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When is a life worth living? A dynastic efficiency criterion for fertility decisions*

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Abstract

This paper extends the Pareto efficiency to setups with endogenous fertility. Adding an extra child will be a social improvement if her life is worth living. For any criterion for lives worth living, an allocation is efficient when it is Pareto efficient (i.e. with a fixed population) and when no one with a life (not) worth living can be added (removed) without reducing someone's well-being. I also define the ALW (All Lives Worth Living) equity criterion which requires that all agents have lives worth living. The first welfare theorem stands and I connect with results in the literature (e.g. Golosov, Jones, and Tertilt (2007)). Furthermore, I show that binding constraints on bequests are not inefficient if they are not too high: if the bequest from a parent to her child is never constrained to be higher than what is necessary for the child and her descendants to have lives worth living, then the equilibrium is efficient. Criteria for lives worth living necessarily convey value judgements. As a benchmark, I propose a Dynastic criterion relying solely on parents' revealed preferences: a child's life is worth living when, *ceteris paribus*, her altruistic parent is better off with her being born. Then, setting constraints on bequests *exactly* such that parents may be paid back for the raising costs implies that the equilibrium is efficient and that all children have lives worth living. Finally, putting these concepts at work, I explore real world policy implications. I show that 1) direct population control (e.g. China's one-child policy) may be efficient and 2) with external effects to childbearing, Pigouvian taxes restore efficiency.

Keywords: Pareto efficiency, fertility, altruism, bequests.

JEL Classification: D61 H21 J13

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(...) the sole evidence it is possible to produce that anything is desirable, is that people do actually desire it.

John Stuart Mill
in Utilitarianism (1863)

1 Introduction

The single most important decision of any agent's life is not up to the agent herself but to the previous generation: shall she live or not? This decision is inspired by parental love and altruistic feelings but is also influenced by financial constraints and incentives partially shaped by public policies. How should one take into account human beings added or subtracted by such public policies when designing them? Agents deciding on a public policy must answer this question. Ignoring it would mean considering that these lives have no weight in the design of the policy, which is one way to answer the question.

To illustrate this, imagine a society has to choose between two possible allocations of resources where all agents are indifferent between the first and the second allocations except one agent i who is born only in the second allocation. Which allocation should be chosen? Should agent i live or not? The answer I adopt in this work is that agent i should live if and only if her life is "good enough". In other words, agent i should live if and only if her utility level is above a threshold called the Critical Level by the literature. The question of the right value for the Critical Level is still open.

When it comes to choosing among allocations of resources, a usual economist's stance is to avoid making any choice oneself by relying on agents' revealed preferences. Here this stance would imply offering directly to agent i the choice between the two allocations. Agent i would then chose to live if and only if her utility level is "high enough". But agent i doesn't exist before she is born and so she simply can't choose. We are then back to our previous problem: the generation of the parents of agent i (and before) must determine agent i's Critical Level.

In this work I propose to answer these questions by relying solely on parents' preferences: an added human being would be desirable if and only if *ceteris paribus* it makes her parents happier. So, instead of relying on some common or consensual value judgements e.g. agent i should live if she is well fed and healthy, has fundamental human rights and has access to education, I propose to rely solely on each parent's preferences which can be revealed by their fertility decisions. This is the only way to define a non-trivial Critical Level without imposing some arbitrary value judgement from the outside.

I make several contributions in this paper.

First, I introduce the $\mathcal{C}$-efficiency which is a large family of criteria to assess the efficiency of allocations when populations are endogenous. Each criterion relies on a different choice of endogenous Critical Levels for each potential future agents. These efficiency criteria are a bridge between the usual Pareto efficiency used in Public Economics and the Critical Level Utilitarianism established in the Social Choice & Welfare literature. The $\mathcal{C}$-efficiency nests some other efficiency criteria introduced in Golosov, Jones, and Tertilt (2007), namely the $\mathcal{P}$-efficiency (under some conditions) and the $\mathcal{A}$-efficiency.

Second, I introduce a special case of the $\mathcal{C}$-efficiency that I call $\mathcal{D}$-efficiency. I define Critical Levels called $\mathcal{D}$ynastic Critical Levels which are based on parents' preferences. They are built implicitly: a child utility level is above her Dynastic Critical Level if and only if, *ceteris paribus*, her parent has a higher utility level when her child is born than when she is not.

Third, I establish a theoretical result: binding constraints on bequests are not inefficient as long as they are not too high. This implies that a departure from *laissez-faire* is not necessarily inefficient even without any external effect. Binding constraints on bequests make children more costly to parents so they have less children. But, as long as such a *not born* child or at least one of her descendants would have had a life not worth living, it is not inefficient to prevent him from living with constraints on bequests.

Fourth, I introduce a new equity criterion called All Lives Above their Critical levels or ALAC. An allocation is ALAC when all agents have utility levels not below their Critical Levels i.e. all agents have live worth living. When Critical Levels are exogenous (i.e. constant), it is possible to define constraints on bequests such that the equilibrium plan is $\mathcal{C}$-efficient and ALAC. In the special case of Dynastic Critical Levels, it is possible to build constraints on bequests such that the equilibrium plan is ALAC. Constraints on bequests must be built such that financial incentives don't play any role in the desire of having children i.e. parents cannot get back more than the raising costs out of their children. Hence parents have children only because they get *direct* utility out of them which exactly means that children have lives above their Dynastic Critical Levels.

Fifth, I look at two types of situations in which deviating from spontaneous fertility decisions of parents might be desirable. First the monitoring of population sizes and second the existence of external effects of children. Surprisingly, I find that implementing direct constraints on the number of children such as the Chinese One Child policy is not $\mathcal{C}$-inefficient as long as all children have utility levels strictly above their Critical Levels (which can be achieved through high constraints on bequests). Thus, a cap-trade scheme for breeding permits is not required to efficiently monitor population. When children have negative external effects, a Pigouvian tax is enough to restore the $\mathcal{C}$-efficiency in equilibrium if the revenue is distributed to the initial generation.

Related Literature

The impact of population growth on economic outcomes has been studied by Samuelson (1975). However, children are assumed to be costless and children don't enter into their parents' preferences. Since parents are indifferent, fertility decisions may be fixed by the government which makes the trade-off between dilution of capital and increase in labor supply. The seminal idea that a fertility decision could be a classic economic decision that balances raising costs with joy of having children was proposed by Garry Becker (e.g. Becker and Barro (1988) and Barro and Becker (1989)).

Ranking allocations with different populations is an old question. For instance, Mill (1848) proposed to average utility levels. In contrast, Bentham (1823) proposed to sum up utility levels. Both need to make interpersonal comparisons of utility levels which I don't need in this work. The question of the "right" zero utility level in Bentham (1823) is equivalent to the question of the "right" Critical Level addressed here. Blackorby, Bossert, and Donaldson (1995) showed that the Benthamite utilitarian framework with critical utility levels met some desirable axioms when children had no impact on their parents' welfare. However, the anonymity axiom forces all potential agents to have the same constant critical utility levels. I depart from this requirement in this work.

Michel and Wigniolle (2007) and Conde-Ruiz, Giménez, and Pérez-Nievas (2010) use a representative agent for each generation, whatever the number of agents in each generation they will always be treated symmetrically and only the average utility matters. This is a type of Millian efficiency. Chu and Koo (1990) identify agents by their generations and by their types. They use Stochastic Dominance i.e. what matters is the probability distribution of welfare, whatever the size of each generation. This is also a Millian criterion. Under some conditions, they show that reducing the fertility of the poorest implies a Stochastic Dominating welfare distribution. However Cordoba and Liu (2016) noted that they didn't take into account the impact of fertility constraints on welfare and that their results heavily relied on neglecting generations' sizes. In Phelan and Rustichini (2018) agents are identified by their generations, by their own types and by all their ancestors' types. Parents don't know their descendants' types which will be realized in the future. Each altruistic agent cares about her descendants' expected instantaneous utility levels. They derive optimal taxation results in a finite horizon model. In this work, I follow Golosov, Jones, and Tertilt (2007) who use a narrower identification: each agent is identified by his node in his dynastic tree i.e. by his parent and his rank in his brotherhood.

Golosov, Jones, and Tertilt (2007) allow for altruism from parents to children. They introduced the $\mathcal{A}$-efficiency and the $\mathcal{P}$-efficiency which I discussed in this work. Schoonbroodt and Tertilt (2014) use their efficiency criterion and show that binding constraints on bequests necessarily lead to inefficient outcomes, I generalize this result to setups where all lives are not necessarily worth living. They also show that fertility dependent Pays As You Go pension schemes restore efficiency. In Conde-Ruiz, Giménez, and Pérez-Nievas (2010), parents only care about the quantity of children i.e. there is no altruism so no bequest motive. The only transfers from a generation to another are made through a Pays As You Go pension system. Efficient pensions must be a linear function of the number of children and such pensions can decentralize any efficient allocation.

In this work, I assume that fertility decisions respond to financial constraints and incentives. This has been studied by Boyer (1989), Laroque and Salanié (2004), Landais (2007), Laroque and Salanié (2008) and Laroque and Salanié (2014). See McDonald (2002) for a review of fertility enhancing policies. The question of a cap and trade scheme for breeding permits has been studied by De La Croix and Gosseries (2009). Of course financial incentives are not alone in explaining fertility decisions and, for instance, preferences themselves could be modified endogenously through social, cultural, educational or religious changes. The utility functions needed to rationalize observed fertility behaviours has been studied (e.g. Jones and Schoonbroodt (2010)).

In this paper I build Critical Levels using parents' preferences. As long as they deem their children desirable *per se* and not for instrumental reasons (e.g. old age security) then society should respect parents' points of view and only correct for externalities. This idea was already present in Friedman (1972) who advocated for *laissez-faire* in Population for two main reasons. First, according to him, positive and negative externalities to childbearing almost cancel out. Second, since parents are altruistic towards their children, they consider their welfare in their fertility decisions. These were just casual arguments that I try to formalize here. Nowadays, externalities to childbearing are significant (e.g. Lee and Miller (1990), Harford (1998) and Bohn and Stuart (2015)).

Belan and Moussault (2018) offer to parents the possibility to transfer time and money to their children children. High constraints on bequests could become inefficient because parents have to make large financial transfers without large time transfers. Parents and children could all be better off with smaller financial transfers and larger time transfers.

Following this introduction, my work is organized as follows. In Section 2, I lay out the dynastic setup and define agents' preferences. In Section 3, I introduce the $\mathcal{C}$-efficiency and a new sub-case called $\mathcal{D}$-efficiency which uses Critical Levels only based on parents' preferences. Section 4 describes the economy and how equilibria form. In Section 5, I introduce a new equity criterion called ALAC (All Lives are Above Critical levels). In Section 6, I derive sufficient conditions on constraints on bequests to guarantee that the equilibrium allocation is $\mathcal{C}$-efficient. Section 7 studies two public economic questions: population control and externalities. The last section concludes.

2 The dynastic set-up

2.1 Agents

Consider a discrete time model where each generation lives for two periods : childhood and adulthood. At $t = 0$ there are N initial agents who are identified by $i_0 \in \{1, \dots, N\}$ and their children. At $t = 1$, the only agents alive are the children of the initial agents who are now adults, and their own children. At $t = 2$, the only agents alive are the children of the agents of period $t = 1$, i.e. the grand-children of the initial agents, who are now adults, along with their own children. The same goes for the following periods $t = 3, 4, 5...\infty$

Each child has a single parent. $M \in \mathbb{N}$ is the maximum number of children any agent can have. For each agent i, f_i is the set of children agent i chooses to have among the set of all her potential children. During her one period adult life, each agent i raises her children f_i and her household consumes $c_i > 0$.

I denote I the set of all potential agents. Depending on the fertility decisions $(f_i)_{i \in I}$ some agents are born and some are not. I denote f^i the set containing agent i and all her born descendants. I call f^i a *dynastic* fertility plan. I denote $f := \bigcup_{i_0 \in \{1, \dots, N\}} f^{i_0}$ the set containing all born agents. I call f a fertility plan.

