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Trade Wars, Currency Wars*

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Abstract

Countries distort trade patterns ('trade wars') to gain strategic advantage relative to one another. At the same time, monetary policies are set independently and have spillover effects on partner countries ('currency wars'). We combine these two scenarios, and show that they interact in deep and interesting ways. The stance of monetary policy has substantial effects on the equilibrium degree of protection in a Nash equilibrium of the monetary and trade policy game. Trade wars lead to higher equilibrium inflation rates. Cooperation in monetary policy leads to both higher inflation and greater degree of trade protection. By contrast, when monetary policy is constrained by pegged exchange rates or the zero lower bound on interest rates, equilibrium tariffs are lower. Finally, when one country has the dominant currency in trade, it gains a large advantage in a trade war.

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1 Introduction

This paper analyzes the strategic interaction between governments in a model in which both trade and monetary policy decisions are endogenously determined in a non-cooperative environment. There is a long literature on non-cooperative trade policy beginning at least with Johnson (1953) (see other references below). These models analyze the way countries distort trade flows to gain a strategic advantage in a 'trade war'. Likewise, there are many studies of non-cooperative monetary policy in the international macro literature (references below). In recent years, the importance of international spillovers of monetary policy has sparked a debate on 'currency wars'. But in reality, the two policies must interact. Both trade and monetary policy work by affecting relative prices, which have market and welfare spillovers across countries. Monetary policy has real effects if some nominal prices are sticky. The impact of trade policy such as tariffs will depend on how prices adjust and the exchange rate responds. This paper shows that the equilibrium degree of protection in a non-cooperative environment depends critically on the stance of monetary policy, and conversely, the equilibrium monetary policy choice depends on the outcome of trade wars between countries.

Traditional studies on trade policy focused on the different degrees of bilateral or multilateral cooperation imposed by trade agreements within regions or more widely orchestrated by the WTO.[1] But in recent years, we have seen a progressive breakdown of existing trade agreements and a rise in protectionism among many countries. A particular aspect of this trend is the tendency to engage in protectionism to achieve short-term macroeconomic objectives.

In the aftermath of the Global Financial Crisis, many writers described the spillovers of non-cooperative monetary policy across countries as an outcome of 'currency wars'.[2] Much of the discussion of currency wars emphasized the impact of monetary policy shocks from large countries to small countries, and the need for some form of monetary policy coordination. But with the increasing aggressive use of protectionist policies, it becomes important to re-focus on the consequences of 'trade wars', and to understand the interaction between currency wars and trade wars.[3]

Our paper explores the relationship between currency wars and trade wars within a standard two country New Keynesian DSGE model. Aside from the exploration of endogenous, non-cooperative trade policy, the model is unoriginal. Households consume and supply labor, and monopolistic competitive firms maximize profits subject to costs of price adjustment. For the baseline model, prices are set in producer currencies. Policy-makers in each country are assumed to have the choice of inflation rates through endogenous monetary policy but also trade

[1] For a recent survey, see Bagwell and Staiger (2016).

[2] For instance, Mishra and Rajan (2018) argue that " Aggressive monetary policy actions by one country can lead to significant adverse cross-border spillovers on others, especially as countries contend with the zero lower bound. If countries do not internalize these spillovers, they may undertake policies that are collectively suboptimal. "

[3] For recent evaluation of the policy issues, see for instance Bénassy-Quéré, Bussière, and Wibaux (2018), Fajgelbaum et al. (2019), and Eichengreen (2019).

protection in the form of tariffs. [4]

The paper first derives some general results on the joint determination of inflation and tariff rates in a discretionary Nash equilibrium. Tariffs are determined by an amended version of the optimal monopoly tariff formula, depending on both the elasticity of foreign excess demand for the home export, as well as equilibrium domestic inflation rates. Both tariffs and inflation rates are determined as a trade-off between internal distortions and external terms-of-trade manipulation in a manner familiar from previous literature. A key determinant of both tariffs and inflation is the presence of markups of price over marginal cost at the firm level. Positive markups tend to imply lower tariffs but higher inflation rates. But with positive markups, both tariffs and inflation rates are higher in the joint currency and trade war game than when currency wars and trade wars are analyzed separately. Eliminating monopoly markups through production subsidies reduces the inflation bias, and leads to an equilibrium with zero inflation. But a by-product of this is a large rise in protection, as tariff setters focus only on terms-of-trade manipulation.

A common theme in the literature on international monetary policy is the possibility of gains from international cooperation. Most quantitative studies have found these gains to be very small. In fact, with discretionary monetary policy, Rogoff (1985) showed that cooperation may actually reduce welfare by leading to excessive inflation in an economy with markup distortions. In our analysis, we find that cooperation leads to both an increase in inflation *and* a rise in trade protection. Also, in contrast to Rogoff (1985), we find that monetary policy cooperation can be self-defeating, even in the absence of monopolistic markups. When monetary authorities cooperate across countries, this opens up a strategic interaction between a global monetary authority and national trade authorities which may both increase inflation *and* increase protection.

An important dimension of the debate on currency wars is the asymmetry in country size. Conventional wisdom suggests that self-oriented policy-making by large countries imposes negative spillovers on smaller countries. Surprisingly, we find the opposite: an increase in the size of a country generally reduces its welfare – large countries do not gain an advantage in the currency war. This is because larger countries generally choose higher discretionary inflation rates. But when trade wars and currency wars are combined, size wins out. With endogenous trade policy and monetary policy, larger countries are better off, since they do better in the trade war.

The baseline model assumes discretionary policy-making. Tariffs and inflation are chosen without commitment to future policy actions But we also consider the possibility that trade policy engenders some degree of commitment. We explore a simplified version of the model where policy-makers choose tariffs non-cooperatively, but can commit to tariff rates in advance of the currency war. In this case, equilibrium tariff rates are much lower than in the baseline case. With commitment, trade authorities take account of the impact of higher tariffs on inflation choices of the monetary authorities, tempering the incentive for terms-of-trade manipulation.

[4] We use the label 'currency wars' as a description of a situation where countries follow independent monetary policies which may have negative international spillovers. Other senses of the term may suggest that countries are attempting to target a level of the real exchange rate. That is not the case in our model.

In an economy with sticky prices, the nominal exchange rate becomes a critical channel through which both monetary and trade policy operate. How do our results change if exchange rates are fixed by one country? Here again, we find a very different outcome between the currency war and the currency and trade war equilibrium. Under pegged exchange rates where inflation is the only policy instrument, only one country has an independent choice of instrument, and in a symmetric equilibrium inflation rates are equalized across countries. But allowing for independent choice of tariffs, constrained by a fixed exchange rate, optimal tariffs are much lower than under flexible exchange rates. Under fixed exchange rates, terms-of-trade manipulation can be done only by generating differences in inflation rates, which in itself imposes additional costs.

We then extend the analysis to a situation where countries lose control over monetary policy due to the zero lower bound (ZLB) constraint on nominal interest rates. An equilibrium constrained by the ZLB delivers lower rates of inflation, lower consumption and output, and lower welfare than the outcome under currency wars. Further, because inflation rates become endogenous at the ZLB rather than a policy variable, trade policy-makers take account of their choice of tariffs in the two countries. As in the case of commitment in trade policy, this leads them to limit the size of their tariff choices.

The main results above are derived under the standard assumption of 'producer currency pricing' (PCP) as in the classic model of Galì and Monacelli (2005). We finally extend the analysis to 'dominant currency pricing' (DCP hereafter), as in Gopinath et al. (2020) and Mukhin (2018). In this setting, both countries set their export prices in that of the home country's currency (the dominant currency). This introduces a key asymmetry in the analysis, since import prices in the home country are no longer directly tied to exchange rate changes. We find that this asymmetry leads to higher inflation rates in the dominant currency, and a large positive gap between home and foreign tariff rates. Since under DCP, the foreign country tariff can only improve its terms of trade by generating inflation in its export goods prices, tariffs are a much less effective tool for this country. We find that the equilibrium of the trade and currency war under DCP dramatically favours the dominant currency issuer. Hence, in this perspective, a benefit of a dominant currency is that allows for a significant advantage in trade wars.

One key message of the paper is that, depending on the different assumptions about exchange rate regimes and pricing currency, trade wars may imply very high rates of protection in standard DSGE macro models. This may seem unrealistic, given that in recent history, observed tariffs among advanced economies have been much lower. But these historical observations refer to a period where WTO rules and other bilateral agreements were in place. By contrast, our paper explores the consequences of a breakdown of cooperation in trade policy. In this case, the tariff rates may not be so unrealistic. For instance, in a calibrated CGE trade model, Ossa (2014) finds that average tariffs would be over 60 percent in a full-scale world 'tariff war'. Also, recent experience suggests that tariff rates may go much higher. As an example, average US tariff on China, as measured by Bown (2019), rose from 8 percent in early 2018 to 26 percent at the end of

2019.

The interaction between trade policy and the macroeconomy has long been a subject of interest to economists (e.g. Eichengreen (1981) and Krugman (1982)). But recent events have seen a revival of interest in this area and an attempt to formalize the relationship within the modern macroeconomic toolkits. A number of recent papers look at the effects of trade policy in dynamic open-economy macroeconomic models. Barattieri, Cacciatore, and Ghironi (2021) investigate empirically the impact of exogenous changes in tariffs in an SVAR framework, and show that they act as negative supply shocks, depressing GDP and raising inflation with little effects on the trade balance. A similar mechanism applies to our paper. Barattieri, Cacciatore, and Ghironi (2021) develop a small open economy model with firm entry and endogenous tradability that successfully rationalizes the empirical evidence. We adopt an alternative approach, considering tariffs as endogenous, exploring the consequences of alternative strategic settings for both monetary policy and tariff formation. Another paper by Erceg, Prestipino, and Raffo (2018) looks at the impact of trade policies in the form of import tariffs and export subsidies. They find that the effects critically depend on the response of the real exchange rate, and that in turn depends on the expectations about future policies and potential retaliation from trade partners. A recent paper by Furceri et al. (2019) examines the macroeconomic consequences of tariff shocks, and shows that these shocks are generally contractionary. Lindé and Pescatori (2019) study the conditions under which Lerner symmetry holds, and how this affects the macroeconomic costs of a trade war.

Two papers that are more closely ours are Jeanne (2020) and Bergin and Corsetti (2020). Jeanne (2020) is closest in spirit to our paper. He explores the interaction between 'currency wars' and 'trade wars' in an analytical framework with a continuum of small open economies with downward nominal wage rigidity and in some cases a global liquidity trap, and explores the benefits of international cooperation. By contrast, our study is focused on a more conventional two country DSGE model, where countries are large, and focuses on a discretionary Ramsey approach to policy-making.[5] Bergin and Corsetti (2020) develop a rich multi-country DSGE model with global value chains and look at the optimal response of monetary policy to exogenous tariff shocks. In addition, they focus on cooperative determination of monetary policy and consider tariffs as exogenous. Another relevant paper is Caballero, Farhi, and Gourinchas (2015), who investigate the interaction between an environment of low interest rate, financial imbalances and currency wars. Our paper does consider a binding ZLB constraint as one of the possible cases but deals with more generic environments, and considers the joint endogenous formation of tariff and monetary policies while neglecting imbalances by assuming balanced-trade restrictions. We thus consider our paper as an important complement, with a focus on the interaction between trade and monetary policy. In particular, we find that international cooperation may be

[5]This type of approach echoes the approach of Chari, Nicolini, and Teles (2018) or Auray, Eyquem, and Gomme (2018), although these papers focus on flex-price environments.

significantly welfare reducing in this environment.[6]

Focusing more closely on the endogenous determination of trade policies, we noted above that there is a large empirical literature investigating the link between trade restrictions and the economic cycle, and separately, the effect of real exchange rate undervaluation on trade policy (*e.g.* Oatley (2010), Gunnar and Francois (2006), Bown and Crowley (2013), among others). In a theoretical model Eaton and Grossman (1985) study optimal tariffs when international asset markets are incomplete and show that they can be used to partly compensate the lack of consumption insurance. Bergin and Corsetti (2008) also consider tariffs as policy instruments in addition to monetary policy but their focus is not specifically on tariffs, rather on the implications of monetary policy on the building of comparative advantages. Campolmi, Fadinger, and Forlati (2014) offer a detailed analysis of optimal non-cooperative policies with a large set of instruments, including tariffs.[7] In a rich model with endogenous location of firms and an extensive margin of trade, they show that the terms-of-trade externality remains the dominating incentive to apply positive tariffs. Bagwell and Staiger (2003) propose a trade model featuring potential terms-of-trade manipulation by governments, and trade agreements as means to restrict this policy option. Our paper is complementary to theirs. Most importantly, we incorporate endogenous tariff formation within a standard open-economy macroeconomic model, showing the importance of price stickiness, the exchange rate regime, the extent of cooperation, ZLB constraints on nominal interest rates or dominant currency pricing for the equilibrium degree of trade protection.

The rest of the paper is organized as follows. Section 2 sets out a basic model and establishes a number of analytical results. Section 3 presents an extended model with intermediate goods in production and trade, which is solved quantitatively, and develops the main results of the paper under currency wars and trade wars, where we explore the impact of cooperation, partial commitment in trade policy and country size. Section 4 analyzes the model under a variety of set-ups in which monetary policy is constrained in a number of ways, due to either an exchange rate peg, the zero lower bound constraint, or dominant currency pricing.

2 The Model

We first describe a simplified two-country model which allows us to derive a number of qualitative results. The full quantitative model is described in Section 3 below. There are two

[6] Corsetti and Pesenti (2001) show how national welfare in open economies may depend on a terms of trade externality, using a two-country model with monopolistic competition. There are many papers analyzing optimal monetary policy in different open economy frameworks, among them Benigno and Benigno (2003), Galì and Monacelli (2005), Faia and Monacelli (2008), de Paoli (2009), Auray, Eyquem, and de Blas (2011), Bhattarai and Egorov (2016), Groll and Monacelli (2020), Fujiwara and Wang (2017), or more recently Egorov and Mukhin (2019).

[7] More generally, our paper also relates to the literature on tax and structural reforms to manipulate the exchange, which includes Correia, Nicolini, and Teles (2008), Hevia and Nicolini (2013), Farhi, Gopinath, and Itskhoki (2014), Eggertsson, Ferrero, and Raffo (2014), Cacciatore et al. (2016), Auray, Eyquem, and Ma (2017) or Barbiero et al. (2019).

countries, Home and Foreign, where agents supply labor and consume each other's goods. The world is populated with a unit mass of agents and for now, countries are equally sized. For now also, firms set prices in the currency of the producer (PCP), but are constrained by Rotemberg-style price adjustment costs.

2.1 Households

Agents in the Home country have preferences over consumption and hours given by

$$U_t = u(C_{h,t}, C_{f,t}) - \ell(H_t) \tag{1}$$

where u is a continuous, twice differentiable, and satisfies $u_{c_{ii}} < 0$ and $u_{c_{ij}} \geq 0$, for $i = h,\ f,\ i \neq j$. Consumption of the Home (Foreign) is $C_{h,t}$ ($C_{f,t}$). $\ell(.)$ is a continuous differentiable function of hours worked, satisfying $\ell'(.) > 0$, and $\ell''(.) > 0$.

The Home country budget constraint is:[8]

$$P_{h,t}C_{h,t} + (1+\tau_t)S_t P^*_{f,t}C_{f,t} = W_t H_t + \Pi_t + TR_t \tag{2}$$

where $P_{h,t}$ ($P^*_{f,t}$) is the Home (Foreign) goods price in Home (Foreign) currency, S_t is the exchange rate, τ_t is an import tariff imposed by the Home government, W_t is the Home nominal wage, Π_t represents the profits of the Home firm and TR_t is a lump-sum transfer from the Home government. Optimal choices over consumption and hours for the Home consumer lead to the conditions:

$$u_{c_{h,t}} \frac{(1+\tau_t)S_t P^*_{f,t}}{P_{h,t}} = u_{c_{f,t}} \tag{3}$$

$$\ell'(H_t) = u_{c_{h,t}} \frac{W_t}{P_{h,t}} \tag{4}$$

2.2 Firms

Firms in the Home economy produce differentiated goods.[9] The aggregate Home good is a composite of these differentiated goods, where the elasticity of substitution between individual goods is denoted as $\epsilon > 1$. For now, assume that the firm's production depends only on domestic labor. Output of firm i in the Home country is:

$$Y_{i,h,t} = A_t H_{i,t} \tag{5}$$

[8] We assume no financial market trading across countries, which implies that trade is balanced.
[9] We describe the situation of Home firms, noting that Foreign firms behave similarly.

where A_t is a measure of aggregate productivity. The profits of Home firm i are then represented as:

$$\Pi_{i,t} = \left((1+s)P_{i,h,t} - \frac{W_t}{A_t} \right) Y_{i,h,t} \tag{6}$$

where $\Pi_{i,t}$ is the price set by firm i in the Home country and s a sales subsidy. Firm i chooses its price to maximize the present value of its expected profits, net of price adjustment costs:

$$E_t \sum_{j=0}^{\infty} \omega_{t+j} \left(\Pi_{i,t+j} - \xi \left(\frac{P_{i,h,t+j}}{P_{i,h,t+j-1}} \right) P_{h,t+j} Y_{h,t+j} \right) \tag{7}$$

where ω_t is the firm's nominal stochastic discount factor, and $\xi(.)$ represents a price adjustment cost function for the firm. We assume that $\xi'(.) > 0$ and $\xi''(.) > 0$. Price adjustment costs are proportional to the nominal value of Home output, to be consistent with the nominal profit objective function of the firm. The first-order condition for profit maximization gives:

$$\begin{aligned} (1+s)Y_{i,h,t} &= \epsilon \left(P_{i,h,t}(1+s) - \frac{W_t}{A_t} \right) \frac{Y_{i,h,t}}{P_{i,h,t}} + \xi' \left(\frac{P_{i,h,t}}{P_{i,h,t-1}} \right) \frac{1}{P_{i,h,t}} P_{h,t} Y_{h,t} \\ &+ E_t \omega_{t+1} \xi' \left(\frac{P_{i,h,t+1}}{P_{i,h,t}} \right) \frac{P_{i,h,t+1}}{P_{i,h,t}^2} P_{h,t+1} Y_{h,t+1} = 0 \end{aligned} \tag{8}$$

2.3 Economic Policy

There are three separate levers of policy in this model. Fiscal policy may be used to subsidize monopoly firms (s). Trade policy may be used to levy tariffs on imports (τ_t and τ_t^*), and monetary policy may be used to either target inflation rates or exchange rates. In the case where firms are subsidized ($s \neq 0$), we follow the literature in assuming that a fiscal authority sets the subsidy to offset the steady-state monopoly markup ($s = \frac{1}{\epsilon - 1}$). But we also allow for the possibility that the monopoly markup remains as a pre-existing distortion in the economy ($s = 0$). As we will see, this may have an important implications for both optimal monetary policy and trade policy.

