D yields

$$\begin{aligned}
D &\geq \int_0^\infty e^{-(\rho-n+\varepsilon)t}\Big[\left(\dot{\tilde{a}}_t-\dot{\tilde{a}}_t^*\right)u'(\tilde{c}_t^*)-(\tilde{a}_t-\tilde{a}_t^*)\left[(r_t-n)\,u'(\tilde{c}_t^*)+\gamma'(\tilde{a}_t^*-b_t+\Delta_t)\right] \\
&\qquad -(i_t-\pi_t-r_t)(\tilde{b}_t^h-\tilde{b}_t^{h*})u'(\tilde{c}_t^*)\Big]\,dt \\
&\quad +\int_0^\infty \varepsilon e^{-\varepsilon T}\Big[\int_T^\infty e^{-(\rho-n)t}\left[\left[(\dot{\bar{a}}_t(T)-\dot{\bar{a}}_t^*(T))\,u'(\bar{c}_t^*(T))\right]\right. \\
&\qquad \left. -(\bar{a}_t(T)-\bar{a}_t^*(T))\left[(r_t-n)\,u'(\bar{c}_t^*(T))+\gamma'(\bar{a}_t^*(T)-b_t)\right]\right]dt\Big]\,dT. \qquad \text{(A8)}
\end{aligned}$$

Integrating by parts, we have

$$\begin{aligned}
&\int_0^\infty e^{-(\rho-n+\varepsilon)t}\left(\dot{\tilde{a}}_t-\dot{\tilde{a}}_t^*\right)u'(\tilde{c}_t^*)dt \\
&\qquad = \left[\lim_{t\to\infty} e^{-(\rho-n+\varepsilon)t}\,(\tilde{a}_t-\tilde{a}_t^*)\,u'(\tilde{c}_t^*)\right]-(\tilde{a}_0-\tilde{a}_0^*)\,u'(\tilde{c}_0^*) \\
&\qquad\qquad -\int_0^\infty (\tilde{a}_t-\tilde{a}_t^*)\,d\left[e^{-(\rho-n+\varepsilon)t}u'(\tilde{c}_t^*)\right], \\
&\qquad = \left[\lim_{t\to\infty} e^{-(\rho-n+\varepsilon)t}\,(\tilde{a}_t-\tilde{a}_t^*)\,u'(\tilde{c}_t^*)\right] \\
&\qquad\qquad -\int_0^\infty e^{-(\rho-n+\varepsilon)t}\,(\tilde{a}_t-\tilde{a}_t^*)\left[-(\rho-n+\varepsilon)u'(\tilde{c}_t^*)+u''(\tilde{c}_t^*)\dot{\tilde{c}}_t^*\right]dt,
\end{aligned}$$

where, to obtain the last line, I have used the fact that initial wealth is exogenously given, implying that $\tilde{a}_0 = \tilde{a}_0^*$. Similarly, we have

$$\begin{aligned}
&\int_T^\infty e^{-(\rho-n)t}\left(\dot{\bar{a}}_t(T) - \dot{\bar{a}}_t^*(T)\right) u'(\bar{c}_t^*(T))dt \\
&\quad = \left[\lim_{t\to\infty} e^{-(\rho-n)t}\left(\bar{a}_t(T) - \bar{a}_t^*(T)\right) u'(\bar{c}_t^*(T))\right] - e^{-(\rho-n)T}\left(\bar{a}_T(T) - \bar{a}_T^*(T)\right) u'(\bar{c}_T^*(T)) \\
&\qquad\qquad - \int_T^\infty e^{-(\rho-n)t}\left(\bar{a}_t(T) - \bar{a}_t^*(T)\right)\left[-(\rho-n)u'(\bar{c}_t^*(T)) + u''(\bar{c}_t^*(T))\dot{\bar{c}}_t^*(T)\right] dt.
\end{aligned}$$

Substituting these two equations into (A8) yields

$$\begin{aligned}
D \;\geq\; & \int_0^\infty e^{-(\rho-n+\varepsilon)t}(\tilde{a}_t - \tilde{a}_t^*)\Big[(\rho-n+\varepsilon)u'(\tilde{c}_t^*) - u''(\tilde{c}_t^*)\dot{\tilde{c}}_t^* \\
&\qquad\qquad - (r_t - n)\,u'(\tilde{c}_t^*) - \gamma'(\tilde{a}_t^* - b_t + \Delta_t)\Big] dt \\
& - \int_0^\infty e^{-(\rho-n+\varepsilon)t}(i_t - \pi_t - r_t)(\tilde{b}_t^h - \tilde{b}_t^{h*})u'(\tilde{c}_t^*)dt \\
& - \int_0^\infty \varepsilon e^{-\varepsilon T}\left[e^{-(\rho-n)T}\left(\bar{a}_T(T) - \bar{a}_T^*(T)\right) u'(\bar{c}_T^*(T))\right] dT \\
& + \int_0^\infty \varepsilon e^{-\varepsilon T}\Bigg[\int_T^\infty e^{-(\rho-n)t}(\bar{a}_t(T) - \bar{a}_t^*(T))\left[(\rho-n)u'(\bar{c}_t^*(T)) - u''(\bar{c}_t^*(T))\dot{\bar{c}}_t^*(T)\right. \\
&\qquad\qquad \left. - (r_t - n)\,u'(\bar{c}_t^*(T)) - \gamma'(\bar{a}_t^*(T) - b_t)\right] dt\Bigg] dT \\
& + \lim_{t\to\infty}\Bigg[e^{-(\rho-n+\varepsilon)t}\left(\tilde{a}_t - \tilde{a}_t^*\right) u'(\tilde{c}_t^*) \\
&\qquad\qquad + \int_0^\infty \varepsilon e^{-\varepsilon T}\left[e^{-(\rho-n)t}\left(\bar{a}_t(T) - \bar{a}_t^*(T)\right) u'(\bar{c}_t^*(T))\right] dT\Bigg].
\end{aligned} \tag{A9}$$

