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Reallocation with Priorities

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Abstract

We consider a reallocation problem with priorities where each agent is initially endowed with a house and is willing to exchange it but each house has a priority ordering over the agents of the market. In this setting, it is well known that there is no individually rational and stable mechanism. As a result, the literature has introduced a modified stability notion called μ_0-stability. In contrast to college admission problems, in which priorities are present but there is no initial endowment, we show that the modified Deferred Acceptance mechanism identified in the literature is not the only individually rational, strategy-proof and μ_0-stable mechanism. By introducing a new axiom called the independence of irrelevant agents and using the standard axiom of unanimity, we show that the modified Deferred Acceptance mechanism is the unique mechanism that is individually rational, strategy-proof, μ_0-stable, unanimous and independent of irrelevant agents.

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1 Introduction

In many applied matching problems, indivisible resources have to be reallocated. In theory, agents are initially endowed with an indivisible object (following the standard terminology in the literature, we call these objects *houses*), monetary transfers

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are not allowed, and we use agents' preferences for houses and the initial allocation to determine a new allocation. In practice, we often encounter a situation where additional priority information is used to discriminate between agents. Example applications include campus housing (Guillen and Kesten, 2012), reassignment of workers to positions (Compte and Jehiel, 2008; Dur and Ünver, 2019), teacher assignment (Pereyra, 2013; Combe, Tercieux, and Terrier, 2016) or school choice with a default option (e.g., a neighborhood school). The problem can also occur if some or all of the houses are initially unallocated and an initial allocation of the unallocated houses is generated by a lottery (Sönmez and Ünver, 2005) or as a second stage of an assignment procedure, where an initial allocation is generated by a matching mechanism.

Ideally, a good reallocation mechanism should satisfy a combination of desirable properties: a minimal requirement for any such mechanism should be *individual rationality* (IR); i.e., each agent should be weakly better off after reallocation. Additionally, the designer would like to achieve incentive compatibility in the sense of *strategy-proofness* (SP), efficiency and some form of fairness. The Top Trading Cycle (TTC) mechanism defined by Shapley and Scarf, 1974 and attributed to David Gale is an IR, SP and Pareto efficient (PE) mechanism, and in fact the only such mechanism (Ma, 1995). Given indivisibilities and the absence of monetary transfers, fairness is generally harder to achieve. For example, minimal fairness requirements such as the equal treatment of equals or envy-freeness will be violated by any reallocation mechanism. However, such solutions completely disregard the priority rankings of the houses. Reallocation problems with priorities can be seen as hybrids between the classical marriage and housing market problems. Then, fairness can be understood in the sense that there is no justified envy; i.e., no agent should prefer a house allocated to a lower priority agent to his allotment.

With initial endowments and priorities, it is well known that there is no matching that is both IR and stable in the sense that no agent has justified envy. To ensure the compatibility between the two notions of IR and stability, the concept of stability has been relaxed to exclude blocking pairs caused by a house that is assigned to its initial owner. With this relaxed notion, which is called μ_0-stability,[1] a simple variation of the Deferred Acceptance (DA) mechanism has been identified: it starts to simply rank the initial owners at the top of the priority ordering of their initial house and runs the standard DA mechanism over these modified priorities. This

[1] This terminology is borrowed from Compte and Jehiel (2008). Pereyra (2013) called such matchings *acceptable matchings* and Guillen and Kesten (2012) simply called them *fair matchings*.

mechanism, which is called DA^*,[2] is IR, SP and μ_0-stable.

Our purpose is to give a normative justification for the use of the DA^* mechanism by providing an axiomatic characterization of it. In a model without initial endowments, the classic *Deferred Acceptance* (DA) mechanism is the unique IR (individual rationality is now understood in the sense that applicants obtain an allotment weakly preferred to being unmatched), SP and *stable* mechanism (see Alcalde and Barberà (1994), Theorem 3). However, the DA^* mechanism in the case of initial endowments is *not* the only mechanism that is IR, SP and μ_0-stable. For example, the trivial mechanism that assigns each agent his initial house is IR, SP and trivially μ_0-stable since blocking pairs are not considered when each agent is assigned his initial house. More generally, we can define a class of IR, SP and μ_0-stable mechanisms where a subgroup of agents is always assigned their initial houses, and we run DA^* on the remaining houses and agents. To rule out these trivial mechanisms, one may require some limited form of efficiency such as *unanimity*: a mechanism should assign each agent his top choice whenever that is possible. We show that there are mechanisms other than DA^* that are IR, SP, μ_0-stable and unanimous. However, these mechanisms are problematic in the sense that the assignment can depend on "irrelevant" preference information. More specifically, an agent's change of ranking of a house can influence the allocation of the other houses, even though the allocation of the former house has not changed. We introduce a new axiom called *Independence of Irrelevant Agents* (IIAg) that rules out the possibility of irrelevant ranking information from an agent influencing the allocation. Moreover, we show that in a reallocation problem with priorities, DA^* is the unique IR, SP, μ_0-stable, unanimous and IIAg mechanism.

Related literature. We build upon the classical housing market setting of Shapley and Scarf (1974) where all of the agents are initially assigned to houses and are willing to exchange them. Our characterization result can be trivially extended to the case where there are initially vacant houses and unassigned agents, which was introduced by Abdulkadiroglu and Sonmez (1999). We discuss this extension in Section 5. Moreover, we add the feature that each house now has a priority ordering over the agents, which makes the model closer to the standard marriage market of Gale and Shapley (1962). The reallocation problem with priorities can be seen as a

[2] This terminology is borrowed from Combe, Tercieux, and Terrier (2016). Pereyra (2013) called this mechanism the *teacher proposing Deferred Acceptance* algorithm. We used the former terminology to highlight that this mechanism differs from the standard DA run over the primitive priorities since the latter is not IR.

hybrid of the two extensions.

Guillen and Kesten (2012) were the first to notice that the NH4 mechanism used for off-campus housing allocation at MIT is equivalent to DA^*. In their framework, houses have a common priority over agents. These authors performed an experiment to compare DA^* with the TTC mechanism and found that the participation under DA^* is significantly higher. Still, these authors' model differs from ours since we allow houses to have different priority orderings over agents. Our goal is also different, as we seek to provide a characterization of DA^*.