2.3.1 Monetary Policy

In the baseline model, it is assumed that the monetary authorities choose an inflation rate for the domestic good. Implicitly, we are assuming that the authorities can implement a desired inflation rate through an interest rate policy, but we abstract from the details of this policy. In a later section, we look at the case where monetary policy is represented by an exchange-rate target on the part of the Foreign government, leaving the Home country to independently choose an inflation rate.

2.3.2 Trade Policy

Trade policy is represented by tariffs chosen by each country. Tariff rates are chosen to maximize domestic welfare. In this scenario, countries engage in a 'trade war', where equilibrium tariff rates are determined in a Nash equilibrium. But in an economy with sticky prices and optimally determined monetary policy, an important determinant of the outcome of trade wars is the relationship between the domestic monetary authority and the trade authority. In the Nash equilibrium of the game between countries (as described below), we assume that both inflation and tariffs are chosen simultaneously by a domestic policy-maker to maximize domestic welfare. But the result would be the same if we thought of monetary and trade policy as determined separately by a monetary and fiscal authority.

In our model, the only motive to levy tariffs is to affect the terms of trade. While the literature has explored many other reasons for countries to apply trade restrictions, Bagwell and Staiger (2010) argue that terms of trade manipulation is the most important and empirically relevant driver of tariffs.[10]

We also explore the implications of cooperation in monetary policy, assuming that tariffs are still chosen independently. This assumption is natural, since cooperative tariffs would always be zero in a symmetric equilibrium of our model, where trade policy to be determined jointly by policy-makers. In all cases, regardless of the assumptions about trade and monetary policy, we assume that policy is discretionary. This means that policy-makers maximize *current* welfare, taking as given that future policy-makers will behave in a similar fashion.

2.3.3 Government Budget constraint

While the assumptions about the stance of policy differs, the representation of the consolidated government budget constraint is the same in all situations. The government in each country balances its budget. Tariffs generate revenues, while subsidies represent a cost paid to domestic firms. The difference is rebated back to domestic households in the form of lump-sum transfers. Hence, for the Home country we have

$$TR_t = \tau_t S_t P^*_{f,t} C_{f,t} - s P_{h,t} Y_{h,t} \tag{9}$$

where the last expression on the right hand side represents total subsidies paid to all domestic firms.

[10] Broda, Limao, and Weinstein (2008) find that countries systematically set higher tariffs on imports with more inelastic supply schedules.

2.4 The Competitive Equilibrium

The full description of the competitive equilibrium for this economy is available in Appendix A. Monetary policy is represented by the PPI inflation rates set by each policy-maker, $\pi_{h,t} = \frac{P_{h,t}}{P_{h,t-1}}$ and $\pi^*_{f,t} = \frac{P^*_{f,t}}{P^*_{f,t-1}}$. In addition, we can define the terms of trade as $\mathcal{S}_t = \frac{S_t P^*_{f,t}}{P_{h,t}}$. Then, Appendix A shows that, conditional on monetary policies $\left\{\pi_{h,t}, \pi^*_{f,t}\right\}$ and tariff policies $\{\tau_t, \tau^*_t\}$, the equilibrium can be written in the form of 7 equations in the 7 variables H_t, H^*_t, $C_{ht}, C_{f,t}$ $C^*_{h,t}$, $C^*_{f,t}$ and $\mathcal{S}_t$.

$$\text{Balance of Payments} \; : \; C^*_{h,t} = \mathcal{S}_t C_{f,t} \tag{10}$$

$$\text{Home Market clearing} \; : \; A_t H_t (1 - \frac{\phi}{2}(\pi_{h,t} - 1)^2) = C_{h,t} + C^*_{h,t} \tag{11}$$

$$\text{Foreign Market clearing} \; : \; A_t H^*_t (1 - \frac{\phi}{2}(\pi^*_{f,t} - 1)^2) = C_{f,t} + C^*_{f,t} \tag{12}$$

$$\text{Home labor market} \; : \; \ell'(H_t) = A_t u_{c_{h,t}} E_t \Psi(\pi_{h,t}, \pi_{h,t+1}, \theta) \tag{13}$$

$$\text{Foreign labor market} \; : \; \ell'(H^*_t) = A^*_t u_{c^*_{f,t}} E_t \Psi(\pi^*_{f,t}, \pi^*_{f,t+1}, \theta^*) \tag{14}$$

$$\text{Foreign optimal spending} \; : \; u_{c^*_{h,t}} \mathcal{S}_t = (1 + \tau^*_t) u_{c^*_{f,t}} \tag{15}$$

$$\text{Home optimal spending} \; : \; u_{c_{h,t}} (1 + \tau_t) \mathcal{S}_t = u_{c_{f,t}} \tag{16}$$

where we have used the assumption that the quadratic cost of price adjustment is $\frac{\phi}{2}(\pi_{h,t} - 1)^2$ for the Home country and analogously for the Foreign country. We define $\Psi_t = \theta + \phi\pi_{h,t}(\pi_{h,t} - 1) - \beta\pi_{h,t+1}(\pi_{h,t+1} - 1)$ (and analogously for Foreign), which represents the impact of price adjustment costs on the firm's profit maximization, and $\theta = \frac{(1+s)(\epsilon-1)}{\epsilon}$ is a subsidy-adjusted measure of the monopoly distortion, with $\theta = 1$ if an optimal subsidy $s = \frac{1}{\epsilon - 1}$ is in place. If current and future inflation is zero, and the optimal subsidy is in place, then $\Psi = 1$.[11]

2.5 Optimal Monetary Policy: Currency Wars

We first analyze the determination of optimal inflation rates in a discretionary Nash equilibrium. Each government chooses its inflation rate to maximize domestic welfare, subject to competitive equilibrium conditions, taking the inflation rate of the other government as given. Since there is no trade in financial assets, there are no endogenous state variables in the model, so a discretionary (time-consistent) Nash equilibrium can be described simply by each government's choice of current-valued variables, taking future inflation rates, consumption levels, output levels and terms of trade as given (assuming $\tau_t = \tau^*_t = 0$).[12] Let the current state be defined as $\mathcal{Z}_t = (A_t, A^*_t)$, representing aggregate TFP. Define the firm's value function as

[11]Here we simplify by assuming the firm's discount factor for the expected future inflation cost is constant at β. This makes little difference to the example.

[12]In the extensive description of the model, nominal price levels are state variables. But since monetary policy is implemented by the choice of inflation rates, current policy makers take future inflation as chosen by future policy, so current price levels have no relevance for the evaluation of future welfare.

$v(\mathcal{Z}_t)$. In a discretionary Nash equilibrium in monetary policy the Home government chooses $\{C_t, C_t^*, H_t, H_t^*, \mathcal{S}_t, \pi_{h,t}\}$ to maximize

$$v(\mathcal{Z}_t) = u(C_{h,t}, C_{f,t}) - \ell(H_t) + \beta E_t v(\mathcal{Z}_{t+1}), \tag{17}$$

subject to Equations (10)-(16), while the Foreign firm chooses $\{C_t, C_t^*, H_t, H_t^*, \mathcal{S}_t, \pi_{f,t}^*\}$ to maximize

$$v^*(\mathcal{Z}_t) = u(C_{h,t}^*, C_{f,t}^*) - \ell(H_t^*) + \beta E_t v^*(\mathcal{Z}_{t+1}), \tag{18}$$

subject to Equations (10)-(16). Since the model is symmetric, we state results for the Home country alone. The Nash equilibrium implies equivalent results for the Foreign country. Let $\xi_{1,t}$ .. $\xi_{7,t}$ denote the Home country Lagrange multipliers on constraints (10)-(16) respectively. Then Appendix A shows that, in a steady-state Nash equilibrium of the currency war game, the inflation rate of the Home government is implicitly characterized by the condition:

$$\Psi = \frac{1 - \frac{\phi}{2}(\pi_h - 1)^2 - \frac{(\pi_h - 1)}{(2\pi_h - 1)}\psi\Psi}{1 - \frac{AH(\pi_h - 1)}{u_{c_h}(2\pi_h - 1)}u_{c_{hh}}\Psi - \frac{\xi_7}{\xi_2}(\mathcal{S}u_{c_{hh}} - u_{c_{hf}})} \tag{19}$$

where $\psi = \frac{H\ell''(H)}{\ell'(H)}$ is the inverse of the Frisch elasticity of labor supply, and $u_{c_{hh}}$ indicates the second derivative of the utility function and $\Psi = \theta + \phi\pi_h(\pi_h - 1)(1 - \beta)$. From this expression, we do obtain the following results (detailed in Appendix A):

Result 1. *In general, a Nash equilibrium in the currency war game between countries will not exhibit price stability (zero inflation).*

Proof. Appendix A shows that $\pi_h = 1$, is a generically not a solution to Equation (19).

Result 2. *When $s = \frac{1}{\epsilon - 1}$, the steady-state Nash equilibrium of the currency war game implies that both countries choose negative inflation rates.*

Proof. When $s = \frac{1}{\epsilon - 1}$, Appendix A shows that $\frac{\xi_7}{\xi_2} > 0$. Then the only solution to (19) must involve $\pi_h < 1$ (and therefore also $\pi_f^* < 1$).

This case recalls well-known results in the open economy macro literature. Corsetti and Pesenti (2001) and Clarida, Gali, and Gertler (2002) characterize the optimal monetary policy in an open economy as a tension between the desire to eliminate domestic distortions associated with monopoly pricing on the one hand, and the desire to manipulate the terms of trade for strategic advantage on the other hand. These conflicting objectives would involve either a positive or a negative inflation rate. When the optimal subsidy is in place, each policy maker focuses only on the terms-of-trade objective, and inflation is negative in a Nash equilibrium of the currency war game.

This example is also reminiscent of Rogoff (1985), which shows that international policy cooperation can reduce welfare in the case of monopoly distortions, since it eliminates the terms-of-trade externality, and may lead to excessive Nash inflation in the currency war game. In the quantitative analysis below, we confirm this result. With an optimal subsidy in place, however, international cooperation eliminates disinflation and raises welfare relative to the currency war Nash equilibrium. It is straightforward to extend Result 2 to show that in a symmetric cooperative equilibrium inflation rates are zero in each country.

2.6 Optimal Monetary and Trade Policy: Trade Wars and Currency Wars

We now extend the analysis to the game where governments choose both inflation and tariff rates. The problem of the Home government is to choose $\{C_t, C_t^*, H_t, H_t^*, \mathcal{S}_t, \pi_{h,t}, \tau_t\}$ to maximize (17) subject to Equations (10)-(16).

In this extended environment, Appendix A shows that the determination of tariffs and inflation in the Home economy may be represented by the conditions:

$$\frac{1}{1+\tau_t} = \frac{1 - \frac{A_t H_t (\pi_{h,t}-1)}{u_{c_{h,t}}(2\pi_{h,t}-1)} u_{c_{hh,t}} \Psi(\pi_{h,t}, E_t \pi_{h,t+1})}{\frac{\eta_t}{\eta_t - 1} - \frac{A_t H_t (\pi_{h,t}-1)}{\mathcal{S}_t u_{c_{h,t}}(2\pi_{h,t}-1)} u_{c_{hf,t}} \Psi(\pi_{h,t}, E_t \pi_{h,t+1})} \tag{20}$$

$$\Psi(\pi_{h,t}, E_t \pi_{h,t+1}) = \frac{1 - \frac{\phi}{2}(\pi_{h,t}-1)^2 - \frac{(\pi_{h,t}-1)}{(2\pi_{h,t}-1)} \psi \Psi(\pi_{h,t}, E_t \pi_{h,t+1})}{1 - \frac{A_t H_t (\pi_{h,t}-1)}{u_{c_{h,t}}(2\pi_{h,t}-1)} u_{c_{hh,t}} \Psi(\pi_{h,t}, E_t \pi_{h,t+1})} \tag{21}$$

where η_t is the foreign country's general equilibrium elasticity of demand for home goods.[13]

From Equation (20), we see that the Home country's optimal tariff follows an amended 'monopoly tariff' formula. The pure monopoly tariff would be set at $\tau_t = \frac{1}{\eta_t - 1}$, the reciprocal of the demand elasticity minus one. But in the presence of inflation and monopoly distortions in domestic production, the tariff in general will be lower than this value. If inflation is positive, then $\tau_t < \frac{1}{\eta_t - 1}$, since with positive inflation output is inefficiently low, and a tariff reduces output further. Equations (20) and (21) then imply:

Result 3. *In a steady-state Nash equilibrium of the trade and currency war game, when $s = s^* = \frac{1}{\eta - 1}$, the Home and Foreign country will set inflation rates to zero and tariffs are given by the pure monopoly tariff formula $\frac{1}{\eta - 1}$ and $\frac{1}{\eta^* - 1}$.*

[13] In particular,

$$\eta_t = \frac{\frac{(u^*_{c_{ff,t}}(1+\tau_t^*) - u^*_{c_{hf,t}} \mathcal{S}_t) c^*_{h,t}}{u^*_{c_{h,t}} \mathcal{S}_t^2 \left(1 - \frac{A_t^{*2}(1-\varphi_t^*)\Psi_t^* u^*_{c_{ff,t}}}{\ell''(H_t^*)}\right)} - 1}{\frac{(u^*_{c_{ff,t}}(1+\tau_t^*) - u^*_{c_{hf,t}} \mathcal{S}_t) c^*_{h,t}}{u^*_{c_{h,t}} \mathcal{S}_t^2 \left(1 - \frac{A_t^{*2}(1-\varphi_t^*)\Psi_t^* u^*_{c_{ff,t}}}{\ell''(H_t^*)}\right)} \left(1 - \mathcal{S}_t \frac{A_t^{*2}(1-\varphi_t^*)\Psi_t^* u^*_{c_{hf,t}}}{\ell''(H_t^*)}\right) + \frac{(u^*_{c_{hh,t}} \mathcal{S}_t - u^*_{c_{hf,t}}(1+\tau_t^*)) c^*_{h,t}}{u^*_{c_{h,t}} \mathcal{S}_t}}$$

where $\varphi_t^* \equiv \frac{\phi}{2}(\pi^*_{f,t} - 1)^2$.

Proof. From Equation (21), with $s = \frac{1}{\epsilon - 1}$, we obtain $\pi_h = 1$ in a steady state. Then from Equation (20) we obtain $\tau = \frac{1}{\eta - 1}$ and likewise for the Foreign country.

Result 4. *In the presence of monopoly distortions, the stationary Nash equilibrium of the trade and currency war game exhibits positive inflation rates and tariffs rates lower than the monopoly tariff.*

Proof. When $s = 0$, the left-hand side of Equation (21) is less than unity in a steady state with $\pi_h = 1$, while the right-hand side equals unity. Since the left-hand side (right-hand side) is increasing (decreasing) in π_h, the solution then must involve $\pi_h > 1$. Then from Equation (20), since $u_{c_{hh}} < 0$ and $u_{c_{hf}} \geq 0$, the right-hand side must be greater than $\frac{1}{\frac{\eta}{\eta-1}}$, so $\tau < \frac{1}{\eta - 1}$, and analogously for the Foreign country.

Result 4 above points out the interrelationship between trade policy and monetary policy in a distorted economy. Monopoly distortions tend to reduce the degree of protectionism, while increasing the inflation rate. By contrast to the currency war example, eliminating monopoly distortions fully removes deflation bias and leads to zero inflation but a by-product is that equilibrium tariff rates increase as shown by Result 3. In the quantitative analysis below, we show that the rise in tariffs following the removal of domestic distortions may be large.

The above analysis suggests that trade wars lead to higher equilibrium inflation rates in a currency war. We can also ask how trade wars would play out with an alternative scenario for monetary policy. We explore this by assuming that monetary authorities follow a passive policy of zero inflation, limiting strategic interaction to trade policy alone. This case is analyzed in Appendix A. There we establish the following.

Result 5. *When monetary policy is limited to price stability, ($\pi_{h,t} = \pi^*_{f,t} = 0$), the optimal tariff in the steady-state Nash equilibrium of the trade war game is described by:*

$$\frac{1}{1+\tau} = \frac{1 + \Omega u_{c_{hh}} \theta}{\frac{\eta_t}{\eta_t - 1} + \Omega u_{c_{hf}} \theta} \tag{22}$$

where $\Omega = \frac{(\theta - 1)A}{\frac{\ell'}{A} - u_{c_{hh}} A \theta^2}$, *and* $\theta = \frac{(1+s)(\epsilon - 1)}{\epsilon} \leq 1$.

Proof. See Appendix A.

From Equation (22) we conclude that, in a distorted economy where $\theta < 1$ and without active monetary policy, the steady-state Nash equilibrium tariff in the currency war game will be less than the pure monopoly tariff rate. Intuitively, this is because policy-makers take account of the distortionary impacts of the tariff on domestic production, which is inefficiently low when $\theta < 1$. In the quantitative analysis below, we compare the equilibrium tariff rates in the model with constrained, zero-inflation policies with those of an economy where policy-makers choose inflation rates optimally. We find that tariffs are significantly lower in the constrained case. When tariffs are the only instrument to exploit terms-of-trade externalities and respond to domestic distortions, the optimal degree of protection is reduced.[14]

In the case of the currency war above, we noted that international monetary policy cooperation would raise welfare in the absence of monopoly distortions. We find that this is not the case in the currency and trade war equilibrium. Indeed, we may state the following.

