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Open Economy Secular Stagnation and Financial Integration

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Abstract

What is the optimal policy response to secular stagnation within a small open economy? Secular stagnation is characterized by a persistent lack of demand, resulting in under-employment. Within a small open economy, the degree of financial integration determines the nature of the secular stagnation equilibrium. Under perfect capital mobility, stagnation is due to downward nominal wage rigidities; while under financial autarky, it is due to an excessively high real interest rate. I characterize the planner's optimal allocation of resources and solve for the tax policy that implements it within a stagnating economy. Under perfect financial integration, payroll taxes should be falling and labor income taxes rising such as to relax the downward nominal wage rigidity; while under financial autarky, the opposite policy should be implemented such as to generate inflation. Alternatively, full employment can be achieved by setting a sufficiently inflation target and by relying on an exchange rate policy to import inflation from abroad. Policy options are more limited under a fixed exchange rate regime, where efficiency can either be reached through a coordinated policy response or by abandoning the peg.

Keywords: Financial integration, Liquidity trap, Secular stagnation

JEL Classification: E31, E63, F38, F41

1 Introduction

Since the mid-1990s, Japan has experienced secular stagnation, characterized by a binding zero lower bound on the nominal interest rate, very low inflation, and depressed economic activity. Following the Great Recession of 2008, most of the industrialized world has been walking in the footsteps of Japan. Within the eurozone, the situation is even more severe since European countries have now lost their two main variables of

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macroeconomic adjustment: exchange rates are fixed within the eurozone and the interest for the whole of the currency union is stuck at the zero lower bound. This results in persistent macroeconomic imbalances.

How should the government of a small open economy respond to secular stagnation? How is this response affected by the state of the world economy, by the degree of financial market integration, or by the exchange rate regime? This paper provides a simple model of open economy secular stagnation such as to answer these questions by characterizing optimal policies that can bring the economy back to the efficient allocation with full employment.

Throughout my analysis, I rely on a representative-household model of demand-driven secular stagnation. Goods are differentiated across countries, as in Gali and Monacelli (2005), but for simplicity firms do not have market power and prices are perfectly flexible. Nominal wages are downward rigid, as in Schmitt-Grohé and Uribe (2016). Building on Michau (2018), I assume that households have a preference for wealth. They also have a preference for domestic over foreign assets. The strength of this preference determines the degree of financial market integration, which offers a parsimonious way to embody a range of possibilities.

Before solving for the optimal policy, I characterize the steady state equilibria of our small open economy under *laissez-faire*. Unsurprisingly, it is always possible for the domestic economy to be in an equilibrium that is symmetric to the rest of the world, where our small open economy and the rest of the world are either both at full employment or both mired in secular stagnation (i.e. with under-employment). Under flexible exchange rate, asymmetric equilibria are also possible. Thus, within a stagnating world, our small open economy can be at full employment thanks to a depreciation of its domestic currency. Conversely, within a world economy at full employment, it is possible for the domestic economy to be depressed due to the combination of low inflation and an over-valued exchange rate.

The nature of secular stagnation is fundamentally different depending on the extent of financial market integration and on the exchange rate regime. Under either perfect financial integration or fixed exchange rate, the steady state inflation rate is determined internationally. Stagnation is therefore due to excessively low wage growth resulting in the binding downward nominal wage rigidity. If wages are more downward flexible, the domestic economy becomes more competitive at the expense of its trade partners. Conversely, under financial autarky and flexible exchange rate, stagnation is due to an excessively high real interest rate resulting from a binding zero lower bound. In that case, as in a closed economy, the paradox of flexibility holds: more flexible wages reduce the rate of inflation, which increases the real interest rate and further depresses the economy. This effect of wage flexibility is magnified by an appreciation of the domestic currency, which further depresses aggregate demand. Despite this fundamental difference in the nature of secular stagnation, higher inflation is always desirable, either to overcome the downward nominal wage rigidity or the zero lower bound.

While many of these insights were already present in the literature (Eggertsson, Mehrotra, Singh, and Summers 2016; Corsetti, Mavroeidi, Thwaites, and Wolf 2018; Caballero, Farhi, and Gourinchas 2021), my analysis combines them within a single parsimonious framework.

To characterize the optimal allocation of resources, I set up and solve a planner's problem. In addition to the standard resource constraints, there are two demand functions that the planner must take as given: the foreign demand for domestic goods as function of the international terms of trade and the foreign demand for domestic bonds as a function of their rate of return.

I then characterize the tax policy that implements the planner's optimal allocation within a decentralized economy. I first focus on a flexible exchange rate regime. Both goods and assets are differentiated across countries. Hence, the optimal policy consists in taxing exports and foreign capital inflows such as to extract monopolistic rents from the rest of the world. In addition, the inflation target must be sufficiently high for the downward nominal wage rigidity and the zero lower bound to be non-biding. If inflation is on target, then this policy is sufficient to implement the optimal allocation.

However, this policy does not rule out the possibility that inflation falls below target, resulting in a zero nominal interest rate and under-employment. This outcome can be avoided through a dynamic tax policy that forces the under-employment equilibrium to coincide with the planner's optimal allocation. Under flexible exchange rate, the nature of this policy is diametrically opposed depending on whether capital markets are financially integrated or not. When capital mobility is perfect, payroll taxes need to be falling over time, such as to relax the downward nominal wage rigidity. Labor income taxes need to be concurrently rising such as to keep labor supply undistorted. Conversely, under financial autarky, payroll taxes need to be rising such as to increase the cost of production and, hence, domestic inflation. This reduces the real interest rate, which stimulates aggregate demand. Labor income taxes now need to be concurrently falling over time to keep labor supply undistorted. These diverging policy recommendations are fundamentally related to the paradox of flexibility: when it does not hold, the optimal policy consists in relaxing the downward wage rigidity; while, when it does, the optimal policy consists instead in raising inflation.

To overcome under-employment, an alternative consists in forcing inflation to be on target through the adoption of an exchange rate policy, as initially suggested by Svensson (2001, 2003). If foreign inflation is sufficiently high, a peg allows the domestic economy to import inflation from abroad. But, if foreign inflation is depressed, then a crawling peg is necessary whereby a gradual depreciation of the domestic currency generates domestic inflation.

Government support through either the dynamic tax policy or the exchange rate policy needs to be continued until households coordinate their expectations on remaining in the full employment equilibrium, at which point the economy can remain there without any further support.

Within a fixed exchange rate regime, in steady state, the rate of inflation is determined abroad. Hence, if the world economy is depressed with a persistently low rate of inflation, then domestic inflation is so low that the downward nominal wage rigidity is constantly binding, resulting in enduring under-employment. To overcome the problem, payroll taxes need to be falling forever. The alternative is to raise inflation either by unilaterally abandoning the peg or by cooperating with the other countries sharing the peg, which may form a currency union.

A fixed exchange rate regime generates the additional difficulty that the initial exchange rate may be over-valued, resulting in a depressed domestic economy. This problem can be overcome through a fiscal devaluation *à la* Farhi, Gopinath, and Itskhoki (2014) consisting in a downward jump in the level of firms' payroll taxes compensated by an upward jump in consumption taxes or, alternatively, by an upward jump in labor income taxes. Fiscal devaluation can also be achieved through a joint increase in export subsidies and import taxes.

Related Literature. Keynes (1936) explained how an economy can be depressed due to a lack of demand. Hansen (1939) formulated the secular stagnation hypothesis, according to which the lack of demand can be a permanent state of affairs. A number of papers have now provided complete microfoundations for this possibility within a closed economy (Ono 1994, 2001, Michau 2018, Eggertsson, Mehrotra, and Robbins 2019).[1] These microfoundations entail the paradox of flexibility, which shows that under-employment is not a partial equilibrium phenomenon, due to excessively high wages, but a general equilibrium phenomenon resulting from the interaction between the financial market, the market for goods, and the market for labor: the binding zero lower bound results in an excessively high real interest rate in financial markets, which depresses households' demand for goods, and hence firms' demand for labor. This immediately raises the question about the possibility of secular stagnation within an open economy, where general equilibrium effects are muted by international flows of capital and of goods.

Relying on two-country models, Eggertsson, Mehrotra, Singh, and Summers (2016) and Caballero, Farhi, and Gourinchas (2021) have shown that secular stagnation is not only possible within an open economy, but that financial integration allows it to spread

[1] See also Benigno and Fornaro (2018), Geerolf (2019), Michaillat and Saez (2021), and Kocherlakota (2021) for closely related contributions.

across borders, thereby making stagnation a "contagious malady". Indeed, the lack of demand induces an excess of savings that tends to be channeled to foreign countries, resulting in a domestic trade surplus that absorbs some of the foreign demand.[2] They argued that, under perfect financial integration, policies stimulating competitiveness, such as exchange rate policies or structural reforms, generate negative externalities on trade partners; while policies stimulating aggregate demand, such as fiscal policy, generate positive externalities. Importantly, Caballero, Farhi, and Gourinchas (2021) emphasized that, when both countries are depressed, the nominal exchange is indeterminate, which raises the scope for currency wars.

Corsetti, Mavroeidi, Thwaites, and Wolf (2018) extended this analysis to the case of a small open economy. They showed that, under a strong complementarity between domestic and foreign goods, escaping stagnation through a depreciation of the domestic currency can be welfare reducing.

While these papers investigate the effects of various policies within the secular stagnation equilibrium, my contribution is to solve for optimal policies that end stagnation and restore efficiency. I therefore set up a planner's problem; characterize the optimal allocation of resources; and explain how it can be implemented within the decentralized economy through taxes, subsidies, and monetary policy.

The possibility of open-economy stagnation has also been documented by Ono (2006, 2017) for a two-country world and by Hashimoto and Ono (2020) for a small open economy. Unlike other papers on the topic (including this one), they find that the paradox of flexibility holds under perfect financial integration and flexible exchange rate. This finding results from their assumption that the lack of demand is due to an insatiable preference for liquidity. Hence, their liquidity trap is characterized by a strictly positive nominal interest rate. But, under perfect financial integration, the domestic real interest rate is determined abroad. Hence, more flexible wages reduce inflation, which lowers the nominal interest rate, which induces households to demand more money and less consumption. By contrast, in my framework, at the zero lower bound, inflation is determined by the world interest rate. Hence, more flexible wages cannot affect domestic inflation and triggers instead a depreciation of the domestic currency that stimulates economic activity.

Imperfect financial integration is important, but has traditionally been difficult to incorporate into policy analysis. Eggertsson, Mehrotra, Singh, and Summers (2016) assumed an upper bound to foreign asset holdings, but focused their policy analysis

[2]Focusing on a temporary liquidity trap, Fujiwara, Nakajima, Sudo, and Teranishi (2009), Acharya and Bangui (2018), and Fornaro and Romei (2019) have also found that capital flows tend to export the lack of demand to trading partners. However, Cook and Devereux (2013, 2016) have shown that a strong home bias in consumption can reverse this effect, due to domestic deflation triggering a perverse appreciation of the domestic currency.

on the case of perfect capital mobility where the constraint is not binding. By assuming a preference for domestic over foreign assets, my analysis offers a convenient way to allow for various degrees of financial integration, which turns out to have a major impact on both the properties of the secular stagnation equilibrium under *laissez-faire* and the policy recommendations.

Relying on the new Keynesian framework, Correia, Nicolini, and Teles (2008) and Adao, Corriea, and Teles (2009) have proved fundamental equivalence results between monetary and tax policies. This implies that, when monetary policy is out of ammunitions, either due to a binding zero lower bound or to a fixed exchange rate, taxes and subsidies can be used as instruments of macroeconomic stabilization. In a closed economy, Feldstein (2002) was the first to advocate for a rising path of the Value Added Tax to stimulate aggregate demand at the zero lower bound. This insight was subsequently formalized by Correia, Farhi, Nicolini, and Teles (2013) who showed that a dynamic tax policy can implement the first-best allocation within a new Keynesian economy, despite a binding zero lower bound. Building on this approach, Michau (2018) characterized the optimal tax policy in the context of secular stagnation. Similarly, in an open economy, Farhi, Gopinath, and Itskhoki (2014) showed that a fiscal devaluation can overcome an over-valued fixed exchange rate. More recently, D'Acunto, Huang, and Weber (2021) have provided empirical evidence for the effectiveness of this type of unconventional fiscal policies.

My paper applies this method to the context of secular stagnation within a small open economy, with either fixed or flexible exchange rate. It recover some insights from previous closed and open economy analyses, but finds that the nature of the optimal tax policy is reversed depending on the extent of financial integration.

For a small open economy, an alternative to these sophisticated tax policies is to rely on a dynamic exchange rate policy to import inflation from abroad. My investigation of this possibility builds on Svensson (2001, 2003), who was the first to advocate for such policies in the context of a temporary liquidity trap.

Jeanne (2021) showed that a global liquidity trap can trigger trade and currency wars. While my analysis relies on a similar framework, I assume a larger set of policy instruments, which makes it possible to raise inflation such as to escape stagnation and restore efficiency.

Schmitt-Grohé and Uribe (2016) showed that, under perfect financial integration, the combination of a fixed exchange rate and a downward wage rigidity can result in a deep depression. The first-best allocation can nevertheless be implemented either through a devaluation of the exchange rate or though taxes and subsidies. As a second-best policy, capital controls that lean against the wind can significantly mitigate the severity of depression. Farhi and Werning (2012, 2014) advocated for a similar policy within a sticky-price new Keynesian economy. Devereux and Yetman (2014) also investigated the interaction between monetary policy and capital controls at the zero lower bound.

The paper begins with a careful exposition of the model of our small open economy together with a definition of equilibrium. Section 3 derives the steady state equilibria for the world economy, assuming that all countries are identical. Focusing on a flexible exchange rate regime, Section 4 reviews the steady state equilibrium possibilities depending on the state of the world economy and on the degree of financial integration. The following section sets up and solves the planner's problem for our small open economy. Section 6 describes the tax policy that implements the planner's optimal allocation under floating exchange rate. Section 7 replicates the analysis for a fixed exchange rate regime. The paper ends with a conclusion.

2 Economy

The setup of the economy consists of a representative household, a representative firm, the government, and a downward nominal wage rigidity. I present each in turn, before defining the equilibrium of the economy.

2.1 Households

Goods and prices. There is a continuum of mass one of countries, indexed by $i \in [0,1]$. Each country i produces a single consumption good, as in Gali and Monacelli (2005). My analysis focuses on a small open economy. Foreign consumption c_t^F is composed of a basket of consumption goods from all over the world given by

$$c_t^F = \left[\int_0^1 \left(c_t^i\right)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{1}$$

where c_t^i is the quantity imported from country i and $\varepsilon \in (1,\infty)$ is the elasticity of substitution between goods produced in two different countries. The domestic consumption basket is composed of goods produced at home c_t^H and in foreign countries c_t^F. It's consumption value c_t is given by

$$c_t = \left[\omega^{\frac{1}{\theta}}\left(c_t^H\right)^{\frac{\theta-1}{\theta}} + (1-\omega)^{\frac{1}{\theta}}\left(c_t^F\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \tag{2}$$

where $\theta \in [0,\infty)$ is the elasticity of substitution between domestic and foreign goods and $\omega \in [0,1]$ is the home bias in consumption.

Let P_t^H denote the domestic price of the domestic good and P_t^F the domestic price of the foreign basket. For simplicity, I assume that prices are perfectly flexible and that all foreign countries are identical. Hence, if P_t^i denotes the domestic price of imported goods from country i, we have $P_t^i = P_t^F$ and $c_t^i = c_t^F$ for all i. The domestic demand for home and foreign goods are respectively given by

$$c_t^H = \omega \left(\frac{P_t^H}{P_t}\right)^{-\theta} c_t \text{ and } c_t^F = (1-\omega)\left(\frac{P_t^F}{P_t}\right)^{-\theta} c_t, \tag{3}$$

where the consumer price index is defined as

$$P_t = \left[\omega \left(P_t^H\right)^{1-\theta} + (1-\omega)\left(P_t^F\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{4}$$

Let $\tilde{P}_t^H$ and $\tilde{P}_t^F$ denote the (domestic currency) price of the domestic and foreign goods, respectively, in international markets. The difference between international and domestic prices is due to taxes. Imported goods are subject to an import tariff τ_t^I, while exported goods are subsidized at rate s_t^X. In addition, domestic consumption is subject to a consumption tax (a Value Added Tax) τ_t^C. We therefore have

$$P_t^F = (1+\tau_t^C)(1+\tau_t^I)\tilde{P}_t^F, \tag{5}$$

$$P_t^H = (1+\tau_t^C)(1+s_t^X)\tilde{P}_t^H, \tag{6}$$

where $(1+s_t^X)\tilde{P}_t^H$ is the price received by a domestic producer for selling one unit of goods (either at home or abroad).