For any agent i, the consumption decision is denoted c_i. I denote $c^i := (c_j)_{j \in f^i}$ the set of all consumption decisions taken by agent i's dynasty (including c_i). I call c^i a *dynastic* consumption plan. I denote $c := (c_j)_{j \in f}$ the set of consumption decisions taken by born agents. I call c a consumption plan.

(f, c) is simply called a plan. I assume that there is no other degree of freedom so that a plan (f, c) fully describes the allocation of resources.

I denote $I(t)$ the set of all potential agents who are adults in period t and, conversely, I denote $t(i)$ the period during which agent i is adult (if born). The set of agent i's descendants born under the dynastic plan f^i who are adults in period t is denoted $f^i(t)$. The set of all agents born under the plan f and adults in period t is denoted $f(t)$.

2.2 Preferences

Agents derive utility from consumption and children. I assume parents are *perfectly altruistic* i.e. for a fixed choice of children, the happier the children the happier their parents.[1] I assume that preferences are well represented by continuous and bounded utility functions so that the utility level U_i obtained by any agent i is n by $U_i = u_i(c_i, \{U_j\}_{j\in f_i})$ where $\{U_j\}_{j\in f_i}$ are the utility levels which agent i's born children will obtain. I assume that $u_i(\cdot)$ is increasing and *strictly* concave with respect to consumption c_i. To be able to assess future utility levels, I assume that agents know the preferences of their descendants and are able to compute their utility levels under any plan (f, c) . Thus, for any agent i, the utility level U_i can be computed as a function of all consumption and fertility levels of all her descendants and herself i.e. the *dynastic* plan (f^i, c^i) . So $U_i = u_i(c_i, \{U_j\}_{j\in f_i}) = \breve{u}_i(f^i, c^i)$.

3 The efficiency criterion

In this section I introduce a general extension of Pareto efficiency criterion to setups with endogenous populations called $\mathcal{C}$-efficiency, which I compare to the $\mathcal{A}$-efficiency and $\mathcal{P}$-efficiency introduced by Golosov, Jones, and Tertilt (2007). I also introduce my novel criterion for a life worth living and the $\mathcal{D}$-efficiency which follows.

3.1 $\mathcal{C}$-efficiency

In this subsection, I define my extension of Pareto efficiency criterion to setups with endogenous populations: the $\mathcal{C}$-efficiency. The $\mathcal{C}$ stands for "Critical" because I use Critical Levels denoted α_i. Using Critical Level α_i means this: imagine that agent i can be added without any net effects on other agents utility levels, then her life is a strict social improvement if and only if $U_i > \alpha_i$. One novelty of this work is to allow for endogenous critical levels $\alpha_i(\cdot)$. In all generality, I allow the critical levels $\alpha_i(\cdot)$ to be function of the whole plan (f, c). Henceforth, I will say that the life of agent i in the plan (f, c) is worth living if and only if $\breve{u}_i(f^i, c^i) \geq \alpha_i(f, c)$. When this inequality is strict her life will be said to be *strictly* worth living.

[1] Here I follow Golosov, Jones, and Tertilt (2007). Perfect altruism allows me to avoid inefficiencies that can be overcome with complex commitment devices. These inefficiencies were studied by Conde-Ruiz, Giménez, and Pérez-Nievas (2014).

The endogeneity of Critical Levels captures the fact that, depending on the way a society evolves (typically if it becomes rich or poor), the normative view for lives worth living might evolve. Note that $(f^{\backslash j}, c^{\backslash j})$ also contains predictions of decisions that will take place long after agent j has lived. For instance, the anticipation of very low utility levels in the future could make today's society less demanding when it comes to judging whether or not current lives are worth living.

I can now introduce the $\mathcal{C}$-domination and the $\mathcal{C}$-efficiency:

Definition 1. ($\mathcal{C}$-domination)
For given critical levels $\{\alpha_j(\cdot)\}$, I say that a plan (f, c) is $\mathcal{C}$-**dominated** by $(\hat{f}, \hat{c})$ when:

1. Agents i born under plan f **and** plan $\hat{f}$ are better off with $(\hat{f}, \hat{c})$
   I.e. $\forall i \in f \cap \hat{f} \quad u_i(c_i, \{U_j\}_{j \in f_i}) \leq u_i(\hat{c}_i, \{\hat{U}_j\}_{j \in \hat{f}_i})$
2. Agents i born **only** under plan $\hat{f}$ have lives worth living in $(\hat{f}, \hat{c})$
   I.e. $\forall i \in \hat{f} \backslash f \quad u_i(\hat{c}_i, \{\hat{U}_j\}_{j \in \hat{f}_i}) \geq \alpha_i(\hat{f}, \hat{c})$
3. Agents i born **only** under plan f don't have lives *strictly* worth living in (f, c)
   I.e. $\forall i \in f \backslash \hat{f} \quad u_i(c_i, \{U_j\}_{j \in f_i}) \leq \alpha_i(f, c)$

I say that (f, c) is *strictly* $\mathcal{C}$-dominated by $(\hat{f}, \hat{c})$ when there is at least one agent for whom the inequality is strict.

Finally, I am able to introduce the definition of my extension of Pareto efficiency to setups with endogenous fertility.

Definition 2. ($\mathcal{C}$-efficiency)
A *feasible* plan (f, c) is $\mathcal{C}$**-efficient** when it is not strictly $\mathcal{C}$-dominated by another *feasible* plan $(\hat{f}, \hat{c})$.

Of course when fertility decisions are exogenous, this definition is exactly equivalent to the usual Pareto efficiency.

3.2 Previous extensions of Pareto efficiency

In this subsection, I look at two previous extensions of Pareto efficiency criterion to setups with endogenous fertility. They were introduced by Golosov, Jones, and Tertilt (2007). When comparing two allocations, either they rely only on preferences of agents born in both allocations (for the $\mathcal{A}$-efficiency) or they give preferences to unborn agents as well as born agents (for the $\mathcal{P}$-efficiency). I will discuss how these extensions relate to my $\mathcal{C}$-efficiency.

3.2.1 $\mathcal{A}$-efficiency

The $\mathcal{A}$-efficiency was introduced by Golosov, Jones, and Tertilt (2007). It is called $\mathcal{A}$-efficiency because when two plans are compared, only utilities of agents $\mathcal{A}$**live** in both plans are compared. A feasible plan (f, c) is said to be $\mathcal{A}$-efficient if there is no other feasible plan $(\hat{f}, \hat{c})$ such that:

1. For all agents i born in f and in $\hat{f}$ $\quad u_i(\hat{f}^i, \hat{c}^i) \geq u_i(f^i, c^i)$

2. There is an agent i born in f and in $\hat{f}$ such that $\quad u_i(\hat{f}^i, \hat{c}^i) > u_i(f^i, c^i)$

This definition implies that, when implementing a public policy which has an effect on fertility decisions, the added or subtracted lives should not have any weight (neither positive nor negative) in the decision. $\mathcal{A}$-efficiency means that any number of lives can be added or subtracted, if other agents are indifferent then it is not a *strict* improvement nor a *strict* deterioration. Using the terms of the $\mathcal{C}$-efficiency, it means that for any agent i, her utility level $u_i(f, c)$ is always equal to her critical level $\alpha_i(f, c)$. This means that the $\mathcal{A}$-efficiency is a special case of the $\mathcal{C}$-efficiency. Indeed we have the following result:

Lemma 1. ($\mathcal{A}$-efficiency equivalence)
If $\alpha_i(f, c) = u_i(f, c)$ for all agents i and for all plans (f, c), then the $\mathcal{C}$-efficiency is equivalent to the $\mathcal{A}$-efficiency i.e. any plan (f, c) is $\mathcal{C}$-efficient if and only if it is $\mathcal{A}$-efficient.

See the proof in the Appendix B

To a government who wants to avoid making any value judgment the $\mathcal{A}$-efficiency is appealing because there is no need to take any position on the "quality" of added or subtracted lives. While this seems quite agnostic, judging that an added life is never a good thing nor a bad thing is a value judgement, as the next example illustrates. Imagine there are only two feasible allocations. The second allocation is a deviation of the first in which an extra agent is born and works for all the other agents and is given very little food and comfort. All the other agents are better off in the second allocation because they enjoy more goods thanks to her work. However, to achieve this the extra agent had a very poor life. Which allocation should be chosen? Depending on the "quality" of the life offered to the extra agent, it is probably possible to find readers who would favour the first allocation and some who would favour the second, the answer is a value judgement... In contrast, relying on the $\mathcal{A}$-efficiency would mean choosing the second allocation anytime, because the second allocation $\mathcal{A}$-dominates the first, however miserable the extra agent is. There is indeed a value judgement embedded inside the $\mathcal{A}$-efficiency.

Let me leave fertility matters to discuss briefly a subject also related to endogenous populations. Imagine a group of agents deciding on the number of *external* agents to accept in the group. Just like children, *external* agents are *potential* agents but they want to join the group so, if the group is indifferent, it is a social improvement to have external agents joining the group. So it makes no doubt that, if it makes everyone in the group better off, then an external agent should be accepted. In sharp contrast, even if it makes everyone better off, it is not necessarily an improvement to have an additional child.

Note that the group is basing its decisions to maximize their well fare without tkaing into account that external agents leaving their original groups could have detrimental effects..

3.2.2 $\mathcal{P}$-efficiency

The $\mathcal{P}$-efficiency was also introduced by Golosov, Jones, and Tertilt (2007). It is called $\mathcal{P}$-efficiency because, when two allocations are compared, utilities of all $\mathcal{P}$**otential** agents are compared. To be well defined, the $\mathcal{P}$-efficiency requires that, for any plan (f, c), any utility function u_i takes a value, even when agent i is not born under the fertility plan f.[2]
A feasible plan (f, c) is said to be $\mathcal{P}$-efficient if there is no other feasible plan $(\hat{f}, \hat{c})$ such that:

1. For all agents i (born or not) $\quad u_i(\hat{f}, \hat{c}) \geq u_i(f, c)$
2. There is an agent i (born or not) such that $\quad u_i(\hat{f}, \hat{c}) > u_i(f, c)$

The set of agents' preferences used by Golosov, Jones, and Tertilt (2007) is broader than mine because they allow the utility function u_i to take values for all plans (f, c) even though agent i is not born in some fertility plans f. Furthermore, the utility level of agent i may vary across plans in which agent i is not born i.e., among two plans in which i is not born, she might strictly prefer one over the other. This means that the $\mathcal{P}$-efficiency doesn't necessarily nest the Pareto criterion with exogenous population. Indeed, take two allocations with the same sets of born agents such that the first Pareto dominates the secon i.e. all born agents prefer the second over the first allocation. If an unborn agent strictly prefers the second then the first allocation doesn't $\mathcal{P}$-dominates

[2]In Subsection 2.2 $u_i(\cdot)$ is defined only when agent i is born. Here and only here, I assume that $u_i(\cdot)$ is defined over all plan (f, c) to introduce the $\mathcal{P}$-efficiency.

the second allocation. However, the Assumption 4 a) in Golosov, Jones, and Tertilt (2007) restricts the values taken by $u_i(\cdot)$ when agent i is not born to only one value denoted $\overline{u}_i$. With this Assumption 4 a), Pareto domination implies $\mathcal{P}$-domination. Furthermore, *ceteris paribus*, if one agent i is added (substracted) then it is an improvement (deterioration) only if $U_i \geq \overline{u}_i$. In the context of the $\mathcal{C}$-efficiency, it means that for any agent i, her critical level α_i is a constant equal to $\overline{u}_i$. This means that the $\mathcal{P}$-efficiency is a special case of the $\mathcal{C}$-efficiency and I have the following equivalence result:

Lemma 2. ($\mathcal{P}$-efficiency equivalence)
If $\alpha_i = \overline{u}_i$ for all agents i, then the $\mathcal{C}$-efficiency is equivalent to the $\mathcal{P}$-efficiency i.e. a plan (f, c) is $\mathcal{C}$-efficient if and only if it is $\mathcal{P}$-efficient.