Result 6. *In a stationary Nash equilibrium of the currency and trade war game when each country uses a subsidy to offset monopoly distortions, international monetary policy cooperation will lead to positive rates of inflation and and tariff rates above the monopoly tariff level.*

Proof. Appendix A shows that the inflation rate of the Home (and Foreign) country in the steady state of a symmetric cooperative equilibrium is characterized by the condition

$$\Psi = \frac{1 - \frac{\phi}{2}(\pi_h - 1)^2 - \frac{(\pi_h - 1)}{(2\pi_h - 1)}\psi\Psi}{1 - \frac{AH(\pi_h - 1)}{u_{c_{h,t}}(2\pi_h - 1)}u_{c_{hh}}\Psi - \frac{\xi_6}{\xi_2}(u_{c_{hh}}(1+\tau) - u_{c_{hf}})} \tag{23}$$

where

$$\frac{\xi_6}{\xi_2} = \frac{u_{c_h}\tau}{\xi_2(u_{c_{hh}}(1+\tau) + u_{c_{ff}} - 2u_{c_{hf}})} + \frac{(u_{c_{hh}} - u_{c_{hf}})\Psi AH(\pi_h - 1)}{u_{c_h}(2\pi_h - 1)} \tag{24}$$

with $\xi_2 = \frac{\ell'(H)}{A(1 - \frac{\phi}{2}(\pi_h - 1)^2) - \frac{\ell''(H)H(\pi_h - 1)}{u_{c_h}(2\pi_h - 1)}}$.

Let us start with inflation. Assume that $\theta = 1$, so that the optimal subsidy is applied to correct the monopoly distortion. If $\pi_h = 1$ the left-hand side of Equation (23) is unity, while the right-hand side is greater than unity, using Equation (24), as long as there is a positive tariff rate, *i.e.* $\tau > 0$. Since the left-hand side is increasing in π_h and the right-hand side is decreasing in π_h, it must be that the equilibrium cooperative inflation rate is greater than zero when $\theta = 1$ and $\tau > 0$.

Now focus on tariffs. Appendix A shows that in a symmetric steady state with international cooperation in monetary policy, the Home (and Foreign) tariff rate is given by:

$$\frac{1}{1+\tau} = \frac{1 + \frac{\xi_4}{\xi_2}u_{c_{hh}}\Psi}{\frac{\eta}{\eta - 1} + \frac{\xi_4}{\xi_2}u_{c_{hf}}\Psi} \tag{25}$$

where again, the Foreign demand elasticity is η. In addition, it is shown that $\xi_4 = \frac{\ell'(H) - Au_{c_h}(1 - \frac{\phi}{2}(\pi_h - 1)^2)}{\frac{\ell'(H)}{A} - u_{c_{hh}}\Psi A(1 - \frac{\phi}{2}(\pi_h - 1)^2)}$ and $\xi_2 = u_{c_h} - u_{c_{hh}}\Psi\frac{(\ell'(H) - Au_{c_h}(1 - \frac{\phi}{2}(\pi_h - 1)^2))}{\frac{\ell'(H)}{A} - u_{c_{hh}}\Psi A(1 - \frac{\phi}{2}(\pi_h - 1)^2)}$. When $\theta = 1$, then from Equation (14), in a steady state, we have $\ell'(H_t) - A_t u_{c_{h,t}}(1 - \frac{\phi}{2}(\pi_{h,t-1})^2) = Au_{c_h}\left(\phi\pi_h(\pi_h - 1) + \frac{\phi}{2}(\pi_h - 1)^2\right) > 0$. Since $\frac{\xi_4}{\xi_2} > 0$ it follows from Equation (25) that in the case $\theta = 1$, and monetary policy is determined cooperatively, the tariff rate exceeds the monopoly tariff rate.

[14]This case is actually equivalent to that of a flexible price economy (where $\phi = 0$), since zero inflation rates in this model replicate the flexible price equilibrium.

To follow the intuition for Result 6, look at Equation (14). When $\theta = 1$ and inflation is zero, output is determined by $\ell'(h) = Au_{c_h}$ in the Home country and similarly in the Foreign country. The presence of tariffs distort the pattern of consumption in both countries, reducing output, for a given inflation rate. The cooperative policy-makers increase inflation above zero, raising equilibrium output. *Ceteris paribus* however, this tends to reduce the terms of trade for each country, and non-cooperative tariff authorities respond by increasing tariff rates.

Result 6 represents an interesting addition to the classic Rogoff (1985) result. Result 3 and 4 above showed that with an optimal monopoly subsidy, the Nash equilibrium implied zero inflation and tariff rates equal to the monopoly tariff level. But if the policy choice is separated so that monetary policy is chosen cooperatively, policy-makers will increase inflation rates to offset the distortion generated by tariffs. Simultaneously, acting individually, tariff setters will increase their tariffs since the perceived external monopoly strength increases as inflation is higher. In the quantitative analysis below, we show that, even in the case of an optimal domestic monopoly subsidy, welfare may be reduced by international monetary policy cooperation, since it leads to higher tariffs and higher inflation rates.

3 Quantitative Results

We now extend the analysis to a more general model allowing for CES preferences, production using intermediate goods, trade in intermediate goods, differentials in country size, and home bias in preferences and technology. In particular, we assume that period utility is now

$$U_t = \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{\chi}{1+\psi} H_t^{1+\psi} \tag{26}$$

where:

$$C_t = \left(\varepsilon^{\frac{1}{\lambda}} C_{h,t}^{1-\frac{1}{\lambda}} + (1-\varepsilon)^{\frac{1}{\lambda}} C_{f,t}^{1-\frac{1}{\lambda}} \right)^{\frac{1}{1-\frac{1}{\lambda}}} \tag{27}$$

where $\varepsilon \geq n$, representing the possibility of home bias in preferences.[15] Given this, the true price index for the Home consumer becomes

$$P_t = \left(\varepsilon P_{h,t}^{1-\lambda} + (1-\varepsilon)((1+\tau_t) S_t P_{f,t}^*)^{1-\lambda} \right)^{1/(1-\lambda)}. \tag{28}$$

Firms now use domestic and imported intermediate goods in production, so the production function for home firm i becomes:

$$Y_{i,h,t} = A_t H_{i,t}^{1-\alpha} X_{i,t}^{\alpha}. \tag{29}$$

Here, $X_{i,t}$ represents the use of intermediate goods on the part of the Home firm i and $H_{i,t}$

[15]Letting $0 \leq x \leq 1$ represents the degree of home bias in preferences, where $x = 0$ ($x = 1$) represents zero (full) home bias, we can define $\varepsilon = n + x(1-n)$.

the use of labor. We allow that intermediate good inputs are composed of Home and Foreign goods in a different composition than that of the consumption aggregator. Namely

$$X_{i,t} = \left(\varepsilon_x^{\frac{1}{\lambda}} X_{i,h,t}^{1-\frac{1}{\lambda}} + (1-\varepsilon_x)^{\frac{1}{\lambda}} X_{i,f,t}^{1-\frac{1}{\lambda}} \right)^{\frac{1}{1-\frac{1}{\lambda}}}. \tag{30}$$

The full description of the competitive equilibrium is set out in Appendix B. Using the notation of Section 2 above, and the definition of the true price index, we define $\frac{P_{h,t}}{P_t} = \frac{1}{\mathcal{P}_t}$, where $\mathcal{P}_t = \left(\varepsilon + (1-\varepsilon)((1+\tau_t)\mathcal{S}_t)^{1-\lambda}\right)^{\frac{1}{1-\lambda}}$, and likewise $\mathcal{P}_{x,t} = \left(\varepsilon_x + (1-\varepsilon_x)((1+\tau_t)\mathcal{S}_t)^{1-\lambda}\right)^{\frac{1}{1-\lambda}}$. Then, Appendix B shows that, conditional on monetary policies $\left\{\pi_{h,t}, \pi^*_{f,t}\right\}$ and tariff policies $\{\tau_t, \tau^*_t\}$, the equilibrium can be written in the form of 5 equations in the 5 variables $Y_{h,t}$, $Y^*_{f,t}$, C_t, C^*_t and $\mathcal{S}_t$. These are expressed as follows:

$$Y_{h,t}\left(1-\xi(\pi_{h,t})\right) = \mathcal{S}_t \left(\frac{(1+\tau_t)\mathcal{S}_t}{\mathcal{P}_t}\right)^{-\lambda} D^f_{h,t} + \left(\frac{1}{\mathcal{P}_t}\right)^{-\lambda} D^h_{h,t} \tag{31}$$

$$(1+s) = \epsilon\left((1+s) - \mathcal{P}_{x,t}\alpha^{-1}\mathcal{X}_t\right) + \xi'\left(\pi_{h,t}\right)\pi_{h,t} + E_t\omega_t\xi'\left(\pi_{h,t+1}\right)\pi_{h,t+1} \tag{32}$$

$$(1+s^*) = \epsilon\left((1+s^*) - \mathcal{P}^*_{x,t}\mathcal{S}_t^{-1}\alpha^{-1}\mathcal{X}^*_t\right) + \xi'\left(\pi^*_{f,t}\right)\pi^*_{f,t} + E_t\omega^*_t\xi'\left(\pi^*_{f,t+1}\right)\pi^*_{f,t+1} \tag{33}$$

$$Y_{h,t}\left(1-\xi\left(\pi_{h,t}\right)\right) = \left(\frac{1}{\mathcal{P}_t}\right)^{-\lambda} D^h_{h,t} + \frac{1-n}{n}\left(\frac{(1+\tau^*_t)}{\mathcal{P}^*_t}\right)^{-\lambda} D^h_{f,t} \tag{34}$$

$$Y^*_{f,t}\left(1-\xi\left(\pi^*_{f,t}\right)\right) = \left(\frac{\mathcal{S}_t}{\mathcal{P}^*_t}\right)^{-\lambda} D^f_{f,t} + \frac{n}{1-n}\left(\frac{(1+\tau_t)\,\mathcal{S}_t}{\mathcal{P}_t}\right)^{-\lambda} D^f_{h,t} \tag{35}$$

where

$$D^f_{h,t} = (1-\varepsilon)C_t + (1-\varepsilon_x)\left(\frac{\mathcal{P}_t}{\mathcal{P}_{x,t}}\right)^{-\lambda}\mathcal{X}_t \text{ and } D^h_{h,t} = \varepsilon C_t + \varepsilon_x\left(\frac{\mathcal{P}_t}{\mathcal{P}_{x,t}}\right)^{-\lambda}\mathcal{X}_t \tag{36}$$

$$D^h_{f,t} = (1-\varepsilon^*)C^*_t + (1-\varepsilon^*_x)\left(\frac{\mathcal{P}^*_t}{\mathcal{P}^*_{x,t}}\right)^{-\lambda}\mathcal{X}^*_t \text{ and } D^f_{f,t} = \varepsilon^* C^*_t + \varepsilon^*_x\left(\frac{\mathcal{P}^*_t}{\mathcal{P}^*_{x,t}}\right)^{-\lambda}\mathcal{X}^*_t \tag{37}$$

stand for aggregate Home/Foreign demands for Home/Foreign goods, and

$$\mathcal{X}_t = \alpha\left(\frac{\mathcal{P}_t C_t^{\sigma} Y_{h,t}^{\psi}}{\mathcal{P}_{x,t} A_t^{(1+\psi)/(1-\alpha)}}\right)^{\frac{1-\alpha}{1-\alpha+2\alpha}} Y_{h,t}\Phi_1 \text{ and } \mathcal{X}^*_t = \alpha\left(\frac{\mathcal{P}^*_t C_t^{*\sigma} Y_{f,t}^{\psi}}{\mathcal{P}^*_{x,t} A_t^{*(1+\psi)/(1-\alpha)}}\right)^{\frac{1-\alpha}{1-\alpha+2\alpha}} Y^*_{f,t}\Phi_1 \tag{38}$$

for the Home and Foreign aggregate use of intermediate goods, with $\Phi_1 \equiv \left(\frac{1}{(\alpha^{\alpha})^2}\right)^{\frac{1}{1-\alpha+2\alpha}}$. Equation (31) represents the concentrated Home budget constraint, taking account of the demand for intermediate goods on the part of Home firms. Then, Equations (32) and (33) are the Home and Foreign inflation equations, after substituting for the marginal cost functions, while Equations (34) and (35) are the goods market clearing conditions for the two countries.

3.1 Calibration

We now derive the solution to the optimal policy games in the full model. The model is calibrated to an annual frequency. The discount factor of households is $\beta = 0.96$, consistent with a real interest rate of 4% *per annum*. Both countries are of similar size in the baseline calibration so that $n = 0.5$. Further, we assume a home bias parameter $x = 0.7$ which implies $\varepsilon = \varepsilon_x = (1 - \varepsilon^*) = (1 - \varepsilon_x^*) = 0.85$. With zero tariffs, this number is associated with a 30% total trade openness ratio, as in U.S. data. We consider a baseline value of $\sigma = 1$, implying a log utility for consumption, but also examine alternative values of σ. The Frisch elasticity is $\psi^{-1} = 1$ and we normalize $\chi = 1$. The elasticity of substitution between varieties is $\epsilon = 6$, consistent with a 20% steady-state price-cost mark-up when not corrected by a steady-state subsidy and the Rotemberg parameter is $\phi = 40$. Following Itskhoki and Mukhin (2021), we consider a $\alpha = 0.5$ share of intermediate goods in production. Last, the trade elasticity is $\lambda = 3$. This is on the higher end of the range estimated by Feenstra et al. (2018), but is more appropriate for the evaluation of trade policy.

3.2 Currency Wars

Table 1 describes the steady state outcome of the Nash and Cooperative equilibrium where policy-makers choose only inflation rates. As described above, in the Nash equilibrium each country faces a trade-off between choosing a positive rate of inflation in order to eliminate the monopoly pricing distortion on economic activity, and choosing disinflation to reduce output and appreciate the terms of trade against their trading partner, thus partly substituting for the absence of direct trade policy instruments. For the particular calibration in Table 1, the first motive dominates, and the Nash equilibrium inflation rate is 4.02 percent. By contrast, with cooperative monetary policy the terms of trade motive is eliminated, and each country chooses a much higher positive rate of inflation of 4.84 percent. As expected, monetary policy cooperation is welfare reducing, confirming the Rogoff (1985).

If subsidies are in place to offset the monopoly distortion, then Table 1 confirms Result 2 above in Section 2. Each country follows a deflationary monetary policy, since the terms-of-trade motive then fully dominates the incentives for inflation in each country. By contrast, if optimal subsidies are in place, and monetary policy is chosen cooperatively, inflation rates are zero, then the equilibrium is first-best, since all distortions are eliminated and inflation is zero.

We conclude from these results that 'currency wars' may be either good or bad. If there is a pre-existing monopoly distortion, cooperation in monetary policy may be undesirable, whereas with optimal subsidies in place, cooperation supports the first best outcome.

Table 1: Currency wars

	No subsidy ($s = 0$)			Subsidy ($s = 1/(\epsilon - 1)$)		
	Base.	Coop M.	Flex. P	Base.	Coop M.	Flex. P
$\pi_h = \pi_f^*$	1.0402	1.0484	–	0.9898	1.0000	–
$\tau = \tau^*$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
S	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$C = C^*$	0.2026	0.2005	0.2054	0.2488	0.2500	0.2500
$H = H^*$	0.8799	0.8941	0.8452	0.9994	1.0000	1.0000
Welfare loss (%)	9.2647	11.3257	5.2142	0.4189	0.0000	0.0000

Note: Base. denotes the baseline Nash equilibrium, Coop M. the equilibrium where monetary policy is chosen cooperatively and Flex. P the allocations under flexible prices.

3.3 Currency Wars and Trade Wars

We now compare the previous results to a situation where policy involves the choice of both tariffs and inflation rates in a Nash discretionary equilibrium. Table 2 illustrates the allocations and welfare effects of the combined currency war and trade war. We start with the unilateral case of the Table, which shows the outcome where both countries choose inflation rates non-cooperatively, but the Home country chooses an optimal tariff unilaterally. Under the baseline calibration, without subsidy, the Home country chooses a tariff rate of 41 percent. This generate a 18.7 percent appreciation in its terms of trade and raises Home welfare at the expense of Foreign welfare, comparing the 4th column of the left panel of Table 2 with the corresponding currency war in Table 1. In addition, the tariff raises relative Home consumption, but reduces relative Home output.

Table 2: Trade and Currency Wars

	No subsidy ($s = 0$)					Subsidy ($s = 1/(\epsilon - 1)$)			
	Base.	Coop M.	Flex P.	Unil.	Commit.	Base.	Coop M.	Flex P.	Unil.
π_h	1.047	1.053	–	1.047	1.045	1.000	1.007	–	1.000
π_f^*	1.047	1.053	–	1.043	1.045	1.000	1.007	–	0.994
τ	0.405	0.418	0.354	0.410	0.168	0.518	0.521	0.518	0.525
τ^*	0.405	0.418	0.354	0.000	0.168	0.518	0.521	0.518	0.000
S	1.000	1.000	1.000	0.813	1.000	1.000	1.000	1.000	0.775
C	0.190	0.188	0.195	0.200	0.196	0.234	0.235	0.234	0.249
C^*	0.190	0.188	0.195	0.192	0.196	0.234	0.235	0.234	0.234
H	0.873	0.884	0.829	0.864	0.874	0.976	0.979	0.976	0.962
H^*	0.873	0.884	0.829	0.884	0.874	0.976	0.979	0.976	0.999
Home welf. loss (%)	14.438	16.116	8.666	9.255	11.619	3.994	4.199	3.994	−3.252
Foreign welf. loss (%)	14.438	16.116	8.666	14.473	11.619	3.994	4.199	3.994	6.129

Note: Base. denotes the baseline Nash equilibrium, Coop M. the equilibrium where monetary policy is chosen cooperatively, Flex. P the allocations under flexible prices, Unil. stands for the case with non-cooperative monetary policy and the Home Nash policy-maker sets its tariff while the Foreign tariff is nul. Commit. denotes the case where tariffs are set non-cooperatively but taking into account their effects on monetary policy.

The remaining columns of Table 2 show the results for a trade war, where both countries choose an optimal tariff rate, in addition to an optimal inflation rate, in a discretionary Nash equilibrium. Without optimal subsidies, the trade war leads to mutual tariff rates of 40.5 percent. In the symmetric Nash equilibrium, there is no change in the terms of trade, but the rise in domestic prices leads to a shift back in labor supply which reduces equilibrium employment and output. At the same time, the fall in consumption of imported goods distorts the composition of consumption and leads to a fall in aggregate consumption in both countries. Thus, the trade war has large negative effects on real activity.