By the households' flow of funds constraint (10), at time T, when the price level jumps, we have

$$\bar{a}_T(T) = \tilde{a}_T - \tilde{b}_T^h \frac{\Delta_T}{b_T}.$$

This implies

$$\begin{aligned}
&\int_0^\infty \varepsilon e^{-\varepsilon T}\left[e^{-(\rho-n)T}\left(\bar{a}_T(T) - \bar{a}_T^*(T)\right) u'(\bar{c}_T^*(T))\right] dT \\
&\qquad = \int_0^\infty e^{-(\rho-n+\varepsilon)T}\left[(\tilde{a}_T - \tilde{a}_T^*) - (\tilde{b}_T^h - \tilde{b}_T^{h*})\frac{\Delta_T}{b_T}\right]\varepsilon u'(\bar{c}_T^*(T))dT.
\end{aligned}$$

Substituting this expression into the third term of the right-hand side of the inequality

(A9) and rearranging terms yields

$$
\begin{aligned}
D \geq & \int_0^\infty e^{-(\rho-n+\varepsilon)t}(\tilde{a}_t - \tilde{a}_t^*)u'(\tilde{c}_t^*) \\
& \qquad \left[\rho + \varepsilon - \frac{u''(\tilde{c}_t^*)}{u'(\tilde{c}_t^*)}\dot{\tilde{c}}_t^* - r_t - \frac{\gamma'(\tilde{a}_t^* - b_t + \Delta_t)}{u'(\tilde{c}_t^*)} - \varepsilon\frac{u'(\bar{c}_t^*(t))}{u'(\tilde{c}_t^*)}\right] dt \\
& + \int_0^\infty e^{-(\rho-n+\varepsilon)t}(\tilde{b}_t^h - \tilde{b}_t^{h*})u'(\tilde{c}_t^*)\left[\varepsilon\frac{\Delta_t}{b_t}\frac{u'(\bar{c}_t^*(t))}{u'(\tilde{c}_t^*)} - i_t + \pi_t + r_t\right] dt \\
& + \int_0^\infty \varepsilon e^{-\varepsilon T}\left[\int_T^\infty e^{-(\rho-n)t}(\bar{a}_t(T) - \bar{a}_t^*(T))u'(\bar{c}_t^*(T))\right. \\
& \qquad \left.\left[\rho - \frac{u''(\bar{c}_t^*(T))}{u'(\bar{c}_t^*(T))}\dot{\bar{c}}_t^*(T) - r_t - \frac{\gamma'(\bar{a}_t^*(T) - b_t)}{u'(\bar{c}_t^*(T))}\right] dt\right] dT \\
& + \lim_{t\to\infty}\left[e^{-(\rho-n+\varepsilon)t}\left(\tilde{a}_t - \tilde{a}_t^*\right)u'(\tilde{c}_t^*)\right. \\
& \qquad \left. + \int_0^\infty \varepsilon e^{-\varepsilon T}\left[e^{-(\rho-n)t}\left(\bar{a}_t(T) - \bar{a}_t^*(T)\right)u'(\bar{c}_t^*(T))\right] dT\right].
\end{aligned}
$$

By the Euler equation before the price level jump (A2), the expression for the risk premium (A4), the Euler equation after the price level jump (A3), respectively, the first three terms must be equal to zero, which yields

$$D \geq \lim_{t\to\infty} \mathbb{E}_0\left[e^{-(\rho-n)t}u'(c_t^*)(a_t - a_t^*)\right].$$

By the transversality condition (A6), this simplifies to

$$D \geq \lim_{t\to\infty} \mathbb{E}_0\left[e^{-(\rho-n)t}u'(c_t^*)a_t\right].$$

But the borrowing constraint (11) implies that a_t cannot be negative. This establishes that, for any feasible allocation $(\tilde{c}_t, \tilde{a}_t, \tilde{b}_t^h, (\bar{c}_s(t), \bar{a}_s(t))_{s=t}^\infty)_{t=0}^\infty$, we have $D \geq 0$, with D defined by (A7). Hence, the feasible allocation $(\tilde{c}_t^*, \tilde{a}_t^*, \tilde{b}_t^{h*}, (\bar{c}_s^*(t), \bar{a}_s^*(t))_{s=t}^\infty)_{t=0}^\infty$ that satisfies (A2), (A3), (A4), and (A6) must be welfare maximizing.