See the proof in the Appendix B

Unlike $\mathcal{P}$-efficiency, $\mathcal{C}$-efficiency doesn't require the utility of an agent in states of the world where she is not born. However since $\mathcal{C}$-efficiency allows for endogenous Critical Levels $(\alpha_i(\cdot))$, it is possible that agent i prefers a plan (f, c) over a plan $(\tilde{f}, \tilde{c})$ (i.e. $\breve{u}_i(f^i, c^i) \geq \breve{u}_i(\tilde{f}^i, \tilde{c}^i)$) and agent i has a life worth living in $(\tilde{f}, \tilde{c})$ and not in (f, c) (i.e. $\breve{u}_i(f^i, c^i) < \alpha_i(f, c)$ and $\breve{u}_i(\tilde{f}^i, \tilde{c}^i) \geq \alpha_i(\tilde{f}, \tilde{c})$).

3.3 Dynastic efficiency or $\mathcal{D}$-efficiency

In this subsection, I introduce a new norm for Critical Levels which aims at relying only on the revealed preferences of the agents who actually decide who live and who don't: the parents. They are called *dynastic* Critical Levels and are denoted $(\alpha_i^D)_i$.

In my model, parents make fertility decisions to maximize their utility. I assumed altruistic preferences so their children's expected "lives qualities" play a role in their choices. I use this to build new Critical Levels $(\alpha_i^D)_i$. I say that the life of a child with utility level is worth living if her parent is happier with this additional child even though her being born brings in no financial benefit (nor raising costs). Formally:

Definition 3. (Dynastic Critical Levels $\{\alpha_j^{\mathcal{D}}\}_j$)
For any agent j born under f, the critical level $\alpha_j^D(f^{\setminus j}, c^{\setminus j})$ is defined by the utility level agent j must have in order to make her parent i indifferent between having her and getting utility level $u_i(\ c_i\ , \{u_k(f,c)\}_{k\in f_i\setminus\{j\}} \cup \{\alpha_j^D(f^{\setminus j}, c^{\setminus j})\})$ and not having her and getting utility level $u_i(\ c_i\ , \{u_k(f,c)\}_{k\in f_i\setminus\{j\}})$.
All other things, i.e. parent i's consumption level c_i and the other children's utility levels $\{u_k(f,c)\}_{k\in f_i\setminus\{j\}}$, are unchanged.
If her parent i always prefers not to have her, however high the utility level agent j gets,
then $\alpha_j^D(f^{\setminus j}, c^{\setminus j}) = +\infty$
If her parent i always prefers to have her, however low the utility level agent j gets,
then $\alpha_j^D(f^{\setminus j}, c^{\setminus j}) = -\infty$

Since $u_i(\cdot)$ is continuous, there are no possible other cases and so $\alpha_j^D(\cdot)$ is well defined. Note that $\alpha_j^D(\cdot)$ is independent of agent j's dynastic plan (f^j, c^j) so $\alpha_j^D(\cdot)$ is indeed a function of the *environmental* plan $(f^{\setminus j}, c^{\setminus j})$.

I simply define the $\mathcal{D}$-domination and the $\mathcal{D}$-efficiency as sub-cases of the $\mathcal{C}$-domination and the $\mathcal{C}$-efficiency when the Dynastic critical levels $(\alpha_i^D(\cdot))_i$ are used.

4 The economy

In the previous section, I have defined preferences of agents over plans and introduced the $\mathcal{C}$-efficiency and the $\mathcal{D}$-efficiency for *feasible* plans. In this section, I describe what these *feasible* plans are in a very simple economy, namely an endowment economy with a linear saving technology.

4.1 Technology

I assume that each adult i is endowed with a positive amount of synthetic good denoted e_i. Within each period, goods can be moved from one agent to another without any cost. Goods can be used three ways: they can be consumed, used to raise children or stored for the next generation at a constant rate of return r. I denote $Cost_j$ the cost of raising agent j [3]. Each agent j receives a bequest b_j from her parent. For simplicity, I assume that initial agents have not received any bequest from the previous generation

[3]Children have impacts on multiple dimensions of parents lives and it is not easy to synthesise all the costs in one number $Cost_j$ e.g. cost of time spent with children. What to include in this cost is left to Society to decide.

so that the total amount of goods available at time $t = 0$ is $\sum_{i \in I(0)} e_i$.

A plan (f, c) is said to be *feasible* when no agent has more than M children and the inter-temporal resource constraint of the whole economy is met i.e. the discounted sum of endowments must be larger than the discounted sum of consumption and raising cost.[4]

Formally, $0 \leq \sum_{i \in f(0)} e_i - c_i + \sum_{t>0} \sum_{j \in f(t)} \frac{e_j - c_j - Cost_j(1+r)}{(1+r)^t}$

The inter-temporal budget constraint of the dynasty of agent i alive at time $t(i)$ follows the same spirit: $0 \leq e_i - c_i + \sum_{s>0} \sum_{j \in f^i(t(i)+s)} \frac{e_j - c_j - Cost_j(1+r)}{(1+r)^t}$

To make the problem more tractable I assume that the discounted sum of ressources is never infinite:

Assumption 1. (Finite Discounted Sum of resources) For all feasible plans (f, c),

$$\sum_{t>0} \sum_{j \in f(t)} \frac{e_j}{(1+r)^t} < +\infty$$

4.2 Constraints on bequests

Agents maximize their utility under their budget constraints by choosing how much to consume, how many children they have and how much they bequest to each of their children. Agents are also subject to another type of constraints: the bequests they leave to their children cannot be too low i.e. for any child j there is a minimum bequest denoted $\underline{b}_j$. Typically, parents won't be allowed to leave too negative bequests. This is a way to protect descendants' property rights on their endowments. These constraints on bequests will be a policy tool.[5]

4.3 Equilibrium

I show in Appendix A that, thanks to the perfect altruism assumption, agents take actions as if initial agents could decide the whole dynastic plan. The first welfare theorem holds i.e., without constraints on bequests, the equilibrium allocation is efficient.

[4]I work in partial equilibrium so I can ignore each *per period* constraint. My results would hold with capital accumulation and endogenous interest rates as long as atomicity is assumed.

[5]More usual tools are taxes/subsidies on children and inter-generational transfers. There is a transfers/taxes/subsidies scheme that gives the exact same incentives as constraints on bequests. Using only the constraints on bequests allows us to avoid any redistribution question across dynasties.

Proposition 1. **(*Laissez-faire* is $\mathcal{C}$-efficient)**
For any preferences $\{u_i(\cdot)\}_i$, for any endowments $\{e_i\}_i$, for any critical levels $\{\alpha_i(\cdot)\}_i$,
If for all agents i there is no constraints on bequests i.e. $\underline{b}_i = -\infty$,
Then the equilibrium $(\dot{f}, \dot{c})$ allocation is $\mathcal{C}$-efficient.

The proof is postponed since it is a special case of the Proposition 2. To be able to build this more general result I need to introduce the ALAC criterion.

5 ALAC: All Lives are Above their Critical levels

In the usual setup with an exogenous population, an efficient allocation can be very unequal and give very poor lives to some agents. Agents facing a collective choice could want to rule out some allocations because they are not *equitable*, even though they are efficient. In this section I propose a very general equity criterion called ALAC which means "All Lives are Above their Critical levels".

5.1 Definition

A plan is ALAC when all agents have lives worth living. Formally,

Definition 4. **(All Lives Above their Critical levels: ALAC)**
A plan (f, c) is **ALAC** when all agents i born under the fertility plan f have lives worth living i.e. their utility levels $u_i(f, c)$ are not *strictly* below their critical levels $\alpha_i(f, c)$.

It is possible that a $\mathcal{C}$-efficient plan gives a life not worth living to an agent i i.e. her utility level is strictly bellow her Critical Level. If, for instance, one agent is born only in order to be exploited by her parent, in other words she has very low consumption level because all of her goods are taken by her parent who enjoys a high consumption level, then the corresponding plan (f, c) is not necessarily $\mathcal{C}$-inefficient because the sacrifice of the poor child is necessary to her parent's high consumption level. This is consistent with the usual Pareto efficiency with an exogenous population where some agents may be very poor as long as it benefits others.

An ALAC plan guarantees that any agent i gets at least a utility level equal to her critical level which restricts the scope for inequality. Depending on the $(\alpha_i(\cdot))_i$, there are many feasible ALAC plans, the ALAC crierion doesn't single out one plan. A necessary condition to have an ALAC dynastic plan is to have enough resources to be able to offer lives worth living to everyone in the dynasty. I explore this in the next subsection.

5.2 The minimum ALAC bequests

In this subsection, I define the minimum amount of resources that has to be left to an agent i so that she and all her descendants have enough resources to have lives worth living.

Definition 5. (Minimum ALAC bequest)
For any born agent i and for any *environmental* plan $(f^{\setminus i}, c^{\setminus i})$, the *minimum ALAC bequest* $b_i^m(f^{\setminus i}, c^{\setminus i})$ is the minimum amount of bequest agent i must receive so that she and all her descendants have enough resources to have lives worth living when the *environmental* plan is $(f^{\setminus i}, c^{\setminus i})$. Formally,

$$b_i^m(f^{\setminus i}, c^{\setminus i}) := \min_{(f^i, c^i) \in A_i(f^{\setminus i}, c^{\setminus i})} \{\, c_i - e_i + \sum_{s \in \mathbb{N}^*} \sum_{j \in f^i(t+s)} \frac{c_j + Cost_j(1+r) - e_j}{(1+r)^s} \,\}$$

Where $A_i(f^{\setminus i}, c^{\setminus i}) := \{\, (f^i, c^i) \mid \forall j \in f^i,\ \ u_j(f^j, c^j) \geq \alpha_j(f^{\setminus j}, c^{\setminus j}) \,\}$
I.e. $A_i(f^{\setminus i}, c^{\setminus i}) \ = \ \{\, \textit{All ALAC dynastic plans when the environmental plan is } (f^{\setminus i}, c^{\setminus i}) \,\}$

I show that the minimum is indeed reached in Appendix B.

So, if agent i receives more than $b_i^m(f^{\setminus i}, c^{\setminus i})$, then there exists a budget balance ALAC dynastic plan for agent i.

I will use *minimum ALAC bequests* in the next section on equilibria. They will be used to define a higher bound on the constraints on bequests which necessarily lead to a $\mathcal{C}$-efficient outcome. Before this let us quickly discuss the special case of the ALAC criterion used with Dynastic critical levels.

5.3 $\mathcal{D}$ynastic ALAC

The ALAC criterion is well defined for any family of critical levels $\{\alpha_i(\cdot)\}_i$. It has two interesting properties. First, the existence of an ALAC allocation is guaranteed because it is sufficient not to have any child. Second, interpersonal comparisons of utility levels are not used. In the particular case where the dynastic critical levels $\{\alpha_i^D(\cdot)\}_i$ are used, we get an equity criterion with a third interesting property: by construction the dynastic ALAC criterion is based only on revealed preferences. As far as I know there is no other equity criterion that has these three properties.

An extensively used definition for equity is envy-freeness: an allocation is envy-free where there is no envious agent i.e. an agent who would rather have what another agent has. Notice that this equity criterion is only based on preferences and no interpersonal comparisons of utility levels are used. There are many refinements of this definition (e.g. Varian (1974)) but in general envy-freeness is not well suited to a model in which several generations are involved. Indeed, with technical progress and natural resources, some periods offer conditions that no other periods offer and, if preferences are heterogeneous enough, there will always be a pair of agents who would like to switch their positions and they cannot switch if they live in different periods. So, envy-freeness cannot be achieved through market exchanges or government transfers. This is a reason why, when different generations are involved, most equity criteria rely on utilitarianism along with a discount factor (e.g. Stern (2007) on climate change) which means that interpersonal comparisons of utility levels are used.