Table 2 also shows, however, that the trade war causes a change in equilibrium inflation rates. Absent the trade war, Nash equilibrium inflation rates were 4.02 percent (see Table 1), which as described above, represented a balance between the desire to eliminate monopoly distortions and the desire to improve the national terms of trade. When countries engage in the trade war, optimal tariffs focus on the second objective – terms-of-trade manipulation – and monetary authorities redirect inflation rates towards the first objective. As a result, inflation rates are quite higher – at 4.7 percent – in the equilibrium with both trade and currency wars.

Table 2 further indicates that the trade war has major implications for welfare. Comparing the Nash discretionary equilibrium of the combined trade and currency wars with that of the Nash equilibrium under the currency war alone (Table 2 compared with similar cases in Table 1) leads to a fall in welfare. Without subsidies, the welfare losses from a currency war are 9.26% of first-best equivalent consumption while the welfare losses from combined trade and currency wars jump to 14.44 percent, a difference of more than 5 percentage points of first-best consumption equivalent.

The second column of Table 2, still in the case of zero subsidies, documents the outcome where policy-makers cooperate on monetary policy, but follow a trade war in the choice of tariffs. As we would anticipate, given the results of Table 1, monetary policy cooperation is again counter-productive. But this is now for two reasons. First, as before, the equilibrium inflation rates increase from 4.7 percent to 5.3 percent, as monetary policy focuses only on eliminating domestic distortions and ignores the impact on the terms of trade. Second, this adjustment in the focus of monetary policy leads to a redirection of tariffs: the trade war becomes more intense, as independent policy-makers increase tariffs to more fully exploit a terms-of-trade advantage. Tariff rates increase to 41.8 percent – against 40.5 percent when monetary policy is non-cooperative – and aggregate consumption falls by 1 percent. We conclude again that eliminating currency wars is undesirable, not just due to higher inflation, but because it also leads to an increase in trade protection.

3.3.1 Equilibrium with zero markups

When markups are removed by a production subsidy, Result 1 in the simple model of Section 2 showed that inflation rates in the currency and trade war Nash equilibrium were zero. This is confirmed in Table 2: imposing an optimal subsidy full eliminates inflation. But the consequence is a substantial increase in protection, as removing the markup distortion leads to a rise in equilibrium tariffs from 40.5 percent to 51.8 percent. Instead of using deflationary monetary policy as in the currency wars case in Table 1, governments now increase tariff rates, consistently with the analytical results of Section 2. Intuitively, in the distorted economy, tariffs are set as a compromise between improving the terms of trade and limiting the distortionary effects on domestic output, which is already inefficiently low due to the presence of markups. Removing markups means that both governments implement the monopoly tariff level as shown in Section 2.

The 7th column of Table 2 quantitatively echoes the conclusions of Result 6 above. Monetary policy cooperation reduces welfare when tariffs are determined non-cooperatively, *even* in the presence of optimal subsidies. Indeed, comparing the 6th and 7th column of Table 2, cooperation in monetary policy leads to a rise in equilibrium inflation rates, and a rise in equilibrium tariff rates. As shown in Section 2 , inflation rates rise as cooperative policy-makers attempt to offset the distortion in the composition of global consumption generated by tariffs. But at the same time, this would reduce the terms of trade for each country and thus leads individual tariff setters to raise their tariff rates in a Nash equilibrium. Thus, eliminating the currency war (without eliminating the trade war) is counter-productive, even in the absence of monopoly pricing distortions.

3.3.2 Flexible price equilibrium

Result 4 above showed that in the simplified model, when monetary policy was constrained to stabilize prices, tariffs were lower than the monopoly tariff rate if there were positive monopoly markups.

The 3rd column of Table 2 shows that without production subsidies, equilibrium Nash tariffs are substantially lower under zero-inflation monetary policy. We noted that this outcome is identical to one without any price rigidities (*i.e.* $\phi = 0$), since in this model, the equilibrium under zero inflation is equivalent to a flexible price economy.

With monopoly markup distortions, and when prices are fully flexible, inflation has no traction in either reducing distortions or affecting the terms of trade. Hence, tariffs must be used as a compromise between the two objectives, and in a Nash equilibrium protection is less than in an economy with sticky prices. By contrast, without the markup distortion, tariff rates are exactly the same whether prices are flexible or sticky. In this case, as shown in Results 3 and 4, tariffs are entirely focused on the terms-of-trade externality.

These results indicate that trade wars imply very high rates of protection in standard DSGE macro models. We might thus question the relevance of this analysis, given that in recent history, observed tariffs among advanced economies have been much lower. For instance, the average degree of trade restriction (including both tariff and non-tariff barriers), reported by UNCTAD (2013) for advanced economies is approximately 10 percent. But it is important to note that these observations are taken from a period where WTO rules and other bilateral agreements governed the size of tariffs. The interpretation we follow here is to explore the consequences of a full scale breakdown of cooperation in trade policy. In this case, the tariff rates may not be so unrealistic. In fact, in a calibrated multi-country trade model, Ossa (2014) finds that average tariffs would be over 60 percent in a full-scale world 'tariff war'. In addition, we note that in the case of US China trade, average US tariff rates as measured by Bown (2019) rose from 8 percent in early 2018 to 26 percent at the end of 2019.

3.4 Commitment in trade policy

So far it has been assumed that both inflation and tariffs are chosen simultaneously by domestic policy-makers to maximize national welfare. A central assumption is that policy is discretionary, so that policy-makers cannot bind the hands of future policy-makers, rather take these future actions as given. But it could be argued that trade policy embodies more commitment than monetary policy. Trade policy is typically enacted by legislation, and this is not as easily changed as monetary policy decisions, which can be altered at the whim of an independent central bank.

In this subsection, we analyze a simplified game where trade policy is determined in a non-cooperative game between policy-makers, but assuming that the trade policy-makers can commit to their tariff choices. The general case where trade policy is made with commitment and monetary policy is discretionary in the two country setting involves a complicated dynamic interaction. We focus instead on a much simplified setting where trade authorities commit to a single tariff rate that remains constant. Moreover, we assume that in choosing tariffs, the trade authorities internalize the endogenous response of inflation rates to tariffs in the currency war game between monetary authorities.

Therefore, in the initial period trade authorities choose a tariff rate, taking the tariff rate of the other authority as given, but taking into account the equilibrium of the monetary policy game played by the monetary authorities, within each period. We focus on a steady state of this tariff game with commitment. Given the initial tariff rate, monetary authorities choose their inflation rate in a currency war, without commitment. With constant tariff rates, which are equal in a symmetric equilibrium, inflation rates are constant over time, and also equal across countries.

The optimal tariff rates for this game can be chosen simply as a Nash equilibrium in τ and τ^* where each trade authority chooses to maximize one-period domestic utility, taking account of the competitive equilibrium, and internalizing the response of inflation in both countries to their tariff rate, but taking as given the tariff rate of the other country.

Somewhat more formally, define $V(\tau_t, \tau_t^*)$ and $V^*(\tau_t, \tau_t^*)$ as follows:

$$V(\tau_t, \tau_t^*) = \text{Max}_{\{C_t, C_t^*, Y_{h,t}, Y_{f,t}^*, \mathcal{S}_t, \pi_{h,t}\}} \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{\chi}{1+\psi}\left(\frac{Y_{h,t}}{A_t \mathcal{X}_t^{\alpha}}\right)^{\frac{1+\psi}{1-\alpha}} \tag{39}$$

subject to (31)-(35).

$$V^*(\tau_t, \tau_t^*) = \text{Max}_{\{C_t, C_t^*, Y_{h,t}, Y_{f,t}^*, \mathcal{S}_t, \pi_{f,t}\}}, \ \frac{C_t^{*1-\sigma} - 1}{1-\sigma} - \frac{\chi}{1+\psi}\left(\frac{Y_{f,t}}{A_t^* \mathcal{X}_t^{*\alpha}}\right)^{\frac{1+\psi}{1-\alpha}} \tag{40}$$

subject to (31)-(35).

Then a Nash equilibrium with commitment in tariff policy, τ_t^N, τ_t^{*N} is defined by the equilibrium to the conditions:

$$\text{Max}_{\tau_t} V(\tau_t, \tau_t^{*N}) \tag{41}$$

$$\text{Max}_{\tau_t^*} V^*(\tau_t^N, \tau_t^*) \tag{42}$$

The 5th column of Table 2 illustrates the equilibrium of this game when markup are not offset by subsidies. The most striking feature of the corresponding results is that tariff rates are significantly lower than those in the baseline case of the simultaneous-move game. The Nash tariff rates for the calibrated model are 16.8 percent, compared with 40.5 percent in the baseline model. At the same time, the equilibrium inflation rates are lower, and consumption, output, and welfare for each country is higher.

What accounts for the difference between the commitment equilibrium and the baseline case? The key factor is that the trade authorities take account of the endogenous increase in inflation that will follow from a higher round of tariffs facing the monetary policy-makers in the second stage of the game. Because this inflation will be costly due to price adjustment costs, but have little benefit in terms of higher output, the trade authorities endogenously choose lower equilibrium tariff rates. Individually, monetary authorities choose a rate of inflation taking future inflation rates as given. In a steady-state equilibrium, the future inflation rate is equal to the current inflation rate, so that from the firm's first order condition in the Home country, we have

$$(1+s) - \epsilon\left((1+s) - mc\right) - \xi'\left(\pi_h\right)\pi_h + \beta\xi'\left(\pi_h\right)\pi_h = 0$$

Since the trade authorities take account of the sequence of their tariff choices on π, they individually choose a lower degree of protection than in the tariff game without commitment, where both tariffs and inflation are taken as given.

This example highlights the implications of a loss of commitment in trade policy. Even in the absence of any international trade agreements, when tariffs are chosen without commitment, at the same frequency as monetary policy, there may be significant losses in welfare.

3.5 Country-size effects and alternative parameter values

All the previous derivations assumed equally-sized countries. But discussions of currency and trade wars are often focused on the role of large countries relative to small countries. Particularly in the discussion of monetary policy spillovers, it is often argued that smaller countries

Table 3: Effects of country size on combined Trade and Currency Wars

	Currency war		Curr.+Trade war	
	Base.	$n = 0.75$	Base.	$n = 0.75$
π_h	1.0402	1.0445	1.0474	1.0479
π_f^*	1.0402	1.0357	1.0474	1.0467
τ	0.0000	0.0000	0.4053	0.4179
τ^*	0.0000	0.0000	0.4053	0.3934
S	1.0000	1.0009	1.0000	0.9809
C	0.2026	0.2016	0.1900	0.1955
C^*	0.2026	0.2036	0.1900	0.1840
H	0.8799	0.8869	0.8733	0.8837
H^*	0.8799	0.8732	0.8733	0.8629
Home welf. loss (%)	9.2647	10.2871	14.4376	12.7529
Foreign welf. loss (%)	9.2647	8.2732	14.4376	16.3782

are more exposed to the negative effects of policy spillovers from larger countries.

In the baseline model without endogenous policy choice, country size is irrelevant for real outcomes such as consumption, output, terms of trade or welfare. [16] But size may matter when countries engage in currency wars or trade wars. Table 3 illustrates the importance of large versus small countries in the case of currency wars, and currency and trade wars.

The first two panels on the left-hand side illustrate the impact of an increase in the size of the Home country from 50 percent to 75 percent of the world economy in the case of a currency war alone, with zero tariffs. Contrary to received wisdom, the Home (large) country actually suffers relative to the equal size benchmark. The reason is again related to the trade-off between terms-of-trade manipulation and inflation. When the Home country is larger, it behaves more like a closed economy and focuses more on inflationary stimulus to offset the monopoly distortion. In a discretionary equilibrium, this leaves the Home country worse off. The Foreign country, by contrast, focuses more on terms-of-trade manipulation. In equilibrium, the Home's inflation rate rises, and Foreign's falls. So, in the currency war, country size is welfare reducing.

The two right-hand panels of Table 3 illustrate the impact of country size in the case of combined currency and trade wars. Relative to the equal-size Nash equilibrium, the Home tariff rises and Foreign's falls. Because the larger country's consumption basket is more weighted towards its own goods, the cost of a tariff on domestic consumption is less, while conversely, that for the Foreign country is greater. The result is that the (large) Home country is more protectionist, obtains a significant terms-of-trade advantage, and gains in welfare relative to the Foreign country. Country size is thus an advantage in the combined currency and trade war environment, but a disadvantage in the currency war alone.

Table 7 in Appendix C illustrates the outcome under alternative parameter values for the trade

[16] This is because as country size varies, so also does the range of goods that each country produces, so size has no implications for the terms of trade.

and currency wars. For the degree of protection, not surprisingly the most important parameter is the trade elasticity. Our calibration uses $\lambda = 3$, which is on the high side of the trade elasticities used in the aggregate macro literature. But elasticities in the trade literature tend to be higher. For a value of $\lambda = 6$ we find that the symmetric Nash equilibrium of the current and trade war implies a tariff rate of 16.4 percent, substantially lower than that of Table 2. The consequent welfare impacts of the trade war are then less. But the main qualitative implications are the same as above.

4 Constraints on monetary policy

In this section we explore the effects of three situations where monetary policy is subject to some kind of constraint, and the interaction between monetary and trade policy susceptible of being significantly altered. First we consider a situation of fixed exchange rates, in which the Foreign economy loses its monetary policy independence by pegging its currency to the Home economy. Second, we investigate how trade and monetary policies are affected when both economies are hit by a large discount factor shock that leads both economies to hit the zero lower bound (ZLB) on nominal interest rates. In this case, the determination of both inflation rates becomes endogenous. Third, we consider a world economy with dominant currency pricing (DCP), and where the Home economy issues the dominant currency.

4.1 Fixed Exchange Rates

Now assume that the Foreign economy has an exchange-rate target, so it cedes control over its domestic inflation rate, leaving the Home country to independently choose an inflation rate. In this case, only the Home policy-maker has an independent monetary instrument. If the Foreign country targets the nominal exchange rate, we must have

$$\pi_{h,t} = \pi^*_{f,t} \frac{\mathcal{S}_{t-1}}{\mathcal{S}_t}. \tag{43}$$

This adds a state variable to the model in the form of the lagged terms of trade. Since the nominal exchange rate is pegged, the terms of trade can adjust *only* via differences in inflation rates. In addition, because the Foreign country is pegging the nominal exchange rate, it loses control of $\pi^*_{f,t}$, so the Home country takes (43) as a constraint in its choice of $\pi_{h,t}$.

Under a fixed exchange rate regime, we must explicitly account for the initial conditions faced by the policy-makers in the form of the lagged terms of trade $\mathcal{S}_{t-1}$. Since the peg itself represents the monetary policy of the Foreign country, we describe a fixed exchange rate problem as the problem of the Home country. In this case, the Home country will choose $\pi_{h,t}$ to maximize its

Table 4: Trade war under fixed exchange rates

	Currency war				Curr.+Trade war			
	No subsidy ($s=0$)		Subsidy ($s=1/(\epsilon-1)$)		No subsidy ($s=0$)		Subsidy ($s=1/(\epsilon-1)$)	
	Flex ER	Fixed ER	Flex ER	Fixed ER	Flex ER	Fixed ER	Flex ER	Fixed ER
π_h	1.0402	1.0484	0.9898	1.0000	1.047	1.053	1.000	1.007
π_f^*	1.0402	1.0484	0.9898	1.0000	1.047	1.053	1.000	1.007
τ	0.0000	0.0000	0.0000	0.0000	0.405	0.292	0.518	0.410
τ^*	0.0000	0.0000	0.0000	0.0000	0.405	0.292	0.518	0.410
S	1.0000	1.0000	1.0000	1.0000	1.000	1.000	1.000	1.000
C	0.2026	0.2005	0.2488	0.2500	0.190	0.191	0.234	0.237
C^*	0.2026	0.2005	0.2488	0.2500	0.190	0.191	0.234	0.237
H	0.8799	0.8841	0.9994	1.0000	0.873	0.885	0.976	0.979
H^*	0.8799	0.8841	0.9994	1.0000	0.873	0.885	0.976	0.979
Home welf. loss (%)	9.2647	11.326	0.4189	0.0000	14.438	14.904	3.994	3.183
Foreign welf. loss (%)	9.2647	11.326	0.4189	0.0000	14.438	14.904	3.994	3.183

value $v(\mathcal{S}_{t-1})$. The problem can be stated as

$$\text{Max}_{\{C_t,C_t^*,Y_{h,t},Y_{f,t}^*,\mathcal{S}_t,\pi_{h,t},\pi_{f,t}\}} \quad v(\mathcal{S}_{t-1}) = \frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{\chi}{1+\psi}\left(\frac{Y_{h,t}}{A_t\mathcal{X}_t^{\alpha}}\right)^{\frac{1+\psi}{1-\alpha}} + E_t\beta v(\mathcal{S}_t) \tag{44}$$

subject to (31)-(35) and (43).

Table 4 shows the outcome of the currency and trade war in the case of fixed exchange rates. When exchange rates are pegged by the Foreign country, only the Home country has an independent monetary policy. Absent tariffs, the left panel shows that the Home country will choose an inflation rate of 4.84 percent under our calibration, and the equilibrium is perfectly symmetric. Given unitary initial terms of trade, so that $\mathcal{S}_{t-1} = 1$, the Home country can only improve its terms of trade by a higher rate of inflation, relative to the Foreign country. This contrasts with the flexible exchange rate case, where, for a given Foreign rate of inflation, the terms of trade can be improved by a contractionary monetary policy and an exchange rate appreciation, giving rise to a downward bias in inflation rates in both countries. With a fixed exchange rate, the Home country instead focuses on removing the monopoly distortion for a given terms of trade. This leads to a symmetric equilibrium where both countries inflation rates are positive, and the terms of trade is unchanged. In fact, in comparing Table 4 with Table 1, we see that the fixed exchange rate case is identical to the equilibrium of the currency war with cooperation in monetary policy. This then implies that in welfare terms, the currency war equilibrium dominates the equilibrium with fixed exchange rates, absent the trade war.