Let E_t denote the nominal exchange rate, i.e. the domestic price of one unit of foreign currency, and P_t^* the foreign consumer price index in units of foreign currency. By the law of one price, any consumption good must have the same value across all countries. This implies

$$\tilde{P}_t^F = E_t P_t^*. \tag{7}$$

Note that the law of one price entails producer currency pricing.

The foreign demand for domestically produced goods amounts to

$$c_t^{H*} = (1-\omega)\left(\frac{\tilde{P}_t^H}{\tilde{P}_t^F}\right)^{-\varepsilon} c_t^*, \tag{8}$$

where c_t^* denotes the level of foreign demand.[3]

[3]Domestic demand for imports from country i is given by $c_t^i = \left(\frac{P_t^i}{P_t^F}\right)^{-\varepsilon} c_t^F = (1-\omega)\left(\frac{P_t^i}{P_t^F}\right)^{-\varepsilon}\left(\frac{P_t^F}{P_t}\right)^{-\theta} c_t$. Thus, by symmetry (and aggregating over the unit mass of foreign countries), the foreign demand for domestic goods is equal to $c_t^{H*} = (1-\omega)\left(\frac{\tilde{P}_t^H/E_t}{P_t^{F*}}\right)^{-\varepsilon}\left(\frac{P_t^{F*}}{P_t^*}\right)^{-\theta} c_t^*$. But, as all foreign countries are identical, we have $P_t^{F*} = P_t^*$ and, hence, by (7), $P_t^{F*} = \tilde{P}_t^F/E_t$. This gives equation (8).

The international terms of trade are defined as

$$S_t = \frac{\tilde{P}_t^F}{\tilde{P}_t^H}. \tag{9}$$

I also define the real exchange rate as the ratio of the price of foreign goods in international markets relative to the domestic price level, which gives

$$Q_t = \frac{E_t P_t^*}{P_t} = \frac{\tilde{P}_t^F}{P_t}. \tag{10}$$

The definition of the consumer price index (4) implies that the terms of trade and the real exchange rate are related by

$$(1+\tau_t^C)(1+\tau_t^I)Q_t = \left[\omega\left(\frac{1+\tau_t^I}{1+s_t^X}S_t\right)^{\theta-1} + 1 - \omega\right]^{\frac{1}{\theta-1}}. \tag{11}$$

In the special case where the domestic basket is characterized by Cobb-Douglas preferences, with $\theta = 1$, we have $(1+\tau_t^C)(1+\tau_t^I)Q_t = \left(\frac{1+\tau_t^I}{1+s_t^X}S_t\right)^{\omega}$.

The household's intertemporal problem. Now that we have characterized the demand for different goods at a given point in time, let us focus on the intertemporal dimension of the household's problem. Time is continuous. In the domestic economy, there is a unit mass of infinitely lived households. They discount the future at rate ρ, with $\rho > 0$. They derive utility $u(c_t)$ from consuming c_t at time t, with $u'(\cdot) > 0$, $u''(\cdot) < 0$, and $\lim_{c \to 0} u'(c) = \infty$. They also incur disutility $v(l_t^s)$ from supplying l_t^s units of labor, with $v'(\cdot) > 0$, $v''(\cdot) > 0$, $v'(0) = 0$, and $\lim_{l^s \to \bar{l}} v'(l^s) = \infty$ where $\bar{l}$ is the maximum feasible supply of labor, which can be arbitrarily large.

Households also derive utility from holding wealth. Let A_t denote the representative household's nominal wealth and $a_t = A_t/P_t$ its real wealth. Throughout my analysis, I consider that claims on foreign goods are the international store of value. Hence, households value their wealth in units of foreign goods at international prices, as given by $A_t/\tilde{P}_t^F = a_t/Q_t$. Note that, from the perspective of our small domestic economy, units of foreign goods can be seen as perfectly substitutable with units of gold. As $\tilde{P}_t^F$ is beyond the government's control, this specification rules out any temptation to adjust taxes such as to raise households' perceived wealth.

Real government debt $b_t = B_t/P_t$ must eventually be redeemed through taxes. Hence, households perceive their real net wealth to be equal to $(a_t - b_t)/Q_t$. This specification implies that households are Ricardian. Indeed, a lump-sum transfer that is eventually paid for by a lump-sum tax raises both a_t and b_t, while leaving their net wealth $(a_t - b_t)/Q_t$ unchanged, consistently with the Ricardian equivalence proposition. The representative household therefore derives utility $\gamma((a_t - b_t)/Q_t)$ from holding net wealth $(a_t - b_t)/Q_t$, with $\gamma'(\cdot) > 0$, $\gamma''(\cdot) < 0$, $\lim_{a\to\infty} \gamma'(a) = 0$, and $\lim_{a\to\underline{a}} \gamma'(a) = \infty$ where $\underline{a} < 0$ can be arbitrarily low.

Finally, households also have a preference over their portfolio composition. Their wealth A_t consists of domestic bonds D_t^H and of foreign bonds $E_t D_t^F$. Let $d_t^H = D_t^H/P_t$ and $d_t^F = E_t D_t^F/P_t$ denote the real value of domestic and foreign bonds held by the representative household, respectively. Even though both assets are risk-free, I assume that households have a preference for domestic assets, which allows for different rates of return across countries. More specifically, their disutility from holding foreign assets is given by $-\psi/2\left(E_t D_t^F/\tilde{P}_t^F\right)^2$, which can equivalently be written as $-\psi/2\left(d_t^F/Q_t\right)^2$, where ψ is the intensity of this preference for portfolio composition.[4] This component of utility is a convenient way to introduce imperfect financial integration within my model. I will subsequently focus on two special cases: $\psi = 0$ for perfect financial integration and $\psi = \infty$ for financial autarky.

The household's intertemporal utility is given by

$$\int_0^\infty e^{-\rho t}\left[u(c_t) - v(l_t^s) + \gamma\left(\frac{a_t - b_t}{Q_t}\right) - \frac{\psi}{2}\left(\frac{d_t^F}{Q_t}\right)^2\right] dt. \tag{12}$$

Labor income is equal to $(1-\tau_t^L) w_t l_t^s$, where w_t denotes the real wage and τ_t^L the labor income tax. The household also receives dividends ξ_t from firm ownership and pays a lump-sum tax τ_t. Its nominal consumption is equal to $P_t^H c_t^H + P_t^F c_t^F = P_t c_t$. Hence, its real consumption is equal to c_t. Finally, the nominal returns on its portfolio $A_t = D_t^H + E_t D_t^F$ are equal to $i_t D_t^H + (i_t^* + \dot{E}_t/E_t - \tau_t^F) E_t D_t^F$, where i_t and i_t^* denote the domestic and foreign nominal interest rates, respectively, and τ_t^F is a tax on capital outflows (or, if $D_t^F < 0$, a subsidy on capital inflows) paid by domestic residents. Hence, the real returns on wealth $a_t = A_t/P_t$ are equal to $i_t d_t^H + (i_t^* + \dot{E}_t/E_t - \tau_t^F) d_t^F - \pi_t a_t$, which can also be written as $r_t a_t + (i_t^* + \dot{E}_t/E_t - i_t - \tau_t^F) d_t^F$, where $r_t = i_t - \pi_t$ denotes the domestic real interest rate. From the definition of the real exchange rate (10), we have $i_t^* + \dot{E}_t/E_t - i_t = r_t^* + \dot{Q}_t/Q_t - r_t$. The wealth of the representative household therefore

[4] A more general specification would be

$$-\frac{\psi}{2}\left[\frac{d_t^F - (1-\kappa)(a_t - b_t)}{Q_t}\right]^2,$$

where $\kappa \in [0,1]$ is the portfolio's home bias. With $\kappa = 1$, the household would ideally like to hold domestic assets exclusively; while with $\kappa = 0$, the household would like its net wealth to be perfectly diversified internationally (but is happy to hold domestic asset to pay for government liabilities b_t). For simplicity, I focus on the $\kappa = 1$ case throughout my analysis.

evolves according to

$$\dot{a}_t = r_t a_t + \left(r_t^* + \frac{\dot{Q}_t}{Q_t} - r_t - \tau_t^F \right) d_t^F + (1 - \tau_t^L) w_t l_t^s + \xi_t - \tau_t - c_t. \tag{13}$$

The household maximizes its intertemporal utility (12) with respect to c_t, l_t^s, a_t, and d_t^F subject to its flow of funds constraint (13) with a_0 given.[5] By the maximum principle, the solution to the household's problem is characterized by a consumption Euler equation

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\sigma_u(c_t)} \left[r_t - \rho + \frac{\gamma' \left((a_t - b_t)/Q_t \right)}{Q_t u' (c_t)} \right], \tag{14}$$

where $\sigma_u(c_t) = -c_t u'' (c_t) / u' (c_t)$, a labor supply function

$$v' (l_t^s) = (1 - \tau_t^L) w_t u' (c_t), \tag{15}$$

and a demand for foreign assets

$$\psi \frac{d_t^F / Q_t}{Q_t u' (c_t)} = r_t^* + \frac{\dot{Q}_t}{Q_t} - r_t - \tau_t^F. \tag{16}$$

Let D_t^{H*} denote the domestic bonds held by foreign households and let $d_t^{H*} = D_t^{H*}/P_t$ be their real value. Assuming that foreign households have similar preferences over portfolio composition, the foreign demand for domestic assets is given by

$$\psi \frac{d_t^{H*} / Q_t}{u'(c_t^*)} = r_t + s_t^F - \frac{\dot{Q}_t}{Q_t} - r_t^*, \tag{17}$$

where s_t^F is a subsidy on capital inflows (or, if $D_t^{H*} < 0$, a tax on capital outflows) received by foreign residents. This is formally shown in Appendix A.

The tax τ_t^F on domestic residents and the subsidy s_t^F on foreign residents are the two instruments of capital controls. Clearly, in the special case of perfect financial integration, where $\psi = 0$, the tax τ_t^F and the subsidy s_t^F have to be equal to each other to prevent the occurrence of arbitrage opportunities, at an unbounded cost for public funds. This explains why Schmitt-Grohé and Uribe (2016) and Farhi and Werning (2012, 2014), who investigate optimal capital controls under perfect capital mobility, only consider a single instrument of capital controls.

[5] For simplicity, I do not impose any borrowing limit as the household's preference for wealth, which satisfies $\lim_{a \to \underline{a}} \gamma' (a) = \infty$ for some $\underline{a} < 0$, already rules out endless debt accumulation.

2.2 Firms

For simplicity, I assume that labor is the only factor of production. A representative firm employs L_t^d units of labor to produce $f(L_t^d)$ units of domestic consumption goods. This aggregate production function is characterized by non-increasing returns, with $f'(\cdot) > 0$, $f''(\cdot) \leq 0$, and $f(0) = 0$. Allowing for the possibility of decreasing returns facilitates the exposition of the model, as it makes the production function $f(\cdot)$ clearly apparent throughout its derivation. However, the special case of constant returns to scale, with $f(L_t^d) = L_t^d$, does not cause any difficulty.

The firms chooses labor demand L_t^d such as to maximize profits

$$\frac{P_t^H}{1+\tau_t^C} f\left(L_t^d\right) - \left(1+\tau_t^P\right) W_t L_t^d, \tag{18}$$

where $W_t = P_t w_t$ denotes the nominal wage and τ_t^P is the payroll tax paid by the employer. Domestic producers are competitive. Hence, the revenue from selling to a domestic consumer $P_t^H/(1+\tau_t^C)$ must be equal to the revenue from selling to a foreign consumer $(1+s_t^X)\tilde{P}_t^H$, consistently with equation (6). In equilibrium, the real wage must be equal to the real marginal product of labor, which gives

$$w_t = \frac{1+s_t^X}{1+\tau_t^P} \frac{\tilde{P}_t^H}{P_t} f'\left(L_t^d\right). \tag{19}$$

Real aggregate profits are therefore equal to

$$\xi_t = \left(1+s_t^X\right) \frac{\tilde{P}_t^H}{P_t} [f\left(L_t^d\right) - f'\left(L_t^d\right) L_t^d]. \tag{20}$$

Profits are strictly positive whenever the production function is characterized by decreasing returns to scale.

2.3 Downward nominal wage rigidity

Recall from (15) that households' desired labor supply l_t^s is such the (money-metric) marginal disutility of labor $P_t v'(l_t^s)/[(1-\tau_t^L)u'(c_t)]$ is equal to the nominal wage W_t. I now impose a downward rigidity on the nominal wage W_t and assume that, when desired labor supply l_t^s exceeds firms' labor demand L_t^d, workers end up supplying whatever quantity of labor L_t^d firms demand.

The profit maximizing behavior of firms implies, by (19), that the nominal wage W_t is always equal $[(1+s_t^X)/(1+\tau_t^P)]\tilde{P}_t^H f'(L_t^d)$. Hence, the growth rate of nominal wages for a given employment level L_t^d, denoted by π_t^W, is given by

$$\pi_t^W = \frac{\dot{s}_t^X}{1+s_t^X} - \frac{\dot{\tau}_t^P}{1+\tau_t^P} + \tilde{\pi}_t^H, \tag{21}$$

where $\tilde{\pi}_t^H = \dot{\tilde{P}}_t^H/P_t^H$. Note that rising export subsidies s_t^X or falling payroll taxes τ_t^P increase the marginal product of labor and, hence, the growth rate of nominal wages. I now impose the downward nominal wage rigidity given by

$$L_t^d \leq l_t^s \tag{22}$$

and

$$\left(1+\pi_t^W dt\right) W_t \geq (1+\pi^R dt) W_t + \alpha dt \left[\frac{P_t v'(L_t^d)}{(1-\tau_t^L) u'(c_t)} - W_t\right], \tag{23}$$

with complementary slackness. Thus, for a given employment level L_t^d, workers do not want their nominal wages to grow by less than the reference rate of inflation π^R.[6] However, the resulting under-employment generates a wedge between the (money-metric) marginal disutility of labor $P_t v'(L_t^d)/[(1-\tau_t^L)u'(c_t)]$ and the nominal wage $W_t = P_t v'(l_t^s)/[(1-\tau_t^L)u'(c_t)]$, which induces workers to accept a growth rate for their nominal wage π_t^W that falls below the reference rate of inflation π^R. Thus, α is the wage flexibility parameter. Wages are completely downward rigid when $\alpha = 0$, and become perfectly flexible as $\alpha \to \infty$. The complementary slackness condition implies that, if there is under-employment $L_t^d < l_t^s$, then the downward wage rigidity (23) must be binding. Conversely, if the wage rigidity is slack, then there must be full employment $L_t^d = l_t^s$.

The following lemma, proved in Appendix B, shows how the downward nominal wage rigidity can be simplified.

Lemma 1 *The downward nominal wage rigidity given by (22) and (23) with complementary slackness can be written as*

$$v'(L_t^d) = \begin{cases} v'(l_t^s) & \text{if } \pi_t^W \in \left[\pi^R, +\infty\right) \\ \left[1 - \frac{\pi^R - \pi_t^W}{\alpha}\right] v'(l_t^s) & \text{if } \pi_t^W \in \left[\pi^R - \alpha, \pi^R\right] \end{cases}, \tag{24}$$

where

$$\pi_t^W = \frac{\dot{s}_t^X}{1+s_t^X} - \frac{\dot{\tau}_t^P}{1+\tau_t^P} - \frac{\dot{S}_t}{S_t} + \frac{\dot{Q}_t}{Q_t} + \pi_t. \tag{25}$$

If inflation π_t (and, hence, π_t^W) is sufficiently high, then the wage rigidity is not binding, resulting in full employment. Conversely, when inflation π_t is depressed, the rigidity is

[6] For simplicity, I assume that, when the employment level L_t^d changes, nominal wages adjust in line with the resulting evolution of the marginal product of labor $f'\left(L_t^d\right)$. This assumption is not needed under a constant marginal product of labor, where $f\left(L_t^d\right) = L_t^d$.

binding, resulting in under-employment. In that case, as α increases and wages become more flexible, the gap between labor demand L_t^d and labor supply l_t^s shrinks. We shall see that this will not always be true in general equilibrium, where we endogenize π_t.