6 Equilibria

In this section I look at equilibria and whether or not they are $\mathcal{C}$-efficient.

6.1 $\mathcal{C}$-efficient equilibria

In equilibrium, binding constraints on bequests have two effects. First, for a given number of children, parents will have to leave higher bequests than what they would optimally choose. This forced transfer is never inefficient because children are not altruistic towards their parents, so children don't mind these constraints. This effect exist with exogenous fertility as well. Second, parents will choose to have less children because children appear as a less interesting saving/investment device and as a more costly consumption good. This second effect is specific to setups with endogenous fertility and may lead to $\mathcal{C}$-inefficient equilibria. Indeed, imagine a potential child who has an endowment large enough to compensate her parent for having her, to have a life worth living and to bequest enough so that all her descendants have lives worth living. Not having this child born is clearly a $\mathcal{C}$-inefficiency but it may take place in equilibrium if the constraints on bequests are too high such that parents cannot be compensated by their children. Note that a necessary condition to create this type of $\mathcal{C}$-inefficiency, is that the constraint on bequests is higher than the *minimum ALAC bequest* defined in the previous section. So if constraints on bequests are not too high, no such $\mathcal{C}$-inefficiency occurs in equilibrium.

A second type of $\mathcal{C}$-inefficiency may occur if some dynasties face constraints on bequest which are growing generation after generation such that the dynasty is forced to over-accumulate i.e. a $\mathcal{C}$-improvement is possible where all agents receive less from their parents and give less to their children. So if constraints on bequests are not growing too fast, no such $\mathcal{C}$-inefficiency occurs in equilibrium.

Without external effect between dynasties, no other efficiency can occur and so we have the following Proposition:

Proposition 2. ($\mathcal{C}$-efficient constraints)
For any preferences $\{u_i(\cdot)\}_i$, for any endowments $\{e_i\}_i$, for any critical levels $\{\alpha_i(\cdot)\}_i$, If for all agents i $\underline{b}_i \leq \inf_{(f^{\setminus i}, c^{\setminus i})} [b_i^m(f^{\setminus i}, c^{\setminus i})]$ and if $\frac{M^t}{(1+r)^t} \max_{k \in I(t)} \{\underline{b}_k, 0\} \rightarrow 0$ as $t \rightarrow \infty$
Then the equilibrium allocation $(\dot{f}, \dot{c})$ is $\mathcal{C}$-efficient.

See the proof in the Appendix B

This result seems at odd with the idea that binding constraints of any kind result in inefficient allocations (when there is no external effect). In a very similar economy, Schoonbroodt and Tertilt (2014) finds that equilibria are $\mathcal{A}$-efficient or $\mathcal{P}$-efficient if and only constraints on transfers are not binding.
This comes from the difference in efficiency concepts. The $\mathcal{A}$-efficiency and the $\mathcal{P}$-efficiency that they use assume that *not being born* is never socially preferred over *being born* I.e. all lives are worth living. Binding constraints on bequest makes parents unhappy and leads to lower fertility so that agents with lives worth living are prevented from living. With the $\mathcal{C}$-efficiency, there are lives not worth living. If a binding constraint is higher than $\inf_{(f^{\setminus i}, c^{\setminus i})} [b_i^m(f^{\setminus i}, c^{\setminus i})]$ then it is possible that a sub-dynasty since some lives are not so that So Here children to the fact that children are themselves agents who are taken into account in the definition of the $\mathcal{C}$-efficiency.

The second condition is similar to a transversality condition and it would also appear with exogenous fertility decisions. But, as long as constraints on bequests are non positive, this problem cannot arise.

The *Laissez-faire* case, i.e. without any constraints on bequests, is a particular case of the Proposition 2 and also leads a $\mathcal{C}$-efficient equilibrium.

6.2 Constant critical levels $\{\alpha_j\}_j$

In the previous subsection I have shown that some equilibria were $\mathcal{C}$-efficient plans. But do ALAC equilbria exist ? In this subsection I assume that critical levels $\{\alpha_j\}_j$ are constant and so all *minimum ALAC bequests* b_i^m are also constant across *environmental* plans.

In an ALAC plan, it is necessary that all received bequests are higher than the *minimum ALAC bequest*, otherwise, by definition, some agent in the dynasty won't have a life worth living. So, one way to make sure that, for all agents i, the equilibirum bequest $\dot{b}_i$ is not below the *minimum ALAC bequest* b_i^m is to set the constraints on bequests $\underline{b}_i$ not lower than b_i^m . Furthermore, the constraints on bequests should not be strictly higher than the *minimum ALAC bequests*. Indeed, imagine a parent i who has received exactly b_i^m and who is facing a binding constraint $\underline{b}_j > b_j^m$ for her child j. Then agent i cannot have a life worth living. Indeed, assume she has a life worth living in equilibrium. Then, since the constraint is binding, she could be even better off by bequesting $\underline{b}_j - \varepsilon$ and, by definition of b_j^m, it would still be possible for the sub-dynasty of agent j to achieve an ALAC dynastic plan. And then agent i would have a utility level strictly above her Critical Level. So, by receiving a bit less than b_i^m and by consuming a bit less she would still have a life worth living and so would all her descendants. This is a contradiction with the definition of b_i^m. So setting $\underline{b}_i := b_i^m$ for all agents i avoids this problem. It even leads to an ALAC equilibrium as the next Proposition shows;

Proposition 3. (ALAC constraints)
For any preferences $\{u_i(\cdot)\}_i$, for any endowments $\{e_i\}_i$ and for any **constant** Critical Levels $\{\alpha_i\}_i$. If for all agents i $\underline{b}_i = b_i^m$, then the equilibrium plan $(\dot{f}, \dot{c})$ is ALAC.

See the proof in the Appendix B

Thus it is possible to protect enough the next generations' wealth through constraints on bequests so that they enjoy utility levels above their Critical Levels. But, quite surprisingly, if some agents have their wealth even more protected, then the equilibrium plan might not be ALAC. This is because some agents will receive the same bequests from their parents but will have to bequest more to their children so they will have lower utility levels which could be below their Critical Levels.

A very convenient feature of constant Critical Levels is that, using propositions 2 and 3, one can get a $\mathcal{C}$-efficient **and** ALAC equilibrium using the right constraints on bequests.

Corollary 1. **($\mathcal{C}$-efficient and ALAC constraint)**
For any preferences $\{u_i(\cdot)\}_i$, for any endowments $\{e_i\}_i$ and for any **constant** Critical Levels $\{\alpha_i\}_i$. If for all agents i $\underline{b}_i = b_i^m$ and if $\frac{M^t}{(1+r)^t} \max_{k \in I(t)} \underline{b}_k^+ \to 0$ as $t \to \infty$
Then the equilibrium plan $(\dot{f}, \dot{c})$ is $\mathcal{C}$-efficient and ALAC.

In the next subsection I discuss why the previous result doesn't hold when the Critical Levels $\{\alpha_i\}_i$ are endogenous.

6.3 Endogenous constraints on bequests

One novelty of this work is to allow for endogenous Critical Levels and this leads to endogenous *minimal ALAC bequests*. So, in order to meet the conditions of the Proposition 3 and the Corollary 1 i.e. $\forall i \quad \underline{b}_i = b_i^m$, constraints on bequests have to be endogenous as well.

Endogenous constraints on bequests could be implemented in reality. Indeed, there are examples in History where some inherited goods such as family estates or *privilèges* could not be sold and had to be passed on to the next generations. So, for a dynasty, being rich at some point in time might lead to more constraints on bequests in the future. At a broader level, more developed countries tend to protect more the property rights of children than less developed countries through the banishment of child labor or through mandatory schooling for instance. Here again, becoming rich implies that children property rights are more protected. In our setup, this would mean that the constraints on the bequest left to agent j would be a function of the *environmental* plan $(f^{\backslash j}, c^{\backslash j})$.

Unfortunately, when the constraints on bequests are functions of ancestors' choices, these ancestors might be tempted to deviate from efficiency in order to manipulate the constraints on bequests faced by their descendants. I discuss two examples.

Assume that the constraints on bequests faced by the descendants of agent i depend negatively on $Card\{f_i\}$, the number of children agent i has. Then agent i could decide to have an extra child j to relax the constraint on bequests faced by her descendants even though the child j doesn't bring any direct benefit to agent i. This happens when the utility of the forgone consumption equivalent to $(Cost_j + b_j)$ is not worth the extra utility brought by U_j to her parent i. If this child j and all her descendants have lives strictly not worth living, then agent i's decision to have this extra child j is $\mathcal{C}$-inefficient. Agent i has created a sub-dynasty of agents with lives not worth living just to make possible a transfer from her grand children to her children.

Conversely, if the constraints on bequests faced by the descendants of agent i depend positively on $Card\{f_i\}$, then agent i could decide to have less children even though it is $\mathcal{C}$-inefficient just to relax the constraint on bequests faced by her descendants.

If the constraints on bequests faced by the descendants of agent j depend positively on her consumption c_j, then agent i, j's parent, could decide not to bequest an extra ε to j because if $b_{k\in f_j}$ are constrained then the extra ε could not be totally consumed by j because $\{\underline{b}_k\}_{k\in f_j}$ would increase. So, even though $\frac{\partial u_i}{\partial c_i} < (1+r)\frac{\partial u_i}{\partial U_j}\frac{\partial u_j}{\partial c_j}$ i.e. a transfer from c_i to c_j would be a Pareto improvement, this transfer is not possible because of the increase of the binding constraints on bequests to the children of agent j. Note that this inefficiency would also exist with exogenous fertility.

These examples establish why, in general, it won't be possible to have endogenous binding constraints on bequests which lead to a $\mathcal{C}$-efficient equilibrium.

6.4 Properties of the $\mathcal{D}$-efficiency

In the previous subsection, I have discussed why endogenous constraints on bequests led to inefficiencies (if binding). Furthermore, in general there is no constraint on bequests that guarantees an ALAC equilibrium plan either. In other words, I cannot find the equivalent result of the Proposition 3 with endogenous critical levels. One trivial exception being when constraints are so high that they cannot be met. Then there will be no children, which is an ALAC outcome.

However, in the case of dynastic critical levels, it is possible to make sure that all lives are worth living. Remember that a child's utility level is above her dynastic critical level when her parent are better off having her with no impact on her budget. By making sure that the investment/saving motive is not playing in favour of the existence of a child, the existence of the child will be driven only by the *child as a consumption good* motive i.e. this child will necessarily have a life worth living. To implement this, it is sufficient to constraint bequest to be above what a child cost to raise. We have the following result.

Proposition 4. (ALAC constraints)
For any preferences $\{u_i(\cdot)\}_i$ and for any endowments $\{e_i\}_i$, if for all agents i $\underline{b}_i \geq -Cost_j$, then the equilibrium plan $(\dot{f}, \dot{c})$ is **Dynastic** ALAC.

See the proof in the Appendix B

This result is rather convenient because the constraints on bequests are built on observables and not on preferences of future agents. With constant Critical Levels, I was able to build constraints on bequests which lead to an ALAC equilibrium but these constraints are equal to the minimal ALAC bequests $\{b_i^m\}$ which are built using future preferences and not on observables (see Proposition 2).

Unfortunately, I am not able to derive binding constraints on bequests built on observables such that the equilibrium plan is $\mathcal{D}$-efficient.

7 Public policies

7.1 Population control

Population control is an important topic for many reasons. First, a large population has long been a source of military and economic power for a country. Second, a growing population is a drag because of the raising and education costs and it dilutes the physical and natural capitals but it also dilutes the efforts of young agents needed to support the elderly and pay back any public debt (See Samuelson (1975) and Conde-Ruiz, Giménez, and Pérez-Nievas (2010)). Third, scarce natural ressources and climate change are major issues and the larger the world population, the larger these problems (See Harford (1998)).