The right panel of Table 4 compares trade wars under an exchange rate peg to the flexible exchange rate case. This panel also identifies a fully symmetric outcome, where the existing terms of trade facing each policy-maker is unity. The Home country chooses its inflation rate and its tariff rate, and the Foreign country chooses only its tariff rate. In a symmetric equilibrium both inflation rates and tariff rates are equal. What is most striking about this outcome is the large difference between non-cooperative tariff rates relative to the flexible exchange rate case. In the Nash equilibrium tariff rates in each country are only 29.2 percent, compared to 40.5 percent in the flexible exchange rate equilibrium.

What is the intuition for the substantial difference between fixed and flexible exchange rates with respect to equilibrium tariff rates? This can be best explained by focusing on equation (43), repeated here.

$$\pi_{h,t} = \pi^*_{f,t} \frac{\mathcal{S}_{t-1}}{\mathcal{S}_t}.$$

Under the fixed exchange rate regime, the Home country is choosing both its tariff rate and its own inflation rate. If it chooses its tariff rate to appreciate the terms of trade, then this implies, given $\mathcal{S}_{t-1}$, that it must be increasing its inflation rate, relative to the Foreign country inflation rate. But the fact that the authority is simultaneously choosing $\pi_{h,t}$ subject to the costs of inflation adjustment effectively reduces the benefits of an appreciated terms of trade. In a symmetric equilibrium where $\mathcal{S}_{t-1} = 1$ these factors exactly offset, so that it chooses an inflation rate identical to the Foreign rate, and a tariff rate identical to the Foreign tariff rate. For our calibration, the reduced benefit of tariff hikes under a peg leads to a substantially lower equilibrium tariff rates. Moreover, in welfare terms there is little difference between fixed and flexible exchange rates under the trade and currency war, given the lower rate of protection in the former, while, as noted, the currency war outcome under fixed exchange rates is significantly worse in welfare terms.[17]

4.2 The Zero Lower Bound

One of the principal sources of the debate on currency wars was the fall in policy interest rates in the US and Europe following the Great Financial Crisis. Caballero, Farhi, and Gourinchas (2015) and Jeanne (2020) develop models of trade and currency wars at the zero lower bound (ZLB) of nominal interest rates. We now address this issue within the context of our model. We assume that monetary policy is temporarily constrained, and inflation rates are determined endogenously, given expectations about future monetary policy as well as the current stance of trade policy. In this case, the only policy tool available during the zero-bound period is that of trade policy.[18]

When there is no constraint on nominal interest rate adjustment, it is not necessary to incorporate household's Euler equations to determine equilibria. But in order to capture the ZLB

[17] There is an important caveat to these results. Indeed, there exists a continuum of equilibrium Nash tariff rates conditioned on different values of $\mathcal{S}_{t-1}$. If we take an initial value $\mathcal{S}_{t-1} < 1$, then the Home country will choose a tariff rate higher than that of the Foreign country, so that in equilibrium $\mathcal{S}_t = \mathcal{S}_{t-1} < 1$, and equilibrium inflation rates are, again, equalized. Likewise for $\mathcal{S}_{t-1} > 1$, then the Home country chooses a lower tariff rate than the Foreign country, and again $\mathcal{S}_t = \mathcal{S}_{t-1} > 1$, with identical inflation rates. Thus, there is a continuum of Nash equilibrium tariff rates in which the Home country is more or less protectionist than the Foreign country, and each delivers a more or less appreciated terms of trade for the Home country. See Auray, Devereux, and Eyquem (2020) for a further analysis of this case.

[18] Since we are assuming that all monetary policy-makers lack commitment, we do not explore the consequences of Forward Guidance in monetary policy announcements.

constraint, we have to incorporate households inter-temporal choices. Because we are in financial autarky, net national saving is zero, so household's Euler equations must be consistent with zero current-account balance. Nonetheless, when nominal interest rates are constrained at zero, households inter-temporal savings decisions have an impact on aggregate demand and economic activity. In the case of the Home economy, defining R_t as the gross nominal interest rate, the Euler equation is:

$$1 = \beta \exp\left(-\zeta_t\right) E_t \frac{C_t^{\sigma} P_t}{C_{t+1}^{\sigma} P_{t+1}} R_t. \tag{45}$$

We assume that outside of the ZLB, the interest rate is determined by the Taylor rule:

$$R_t^{Taylor} = \frac{1}{\beta} \exp\left(\zeta_t\right) \left(\frac{\pi_{h,t}}{\bar{\pi}}\right)^{\sigma_\pi} \tag{46}$$

where $\bar{\pi}$ is a target rate of inflation, which is set to mimic the steady state of the Nash equilibrium in the policy game defined above, and ζ_t is a time preference shock. We also assume that $\sigma_\pi > 1$. We will assume an 'MIT' shock process for the ζ_t shock. Initially, $\zeta = 0$, but then $\zeta < 0$ occurs without anticipation, and continues with probability μ, while it reverts to zero with probability $1 - \mu$. We assume identical ζ_t shocks in each country. We focus on a $\zeta_t < 0$ that is large enough in absolute value that, without restriction, $R_t < 1$ would be required to satisfy (45) and (46). In this case, we need to impose the interest rate non-negativity constraint:

$$R_t = \max\left(R_t^{Taylor}, 1\right). \tag{47}$$

Table 5 illustrates the impact of the zero lower bound on the trade war. In our numerical computation, the ZLB is generated by a 15% fall in the subjective discount rate of the private sector and we assume this persists with probability 0.5. As discussed above, in this case, the monetary authority has no control of current rates of inflation, and inflation is determined by aggregate demand, given forward looking consumers and the expectation that the economy will revert to the Nash equilibrium of the currency and trade war as described in Table 2. In the absence of trade policy, the ZLB outcome leads to an equilibrium with large deflation rates, with consumption and output significantly below the Nash equilibrium of the currency war levels.

As shown in Table 5, when countries engage in a trade war under the zero lower bound, the outcome is substantially worse. Each country levies tariffs in the Nash equilibrium, but this leads to essentially unchanged inflation rates, but results in lower levels of consumption, output, and welfare. Although the trade war worsens the conditions of the ZLB, the equilibrium tariff rates are actually lower; 27 percent compared to 40.5 percent in the baseline case with active monetary policy and flexible exchange rates. The reasoning behind this is similar to the example of commitment in trade policy discussed above. In the environment of the ZLB, trade policy-makers take account of their choice of tariffs on the endogenous rates of inflation in the

Table 5: Trade and Currency Wars at the ZLB

	Curr. war		Curr.+Trade war	
	Base.	ZLB	Base.	ZLB
π_h	1.0402	0.9812	1.047	0.975
π_f^*	1.0402	0.9812	1.047	0.975
τ	0.0000	0.0000	0.405	0.270
τ^*	0.0000	0.0000	0.405	0.270
S	1.0000	1.0000	1.000	1.000
C	0.2026	0.1773	0.190	0.170
C^*	0.2026	0.1773	0.190	0.170
H	0.8799	0.7698	0.873	0.756
H^*	0.8799	0.7698	0.873	0.756
Home welf. loss (%)	9.2647	13.0390	14.438	15.970
Foreign welf. loss (%)	9.2647	13.0390	14.438	15.970

two countries. This leads them to limit the size of their tariff choices relative to the case where inflation and tariff rates are chosen simultaneously. ZLB constraints thus make trade wars less rather than more intense.

4.3 Dominant Currency Pricing

Recent evidence has pointed to the role of the US dollar as an invoice currency for pricing exports for a large share of the world economy (see Gopinath et al. (2020) and Mukhin (2018)). In terms of our model, this would imply that one country (say the Home country) sets the price of both its exports and domestic sales in its own currency, while the Foreign country sets its domestic sales price in its own currency, while setting its export price in the currency of the Home country. Gopinath et al. (2020) denote this practice as one of dominant currency pricing. In this section we explore the implications of DCP for the currency and trade war equilibrium.

The model under DCP differs in only a few features. The nominal exchange rate is still flexible, but the impact of exchange rate changes on the terms of trade is muted, in particular for the Home country, since both its exports and imports are priced in its own currency. As we show below, this has significant implications for the equilibrium of the policy game and its outcome.

The true price index for the Home consumers under DCP now becomes:

$$P_t = \left(\varepsilon P_{h,t}^{1-\lambda} + (1-\varepsilon)((1+\tau_t)P_{f,t})^{1-\lambda}\right)^{1/(1-\lambda)} \tag{48}$$

where $P_{f,t}$ is the price of the Foreign good set in Home currency. By contrast, the price index for the Foreign economy is unchanged compared to the baseline PCP model, since the Home country firm sets all prices in Home currency.

The optimal pricing policy of Home firms is as before, but Foreign firms charge separate prices to the domestic (in Foreign currency) and Home (in Home currency) firms and households respectively buying intermediate and final goods.

The profits of a representative Foreign firm i are then represented as:

$$\Pi_{i,t}^{*} = \Big((1+s^{*})(P_{i,f,t}^{*}Y_{i,f,t}^{*} + S_t^{-1}P_{i,f,t}Y_{i,f,t}) - MC_{f,t}^{*}(Y_{i,f,t}^{*} + Y_{i,f,t})\Big) \tag{49}$$

where $MC_{f,t}^{*}$ is defined in the same way as before. The first order conditions for profit maximization for the Foreign firm i selling to the Home country can be described as

$$\begin{aligned}(1+s^{*})S_t^{-1}Y_{i,f,t} \quad &- \quad \epsilon(S_t^{-1}P_{i,f,t}(1+s^{*}) - MC_t^{*})\frac{Y_{i,f,t}}{P_{i,f,t}} - \xi'\left(\frac{P_{i,f,t}}{P_{i,f,t-1}}\right)\frac{1}{P_{i,f,t-1}}S_t^{-1}P_{f,t}Y_{f,t} \\ &+ \quad E_t\omega_{t+1}^{*}\xi'\left(\frac{P_{i,f,t+1}}{P_{i,f,t}}\right)\frac{P_{i,f,t+1}}{P_{i,f,t}^{2}}S_{t+1}^{-1}P_{f,t+1}Y_{f,t+1} = 0\end{aligned} \tag{50}$$

Note that the Foreign firm incurs costs of price adjustment for sales to the Home country that are separate from those pertaining to sales to the domestic consumers and firms.

The essential new element that DCP brings to the analysis relates to the terms of trade. In fact, we now have two separate terms of trade. For the Home country, the relative price of imports to exports is now $\mathcal{S}_t = \frac{P_{f,t}}{P_{h,t}}$, where both prices are expressed in Home currency. The terms of trade for the Foreign country is expressed as before; $\mathcal{S}_t^{*} = \frac{S_tP_{f,t}^{*}}{P_{h,t}}$. The two measures may differ due to deviations of the law of one price for the foreign good, since in general with price adjustment costs, $P_{f,t}$ will not always equal $S_tP_{f,t}^{*}$. More critically, $\mathcal{S}_t$ can be adjusted only through nominal price adjustment, while $\mathcal{S}_t^{*}$ adjusts to nominal exchange rate changes for given nominal prices. This effectively means that the Home country terms of trade $\mathcal{S}_t$ displays the same type of persistence as in the case of fixed exchange rates. Since $\mathcal{S}_t = \frac{P_{f,t}}{P_{h,t}}$, we have

$$\mathcal{S}_t = \mathcal{S}_{t-1}\frac{\pi_{f,t}}{\pi_{h,t}}. \tag{51}$$

Thus, the home terms of trade adjusts according to the differential between the Foreign export price inflation and the Home inflation rate.

In the analysis so far, we have assumed that the monetary policy instrument for each country is the PPI inflation rate. In the case of DCP we continue to assume that each country targets its PPI inflation rate in domestic currency. But then from Equation (51), the foreign exported goods inflation rate $\pi_{f,t}$ is an endogenous variable.

The policy game under DCP is defined in the same way as before, where in the currency war game the Home and Foreign policy-makers choose $\pi_{h,t}$ and $\pi_{f,t}^{*}$ respectively, and with both trade and currency wars they choose both inflation rates and tariff rates.

Table 6 describes the equilibrium of the policy game under DCP. First, focusing on the currency war outcome, we see that the equilibrium is asymmetric, with the Home policy-maker

Table 6: Trade and Currency Wars under Dominant Currency Pricing (DCP)

	Curr. war		Curr.+Trade war	
	Base.	DCP	Base.	DCP
π_h	1.0402	1.0515	1.0474	1.0570
π_f^*	1.0402	1.0337	1.0474	1.0303
π_f	1.0402	1.0515	1.0474	1.0570
τ	0.0000	0.0000	0.4053	0.2576
τ^*	0.0000	0.0000	0.4053	0.0106
$\mathcal{S}^*$	1.0000	1.0054	1.0000	0.8833
$\mathcal{S}$	1.0000	0.9993	1.0000	0.8752
C	0.2026	0.1997	0.1900	0.1972
C^*	0.2026	0.1941	0.1900	0.1855
H	0.8799	0.9001	0.8733	0.8906
H^*	0.8799	0.9159	0.8733	0.9210
Home welf. loss (%)	9.2647	12.1885	14.4376	12.5175
Foreign welf. loss (%)	9.2647	15.8480	14.4376	19.9588

choosing a larger inflation rate (5.15%) than in the PCP case (4.02%), and the Foreign country choosing a smaller inflation rate (3.37%). For the currency war these results very much look like the PCP case with asymmetric country size but for a different reason: running up inflation is the only way for the Home country to improve its terms of trade. This policy gives the dominant currency issuer an edge in terms of welfare compared to the Foreign country, but both countries are worse off than in the case of a currency war under flexible exchange rates and PCP.

When we allow for both currency wars and trade wars under the DCP specification, Table 6 shows a more substantial asymmetry. The Foreign country sets a very low tariff, around 1%, while the Home country imposes a 26% tariff, much larger than the Foreign country but lower than the PCP tariff. This leads to an equilibrium where the terms of trade are substantially in favor of the Home country. The logic behind this follows from the fact that for the Foreign country to improve its terms of trade *via* a tariff, it must engage in costly inflation in its exported goods price. But in the Nash equilibrium of the trade and currency war, inflation is already high. Increasing exported goods inflation even further would be self-defeating. In fact, it is optimal to moderate inflation through a very small tariff. This leads to a terms-of-trade benefit for the Home country. Then, for the Home country, given an equilibrium terms of trade substantially in its favor, there is little benefit in levying a large tariff. As a result, the presence of DCP leads to a significant asymmetry in welfare outcomes in favor of the dominant currency issuer.[19]

[19] It is important to note that this effect of DCP is purely due to the currency of price-setting and the presence of sticky prices. If prices were fully flexible, then both countries would levy tariffs in a Nash equilibrium at an equal rate given by Table 2.

5 Conclusions

This paper is primarily a theoretical exploration of the links between trade policy and monetary policy from the point of view of international strategic policy interaction. There is a large literature both on international macroeconomic policy coordination/non-coordination on the one hand and the determinants of trade policy and tariff setting in strategic environments on the other hand. In our labeling, we denote the first topic as pertaining to 'currency wars', and the second related to 'trade wars'. Our paper represents a first pass at combining 'currency wars' and 'trade wars' within a simple New Keynesian open economy framework. In the introduction, we argued that contemporary developments in global economic policy made the interaction of these two dimensions of policy-making of much greater relevance than in the past. The results of our analysis show that in many ways, currency wars and trade wars are very closely linked to one another, and differences in policy settings can lead to major differences in macroeconomic outcomes, the overall degree of trade protection, and welfare.

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A The Analytical Model

Using Equations (2), (6) and (9) of the main text, we obtain the balance of payments condition

$$C^*_{h,t} = \mathcal{S}_t C_{f,t} \tag{A.1}$$

Goods market clearing conditions for the home and foreign country are represented by:

$$A_t H_t (1 - \frac{\phi}{2}(\pi_{h,t} - 1)^2) = C_{h,t} + C^*_{h,t} \tag{A.2}$$

$$A^*_t H^*_t (1 - \frac{\phi}{2}(\pi^*_{f,t} - 1)^2) = C_{f,t} + C^*_{f,t} \tag{A.3}$$

Using Equations (4), (8) and the equivalent for the Foreign country, we obtain the labor market equilibrium conditions:

$$\frac{\ell'(H_t)}{A_t} = u_{c_{h,t}} E_t \Psi(\pi_{h,t}, \pi_{h,t+1}, \theta) \tag{A.4}$$

$$\frac{\ell'(H^*_t)}{A^*_t} = u_{c^*_{f,t}} E_t \Psi(\pi^*_{f,t}, \pi^*_{f,t+1}, \theta^*) \tag{A.5}$$

Finally, using Equation (3) and the equivalent for the Foreign country we obtain:

$$\frac{u_{c^*_{h,t}}}{u_{c^*_{f,t}}} = \frac{(1 + \tau^*_t)}{\mathcal{S}_t} \tag{A.6}$$

$$\frac{u_{c_{h,t}}}{u_{c_{f,t}}} = \frac{1}{(1 + \tau_t)\mathcal{S}_t} \tag{A.7}$$

Equations (A.1)-(A.7) represent the competitive equilibrium of the simplified model, conditional on monetary and tariff policy, which can be implicitly solved for H_t, H^*_t, $C_{ht}, C_{f,t}$ $C^*_{h,t}$, $C^*_{f,t}$, and $\mathcal{S}_t$.

A.1 Currency Wars: Optimal inflation choice

The policy-maker in the Home economy chooses inflation, taking the actions of both Foreign policy-maker and future policy-makers (both domestic and foreign) as given. We take the firm's production subsidy as given and constant. In this problem, we abstract from tariffs altogether, and assume that there is free trade, so that $\tau_t = \tau^*_t = 0$. The point is to show that the inflation choice of governments will partly attempt to manipulate the terms of trade in the absence of tariffs.