2.4 Government

At time t, the government collects taxes on consumption τ_t^C, on imports τ_t^I, on capital outflows τ_t^F, on labor τ_t^L, and on payrolls τ_t^P. It also subsidizes exports at rate s_t^X, capital inflows s_t^F from foreigners, and collects a lump-sum tax τ_t, which may be negative. Hence, real government debt $b_t = B_t/P_t$ evolves according to

$$\begin{aligned}\dot{b}_t &= r_t b_t - \tau_t^C \left[(1+s_t^X)\frac{\tilde{P}_t^H}{P_t} c_t^H + (1+\tau_t^I)\frac{\tilde{P}_t^F}{P_t} c_t^F \right] \\ &\quad -\tau_t^I \frac{\tilde{P}_t^F}{P_t} c_t^F - \tau_t^F d_t^F + s_t^F d_t^{H*} - (\tau_t^L + \tau_t^P) w_t L_t^d + s_t^X \frac{\tilde{P}_t^H}{P_t} c_t^{H*} - \tau_t.\end{aligned} \tag{26}$$

Throughout my analysis, I rule out the possibility that the government runs a Ponzi scheme. I therefore impose the government's no-Ponzi condition

$$\lim_{t\to\infty} e^{-\int_0^t r_s ds} b_t \leq 0, \tag{27}$$

which I assume to be binding throughout my analysis.

Monetary policy targets an inflation rate $\bar{\pi}$, but is subject to the zero lower bound on the nominal interest rate. Hence, we must have[7]

$$\pi_t \leq \bar{\pi} \text{ and } i_t \geq 0 \text{ with complementary slackness.} \tag{28}$$

Throughout my analysis, I consider that $\bar{\pi} > \pi^R$. So, the inflation target is sufficiently high for the downward nominal wage rigidity (24) to be non-binding in steady state under *laissez-faire*.

For now, the exchange rate is assumed to be flexible. As the nominal wage W_t is the only nominal variable of the economy that can be sticky, let us define the nominal exchange rate residually from the firm's wage equation (19) with $\tilde{P}_t^H = \tilde{P}_t^F/S_t = E_t P_t^*/S_t$, which gives

$$E_t = \frac{1+\tau_t^P}{1+s_t^X}\frac{S_t}{P_t^*}\frac{W_t}{f'\left(L_t^d\right)}. \tag{29}$$

Thus, at time 0, the nominal exchange rate can jump such as to allow the terms of trade

[7] This monetary policy can be seen as resulting from a Taylor rule

$$i_t = \max\left\{\bar{r} + \bar{\pi} + \phi\left[\pi_t - \bar{\pi}\right], 0\right\},$$

where $\bar{r}$ is the targeted real interest rate, with an infinitely high Taylor coefficient ϕ.

S_t to be consistent with the other equilibrium equations of the economy, despite the potentially binding nominal wage rigidity.[8] By contrast, as we shall see in Section 7, a fixed exchange rate regime can result in a persistently over-valued currency and in an uncompetitive economy.

2.5 Equilibrium

Domestic production $f(L_t^d)$ is either consumed by domestic households, in quantity c_t^H, or by foreigners, in quantity c_t^{H*}. This yields the domestic goods market clearing condition

$$f(L_t^d) = c_t^H + c_t^{H*}. \tag{30}$$

Similarly, domestic government bonds B_t are either held by domestic households, in quantity D_t^H, or by foreigners, in quantity D_t^{H*}. This yields the domestic asset market clearing condition

$$B_t = D_t^H + D_t^{H*}. \tag{31}$$

The net foreign asset position is equal to the domestic holdings of foreign assets net of the foreign holdings of domestic assets, which gives

$$nfa_t = \frac{E_t D_t^F - D_t^{H*}}{P_t}. \tag{32}$$

But, recall that the wealth of domestic households is given by $a_t = (D_t^H + E_t D_t^F)/P_t$, while public debt is equal to $b_t = (D_t^H + D_t^{H*})/P_t$. We must therefore have $nfa_t = a_t - b_t$. Using the households' flow of funds constraint (13) with labor income equal to $w_t L_t^d$ (rather than $w_t l_t^s$), the wage rate given by (19), profits by (20), and $c_t = [P_t^F c_t^F + P_t^H c_t^H]/P_t$ where P_t^F and P_t^H are given by (5) and (6), together with the government's flow of funds constraint (26) and the goods market clearing condition (30), we obtain:

$$\dot{nfa}_t = r_t nfa_t + \left(r_t^* + \frac{\dot{Q}_t}{Q_t} - r_t \right) d_t^F - s_t^F d_t^{H*} + \frac{\tilde{P}_t^H}{P_t} c_t^{H*} - \frac{\tilde{P}_t^F}{P_t} c_t^F. \tag{33}$$

The right-hand side is the current account surplus, equal to the sum of the net investment income and of the trade surplus.

Let $\widetilde{nfa}_t = nfa_t/Q_t$ denote the net foreign asset position in units of foreign consumption goods (since $Q_t = \tilde{P}_t^F/P_t$). Similarly, let $\tilde{d}_t^F = E_t D_t^F/\tilde{P}_t^F = d_t^F/Q_t$ and $\tilde{d}_t^{H*} = D_t^{H*}/\tilde{P}_t^F = d_t^{H*}/Q_t$ denote domestic holdings of foreign assets and the foreign

[8] When the downward nominal wage rigidity is non-binding, we can incorporate money-in-the-utility-function such as to pin down the level of the nominal variables E_t, P_t, or W_t from the nominal money supply.

holdings of domestic assets, respectively, in units of foreign consumption goods.

The equilibrium of the domestic economy $(c_t, c_t^H, c_t^F, L_t^d, l_t^s, \widetilde{nfa}_t, \tilde{d}_t^F, S_t, Q_t, r_t, \pi_t, \pi_t^W)_{t=0}^{\infty}$ is fully characterized by the value of the consumption basket (2)

$$c_t = \left[\omega^{\frac{1}{\theta}} \left(c_t^H\right)^{\frac{\theta-1}{\theta}} + (1-\omega)^{\frac{1}{\theta}} \left(c_t^F\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}; \tag{34}$$

the domestic demand for home goods (3)

$$c_t^H = \omega \left((1+\tau_t^C)(1+s_t^X)\frac{Q_t}{S_t}\right)^{-\theta} c_t; \tag{35}$$

the relative domestic demand for foreign goods, which follows from (3),

$$\frac{c_t^F}{c_t^H} = \left(\frac{1+s_t^X}{1+\tau_t^I}\frac{1}{S_t}\right)^{\theta} \frac{1-\omega}{\omega}; \tag{36}$$

the household's consumption Euler equation (14)

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\sigma_u(c_t)}\left[r_t - \rho + \frac{\gamma'(\widetilde{nfa}_t)}{Q_t u'(c_t)}\right], \tag{37}$$

the interest parity condition implied by the demand for foreign assets (16)

$$r_t = r_t^* + \frac{\dot{Q}_t}{Q_t} - \frac{\psi}{Q_t u'(c_t)}\tilde{d}_t^F - \tau_t^F; \tag{38}$$

the portfolio composition implied by the domestic demand for foreign assets (16) and the foreign demand for domestic assets (17)

$$\tau_t^F - s_t^F = -\frac{\psi}{Q_t u'(c_t)}\tilde{d}_t^F - \frac{\psi}{u'(c_t^*)}[\tilde{d}_t^F - \widetilde{nfa}_t]; \tag{39}$$

the domestic goods market clearing condition (30) together with the foreign demand for domestic goods (8)

$$f(L_t^d) = c_t^H + (1-\omega)\, S_t^{\varepsilon} c_t^*; \tag{40}$$

the national accounting identity (33), together with the foreign demand for domestic goods (8) and the foreign demand for domestic assets (17) where $\tilde{d}_t^{H*} = \tilde{d}_t^F - \widetilde{nfa}_t$, which gives

$$\dot{\widetilde{nfa}}_t = r_t^* \widetilde{nfa}_t - \frac{\psi}{u'(c_t^*)}\left(\tilde{d}_t^F - \widetilde{nfa}_t\right)^2 + (1-\omega)\, S_t^{\varepsilon-1} c_t^* - c_t^F, \tag{41}$$

with the initial net foreign asset position given by $\widetilde{nfa}_0 = D_0^F/P_0^* - D_0^{H*}/(E_0 P_0^*)$; the downward nominal wage rigidity constraint (24)

$$v'(L_t^d) = \begin{cases} v'(l_t^s) & \text{if } \pi_t^W \in \left[\pi^R, +\infty\right) \\ \left[1 - \frac{\pi^R - \pi_t^W}{\alpha}\right] v'(l_t^s) & \text{if } \pi_t^W \in \left[\pi^R - \alpha, \pi^R\right] \end{cases}, \tag{42}$$

where

$$\pi_t^W = \frac{\dot{s}_t^X}{1+s_t^X} - \frac{\dot{\tau}_t^P}{1+\tau_t^P} - \frac{\dot{S}_t}{S_t} + \frac{\dot{Q}_t}{Q_t} + \pi_t; \tag{43}$$

the household's optimality condition for labor supply (15)

$$v'\left(l_t^s\right) = \left(1+s_t^X\right) \frac{1-\tau_t^L}{1+\tau_t^P} \frac{Q_t}{S_t} f'\left(L_t^d\right) u'\left(c_t\right); \tag{44}$$

and, finally, the monetary policy of inflation targeting (28)

$$\pi_t \leq \bar{\pi} \text{ and } r_t + \pi_t \geq 0 \text{ with complementary slackness.} \tag{45}$$

So, equations (34)-(45) jointly define the equilibrium of the economy.

Let us now solve for the equilibria of the economy under *laissez-faire*, i.e. with $\tau_t^C = \tau_t^I = \tau_t^L = \tau_t^F = \tau_t^P = s_t^F = s_t^X = 0$. But, before that, we need to characterize the equilibrium possibilities for the world economy.

3 World economy

Foreign countries are all identical, while our open economy is infinitely small. For simplicity, I assume that all foreign countries are in a symmetric equilibrium. Hence, the equilibrium for the whole world is jointly characterized by the above equations (34)-(45) with $c_t = c_t^*$, $r_t = r_t^*$, $\pi_t = \pi_t^*$, $Q_t = S_t = 1$, and $\widetilde{nfa}_t = \tilde{d}_t^F = 0$. Throughout my analysis, I assume that the world is in steady state. As in the closed economy analysis of Michau (2018), there are two steady state: a neoclassical and a secular stagnation steady state.

Neoclassical steady state. The neoclassical steady state $\left(c^{*n}, l^{s*n}, L^{d*n}, r^{*n}\right)$ is characterized by full employment $L^{d*n} = l^{s*n}$. Thus, consumption c^{*n} and labor supply L^{d*n} are jointly determined by the labor supply function $v'\left(L^{d*n}\right) = f'\left(L^{d*n}\right) u'\left(c^{*n}\right)$ and the goods market clearing equation $f\left(L^{d*n}\right) = c^{*n}$. The corresponding real interest rate can be deduced from the steady state Euler equation

$$r^{*n} = \rho - \frac{\gamma'(0)}{u'\left(c^{*n}\right)}, \tag{46}$$

where r^{*n} corresponds to the natural real interest rate of the world economy.

This neoclassical steady state exists if and only if the natural real interest rate satisfies $r^{*n} \geq -\bar{\pi}$. Indeed, the monetary policy of inflation targeting (45) rules out any equilibrium with a real interest rate below $-\bar{\pi}$. I henceforth assume that the condition $r^{*n} \geq -\bar{\pi}$ is satisfied. Note that, on the nominal side, the neoclassical steady state is compatible with either inflation on target, where $\pi^{*n} = \bar{\pi}$ and $i^{*n} = r^{*n} + \bar{\pi} \geq 0$, or a binding zero lower bound (*à la* Benhabib, Schmitt-Grohé, and Uribe 2001, 2002), where $\pi^{*n} = -r^{*n} \leq \bar{\pi}$ and $i^{*n} = 0$.

Secular stagnation steady state. The secular stagnation steady state $(c^{*s}, l^{s*s}, L^{d*s}, r^{*s}, i^{*s}, \pi^{*s})$ is characterized by under-employment $L^{d*n} < l^{s*n}$ and a binding zero lower bound $i^{*s} = 0$. The inflation rate is given by the binding downward wage rigidity

$$\pi^{*s} = \pi^R + \alpha \left[\frac{v'\left(L^{d*s}\right)}{v'\left(l^{s*s}\right)} - 1 \right]. \tag{47}$$

This pins down the real interest rate $r^{*s} = -\pi^{*s}$, which, by the Euler equation, determines the households' demand for consumption $1/u'\left(c^{*s}\right) = \left(\rho + \pi^{*s}\right)/\gamma'(0)$ and, by the goods market clearing condition, the firms' demand for labor $f\left(L^{d*s}\right) = c^{*s}$. Finally, labor supply is given by the labor supply function $v'\left(l^{s*s}\right) = f'\left(L^{d*s}\right) u'\left(c^{*s}\right)$. To have under-employment, i.e. $L^{d*s} < l^{s*s}$, the real interest rate $-\pi^{*s}$ must be above the natural rate r^{*n}. A sufficient condition is $r^{*n} < -\pi^R$.

It can be shown that a sufficient condition for the existence of this secular stagnation steady state is that aggregate demand is sufficiently depressed, with $r^{*n} < -\pi^R$, and that nominal wages are sufficiently downward rigid, with $\alpha < \rho + \pi^R$. I henceforth assume that these two conditions are satisfied.

Importantly, as wages become more flexible, i.e. as α rises, inflation is even lower, the real interest rate even higher, demand even more depressed and under-employment even more severe. This is the paradox of flexibility, which shows that the fundamental cause of secular stagnation is not the wage rigidity, but the existence of money, which prevents the nominal and real interest rates from falling sufficiently. The downward wage rigidity is only necessary to put a break on the deflationary spiral, that would otherwise be so strong as to prevent the existence of the secular stagnation steady state. As we shall now see, this paradox of flexibility holds for a closed economy or for the entire world, but not necessarily for a small open economy.

4 Small open economy steady states

Let us now solve for the steady state equilibria of our small open economy under *laissez-faire*. For clarity of exposition, I focus on two polar cases: perfect financial integration, where $\psi = 0$, and financial autarky, where $\psi = +\infty$. In each case, I consider both the possibility that the world is in the neoclassical steady state and that it is in the secular stagnation steady state. Trivially, it is always possible for our small open economy to be in the same equilibrium as the rest of the world. But, sometimes, it is also possible for the small open economy to be in a different equilibrium.

4.1 Perfect financial integration

Under perfect financial integration, with $\psi = 0$, the interest parity condition (38) implies that the steady state real interest rate is determined internationally, such that $r = r^*$.

Neoclassical world. If the world is in the neoclassical steady state, then by symmetry our small open economy can be in an identical steady state, with $c = c^{*n}$, $l^s = l^{s*n}$, $L^d = L^{d*n}$, $r = r^{*n}$, $\widetilde{nfa} = \tilde{d}^F = 0$, and $Q = S = 1$. By the wage rigidity (42) and the monetary policy rule (45), with full employment, we can either have inflation on target, with $\pi = \bar{\pi}$ and $i = r^{*n} + \bar{\pi} \geq 0$, or a binding zero lower bound, with $i = 0$ and $\pi = -r^{*n} \in (\pi^R, \bar{\pi}]$.

In the previous section, we have been assuming that aggregate demand is so depressed that the whole world can alternatively be in secular stagnation. We therefore have $r^* = r^{*n} < -\pi^R$. This implies that our small open-economy cannot be the only one suffering from under-employment. Indeed, whether inflation is on target $\pi = \bar{\pi} > \pi^R$ or at the zero lower bound $\pi = -r^{*n} > \pi^R$, inflation is too high for the downward nominal wage rigidity to be binding.

Paradoxically, if we had a more vibrant demand with $r^{*n} > -\pi^R$ (which would prevent the existence of the secular stagnation steady state for the world economy), then our small open economy could be the only one suffering from under-employment with a binding zero lower bound. An appreciation of the real exchange rate would make the domestic economy uncompetitive.