Pereyra (2013) also studied DA^* in the context of teacher assignments. Our model can be seen as a one-to-one version of his, i.e., where each school has only one initially assigned teacher. This author's focus is on the relaxation of the stability notion in the presence of IR. He defines a matching as acceptable if it is IR, and the only justified envies are the ones where a teacher prefers a school and has a higher priority than an initial teacher of that school who is assigned to it (we call these matchings μ_0-stable). This author's main result shows that an acceptable matching minimizes the remaining blocking pairs in the sense of inclusion if and only if it is the matching produced by the DA^* mechanism. This property can be seen as characterizing the DA^* mechanism. In the same vein, an alternative to our characterization is to require the mechanism to always return a μ_0-*optimally stable* matching, i.e., a μ_0-stable matching that every agent prefers to any other μ_0-stable matching. In the standard setting without endowment, it is well known that this property alone is enough to characterize DA, and it would be the same for DA^* in our setting. Our characterization provides another approach that uses standard axioms in the literature and allows us to clearly use the properties of IR, μ_0-stable and strategy-proof mechanisms. This update is important for two reasons. First, μ_0-optimal stability is not independent of the strategy-proofness axiom since it implies this axiom. For this reason, the characterization of Alcalde and Barberà (1994) does not rely on this stability and proves that with the weaker axiom of strategy-proofness, DA is still the only stable and SP mechanism. In our setting with an initial endowment and priorities, the set of IR, SP and μ_0-stable mechanisms is not a singleton anymore, hence, understanding the structure of such mechanisms is important. Our characterization and the related examples we provide help to reveal how one can build different mechanisms in this class. Second, in many applications, constrained efficiency is not the main policy objective. Policy makers may want to trade off the welfare of agents with other objectives, such as distributional objectives or the welfare of entities outside the model that are encoded into the priorities of the objects. For instance, this scenario occurs if one considers teachers' assignments, which are discussed in Combe, Tercieux, and Terrier (2016), or tuition exchanges as in Dur and Ünver (2019). Thus, a policymaker may be willing to only consider IR, SP and μ_0-stable mechanisms but select from among them a mechanism that respects other desiderata. Our results help to clearly identify which necessary properties to trade off when one picks different mechanisms in this class.

In Section 2, we introduce the reallocation problem with priorities and the standard axioms of the literature. Then, in Section 3, we provide an example to show that the DA^* mechanism is not the only IR, SP, μ_0-stable and unanimous mechanism. Then, we introduce our new axiom of Independence of Irrelevant Agents. Section 4 provides our main characterization result for DA^*, and Section 5 discusses how to extend our result to the case with initially vacant houses and unassigned agents and provides possible directions for future research.

2 Model and Definitions

A reallocation problem with priorities first starts with a standard housing market problem as proposed by Shapley and Scarf (1974). Let I be a finite set of **agents**, and let H be a finite set of **houses** such that $|I| = |H|$. Agents have strict **preferences** over houses that are modeled by a linear order over H.[3] We denote by $\mathcal{R}$ the set of all profiles of strict preferences $R = (R_i)_{i \in I}$ such that for each $i \in I$, R_i is a linear order over H. Following Guillen and Kesten (2012), the main departure from the standard housing market problem is that each house h has a strict **priority ordering** $\succ_h$ over agents, which is a linear order over I. We use standard notions: for a set of houses $H' \subset H$ and a preference profile R, we denote by $R|_{H'}$ the profile of linear orders over the subset H' implied by R. For a set of agents $I' \subset I$ and a preference profile R, we let $R_{I'}$ be the restriction of R to the agents in I'. For a preference profile R and an agent i, R_{-i} will be a shorthand notation for $R_{I \setminus \{i\}}$.

A **matching** is a bijection between I and H. Denote the set of all matchings by $\mathcal{M}$. We assume that there is an **initial matching** $\mu_0 \in \mathcal{M}$ that we may want to improve upon through reallocation.[4] A (reallocation) **mechanism** φ assigns a

[3] A linear order over H is a binary relation R_i that is *antisymmetric* (for each $h, h' \in H$, if $h \, R_i \, h'$ and $h' \, R_i \, h$, then $h = h'$), *transitive* (for each $h, h', h'' \in H$, if $h \, R_i \, h'$ and $h' \, R_i \, h''$, then $h \, R_i \, h''$), and *complete* (for each $h, h' \in H$, $h \, R_i \, h'$ or $h' \, R_i \, h$). By P_i, we denote the asymmetric part of R_i. Hence, given $h, h' \in H$, $h \, P_i \, h'$ means that h is *strictly preferred* to h'; $h \, R_i \, h'$ means that $h \, P_i \, h'$ or $h = h'$ and that h is *weakly preferred* to h'.

[4] As mentioned, our analysis can easily be extended to the case where some houses are vacant and some agents are initially unassigned. For simplicity, we only consider the case where all of the agents are initially assigned to a house and vice versa. See Section 5 for the extension.

matching to each preference profile, i.e., it is a mapping $\varphi : \mathcal{R} \to \mathcal{M}$. We are interested in designing mechanisms that have certain desirable properties. In the context of reallocation, the existing rights should be respected by ensuring that every agent is as least as well off as under their initial assignment. Formally,

Axiom (Individual Rationality)**.** *A mechanism φ is individually rational (IR) with respect to the initial matching μ_0 if for each $R \in \mathcal{R}$ and $i \in I$ we have*

$$\varphi_i(R)\, R_i\, \mu_0(i).$$

Here, we assume that all of the agents find their initial houses acceptable.[5] For a matching μ, a pair $(i, h) \in I \times H$ is called a blocking pair of μ if $hP_i\mu_i$ and $i \succ_h \mu^{-1}(h)$. A matching is stable if it does not have any blocking pair. Individual rationality can in general be in conflict with the priorities so that there could be no matching that is both IR and stable. However, we can require a relaxed notion of stability:

Axiom (μ_0-Stability)**.** *A matching μ is μ_0-stable with respect to preferences R and priorities $\succ$ if for each $i \in I$ and $h \in H$ we have the following: if $h\, P_i\, \mu(i)$ and $i \succ_h \mu^{-1}(h)$, then $\mu_0(\mu^{-1}(h)) = h$. Mechanism φ is μ_0-stable if it assigns to each profile $R \in \mathcal{R}$ a μ_0-stable matching with respect to R and $\succ$.*

In other words, μ_0-stability only allows blocking pairs if they are caused by an agent staying at his initial house. Additionally, we use the incentive compatibility property of strategy-proofness:

Axiom (Strategy-Proofness)**.** *A mechanism φ is strategy-proof if for agent $i \in I$ and profiles R, R' with $R'_{-i} = R_{-i}$ we have*

$$\varphi_i(R)\, R_i \varphi_i(R').$$

Ideally, a mechanism is individually rational, strategy-proof, respecting of priorities (in the sense of μ_0-stability) and Pareto-efficient. Generally, these properties are incompatible (Ergin, 2002). Thus, we have to content ourselves with a weaker notion of efficiency. For each $R \in \mathcal{R}$ and $i \in I$, we denote by $top(R_i)$ the highest ranked house according to R_i. We call a profile $R \in \mathcal{R}$ **unanimous** if for $i \neq j$ we have $\mathrm{top}(R_i) \neq \mathrm{top}(R_j)$. A mechanism is unanimous if it assign everyone their top house for unanimous profiles.