Define the terms of trade as $\mathcal{S}_t = \frac{S_t P^*_{f,t}}{P_{h,t}}$. The policy problem for the Home government is defined in the form of a value function:

$$v(\mathcal{Z}_t) = \text{Max}_{\{C_{h,t}, C_{f,t},\ H_t, \mathcal{S}_t, \pi_t, C^*_{h,t}, C^*_{f,t}, H^*_t\}}\ u(C_{h,t}, C_{f,t}) - \ell(H_t) + E_t \beta v(\mathcal{Z}_{t+1}) \tag{A.8}$$

subject to (A.1)-(A.7). Let $\xi_{1,t}$, ...,$\xi_{7,t}$ denote the Lagrange multipliers on the constraints (A.1)-(A.7). The first-order conditions for the discretionary policy-maker are then listed as:

$$C_{h,t} \quad : \quad u_{c_{h,t}} = \xi_{2,t} + \xi_{4,t} u_{c_{hh,t}} \Psi_t - \xi_{7,t}(\mathcal{S}_t u_{c_{hh,t}} - u_{c_{hf,t}}) \tag{A.9}$$

$$C_{f,t} \quad : \quad u_{c_{f,t}} = \xi_{1,t}\mathcal{S}_t + \xi_{3,t} + \xi_{4,t} u_{c_{hf,t}} \Psi_t - \xi_{7,t}(\mathcal{S}_t u_{c_{hf,t}} - u_{c_{ff,t}}) \tag{A.10}$$

$$H_t \quad : \quad \ell'(H_t) = \xi_{2,t} A_t \left(1 - \frac{\phi}{2}(\pi_{h,t} - 1)^2\right) + \xi_{4,t} \frac{\ell''(H_t)}{A_t} \tag{A.11}$$

$$\mathcal{S}_t \quad : \quad -\xi_{1,t} \frac{C^*_{h,t}}{\mathcal{S}_t} + \xi_{6,t} u^*_{c_{h,t}} + \xi_{7,t} u_{c_{h,t}} = 0 \tag{A.12}$$

$$\pi_{h,t} \quad : \quad -\xi_{2,t} A_t H_t \phi(\pi_{h,t} - 1) - \xi_{4,t} u_{c_{h,t}} \phi(2\pi_{h,t} - 1) = 0 \tag{A.13}$$

$$C^*_{h,t} \quad : \quad \xi_{1,t} - \xi_{2,t} - \xi_{5,t} u^*_{c_{hf,t}} \Psi^*_t + \xi_{6,t}(\mathcal{S}_t u^*_{c_{hh,t}} - u^*_{c_{hf,t}}) = 0 \tag{A.14}$$

$$C^*_{f,t} \quad : \quad -\xi_{3,t} + \xi_{6,t}(u^*_{c_{hf,t}} \mathcal{S}_t - u^*_{c_{ff,t}}) - \xi_{5,t} u^*_{c_{ff,t}} \Psi^*_t = 0 \tag{A.15}$$

$$H^*_t \quad : \quad \xi_{3,t} A^*_t \left(1 - \frac{\phi}{2}(\pi^*_{f,t} - 1)^2\right) + \xi_{5,t} \frac{\ell''(H^*_t)}{A^*_t} = 0 \tag{A.16}$$

Using Equations (A.9) and (A.11) along with Equation (A.4), we can obtain:

$$\Psi_t = \frac{1 - \frac{\phi}{2}(\pi_{h,t} - 1)^2 - \frac{(\pi_{h,t}-1)}{(2\pi_{h,t}-1)} \psi \Psi_t}{1 - \frac{A_t H_t (\pi_{h,t}-1)}{u_{c_{h,t}}(2\pi_{h,t}-1)} u_{c_{hh,t}} \Psi_t - \frac{\xi_{7,t}}{\xi_{2,t}}(\mathcal{S}_t u_{c_{hh,t}} - u_{c_{hf,t}})} \tag{A.17}$$

where $\psi = \frac{H\ell''(H)}{\ell'(H)}$ is the inverse of the Frisch elasticity of labor supply.

Proof of Result 1. Assume that in a steady state, $\pi_h = 1$. Then from Equation (A.17) it must be that:

$$\theta = \frac{1}{1 - \frac{\xi_{7,t}}{\xi_{2,t}}(\mathcal{S}_t u_{c_{hh,t}} - u_{c_{hf,t}})} \tag{A.18}$$

(where $\theta = \frac{(1+s)(\epsilon-1)}{\epsilon} \leq 1$) which in a symmetric equilibrium implies a unique particular value of $\frac{\xi_{7,t}}{\xi_{2,t}}$. But this is generally inconsistent with the solution of Equations (A.9)-(A.16).

Proof of Result 2. Assume that $\theta = 1$ (so the optimal subsidy is applied). Then in a symmetric equilibrium $u_{c_h} = u_{c_f}$ so that from Equations (A.9) and (A.10) we have:

$$1 - \frac{\xi_{1,t}\mathcal{S}_t + \xi_{3,t}}{\xi_{2,t}} + \frac{\xi_{4,t}}{\xi_{2,t}}(u_{c_{hh,t}} - u_{c_{hf,t}})\Psi_t = \frac{\xi_{7,t}}{\xi_{2,t}}(\mathcal{S}_t u_{c_{hh,t}} + u_{c_{ff,t}} - 2u_{c_{hf,t}}). \tag{A.19}$$

The expression $\frac{\xi_{1,t}\mathcal{S}_t + \xi_{3,t}}{\xi_{2,t}}$ is a measure of the Foreign elasticity of demand for the Home good (see below) which is greater than unity by assumption. If $\pi_h = 0$, then $\xi_4 = 0$ and from (A.19) we must have $\xi_7 > 0$. When $\theta = 1$ this must imply that beginning at $\pi_h = 0$, the left-hand side of Equation (A.17) falls, so π_h must fall to ensure that (A.17) is satisfied.

A.2 Optimal policy with both tariffs and inflation as instruments.

The policy problem for the home government is defined in the form of a value function:

$$v(\mathcal{Z}_t) = \text{Max}_{\{C_{h,t},C_{f,t},H_t,\mathcal{S}_t,\pi_{h,t},C^*_{h,t},C^*_{f,t},H^*_t,\tau_t\}} u(C_{h,t},C_{f,t}) - \ell(H_t) + E_t\beta v(\mathcal{Z}_{t+1}) \tag{A.20}$$

subject to

$$\begin{aligned}
\text{Balance of Payments} &: \; C^*_{h,t} = \mathcal{S}_t C_{f,t} && \text{(A.21)}\\
\text{Market clearing Home} &: \; A_t H_t \left(1 - \frac{\phi}{2}(\pi_{h,t}-1)^2\right) = C_{h,t} + C^*_{h,t} && \text{(A.22)}\\
\text{Market clearing Foreign} &: \; A^*_t H^*_t \left(1 - \frac{\phi}{2}(\pi^*_{f,t}-1)^2\right) = C_{f,t} + C^*_{f,t} && \text{(A.23)}\\
\text{Labor market equilibrium Home} &: \; \frac{\ell'(H_t)}{A_t} = u_{c_{h,t}} E_t \Psi(\pi_{h,t},\pi_{h,t+1},\theta) && \text{(A.24)}\\
\text{Labor market equilibrium Foreign} &: \; \frac{\ell'(H^*_t)}{A^*_t} = u_{c^*_{f,t}} E_t \Psi(\pi^*_{f,t},\pi^*_{f,t+1},\theta^*) && \text{(A.25)}\\
\text{Optimal spending Foreign} &: \; \frac{u_{c^*_{h,t}}}{u_{c^*_{f,t}}} = \frac{1+\tau^*_t}{\mathcal{S}_t} && \text{(A.26)}\\
\text{Optimal spending Home} &: \; \frac{u_{c_{h,t}}}{u_{c_{f,t}}} = \frac{1}{\mathcal{S}_t(1+\tau_t)} && \text{(A.27)}
\end{aligned}$$

Since the policy-maker has free choice over τ_t, constraint (A.27) will not bind in equilibrium, so we can ignore it in the policy problem. Denote $\xi_{1,t}$,...,$\xi_{6,t}$ as the Lagrange multipliers on the constraints (A.21)-(A.26) respectively. The first-order conditions for the discretionary policy-maker are then listed as:

$$\begin{aligned}
C_{h,t} &: \; u_{c_{h,t}} = \xi_{2,t} + \xi_{4,t} u_{c_{hh,t}} \Psi_t && \text{(A.28)}\\
C_{f,t} &: \; u_{c_{f,t}} = \xi_{1,t}\mathcal{S}_t + \xi_{3,t} + \xi_{4,t} u_{c_{hf,t}} \Psi_t && \text{(A.29)}\\
H_t &: \; \ell'(H_t) = \xi_{2,t} A_t \left(1 - \frac{\phi}{2}(\pi_{h,t}-1)^2\right) + \xi_{4,t}\frac{\ell''(H_t)}{A_t} && \text{(A.30)}\\
\mathcal{S}_t &: \; -\xi_{1,t}\frac{c^*_{h,t}}{\mathcal{S}_t} + \xi_{6,t} u^*_{c_{h,t}} = 0 && \text{(A.31)}\\
\pi_{h,t} &: \; -\xi_{2,t} A_t H_t \phi(\pi_{h,t}-1) - \xi_{4,t} u_{c_{h,t}} \phi(2\pi_{h,t}-1) && \text{(A.32)}\\
C^*_{h,t} &: \; \xi_{1,t} - \xi_{2,t} - \xi_{5,t} u^*_{c_{hf,t}} \Psi^*_t + \xi_{6,t}(\mathcal{S}_t u^*_{c_{hh,t}} - u^*_{c_{hf,t}}(1+\tau^*_t)) = 0 && \text{(A.33)}\\
C^*_{f,t} &: \; -\xi_{3,t} + \xi_{6,t}(u^*_{c_{hf,t}}\mathcal{S}_t - u^*_{c_{ff,t}}(1+\tau^*_t)) - \xi_{5,t} u^*_{c_{ff,t}} \Psi^*_t && \text{(A.34)}\\
H^*_t &: \; \xi_{3,t} A^*_t \left(1 - \frac{\phi}{2}(\pi^*_{f,t}-1)^2\right) + \xi_{5,t}\frac{\ell''(H^*_t)}{A^*_t} = 0 && \text{(A.35)}
\end{aligned}$$

From Equation (A.31), we have:

$$\xi_{6,t} = \xi_{1,t}\frac{c^*_{h,t}}{\mathcal{S}_t u^*_{c_{h,t}}} \tag{A.36}$$

and from Equation (A.35):

$$\xi_{5,t} = -\xi_{3,t}\frac{A^*_t\left(1-\frac{\phi}{2}(\pi^*_{f,t}-1)^2\right)}{\frac{\ell''(H^*_t)}{A^*_t}} \tag{A.37}$$

Use these in Equation (A.34) to get:

$$-\xi_{3,t}+\xi_{1,t}\frac{c^*_{h,t}}{\mathcal{S}_t u^*_{c_{h,t}}}(u^*_{c_{hf,t}}\mathcal{S}_t - u^*_{c_{ff,t}}(1+\tau^*_t)) + \xi_{3,t}\frac{A^*_t\left(1-\frac{\phi}{2}(\pi^*_{f,t}-1)^2\right)}{\frac{\ell''(H^*_t)}{A^*_t}}u^*_{c_{ff,t}}\Psi^*_t = 0 \tag{A.38}$$

which gives:

$$\xi_{3,t} = \frac{\xi_{1,t}\frac{c^*_{h,t}}{\mathcal{S}_t u^*_{c_{h,t}}}(u^*_{c_{hf,t}}\mathcal{S}_t - u^*_{c_{ff,t}}(1+\tau^*_t))}{1-\frac{A^*_t\left(1-\frac{\phi}{2}(\pi^*_{f,t}-1)^2\right)}{\frac{\ell''(H^*_t)}{A^*_t}}u^*_{c_{ff,t}}\Psi^*_t} \tag{A.39}$$

From Equations (A.33) and (A.36), we have:

$$\begin{aligned}
\xi_{2,t} &= \xi_{1,t}+\xi_{1,t}\frac{c^*_{h,t}}{\mathcal{S}_t u^*_{c_{h,t}}}(\mathcal{S}_t u^*_{c_{hh,t}} - u^*_{c_{hf,t}}(1+\tau^*_t)) - \xi_{5,t}u^*_{c_{hf,t}}\Psi^*_t \\
&= \xi_{1,t}+\xi_{1,t}\frac{c^*_{h,t}}{\mathcal{S}_t u^*_{c_{h,t}}}(\mathcal{S}_t u^*_{c_{hh,t}} - u^*_{c_{hf,t}}(1+\tau^*_t)) + \xi_{3,t}\frac{A^*_t\left(1-\frac{\phi}{2}(\pi^*_{f,t}-1)^2\right)}{\frac{\ell''(H^*_t)^*}{A_t}}u^*_{c_{hf,t}}\Psi^*_t \\
&= \xi_{1,t}+\xi_{1,t}\frac{c^*_{h,t}}{\mathcal{S}_t u^*_{c_{h,t}}}(\mathcal{S}_t u^*_{c_{hh,t}} - u^*_{c_{hf,t}}(1+\tau^*_t)) + \frac{\xi_{1,t}\frac{c^*_{h,t}}{\mathcal{S}_t u^*_{c_{h,t}}}(u^*_{c_{hf,t}}\mathcal{S}_t - u^*_{c_{ff,t}}(1+\tau^*_t))}{1-\frac{A^*_t\left(1-\frac{\phi}{2}(\pi^*_{f,t}-1)^2\right)}{\frac{\ell''(H^*_t)}{A^*_t}}u^*_{c_{ff,t}}\Psi^*_t}\frac{A^*_t\left(1-\frac{\phi}{2}(\pi^*_{f,t}-1)^2\right)}{\frac{\ell''(H^*_t)}{A^*_t}}u^*_{c_{hf,t}}\Psi^*_t
\end{aligned} \tag{A.40}$$

From Equation (A.39), we have:

$$\xi_{1,t}\mathcal{S}_t+\xi_{3,t} = \xi_{1,t}\mathcal{S}_t + \frac{\xi_{1,t}\frac{c^*_{h,t}}{\mathcal{S}_t u^*_{c_{h,t}}}(u^*_{c_{hf,t}}\mathcal{S}_t - u^*_{c_{ff,t}}(1+\tau^*_t))}{1-\frac{A^*_t\left(1-\frac{\phi}{2}(\pi^*_{f,t}-1)^2\right)}{\frac{\ell''(H^*_t)}{A^*_t}}u^*_{c_{ff,t}}\Psi^*_t} \tag{A.41}$$

So, we get:

$$\begin{aligned}
\frac{\xi_{1,t}\mathcal{S}_t+\xi_{3,t}}{\xi_{2,t}} &= \frac{\mathcal{S}_t + \frac{\frac{c^*_{h,t}}{\mathcal{S}_t u^*_{c_{h,t}}}(u^*_{c_{hf,t}}\mathcal{S}_t - u^*_{c_{ff,t}}(1+\tau^*_t))}{1-\frac{A^*_t\left(1-\frac{\phi}{2}(\pi^*_{f,t}-1)^2\right)}{\frac{\ell''(H^*_t)}{A^*_t}}u^*_{c_{ff,t}}\Psi^*_t}}{1+\frac{c^*_{h,t}}{\mathcal{S}_t u^*_{c_{h,t}}}(\mathcal{S}_t u^*_{c_{hh,t}} - u^*_{c_{hf,t}}(1+\tau^*_t)) + \frac{\frac{c^*_{h,t}}{\mathcal{S}_t u^*_{c_{h,t}}}(u^*_{c_{hf,t}}\mathcal{S}_t - u^*_{c_{ff,t}}(1+\tau^*_t))}{1-\frac{A^*_t\left(1-\frac{\phi}{2}(\pi^*_{f,t}-1)^2\right)}{\frac{\ell''(H^*_t)}{A^*_t}}u^*_{c_{ff,t}}\Psi^*_t}\frac{A^*_t\left(1-\frac{\phi}{2}(\pi^*_{f,t}-1)^2\right)}{\frac{\ell''(H^*_t)}{A^*_t}}u^*_{c_{hf,t}}\Psi^*_t} \\
&= \mathcal{S}_t\frac{\eta_t}{\eta_t-1}
\end{aligned} \tag{A.42}$$

where η_t is the Foreign country's general equilibrium elasticity of demand for Home goods, which is:

$$\eta_t = \frac{\frac{(u^*_{c_{ff,t}}(1+\tau^*_t)-u^*_{c_{hf,t}}\mathcal{S}_t)c^*_{h,t}}{u^*_{c_{h,t}}\mathcal{S}^2_t(1-\frac{A^{*2}_t(1-\varphi^*_t)\Psi^*_t u^*_{c_{ff,t}}}{\ell''(H^*_t)})} - 1}{\frac{(u^*_{c_{ff,t}}(1+\tau^*_t)-u^*_{c_{hf,t}}\mathcal{S}_t)c^*_{h,t}}{u^*_{c_{h,t}}\mathcal{S}^2_t(1-\frac{A^{*2}_t(1-\varphi^*_t)\Psi^*_t u^*_{c_{ff,t}}}{\ell''(H^*_t)})}(1-\mathcal{S}_t\frac{A^{*2}_t(1-\varphi^*_t)\Psi^*_t u^*_{c_{hf,t}}}{\ell''(H^*_t)}) + \frac{(u^*_{c_{hh,t}}\mathcal{S}_t - u^*_{c_{hf,t}}(1+\tau^*_t))c^*_{h,t}}{u^*_{c_{h,t}}\mathcal{S}_t}} \tag{A.43}$$

and $\varphi^*_t \equiv \frac{\phi}{2}(\pi^*_{f,t}-1)^2$. From Equation (A.32) we have:

$$\frac{\xi_{4,t}}{\xi_{2,t}} = -\frac{A_t H_t \phi(\pi_{h,t}-1)}{u_{c_{h,t}}\phi(2\pi_{h,t}-1)} \tag{A.44}$$