Stagnating world. If the world is in the secular stagnation steady state, then again, by symmetry, our small open economy can be in an identical steady state, with $c = c^{*s}$, $l^s = l^{s*s}$, $L^d = L^{d*s}$, $i = 0$, $\pi = \pi^{*s}$, $r = r^{*s} = -\pi^{*s}$, $\widetilde{nfa} = \tilde{d}^F = 0$, and $Q = S = 1$.

Interestingly, if the domestic economy has more flexible wages than foreign economies, i.e. if it has a higher α, then it must be less depressed. This can readily be seen from the downward nominal wage rigidity (42) with a fixed inflation rate $\pi = \pi^{*s} = -r^{*s} < \pi^R$, where a higher α reduces the gap between labor supply and labor demand. As wages become more downward flexible, domestic supply increases, which induces a fall in the price of domestic goods such as to stimulate their demand from consumers at home and abroad. So, under stagnation, more flexible wages allow the domestic economy to be more competitive, at the expense of the rest of the world. The paradox of flexibility does not hold because of perfect financial integration, which eliminates the general equilibrium effect of more flexible wages on inflation and on the real interest rate. This result was originally pointed out by Caballero, Farhi, and Gourinchas (2021).

Our small open economy does not have to be in this secular stagnation steady state. Alternatively, it can also be at full employment.

Proposition 1 *Under perfect financial integration, despite the world being in secular stagnation, a full employment steady state with $S \in (1, \infty)$ exists for our small open economy provided that r^{*s} is either negative or sufficiently close to zero.*[9]

A real devaluation of the domestic currency, whereby $Q > 1$ and $S > 1$, allows the small open economy to produce at full capacity despite a depressed world economy. Inflation must be on target as, otherwise, a binding zero lower bound with $\pi = -r^{*s} < \pi^R$ would imply a binding downward wage rigidity and under-employment.

It can be shown that, if the elasticities of substitution θ and ε are sufficiently close to zero, domestic consumption c and the net foreign asset position $\widetilde{nfa}$ can both be decreasing in S. Hence, the full employment steady state is not necessarily preferable to the stagnation steady state. To achieve full employment, the terms of trade must deteriorate, which adversely affect households when domestic and foreign goods are poor substitutes. This was originally pointed out by Corsetti, Mavroeidi, Thwaites, and Wolf (2018).

4.2 Financial autarky

Under financial autarky, with $\psi = \infty$, the interest parity condition (38) and the portfolio composition (39) imply that the demand for foreign assets $\tilde{d}^F$ and the net foreign asset position $\widetilde{nfa}$ are always both equal to zero. Substituting this into the national accounting identity (41) implies that trade must always be balanced, resulting in $c^{H*}/S = c^F$ (which can equivalently be written as $\tilde{P}^H c^{H*} = \tilde{P}^F c^F$).

Neoclassical world. If the world is in the neoclassical steady state, then again, by symmetry, our small open economy can be in an identical steady state, with $c = c^{*n}$, $l^s = l^{s*n}$, $L^d = L^{d*n}$, $r = r^{*n}$, and $Q = S = 1$. As in the case of perfect financial integration, we can either have inflation on target, with $\pi = \bar{\pi}$ and $i = r^{*n} + \bar{\pi} \geq 0$, or a binding zero lower bound, with $i = 0$ and $\pi = -r^{*n} \in (\pi^R, \bar{\pi}]$.

Under financial autarky, our small open economy can be in a secular stagnation steady state despite the world economy producing at full capacity.

[9] For a constant marginal utility of wealth, i.e. $\gamma''(\cdot) = 0$, it can easily be shown that this steady state exists for any value of r^{*s}.

Proposition 2 *Under financial autarky, despite the world producing at full capacity, a secular stagnation steady state with* $i = 0$ *and* $S \in (0, 1)$ *exists for our small open economy provided that we either have* $\theta > 1$ *or* $\varepsilon > 1 - \theta + 1/\sigma_u(0)$ *(where* $\sigma_u(c) = -cu''(c)/u'(c)$*).*

If domestic demand is depressed, then households' demand for imports is also depressed. Under financial autarky, this cannot generate a current account surplus and a corresponding increase in the net foreign asset position. Instead, it must induce an increase in the relative price of domestic goods, which reduces the foreign demand for domestic goods such as to restore trade balance.[10] So, under financial autarky, the depression in domestic demand depresses the foreign demand for domestic production! With a binding zero lower bound, this results in a secular stagnation steady state with under-employment, following exactly the same mechanism as in a closed economy.

Under financial autarky, we recover the paradox of flexibility.

Lemma 2 *Under financial autarky, if either* $\theta > 1$ *or* $\varepsilon > 1 - \theta + 1/\sigma_u(0)$ *and the secular stagnation steady state is unique, an increase in wage flexibility* α *reduces employment* L^d *and raises labor supply* l^s*.*

Under financial autarky, the real interest is determined domestically and, hence, aggregate demand behaves as if the economy was closed: greater wage flexibility reduces inflation, which raises the real interest rate and depresses aggregate demand. In addition, trade must be balanced at each point in time. Hence, a decrease in the domestic demand for imports must trigger an appreciation of the domestic currency such as to reduce exports. This amplifies the contraction in demand. On the supply side, the currency appreciation raises the wage rate and, hence, the labor supply.

Stagnating world. Once again, if the world is in the secular stagnation steady state, then, by symmetry, our small open economy can be in an identical steady state, with $c = c^{*s}$, $l^s = l^{s*s}$, $L^d = L^{d*s}$, $i = 0$, $\pi = \pi^{*s}$, $r = r^{*s} = -\pi^{*s}$, and $Q = S = 1$. It can be shown that the paradox of flexibility also hold in that case: if the domestic economy has more flexible wages than foreign economies, then it must be more depressed.

Our small open economy can also be at full employment.

Proposition 3 *Under financial autarky, despite the world being in secular stagnation, a full employment steady state with* $S \in (1, \infty)$ *exists for our small open economy.*

As in a closed economy, a decrease in the real interest rate stimulates aggregate demand such as to restore full employment. However, as domestic demand increases, the relative price of domestic goods declines (i.e. S increases) such as to stimulate exports, which is

[10] Our assumption $\varepsilon > 1$ is sufficient for the Marshall-Lerner condition to hold.

necessary to maintain trade balance. Hence, to reach full employment, the real interest rate does not need to fall by as much as in a closed economy, since its effect on demand is amplified by the endogenous response of the terms of trade.

5 Planner's problem

Clearly, the possibility of secular stagnation is inefficient. But, full employment achieved through a deterioration of the terms of trade $S_t = \tilde{P}_t^F / \tilde{P}_t^H$ is not necessarily efficient either. Hence, to determine the optimal policy response to stagnation, I now set up and solve the planner's problem. In the following section, I shall then characterize the combination of taxes and subsidies that implement the planner's optimal allocation.

To set up the planner's problem, we should not assume that the market mechanism operates within the domestic economy. However, the planner is constrained by the terms of trade with the rest of the world. More specifically, one unit of domestic goods can be exchanged on international markets against $S_t = \tilde{P}_t^F / \tilde{P}_t^H$ units of foreign goods. Hence, the trade surplus expressed in units of foreign goods, equal to $\tilde{P}_t^H c_t^{H*} / \tilde{P}_t^F - c_t^F = c_t^{H*}/S_t - c_t^F$, raises one-for-one the net foreign asset position expressed in units of foreign goods $\widetilde{nfa}_t$. This net foreign asset position $\widetilde{nfa}_t$ is equal to foreign assets held by domestic households $\tilde{d}_t^F = E_t D_t^F / \tilde{P}_t^F$, which yield a real return r^*, net of the domestic assets held by foreigners $\tilde{d}_t^{H*} = D_t^{H*} / \tilde{P}_t^F$, which yield some return denoted by r_t^{H*} and measured in units of foreign goods. Hence, the domestic economy's net foreign asset position evolves according to

$$\dot{\widetilde{nfa}}_t = r^* \tilde{d}_t^F - r_t^{H*} \tilde{d}_t^{H*} + \frac{c_t^{H*}}{S_t} - c_t^F. \tag{48}$$

The planner must take as given the foreign demand for domestic goods c_t^{H*} as well as the foreign demand for domestic assets $\tilde{d}_t^{H*}$. By (8), we have $c_t^{H*} = (1-\omega) S_t^{\varepsilon} c^*$. Also, by (17) with the return $r_t + s_t^F - \dot{Q}_t / Q_t$ replaced by r_t^{H*}, we have $\tilde{d}_t^{H*} = (r_t^{H*} - r^*) u'(c^*)/\psi$. Substituting these expressions, together with $\widetilde{nfa}_t = \tilde{d}_t^F - \tilde{d}_t^{H*}$, into (48) yields

$$\dot{\widetilde{nfa}}_t = r^* \widetilde{nfa}_t - \frac{\psi}{u'(c^*)} \left(\tilde{d}_t^F - \widetilde{nfa}_t \right)^2 + (1-\omega) S_t^{\varepsilon - 1} c^* - c_t^F. \tag{49}$$

The objective of the planner is to maximize the welfare of the representative household, given by (12) with c_t given by (2), the effective labor supply now denoted by L_t, net wealth equal to $\widetilde{nfa}_t$, and holdings of foreign assets equal to $\tilde{d}_t^F$. In addition to the international terms of trade constraint (49), the planner must also satisfy the resource constraint for the production of domestic goods $f(L_t) = c_t^H + c_t^{H*}$ with $c_t^{H*} = (1-\omega) S_t^{\varepsilon} c^*$.

The planner's problem therefore consists in choosing $(c_t, c_t^H, c_t^F, L_t, \widetilde{nfa}_t, \tilde{d}_t^F, S_t)_{t=0}^{\infty}$

such as to maximize intertemporal utility

$$\int_0^\infty e^{-\rho t}\left[u\left(c_t\right)-v\left(L_t\right)+\gamma\left(\widetilde{nfa}_t\right)-\frac{\psi}{2}\left(\tilde{d}_t^F\right)^2\right]dt \tag{50}$$

subject to the constraints

$$c_t=\left[\omega^{\frac{1}{\theta}}\left(c_t^H\right)^{\frac{\theta-1}{\theta}}+(1-\omega)^{\frac{1}{\theta}}\left(c_t^F\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \tag{51}$$

$$f(L_t)=c_t^H+(1-\omega)\,S_t^{\varepsilon}c^*, \tag{52}$$

$$\dot{\widetilde{nfa}}_t=r^*\widetilde{nfa}_t-\frac{\psi}{u'(c^*)}\left(\tilde{d}_t^F-\widetilde{nfa}_t\right)^2+(1-\omega)\,S_t^{\varepsilon-1}c^*-c_t^F, \tag{53}$$

$$\widetilde{nfa}_0=\frac{D_0^F}{P_0^*}-\frac{D_0^{H*}}{E_0P_0^*}, \tag{54}$$

where D_0^F, D_0^{H*}, P_0^*, and E_0 are exogenously given. In particular, by taking E_0 as given, I am assuming that the government rules out a surprise depreciation of the domestic currency such as to raise its initial net foreign asset position $\widetilde{nfa}_0$ (assuming that $D_0^{H*}>0$).

By the maximum principle, the planner's optimality conditions are given by

$$c_t^H=\omega\left(\frac{v'\left(L_t\right)}{f'\left(L_t\right)u'\left(c_t\right)}\right)^{-\theta}c_t, \tag{55}$$

$$\frac{c_t^F}{c_t^H}=\left(\frac{\varepsilon-1}{\varepsilon}\frac{1}{S_t}\right)^{\theta}\frac{1-\omega}{\omega}, \tag{56}$$

$$\left(\frac{c_t^F}{(1-\omega)c_t}\right)^{1/\theta}\frac{\tilde{d}_t^F}{u'\left(c_t\right)}=\frac{-2}{u'(c^*)}[\tilde{d}_t^F-\widetilde{nfa}_t], \tag{57}$$

$$\frac{\dot{c}_t}{c_t}=\frac{1}{\sigma_u(c_t)}\left[r^*-\rho+\left(\frac{c_t^F}{(1-\omega)c_t}\right)^{1/\theta}\frac{\gamma'\left(\widetilde{nfa}_t\right)-\psi\tilde{d}_t^F}{u'\left(c_t\right)}-\frac{1}{\theta}\left(\frac{\dot{c}_t^F}{c_t^F}-\frac{\dot{c}_t}{c_t}\right)\right]. \tag{58}$$

The planner's optimal allocation of resources is therefore the solution to equations (51)-(58).

6 Optimal policy

In this section, I characterize the tax policy that induces the decentralized equilibrium of the market economy with flexible exchange rate, given by (34)-(45), to coincide with the solution to the planner's problem, given by (51)-(58). Throughout my analysis, I now treat the inflation target $\bar{\pi}$ as a policy instrument. Within *all* the formulas of this section, the allocation $(c_t, c_t^H, c_t^F, L_t, \widetilde{nfa}_t, \tilde{d}_t^F, S_t)_{t=0}^\infty$ corresponds to the solution to the planner's problem.

The first-best allocation is characterized by full employment. Substituting the labor supply condition (44) with $l_t^s = L_t^d = L_t$ within the domestic demand for home goods (35) yields

$$c_t^H = \omega \left((1+\tau_t^C) \frac{1+\tau_t^P}{1-\tau_t^L} \frac{v'(L_t)}{f'(L_t)\, u'(c_t)} \right)^{-\theta} c_t. \tag{59}$$

This coincides with the planner's optimality condition (55) if and only if

$$1+\tau_t^C = \frac{1-\tau_t^L}{1+\tau_t^P}. \tag{60}$$

This shows that, to implement the optimal allocation, taxes must not distort labor supply.

Comparing the relative demand for foreign goods of the decentralized equilibrium (36) and of the planner's optimal allocation (56) reveals that international trade needs to be distorted by

$$\frac{1+s_t^X}{1+\tau_t^I} = 1 - \frac{1}{\varepsilon}. \tag{61}$$

Even though domestic producers are competitive, they collectively have a monopoly over the production of the domestic variety of goods that the entire world demands. Trade policy should exploit this monopoly power by reducing the price of domestic goods at home, relative to what it is abroad. This requires taxing exports or imports such that $P_t^H/P_t^F = (1-1/\varepsilon)\tilde{P}_t^H/\tilde{P}_t^F < \tilde{P}_t^H/\tilde{P}_t^F$. This strengthens the home bias in consumption relative to *laissez-faire*.

In the decentralized equilibrium, the domestic demand for foreign goods, which can be deduced from (35) and (36), implies that the real exchange rate is given by

$$\frac{1}{Q_t} = (1+\tau_t^C)(1+\tau_t^I) \left(\frac{c_t^F}{(1-\omega)c_t} \right)^{1/\theta}. \tag{62}$$

This can be substituted into equation (39) that implicitly characterizes portfolio composition within the decentralized economy. To coincide with the planner's optimal portfolio (57), we must have

$$\tau_t^F - s_t^F = \left[2(1+\tau_t^C)(1+\tau_t^I) - 1 \right] \frac{\psi}{u'(c^*)} [\tilde{d}_t^F - \widetilde{nfa}_t]. \tag{63}$$

For a given net foreign asset position $\widetilde{nfa}_t = \tilde{d}_t^F - \tilde{d}_t^{H*}$, whenever a domestic household buys an additional unit of foreign asset, foreigners must be induced to buy a unit of domestic asset. Under imperfect financial integration, i.e. with $\psi > 0$, this requires an increase in the domestic rate of return. This is costly to the domestic economy whenever $\tilde{d}_t^{H*} = \tilde{d}_t^F - \widetilde{nfa}_t > 0$. To offset this pecuniary externality, domestic capital outflows must be taxed more than foreign capital inflows are subsidized.

From the equilibrium real exchange rate (62), we have

$$\frac{\dot{Q}_t}{Q_t} = -\frac{1}{\theta}\left(\frac{\dot{c}_t^F}{c_t^F} - \frac{\dot{c}_t}{c_t}\right) - \frac{\dot{\tau}_t^C}{1+\tau_t^C} - \frac{\dot{\tau}_t^I}{1+\tau_t^I}. \tag{64}$$

Substituting this expression and the interest parity condition (38) into the Euler equation (37) yields the intertemporal allocation of consumption of the decentralized economy. To coincide with the planner's consumption Euler equation, given by (58), we must have

$$\begin{aligned}\tau_t^F \;=\; & \left[(1+\tau_t^C)(1+\tau_t^I) - 1\right]\left[\rho - r^* + \sigma_u(c_t)\frac{\dot{c}_t}{c_t} + \frac{1}{\theta}\left(\frac{\dot{c}_t^F}{c_t^F} - \frac{\dot{c}_t}{c_t}\right)\right] \\ & - \frac{\dot{\tau}_t^C}{1+\tau_t^C} - \frac{\dot{\tau}_t^I}{1+\tau_t^I}.\end{aligned} \tag{65}$$

The tax system should not create intertemporal distortions. Hence, the effects of various taxes on the timing of consumption should offset each other.