[5] Since we only consider IR mechanisms and we assume that the initial houses are acceptable, there is no loss of generality in assuming that preferences are linear orderings over H, which implies we are not ranking the possibility of remaining unassigned. We follow the standard housing market model as introduced by Shapley and Scarf (1974).

Axiom (Unanimity)**.** *A mechanism φ is unanimous if for each unanimous profile R we have $\varphi_i(R) = top(R_i)$.*

Ownership-Adjusted Deferred Acceptance

The Deferred Acceptance algorithm Gale and Shapley (1962) is strategy-proof, respects priorities and is unanimous; more generally, it is constrained optimal in the sense that it selects (the unique) stable matching that is not Pareto dominated within the set of stable matchings. The mechanism can easily be adapted to respect initial ownership rights by treating owners as if they have top priority in their initial houses. Formally, the **ownership-adjusted Deferred Acceptance mechanism** for preferences R, priorities $\succ$ and initial matching μ_0 proceeds in rounds where in each round the following steps are performed.

1. Each agent i applies to his favorite house according to R_i that has not previously rejected i.

2. Each house h tentatively accepts $\mu_0(h)$ if $\mu_0(h)$ has applied to it. Otherwise, the house tentatively accepts the highest priority agent according to $\succ_i$ among the agents that have applied to it and rejects all other applicants.

We omit the priorities in our notations since the context will always be clear, and we denote the final matching by $DA^*(R)$.

3 Example and Independence of Irrelevant Agents

While Deferred Acceptance is the unique stable and strategy-proofness mechanism in the model without initial endowments Alcalde and Barberà (1994), there are many different individually rational, μ^0-stable and strategy-proof mechanisms. For example, one such mechanism is the trivial mechanism that assigns each agent to her initial house independently of the submitted preferences. More surprisingly, perhaps, there even exist IR, SP, μ_0-stable and unanimous mechanisms that differ from DA^*:

Example 1. *Consider five agents $I = \{a, b, c, d, e\}$, five houses $H = \{h_a, h_b, h_c, h_d, h_e\}$ and an initial matching μ_0 s.t. $\mu_0(k) = h_k$ for $k \in I$. Consider a priority relation*

$\succ$ *such that we have the following:*

$$
\begin{array}{llllll}
\succ_{h_a}: & a & b & c & d & e \\
\succ_{h_b}: & b & a & c & d & e \\
\succ_{h_c}: & c & a & b & d & e \\
\succ_{h_d}: & d & a & b & c & e \\
\succ_{h_a}: & e & a & b & c & d
\end{array}
$$

We define φ as an IR, strategy-proof, μ_0-stable and unanimous mechanism that is not DA^* with priorities $\succ$ as follows.

Denote by $\mathcal{R}' \subseteq \mathcal{R}$ the set of preference profiles R such that

$$
\begin{array}{lllllll}
R_a: & h_b & P_a & h_a & \ldots & & \\
R_b: & h_c & P_b & h_b & P_b \ldots & & \\
R_c: & h_b & P_c & h_a & P_c & h_c & P_c \ldots
\end{array}
$$

Define a matching μ by

$$\mu(a) = h_b, \quad \mu(b) = h_c, \quad \mu(c) = h_a, \quad \mu(d) = h_d, \quad \mu(e) = h_e.$$

$$\varphi(R) := \begin{cases} DA^*(R), & \text{if } R \notin \mathcal{R}', \\ \mu, & \text{if } R \in \mathcal{R}'. \end{cases}$$

The mechanism φ is unanimous since DA^* is unanimous, and profiles in $\mathcal{R}'$ are not unanimous since agents a and c both rank house h_b first at these profiles. Moreover, φ is μ_0-stable, since DA^* is μ_0-stable and μ is a μ_0-stable matching for each $R \in \mathcal{R}'$ (as d and e have lower priority than a, b and c at houses h_a, h_b and h_c). For strategy-proofness, note that by the strategy-proofness of DA^*, only a, b and c can possibly manipulate φ. However, note that for each profile $R \in \mathcal{R}'$ we have $\varphi_i(R) = DA_i^*(R)$ for $i \in \{a, b, c\}$. Thus, strategy-proofness follows from the strategy-proofness of DA^*. Finally, note that $\varphi \neq DA^*$. Indeed, we select a profile $R \in \mathcal{R}'$ such that

$$
\begin{array}{lllll}
R_d: & h_e & P_d & h_d & P_d \ldots \\
R_e: & h_d & P_e & h_e & P_e \ldots
\end{array}
$$

In this case, $DA^*(R)$ assigns h_e to d and h_d to e, whereas $\varphi(R)$ assigns h_d to d and h_e to e.

The mechanism φ also satisfies other desirable properties that have been discussed in the context of axiomatizations of the Deferred Acceptance mechanism. The mechanism is, for example, *weakly Maskin monotonic* in the sense of Kojima and Manea (2010), and it is weakly Pareto efficient. However, the mechanism has an important and less appealing feature: in the last preference profile that is considered in the example where $\varphi(R) \neq DA^*(R, \succ)$, the mechanism φ does not allow agents d and e to exchange their houses. However, if agent c reports the profile $R'_c : h_a \quad P'_c \quad h_c$, then φ allows d and e to exchange houses under (R'_c, R_{-c}). Therefore, at profile R, mechanism φ forbids the exchange between d and e because of the preference profile of c and his ranking of house h_b. The exchange is forbidden even though this house is not a part of the exchange between d and e, not even indirectly (as would be the case if, for example, h_b would be allocated to a different agent so that the original recipient of h_b could now be assigned h_d or h_e). Thus, the assignment switches based on the preference information of an agent that seems to be *irrelevant* for the allotment of h_d and h_e.

Based on the intuition of the mechanism in Example 1, let us define a new axiom. For an agent i, we have a house h and two profiles R_i and $\tilde{R}_i$. We say that profile $\tilde{R}_i$ is a *monotonic transformation* of R_i at h if it starts from profile R_i and ranks house h higher while keeping the same ordinal ranking for the other houses. Formally, $\tilde{R}_i|_{H\setminus\{h\}} = R_i|_{H\setminus\{h\}}$ and $(h'\tilde{R}_i h \Rightarrow h' R_i h)$.