So that using Equations (A.28) and (A.29) we have:

$$\begin{aligned}
\frac{u_{c_{h,t}}}{u_{c_{f,t}}} &= \frac{\xi_{2,t}+\xi_{4,t}u_{c_{hh,t}}\Psi_t}{\xi_{1,t}\mathcal{S}_t+\xi_{3,t}+\xi_{4,t}u_{c_{hf,t}}\Psi_t} \\
&= \frac{1+\frac{\xi_{4,t}}{\xi_{2,t}}u_{c_{hh,t}}\Psi_t}{\frac{\xi_{1,t}\mathcal{S}_t+\xi_{3,t}}{\xi_{2,t}}+\frac{\xi_{4,t}}{\xi_{2,t}}u_{c_{hf,t}}\Psi_t} \\
&= \frac{1-\frac{A_tH_t\phi(\pi_{h,t}-1)}{u_{c_{h,t}}\phi(2\pi_{h,t}-1)}u_{c_{hh,t}}\Psi_t}{\frac{\mathcal{S}_t\eta_t}{\eta_t-1}-\frac{A_tH_t\phi(\pi_{h,t}-1)}{u_{c_{h,t}}\phi(2\pi_{h,t}-1)}u_{c_{hf,t}}\Psi_t}
\end{aligned} \tag{A.45}$$

Then, using the competitive equilibrium condition:

$$\frac{u_{c_{h,t}}}{u_{c_{f,t}}} = \frac{1}{\mathcal{S}_t(1+\tau_t)} \tag{A.46}$$

we have:

$$\frac{1}{1+\tau_t} = \frac{1-\frac{A_tH_t\phi(\pi_{h,t}-1)}{u_{c_{h,t}}\phi(2\pi_{h,t}-1)}u_{c_{hh,t}}\Psi_t}{\frac{\eta_t}{\eta_t-1}-\frac{A_tH_t\phi(\pi_{h,t}-1)}{\mathcal{S}_tu_{c_{h,t}}\phi(2\pi_{h,t}-1)}u_{c_{hf,t}}\Psi_t} \tag{A.47}$$

From Equation (A.32) we have:

$$\xi_{4,t} = -\xi_{2,t}\frac{A_t h_t \phi(\pi_{h,t}-1)}{u_{c_{h,t}}\phi(2\pi_{h,t}-1)} \tag{A.48}$$

Using this with (A.28) and (A.30) we arrive at the following description for the labor market condition:

$$\frac{\ell'(H_t)}{u_{c_{h,t}}} = \frac{\left(A_t\left(1-\frac{\phi}{2}(\pi_{h,t}-1)^2\right) - \frac{A_tH_t\phi(\pi_{h,t}-1)}{u_{c_{h,t}}\phi(2\pi_{h,t}-1)}\frac{\ell''(H_t)}{A_t}\right)}{\left(1-\frac{A_tH_t\phi(\pi_{h,t}-1)}{u_{c_{h,t}}\phi(2\pi_{h,t}-1)}u_{cc_{h,t}}\Psi_t\right)} \tag{A.49}$$

which, from Equation (A.24) gives:

$$\Psi(\pi_{h,t}, E_t\pi_{h,t+1}) = \frac{\left(\left(1-\frac{\phi}{2}(\pi_{h,t}-1)^2\right) - \frac{\phi(\pi_{h,t}-1)}{\phi(2\pi_{h,t}-1)}\frac{H_t\ell''(h_t)}{\ell'(H_t)}\Psi_t\right)}{\left(1-\frac{A_tH_t\phi(\pi_{h,t}-1)}{u_{c_{h,t}}\phi(2\pi_{h,t}-1)}u_{cc_{h,t}}\Psi_t\right)} \tag{A.50}$$

This implicitly determines the inflation rate in the Home country.

Also, this analysis pertains only to the Home country's tariff decisions. The other country's decision is exactly analogous. Then tariffs will be determined simultaneously in the Markov Nash game between countries.

Results 3 and 4 may be obtained from Equations (A.47) and (A.50). If monopoly distortions are zero, then inflation will be zero and the tariffs will follow the optimal monopoly tariff rule.

Proof of Result 5. Either in the case of zero inflation, or purely flexible prices, we can set $\phi = 0$, and from Equation (A.30) together with Equations (A.28) and (A.29) and the definition of η_t from above, we obtain the implicit tariff formula as:

$$\frac{1}{1+\tau_t} = \frac{1+\Omega_t u_{c_{hh,t}}\Psi_t}{\frac{\eta_t}{\eta_t-1}+\Omega_t u_{c_{hf,t}}\Psi_t} \tag{A.51}$$

where $\Omega_t = \frac{(\theta-1)A_t}{\frac{\ell''(H_t)}{A_t}-u_{c_{hh,t}}A_t\theta^2}$, and $\theta = \frac{(1+s)(\epsilon-1)}{\epsilon} \leq 1$. From Equation (A.51), we conclude that in a distorted economy, where $\theta < 1$ with flexible prices ($\phi = 0$), the Nash equilibrium tariff in the currency war game will be less than the pure monopoly tariff rate. Intuitively, this is because policy-makers take account of the distortionary impacts of the tariff on domestic production, which is inefficiently low when $\theta < 1$.

Proof of Result 6 - Part 1

Let inflation be determined cooperatively and tariffs non-cooperatively. We define the terms of trade as $\mathcal{S}_t = \frac{S_tP^*_{f,t}}{P_{h,t}}$. The policy problem for the cooperative government is defined in the form of a value function:

$$v(\mathcal{Z}_t) = \text{Max}_{\{C_{h,t},C_{f,t},H_t,H^*_t,C^*_{h,t},C^*_{f,t},\mathcal{S}_t,\pi_{h,t},\pi^*_{f,t}\}} u(C_{h,t},C_{f,t}) - \ell(H_t) + u(C^*_{h,t},C^*_{f,t}) - \ell(H^*_t) + E_t\beta v(\mathcal{Z}_{t+1}) \tag{A.52}$$

subject to

$$\begin{aligned}
\text{Balance of Payments} \quad &: \quad C^*_{h,t} = \mathcal{S}_t C_{f,t} && \text{(A.53)}\\
\text{Market clearing Home} \quad &: \quad A_t H_t \left(1 - \frac{\phi}{2}(\pi_{h,t}-1)^2\right) = C_{h,t} + C^*_{h,t} && \text{(A.54)}\\
\text{Market clearing Foreign} \quad &: \quad A^*_t H^*_t \left(1 - \frac{\phi}{2}(\pi^*_{f,t}-1)^2\right) = C_{f,t} + C^*_{f,t} && \text{(A.55)}\\
\text{Labor market equilibrium Home} \quad &: \quad \frac{\ell'(H_t)}{A_t} = u_{c_{h,t}} E_t \Psi(\pi_{h,t}, \pi_{h,t+1}, \theta) && \text{(A.56)}\\
\text{Labor market equilibrium Foreign} \quad &: \quad \frac{\ell'(H^*_t)}{A^*_t} = u_{c^*_{f,t}} E_t \Psi(\pi^*_{f,t}, \pi^*_{f,t+1}, \theta^*) && \text{(A.57)}\\
\text{Optimal spending Home} \quad &: \quad \frac{u_{c_{h,t}}}{u_{c_{f,t}}} = \frac{1}{\mathcal{S}_t(1+\tau_t)} && \text{(A.58)}\\
\text{Optimal spending Foreign} \quad &: \quad \frac{u_{c^*_{h,t}}}{u_{c^*_{f,t}}} = \frac{1+\tau^*_t}{\mathcal{S}_t} && \text{(A.59)}
\end{aligned}$$

The first-order conditions for the discretionary policy-maker are then listed as:

$$\begin{aligned}
C_{h,t} \quad &: \quad u_{c_{h,t}} = \xi_{2,t} + \xi_{4,t} u_{c_{hh,t}} \Psi_t - \xi_{6,t}(u_{c_{hh,t}} \mathcal{S}_t (1+\tau_t) - u_{c_{hf,t}}) && \text{(A.60)}\\
C_{f,t} \quad &: \quad u_{c_{f,t}} = \xi_{1,t} \mathcal{S}_t + \xi_{3,t} + \xi_{4,t} u_{c_{hf,t}} \Psi_t + \xi_{6,t}(u_{c_{ff,t}} - u_{c_{hf,t}}(1+\tau_t)\mathcal{S}_t) && \text{(A.61)}\\
H_t \quad &: \quad \ell'(H_t) = \xi_{2,t} A_t \left(1 - \frac{\phi}{2}(\pi_{h,t}-1)^2\right) + \xi_{4,t} \frac{\ell''(H_t)}{A_t} && \text{(A.62)}\\
\mathcal{S}_t \quad &: \quad -\xi_{1,t} \frac{c^*_{h,t}}{\mathcal{S}_t} + \xi_{6,t} u_{c_{h,t}} (1+\tau_t) + \xi_{7,t} u^*_{c_{h,t}} = 0 && \text{(A.63)}\\
\pi_{h,t} \quad &: \quad -\xi_{2,t} A_t H_t \phi(\pi_{h,t}-1) - \xi_{4,t} u_{c_{h,t}} \phi(2\pi_{h,t}-1) = 0 && \text{(A.64)}\\
C^*_{h,t} \quad &: \quad u^*_{c_{h,t}} + \xi_{1,t} - \xi_{2,t} - \xi_{5,t} u^*_{c_{hf,t}} \Psi^*_t + \xi_{7,t}(\mathcal{S}_t u^*_{c_{hh,t}} - u^*_{c_{hf,t}}(1+\tau^*_t)) = 0 && \text{(A.65)}\\
C^*_{f,t} \quad &: \quad u^*_{c_{f,t}} - \xi_{3,t} + \xi_{7,t}(u^*_{c_{hf,t}} \mathcal{S}_t - u^*_{c_{ff,t}}(1+\tau^*_t)) - \xi_{5,t} u^*_{c_{ff,t}} \Psi^*_t = 0 && \text{(A.66)}\\
H^*_t \quad &: \quad -\ell'(H^*_t) + \xi_{3,t} A^*_t \left(1 - \frac{\phi}{2}(\pi^*_{f,t}-1)^2\right) + \xi_{5,t} \frac{\ell''(H^*_t)}{A^*_t} = 0 && \text{(A.67)}\\
\pi^*_{f,t} \quad &: \quad -\xi_{3,t} A^*_t H^*_t \phi(\pi^*_{f,t}-1) - \xi_{5,t} u^*_{c_{f,t}} \phi(2\pi^*_{f,t}-1) = 0 && \text{(A.68)}
\end{aligned}$$

Result 6 states that, when tariffs are positive, the cooperative policy-maker will depart from a zero inflation policy even when the monopoly distortion in prices is offset by an optimal subsidy. To prove it, start by assuming the opposite. Say the cooperative policy-maker sets inflation to zero in both countries, *i.e.* $\pi_{h,t} = \pi^*_{f,t} = 0$. Then by Equation (A.64) and Equation (A.68) we must have $\xi_4 = \xi_5 = 0$. But if the firms receive an optimal subsidy, then we must have: $\ell'(H_t) = A_t u_{c_{h,t}}$ and $\ell'(H^*_t) = A^*_t u^*_{c_{f,t}}$, so that from Equations (A.62) and (A.67), we must have $\xi_{2,t} = u_{c_{h,t}}$ and $\xi_{3,t} = u^*_{c_{f,t}}$. Then from Equations (A.60) and (A.66), we must have $\xi_{6,t} = \xi_{7,t} = 0$. And from Equation (A.63), we must have $\xi_{1,t} = 0$. This then implies from Equations (A.60) and (A.65), and also from Equations (A.61) and (A.66), that $u_{c_{h,t}} = u^*_{c_{h,t}}$ and $u_{c_{f,t}} = u^*_{c_{f,t}}$. But this violates the optimal spending equations (A.58) and (A.59), which together imply

$$\frac{u_{c_{h,t}}}{u^*_{c_{h,t}}} = \frac{u_{c_{f,t}}}{u^*_{c_{f,t}}} \frac{1}{(1+\tau_t)(1+\tau^*_t)} \tag{A.69}$$

Thus, we have a contradiction. So cooperative policy-making with non-cooperative tariff setting will not close the output gap, even if an optimal subsidy is in place.

Intuitively, the cooperative planner will depart from zero inflation if tariffs are positive, because there is a distortion preventing full consumption risk-sharing across countries. We can see this more clearly as follows.

We may show more directly how this impacts on the equilibrium rate of inflation. Using Equations (A.63), (A.62), and (A.60) we obtain:

$$\frac{\ell'(H_t)}{u_{c_{h,t}}} = \frac{A_t\left(1-\frac{\phi}{2}(\pi_{h,t}-1)^2\right) - \frac{\ell''(H_t)H_t(\pi_{h,t}-1)}{u_{c_{h,t}}(2\pi_{h,t}-1)}}{1-\frac{A_tH_t(\pi_{h,t}-1)}{(2\pi_{h,t}-1)}u_{c_{hh,t}}\Psi_t - \frac{\xi_{6,t}}{\xi_{2,t}}u_{c_{hh,t}}} \tag{A.70}$$

Using Equation (A.56), we can write this as an equation determining the inflation rate (also imposing a symmetric equilibrium with $\mathcal{S}_t = 1$):

$$\Psi_t = \frac{1-\frac{\phi}{2}(\pi_{h,t}-1)^2 - \frac{\ell''(H_t)H_t(\pi_{h,t}-1)}{A_tu_{c_{h,t}}(2\pi_{h,t}-1)}}{1-\frac{A_tH_t(\pi_{h,t}-1)}{u_{c_{h,t}}(2\pi_{h,t}-1)}u_{c_{hh,t}}\Psi_t - \frac{\xi_{6,t}}{\xi_{2,t}}(u_{c_{hh,t}}(1+\tau_t)-u_{c_{hf,t}})} \tag{A.71}$$

where in a symmetric equilibrium it can be shown that:

$$\frac{\xi_{6,t}}{\xi_{2,t}} = \frac{u_{c_{h,t}}\tau_t}{\xi_{2,t}(u_{c_{hh,t}}(1+\tau_t)+u_{c_{ff,t}}-2u_{c_{hf,t}})} + \frac{(u_{c_{hh,t}}-u_{c_{hf,t}})\Psi_tA_tH_t(\pi_{h,t}-1)}{u_{c_{h,t}}(2\pi_{h,t}-1)} \tag{A.72}$$

with $\xi_{2,t} = \frac{\ell''(H_t)}{A_t\left(1-\frac{\phi}{2}(\pi_{h,t}-1)^2\right)-\frac{\ell''(H_t)H_t(\pi_{h,t}-1)}{u_{c_{h,t}}(2\pi_{h,t}-1)}}$

Now take Equation (A.71), and impose a steady state. Assume that $\theta = 1$, and then assume that inflation was zero, so $\pi_h = 1$. Then the left-hand side of Equation (A.71) is unity, while the right-hand side is greater than unity, using Equation (A.72) as long as there is a positive tariff rate, *i.e.* $\tau > 0$. Since the left-hand side is increasing in π_h and the right-hand side is decreasing in π_h, it must be that the equilibrium cooperative inflation rate is greater than zero when $\theta = 1$ and $\tau > 0$.

Proof of Result 6 - part 2

The policy problem for the home tariff setter when inflation is chosen by the cooperative planner is:

$$v(\mathcal{Z}_t) = \text{Max}_{\{C_{h,t},C_{f,t},H_t,\mathcal{S}_t,C^*_{h,t},C^*_{f,t},H^*_t,\ \tau_t\}} u(C_{h,t},C_{f,t}) - \ell(H_t) + E_t\beta v(\mathcal{Z}_{t+1}) \tag{A.73}$$

subject to

$$\begin{aligned}
\text{Balance of Payments} &: \quad C^*_{h,t} = \mathcal{S}_t C_{f,t} && \text{(A.74)}\\
\text{Market clearing Home} &: \quad A_t H_t (1 - \frac{\phi}{2}(\pi_{h,t} - 1)^2) = C_{h,t} + C^*_{h,t} && \text{(A.75)}\\
\text{Market clearing Foreign} &: \quad A^*_t H^*_t (1 - \frac{\phi}{2}(\pi^*_{f,t} - 1)^2) = C_{f,t} + C^*_{f,t} && \text{(A.76)}\\
\text{Labor market equilibrium Home} &: \quad \frac{\ell'(H_t)}{A_t} = u_{c_{h,t}} E_t \Psi(\pi_{h,t}, \pi_{h,t+1}, \theta) && \text{(A.77)}\\
\text{Labor market equilibrium Foreign} &: \quad \frac{\ell'(H^*_t)}{A^*_t} = u_{c^*_{f,t}} E_t \Psi(\pi^*_{f,t}, \pi^*_{f,t+1}, \theta^*) && \text{(A.78)}\\
\text{Optimal spending Foreign} &: \quad \frac{u_{c^*_{h,t}}}{u_{c^*_{f,t}}} = \frac{1+\tau^*_t}{\mathcal{S}_t} && \text{(A.79)}\\
\text{Optimal spending Home} &: \quad \frac{u_{c_{h,t}}}{u_{c_{f,t}}} = \frac{1}{\mathcal{S}_t(1+\tau_t)} && \text{(A.80)}
\end{aligned}$$

Again, Equation (A.80) will not bind. The first-order conditions for the discretionary policy-maker are then listed as:

$$\begin{aligned}
C_{h,t} &: \quad u_{c_{h,t}} = \xi_{2,t} + \xi_{4,t} u_{c_{hh,t}} \Psi_t && \text{(A.81)}\\
C_{f,t} &: \quad u_{c_{f,t}} = \xi_{1,t}\mathcal{S}_t + \xi_{3,t} + \xi_{4,t} u_{c_{hf,t}} \Psi_t && \text{(A.82)}\\
H_t &: \quad \ell'(H_t) = \xi_{2,t} A_t \left(1 - \frac{\phi}{2}(\pi_{h,t} - 1)^2\right) + \xi_{4,t} \frac{\ell''(H_t)}{A_t} && \text{(A.83)}\\
\mathcal{S}_t &: \quad -\xi_{1,t} \frac{c^*_{h,t}}{\mathcal{S}_t} + \xi_{6,t} u^*_{c_{h,t}} = 0 && \text{(A.84)}\\
C^*_{h,t} &: \quad \xi_{1,t} - \xi_{2,t} - \xi_{5,t} u^*_{c_{hf,t}} \Psi^*_t + \xi_{6,t}(\mathcal{S}_t u^*_{c_{hh,t}} - u^*_{c_{hf,t}}(1+\tau^*_t)) = 0 && \text{(A.85)}\\
C^*_{f,t} &: \quad -\xi_{3,t} + \xi_{6,t}(u^*_{c_{hf,t}} \mathcal{S}_t - u^*_{c_{ff,t}}(1+\tau^*_t)) - \xi_{5,t} u^*_{c_{ff,t}} \Psi^*_t = 0 && \text{(A.86)}\\
H^*_t &: \quad \xi_{3,t} A^*_t \left(1 - \frac{\phi}{2}(\pi^*_{f,t} - 1)^2\right) + \xi_{5,t} \frac{\ell''(H^*_t)}{A^*_t} = 0 && \text{(A.87)}
\end{aligned}$$