C Intertemporal government budget constraint

With a preference for wealth, the stochastic discount factor is given by

$$\Lambda_t = e^{-\int_0^t\left(\rho - n - \frac{\gamma'(\Delta_u)}{u'(c_u)}\right)du}u'(c_t).$$

Applying Itô's lemma with jumps (from appendix A) yields

$$\begin{aligned} d\Lambda_t &= -\left(\rho - n - \frac{\gamma'(\Delta_t)}{u'(c_t)}\right)\Lambda_t dt + e^{-\int_0^t \left(\rho - n - \frac{\gamma'(\Delta_u)}{u'(c_u)}\right)du} u''(c_t)\dot{c}_t dt \\ &\qquad + e^{-\int_0^t \left(\rho - n - \frac{\gamma'(\Delta_u)}{u'(c_u)}\right)du} \left[u'(\bar{c}_t) - u'(c_t)\right] dJ_t, \\ &= -\left(\rho - n - \frac{\gamma'(\Delta_t)}{u'(c_t)}\right)\Lambda_t dt + \frac{u''(c_t)}{u'(c_t)}\dot{c}_t \Lambda_t dt + \left[\frac{u'(\bar{c}_t)}{u'(c_t)} - 1\right]\Lambda_t dJ_t. \end{aligned}$$

where $\bar{c}_t$ denote consumption immediately after the price level jump (which was denoted more preciesly by $\bar{c}_t(t)$ in appendix B). Using the Euler equation (12), together with the fact that $\mathbb{E}_t[dJ_t] = \varepsilon dt$, yields $\mathbb{E}_t[d\Lambda_t] = -(r_t - n)\Lambda_t dt$, as expected for the stochastic discount factor.

Using the definition of the present value of surpluses Φ_t given by (18), which is not directly affected by the price level jump, we have

$$\begin{aligned} d(\Lambda_s \Phi_s) &= \Lambda_s d\Phi_s + \Phi_s d\Lambda_s, \\ &= \left[(r_s - n)\Phi_s - \tau_s\right]\Lambda_s ds + \Phi_s d\Lambda_s. \end{aligned}$$

Taking expectations and using the fact that $\mathbb{E}_s[d\Lambda_s] = -(r_s - n)\Lambda_s ds$ yields

$$\begin{aligned} \mathbb{E}_s\left[d(\Lambda_s \Phi_s)\right] &= \left[(r_s - n)\Phi_s - \tau_s\right]\Lambda_s ds - (r_s - n)\Phi_s \Lambda_s ds, \\ &= -\tau_s \Lambda_s ds. \end{aligned}$$

Taking expectation at time t with $t \leq s$, using the law of iterated expectations, integrating with respect to s from time t to infinity, and using the limit condition $\lim_{T\to\infty} \mathbb{E}_t\left[\Lambda_T \Phi_T\right] = 0$ yields

$$\Lambda_t \Phi_t = \mathbb{E}_t\left[\int_t^\infty \Lambda_s \tau_s ds\right].$$

This gives expression (19) for the present value of real primary surpluses.

Recall from equation (2) that the Ponzi debt scheme is defined by $\Delta_t = b_t - \Phi_t$. From the evolution of public debt b_t and of the present value of surpluses Φ_t, respectively given by (4) and (18), and then using the expression for the risk premium (13), we obtain

$$\begin{aligned} d\Delta_t &= \left[(i_t - \pi_t - n)b_t - \tau_t\right]dt - \Delta_t dJ_t - \left[(r_t - n)\Phi_t - \tau_t\right]dt, \\ &= \left[\left(r_t - n + \varepsilon \frac{\Delta_t}{b_t}\frac{u'(\bar{c}_t)}{u'(c_t)}\right)b_t - (r_t - n)\Phi_t\right]dt - \Delta_t dJ_t, \\ &= \left[r_t - n + \varepsilon \frac{u'(\bar{c}_t)}{u'(c_t)}\right]\Delta_t dt - \Delta_t dJ_t, \end{aligned}$$

which corresponds to equation (20).

Itô's lemma with jumps implies that $d(\Lambda_s \Delta_s) = \Lambda_s d\Delta_s + \Delta_s d\Lambda_s + d\Lambda_s d\Delta_s$. From the above expressions for $d\Lambda_s$ and $d\Delta_s$, we therefore have

$$\begin{aligned} d(\Lambda_s \Delta_s) &= \left[r_s - n + \varepsilon \frac{u'(\bar{c}_s)}{u'(c_s)} \right] \Delta_s \Lambda_s ds - \Delta_s \Lambda_s dJ_s + \Delta_s d\Lambda_s - \left[\frac{u'(\bar{c}_s)}{u'(c_s)} - 1 \right] \Delta_s \Lambda_s dJ_s, \\ &= \left[r_s - n + \varepsilon \frac{u'(\bar{c}_s)}{u'(c_s)} \right] \Delta_s \Lambda_s ds + \Delta_s d\Lambda_s - \frac{u'(\bar{c}_s)}{u'(c_s)} \Delta_s \Lambda_s dJ_s. \end{aligned}$$