Two tools

Several tools might be implemented to influence parents fertility decisions. One tool involves modifying the private cost and benefits of bearing children, the examples are numerous: free education, cash subsidies, housing subsidies or less taxes for large households, limiting child labour, non negative estate (See McDonald (2002) for a list). Another type of tools directly limits the number of children of each household (e.g. One Child Policy in China).

A plan is $\mathcal{D}$-efficient *conditional* on generations' sizes being equal to N or simply *conditional* $\mathcal{D}$-efficient when there is no other plan which meets the generations' sizes restrictions and which is a *strict* $\mathcal{D}$-improvement.

$\mathcal{D}$-efficient population control

A government wants to fix the population size of each generation at N. Let us assume that, without any government intervention, the population would be greater than N. Thus, the government has to refrain his people from having children (as in Bohn and Stuart (2015)). In my setup, there are three policy tools: taxes on children, constraints on bequests and constraints on number of children. What are the combination of these three tools that induce an *conditional* $\mathcal{D}$-efficient equilibrium plan?

Notice that if all agents have utility levels *strictly* above their critical levels $\alpha_i^D(\cdot)$ and if there is no mutually beneficial bequest then this plan is *conditional* $\mathcal{D}$-efficient. So, if parents cannont make profit out of their children (i.e. for all agents i $\underline{b}_i > -cost_i$) and if a *generation dependent* tax on children is implemented to meet the generations' sizes requirements, then the resulting allocation would be *conditional* $\mathcal{D}$-efficient.

More surprisingly, if for all agents i, $\underline{b}_i > -cost_i$, a cap on the number of children is implemented and a *generation dependent* tax on children is implemented to meet the generations' sizes requirements, then the resulting allocation would be *conditional $\mathcal{D}$-efficient*. This is at odd with the idea that a (shadow) price for a scarce resource (here breeding rights at each period) must necessary be constant across potential buyers (here parents of the same generations) to get an efficient allocation.

7.2 Externalities

One focus of the debates on fertility monitoring and on the limits of global population is the importance of limited natural resources and pollution. For instance, our Earth has a limited yearly supply of fresh water. If humans are sufficiently numerous they will face a binding water constraint and any extra human will make this constraint even more binding. If humans are even more numerous their lives will be a constant struggle for water. Any extra human with claims to some water will make some others worst off as far as water supply is concerned.

Limited renewable resources

In this subsection, I will assume that each agents is endowed with a strictly personal endowment e_i^p and a fraction of a common endowment E renewed at each period and evenly shared among all agents of each generation i.e.

$$\forall f \;\; \forall t \;\; \forall i \in f(t) \;\; e_i := e_i^p + \frac{E}{Card\{f(t)\}}$$

So, an extra child deprives each other born agent of her generation of approximately $\frac{E}{Card\{f(t)\}^2}$. From the point of view of a parent, an extra child j might be a good private decision because she would weight $Cost_j$ against $e_j^p + \frac{E}{Card\{f(t(j))\}}$ whereas a social planner would balance $Cost_j$ and e_j^p since $\frac{E}{Card\{f(t(j))\}}$ is not a net gain for society.

Pigouvian Tax

Facing the previous situation, a quite natural policy would be to implement a Pigouvian tax. One could tax children endowments in order to internalize the negative effects they have on the others i.e. leave each child j with their net contribution to society wealth i.e. their strictly personal endowment e_j^p. This could be considered as an abuse of the name "Pigouvian" because the parent who is taking the fertility decision is not the one taxed and internalizes the effect of the tax only to the extent she is altruistic. If parents were not altruistic, and if the constraints on bequests were binding then the tax would not have any effect on parents' decisions even though this tax makes sure that a child is born if and only her social costs are larger than her social benefits. This echoes the tax on adults implemented by Conde-Ruiz, Giménez, and Pérez-Nievas (2010) to offset the cost of the education freely provided when these adults were children.

I will denote T_j^E the lump-sum Pigouvian tax borne by child j. Note that all agents of the same generation $t(j)$ are taxed similarly and that the taxes raised that way would always sum up to the common pool of resource E available at each period. So the total discounted tax revenues sum up to $\frac{E}{r}$ and can be distributed lump-sum to the initial agents. I then have the following result:

Proposition 5. (Pigouvian Taxes)
For any preferences $\{u_i(\cdot)\}_i$, for any endowments $\{e_i\}_i$ and for any critical levels $\{\alpha_i(\cdot)\}_i$, if for all agents i $\underline{b}_i \leq \inf_{(f^{\setminus i}, c^{\setminus i})} [b_i^m(f^{\setminus i}, c^{\setminus i})]$, if $\frac{M^t}{(1+r)^t} \max_{k \in I(t)} \underline{b}_k^+ \to 0$ as $t \to \infty$ and if there is a tax schedule (T_t) of taxes on children born in period t such that $T_t = \frac{E}{Card\{\dot{f}(t)\}}$, then the equilibrium plan $(\dot{f}, \dot{c})$ is $\mathcal{C}$-efficient.

See the proof in the Appendix B

A more usual Pigouvian tax would be a child tax $T_t' := \frac{E}{Card\{\dot{f}(t+1)\}(1+r)}$ born by parents of period t. It would also yield a $\mathcal{C}$-efficient equilibrium plan but not necessarily the same pla.

My specification of a child externality is very special for two reasons. First, children of a given generation have all the same external effects. Second, any child has the same effects (in terms of consumption loss) on all the other children in the same generation. In reality, some children have a bigger negative externality than others (typically depending on their consumption) and this would call for a differentiated Pigouvian tax.

In contrast with Harford (1998) and Bohn and Stuart (2015) who assumed that each born agent had a claim on the government rent associated with the tax, here it is not necessary to take into account the externalities of grand-children, grand-grand-children etc when taxing children. In my setup, the government rent is fully given to the initial agents. So there is no need for current parents to be taxed to internalize the future claims of their descendants on the government rent. Both views are difficult to implement: my view needs well defined property rights for the common resource and their view needs to be able to compute the external effects of all the dynasties which could have different preferences.

8 Conclusion

In this work, I have introduced a general extension of Pareto efficiency when fertility decisions are endogenous called $\mathcal{C}$-efficiency. The first welfare theorem holds. I have also introduced the ALAC (All Lives Above Critical levels) equity criterion.

I have shown that constraints on bequests are inefficient but only if too high. Externalities from childbearing (e.g. common wealth dilution, pollution, fiscal externality) may also be corrected through the use of Pigouvian taxes if the revenue is given to the initial generation. This last condition is an argument in favor of grandfathering pollution permits so that a growing country will be somehow punished compared to a country with constant population. I have derived these results with very simple external effects where all children belonging to the same generation have exactly the same effects. For instance, all potential individuals are far from being equal polluters. I have derived another interesting result: under mild conditions, direct population control (e.g. China's one-child policy) is not inefficient.

Throughout the paper I have assumed perfect altruism, relaxing this would lead to inefficiencies as shown by Conde-Ruiz, Giménez, and Pérez-Nievas (2014). I have also assumed a totally risk-free environment where offspring's tastes and endowments are known from the beginning of time.

The identification scheme with dynastic trees used in this work (directly inspired by Golosov, Jones, and Tertilt (2007)) is quite narrow but still, one could argue that an individual raised in two different environments isn't actually the same individual and should be identified differently in these two environments. Maybe future works could, just as I did here, rely on parents' points of view to decide whether or not this individual should be identically identified across environments.

I use endogenous Critical Levels throughout the paper. Any society has to choose values for them and I have proposed the Dynastic Critical Levels which pin them down using parents' preferences. However, once this question has been settled, one is still far from having the necessary tools to choose an optimal fertility policy. For instance, when choosing among different outcomes implementable with a tax schedule, all one needs are generalized Pareto weights (see Saez and Stantcheva (2016)) i.e. the relative values society gives to the same amount attributed to different individuals. Here, in contrast, society has to be able to attribute relative values to different individuals' lives and to same amount attributed to these individuals. A Critical Level only provides a sign to the value of a life. It could be interesting to study the implicit values used in actual policies.

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A Equilibrium Proof

A.1 Agents behaviours

Agents choose how much they bequest to their children but they don't choose how their children are going to use their resources. I assume that any agent i have a strategy σ_i that maps how much she receives from her parent to the decisions she makes.[6] Formally $\sigma_i : b_i \mapsto (c_i, f_i, \{b_j\}_{j\in f_i})$. The collection of all the agents' strategies is called a strategy profile and his denoted σ. I will call equilibrium strategy profiles the strategy profiles which are Sub Game Perfect.

Since any agent i knows the strategies of her descendants she knows what her descendants are going to do for any bequest she makes and so agent i is able to compute the utility levels of all her descendants. The utility levels of her children under the strategy profile σ will be denoted $\{U_j^\sigma(\cdot)\}_{j\in f_i}$

Knowing the strategy profile σ, the maximizing agent i who has received a bequest b_i solves the following problem denoted $P_i^\sigma(b_i)$:

$$\sup_{c_i, f_i, \{b_j\}_{j\in f_i}, \{U_j\}_{j\in f_i}} \quad u_i(c_i, \{U_j\}_{j\in f_i})$$

$$subject\ to \quad U_j = U_j^\sigma(b_j) \quad for\ all\ born\ children\ j \in f_i$$

$$to \quad (1+r)b_i + e_i \ \geq \ c_i + \sum_{j\in f_i}[b_j + Cost_j]$$

$$and\ to \quad b_j \geq \underline{b}_j \quad for\ all\ born\ children\ j \in f_i$$

I want to characterize a strategy profile σ that leads to a Sub Game Perfect Equilibrium.

A.2 Relaxed problem $P_i^\star$

Consider the relaxed problem where an agent i maximizes her utility and is able to choose the full dynastic plan (c^i, f^i) subject to all the budget constraints and to all the constraints on bequests of her dynasty. This problem is denoted $(P_i^\star)(b_i)$:

[6] Strategies could take more inputs but in Sub Game Perfect Equilibria only bequests make sense because only they influence the choice sets faced by children

$$U_i^\star(b_i) := \sup_{c^i, f^i, b^i, U^i} U_i$$

$$subject\ to \quad U_k = u_k(c_k, \{U_j\}_{j \in f_k}) \ \ for\ i\ and\ all\ her\ descendants\ born\ in\ f^i$$

$$to \quad (1+r)b_k + e_k \ \geq \ c_k + \sum_{j \in f_k} [b_j + Cost_j] \ \ for\ i\ and\ all\ her\ born\ descendants\ born\ in\ f^i$$

$$and\ to \quad b_k \geq \underline{b}_k \ \ for\ all\ descendants\ of\ i$$

Lemma 3. (Sup is Max)
For any agent i and for any bequest b_i, the sup of $P_i^\star(b_i)$ is reached.

See the proof in the Appendix B

Since the supremum of the problem $P_i^\star(b_i)$ is always reached, I can define the following policy functions $\sigma_i^\star \ : \ b_i \mapsto (f_i^\star(b_i), c_i^\star(b_i), \{b_j^\star(b_i)\}_{j \in f_i^\star(b_i)})$ that correspond to a maximum (truncated) solution of the problem $P_i^\star(b_i)$.

When there are several dynastic plans that are solutions of $(P_i^\star)(b_i)$, I will assume the following Tie Breaking Rule: if a maximum solution is $\mathcal{C}$-dominated by another maximum solution then it is never chosen.

I denote $\sigma^\star$ the strategy profile where all agents i follow their strategies $\sigma_i^\star$.