Axiom (Independence of Irrelevant Agents)**.** *A mechanism φ is Independent of Irrelevant Agents (IIAg) if $\forall i \in I$, $\forall h \in H$, and $R, \tilde{R} \in \mathcal{R}$ such that $\tilde{R}_{-i} = R_{-i}$, $\tilde{R}_i$ is a monotonic transformation of R_i at h and $\varphi_h(R) \neq i$, we have*

$$\varphi_h(\tilde{R}) = \varphi_h(R) \Rightarrow \varphi(\tilde{R}) = \varphi(R).$$

The axiom states that if an agent does not obtain a house and improves the ranking of that house but this change is *irrelevant* for the allocation of that house, i.e., if it does not change the allocation of that house, then the whole allocation must remain the same.

The axiom has a similar flavor to the *independence of irrelevant alternatives* axiom (see, for instance, Arrow, 2012). In standard social choice theory, a mechanism is independent of irrelevant alternatives if whenever a social choice μ is chosen by the mechanism and the preference profile of the agents is R, then μ continues to be chosen at any profile R' where all of the agents rank μ weakly higher than under profile R.[6] Our IIAg axiom restricts social choice in a similar way. However, there

[6] In standard social choice, agents have complete linear orderings over the set of *alternatives*, which in our context would be equivalent to matchings.

are two important differences. We are not in a standard social choice model since agents have preferences only over their final allocated houses and not over the entire set of matchings.[7] Furthermore, we have an additional condition that checks whether the assignment of the house has changed once it has been upgraded. In the social choice context, the independence of irrelevant alternatives axiom states that once the choice has been made, the pieces of information of the houses ranked above the choice by each agent (which have therefore been disregarded by the mechanism) should not impact the initial choice that was made. Our IIAg axiom states that if the improvement of the ranking of a house by an agent is irrelevant to determining the allocation of that house, then it should not impact the overall assignment and the choice rule must remain consistent with the agent's first decision.

4 Characterization

Before stating our main theorem, we discuss the axioms. We are interested in defining reasonable mechanisms that are IR, SP and μ_0-stable. As we have seen, one trivial solution is to consider constant mechanisms that always retain a subset of agents from their initial allocation. Unanimity allows us to rule out such constant mechanisms. Intuitively, one way to construct a new IR, SP, μ_0-stable and unanimous mechanism is to force some group of agents to stay at their initial allocation under some preference profiles when they would otherwise move under DA^*, but to let them move under other preference profiles, typically the unanimous ones.[8] Intuitively, to maintain strategy-proofness for these agents, the decision of whether to hold them at their initial allocation cannot depend on their reported preferences. Thus, this decision must be taken by using the change in the preferences from another "irrelevant" agent, which is exactly illustrated by our Example 1.

Thus, one may wonder what mechanism is left once we rule out such group variations based on irrelevant agents. The answer to this question is exactly our

[7] Their preferences over matchings exhibit ties since they are indifferent with respect to any two matchings that assign them to the same house. This feature is why the matching framework allows one to overcome the so-called *Arrow Impossibility Theorem* or the *Gibbard-Satthethwaite Impossibility Theorem* in social choice.

[8] Of course, one can also select another μ_0-stable matching for these agents instead of keeping them at their initial allocation. However, under some profile, strategy-proofness would force the same mechanism to hold some of these agents to their initial allocations. Indeed, if for some agent i and profile R, we have $DA^*_i(R) P_i \varphi_i(R) P_i \mu_0(i)$, then by reporting the profile R'_i that only ranks $DA^*_i(R)$ above $\mu_0(i)$, strategy-proofness would lead to $DA^*_i(R'_i, R_{-i}) P_i \varphi_i(R'_i, R_{-i}) = \mu_0(i)$. In a way, our proof below will work with such a minimal example.

main result.

Theorem 1. *A mechanism is IR, SP, μ_0-stable, unanimous and independent of irrelevant agents if and only if it is the DA^* mechanism.*

Proof. First, it is standard to show that DA^* is IR, SP, μ_0-stable and unanimous.[9] For IIAg,[10] let $i \in I$, $h \in H$, $R, \tilde{R} \in \mathcal{R}$ with $\tilde{R}_i|_{H\setminus\{h\}} = R_i|_{H\setminus\{h\}}$ and $\tilde{R}_{-i} = R_{-i}$. Suppose that $DA^*_i(R) \neq h$ and let $j \neq i$ be assigned h, i.e., let $DA^*_j(R) = h$. Furthermore, suppose that $DA^*_j(\tilde{R}) = h$.

If $DA^*_i(\tilde{R})\, P_i\, h$, then $DA^*(\tilde{R})$ is also μ_0-stable under R. If $h\, P_i\, DA^*_i(\tilde{R})$ and $DA^*(\tilde{R})$ is not μ_0-stable under R, then since only i changed the ranking of h, i and h block $DA^*(\tilde{R})$. However, because $DA^*_j(R) = h = DA^*_j(\tilde{R})$, we have $DA^*_i(R)\, P_i\, h\, P_i\, DA^*_i(\tilde{R})$, as otherwise i and h would also block $DA^*(R)$. Since $\tilde{R}_i|_{H\setminus\{h\}} = R_i|_{H\setminus\{h\}}$ and $DA^*(\tilde{R})$ is μ_0-stable under $\tilde{R}$, this relation implies $DA^*_i(R)\, \tilde{P}_i\, DA^*_i(\tilde{R})\, \tilde{P}_i\, h$. Then, $DA^*(R)$ is also μ_0-stable under $\tilde{R}$, and $DA^*(\tilde{R})$ is not the agent-optimal μ_0-stable matching under $\tilde{R}$, which is a contradiction. As a result, $DA^*(\tilde{R})$ is μ_0-stable under R. By symmetry, $DA^*(R)$ is μ_0-stable under $\tilde{R}$. Thus, $DA^*(R) = DA^*(\tilde{R})$.