Following the steps from Equation (A.42) we have:

$$\frac{u_{c_{h,t}}}{u_{c_{f,t}}} = \frac{1 + \frac{\xi_{4,t}}{\xi_{2,t}} u_{c_{hh,t}} \Psi_t}{\frac{\xi_{1,t}\mathcal{S}_t + \xi_{3,t}}{\xi_{2,t}} + \frac{\xi_{4,t}}{\xi_{2,t}} u_{c_{hf,t}} \Psi} \tag{A.88}$$

where $\frac{\xi_{1,t}\mathcal{S}_t + \xi_{3,t}}{\xi_{2,t}}$ is the same as in Equation (A.42), with

$$\xi_{4,t} = \frac{\ell'(H_t) - A_t u_{c_{h,t}} \left(1 - \frac{\phi}{2}(\pi_{h,t} - 1)^2\right)}{\frac{\ell''(H_t)}{A_t} - u_{c_{hh,t}} \Psi_t A_t \left(1 - \frac{\phi}{2}(\pi_{h,t} - 1)^2\right)}$$

and

$$\xi_{2,t} = u_{c_{h,t}} - u_{c_{hh,t}} \Psi_t \frac{\ell'(H_t) - A_t u_{c_{h,t}} \left(1 - \frac{\phi}{2}(\pi_{h,t} - 1)^2\right)}{\frac{\ell''(H_t)}{A_t} - u_{c_{hh,t}} \Psi_t A_t \left(1 - \frac{\phi}{2}(\pi_{h,t} - 1)^2\right)}$$

If we assume $\theta = 1$, then from from Equation (A.77), in a steady state, we have

$$\ell'(H_t) - A_t u_{c_{h,t}} \left(1 - \frac{\phi}{2}(\pi_{h,t} - 1)^2\right) = A u_{c_h} \left(\phi \pi_h (\pi_h - 1) + \frac{\phi}{2}(\pi_h - 1)^2\right) > 0 \tag{A.89}$$

Then $\frac{\xi_4}{\xi_2} > 0$, and from Equation (A.42) ,we can describe the optimal home tariff by the condition:

$$\frac{1}{1+\tau} = \frac{1 + \frac{\xi_4}{\xi_2} u_{c_{hh}} \Psi}{\frac{\eta}{\eta - 1} + \frac{\xi_4}{\xi_2} u_{c_{hf}} \Psi} \tag{A.90}$$

where we have used the notation for the steady-state Foreign demand elasticity η. Since $\frac{\xi_4}{\xi_2} > 0$ it follows that in the case $\theta = 1$, and monetary policy is determined cooperatively, the tariff rate exceeds the monopoly tariff rate.

B General Model derivation

We describe a two country model, denoted Home and Foreign, where agents supply labor and consume goods from both countries. The world is populated with a unit mass of agents and Home has share n of these, with Foreign share $1 - n$. We assume that firms set prices in domestic currency (PCP), and adjust prices constrained by Rotemberg-style price adjustment costs. Agents in the Home country have preferences over consumption and hours given by

$$U_t = \frac{C_t^{1-\sigma} - 1}{1 - \sigma} - \frac{\chi}{1 + \psi} H_t^{1+\psi} \tag{B.91}$$

We assume *no* financial market trading across countries, which implies that trade is balanced.

B.1 Households

Absent international asset trading, the Home country budget constraint is

$$P_{h,t} C_{h,t} + (1 + \tau_t) S_t P^*_{f,t} C_{f,t} = W_t H_t + \Pi_t + TR_t \tag{B.92}$$

where $P_{h,t}$ ($P^*_{f,t}$) is the Home (Foreign) goods price in Home (Foreign) currency, S_t is the exchange rate, $C_{h,t}$ ($C_{f,t}$) is the consumption of the Home (Foreign) good, τ_t is an import tariff imposed by the Home government, W_t is the Home nominal wage, Π_t represents the profits of the Home firm and TR_t is a lump sum transfer from the Home government. The elasticity of substitution between Home and Foreign goods is λ.

It is assumed that

$$C_t = \left(\varepsilon^{\frac{1}{\lambda}} C_{h,t}^{1-\frac{1}{\lambda}} + (1-\varepsilon)^{\frac{1}{\lambda}} C_{f,t}^{1-\frac{1}{\lambda}} \right)^{\frac{1}{1-\frac{1}{\lambda}}} \tag{B.93}$$

where $\varepsilon \geq n$, representing the possibility of home bias in preferences.[20]

The true price index for the Home consumer given the above preferences and the price definitions then becomes

$$P_t = \left(\varepsilon P_{h,t}^{1-\lambda} + (1-\varepsilon)((1+\tau_t) S_t P_{f,t}^{*})^{1-\lambda} \right)^{1/(1-\lambda)} \tag{B.94}$$

Optimal consumption of Home and Foreign goods for the Home consumer is

$$C_{h,t} = \varepsilon \left(\frac{P_{h,t}}{P_t} \right)^{-\lambda} C_t \tag{B.95}$$

$$C_{f,t} = (1-\varepsilon) \left(\frac{(1+\tau_t) S_t P_{f,t}^{*}}{P_t} \right)^{-\lambda} C_t \tag{B.96}$$

Optimal labor supply is described by

$$W_t = \chi P_t C_t^{\sigma} H_t^{\psi} \tag{B.97}$$

The preferences, budget constraints, and optimal choices for the Foreign economy are analogous. The presence of home bias in Foreign preferences then implies that the price index for the Foreign economy is

$$P_t^{*} = \left(\varepsilon^{*} P_{f,t}^{1-\lambda} + (1-\varepsilon^{*}) \left((1+\tau_t^{*}) \frac{P_{h,t}}{S_t} \right)^{1-\lambda} \right)^{\frac{1}{1-\lambda}} \tag{B.98}$$

B.2 Firms

A measure n of firms in the Home economy produce differentiated goods. The aggregate Home good is a composite of these differentiated goods, where the elasticity of substitution between individual goods is denoted as $\epsilon > 1$. The production function for firm i in the Home country is

$$Y_{i,h,t} = A_t H_{i,t}^{1-\alpha} X_{i,t}^{\alpha} \tag{B.99}$$

where A_t is an aggregate productivity term. Here, $X_{i,t}$ represents the use of intermediate goods on the part of the Home firm i and $H_{i,t}$ the use of labor. We allow that intermediate good inputs are composed of Home and Foreign goods in a different composition than that of the

[20] Letting $0 \leq x \leq 1$ represent the degree of home bias in preferences, where $x = 0$ ($x = 1$) represents zero (full) home bias, we can define $\varepsilon = n + x(1-n)$.

consumption aggregator. Namely

$$X_{i,t} = \left(\varepsilon_x^{\frac{1}{\lambda}} X_{i,h,t}^{1-\frac{1}{\lambda}} + (1-\varepsilon_x)^{\frac{1}{\lambda}} X_{i,f,t}^{1-\frac{1}{\lambda}} \right)^{\frac{1}{1-\frac{1}{\lambda}}}$$

where $X_{i,j,t}$ is the Home firm i's use of inputs from country $j = h, f$. The profits of Home firm i are then represented as

$$\Pi_{i,t} = ((1+s_{i,t})P_{i,h,t} - MC_t)\, Y_{i,h,t} \tag{B.100}$$

where $MC_t = \frac{(1-\alpha)^{\alpha-1}\alpha^{-\alpha}}{A_t} W_t^{1-\alpha} P_{x,t}^{\alpha}$ denotes the firm's marginal cost, where $P_{x,t}$ is the price index relevant for the firm's use of intermediate inputs, and $s_{i,t}$ represents a subsidy that may be given to the firm to offset the monopoly distortion in pricing.[21]

Cost minimization by the firm implies:

$$A_t(1-\alpha)H_{i,t}^{-\alpha} X_{i,t}^{\alpha} MC_t = W_t \tag{B.101}$$

$$A_t \alpha H_{i,t}^{1-\alpha} X_{i,t}^{\alpha-1} \varepsilon_x^{\frac{1}{\lambda}} \left(\frac{X_{i,h,t}}{X_{i,t}} \right)^{-\frac{1}{\lambda}} MC_t = P_{h,t} \tag{B.102}$$

$$A_t \alpha H_{i,t}^{1-\alpha} X_{i,t}^{\alpha-1} (1-\varepsilon_x)^{\frac{1}{\lambda}} \left(\frac{X_{i,f,t}}{X_{i,t}} \right)^{-\frac{1}{\lambda}} MC_t = (1+\tau_t) S_t P_{f,t}^{*} \tag{B.103}$$

The firm chooses its price to maximize its present value of expected profits, net of price adjustment costs

$$E_t \sum_{j=0}^{\infty} \omega_{t+j} \left(\Pi_{i,t+j} - \xi \left(\frac{P_{i,h,t+j}}{P_{i,h,t+j-1}} \right) P_{h,t+j} Y_{h,t+j} \right) \tag{B.104}$$

where ω_t is the firm's nominal stochastic discount factor, and $\xi(.)$ represents a price adjustment cost function for the firm. We assume that $\xi'(.) > 0$, and $\xi''(.) > 0$. Price adjustment costs are proportional to the nominal value of Home output, to be consistent with the nominal profit objective function of the firm.

The first-order condition for profit maximization for the Home firm i can be described as

$$\begin{aligned}(1+s)Y_{i,h,t} = \epsilon(P_{i,h,t}(1+s_t) \quad &- \quad MC_t)\frac{Y_{i,h,t}}{P_{i,h,t}} + \xi' \left(\frac{P_{i,h,t}}{P_{i,h,t-1}} \right) \frac{1}{P_{i,h,t}} P_{h,t} Y_{h,t} \\ &+ \quad E_t \omega_{t+1} \xi' \left(\frac{P_{i,h,t+1}}{P_{i,h,t}} \right) \frac{P_{i,h,t+1}}{P_{i,h,t}^2} P_{h,t+1} Y_{h,t+1} = 0 \end{aligned} \tag{B.105}$$

B.3 Economic Policy

There are three separate levers of policy in this model. Fiscal policy may be used to subsidize monopoly firms. Trade policy may be used to levy tariffs on imports, and monetary policy may be used to either target inflation rates or exchange rates. In the case where firms are subsidized,

[21] In particular, $P_{x,t} = \left(\varepsilon_x P_{h,t}^{1-\lambda} + (1-\varepsilon_x)((1+\tau_t)S_t P_{f,t}^{*})^{1-\lambda} \right)^{1/(1-\lambda)}$.

we follow the literature in assuming that a fiscal authority chooses a subsidy to offset the steady-state monopoly markup. But we also allow for the possibility that the monopoly markup remains as a pre-existing distortion in the economy. As we see, this may have an important implication for both optimal monetary policy and trade policy.

C The Competitive Equilibrium

When we combine the description of optimal behavior for the Home economy with the analogous conditions for the Foreign economy, and impose market clearing conditions, we obtain a competitive equilibrium which can be described by the following equations:

$$\varepsilon\left(\frac{P_{h,t}}{P_t}\right)^{-\lambda}C_t+\varepsilon_x\left(\frac{P_{h,t}}{P_{x,t}}\right)^{-\lambda}X_t+(1-\varepsilon)\frac{S_tP^*_{f,t}}{P_{h,t}}\left(\frac{(1+\tau_t)S_tP^*_{f,t}}{P_t}\right)^{-\lambda}C_t+(1-\varepsilon_x)\frac{S_tP^*_{f,t}}{P_{h,t}}\left(\frac{(1+\tau_t)S_tP^*_{f,t}}{P_{x,t}}\right)^{-\lambda}X_t = Y_{h,t}-\xi\left(\frac{P_{h,t}}{P_{h,t-1}}\right)Y_{h,t} \quad (C.1)$$

$$(1+s)Y_{i,h,t}-\epsilon(P_{i,h,t}(1+s)-MC_t)\frac{Y_{i,h,t}}{P_{i,h,t}}-\xi\prime\left(\frac{P_{i,h,t}}{P_{i,h,t-1}}\right)\frac{P_{h,t}Y_{h,t}}{P_{i,h,t-1}}+E_t\omega_{t+1}\xi\prime\left(\frac{P_{i,h,t+1}}{P_{i,h,t}}\right)\frac{P_{i,h,t+1}}{P^2_{i,h,t}}P_{h,t+1}Y_{h,t+1} = 0 \quad (C.2)$$

$$A_t(1-\alpha)L_t^{-\alpha}X_t^{\alpha}MC_t = W_t \quad (C.3)$$

$$A_t\alpha L_t^{\alpha}X_t^{\alpha-1}MC_t = P_{x,t} \quad (C.4)$$

$$\chi P_tC_t^{\sigma}H_t^{\psi} = W_t \quad (C.5)$$

$$(1+s^*)Y^*_{i,f,t}-\epsilon(P^*_{i,f,t}(1+s^*)-\frac{MC^*_t}{A^*_t})\frac{Y^*_{i,f,t}}{P^*_{i,f,t}}-\xi\prime\left(\frac{P^*_{i,f,t}}{P^*_{i,f,t-1}}\right)\frac{P^*_{f,t}Y^*_{f,t}}{P^*_{i,f,t-1}}+E_t\omega^*_{t+1}\xi\prime\left(\frac{P^*_{i,f,t+1}}{P^*_{i,f,t}}\right)\frac{P^*_{i,f,t+1}}{P^{*2}_{i,h,t}}P^*_{f,t+1}Y^*_{f,t+1} = 0 \quad (C.6)$$

$$A^*_t(1-\alpha)L_t^{*-\alpha}X_t^{*\alpha}MC^*_t = W^*_t \quad (C.7)$$

$$A^*_t\alpha L_t^{*\alpha}X_t^{*\alpha-1}MC^*_t = P^*_{x,t} \quad (C.8)$$

$$\chi P^*_tC_t^{*\sigma}H_t^{*\psi} = W^*_t \quad (C.9)$$

$$\varepsilon\left(\frac{P_{h,t}}{P_t}\right)^{-\lambda}C_t+\varepsilon_x\left(\frac{P_{h,t}}{P_{x,t}}\right)^{-\lambda}X_t+\frac{(1-n)}{n}\left((1-\varepsilon^*)\left(\frac{(1+\tau^*_t)P_{h,t}}{S_tP^*_t}\right)^{-\lambda}C^*_t+(1-\varepsilon^*_x)\left(\frac{(1+\tau^*_t)P_{h,t}}{S_tP^*_{x,t}}\right)^{-\lambda}X^*_t\right) = Y_{h,t}-\xi\left(\frac{P_{h,t}}{P_{h,t-1}}\right)Y_{h,t} \quad (C.10)$$

$$\varepsilon^*\left(\frac{P_{f,t}}{P^*_t}\right)^{-\lambda}C^*_t+\varepsilon^*_x\left(\frac{P_{f,t}}{P^*_{x,t}}\right)^{-\lambda}X^*_t+\frac{n}{1-n}\left((1-\varepsilon)\left(\frac{(1+\tau_t)S_tP_{f,t}}{P_t}\right)^{-\lambda}C_t+(1-\varepsilon_x)\left(\frac{(1+\tau_t)S_tP_{f,t}}{P_{x,t}}\right)^{-\lambda}X_t\right) = Y^*_{f,t}-\xi\left(\frac{P^*_{f,t}}{P^*_{f,t-1}}\right)Y^*_{f,t} \quad (C.11)$$

Equation (C.1) is the Home country budget constraint after netting out the government budget constraint. Equation (C.2) and equations (C.3)-(C.5) are the profit maximizing and cost minimizing relationships for each Home firm i, and the Home labor supply equations. Equations (C.6) and equations (C.7)-(C.9) are the analogous conditions for the Foreign firm. Then equations (C.10) and (C.11) are the Home and Foreign goods market clearing conditions. The system (C.1)-(C.11) can be simplified and rewritten into the 5 equations (31)-(35) of the text.

D Appendix: Alternative parameter values

Table 7 describes the results of the currency and trade war under alternative parameter values. For a larger trade elasticity, assuming $\lambda = 6$, equilibrium tariffs in the trade war are substantially lower. Tariffs are higher than the baseline when the monopoly markup is lower ($\epsilon = 11$, implying

Table 7: Trade and Currency Wars under alternative parameter values

	Trade and currency war - no subsidy ($s = 0$)						
	Base.	$\lambda = 6$	$\epsilon = 11$	$\varepsilon = \varepsilon_x = 0.75$	$\alpha = 0.2$	$\sigma = 2$	$\psi = 0$
π_h	1.0474	1.0479	1.0170	1.0465	1.0278	1.0433	1.0580
π_f^*	1.0474	1.0479	1.0170	1.0465	1.0278	1.0433	1.0580
τ	0.4053	0.1644	0.4352	0.4164	0.4612	0.4314	0.3912
τ^*	0.4053	0.1644	0.4352	0.4164	0.4612	0.4314	0.3912
S	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
C	0.1900	0.1960	0.2130	0.1826	0.4572	0.3310	0.1674
C^*	0.1900	0.1960	0.2130	0.1826	0.4572	0.3310	0.1674
H	0.8733	0.8845	0.9000	0.8575	0.8946	1.5055	0.8012
H^*	0.8733	0.8845	0.9000	0.8575	0.8946	1.5055	0.8012
Home welf. loss (%)	14.4376	12.5938	6.2893	16.6131	5.5570	86.2889	18.3069
Foreign welf. loss (%)	14.4376	12.5938	6.2893	16.6131	5.5570	86.2889	18.3069

a 10 percent markup), and lower in the case of greater home bias in preferences and production. In addition, a smaller weight of intermediate goods, and a lower elasticity of intertemporal substitution also leads to higher Nash equilibrium tariff rates. A lower Frisch elasticity of substitution in labor supply has minimal effects on equilibrium tariff rates, but leads to a 1 percentage point rise in the equilibrium inflation rate.

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