A.3 Sub-Game Perfect Equilibrium

In most economic settings, utility functions have a finite number of arguments so that their continuity is unambiguously defined. Here dynastic plans such as (f^i, c^i) contain an infinite number of arguments (unless the dynasty stops at some point in time) so I need some structure to handle the infinite time horizon and the infinite number of inputs that goes with it. Henceforth, I will assume that agents don't care much about the distant future as long as it is distant enough. Formally I have:

Assumption 2. (Distant future is negligible)
For any agent i, for any dynastic plan (f^i, c^i) and for any $\varepsilon > 0$, there exists a period T such that any T-deviation $(\tilde{f}^i, \tilde{c}^i)$ of (f^i, c^i) is such that $\mid u_i(\tilde{f}^i, \tilde{c}^i) - u_i(f^i, c^i) \mid \ < \varepsilon$.
Where a T-deviation of (f^i, c^i) is a dynastic plan $(\tilde{f}^i, \tilde{c}^i)$ such that all the decisions of agents born before period T are identical in $(\tilde{f}^i, \tilde{c}^i)$ and in (f^i, c^i)

Thanks to this assumption, the strategy profile $\sigma^\star$ is the equilibrium outcome of this dynastic game, as the following results show.

Lemma 4. (Equilibrium Results)
The strategy profile $\sigma^\star$ is Sub-Game Perfect and focal.
Furthermore, if the strategy profile $\sigma^\star$ is chosen by all agents,
Then any agent j who receives b_j has a utility level equal to $U_j^\star(b_j)$

See the proof in the Appendix

B Other Proofs

Proof of Lemma 1 ($\mathcal{A}$-efficiency equivalence)

Back to main text

Assume that (f,c) is not $\mathcal{C}$-efficient.
This is equivalent to say that there is a plan $(\hat{f},\hat{c})$ such that all the following inequalities hold and one is a strict inequality:

1. Agents i born under plan f **and** plan $\hat{f}$ are better of with $(\hat{f},\hat{c})$
   I.e. $\forall i \in f \cap \hat{f} \quad u_i(c_i, \{U_j\}_{j\in f_i}) \ \leq \ u_i(\hat{c}_i, \{\hat{U}_j\}_{j\in i,\hat{f}_i})$
2. Agents i born **only** under plan $\hat{f}$ have lives worth living in $(\hat{f},\hat{c})$
   I.e. $\forall i \in \hat{f}\backslash f \quad u_i(\hat{c}_i, \{\hat{U}_j\}_{j\in \hat{f}_i}) \ \geq \ \alpha_i(\hat{f},\hat{c})$
3. Agents i born **only** under plan f don't have lives *strictly* worth living in (f,c)
   I.e. $\forall i \in f\backslash \hat{f} \quad u_i(c_i, \{U_j\}_{j\in f_i}) \ \leq \ \alpha_i(f,c)$

Since for all agents i and for all plans (f,c), $\alpha_i(f,c) = u_i(f,c)$, this implies that that 2) and 3) are always equalities so it is equivalent to say there is a plan $(\hat{f},\hat{c})$ such that all the following inequalities hold and one is a strict inequality:

1. Agents i born under plan f **and** plan $\hat{f}$ are better of with $(\hat{f},\hat{c})$
   I.e. $\forall i \in f \cap \hat{f} \quad u_i(c_i, \{U_j\}_{j\in f_i}) \ \leq \ u_i(\hat{c}_i, \{\hat{U}_j\}_{j\in i,\hat{f}_i})$

This is equivalent to say that that (f,c) is not $\mathcal{A}$-efficient.

■

Proof of Lemma 2 ($\mathcal{P}$-efficiency equivalence)

Back to main text

Assume that (f, c) is not $\mathcal{C}$-efficient.
This is equivalent to say that there is a plan $(\hat{f}, \hat{c})$ such that all the following inequalities hold and one is a strict inequality:

1. Agents i born under plan f **and** plan $\hat{f}$ are better of with $(\hat{f}, \hat{c})$
   I.e. $\forall i \in f \cap \hat{f} \quad u_i(c_i, \{U_j\}_{j \in f_i}) \ \leq \ u_i(\hat{c}_i, \{\hat{U}_j\}_{j \in i, \hat{f}_i})$
2. Agents i born **only** under plan $\hat{f}$ have lives worth living in $(\hat{f}, \hat{c})$
   I.e. $\forall i \in \hat{f} \backslash f \quad u_i(\hat{c}_i, \{\hat{U}_j\}_{j \in \hat{f}_i}) \ \geq \ \alpha_i(\hat{f}, \hat{c})$
3. Agents i born **only** under plan f don't have lives *strictly* worth living in (f, c)
   I.e. $\forall i \in f \backslash \hat{f} \quad u_i(c_i, \{U_j\}_{j \in f_i}) \ \leq \ \alpha_i(f, c)$

Since for all agents i and for all plans (f, c), $\alpha_i(f, c) = \overline{u_i}$ and since $\overline{u_i}$ is the utility that agent i gets when not born it is equivalent to say there is a plan $(\hat{f}, \hat{c})$ such that all the following inequalities hold and one is a strict inequality:

1. Agents i born under plan f **and** plan $\hat{f}$ are better of with $(\hat{f}, \hat{c})$
   I.e. $\forall i \in f \cap \hat{f} \quad u_i(c_i, \{U_j\}_{j \in f_i}) \ \leq \ u_i(\hat{c}_i, \{\hat{U}_j\}_{j \in i, \hat{f}_i})$
2. Agents i born **only** under plan $\hat{f}$ have lives worth living in $(\hat{f}, \hat{c})$
   I.e. $\forall i \in \hat{f} \backslash f \quad u_i(\hat{c}_i, \{\hat{U}_j\}_{j \in \hat{f}_i}) \ \geq \ \overline{u_i} = u_i(f, c)$
3. Agents i born **only** under plan f don't have lives *strictly* worth living in (f, c)
   I.e. $\forall i \in f \backslash \hat{f} \quad u_i(c_i, \{U_j\}_{j \in f_i}) \ \leq \ \overline{u_i} = u_i(\hat{f}, \hat{c})$

This is equivalent to say that that (f, c) is not $\mathcal{P}$-efficient.

■

Proof of Lemma 3 (Sup is Max)

Back to main text

Take $(c^i_n, f^i_n, b^i_n, U^i_n)_n$ a sequence of elements of the choice set of $(P^\star_i)(b_i)$ such that $U_{i,n} \to U_i(b_i)$ as $n \to \infty$. The choice set of $(P^\star_i)(b_i)$ is a subset of a countable Cartesian product of bounded closed sets so it is possible to build a sub-sequence that is point-wise converging. By continuity of all the utility functions $(u_k)_k$ the limit of this sub-sequence is in the choice set of $(P^\star_i)(b_i)$ and maximizes U_i i.e. the sup of of $P^\star_i(b_i)$

is reached.

■

Proof of Lemma 4 (Game Theory Results)

Back to main text

1) Let us denote $(\tilde{f}, \tilde{c})$ the plan when all agents follow the strategy profile $\sigma^\star$.
I assume that all agents follow $\sigma^\star$ and that there is an agent i who receives $\tilde{b}_i$ and gets utility level different from $U_i^\star(\tilde{b}_i)$.

Since all strategies of $\sigma^\star$ respect the constraints (on resources and bequests), the dynastic plan $(\tilde{f}^i, \tilde{c}^i)$ belongs to the choice set of $P_i^\star(\tilde{b}_i)$, and since $u_i(\tilde{f}^i, \tilde{c}^i) \neq U_i^\star(b_i)$, I have $u_i(\tilde{f}^i, \tilde{c}^i) < U_i^\star(b_i)$ i.e. $(\tilde{f}^i, \tilde{c}^i)$ is not a maximum solution of $P_i^\star(\tilde{b}_i)$.

I use a recurrence argument to establish that, for any period T, there exists a T-deviation of $(\tilde{f}^i, \tilde{c}^i)$ which is a maximum solution of $P_i^\star(\tilde{b}_i)$.:

$\underline{T=1}$: Since agent i actions $(f_i^\star(\tilde{b}_i), c_i^\star(\tilde{b}_i), \{b_j^\star(\tilde{b}_i)\}_{j \in f_i^\star(\tilde{b}_i)})$ correspond to a maximum solution of $P_i^\star(\tilde{b}_i)$, we can replace all the dynastic plans of her descendants by this maximum solution of $P_i^\star(\tilde{b}_i)$ and get a solution of $P_i^\star(\tilde{b}_i)$.
So there is a 1-deviation of $(\tilde{f}^i, \tilde{c}^i)$ that is solution $P_i^\star(\tilde{b}_i)$.

$\underline{T \longrightarrow T+1}$: Let us assume that there is a T-deviation of $(\tilde{f}^i, \tilde{c}^i)$ that is solution $P_i^\star(\tilde{b}_i)$ and let us show that there is a (T+1)-deviation of $(\tilde{f}^i, \tilde{c}^i)$ that is solution $P_i^\star(\tilde{b}_i)$.

We have assumed that there existed $(\hat{f}^i, \hat{c}^i)$, a T-deviation of $(\tilde{f}^i, \tilde{c}^i)$ that is solution $P_i^\star(\tilde{b}_i)$.

For all $j \in \tilde{f}^i(T)$, the corresponding dynastic plans $(\hat{f}^j, \hat{c}^j)$ are solutions of $P_j^\star(\tilde{b}_j)$.
If not, replacing $(\hat{f}^j, \hat{c}^j)$ by solutions of $P_j^\star(b_j)$,would bring a strictly higher utility to agent j. All her ancestors, including i would get a higher utility level.
This would be a contradiction with the optimality of $(\hat{f}^j, \hat{c}^j)$.

Furthermore, the dynastic plans $(\hat{f}^j, \hat{c}^j)$ can be replaced by any solutions of $P_j^\star(\tilde{b}_j)$, the dynastic plan of i would still bring her $U_i^\star(\tilde{b}_i)$.

We have shown that any T-deviation of $(\tilde{f}^i, \tilde{c}^i)$ where, for all agents $j \in \tilde{f}^i(T)$, the dynastic plans $(\tilde{f}^j, \tilde{c}^j)$ are replaced by any solutions of $P_j^\star(\tilde{b}_j)$ is a solution of $P_i^\star(\tilde{b}_i)$.

Since all agents $j \in \tilde{f}^i(T)$ follow $\sigma^\star$, their actions are already made according to the solutions of $P_j^\star(\tilde{b}_j)$. This means that is not necessary to change the actions of agent of period T to get a solution of $P_i^\star(\tilde{b}_i)$. In other words there exists a (T+1)-deviation of $(\tilde{f}^i, \tilde{c}^i)$ that is solution $P_i^\star(\tilde{b}_i)$.

We have reasoned by recurrence to show that for any period T, there exists a T-deviation of $(\tilde{f}^i, \tilde{c}^i)$ which is a solution of $P_i^\star(\tilde{b}_i)$.

Now take $(\breve{f}^i, \breve{c}^i)$ a solution of $P_i^\star(\tilde{b}_i)$. We have that $u_i(\tilde{f}^i, \tilde{c}^i) < U_i^\star(\tilde{b}_i) = u_i(\breve{f}^i, \breve{c}^i)$. By the Assumption 2 used with $\varepsilon := \frac{U_i^\star(\tilde{b}_i) - u_i(\tilde{f}^i, \tilde{c}^i)}{2}$, there exists a time T such that any T-deviation $(\hat{f}^i, \hat{c}^i)$ of $(\tilde{f}^i, \tilde{c}^i)$ is such that $u_i(\hat{f}^i, \hat{c}^i) < u_i(\breve{f}^i, \breve{c}^i)$.
This is a CONTRADICTION.

2) Assume that the strategy profile $\sigma^\star$ is not Sub-Game Perfect.
Then there exists an agent $j \in f^i$ and a bequest b_j such that j gets strictly higher utility if she deviates unilaterally from $\sigma_j^\star$ while respecting her constraints (resources and bequests). This means that the following actions of her descendants don't respect the constraints because otherwise the corresponding dynastic plan would be in the choice set of $P^\star(b_j)$ and would bring a utility level strictly higher than $U^\star(b_j)$. But $(\sigma_k^\star)$, the strategies of the descendants, respect the constraints on bequests.
This is a CONTRADICTION.