The monetary policy of inflation targeting is given by the complementary slackness condition (45), which entails a ceiling on inflation and the zero lower bound on the nominal interest rate. It implies that we must always have $r_t \geq -\bar{\pi}$. Deducing the real interest rate from the consumption Euler equation (37), this condition can be written as[11]

$$\rho - (1+\tau_t^C)(1+\tau_t^I)\left(\frac{c_t^F}{(1-\omega)c_t}\right)^{1/\theta}\frac{\gamma'(\widetilde{nfa}_t)}{u'(c_t)} + \sigma_u(c_t)\frac{\dot{c}_t}{c_t} \geq -\bar{\pi}. \tag{66}$$

This imposes a lower bound on the inflation target.

Finally, by the downward nominal wage rigidity (42), to have full employment, nominal wage growth π_t^W must be greater or equal to π^R. Two cases can be distinguished, depending on whether inflation is on target or the zero lower bound is binding. Let us first consider the former possibility.

6.1 Optimal tax policy with inflation on target

From expression (43) for π_t^W together with (64) for $\dot{Q}_t/Q_t$ and the optimal trade distortion (61) implying $\dot{s}_t^X/(1+s_t^X) = \dot{\tau}_t^I/(1+\tau_t^I)$, the inequality $\pi_t^W \geq \pi^R$ can be written as

$$-\frac{\dot{\tau}_t^P}{1+\tau_t^P} - \frac{\dot{S}_t}{S_t} - \frac{1}{\theta}\left(\frac{\dot{c}_t^F}{c_t^F} - \frac{\dot{c}_t}{c_t}\right) - \frac{\dot{\tau}_t^C}{1+\tau_t^C} + \bar{\pi} \geq \pi^R. \tag{67}$$

This imposes a second lower bound on the inflation target.

[11] Alternatively, the real interest rate can be taken from the interest parity condition (38). Despite a different formulation, the condition would be exactly equivalent to (66). This follows from the planner's optimal consumption (58) and the corresponding tax distortion (65).

We have therefore shown by construction that, when inflation is on target, the planner's optimal allocation can be implemented within our small open economy through any tax policy $(\tau_t^C, \tau_t^L, \tau_t^P, s_t^X, \tau_t^F, s_t^F)_{t=0}^{\infty}$ that satisfies (60), (61), (63), and (65), provided that the inflation target $\bar{\pi}$ is sufficiently high to satisfy both inequality (66) and (67). This entails the following proposition.

Proposition 4 *When inflation is on target, with*

$$\bar{\pi} \geq \max\left\{-\rho + \left(\frac{c_t^F}{(1-\omega)c_t}\right)^{1/\theta} \frac{\gamma'(\widetilde{nfa}_t)}{u'(c_t)} - \sigma_u(c_t)\frac{\dot{c}_t}{c_t}, \pi^R + \frac{\dot{S}_t}{S_t} + \frac{1}{\theta}\left(\frac{\dot{c}_t^F}{c_t^F} - \frac{\dot{c}_t}{c_t}\right)\right\}, \quad (68)$$

the optimal allocation can be implemented by taxing exports and taxing foreign capital inflows according to

$$s_t^X = -\frac{1}{\varepsilon} \text{ and } s_t^F = -\frac{\psi}{u'(c^*)}\tilde{d}_t^{H*}, \quad (69)$$

while leaving all the other taxes equal to zero.

The inflation target must be sufficiently high to ensure that neither the zero lower bound nor the downward wage rigidity is binding. This entails full employment, where the optimal policy consists in taxing exports such as to reduce $S_t = \tilde{P}_t^F/\tilde{P}_t^H$ and improve the domestic terms of trade. In addition, if financial integration is imperfect, foreign capital inflows should be taxed whenever they are positive such as to reduce the financial revenues paid to foreign investors.

In a nutshell, under imperfect substitutability of goods and assets, the government exploits its monopoly power in the world market for goods and for assets. When inflation is on target, the economy is at full employment and the policy intervention simply consists in exploiting this market power. By contrast, when inflation falls below target, the economy can be stuck into a stagnation equilibrium with persistent under-employment, which requires a much broader policy intervention. Let us now investigate this possibility.

6.2 Optimal tax policy at the zero lower bound

When the zero lower bound is binding, the condition for full employment $\pi_t^W \geq \pi^R$ can now be written as

$$\begin{aligned} &-\frac{\dot{\tau}_t^P}{1+\tau_t^P} - \frac{\dot{S}_t}{S_t} - \frac{1}{\theta}\left(\frac{\dot{c}_t^F}{c_t^F} - \frac{\dot{c}_t}{c_t}\right) - \frac{\dot{\tau}_t^C}{1+\tau_t^C} \\ &-\rho + (1+\tau_t^C)(1+\tau_t^I)\left(\frac{c_t^F}{(1-\omega)c_t}\right)^{1/\theta} \frac{\gamma'(\widetilde{nfa}_t)}{u'(c_t)} - \sigma_u(c_t)\frac{\dot{c}_t}{c_t} \geq \pi^R. \end{aligned} \quad (70)$$

At the zero lower bound, to implement the planner's optimal allocation, the tax policy $(\tau_t^C, \tau_t^L, \tau_t^P, s_t^X, \tau_t^F, s_t^F)_{t=0}^{\infty}$ and the inflation target $\bar{\pi}$ must satisfy (60), (61), (63), (65), as well as inequality (66) and (70).

Let us consider a policy that satisfies the first five restrictions, while ignoring the sixth (70), such as for instance the policy of Proposition 4. In general, by the downward nominal wage rigidity (42), our small open economy can either converge to a full employment or to an under-employment steady state. Intuitively, the distortions to the trade of goods and of assets implied by (61) and (63), respectively, do not modify these possibilities that arose under *laissez-faire*. (But recall from Section 4 that, sometimes, only the under-employment steady state is consistent with a binding zero lower bound.)

If the economy converges to a full employment steady state, then inequality (70) must be non-binding. The optimal allocation is implemented, despite a zero nominal interest rate and inflation falling below target. This corresponds to a liquidity trap *à la* Benhabib, Schmitt-Grohé, and Uribe 2001 and 2002. No further policy intervention is required.

Let us now turn to the more interesting case where the economy is stuck within an under-employment equilibrium. The binding downward nominal wage rigidity (42) implies that, in this equilibrium, $\pi_t^W < \pi^R$. To restore full employment, the tax system must raise the growth rate of wages π_t^W to the reference rate of inflation π^R. Hence, (70) must hold with equality. It follows that the payroll tax τ_t^P, the consumption tax τ_t^C, and the import tax τ_t^I must jointly satisfy

$$\frac{\dot{\tau}_t^P}{1+\tau_t^P} + \frac{\dot{\tau}_t^C}{1+\tau_t^C} + [1-(1+\tau_t^C)(1+\tau_t^I)]\left(\frac{c_t^F}{(1-\omega)c_t}\right)^{1/\theta}\frac{\gamma'(\widetilde{nfa}_t)}{u'(c_t)} \tag{71}$$
$$= -\pi^R - \rho + \left(\frac{c_t^F}{(1-\omega)c_t}\right)^{1/\theta}\frac{\gamma'(\widetilde{nfa}_t)}{u'(c_t)} - \sigma_u(c_t)\frac{\dot{c}_t}{c_t} - \frac{1}{\theta}\left(\frac{\dot{c}_t^F}{c_t^F} - \frac{\dot{c}_t}{c_t}\right) - \frac{\dot{S}_t}{S_t}.$$

This policy forces the under-employment equilibrium to coincide with the planner's optimal allocation. We have therefore established the following result.

Proposition 5 *If the economy is stuck within an under-employment steady state, the optimal allocation can be implemented by taxing exports and taxing foreign capital inflows according to*

$$s_t^X = -\frac{1}{\varepsilon} \text{ and } s_t^F = -\frac{\psi}{u'(c^*)}\tilde{d}_t^{H*}, \tag{72}$$

and by setting a path of payroll taxes satisfying

$$\frac{\dot{\tau}_t^P}{1+\tau_t^P} = -\pi^R - \rho + \left(\frac{c_t^F}{(1-\omega)c_t}\right)^{1/\theta}\frac{\gamma'(\widetilde{nfa}_t)}{u'(c_t)} - \sigma_u(c_t)\frac{\dot{c}_t}{c_t} - \frac{1}{\theta}\left(\frac{\dot{c}_t^F}{c_t^F} - \frac{\dot{c}_t}{c_t}\right) - \frac{\dot{S}_t}{S_t}, \tag{73}$$

while setting labor income taxes τ_t^L such as to keep $(1-\tau_t^L)/(1+\tau_t^P) = 1$ and leaving all the other taxes equal to zero.

The first element of the optimal policy corresponds to the optimal distortion to the trade of goods and of assets, which is the same as when inflation is on target. The dynamic payroll tax policy is specific to under-employment. But, does it imply a rising or falling path of payroll taxes? It depends! For simplicity, let us focus on steady state equilibria and on the polar cases of perfect financial integration, where $\psi = 0$, and of financial autarky, where $\psi = +\infty$.

In steady state with $\psi = 0$, by the planner's consumption Euler equation (58), the optimal policy equation (73) can be written as

$$\frac{\dot{\tau}_t^P}{1+\tau_t^P} = -\pi^R - r^*. \tag{74}$$

But note that, whenever $-\pi^R - r^* > 0$ (such as in a neoclassical world with $r^* = r^{*n} < -\pi^R$), the downward wage rigidity cannot be binding and the economy cannot converge to an under-employment steady state. Indeed, with $\tau_t^P = \tau_t^C = \tau_t^I = 0$, inequality (70) is already satisfied. So, if under-employment arises under perfect financial integration, it must be that $-\pi^R - r^* < 0$ (such as in a stagnating world with $r^* = r^{*s} = -\pi^{*s} > -\pi^R$). Hence, under perfect financial integration, the optimal policy response to under-employment is to set $\dot{\tau}_t^P < 0$. The intuition is that falling payroll taxes increase the marginal product of labor, which is always equal to the wage rate, thereby relaxing the downward nominal wage rigidity. Hence, sufficiently rapidly falling payroll taxes can maintain the economy at full employment. Labor income taxes need to be concurrently rising such as to keep labor supply undistorted, as specified by (60). Note that this policy raises wage inflation π^W, but does not affect price inflation π which, at the zero lower bound, remains equal to $-r^*$.

Under financial autarky, where $\psi = +\infty$, the optimal policy response to under-employment is reversed.

Lemma 3 *Under financial autarky and with the optimal trade distortion $s_t^X = -1/\varepsilon$ and all other taxes and subsidies equal to zero, if a unique secular stagnation steady state exists, then, at the planner's optimal allocation, we must have*

$$-\pi^R - \rho + \left(\frac{c^F}{(1-\omega)c}\right)^{1/\theta} \frac{\gamma'(0)}{u'(c)} > 0. \tag{75}$$

By (73) from Proposition 5, this immediately implies that the optimal policy is to set $\dot{\tau}_t^P > 0$.

Under financial autarky, under-employment results from a lack of demand. Recall that, when domestic demand is depressed, the weak demand for imports triggers an appreciation of the domestic currency such as to reduce the foreign demand for domestic goods, which is necessary to maintain trade balance. This translates into a depressed natural real interest rate, which explains inequality (75). The optimal policy consists in setting a rising path of payroll taxes such as to increase the cost of domestic production, which forces firms to set ever higher prices. At the zero lower bound, higher inflation reduces the real interest rate, which stimulates aggregate demand. Labor income taxes need to be concurrently falling such as not to distort labor supply.

Why is the optimal policy diametrically opposed depending on the degree of financial integration? This is a direct consequence of the paradox of flexibility. Under perfect financial integration, the paradox does not hold. This implies that stagnation is exclusively due to excessively high wage growth. A decreasing path of payroll taxes relaxes the downward nominal wage rigidity, thereby allowing the economy to return to full employment. By contrast, under financial autarky, the paradox of flexibility implies that the fundamental cause of stagnation is an excessively high real interest rate, not excessively high wage growth. Hence, by raising inflation, the optimal policy exploits the paradox of flexibility to overcome under-employment. In fact, the proof of Lemma 3 implies that the paradox of flexibility must hold.

In practice, it may be politically difficult to simultaneously adjust payroll taxes and labor income taxes, especially to lower the former and raise the latter. By our optimal policy formula (71), an alternative is to leave payroll taxes unchanged and adjust consumption taxes over time. The optimal path of τ_t^C is identical to the one that we have just characterized for τ_t^P, while labor income taxes τ_t^L necessitate the same adjustment as before such as to leave labor supply undistorted. In addition, the consumption tax distorts the real exchange rate, given by (62). To offset this effect and to keep $(1+\tau_t^C)(1+\tau_t^I) = 1$, we need to have $\dot{\tau}_t^I/(1+\tau_t^I) = -\dot{\tau}_t^C/(1+\tau_t^C)$. And to maintain the same trade distortion given by $(1+s_t^X)/(1+\tau_t^I) = 1-1/\varepsilon$, we must have $\dot{s}_t^X/(1+s_t^X) = \dot{\tau}_t^I/(1+\tau_t^I)$. In sum, under perfect financial integration, the optimal policy consists in falling consumption taxes and rising labor income taxes, import taxes and export subsidies. The reverse is optimal under financial autarky.

When a government faces under-employment, it could be difficult to know whether the optimal policy consists in implementing rising or falling payroll taxes. A theoretical solution to the problem is to implement rapidly falling payroll taxes. Indeed, if $\dot{\tau}_t^P/(1+\tau_t^P)$ is sufficiently negative, then the inequality (70) must be satisfied. Under financial autarky, despite the paradox of flexibility, the wage rigidity is necessary for the existence of the secular stagnation steady state. Indeed, if wages are too flexible downward, there is no break to the deflationary spiral, which is so strong as to prevent the possibility of stagnation. Nonetheless, the policy of rapidly falling payroll taxes seems reckless since, if $\dot{\tau}_t^P/(1+\tau_t^P)$ is not sufficiently negative, then, under a low degree of financial integration, the policy will make under-employment much worse.

Another problem with the optimal policy of Proposition 5 is that payroll taxes and labor income taxes can hardly be rising or falling forever. One may hope that, after the economy is at full employment for a while, households spontaneously coordinate their expectations on the full employment steady state so that, once the fiscal support is withdrawn, the economy remains there. This may require inflation to jump to target.

Let us now investigate another policy strategy, which consists in forcing inflation to be on target.

6.3 Forcing inflation to be on target

Once the inflation target as been set sufficiently high, such as to satisfy (68) from Proposition 4, the government can avoid a binding zero lower bound and under-employment by importing inflation from abroad such as to hit the target. This can be achieved through an exchange rate policy. I am assuming throughout that the optimal trade distortions, given by (69), are in place.

Under the target $\bar{\pi}$, the price level at time t must be equal $\bar{P}_t = P_0 e^{\bar{\pi}t}$ where P_0 is the initial price level. Recall that the nominal and real exchange rate are related by (10). Hence, the government should set the nominal exchange rate equal to

$$\bar{E}_t = \frac{\bar{P}_t Q_t}{P_t^*}, \tag{76}$$

where Q_t denotes the real exchange rate implied by the planner's optimal allocation as given by (62). The equilibrium of the economy is still characterized by (34)-(45), with the inflation targeting policy (45) replaced by the fixed exchange rate (76). This policy ensures that economy remains at full employment with inflation on target.[12]

This fixed exchange rate evolves over time according to

$$\frac{\dot{\bar{E}}_t}{\bar{E}_t} = \frac{\dot{Q}_t}{Q_t} + \bar{\pi} - \pi^*. \tag{77}$$

If the foreign inflation rate π^* is sufficiently high to satisfy inequality (68) from Proposition 4, then the government can set the domestic inflation target $\bar{\pi}$ equal to π^*. In that case, in steady state, a fixed nominal exchange rate $\bar{E}_t = \bar{E}$ is sufficient to import the prevailing rate of inflation from abroad. Conversely, if foreign inflation is depressed and violates

[12]More formally, if Q_t^P denotes the real exchange rate implied by the planner's optimal allocation (denoted by Q_t throughout this section), then (76) should be written as $\bar{E}_t = \bar{P}_t Q_t^P / P_t^*$. By definition of the real exchange rate (10), we also have $\bar{E}_t = P_t Q_t / P_t^*$, where P_t and Q_t now denote the *equilibrium* price level and real exchange rate, respectively. Hence, the fixed exchange rate policy entails $P_t Q_t = \bar{P}_t Q_t^P$ and, hence, $\pi_t + \dot{Q}_t / Q_t = \bar{\pi} + \dot{Q}_t^P / Q_t^P$. So, in steady state, the economy must be at full employment with inflation on target. Moreover, in the neighborhood of steady state, the downward nominal wage rigidity must be non-binding with the real variables independent of inflation. Hence, $Q_t = Q_t^P$ and, thus, $\pi_t = \bar{\pi}$.