For the other direction, let φ be an IR, SP, μ_0-stable, unanimous, and IIAg mechanism. In the following, for each profile R, we denote by

$$M(R) := \sum_{i\in I} |\{h : h\, P_i\, \mu_0(i)\}|$$

the number of houses ranked above the initial assignment. In addition, we denote by

$$N(R) := \sum_{i\in I} |\{h : h\, P_i\, DA^*_i(R)\}|$$

the number of houses ranked above the DA^* assignment at profile R. Let R be a profile with $\varphi(R) \neq DA^*(R)$ such that

1. for each R' with $\varphi(R') \neq DA^*(R')$, we have $M(R) \leq M(R')$;
2. for each R' with $\varphi(R') \neq DA^*(R')$ and $M(R) = M(R')$, we have $N(R) \leq N(R')$.

[9] See Guillen and Kesten (2012), Pereyra (2013) or Compte and Jehiel (2008).

[10] In the whole proof, because the priority profile $\succ$ is assumed to be fixed, it is omitted from the notations.

Let $\mu := DA^*(R)$ and $\nu := \varphi(R)$.

We prove the result through a sequence of claims. The first claim states that for the profile R, any agent who does not attain his DA^*-outcome under φ only ranks his DA^*-outcome above his initial assignment. This claim only requires the axioms of individual rationality, μ^0-stability and strategy-proofness.

Claim 1. *For each $j \in N$ with $\mu(j) \neq \nu(j)$, only $\mu(j)$ is ranked above the initial match, i.e.,*

$$R_j : \mu(j)\, P_j\, \mu_0(j) = \nu(j) \ldots$$

Proof. Suppose otherwise and let $R'_j : \mu(j)\, P'_j\, \mu_0(j) \ldots$ and $R' := (R'_j, R_{-j})$. By the strategy-proofness of DA^*, we have $DA^*_j(R') = \mu(j)$. Since $DA^*(R)$ is μ_0-optimally stable, i.e., $DA^*(R)$ returns the most preferred μ_0-stable matching for the agents, we have $\mu(j) R_j \nu(j)$. By the strategy-proofness of φ, we have $\varphi_j(R') = \mu_0(j) \neq \mu(j) = DA^*_j(R')$. However, this equation would imply $\varphi(R') \neq DA^*(R')$ and $M(R') < M(R)$, which contradicts (1). Thus, j only ranks $\mu(j)$ above his initial match. □

If $N(R) = 0$, then R is a unanimous profile, and by the unanimity of φ we have $\varphi(R) = DA^*(R)$. Thus, we may assume $N(R) > 0$. Since $N(R) > 0$, there is an agent $i \in I$ and house $h \in H$ with $h\, P_i\, \mu(i)$. In the following, we assume w.l.o.g. that i is the agent with $h\, P_i\, \mu(i)$ who has the highest priority for h among those agents.

Claim 2. *Let $j \in I$ be the agent such that $\mu(j) = h$. Then, $\mu(j) \neq \nu(j)$.*

Proof. Suppose $\mu(j) = h = \nu(j)$. Since $hP_i\mu(i)$, we know that there there is at least one agent k (for example, $k = i$) with $hP_k\mu_0(k)$. Let k be the agent with the lowest priority at h. By the individual rationality of μ, we have $\mu_0(k) \neq h$. Consider the profile $\tilde{R}_k$ where agent k moves h below $\mu_0(k)$, i.e., $\tilde{R}_k|_{H\setminus\{h\}} = \tilde{R}_k|_{H\setminus\{h\}}$ and $\mu_0(k)\, \tilde{P}_k\, h$. Let $\tilde{R} = (\tilde{R}_k, R_{-k})$. Note that $M(\tilde{R}) < M(R)$.

We will now show that $DA^*(\tilde{R}) = DA^*(R)$. First, note that $\tilde{R}$ is a *monotonic transformation* of R at $DA^*(R)$, i.e., for each $h \in H$ and $\ell \in I$, we have $h' \tilde{P}_\ell DA^*(R) \Rightarrow h' P_\ell DA^*(R)$. Note that in our proof, the profile $\tilde{R}_k$ downgrades house h below $\mu_0(k)$ so that there is indeed a monotonic transformation at $DA^*_k(R)$. As shown by Kojima and Manea (2010), DA^* is weakly Maskin monotonic. Thus, it must be the case that for all agents, $DA^*(\tilde{R})$ is weakly preferred to $DA^*(R)$ at profile R. Assume that $DA^*(\tilde{R}) \neq DA^*(R)$. Then, the matching $DA^*(\tilde{R})$ Pareto dominates the matching $DA^*(R)$ at profile R. So it must be the case that the assignment of house h has changed and there is an agent $k' := DA^*_h(\tilde{R}) \neq j = \mu(h)$

who was rejected by agent k at house h under $DA^*(R)$.[11] Thus, $k \succ_h k'$ and $hP_{k'}DA^*_{k'}(R)R_{k'}\mu_0(k')$, contradicting the assumption that k has the lowest priority at h among the agents who strictly prefer h to their initial house. Thus, we have $DA^*(\tilde{R}) = DA^*(R)$.

In that case, by IIAg we either have $\varphi_j(\tilde{R}) \neq \varphi_j(R)$ or $\varphi(\tilde{R}) = \varphi(R)$. In both cases, $\varphi(\tilde{R}) \neq DA^*(R) = DA^*(\tilde{R})$, which is a contradiction since $M(\tilde{R}) < M(R)$. □

Claim 3. *For each $k \neq i$, we have $\mu_0(k)\, R_k\, \mu(i)$, i.e., $\mu(i)$ is unacceptable or the endowment for all other agents.*

Proof. Suppose there is an agent k with $\mu(i)\, P_k\, \mu_0(k)$, and let k be the agent with the lowest priority at $\mu(i)$. Let $\tilde{R}_k$ be a profile such that $\tilde{R}_k|_{H\setminus\{\mu(i)\}} = R_k|_{H\setminus\{\mu(i)\}}$ and $\mu_0(k)\, \tilde{P}_k\, \mu(i)$. This profile is a monotonic transformation of R_k. Note that $M(\tilde{R}) < M(R)$. By Claim 1 applied to agent i and by the assumption that $h\, P_i\, \mu(i)$, we have $\mu(i) = \nu(i)$. Using the same argument that we used in Claim 2, we have $DA^*(\tilde{R}) = DA^*(R)$. Thus, by the IIAg of φ applied to agent k and house $\mu(i)$, either $\varphi(\tilde{R}) = \varphi(R)$ or $\varphi_i(\tilde{R}) \neq \varphi_i(R) = \nu(i) = \mu(i)$. In both cases, we have $\varphi(\tilde{R}) \neq DA^*(\tilde{R}) = DA^*(R)$, which contradicts the notion that $M(R)$ was minimal. □

Now, consider the profile

$$R'_j : \mu(i)\, R'_j\, \mu_0(j)\ \ldots,$$

and let $R' := (R'_j, R_{-j})$.