3) No agent i can benefit from choosing another equilibrium because she cannot receive more than $U_i(b_i)$ so $\sigma^\star$ will always be weakly preferred by all agents. Hence $\sigma^\star$ is focal.

■

Proof that the Minimum ALAC bequest is reached

Back to main text

Take $(f_n^i, c_n^i)_n$ a sequence of elements of $A_i(f^{\setminus i}, c^{\setminus i})$ such that:
$[c_{i,n} - e_i + \sum_{s \in \mathbb{N}^*} \sum_{j \in f_n^i(t+s)} \frac{c_{j,n} + Cost_j(1+r) - e_j}{(1+r)^s}]_n$ is non increasing
and $\quad c_{i,n} - e_i + \sum_{s \in \mathbb{N}^*} \sum_{j \in f_n^i(t+s)} \frac{c_{j,n} + Cost_j(1+r) - e_j}{(1+r)^s} \to b_i^m(f^{\setminus i}, c^{\setminus i})$ as $n \to \infty$.

$A_i(f^{\setminus i}, c^{\setminus i})$ is a subset of a countable Cartesian product of bounded closed sets (Indeed for n large enough , consumption at each time $t+s$ is bounded by $(1+r)^s[b_i^m(f^{\setminus i}, c^{\setminus i}) + \varepsilon + [\max\limits_{0\leq s'\leq s} \max\limits_{i'\in I(t+s')} e_{i'}]\sum\limits_{l\geq 0}\frac{M^l}{(1+r)^l}]$) so it is possible to build a sub-sequence that is point-wise converging. Without loss of generality we assume that $(f_n^i, c_n^i)_n$ is point-wise converging towards $(\tilde{f}^i, \tilde{c}^i)$.

By continuity of the $\alpha_i(\cdot)$ and the fact that they have a finite number of inputs, we have that $(\tilde{f}^i, \tilde{c}^i) \in A_i(f^{\setminus i}, c^{\setminus i})$.

Using Levi-Lebesgue Monotonous Convergence Theorem, we have:

$$\lim_{n\to\infty} c_{i,n} - e_i + \sum_{s\in\mathbb{N}^*}\sum_{j\in f_n^i(t+s)} \frac{c_{j,n}+Cost_j(1+r)-e_j}{(1+r)^s} = \tilde{c}_i - e_i + \sum_{s\in\mathbb{N}^*}\sum_{j\in\tilde{f}^i(t+s)} \frac{\tilde{c}_j+Cost_j(1+r)-e_j}{(1+r)^s}$$

I.e. $\tilde{c}_i - e_i + \sum\limits_{s\in\mathbb{N}^*}\sum\limits_{j\in\tilde{f}^i(t+s)} \frac{\tilde{c}_j+Cost_j(1+r)-e_j}{(1+r)^s} = b_i^m(f^{\setminus i}, c^{\setminus i})$

■

Proof of Proposition 2 ($\mathcal{C}$-efficient constraints)

Back to main text

For an initial agent i, let us show that the dynastic plan $(\dot{f}^i, \dot{c}^i)$ is $\mathcal{C}$-efficient
Let us assume the existence of a dynastic plan $(\tilde{f}^i, \tilde{c}^i)$ which is a *strict* $\mathcal{C}$-improvement of $(\dot{f}^i, \dot{c}^i)$

I want to show the existence of an agent $j \in \tilde{f}^i$ and a consumption plan $\hat{c}^j$ such that $(\tilde{f}^j, \hat{c}^j)$ is in the choice set of $P_j^\star(\dot{b}_j)$ in equilibrium and such that:
$u_j(\tilde{f}^j, \hat{c}^j) > u_j(\tilde{f}^j, \tilde{c}^j) \geq u_j(\dot{f}^i, \dot{c}^i)$

For $\vec{\lambda} = (\lambda_j)_{j\in\tilde{f}^i}$, I define the dynastic consumption plan $\hat{c}^i(\vec{\lambda})$ the following way:
For any $j \in \tilde{f}^i$, and for any $\lambda_j \in [0,1]$,
I define $\hat{c}_j(\lambda_j) := \tilde{c}_j + \lambda_j[\frac{(1+r)^{t(j)+1}}{M^{t(j)}}(S_{t(j)} - \frac{Card\{\tilde{f}_j\}}{M}S_{t(j)+1}) + e_j]$
With $S_t := \sup\limits_{s\geq t}[\frac{M^s}{(1+r)^s}\max\limits_{k\in I(s)} b_k^+]$
We have that $(S_t)_t$ is non increasing and converges to 0.

I then define the dynastic bequest plan $\hat{b}^i(\vec{\lambda})$ and, for all $j \in \tilde{f}^i$, the bequests $\hat{b}_j(\vec{\lambda})$ that correspond to the dynastic consumption plan $\hat{c}^i(\vec{\lambda})$:

$$\vec{\lambda} \mapsto \hat{b}_j(\vec{\lambda}) := \frac{\hat{c}_j(\lambda_j) - e_j + \sum\limits_{s>0}\sum\limits_{k\in\tilde{f}^j(t(j)+s)} \frac{\hat{c}_k(\lambda_k)-e_k+Cost_k(1+r)}{(1+r)^s}}{1+r}$$

First, the plan $(\tilde{f}^i, \hat{c}^i(\vec{1}))$ respects all the constraints on bequests.
Indeed, for any $j \in \tilde{f}^i$

$$(1+r)\hat{b}_j(\vec{1}) := \hat{c}_j(1) - e_j + \sum_{s>0} \sum_{k \in \tilde{f}^j(t(j)+s)} \frac{\hat{c}_k(1) - e_k + Cost_k(1+r)}{(1+r)^s}$$

$$\geq \tilde{c}_j + \frac{(1+r)^{t(j)+1}}{M^{t(j)}}(S_{t(j)} - \frac{Card\{\tilde{f}_j\}}{M} S_{t(j)+1}) + e_j - e_j + \sum_{s>0} \sum_{k \in \tilde{f}^j(t(j)+s)} \frac{\tilde{c}_k + \frac{(1+r)^{t(j)+1}}{M^{t(k)}}(S_{t(j)} - \frac{Card\{\tilde{f}_k\}}{M} S_{t(k)+1}) + e_k - e_k + Cost_k(1}{(1+r)^s}$$

$$\geq \frac{(1+r)^{t(j)+1}}{M^{t(j)}}(S_{t(j)} - \frac{Card\{\tilde{f}_j\}}{M} S_{t(j)+1}) + \sum_{s>0} \sum_{k \in \tilde{f}^j(t(j)+s)} \frac{\frac{(1+r)^{t(k)+1}}{M^{t(k)}}(S_{t(k)} - \frac{Card\{\tilde{f}_k\}}{M} S_{t(k)+1})}{(1+r)^s}$$

So $\hat{b}_j(\vec{1}) \geq \frac{(1+r)^{t(j)}}{M^{t(j)}}(S_{t(j)} - \frac{Card\{\tilde{f}_j\}}{M} S_{t(j)+1}) + \sum_{s>0} \sum_{k \in \tilde{f}^j(t(j)+s)} \frac{\frac{(1+r)^{t(j)+s}}{M^{t(j)+s}}(S_{t(j)+s} - \frac{Card\{\tilde{f}_k\}}{M} S_{t(j)+s+1})}{(1+r)^s}$

$$\geq \frac{(1+r)^{t(j)}}{M^{t(j)}} [\sum_{s \geq 0} \sum_{k \in \tilde{f}^j(t(j)+s)} \frac{1}{M^s}(S_{t(j)+s} - \frac{Card\{\tilde{f}_k\}}{M} S_{t(j)+s+1})]$$

$$\geq \frac{(1+r)^{t(j)}}{M^{t(j)}} \lim_{S \to \infty} [\sum_{S \geq s \geq 0} \sum_{k \in \tilde{f}^j(t(j)+s)} \frac{1}{M^s}(S_{t(j)+s} - \frac{Card\{\tilde{f}_k\}}{M} S_{t(j)+s+1})]$$

$$\geq \frac{(1+r)^{t(j)}}{M^{t(j)}} \lim_{S \to \infty} [\sum_{S \geq s \geq 0} \frac{Card\{\tilde{f}^j(t(j)+s)\}}{M^s} S_{t(j)+s} - \sum_{S \geq s \geq 0} \frac{Card\{\tilde{f}^j(t(j)+s+1)\}}{M^{s+1}} S_{t(j)+s+1}]$$

$$\geq \frac{(1+r)^{t(j)}}{M^{t(j)}} \lim_{S \to \infty} [\frac{Card\{\tilde{f}^j(t(j))\}}{M^0} S_{t(j)} - \frac{Card\{\tilde{f}^j(t(j)+S+1)\}}{M^0} S_{t(j)+S+1}]$$

$$\geq \frac{(1+r)^{t(j)}}{M^{t(j)}} S_{t(j)} \geq \underline{b}_j^+ \geq \underline{b}_j$$

We have used the fact that $0 \leq \frac{Card\{\tilde{f}^j(t(j)+s)\}}{M^s} S_{t(j)+s} \leq S_{t(j)+s} \to 0$ as $s \to \infty$

Second, the plan $(\tilde{f}^i, \hat{c}^i(0)) = (\tilde{f}^i, \tilde{c}^i)$ doesn't respect all the constraints on bequests. Otherwise, $(\tilde{f}^i, \tilde{c}^i)$ would be in the choice set of the problem $P_i^\star$ and since $(\tilde{f}^i, \tilde{c}^i)$ is a *strict* $\mathcal{C}$-improvement of $(\dot{f}^i, \dot{c}^i)$, according to the Tie Breaking Rule, agent i would have chosen $(\tilde{f}^i, \tilde{c}^i)$ instead of $(\dot{f}^i, \dot{c}^i)$.

Third, I want to show the existence of a $\vec{\lambda}^\star$ such that the dynastic bequest plan $\hat{b}^i(\vec{\lambda}^\star)$ meets the constraints on bequests of all the dynasty of agent i and at least one of these constraints is binding.
I start with $\vec{\lambda} = \vec{1}$, I begin to take the λ_j of the children of agent i down to 0, then I take the λ_j of the grand-children of agent i down to 0 and so on... This process stops when one of the constraints on bequests of the dynasty is not met.
To show that this process stops in a finite number of time periods, let us take an agent j such that $\hat{b}_j(\vec{0}) < \underline{b}_j$ and $T > t(j)$. $\vec{\lambda}_T$ is such that $\lambda_k = 0$ if agent k is adult before the period T otherwise $\lambda_k = 1$.

$$\hat{b}_j(\vec{\lambda}_T) = \hat{b}_j(\vec{0}) + \sum_{s \geq T - t(j)} \sum_{k \in \tilde{f}^j(t(j)+s)} \frac{\frac{(1+r)^{t(j)+s+1}}{M^{t(j)+s}}(S_{t(j)+s} - \frac{Card\{\tilde{f}_k\}}{M} S_{t(j)+s+1}) + e_k}{(1+r)^s}$$

We have:

$$\sum_{s \geq T - t(j)} \sum_{k \in \tilde{f}^j(t(j)+s)} \frac{\frac{(1+r)^{t(j)+s+1}}{M^{t(j)+s}}(S_{t(j)+s} - \frac{Card\{\tilde{f}_k\}}{M} S_{t(j)+s+1})}{(1+r)^s}$$

$= \frac{(1+r)^{t(j)}}{M^{t(j)}} \lim_{S \to \infty} [\sum_{S \geq s \geq T-t(j)} \sum_{k \in \tilde{f}^j(t(j)+s)} \frac{1}{M^s} (S_{t(j)+s} - \frac{Card\{\tilde{f}_k\}}{M} S_{t(j)+s+1})]$
$= \frac{(1+r)^{t(j)}}{M^{t(j)}} \lim_{S \to \infty} [\frac{Card\{\tilde{f}^j(t(j)+T)\}}{M^T} S_{t(j)+T} - \frac{Card\{\tilde{f}^j(t(j)+S+1)\}}{M^0} S_{t(j)+S+1}]$
$\leq \frac{(1+r)^{t(j)}}{M^{t(j)}} \frac{Card\{\tilde{f}^j(t(j)+T)\}}{M^T} S_{t(j)+T}$
$\leq \frac{(1+r)^{t(j)}}{M^{t(j)}} S_{t(j)+T} \to 0$ as $T \to \infty$

And $\sum_{s \geq T-t(j)} \sum_{k \in \tilde{f}^j(t(j)+s)} \frac{e_k}{(1+r)^s} \to 0$ as $T \to \infty$
thanks to the Assumption 1.