(68), then we must have $\bar{\pi} > \pi^*$. This implies that, to generate inflation at home, the government must be committed to a crawling peg that depreciates the nominal exchange rate over time.

Importantly, the goal of this policy is to raise inflation such as to overcome the downward nominal wage rigidity and the zero lower bound; it is not to continuously depreciate the domestic currency such as to improve competitiveness. In fact, in steady state, the planner's optimal allocation is characterized by constant terms of trade.

Once households have coordinated their expectations on the full employment equilibrium, the government can revert to a traditional inflation targeting policy, where the possibility of fixing the exchange rate is an off-the-equilibrium threat that prevents inflation from falling below target with a binding zero lower bound. This can be seen as a sophisticated monetary policy *à la* Atkeson, Chari, and Kehoe (2010), where the threat of a regime change from inflation targeting to an exchange rate policy is necessary for global uniqueness. This policy was originally proposed by Svensson (2001, 2003), who was the first to advocate for relying on the exchange rate to escape a liquidity trap.

My analysis assumes that the exchange rate policy is fully credible, which forces the economy to coordinate on the planner's optimal allocation. However, if it is not fully credible, households may expect the economy to remain in stagnation. Under imperfect financial integration, this would force the government to accumulate large amount of foreign exchange reserves, which could prove extremely costly if the government eventually abandons the peg thereby validating households' expectations. Amador, Bianchi, Bocola, and Perri (2020) carefully explore this possibility, as illustrated by the experience of Switzerland over the past decade. Note that, in my model, if the economy does coordinate on the planner's optimal allocation, then the exchange rate policy is costless with no need for any intervention in foreign exchange markets along the equilibrium path.

So far, we have characterized optimal policies under a flexible exchange rate regime. Let us now turn to the case where the domestic economy is committed to a fixed peg, which can result from a common currency.

7 Fixed exchange rate

Let us now assume that the economy has a fixed exchange rate regime with $E_t = \bar{E}$. By definition of the real exchange rate $Q_t = \bar{E}P_t^*/P_t$, inflation is now given by

$$\pi_t = \pi_t^* - \frac{\dot{Q}_t}{Q_t}. \tag{78}$$

Let $\bar{W}_0$ denote the nominal wage at time 0. By the downward wage rigidity, this nominal wage can jump upward but not downward. We must therefore have $W_0 \geq \bar{W}_0$, where

W_0 is the *equilibrium* wage at time 0. From the firm's wage equation (19) together with $\tilde{P}_t^H = \tilde{P}_t^F/S_t = \bar{E}P_t^*/S_t$ and $W_0 \geq \bar{W}_0$, the initial terms of trade are constrained to satisfy

$$S_0 \leq \frac{1+s_0^X}{1+\tau_0^P}\frac{f'\left(L_0^d\right)}{\bar{W}_0}\bar{E}P_0^*. \tag{79}$$

Hence, if the initial nominal wage $\bar{W}_0$ is excessively high and the currency is over-valued, i.e. $\bar{E}$ is excessively low, then the terms of trade are so strong that the domestic economy is uncompetitive, i.e. $S_0 = \tilde{P}_0^F/\tilde{P}_0^H$ is too low.

The equilibrium of the economy under a fixed exchange rate regime is characterized by the same equations as before, given by (34)-(45), with the inflation targeting policy (45) replaced by the new expression for inflation (78) together with the constraint on the initial terms of trade (79).

7.1 Equilibrium possibilities under *laissez-faire*

In a fixed exchange rate regime, the steady state possibilities under *laissez-faire* are more limited than under flexible exchange rate. This is due to the fact that inflation is imported from abroad since, by (43) and (78), we must have $\pi^W = \pi = \pi^*$. Hence, in a neoclassical world producing at full employment, foreign inflation satisfies $\pi^* > \pi^R$. So, the domestic nominal wage rigidity cannot be binding, resulting in full employment. Conversely, in a stagnating world, we have $\pi^* = \pi^{*s} \in [\pi^R - \alpha, \pi^R]$, implying that the domestic economy must also be in a stagnation steady state with under-employment. In sum, if the domestic economy reaches a steady state, it must be symmetric to the rest of the world. With an exogenous inflation rate, the paradox of flexibility cannot hold within the stagnation steady state, regardless of the degree of financial integration. More flexible wages induce a depreciation of the real exchange rate, which allows the economy to be more competitive thereby reducing under-employment.[13]

How does the economy respond when the fixed exchange rate is initially over-valued? This results in a binding downward nominal wage rigidity and, hence, in excessively strong terms of trade, where the excessively low S_0 is pinned down by (79) with equality. This generates under-employment. From the binding wage rigidity, given by (42) and (43), the terms of trade (under *laissez-faire*) subsequently evolve according to

$$\frac{\dot{S}_t}{S_t} = \pi^* - \pi^R + \alpha\left[1 - \frac{v'\left(L_t^d\right)}{v'(l_t^s)}\right], \tag{80}$$

[13] Gali and Monacelli (2016) find that increased wage flexibility can be welfare reducing under a fixed exchange rate regime. However, their analysis focuses on the business cycle and incorporates a welfare loss from price and wage volatility, which drives their result and which is absent from my analysis. In addition, they find that the adverse effect of wage flexibility essentially disappears as prices also become more flexible.

where $l_t^s > L_t^d$. In neoclassical world, $\pi^* > \pi^R$. Hence, S_t increases over time. In a stagnating world, by definition of π^{*s} given by (47), we have

$$\frac{\dot{S}_t}{S_t} = \alpha \left[\frac{v'\left(L_t^{d*s}\right)}{v'(l_t^{s*s})} - \frac{v'\left(L_t^d\right)}{v'(l_t^s)} \right]. \tag{81}$$

The excessively low S_0 results in the domestic economy being less competitive than foreign economies and, hence, in $L_t^d < L_t^{d*s}$ and $l_t^s > l_t^{s*s}$. Thus, S_t increases over time. In both cases, the terms of trade gradually weaken over time, allowing the economy to regain competitiveness and converge to a steady state that is symmetric to the rest of the world, where $S = 1$.

7.2 Optimal policy

To implement the planner's optimal allocation under a fixed exchange regime, the optimal policy requires the same wedges as before to labor supply (60), to trade (61), to portfolio composition (63), and to intertemporal consumption (65). In addition, the zero lower bound cannot be violated, which from (38) and (78) requires

$$r^* + \pi^* - (1+\tau_t^C)(1+\tau_t^I) \left(\frac{c_t^F}{(1-\omega)c_t} \right)^{1/\theta} \frac{\psi}{u'(c_t)} \tilde{d}_t^F - \tau_t^F \geq 0. \tag{82}$$

Also, the downward nominal wage rigidity must be non-binding. From (43) and (78), the condition $\pi_t^W \geq \pi^R$ can be written as

$$\frac{\dot{s}_t^X}{1+s_t^X} - \frac{\dot{\tau}_t^P}{1+\tau_t^P} - \frac{\dot{S}_t}{S_t} + \pi^* \geq \pi^R. \tag{83}$$

Within a neoclassical world, no intervention is needed in steady state. By contrast, in a stagnating world, where $\pi^{*s} < \pi^R$, payroll taxes need to be falling forever to maintain the economy at full employment. Alternatively, export subsidies and import taxes can both be rising forever, such as to maintain constant the trade distortion given by $(1 + s_t^X)/(1+\tau_t^I) = 1 - 1/\varepsilon$. Note that the policy is the same regardless of the degree of financial integration. Now that the paradox of flexibility never holds, the optimal policy always consists in relaxing the downward nominal wage rigidity.

Finally, to implement the planner's allocation, the terms of trade must not be so strong as to make the domestic economy uncompetitive. Formally, the inequality constraint (79) must not be binding, which requires

$$\frac{1+s_0^X}{1+\tau_0^P} \geq \frac{\bar{W}_0}{f'\left(L_0^d\right)} \frac{S_0}{\bar{E} P_0^*}, \tag{84}$$

where L_0^d and S_0 correspond to the planner's allocation. If, initially, the exchange rate is over-valued, the government must implement a fiscal devaluation *à la* Farhi, Gopinath, Itskhoki (2014). This can be achieved through a discrete decrease in the level of payroll taxes, compensated by a rise in consumption or labor income taxes such as to keep labor supply undistorted. Alternatively, the fiscal devaluation can be achieved through a rise in export subsidies, accompanied by a rise in import taxes such as to leave the trade distortion unaffected. Note that, unlike a depreciation of the domestic currency, the fiscal devaluation does not reduce the value of domestic liabilities (in units of foreign goods).

We have therefore established by construction the following result.

Proposition 6 *The planner's optimal allocation can be implemented under a fixed exchange rate regime through any tax policy* $(\tau_t^C, \tau_t^L, \tau_t^P, s_t^X, \tau_t^F, s_t^F)_{t=0}^{\infty}$ *that satisfies (60), (61), (63), (65), (82), (83), and (84).*

The optimal policy consists in distorting trade in goods and in assets as specified in Proposition 4, while implementing a fiscal devaluation if the exchange rate is initially over-valued. If foreign inflation is sufficiently high, this is sufficient to be at full employment; but if foreign inflation is very depressed, payroll taxes must falling forever.

Clearly, to avoid stagnation, it is impractical to have payroll taxes falling forever. There are two alternatives: one is to unilaterally abandon the peg; the other is for the coalition of countries sharing the fixed peg, which may form a currency union, to cooperate (where, for simplicity, I am assuming this coalition to be of zero mass). This brings us back to the floating exchange rate regime, where the optimal policy consists in setting a sufficiently high inflation target, for the individual country or for the coalition as a whole. Bringing inflation to target can either be achieved through the exchange rate policy that imports inflation from the rest of the world or through the payroll tax policy of Proposition 5 hoping that, once the economy is at full employment, inflation expectations jump to target. Note that, under financial autarky, the optimal payroll tax policy for a non-cooperating individual country with the fixed peg or within the currency union is the opposite of the cooperative optimal policy. By contrast, under perfect capital mobility, the non-cooperative and cooperative payroll tax policies are identical. In this last case, there is nonetheless a positive externality from the other countries within the coalition following the same policy as this could induce inflation for the whole coalition to jump to target.

8 Conclusion

This analysis has shown that the nature of secular stagnation within an open economy is fundamentally determined by the extent of financial market integration. Under perfect capital mobility, falling wages enhance the competitiveness of the domestic economy at the expense of the rest of the world; while under financial autarky, falling wages reduce inflation, which causes the real interest rate to rise and the exchange rate to appreciate, both of which depress aggregate demand.

This fundamental difference in the nature of stagnation calls for diametrically opposed policy responses. Under perfect financial integration, payroll taxes should be falling and labor income taxes rising such as to relax the downward nominal wage rigidity; while under financial autarky, the opposite should be done such as to create inflationary pressures.

For a policymaker facing stagnation, it may be difficult to know which of these two cases applies. However, there is one policy that always restores full employment: set the inflation target sufficiently high and raise inflation to target. Under stagnation, two constraints are binding: the downward nominal wage rigidity and the zero lower bound. The fundamental cause of stagnation is the former under perfect financial integration and the latter under financial autarky. But both constraints can be alleviated through sufficiently high inflation. The government can rely on an exchange rate policy to import inflation from abroad. This may require a crawling peg if foreign inflation is below target.

Under a fixed exchange rate regime, such as within a currency union, all countries are sharing the same fate. In particular, they cannot individually raise inflation. Hence, when mired in stagnation, they have a strong incentive to either cooperate or abandon the peg.

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A Foreign Demand for Domestic Assets

Let us solve the portfolio problem of the representative household of country j. I introduce the following notations: $D_t^i(j)$ denotes household j's holdings of bonds from country i, $E_t^i(j)$ is the price of one unit of currency i in currency j, $P_t(j)$ is the domestic price level in country j, $P_t^F(j)$ is the price of the foreign basket in country j, $Q_t(j) = P_t^F(j)/P_t(j)$ is the real exchange rate of country j, $i_t(j)$ is the nominal interest rate in country j, and $c_t(j)$ is the overall consumption level in country j.

Household j's disutility from holding foreign assets is given by

$$\frac{-\psi}{2}\int_0^1 \left(\frac{E_t^i(j)D_t^i(j)}{P_t^F(j)}\right)^2 di.$$

Household j's income from holding foreign, rather than domestic, assets are equal to

$$\int_0^1 \left[i_t(i) + \frac{\dot{E}_t^i(j)}{E_t^i(j)} - i_t(j)\right]\frac{E_t^i(j)D_t^i(j)}{P_t(j)}di.$$

Incorporating these features within household j's intertemporal problem and deriving the first-order condition with respect to $D_t^i(j)$ yields

$$\psi\left(\frac{E_t^i(j)}{P_t^F(j)}\right)^2 D_t^i(j) = u'(c_t(j))\left[i_t(i) + \frac{\dot{E}_t^i(j)}{E_t^i(j)} - i_t(j)\right]\frac{E_t^i(j)}{P_t(j)},$$

which can be written as

$$\frac{E_t^i(j)D_t^i(j)}{P_t(j)} = \frac{[Q_t(j)]^2 u'(c_t(j))}{\psi}\left[i_t(i) + \frac{\dot{E}_t^i(j)}{E_t^i(j)} - i_t(j)\right].$$

If j is the domestic economy, then its demand for foreign assets is given by

$$\begin{aligned} d^F &= \int_0^1 \frac{E_t^i(j)D_t^i(j)}{P_t(j)}di, \\ &= \frac{[Q_t(j)]^2 u'(c_t(j))}{\psi}\int_0^1 \left[i_t(i) + \frac{\dot{E}_t^i(j)}{E_t^i(j)} - i_t(j)\right]di. \end{aligned}$$

But, as j is the domestic economy, we have $Q_t(j) = Q_t$, $c_t(j) = c_t$, $i_t(i) = i_t^* - \tau_t^F$, $i_t(j) = i_t$, and $E_t^i(j) = E_t$. This gives the domestic demand for foreign assets (16) where, by definition of the domestic real exchange rate $Q_t = E_t P_t^*/P_t$, we must have $i_t^* + \dot{E}_t/E_t - i_t = r_t^* + \dot{Q}_t/Q_t - r_t$.

Let us now solve for the world demand for domestic assets. Assuming that j is the domestic economy, the world demand for domestic assets is given by

$$
\begin{aligned}
d_t^{H*} &= \frac{\int_0^1 D_t^j(i)di}{P_t(j)}, \\
&= \frac{1}{P_t(j)} \int_0^1 \frac{P_t(i)[Q_t(i)]^2 u'(c_t(i))}{\psi E_t^j(i)} \left[i_t(j) + \frac{\dot{E}_t^j(i)}{E_t^j(i)} - i_t(i) \right] di.
\end{aligned}
$$

But, as j is the domestic economy, we have $P_t(j) = P_t$, $P_t(i) = P_t^F(i) = P_t^*$, $Q_t(i) = P_t^F(i)/P_t(i) = 1$, $E_t^j(i) = 1/E_t^i(j) = 1/E_t$, $c_t(i) = c_t^*$, $i_t(i) = i_t^*$, and $i_t(j) = i_t + s_t^F$. This implies

$$
d_t^{H*} = \frac{E_t P_t^* u'(c_t^*)}{P_t \psi} \left[i_t + s_t^F - \frac{\dot{E}_t}{E_t} - i_t^* \right].
$$

With the domestic real exchange rate given by $Q_t = E_t P_t^*/P_t$, which implies that $\dot{E}_t/E_t = \dot{Q}_t/Q_t + \pi_t - \pi_t^*$,this yields the foreign demand for domestic assets (17).