Taking expectations and using the fact that $\mathbb{E}_s[d\Lambda_s] = -(r_s - n)\Lambda_s ds$ and $\mathbb{E}_s[dJ_s] = \varepsilon ds$ yields

$$\begin{aligned} \mathbb{E}_s \left[d(\Lambda_s \Delta_s) \right] &= \left[r_s - n + \varepsilon \frac{u'(\bar{c}_s)}{u'(c_s)} \right] \Delta_s \Lambda_s ds - (r_s - n)\Delta_s \Lambda_s ds - \varepsilon \frac{u'(\bar{c}_s)}{u'(c_s)} \Delta_s \Lambda_s ds, \\ &= 0. \end{aligned}$$

Using the law of iterated expectations and integrating from time t to infinity yields

$$\Lambda_t \Delta_t = \lim_{T \to \infty} \mathbb{E}_t \left[\Lambda_T \Delta_T \right],$$

which corresponds to equation (21).

Note that these results imply that

$$\begin{aligned} b_t &= \Phi_t + \Delta_t, \\ &= \mathbb{E}_t \left[\int_t^\infty \frac{\Lambda_s}{\Lambda_t} \tau_s ds \right] + \lim_{T \to \infty} \mathbb{E}_t \left[\frac{\Lambda_T}{\Lambda_t} \Delta_T \right], \\ &= \mathbb{E}_t \left[\int_t^\infty \frac{\Lambda_s}{\Lambda_t} \tau_s ds \right] + \lim_{T \to \infty} \mathbb{E}_t \left[\frac{\Lambda_T}{\Lambda_t} b_T \right], \end{aligned}$$

where, to obtain the last line, I have used the fact that $\lim_{T \to \infty} \mathbb{E}_t \left[\Lambda_T \Phi_T \right] = 0$. This gives the government's intertemporal budget constraint at time t.

D Transversality condition in equilibrium

Using the expression for the stochastic discount factor, we have

$$\mathbb{E}_0 \left[\Lambda_t \Phi_t \right] = \mathbb{E}_0 \left[e^{-\int_0^t \left(\rho - n - \frac{\gamma'(\Delta_u)}{u'(c_u)} \right) du} u'(c_t) \Phi_t \right] \geq \mathbb{E}_0 \left[e^{-(\rho - n)t} u'(c_t) \Phi_t \right],$$

where the inequality follows from the fact that $\int_0^t (\gamma'(\Delta_u)/u'(c_u)) du$ is always non-negative and, hence, $e^{\int_0^t \left(\frac{\gamma'(\Delta_u)}{u'(c_u)} \right) du}$ must always be greater or equal to one, while Φ_t was assumed to be non-negative. But, by definition of Φ_t, we know that $\lim_{t \to \infty} \mathbb{E}_0 \left[\Lambda_t \Phi_t \right] =$

0. Hence, $\lim_{t\to\infty} \mathbb{E}_0 \left[e^{-(\rho-n)t}u'(c_t)\Phi_t\right] = 0$.

Using the asset market clearing condition (17), the household's transversality condition (14) can be written as $\lim_{t\to\infty} \mathbb{E}_0 \left[e^{-(\rho-n)t}u'(c_t)\left(\Phi_t + \Delta_t\right)\right] = 0$. But, we always have $\lim_{t\to\infty} \mathbb{E}_0 \left[e^{-(\rho-n)t}u'(c_t)\Phi_t\right] = 0$. Hence, in equilibrium, the transversality condition (14) can be written as $\lim_{t\to\infty} \mathbb{E}_0 \left[e^{-(\rho-n)t}u'(c_t)\Delta_t\right] = 0$.

Recall from appendix B that a state-contingent allocation can be written as $(\tilde{c}_t, \tilde{a}_t, \tilde{b}^h_t, (\bar{c}_s(t), \bar{a}_s(t))_{s=t}^{\infty})_{t=0}^{\infty}$, where $\tilde{c}_t$, $\tilde{a}_t$, and $\tilde{b}^h_t$ denote consumption, wealth, and government bond holdings at time t conditional on the absence of a jump, while $\bar{c}_t(T)$ and $\bar{a}_t(T)$ denote consumption and wealth at time t conditional on the price level having jumped at time T. Similarly, we could denote by $\tilde{\Delta}_t$ the magnitude of the Ponzi scheme conditional on the absence of a jump, and by $\bar{\Delta}_t(T)$ the Ponzi scheme conditional on a price level jump having taken place at time T. Using equation (A5) from appendix B, we have

$$\mathbb{E}_0 \left[e^{-(\rho-n)t}u'(c_t)\Delta_t\right] = \int_0^t \varepsilon e^{-\varepsilon T}\left[e^{-(\rho-n)t}u'(\bar{c}_t(T))\bar{\Delta}_t(T)\right] dT + e^{-(\rho-n+\varepsilon)t}u'(\tilde{c}_t)\tilde{\Delta}_t.$$

But, after a price level jump, the magnitude of the Ponzi scheme must be equal to zero; which is formally implied by (20). Hence, $\bar{\Delta}_t(T) = 0$. It follows that

$$\lim_{t\to\infty} \mathbb{E}_0 \left[e^{-(\rho-n)t}u'(c_t)\Delta_t\right] = \lim_{t\to\infty} e^{-(\rho-n+\varepsilon)t}u'(\tilde{c}_t)\tilde{\Delta}_t.$$