Claim 4. *Define the matching μ' as follows:*

$$\mu'(i) = h, \quad \mu'(j) = \mu(i), \quad \mu'(k) = \mu(k) \text{ for } k \neq i, j.$$

Then, the matching μ' is μ_0-stable and individually rational under R'.

Proof. Individual rationality for $k \neq i, j$ follows from the individual rationality of μ. Individual rationality for j follows by the definition of R'_j. Individual rationality for i follows by the assumption that $\mu'(i) = h\, P_i\, \mu(i)$ and by the individual rationality of μ.

[11] Using the terminology of Kesten (2010), agent k was *an interrupter* at house h. Agent k is an interrupter at house h if while running $DA^*(R)$, he has been temporarily accepted at house h at Step t and later rejected at $t' > t$ and there has been an agent k' who has been rejected by house h at a Step $\ell \in \{t, t+1, \ldots, t'-1\}$.

Next, we show μ_0-stability. First, consider agent i. Agent i and $\mu(i)$ do not block μ' because $\mu'(i) = h\, P_i\, \mu(i)$. Moreover, for $h' \notin \{\mu(i), h\}$ we have $h' = \mu(k) = \mu'(k)$ for an agent $k \neq i, j$. If i and h' block μ', then $h'\, P_i\, \mu'(i) = h\, P_i\, \mu(i)$ and both i and h' would also block μ under R, contradicting the μ_0-stability of μ under R. Thus, there is no blocking pair involving i. Because agent j obtains his top choice in μ', he cannot be involved in a blocking pair. Finally, we consider $k \neq i, j$. By Claim 3, we have $\mu_0(k)\, R'_k\, \mu(i)$. Moreover, by the individual rationality of μ, we have $\mu(k)\, R'_k\, \mu_0(k)$. Thus, $\mu'(k) = \mu(k)\, R'_k\, \mu_0(k)\, R'_k\, \mu(i)$ and k and $\mu(i)$ do not block μ'. By assumption, i has highest priority for h among those agents who rank h strictly above their assignment under μ. Thus, if $h\, P'_k\, \mu(k) = \mu(k)$, then either $\mu_0(k) = h$ or $i \succ_h k$. The first possibility contradicts the individual rationality of μ under R. In the second case, k and h do not block μ'. Thus, k and h do not block μ'. Finally, k does not block μ' with a house $h' \neq h, \mu(i)$ because otherwise k and h' would block μ under R. □

By the construction of R'_j, $M(R') \leq M(R)$. As μ' is μ_0-stable and individually rational for R', it is Pareto-dominated by $DA^*(R')$. Therefore, we have

$$N(R') \leq \sum_{k \in I} |\{h' : h'\, P'_k\, \mu'(k)\}| < \sum_{k \in I} |\{h' : h'\, P_k\, \mu(k)\}| = N(R),$$

where the inequality in the middle is strict because $\mu'(i) = h\, P_i\, \mu(i)$. Thus, $DA^*(R') = \varphi(R')$ and $\varphi_j(R') = \mu(i)$. Next, let

$$\tilde{R}_j : \; h\, \tilde{P}_j \mu(i)\, \tilde{R}_j\, \mu_0(j).$$

By strategy-proofness applied to R' and $(\tilde{R}_j, R_{-j})$, we have $\varphi_j(\tilde{R}_j, R_{-j}) \in \{\mu(i), h\}$. By strategy-proofness applied to R and $(\tilde{R}_j, R_{-j})$ and by Claim 2, we have $\varphi_j(\tilde{R}_j, R_{-j}) \neq h$. Thus, $\varphi_j(\tilde{R}_j, R_{-j}) = \mu(i)$. Furthermore, note that $DA^*_i(\tilde{R}_j, R_{-j}) = \mu(i) = DA^*_i(R)$. Since $\varphi(\tilde{R}_j, R_{-j})$ is μ_0-stable at $(\tilde{R}_j, R_{-j})$, it is Pareto-dominated by $DA^*(\tilde{R}_j, R_{-j})$. Consequently, we have

$$\mu(i) = DA^*_i(\tilde{R}_j, R_{-j})\, P_i\, \varphi_i(\tilde{R}_j, R_{-j}).$$

Now, suppose i reports

$$\tilde{R}_i : \mu(i)\, \tilde{R}_i\, \mu_0(i) \ldots$$

By strategy-proofness, for $(\tilde{R}_j, R_{-j})$ and $\tilde{R} := (\tilde{R}_i, \tilde{R}_j, R_{-i,j})$, we have $\varphi_i(\tilde{R}) = \mu_0(i)$. By construction, $M(\tilde{R}) \leq M(R)$. Moreover, by the construction of $\tilde{R}$, μ is μ_0-stable and individually rational under $\tilde{R}$. Since $h\, P_i\, \mu(i)$ but $\mu(i)\, \tilde{P}_i\, h$, this fact implies

$$N(\tilde{R}) \leq \sum_{k \in I} |\{h : h\, \tilde{P}_i\, \mu(k)\}| < \sum_{k \in I} |\{h : h\, P_i\, \mu(k)\}| \leq N(R).$$

As a result, $\varphi(\tilde{R}) = DA^*(\tilde{R})$. However, because $\varphi_i(\tilde{R}) = \mu_0(i)$ and μ is a μ_0-stable matching and individually rational matching under $\tilde{R}$, $DA_i^*(\tilde{R})\tilde{R}_i\mu(i)\tilde{P}_i\mu_0(i) = \varphi_i(\tilde{R})$. Therefore, we have a contradiction.

We conclude that $\varphi = DA^*$. □

We conclude this section by showing that the axioms used in Theorem 1 are independent.

Dropping IR. The standard DA (without modifying the priority structure) is an SP, μ_0-stable, unanimous and IIAg mechanism.

Dropping unanimity. The trivial mechanism that assigns every agent to his or her initial house is IR, SP, μ_0-stable and IIAg.

Dropping μ_0-stability TTC is an IR, SP, and IIAg mechanism.

Dropping IIAg. The mechanism in Example 1 is IR, SP, μ_0-stable and unanimous.