B Proof of Lemma 1

Using the labor supply function (15), the downward nominal wage rigidity (23) can be written as

$$
v'\left(L_t^d\right) \leq \left[1 - \frac{\pi^R - \pi_t^W}{\alpha} \right] v'(l_t^s), \tag{A1}
$$

where $\pi_t^W = \dot{s}_t^X/(1+s_t^X) - \dot{\tau}_t^P/(1+\tau_t^P) + \tilde{\pi}_t^H$. Clearly, if $\pi_t^W < \pi^R - \alpha$, this inequality cannot be satisfied. We can therefore consider that $\pi_t^W \geq \pi^R - \alpha$. By definition of the terms of trade (9) and of the real exchange rate (10), we have $\tilde{\pi}_t^H = \dot{Q}_t/Q_t + \pi_t - \dot{S}_t/S_t$.

By complementary slackness, if this wage rigidity constraint (A1) is slack, then $L_t^d = l_t^s$. By (A1) with strict inequality together with $L_t^d = l_t^s$, this implies $\pi_t^W > \pi^R$. Hence, if $\pi_t^W \in \left[\pi^R - \alpha, \pi^R\right]$, then the wage rigidity (A1) must be binding.

By complementary slackness, if $L_t^d < l_t^s$, then the wage rigidity (A1) must be binding. This, together with $L_t^d < l_t^s$, implies that $v'(l_t^s) > v'\left(L_t^d\right) = [1 - (\pi^R - \pi_t^W)/\alpha] v'(l_t^s)$ and, therefore, $\pi_t^W < \pi^R$. Hence, if $\pi_t^W \in \left[\pi^R, +\infty\right)$, then $L_t^d = l_t^s$.

C Proof of Proposition 1

Under perfect financial integration and with a stagnating world, the real interest rate is given by $r = r^{*s} = -\pi^{*s} \in [-\pi^R, \alpha - \pi^R]$. The monetary policy rule (45) implies that we must either have a binding zero lower bound or inflation on target. In the former case, inflation would be equal to $\pi = -r^{*s} \leq \pi^R < \bar{\pi}$ which, by the wage rigidity (42), is inconsistent with full employment. Thus, in a full employment equilibrium, we must have inflation on target $\pi = \bar{\pi}$ with the nominal interest rate therefore equal to $i = r^{*s} + \bar{\pi} \geq -\pi^R + \bar{\pi} > 0$.

The equilibrium is defined by (34)-(45). Substituting (35) and (36) into (34) yields

$$Q = \left[\omega S^{\theta-1} + 1 - \omega\right]^{\frac{1}{\theta-1}}.$$

Also, substituting (40) into (41) and using the fact that (without taxes) $\tilde{P}_t^F c_t^F + \tilde{P}_t^H c_t^H = P_t c_t$ or, equivalently $c_t^F = c_t/Q_t - c_t^H/S_t$, we obtain

$$\frac{c}{Q} = \frac{f\left(L^d\right)}{S} + r^{*s}\widetilde{nfa}.$$

A full employment steady state of the small open economy $(c, L^d, \widetilde{nfa}, Q, S)$ is therefore jointly characterized by

$$Q = \left[\omega S^{\theta-1} + 1 - \omega\right]^{\frac{1}{\theta-1}}; \tag{A2}$$

$$c = \frac{Q}{S} f\left(L^d\right) + r^{*s} Q\widetilde{nfa}; \tag{A3}$$

$$\gamma'\left(\widetilde{nfa}\right) = (\rho - r^{*s}) Q u'(c); \tag{A4}$$

$$v'\left(L^d\right) = \frac{Q}{S} f'\left(L^d\right) u'(c); \tag{A5}$$

$$f(L^d) = \omega\left(\frac{S}{Q}\right)^{\theta} c + (1-\omega) S^{\varepsilon} c^{*s}. \tag{A6}$$

The first of these equations defines the function $Q(S)$. Using this function, the second equation defines $c(\widetilde{nfa}, L^d, S)$. Using these two functions, the third equation defines $\widetilde{nfa}(L^d, S)$. Using all these functions, the fourth equation defines $L^d(S)$. Finally, let us define

$$g(L^d, S) = f(L^d) - \omega\left(\frac{S}{Q(S)}\right)^{\theta} c(\widetilde{nfa}(L^d, S), L^d, S) - (1-\omega) S^{\varepsilon} c^{*s}. \tag{A7}$$

To prove the existence of a steady state with $S \in (1, \infty)$, I show that $g(L^d(S), S)$ is a continuous function of S with $g(L^d(1), 1) > 0$ and $\lim_{S\to\infty} g(L^d(S), S) < 0$.

By definition, $Q(S)$ and $c(\widetilde{nfa}, L^d, S)$ trivially are continuous functions. The function $\widetilde{nfa}(L^d, S)$ is defined by

$$\gamma'\left(\widetilde{nfa}\right) = (\rho - r^{*s}) Q(S) u'\left(\frac{Q(S)}{S} f\left(L^d\right) + r^{*s} Q(S)\widetilde{nfa}\right), \tag{A8}$$

where $\rho - r^{*s} > 0$. The left-hand side is decreasing in $\widetilde{nfa}$; while, when $r^{*s} < 0$, the right-hand side is increasing. Moreover, the left-hand side is greater than the right-hand side for $\widetilde{nfa}$ sufficiently negative, while the opposite is true for $\widetilde{nfa}$ sufficiently large. This shows that, when $r^{*s}<0$, $\widetilde{nfa}$ is uniquely defined as a continuous function of $L^d \in [0,\infty)$ and $S \in (0,\infty)$. It can easily be seen that, when $r^{*s}<0$, $\widetilde{nfa}(L^d,S)$ is a strictly increasing function of L^d.

Substituting the third steady state condition (A4) into the fourth one (A5) yields the following expression, which implicitly defines the function $L^d(S)$,

$$(\rho - r^{*s})S\frac{v'(L^d)}{f'(L^d)} = \gamma'\left(\widetilde{nfa}(L^d,S)\right). \tag{A9}$$

When $r^{*s}<0$, $\widetilde{nfa}(L^d,S)$ is strictly increasing in L^d. Hence, the left-hand side is strictly increasing in L^d and goes from 0 to ∞ as L^d increases from 0 to $\bar{l}$ (recall that $\bar{l}$ is the maximum feasible labor supply defined by $\lim_{L\to\bar{l}} v'(L)=\infty$); while the right-hand side is decreasing in L^d. Thus, L^d is uniquely defined as a continuous function of S for any $S\in(0,\infty)$. It follows that $g(L^d(S),S)$ is indeed continuous when $r^{*s}<0$.

Let us now show that $g(L^d(1),1)>0$. At the world level, the secular stagnation steady state $(c^{*s},l^{s*s},L^{d*s},r^{*s})$ satisfies

$$\begin{aligned} v'(L^{d*s}) \ &< \ v'(l^{d*s}) = f'(L^{d*s})u'(c^{*s}) \\ &= f'(L^{d*s})\frac{\gamma'(0)}{\rho - r^{*s}} = f'(L^{d*s})\frac{\gamma'\left(\widetilde{nfa}(L^d(1),1)\right)}{\rho - r^{*s}}, \end{aligned} \tag{A10}$$

where the first equality follows from the labor supply function (44), the second from the Euler equation (37), and the last from the definition of the function $\widetilde{nfa}(L^d,S)$ given by (A8) which implies that $\widetilde{nfa}(L^{d*s},1)=0$ (since $f\left(L^{d*s}\right)=c^{*s}$). Using the definition of the function $L^d(S)$, given by (A9), we know that $L^d(1)$ is the solution to

$$v'(L^d(1)) = f'(L^d(1))\frac{\gamma'\left(\widetilde{nfa}(L^d(1),1)\right)}{\rho - r^{*s}}.$$

The left-hand side of this expression is increasing in $L^d(1)$ while, when $r^{*s}<0$, the left-hand side is decreasing. This, together with inequality (A10), immediately implies that $L^d(1)>L^{d*s}$.

The characterization of the secular stagnation steady state implies $g(L^{d*s},1)=0$. Substituting the second steady state equation (A3) into the definition of the function $g(L^d,S)$ given by (A7) yields

$$\begin{aligned} g(L^d,S) \ &= \ \left[1-\omega\left(\frac{Q(S)}{S}\right)^{1-\theta}\right]f(L^d) \\ &\quad -(1-\omega)S^{\varepsilon}c^{*s} - \omega S^{\theta}[Q(S)]^{1-\theta}r^{*s}\widetilde{nfa}(L^d,S). \end{aligned} \tag{A11}$$

Thus, when $r^{*s} < 0$, $g(L^d, 1)$ is a strictly increasing function of L^d. This, together with $g(L^{d*s}, 1) = 0$ and $L^d(1) > L^{d*s}$, implies that $g(L^d(1), 1) > 0$.

Finally, we need to show that $\lim_{S\to\infty} g(L^d(S), S) < 0$. By definition of $Q(S)$, we have

$$\lim_{S\to\infty} \frac{Q(S)}{S} = \begin{cases} 0 & \text{if } \theta \leq 1 \\ \omega^{\frac{1}{\theta-1}} & \text{if } \theta > 1 \end{cases},$$

and

$$\lim_{S\to\infty} Q(S) = \begin{cases} (1-\omega)^{\frac{1}{\theta-1}} & \text{if } \theta \leq 1 \\ \infty & \text{if } \theta > 1 \end{cases}.$$

When $\theta \leq 1$, by definition of $\widetilde{nfa}(L^d, S)$ given by (A8), $\lim_{S\to\infty} \widetilde{nfa}(L^d, S)$ is finite and negative and, by definition of $L^d(S)$ given by (A9), $\lim_{S\to\infty} L^d(S) = 0$. When $\theta > 1$, by definition of $\widetilde{nfa}(L^d, S)$, we must have $\lim_{S\to\infty} Q(S)\widetilde{nfa}(L^d, S) = -\infty$ and, by definition of $L^d(S)$, this implies $\lim_{S\to\infty} L^d(S) = 0$.[14] Hence, by definition of the function $g(L^d, S)$ given by (A11), wether $\theta \leq 1$ or $\theta > 1$, we must have $\lim_{S\to\infty} g(L^d(S), S) = -\infty < 0$.

By continuity, the full employment equilibrium with $S \in (1, \infty)$ still exists for r^{*s} positive, but sufficiently close to zero.[15]

D Proof of Proposition 2

By the downward nominal wage rigidity (42), under-employment implies that inflation must be below target. By the monetary policy rule (45), this implies a binding zero lower bound.

The equilibrium is defined by (34)-(45) with $\widetilde{nfa} = 0$ and $r = -\pi$. Hence, a secular stagnation steady state of the small open economy under financial autarky (c, L^d, π, Q, S)

[14] By definition of $\widetilde{nfa}(L^d, S)$ given by (A8), we have

$$Q(S) = \frac{\gamma'\left(\widetilde{nfa}(L^d, S)\right)}{(\rho - r^{*s})u'\left(\frac{Q(S)}{S} f\left(L^d\right) + r^{*s} Q(S)\widetilde{nfa}(L^d, S)\right)}.$$

As the left-hand side tends to infinity when $\theta > 1$, on the right-hand side, with $r^{*s} < 0$, we must either have $\lim_{S\to\infty} \widetilde{nfa}(L^d, S) = \underline{a} < 0$ in the numerator (where $\underline{a}$ is such that $\lim_{a\to\underline{a}} \gamma'(a) = \infty$) or $\lim_{S\to\infty} Q(S)\widetilde{nfa}(L^d, S) = -\infty$ in the denominator, both of which imply $\lim_{S\to\infty} Q(S)\widetilde{nfa}(L^d, S) = -\infty$. Combining the definition of $L^d(S)$ and $\widetilde{nfa}(L^d, S)$ given by (A9) and (A8), we have

$$\frac{v'(L^d(S))}{f'(L^d(S))} = \frac{Q(S)}{S} u'\left(\frac{Q(S)}{S} f\left(L^d(S)\right) + r^{*s} Q(S)\widetilde{nfa}(L^d(S), S)\right),$$

which implies $\lim_{S\to\infty} L^d(S) = 0$.

[15] Within the proof of existence, r^{*s} only has a discontinuous effect in the limit as S tends to infinity. In that case, with $r^{*s} > 0$, we have $\lim_{S\to\infty} \widetilde{nfa}(L^d, S)$ finite and positive when $\theta \leq 1$ and $\lim_{S\to\infty} Q(S)\widetilde{nfa}(L^d, S) = \infty$ when $\theta > 1$. Hence, $\lim_{S\to\infty} r^{*s} Q(S)\widetilde{nfa}(L^d, S)$ remains positive and we still have $\lim_{S\to\infty} g(L^d(S), S) = -\infty < 0$.

is jointly characterized by

$$Q = \left[\omega S^{\theta-1} + 1 - \omega\right]^{\frac{1}{\theta-1}}; \tag{A12}$$

$$c = \frac{Q}{S} f\left(L^d\right); \tag{A13}$$

$$f(L^d) = \omega \left(\frac{S}{Q}\right)^{\theta} c + (1-\omega) S^{\varepsilon} c^{*n}; \tag{A14}$$

$$\gamma'(0) = (\rho + \pi) Q u'(c); \tag{A15}$$

$$v'\left(L^d\right) = \frac{\pi - (\pi^R - \alpha)}{\alpha} \frac{Q}{S} f'\left(L^d\right) u'(c); \tag{A16}$$

where, as in the proof of Proposition 1, (A12) follows from (34), (35) and (36), while (A13) follows from (40) and (41) with $c_t^F = c_t/Q_t - c_t^H/S_t$. The first of these equations defines the function $Q(S)$. Substituting the first two steady state condition (A12) and (A13) into the third one (A14) and rearranging terms yields

$$f(L^d) = S^{\varepsilon} \left[Q(S)\right]^{\theta-1} c^{*n}, \tag{A17}$$

which defines $L^d(S)$. Substituting this expression back into the second steady state condition (A13) yields

$$c = S^{\varepsilon-1} \left[Q(S)\right]^{\theta} c^{*n}, \tag{A18}$$

which defines $c(S)$. From the fourth steady state condition (A15), we have

$$\pi = \frac{\gamma'(0)}{Q(S) u'(c(S))} - \rho, \tag{A19}$$

which defines $\pi(S)$. Finally, let us define

$$g(S) = \frac{v'\left(L^d(S)\right)}{(Q(S)/S) f'\left(L^d(S)\right) u'(c(S))} - \frac{\pi(S) - (\pi^R - \alpha)}{\alpha}, \tag{A20}$$

which is a continuous function of S on $(0, \infty)$. A secular stagnation steady state is characterized as the solution to $g(S) = 0$. To prove the existence of such a steady state with $S \in (0,1)$, I show that $g(1) < 0$ and $g(0) > 0$. In addition, in steady state, we must have $\pi(S) < \pi^R$ as, otherwise, we cannot have under-employment.

We trivially have $f(L^d(1)) = c^{*n}$ and $c(1) = S^{\varepsilon-1} \left[Q(S)\right]^{\theta} c^{*n}$. Also, recall that $f(L^{d*n}) = c^{*n}$ and $v'(L^{d*n}) = f'(L^{d*n}) u'(c^{*n})$. Substituting these expressions into $g(1)$ yields

$$g(1) = \frac{\pi^R - \pi(1)}{\alpha}.$$

We also have $\pi(1) = \gamma'(0)/u'(c^{*n}) - \rho = -r^{*n} > \pi^R$. Hence, $g(1) < 0$.

By definition of $Q(S)$, we have

$$\lim_{S\to 0}\frac{Q(S)}{S}=\begin{cases}\omega^{\frac{1}{\theta-1}} & \text{if } \theta\leq 1\\ \infty & \text{if } \theta>1\end{cases},$$

and

$$\lim_{S\to 0}Q(S)=\begin{cases}0 & \text{if } \theta\leq 1\\ (1-\omega)^{\frac{1}{\theta-1}} & \text{if } \theta>1\end{cases}.$$

Hence, by definition of the function $L^d(S)$ and $c(S)$, given by (A17) and (A18), we have $L^d(0)=0$ and $c(0)=0$. It follows that

$$\begin{aligned} g(0) &= -\frac{\pi(0)-(\pi^R-\alpha)}{\alpha},\\ &= -\frac{1}{\alpha}\frac{\gamma'(0)}{Q(0)u'(0)}+\frac{\rho+\pi^R-\alpha}{\alpha}. \end{aligned} \tag{A21}$$

When $\theta>1$, we trivially have $g(0)=[\rho+\pi^R-\alpha]/\alpha$. But, recall that $\alpha<\rho+\pi^R$ was assumed to ensure the existence of a secular stagnation steady state at the world level. Hence, $g(0)>0$.