E Ruling out explosive Ponzi schemes

The consumption Euler equation before the price level jump (22) can be written as

$$\frac{d\ln\left[u'(c_t)\right]}{dt} = \rho - r_t - \frac{\gamma'(\Delta_t)}{u'(c_t)} - \varepsilon\left(\frac{u'(\bar{c}_t)}{u'(c_t)} - 1\right).$$

Integrating this expression from time t to T yields

$$u'(c_T) = u'(c_t)e^{\int_t^T \left(\rho - r_u - \frac{\gamma'(\Delta_u)}{u'(c_u)} - \varepsilon\left(\frac{u'(\bar{c}_u)}{u'(c_u)} - 1\right)\right)du}.$$

Integrating the dynamics of the Ponzi scheme given by (26) from time t to T yields

$$\Delta_T = \Delta_t e^{\int_t^T \left(r_u - n + \varepsilon\frac{u'(\bar{c}_u)}{u'(c_u)}\right)du}.$$

From these two equations, conditional on the absence of a price level jump, we have

$$\lim_{T\to\infty} e^{-(\rho-n+\varepsilon)(T-t)}u'(c_T)\Delta_T = u'(c_t)\Delta_t \lim_{T\to\infty} e^{-\int_t^T \frac{\gamma'(\Delta_u)}{u'(c_u)}du}.$$

Hence, for the transversality condition (27) to be satisfied, we must have

$$\lim_{T\to\infty}\int_t^T \frac{\gamma'(\Delta_u)}{u'(c_u)}du = \infty. \tag{A10}$$

If the Ponzi scheme is explosive, there must eventually be a strictly positive lower bound x to its growth rate, i.e. $r_u - n + \varepsilon u'(\bar{c}_u)/u'(c_u) \geq x > 0$ for all $u \geq t$ for some arbitrarily large t. We therefore have

$$\Delta_T = \Delta_t e^{\int_t^T \left(r_u - n + \varepsilon \frac{u'(\bar{c}_u)}{u'(c_u)}\right)du} \geq \Delta_t e^{x(T-t)}.$$

Note that, as labor supply is equal to 1, we must have $c_u \leq 1$ for all u. Hence, if the Ponzi scheme is explosive, we have

$$\lim_{T\to\infty}\int_t^T \frac{\gamma'(\Delta_u)}{u'(c_u)}du \leq \frac{1}{u'(1)}\lim_{T\to\infty}\int_t^T \gamma'\left(\Delta_t e^{x(T-t)}\right)du < \infty,$$

where the last inequality follows from our assumption that $\int_0^\infty \gamma'(e^{\lambda t})dt < \infty$ for any $\lambda > 0$. This establishes that the limit (A10), and hence the transversality condition (27), cannot be satisfied for a Ponzi scheme that is explosive conditional on the absence of a price level jump.[21]

F Ponzi-stagnation steady state

A Ponzi-stagnation steady state $(c^{ps}, \pi^{ps}, r^{ps}, i^{ps}, \Delta^{ps})$ is characterized by low inflation $\pi^{ps} = \pi^R$ and $r^{ps} = n - \varepsilon\frac{u'(\bar{c})}{u'(c^{ps})}$. Hence, by the Euler equation, the magnitude of the Ponzi scheme Δ^{ps} is given by

$$\gamma'\left(\Delta^{ps}\right) = (\rho - n + \varepsilon)u'(c^{ps}). \tag{A11}$$

Also, by the Taylor rule $i^{ps} = \max\{r^n + \pi^R + (\phi - 1)[\pi^R - \pi^*], 0\}$ with $\pi^R \leq \pi^*$ and $r^n < -\pi^R$, we must have $i^{ps} = 0$. Finally, from the risk-premium equation (24), we have

$$\varepsilon\frac{\Phi}{\Phi + \Delta^{ps}}\frac{u'(\bar{c})}{u'(c^{ps})} = n + \pi^R, \tag{A12}$$

where we assume that Φ is constant in the Ponzi-stagnation steady state. The steady state values of c^{ps} and Δ^{ps} are jointly determined by (A11) and (A12). The Ponzi-stagnation steady state exists if and only if there exists a solution to these two equa-

[21] A similar proof of the impossibility of explosive bubbles under a preference for wealth was provided by Michau, Ono and Schlegl (2022) for the case of deterministic bubbles, i.e. with $\varepsilon = 0$.

tions that satisfy $c^{ps} < 1$ and $\Delta^{ps} > 0$.[22]

From (A12), this steady state cannot exist when $n + \pi^R \leq 0$. Even when $n + \pi^R > 0$, it does not exist when either ε or Φ is sufficiently close to zero.

If it exists, we can characterize a number of its properties. First, under-employment $c^{ps} < 1$ implies by (A11) that the Ponzi scheme is of a smaller magnitude than in the Ponzi steady state $\Delta^{ps} < \Delta^p$. Second, by (28), (A11), $\Delta^{ps} > 0$, and $c^{ps} < 1$, the Ponzi-stagnation steady state can only exist if $r^n < n - \varepsilon$, i.e. if the Ponzi steady state also exists.