Dropping SP. Consider three agents $I = \{a, b, c\}$, three houses $H = \{h_a, h_b, h_c\}$ and an initial matching μ_0 s.t. $\mu_0(k) = h_k$ for $k \in I$. Consider the priority relation $\succ$ such that

$$\begin{array}{llll} \succ_{h_a}: & a & c & b \\ \succ_{h_b}: & b & a & c \\ \succ_{h_c}: & c & b & a \end{array}$$

Now, let $\mathcal{R}^* \subset \mathcal{R}$ be the set of preference profiles R such that

$$\begin{array}{llll} R_a: & h_b & \dots & \\ R_b: & h_c & \dots & \\ R_c: & h_b & h_a & h_c \end{array}$$

Let φ be the mechanism defined as follows:

$$\varphi(R) = \begin{cases} \mu_0 & \text{if } R \in \mathcal{R}^* \\ DA^*(R) & \text{if } R \notin \mathcal{R}^* \end{cases}$$

It is easy to see that φ is IR and μ_0-stable. Since all of the profiles in $\mathcal{R}^*$ are not unanimous profiles and since φ is the DA^* mechanism outside $\mathcal{R}^*$, φ is a unanimous mechanism. It is also easy to see that φ is not an SP mechanism. At any preference profile $R \in \mathcal{R}$, agent c can manipulate φ in reporting profile $R'_c : h_b, h_c, h_a$.

Here, we show that φ is IIAg. Since we know that DA^* is IIAg, we only need to check two cases. First, when an agent starts at a profile in $\mathcal{R}^*$ and a deviation as defined in the definition of IIAg would lead to a profile outside of $\mathcal{R}^*$. Second, when an agent starts at a profile outside of $\mathcal{R}^*$ and a deviation as defined in the definition of IIAg would lead to a profile inside of $\mathcal{R}^*$. To begin, we take a profile $R \in \mathcal{R}^*$ and fix agent a.

- Assume that $R_a : h_b, h_a, h_c$. If she moves down h_b, then either the matching stays at μ_0 or b and c exchange their houses so that IIAg is trivially respected. If she moves up h_c so that $R'_a : h_c, h_b, h_a$, then note that the matching of $DA^*(R')$ assigns a to h_b, b to h_c and c to h_a. Therefore, IIAg is respected again.

- Assume that $R_a : h_b, h_c, h_a$. We have seen that moving down h_b or moving up h_c so that $R'_a : h_c, h_b, h_a$ forces all of the agents to exchange their houses for IIAg to be respected. If she moves down h_b so that $R'_a : h_c, h_a, h_b$, then since $b \succ_{h_c} a$, $DA^*(R')$ allocates b in h_c and c in h_b and IIAg is still satisfied.

Now, we fix agent b.

- Assume that $R_b : h_c, h_b, h_a$. By moving down h_c, either the matching stays at μ_0 or a and c exchange their houses so that IIAg is respected. If she moves up h_a so that $R'_b : h_a, h_b, h_c$, then because $c \succ_{h_a} b$, $DA^*(R')$ allocates b in h_c and c in h_b, IIAg is still satisfied.

- Assume that $R_b : h_c, h_a, h_b$. By moving down h_c or moving up h_a such that $R'_b : h_a, h_c, h_b$, because $a \succ_{h_b} c$ and $c \succ_{h_a} b$, $DA^*(R')$ would assign a to h_b, b to h_c and c to h_a. Consequently, IIAg would be trivially respected. By reporting $R'_b : h_a, h_b, h_c$, then b would stay at his initial house h_b and so IIAg would be respected whether a and c exchange their houses or not.

Consider agent c.

- Moving down h_b or moving up h_a would make everyone exchange their houses with a assigned to h_b, b assigned to h_c and c assigned to h_a for IIAg to be respected.

- Moving down h_a so that $R'_c : h_b, h_c, h_a$ would make c stay at his initial house since $a \succ_{h_b} c$. In that case, IIAg is trivially respected independently of whether a and b exchange their houses.

We conclude that by starting from any profile $R \in \mathcal{R}^*$, IIAg is respected. Now, we start with a profile $R \notin \mathcal{R}^*$ and fix agent a. We will make a change to R'_a as defined in the definition of IIAg so that the new profile $R' \in \mathcal{R}^*$. In particular, at the initial profile R, $R_c = h_b, h_a, h_c$, agent b ranks house h_c first in R_b and agent a does not rank house h_b first.

- Assume that $R_a : h_a, ...$ Then, a stays at her house h_a. By moving down h_a or moving up h_b, $R'_a : h_b, \ldots$, $\varphi(R') = \mu_0$. So if b and c were also staying at their initial houses under $\varphi(R)$, then IIAg would be trivially satisfied. If b and c were exchanging their houses, the report R'_a would make the assignment of all houses except h_a change so that IIAg would again be satisfied.

- Assume that $R_a : h_c, h_a, h_b$. In that case, since $b \succ_c a$, then $\varphi(R) = DA^*(R)$ does not assign a to h_c. Hence, a stays at her initial house h_a. Then, the same argument as above applies.

- Assume that $R_a : h_c, h_b, h_a$. In that case, one can check that $\varphi(R) = DA^*(R)$ assigns a to h_b, b to h_c and c to h_a. Since any change of preference profile from R_a to R'_a by agent a so that $R' \in \mathcal{R}^*$ would lead to $\varphi(R') = \mu_0$, again IIAg is trivially satisfied.

Now, consider agent b and start at a profile $R \notin \mathcal{R}^*$ where $R_c = h_b, h_a, h_c$, agent a ranks h_b first and agent b does not rank h_c first.

- Assume that $R_b : h_b, ...$ Then, b stays at her initial house h_b under $\varphi(R)$. Thus, a similar argument to the above applies and IIAg is respected.

- Assume that $R_b : h_a, h_b, h_c$. One can check that $\varphi(R) = DA^*(R)$ makes b stay at her initial house h_b so that IIAg is respected if the profile moves to $R' \in \mathcal{R}^*$.

- Assume that $R_b : h_a, h_c, h_b$. Again, one can check that all of the agents exchange their houses under DA^*, which implies that IIAg is trivially respected.

Lastly, consider agent c and start at a profile $R \notin \mathcal{R}^*$ where agent a ranks h_b first and agent b ranks h_c first.

- Assume that $R_c : h_c, ...$ Then, c stays at her initial house h_c under $\varphi(R)$. By a similar argument to the above, IIAg is respected.

- Assume that $R_c : h_a, h_c, h_b$ or $R_c : h_a, h_b, h_c$. In that case, all of the agents exchange their houses under $\varphi(R) = DA^*(R)$ and a is assigned to h_b, b is assigned to h_c and c is assigned to h_a. In that case, any change of preferences from R_a to R'_a so that the new profile $R' \in \mathcal{R}^*$ leads to $\varphi(R) = \mu_0$. Consequently, IIAg is trivially satisfied.