So, as $T \to \infty$, $\hat{b}_j(\vec{\lambda}_T) \to \hat{b}_j(\vec{0}) \ < \ \underline{b}_j$

This means that for T large enough there are constraints on bequests not met by the bequest dynastic plan $\hat{b}^i(\vec{\lambda}_T)$. Let us denote $T^{First} \geq 1$ the smallest of these T.
When going from $\vec{\lambda}_{T^{First}-1}$ to $\vec{\lambda}_{T^{First}}$ and using the continuity of the finite number of bequest functions of the descendants of agent i born before T^{First}, there is a $\vec{\lambda}^\star$ such that all the constraints are met and at least one of them is binding.

There exists $j \in \tilde{f}^i$ such that $\hat{b}_j(\vec{\lambda}^\star) = \underline{b}_j$.
Since $\vec{\lambda} \mapsto \hat{b}_j(\vec{\lambda})$ is strictly increasing, we have
$\tilde{b}_j = \hat{b}_j(0) < \hat{b}_j(\vec{\lambda}) = \underline{b}_j \leq \inf_{(f^{\backslash i}, c^{\backslash i})} [b_i^m(f^{\backslash i}, c^{\backslash i})]$

$(\tilde{f}^i, \tilde{c}^i)$ is a *strict* $\mathcal{C}$-improvement of $(\dot{f}^i, \dot{c}^i)$, So, if agent j is not born in $\dot{f}^i$, then she and all her descendants in $\tilde{f}^j$ must have utility levels not below their critical levels in $(\tilde{f}^i, \tilde{c}^i)$.
By definition of $b_j^m(\cdot)$, we must have $\tilde{b}_j \geq \inf_{(f^{\backslash i}, c^{\backslash i})} [b_i^m(f^{\backslash i}, c^{\backslash i})] \geq \underline{b}_j$
So $\hat{b}_j(\lambda^\star) > \hat{b}_j(0) = \tilde{b}_j \geq \underline{b}_j$ which is impossible.
So agent agent j is born in $\dot{f}^i$.

Since all the constraints on bequests of the descendants of j are met by $(\tilde{f}^j, \hat{c}^j(\lambda^\star))$ and since $\dot{b}_j \geq \underline{b}_j = \hat{b}_j(\lambda^\star)$ the budget constraint is also met by $(\tilde{f}^j, \hat{c}^j(\lambda^\star))$. So the dynastic plan $(\tilde{f}^j, \hat{c}^j(\lambda^\star))$ belongs to the choice set of agent j facing the problem $P_j^\star(\dot{b}_j)$.
And, since $\hat{c}^j(\lambda^\star) \ >> \ \hat{c}^j(0) = \tilde{c}^j \quad u_j(\tilde{f}^j, \hat{c}^j(\lambda^\star)) > u_j(\tilde{f}^j, \tilde{c}^j) \geq u_j(\dot{f}^j, \dot{c}^j)$
So agent j should have chosen $(\tilde{f}^j, \hat{c}^j(\lambda^\star))$ instead of $(\dot{f}^j, \dot{c}^j)$ in equilibrium.
This is a CONTRADICTION.

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Proof of Proposition 3 (Efficient and ALAC Equilibrium)

Back to main text

Let us show that the equilibrium plan $(\dot{f}, \dot{c})$ is ALAC
Take any agent i not in the initial generation, we have shown in Lemma 3 that there existed a dynastic plan $(\tilde{f}^i, \tilde{c}^i)$ that is such that:
$\forall j \in \tilde{f}^i \;\; u_j(\tilde{f}^j, \tilde{c}^j) \geq \alpha_j$
And such that $\tilde{c}_i - e_i + \sum_{s \in \mathbb{N}^*} \sum_{j \in \tilde{f}^i(t+s)} \frac{\tilde{c}_j + Cost_j(1+r) - e_j}{(1+r)^s} = b_i^m$

Let us show that $(\tilde{f}^i, \tilde{c}^i)$ belongs to the choice set of $P_i^\star(\dot{b}_i)$.
First, since $\dot{b}_i \geq \bar{b}_i = b_i^m$, the plan $(\tilde{f}^i, \tilde{c}^i)$ respects the resource constraints.
Second, if $(\tilde{f}^i, \tilde{c}^i)$ doesn't respect the constraint on bequests, then it means that an agent $j \in \tilde{f}^i$ receives a bequest $\tilde{b}_j$ lower than $b_j^m = \bar{b}_i$. So, by definition of $\underline{b}_m^i$, j or one of her descendants k would be such that $u_k(\tilde{f}^k, \tilde{c}^k) < \alpha_k$. But, by definition of $(\tilde{f}^i, \tilde{c}^i)$ all agents of the dynasty of i have utility levels not below their critical levels.
This is a contradiction.

Since $(\tilde{f}^i, \tilde{c}^i)$ belongs to the choice set of $P_i^\star$ we have that:
$U_i^\star(\dot{b}_i) = u_i(\dot{f}^i, \dot{c}^i) \geq u_i(\tilde{f}^i, \tilde{c}^i) \geq \alpha_i$

This is true for all agents i so $(\dot{f}, \dot{c})$ is ALAC

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Proof of Proposition 4 (ALAC constraints)

Back to main text

Let us show that $(\dot{f}, \dot{c})$ is ALAC
Assume that there is an agent j who is strictly below her Dynastic Critical Level. Her parent is denoted agent i. By definition $u_i(\dot{c}_i, \{\dot{U}_j\}_{j \in \dot{f}_i}) < u_i(\dot{c}_i, \{\dot{U}_j\}_{j \in \dot{f}_i \setminus \{j\}})$
Since $\dot{b}_j \geq \underline{b}_j = \geq Cost_j$,
$u_i(\dot{c}_i, \{\dot{U}_j\}_{j \in \dot{f}_i}) < u_i(\dot{c}_i, \{\dot{U}_j\}_{j \in \dot{f}_i \setminus \{j\}}) \leq u_i(\dot{c}_i + Cost_j + \dot{b}_j, \{\dot{U}_j\}_{j \in \dot{f}_i \setminus \{j\}})$
So agent i would have been better off choosing not to have her child j and consume $Cost_j + \dot{b}_j$ more instead.
This is a CONTRADICTION.

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Proof of Proposition 5 (Pigouvian Taxes)

Back to main text

In equilibrium, taxes on children generate $T_t Card\{\dot{f}(t)\} = \frac{E}{Card\{\dot{f}(t)\}} Card\{\dot{f}(t)\} = E$ and the discounted sum of tax revenue $\frac{E}{r}$ is distributed to the initial agents through lump-sum transfers $\{L_i\}_{i \in I(t=0)}$.

Assume there is another feasible plan $(\tilde{f}, \tilde{c})$ which *strictly* $\mathcal{C}$-dominates $(\dot{f}, \dot{c})$.

The resource constraint of the whole economy is:

$$\sum_{i \in I(t=0)} [\tilde{c}_i + \sum_{s \in \mathbb{N}^*} \sum_{j \in \tilde{f}^i(s)} \frac{\tilde{c}_j + Cost_j(1+r)}{(1+r)^s}] \leq \sum_{i \in I(t=0)} [e_i^p + \sum_{s \in \mathbb{N}^*} \sum_{j \in \tilde{f}^i(s)} \frac{e_j^p}{(1+r)^s}] + \sum_{s \in \mathbb{N}^*} \frac{E}{(1+r)^s}$$

Case 1: All initial agents $i \in I(t=0)$ are such that:

$$\tilde{c}_i + \sum_{s \in \mathbb{N}^*} \sum_{j \in \tilde{f}^i(s)} \frac{\tilde{c}_j + Cost_j(1+r)}{(1+r)^s} \leq e_i^p + \sum_{s \in \mathbb{N}^*} \sum_{j \in \tilde{f}^i(s)} \frac{e_j^p}{(1+r)^s} + L_i$$

Since $(\tilde{f}, \tilde{c})$ *strictly* $\mathcal{C}$-dominates $(\dot{f}, \dot{c})$ there is an initial agent i such that $(\tilde{f}^i, \tilde{c}^i)$ *strictly* $\mathcal{C}$-dominates $(\dot{f}^i, \dot{c}^i)$ so $(\tilde{f}^i, \tilde{c}^i)$ cannot belong to the choice set of the problem $P_i^\star$ otherwise agent i would have chosen $(\tilde{f}^i, \tilde{c}^i)$.

The consolidated resource constraint faced by agent i in the problem $P_i^\star$ is:

$$c_i + \sum_{s \in \mathbb{N}^*} \sum_{j \in f^i(s)} \frac{c_j + Cost_j(1+r)}{(1+r)^s} \leq e_i^p + \sum_{s \in \mathbb{N}^*} \sum_{j \in f^i(s)} \frac{e_j^p + \frac{E}{Card\{\dot{f}(s)\}} - T_s}{(1+r)^s} + L_i$$

And since for all period s, $\frac{E}{Card\{\dot{f}(s)\}} = T_s$, so if the dynastic plan $(\tilde{f}^i, \tilde{c}^i)$ doesn't belong to the choice set of the problem $P_i^\star$ it is not because of resource constraint but because of the constraints on bequests.

Just like in the proof of the Proposition 2, using the fact that at least one constraint on bequests is not met by the dynastic plan $(\tilde{f}^i, \tilde{c}^i)$, we can find a descendant of agent i denoted j and a dynastic consumption plan for agent j denoted $\hat{c}^j$ which belongs to the choice set of the problem $P_j^\star(\dot{b}_j)$ such that $u_j(\tilde{f}^j, \hat{c}^j) > u_j(\tilde{f}^j, \tilde{c}^j) \geq u_j(\dot{f}^j, \dot{c}^j)$

So agent j should have chosen $(\tilde{f}^j, \hat{c}^j)$ instead of $(\dot{f}^j, \dot{c}^j)$ in equilibrium.

Case 2: There is an initial agent $i \in I(t=0)$ such that:

$$\tilde{c}_i + \sum_{s \in \mathbb{N}^*} \sum_{j \in \tilde{f}^i(s)} \frac{\tilde{c}_j + Cost_j(1+r)}{(1+r)^s} > e_i^p + \sum_{s \in \mathbb{N}^*} \sum_{j \in \tilde{f}^i(s)} \frac{e_j^p}{(1+r)^s} + L_i$$

Then, since the resources constraint of the whole economy is met by $(\tilde{f}, \hat{c})$ there exists at least one initial agent $i' \in I(t=0)$ such that:

$$\tilde{c}_{i'} + \sum_{s \in \mathbb{N}^*} \sum_{j \in \tilde{f}^{i'}(s)} \frac{\tilde{c}_j + Cost_j(1+r)}{(1+r)^s} \; < \; e^p_{i'} + \sum_{s \in \mathbb{N}^*} \sum_{j \in \tilde{f}^{i'}(s)} \frac{e^p_j}{(1+r)^s} \; + L_{i'}$$

So there is a deviation of $(\tilde{f}^{i'}, \tilde{c}^{i'})$ where agent i' consumes ε more and thus *strictly* $\mathcal{C}$-dominates $(\dot{f}^{i'}, \dot{c}^{i'})$ and meets the consolidated constraint on bequest and we are back to Case 1.

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