Finally, with $\bar{c} \in \{c^{ss}, 1\}$, we must have $c^{ps} > c^{ss}$. Substituting (A12) into (A11) yields $\gamma'(\Delta^{ps}) = (\rho + \pi^R + \varepsilon)u'(c^{ps}) - \varepsilon\frac{\Phi}{\Phi + \Delta^{ps}}u'(\bar{c})$, which defines c^{ps} as an increasing function of Δ^{ps}. Moreover, this relationship together with (30) implies that, if $\Delta^{ps} = 0$, then $(\rho + \pi^R)u'(c^{ss}) + \varepsilon u'(\bar{c}) = (\rho + \pi^R + \varepsilon)u'(c^{ps})$. Thus, if $\Delta^{ps} = 0$, then $c^{ps} \in [c^{ss}, \bar{c}]$. We therefore have a relationship that defines c^{ps} as an increasing function of Δ^{ps} with $c^{ps} \geq c^{ss}$ when $\Delta^{ps} = 0$. This establishes that $c^{ps} > c^{ss}$.

G Welfare function

From the welfare function of the representative household (9), together with the asset market clearing condition (17), the objective of the government is to maximize

$$W = \mathbb{E}_0 \left[\int_0^\infty e^{-(\rho - n)t} \left[u(c_t) + \gamma(\Delta_t) - \psi c \left(\frac{dP_t}{P_t} \right) \right] dt \right].$$

The only source of uncertainty is the time T when the price level jumps, which is exponentially distributed with parameter ε. Let us denote by $(c_t, \pi_t, \Delta_t, \Phi_t)_{t=0}^\infty$ the equilibrium of the economy conditional on the absence of a price level jump and by $(\bar{c}, \bar{\pi})$ the steady state equilibrium afterwards. Using the specification for the cost of inflation

[22] If $c^{ps} = 1$, then $\Delta^{ps} = \Delta^p$ and this steady state coincides with the Ponzi steady state with a binding zero lower bound.

(8), we have

$$\begin{aligned}
W &= \int_0^\infty \varepsilon e^{-\varepsilon T}\left[\int_0^T e^{-(\rho-n)t}\left[u(c_t)+\gamma(\Delta_t)-\psi|\pi_t|\right]dt - e^{-(\rho-n)T}\psi C\left(\frac{\Delta_T}{\Phi_T}\right)\right.\\
&\qquad\qquad\left. + \int_T^\infty e^{-(\rho-n)t}\left[u(\bar{c})+\gamma(0)-\psi|\bar{\pi}|\right]dt\right]dT,\\
&= \int_0^\infty \left[\int_t^\infty \varepsilon e^{-\varepsilon T}dT\right] e^{-(\rho-n)t}\left[u(c_t)+\gamma(\Delta_t)-\psi|\pi_t|\right]dt\\
&\qquad - \int_0^\infty e^{-(\rho-n+\varepsilon)T}\varepsilon\psi C\left(\frac{\Delta_T}{\Phi_T}\right)dT\\
&\qquad + \varepsilon\left[u(\bar{c})+\gamma(0)-\psi|\bar{\pi}|\right]\int_0^\infty e^{-\varepsilon T}\int_T^\infty e^{-(\rho-n)t}dtdT,\\
&= \int_0^\infty e^{-(\rho-n+\varepsilon)t}\left[u(c_t)+\gamma(\Delta_t)-\psi|\pi_t|-\psi\varepsilon C\left(\frac{\Delta_t}{\Phi_t}\right)\right]dt\\
&\qquad + \frac{1}{\rho-n}\frac{\varepsilon}{\rho-n+\varepsilon}\left[u(\bar{c})+\gamma(0)-\psi|\bar{\pi}|\right].
\end{aligned}$$

When there is a Ponzi scheme, at each instant, there is a probability εdt of a price level jump that raises the cost of inflation to $\psi C(\Delta_t/\Phi_t)/dt$.

If the economy is in steady state before the price level jump, then the expected welfare of the representative household is equal to

$$\begin{aligned}
W = \frac{1}{\rho-n+\varepsilon}&\left[u(c)+\gamma(\Delta)-\psi|\pi|-\psi\varepsilon C\left(\frac{\Delta}{\Phi}\right)\right]\\
&+\frac{1}{\rho-n}\frac{\varepsilon}{\rho-n+\varepsilon}\left[u(\bar{c})+\gamma(0)-\psi|\bar{\pi}|\right].
\end{aligned}$$

With $\pi \geq \pi^R \geq 0$, this gives (33).

H Robustness

Figure A1 displays the policy trilemma when $\alpha = 4$ and $\beta = 4$. Figure A2 shows that, under our calibration, when $\alpha = \beta = 1$, the neoclassical steady state is always dominated, effectively resulting in a dilemma.