We conclude that φ is an IR, μ_0-stable, unanimous and IIAg mechanism that is not strategy-proof.

5 Extension with vacant seats and initially unassigned agents

It is a natural question whether and how our results generalize to the house allocation model with existing tenants (Abdulkadiroglu and Sonmez, 1999) in which some of the agents could initially be unassigned and some houses could initially be vacant. We briefly sketch how our characterization and proof generalize in that case.

Note that our model and axioms can be adapted to this case simply by changing the definition of a matching to be a mapping $\mu : I \to H \cup \{\emptyset\}$ such that for $i \neq j$, $\mu(i) = \mu(j)$ implies $\mu(i) = \mu(j) = \emptyset$ and by allowing the possibility that $|I| \neq |H|$. Here, "$\emptyset$" denotes the outside option of being unmatched. In particular, the initial matching μ^0 can be such that some agents are initially unassigned and some houses are initially vacant. All of the axioms are still well defined for this enhanced model.

A careful inspection of the proof of Claim 1 shows that the claim still holds for the enhanced model. Actually, the proof of Claim 1 only requires the axioms of individual rationality, μ^0-stability and strategy-proofness. Next, note that by μ^0-stability and by the rural hospital's theorem for stable matching (McVitie and Wilson, 1970), for each newcomer $i \in N$ with $\mu^0(i) = \emptyset$, we have that $\mu(i) \neq \nu(i)$ implies $\mu(i) \neq \emptyset \neq \nu(i)$ for any two μ^0-stable matchings μ and ν. Thus, by Claim 1, for each newcomer $i \in N$ with $\mu^0(i) = \emptyset$, we have $\mu(i) = \nu(i)$, i.e., for each newcomer we know that the DA^*-outcome and the outcome of any μ^0-stable, individually rational and strategy-proof mechanism is the same. If there are only newcomers and no existing tenants, this statement finishes the proof (in particular, the result by Alcalde and Barberà, 1994 is recovered as a special case). If there are existing tenants in the problem, we can use the same proof as in our characterization for housing markets using the fact that whenever there is an agent i with $\mu(i) \neq \nu(i)$, it was previously established that such an agent must be an existing tenant.

6 Conclusion

Our paper highlights that the problem of reallocation with priorities has distinct differences from its counterparts, namely, the marriage problem and the housing market problem. Thus, there is still significant research to be done to study the specific properties of this problem. In our analysis, we take the priority structure as given and study the properties of the μ_0-stable mechanism DA^*. We follow the path of the school choice literature (Abdulkadiroglu and Sonmez, 2003). In such models, the priorities of one side of the market, e.g., the schools or the houses, are not considered as preferences per se but as part of the design of the mechanism. In that setting, one can see the DA mechanism as a class of mechanisms, where there is one for each profile of priorities. Kojima and Manea (2010) were the first to propose two axiomatic characterizations of the DA mechanisms, and they introduced the axioms of *individually rational monotonicity* and *weak Maskin monotonicity*. Whereas these authors' first results showed that the (student proposing) DA with acceptant and substitutable choice functions is the only non-wasteful and individually rational monotonic mechanism, their second characterization uses the axioms of non-wastefulness, population monotonicity, and weak Maskin monotonicity. Later, Ehlers and Klaus (2016) provided two characterizations of DA using a set of standard axioms: unavailable-type-invariance, individual-rationality, weak non-wastefulness, truncation-invariance, strategy-proofness and either population-monotonicity or resource-monotonicity. It would be an interesting line of future research to investigate whether it is possible to endogenize the priority structure of DA^* with a set of axioms. The key difficulty is that axioms such as population-monotonicity or resource-monotonicity impose constraints when one adds only one additional agent or one additional house. In our reallocation setting, agents are all initially assigned to a house so that both a house and an agent would be added to the market, making comparative static results difficult. Moreover, non-wastefulness has no applicability in our setting since all of houses are initially assigned and each reassignment of resources is non-wasteful by definition.

References

Abdulkadiroglu, A., and T. Sonmez (1999): "House Allocation with Existing Tenants," *Journal of Economic Theory*, 88, 233–260.

——— (2003): "School Choice: A Mechanism Design Approach," *American Economic Review*, 93, 729–747.

Alcalde, J., and S. Barberà (1994): "Top dominance and the possibility of strategy-proof stable solutions to matching problems," *Economic theory*, 4(3), 417–435.

Arrow, K. J. (2012): *Social choice and individual values*, vol. 12. Yale university press.

Combe, J., O. Tercieux, and C. Terrier (2016): "The Design of Teacher Assignment: Theory and Evidence," Discussion paper, mimeo.

Compte, O., and P. Jehiel (2008): "Voluntary Participation and Re-assignment in Two-sided Matching," Paris School of Economics, Unpublished mimeo.

Dur, U. M., and M. U. Ünver (2019): "Two-sided matching via balanced exchange," *Journal of Political Economy*, 127(3), 1156–1177.

Ehlers, L., and B. Klaus (2016): "Object allocation via deferred-acceptance: Strategy-proofness and comparative statics," *Games and Economic Behavior*, 97, 128–146.

Ergin, H. I. (2002): "Efficient Resource Allocation on the Basis of Priorities," *Econometrica*, 70, 2489–2497.

Gale, D., and L. S. Shapley (1962): "College Admissions and the Stability of Marriage," *American Mathematical Monthly*, 69, 9–15.

Guillen, P., and O. Kesten (2012): "Matching Markets with Mixed Ownership: The Case for a Real-life Assignment Mechanism," *International Economic Review*, 53(3), 1027–1046.

Kesten, O. (2010): "School choice with consent," *The Quarterly Journal of Economics*, 125(3), 1297–1348.

Kojima, F., and M. Manea (2010): "Incentives in the Probabilistic Serial Mechanism," *Journal of Economic Theory*, 145, 106–123.

Ma, J. (1995): "Stable matchings and rematching-proof equilibria in a two-sided matching market," *Journal of Economic Theory*, 66, 352–369.

McVitie, D. G., and L. Wilson (1970): "Stable marriage assignments for unequal sets," *BIT*, 10, 295–309.

Pereyra, J. S. (2013): "A Dynamic School Choice Model," *Games and economic behavior*, 80, 100–114.

Shapley, L., and H. Scarf (1974): "On Cores and Indivisibility," *Journal of Mathematical Economics*, 1, 22–37.

Sönmez, T., and M. U. Ünver (2005): "House allocation with existing tenants: an equivalence," *Games and Economic Behavior*, 52(1), 153–185.

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