When $\theta\leq 1$, we need to compute the limit of $Q(S)u'(c(S))$ as S tends to 0. Using L'Hôpital's rule, we have

$$\begin{aligned} \lim_{s\to 0}Q(S)u'(c(S)) &= \lim_{s\to 0}\frac{u'\left(S^{\varepsilon-1}[Q(S)]^{\theta}c^{*n}\right)}{[Q(S)]^{-1}},\\ &= \lim_{s\to 0}\sigma_u(c(S))\left[\frac{\varepsilon-1}{\omega}\left(\frac{Q(S)}{S}\right)^{\theta-1}+\theta\right]Q(S)u'(c(S)),\\ &= \sigma_u(0)[\varepsilon+\theta-1]\lim_{s\to 0}Q(S)u'(c(S)), \end{aligned}$$

where $\sigma_u(c)=-cu''(c)/u'(c)$. Hence, if $\sigma_u(0)[\varepsilon+\theta-1]\neq 1$, $\lim_{s\to 0}Q(S)u'(c(S))$ must either be equal to zero or infinity. When $\theta\leq 1$, the derivative of $Q(S)u'(c(S))$ with respect to S evaluated at 0 is equal to $[1-\sigma_u(0)[\varepsilon+\theta-1]]\omega^{\frac{1}{\theta-1}}u'(0)$. Hence, when $1-\sigma_u(0)[\varepsilon+\theta-1]<0$, this derivative is equal to $-\infty$, implying that $\lim_{s\to 0}Q(S)u'(c(S))=\infty$. Conversely, when $1-\sigma_u(0)[\varepsilon+\theta-1]>0$, this derivative is equal to ∞, implying that $\lim_{s\to 0}Q(S)u'(c(S))=0$. From (A21), this establishes that, when $\theta\leq 1$, we have $g(0)>0$ if and only if $\varepsilon>1-\theta+1/\sigma_u(0)$.

Finally, we need to check that, in steady state, $\pi(S)<\pi^R$. The steady state is characterized by $g(S)=0$ and, from (A20), it must therefore satisfy

$$\frac{v'\left(L^d(S)\right)}{(Q(S)/S)f'\left(L^d(S)\right)u'(c(S))}-1=\frac{\pi(S)-\pi^R}{\alpha}.$$

It can easily be shown that the left-hand side is an increasing function of S and it is equal to 0 when $S = 1$. Hence, it must be negative whenever $S \in (0, 1)$, which immediately implies that $\pi(S) < \pi^R$.

E Proof of Lemma 2

The equilibrium of Proposition 2 can be represented through an aggregate demand and an aggregate supply curve. By (A12) and (A17), we have

$$f(L^d) = [\omega S^{\varepsilon+\theta-1} + (1-\omega)\, S^{\varepsilon}]c^{*n},$$

which implicitly defines $S(L^d)$ (which is the inverse of the function $L^d(S)$ from Proposition 2). The aggregate demand relationship is given by

$$\pi^{AD}(L^d) = \pi(S(L^d)),$$

where the function $\pi(S)$ is given by (A19). We have $\pi^{AD}(L^{d*}) = \pi(S(L^{d*})) = \pi(1) = -r^{*n} > \pi^R$. Similarly, $\pi^{AD}(0) = \pi(S(0)) = \pi(0)$. From the proof of Proposition 2, we know that if either $\theta > 1$ or $\varepsilon > 1-\theta+1/\sigma_u(0)$, we have $\pi(0) = -\rho$. Hence, $\pi^{AD}(0) = -\rho$.

By (A16), the aggregate supply relationship is given by

$$\pi^{AS}(L^d) = \pi^R - \alpha \left[1 - \frac{v'(L^d)}{[Q(S(L^d))/S(L^d)]f'(L^d)u'(c(S(L^d)))}\right], \tag{A22}$$

where $Q(S)$ and $c(S)$ are given by (A12) and (A18). We have $S(L^{d*}) = 1$ and, hence, $\pi^{AS}(L^{d*}) = \pi^R$. We also have $S(0) = 0$ and, hence, $\pi^{AS}(0) = \pi^R - \alpha$.

We have therefore shown that $\pi^{AD}(0) = -\rho < \pi^R - \alpha = \pi^{AS}(0)$ and $\pi^{AD}(L^{d*n}) = -r^n > \pi^R = \pi^{AS}(L^{d*n})$. The aggregate supply curve is strictly increasing. Hence, in (A22), the term in square bracket is decreasing in L^d and equal to 0 when $L^d = L^{d*n}$. This term must therefore be positive whenever $L^d < L^{d*n}$. So, an increase in α must shift the aggregate supply curve $\pi^{AS}(L^d)$ downward for all $L^d \in [0, L^{d*n})$. Assuming that the steady state equilibrium is unique, a simple graphical analysis reveals that, as wages become more flexible, i.e. as α increases, both equilibrium labor demand L^d and inflation π decrease. Indeed, the fact that the aggregate supply curve is increasing implies that, at the unique intersection, the aggregate demand curve must also be increasing.

Recall, by (44), that labor supply is given by $v'(l^s) = [Q(S(L^d))/S(L^d)]f'(L^d)u'(c(S(L^d)))$. Hence, as L^d declines, labor supply l^s increases. So, an increase in wage flexibility widens the gap between labor demand and labor supply, which is the paradox of flexibility.

F Proof of Proposition 3

The equilibrium is defined by (34)-(45) with $\widetilde{nfa} = 0$. Hence, a full employment steady state of the small open economy under financial autarky (c, L^d, r, Q, S) is jointly characterized by

$$Q = \left[\omega S^{\theta-1} + 1 - \omega\right]^{\frac{1}{\theta-1}}; \tag{A23}$$

$$c = \frac{Q}{S} f\left(L^d\right); \tag{A24}$$

$$f(L^d) = \omega \left(\frac{S}{Q}\right)^{\theta} c + (1-\omega) S^{\varepsilon} c^{*s}; \tag{A25}$$

$$\gamma'(0) = (\rho - r) Q u'(c); \tag{A26}$$

$$v'\left(L^d\right) = \frac{Q}{S} f'\left(L^d\right) u'(c); \tag{A27}$$

where (A23) and (A24) are obtained as in the proofs of Proposition 1 and 2. The first of these equations defines the function $Q(S)$. Substituting (A23) and (A24) into (A25) and rearranging terms yields

$$f(L^d(S)) = S^{\varepsilon} \left[Q(S)\right]^{\theta-1} c^{*s}, \tag{A28}$$

which defines $L^d(S)$. Substituting this expression back into (A24) gives

$$c(S) = S^{\varepsilon-1} \left[Q(S)\right]^{\theta} c^{*s}, \tag{A29}$$

which defines $c(S)$. Let us define

$$g(S) = \frac{v'\left(L^d(S)\right)}{(Q(S)/S) f'\left(L^d(S)\right) u'\left(c(S)\right)} - 1, \tag{A30}$$

which is a continuous function of S on $(0, \infty)$. A steady state is characterized as the solution to $g(S) = 0$. To prove the existence of such a steady state with $S \in (1, \infty)$, I show that $g(1) < 0$ and $\lim_{S\to\infty} g(S) > 0$. Finally, I check that the corresponding real interest rate given by (A26) is consistent with the monetary policy rule.

We trivially have $L^d(1) = L^{d*s}$ and $c(1) = c^{*s}$. We also have $Q(1) = 1$. By definition of the secular stagnation steady state at the world level, we therefore have

$$\begin{aligned} g(1) &= \frac{v'\left(L^{d*s}\right)}{(Q(1)/1) f'\left(L^{d*s}\right) u'\left(c^{*s}\right)} - 1, \\ &= \frac{\pi^{*s} - (\pi^R - \alpha)}{\alpha} - 1, \\ &= \frac{\pi^{*s} - \pi^R}{\alpha} < 0. \end{aligned}$$

By (A28) and (A29), we have $\lim_{S\to\infty} L^d(S) = \infty$ and $\lim_{S\to\infty} c(S) = \infty$. We also know that $Q(S)/S$ is a decreasing function of S. Hence, $\lim_{S\to\infty} g(S) = \infty > 0$.

Finally, by the wage rigidity (42) and the monetary policy rule (45), with full employment, we can either have inflation on target, with $\pi = \bar{\pi}$ and $i = r + \bar{\pi} \geq 0$, or a binding zero lower bound, with $i = 0$ and $\pi = -r \in (\pi^R, \bar{\pi}]$. Hence, the existence of a full employment steady state requires $r > -\bar{\pi}$. As we have previously assumed that the inflation target satisfies $r^{*n} \geq -\bar{\pi}$, it is sufficient to show that $r > r^{*n}$.

Hence, I now show that, for the equilibrium $S \in (1, \infty)$, we have $r > r^{*n}$ with r given by (A26). Let $\tilde{Q}(S) = Q(S)/S$. I now introduce the functions $\tilde{L}^d(\tilde{Q})$ and $\tilde{c}(\tilde{Q})$ for equilibrium consumption and labor demand that are jointly characterized by (A24) and (A27) with $Q/S = \tilde{Q}$.[16] By definition of the neoclassical steady state at the world level, $\tilde{c}(1) = c^{*n}$ and $\tilde{L}^d(1) = L^{d*n}$. Implicitly differentiating these functions reveals that $\tilde{c}(\tilde{Q})$ is increasing in $\tilde{Q}$. But, we know that in equilibrium $S > 1$ and therefore $\tilde{Q}(S) < \tilde{Q}(1) = 1$. Thus, $\tilde{c}(\tilde{Q}(S)) < \tilde{c}(\tilde{Q}(1)) = \tilde{c}(1) = c^{*n}$. When $S > 1$, we also have $Q(S) > Q(1) = 1$ (with $Q(S)$ defined by (A23)). Hence, $Q(S)u'(\tilde{c}(\tilde{Q}(S))) > Q(1)u'(\tilde{c}(\tilde{Q}(1))) = u'(c^{*n})$. By (A26), this establishes that, for any equilibrium with $S \in (1, \infty)$, the equilibrium real interest rate is above r^{*n}.

G Proof of Lemma 3

The equilibrium is defined by (34)-(45) with $\widetilde{nfa} = 0$, $r = -\pi$, and all taxes and subsidies equal to zero except $s_t^X = -1/\varepsilon$. Substituting the relative demand for foreign goods (36) with $s_t^X = -1/\varepsilon$ into the definition of the consumption basket (34) yields

$$c = \left[\omega\left(\left(\frac{\varepsilon}{1-\varepsilon}S_t\right)^{\theta}\frac{c_t^F}{1-\omega}\right)^{\frac{\theta-1}{\theta}} + (1-\omega)\left(\frac{c_t^F}{1-\omega}\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}},$$

which can be simplified as

$$\left(\frac{c_t^F}{(1-\omega)c_t}\right)^{-1/\theta} = X_t \tag{A31}$$

where I define X_t by

$$X_t = \left[\omega\left(\frac{\varepsilon}{1-\varepsilon}S_t\right)^{\theta-1} + 1 - \omega\right]^{\frac{1}{\theta-1}}. \tag{A32}$$

[16]Note that, by definition, $\tilde{L}^d(\tilde{Q})$ and $\tilde{c}(\tilde{Q})$ together with $\tilde{Q}(S) = Q(S)/S$ do not correspond to the functions $L^d(S)$ and $c(S)$ defined by (A28) and (A29).

From the national accounting identity (41) under financial autarky, trade must always be balanced, which implies $c_t^F = (1-\omega) S_t^{\varepsilon-1} c^*$. Hence, by (A31), we have

$$c_t = X_t^\theta S_t^{\varepsilon-1} c^*. \tag{A33}$$

Substituting (36) and then $c_t^F = (1-\omega) S_t^{\varepsilon-1} c^*$ into the goods market clearing equation (40) yields

$$f(L_t^d) = \left[\omega \left(\frac{\varepsilon}{1-\varepsilon} \right)^\theta S_t^{\theta+\varepsilon-1} + (1-\omega) S_t^\varepsilon \right] c^*. \tag{A34}$$

Finally, substituting (36) into the labor supply function (44) and using our result (A31) yields

$$v'(l_t^s) = \frac{\varepsilon-1}{\varepsilon} \frac{X_t}{S_t} f'(L_t^d) u'(c_t). \tag{A35}$$

Thus, X_t, c_t, L_t^d, and l_t^s can be defined as a function of S_t by (A32), (A33), (A34), and (A35), respectively.

Let us now focus on an under-employment steady state. From, the consumption Euler equation (37) and a binding zero lower bound, we have

$$\pi = -\rho + \frac{\gamma'(0)}{X u'(c)}, \tag{A36}$$

where I have used the fact that, by (35) and (36), $Q = [c^F/((1-\omega)c)]^{-1/\theta} = X$. Also, from the binding downward nominal wage rigidity (42), we have

$$\pi = \pi^R - \alpha \left[1 - \frac{v'(L^d)}{v'(l^s)} \right]. \tag{A37}$$

Equation (A36) defines the aggregate demand relationship as a function of S, henceforth denoted by $\pi^{AD}(S)$; while equation (A37) defines the aggregate supply relationship as a function of S, henceforth denoted by $\pi^{AS}(S)$. The steady state equilibrium is given by any solution S to $\pi^{AD}(S) = \pi^{AS}(S) \in [\pi^R - \alpha, \pi^R]$. Note that $\pi^{AD}(\cdot)$ and $\pi^{AS}(\cdot)$ are both continuous over $(0, \infty)$.

We have

$$\lim_{S \to 0} X(S) = \begin{cases} (1-\omega)^{\frac{1}{\theta-1}} & \text{if } \theta > 1 \\ 0 & \text{if } \theta \leq 1 \end{cases},$$

and

$$\lim_{S \to \infty} \frac{\varepsilon-1}{\varepsilon} \frac{X(S)}{S} = \begin{cases} \infty & \text{if } \theta \geq 1 \\ \omega^{\frac{1}{\theta-1}} & \text{if } \theta < 1 \end{cases}.$$

Hence, $\pi^{AD}(0) = -\rho$ and $\pi^{AS}(0) = \pi^R - \alpha$. Recall that, in Section 3, to ensure the existence of a secular stagnation steady state for the world economy, we have previously assumed that $\alpha < \rho + \pi^R$. Hence, $\pi^{AD}(0) < \pi^{AS}(0)$.

Let $\tilde{S}$ denote the smallest value of S such that $L^d(S) = l^s(S)$. Since $L^d(0) < l^s(0)$, by continuity of $L^d(\cdot)$ and $l^s(\cdot)$, we trivially have $\tilde{S} > \hat{S}$ where $\hat{S}$ is the under-employment steady state characterized by $\pi^{AD}(\hat{S}) = \pi^{AS}(\hat{S})$. Hence, if the under-employment steady state is unique, by continuity of $\pi^{AD}(\cdot)$ and $\pi^{AS}(\cdot)$, as $\pi^{AD}(0) < \pi^{AS}(0)$, $\pi^{AD}(\hat{S}) = \pi^{AS}(\hat{S})$, and $\tilde{S} > \hat{S}$, we must have $\pi^{AD}(\tilde{S}) > \pi^{AS}(\tilde{S}) = \pi^R$.

The planner's optimal allocation is characterized by the same equations as the decentralized equilibrium, given by (A32)-(A37), with (A37) replaced by $L^d = l^s$. Hence, under the planner's optimal allocation

$$-\rho + \left(\frac{c^F}{(1-\omega)c}\right)^{1/\theta} \frac{\gamma'(0)}{u'(c)} = -\rho + \frac{\gamma'(0)}{Xu'(c)} = \pi^{AD}(\tilde{S}).$$

It immediately follows that

$$-\pi^R - \rho + \left(\frac{c^F}{(1-\omega)c}\right)^{1/\theta} \frac{\gamma'(0)}{u'(c)} = -\pi^R + \pi^{AD}(\tilde{S}) > 0.$$

As in Lemma 2, this proof implies that, if the under-employment steady state is unique, it must satisfy the paradox of flexibility.

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