I Fundamental shock

Recall that J_t is initially equal to zero and can jump to one at any point in time at Poisson rate ε. Rather than being a pure sunspot shock that raises the price level, in accordance with (3), let us now assume that it is a shock to the marginal utility of wealth. More specifically, the representative household's intertemporal utility is now

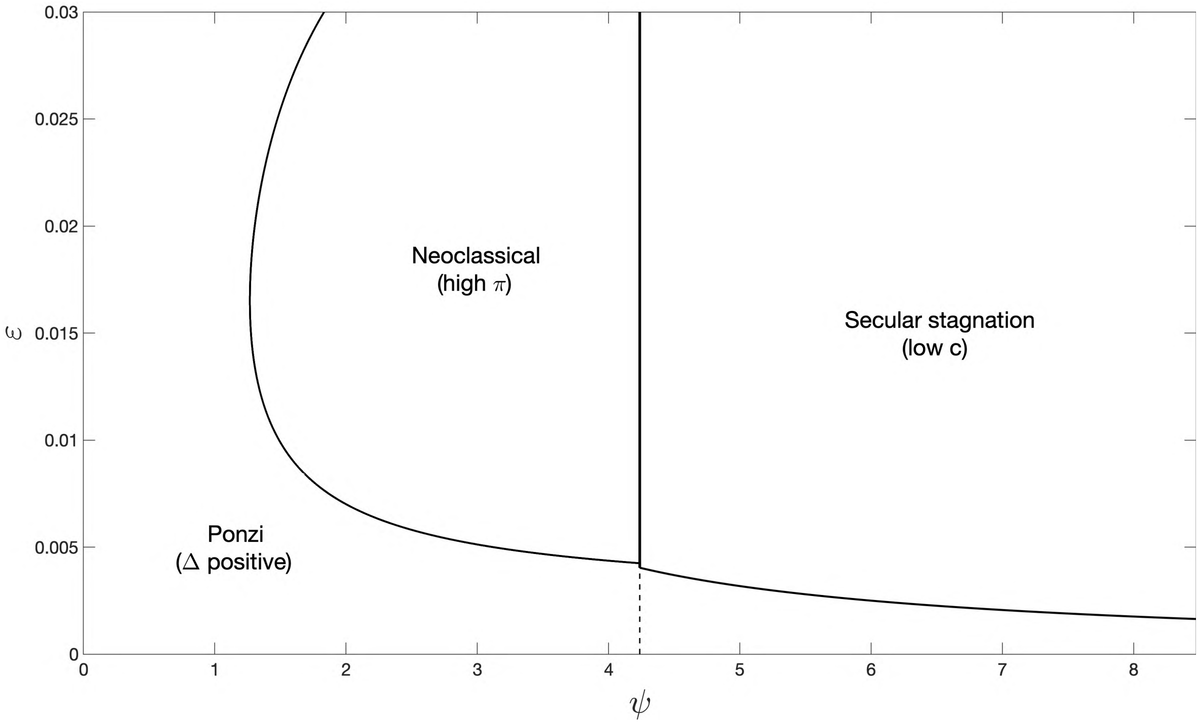

Figure A1: Trilemma for $\alpha = 4$ and $\beta = 4$

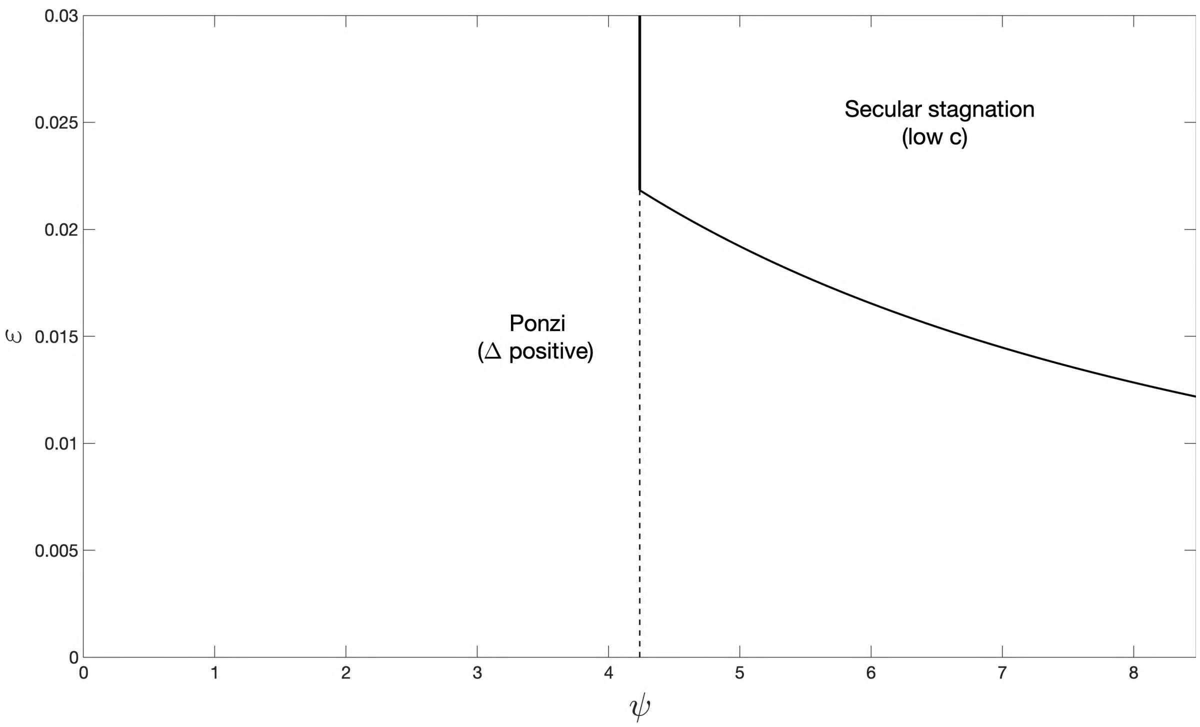

Figure A2: Trilemma (or dilemma) for $\alpha = 1$ and $\beta = 1$

given by

$$\mathbb{E}_0\left[\int_0^{\infty} e^{-(\rho-n)t}\left[u(c_t)+(1-\lambda J_t)\gamma(a_t-b_t+\Delta_t)-\psi c(\pi_t)\right]\,dt\right].$$

The parameter $\lambda \in [0,1]$ is assumed to be sufficiently high to satisfy

$$\rho-(1-\lambda)\frac{\gamma'(0)}{u'(1)} \geq \max\{-\pi^R, n-\varepsilon\}.$$

This implies that, once $J_t = 1$, the natural real interest rate, equal to the left-hand side of this inequality, is too high to allow for the possibility of either a secular stagnation or a Ponzi steady state. Hence, after the realization of the shock, the economy must be in the neoclassical steady state.[23] The equilibrium before the shock occurs remains characterized by equations (22)-(27), but with $\bar{c}_t = 1$.

This establishes that the upward jump in the price level can be driven by a fundamental shock to the economy that reduces the marginal utility of wealth, rather than by a sunspot shock inducing households to run away from the Ponzi scheme of public debt.

The steady states remain unchanged, except for the secular stagnation steady state $(c^{ss}, \pi^{ss}, r^{ss}, i^{ss}, \Delta^{ss})$, which is no longer an absorbing state. This steady state is still characterized by the absence of Ponzi scheme $\Delta^{ss} = 0$ and by a binding downward nominal wage rigidity $\pi^{ss} = \pi^R$. From (22) with $\bar{c}_t = 1$, the Euler equation in steady state is now given by

$$\frac{1}{u'(c^{ss})} = \frac{1}{\gamma'(0)}\left[\rho - r^{ss} - \varepsilon\left(\frac{u'(1)}{u'(c^{ss})}-1\right)\right]. \tag{A13}$$

As $c^{ss} < 1$, we have

$$\frac{\rho - r^n}{\gamma'(0)} = \frac{1}{u'(1)} > \frac{1}{u'(c^{ss})} = \frac{1}{\gamma'(0)}\left[\rho - r^{ss} - \varepsilon\left(\frac{u'(1)}{u'(c^{ss})}-1\right)\right] \geq \frac{\rho - r^{ss}}{\gamma'(0)},$$

which establishes that $r^{ss} > r^n$, where r^n is the natural real interest rate before the occurrence of the fundamental shock. From the risk premium relationship (24) with $\Delta^{ss} = 0$, we have $r^{ss} = i^{ss} - \pi^{ss}$. Hence, from the Taylor rule (25), we have $r^{ss} = \max\{r^n + (\phi-1)[\pi^R - \pi^*], -\pi^R\}$. As $r^{ss} > r^n$ and $\pi^R \leq \pi^*$, we must have $r^{ss} = -\pi^R$ and, hence, $i^{ss} = 0$. From the steady state Euler equation (A13), the output level is

[23] Naturally, I consider that the natural real interest rate r^n of the Taylor rule (25) is equal to $\rho - \frac{\gamma'(0)}{u'(1)}$ before the shock and to $\rho - (1-\lambda)\frac{\gamma'(0)}{u'(1)}$ afterwards.

given by

$$\frac{1}{u'(c^{ss})} = \frac{\rho + \pi^R + \varepsilon}{\gamma'(0) + \varepsilon u'(1)}.$$

Existence of the secular stagnation steady state requires $c^{ss} < 1$ or, equivalently, $r^n < -\pi^R$, which is the same condition as under a permanent lack of demand.

Output c^{ss} is an increasing function of likelihood ε of occurrence of the fundamental shock, with $c^{ss} = 1$ in the limit as ε tends to infinity. When $\varepsilon > 0$, the secular stagnation steady state is the same as under a permanent lack of demand, except that output is higher, i.e. the economy is less depressed. The prospect of an economic recovery raises households' demand for consumption and, hence, output under secular stagnation.

CREST
CENTER FOR RESEARCH
IN ECONOMICS AND STATISTICS

CREST
Center for Research in Economics and Statistics
UMR 9194

5 Avenue Henry Le Chatelier
TSA 96642
91764 Palaiseau Cedex
FRANCE

Phone: +33 (0)1 70 26 67 00
Email: info@crest.science
https://crest.science/

The Center for Research in Economics and Statistics (CREST) is a leading French scientific institution for advanced research on quantitative methods applied to the social sciences.

CREST is a joint interdisciplinary unit of research and faculty members of CNRS, ENSAE Paris, ENSAI and the Economics Department of Ecole Polytechnique. Its activities are located physically in the ENSAE Paris building on the Palaiseau campus of Institut Polytechnique de Paris and secondarily on the Ker-Lann campus of ENSAI Rennes.