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Abstract

The analysis of second price auctions with externalities is utterly modified if the seller is unable to commit not to participate in the mechanism. For the General Symmetric Model (Milgrom and Weber [20]) and standard auction formats, we charaterize the full set of symmetric pure strategy equilibria with shill bids from the seller (using possibly mixed strategy) and compare those equilibria to the optimal one with the ability to commit not to use shill bids. The benefits from the highlighted 'Linkage Principle' are counterbalanced by the 'Shill Bidding Effect' which grows with the winner's curse.

Keywords: Auctions, externalities, linkage principle, shill bidding
JEL classification: D44, D45, D62

Résumé

L'analyse des enchères au second prix avec des externalitiés est considérablement modifiée si le vendeur n'est plus capable de s'engager à ne pas participer à l'enchère. Pour le modèle développé par Milgrom et Weber [20] et pour les formats d'enchères standards, l'ensemble des équilibres symétriques en stratégie pure pour les acheteurs est complètement caractérisé. En revanche, le vendeur participe à l'enchère avec une stratégie mixte. L'ensemble de ces équilibres est comparé avec l'équilibre optimal lorsque le vendeur peut s'engager. Les bénéfices du fameux 'Linkage Principle' sont contrebalancés par ce que nous appelons le 'Shill Bidding Effect', force qui croît avec la malédiction du vainqueur.

Mots-Clés: Auctions, externalities, linkage principle, shill bidding
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1 Introduction

In their General Symmetric Model where private signals are positively correlated through affiliation and where a single item is auctioned, Milgrom and Weber [20] (hereafter MW) derived the so-called 'Linkage Principle', one of the most influential results of the auction literature. A first aspect of this principle is the benefit for the seller ex ante to commit to a policy of publicly revealing her signal. A second aspect is that, due to their relative ability to convey information, the english auction[1] raises a higher revenue than the second price auction which outperforms the first price auction. Ausubel [2] extends MW's results in a multi-unit framework with flat multi-unit demands[2]: Ausubel's dynamic auction for homogenous object outperforms the (static) Vickrey auction.

However, the Linkage Principle is based on an assumption which goes without saying in the auction and more generally mechanism design literature: the seller (or the designer) is able to commit not to participate secretly, under a false name bid for example, in the mechanism. This assumption may be totally unrealistic, especially in online electronic auctions as emphasized by Dobrzynski [6], even if shill bidding is prohibited as on eBay[3]. Nevertheless shill bidding is a pervasive phenomenon in such auctions and is very difficult to detect in practice. How is it possible to prevent the formation of rings of sellers which have no formal acquaintance and whose objective is to shill bid under each other sales? Dobrzynski relates the story of a seller of a painting on eBay who used shill bids under numerous identities and also belonged to such a ring of sellers in order to make potential real buyers believe that it was a masterpiece. Although it was a daub, attempting to copy the style of some Diebenkorn's masterpieces, the seller sold it for over 135,000 $ without pretending any certification. It is clear that the 'real' potential buyers were unaware of the extend of the shill bidding activity which involved about ten different identities who were supposed to be art experts by eBay's reputation mechanism.

The aim of the present paper is to circumvent the degree of validity of the Linkage Principle in the light of the ability for the auctioneer to commit not to participate in the mechanism. The main technical step of the analysis consists in deriving the set of symmetric equilibria in MW's framework when the commitment ability of the seller not to participate in the mechanism is relaxed. The main focus is on the second price auction such

[1]More precisely the english button auction introduced by MW as a model of the traditional english open auction used in auction rooms but which could be a poor description of real-life auctions if no activity rules are present.

[2]Perry and Reny [22] display an example where the first aspect of the principle fails in a multi-unit auction without flat demand.

[3]Family members, roomates and employees of the seller are enclosed in this prohibition (for more details see http://pages.ebay.com/help/policies/seller-shill-bidding.html).

that this is the format considered if not explicitly mentioned. In general, the characterization of an equilibrium of such a Bayesian game between the seller and the buyers is not tractable: that's why we restrict our analysis to an uninformed seller and we also add a suitable (and natural) quasi-concavity assumption. The basic intuition why the second price auction with shill bids raises a smaller revenue than with the ability to commit not to use shill bids follows the same logic as the Linkage Principle: due to the shill bidding activity, the informational linkage between the price paid and the valuation of the item is reduced. Without shill bids, the price paid (given that it is higher than the reserve price) gives the highest signals of his opponents. On the other hand, with shill bidding, the price paid is a 'blurred' signal: it could either reflect the highest signal of his opponents or the shill bid activity of the seller.

A crucial step in the analysis is the no-gap lemma which states that the lowest shill bid and the lowest possible bid of an active bidder must coincide. This lemma should be compared with the opposite property which characterizes second price auctions with informational externalities and no shill bidding activity. MW derive the equilibrium in the general symmetric model with affiliation: with a binding reserve price r, they note that in the second price auction *at equilibrium there will be no bids in a neighborhood of r.* As a corollary, if such a gap exists, the seller would strictly raise her revenue if she could be able to raise secretly a shill bid above r and below the lowest equilibrium bid of an active bidder: it never changes the allocation and strictly raises the price in the event where only one bidder is participating. This incentive to raise secretly the effective reserve price with a shill bid suggests that shill bidding will reduce the level of trade. On the other hand, if the seller can commit to the announced reserve and not to use shill bids, then she can commit to any level of trade (and so to the one that maximizes her revenue). The equilibria derived in the previous literature without shill bids are not candidate equilibrium of the modified auction with the seller's shill bidding activity: the seller must use a mixed shill bid strategy at equilibrium. Moreover, we derive the optimal equilibrium with shill bids and, as the previous intuition suggests, it effectively involves a lower level of trade than the optimal equilibrium with an announced reserve price.

On the other hand, the first price auction is immune to shill bidding. Then we obtain what we call the 'shill bidding effect' a countervailing force to the Linkage Principle in favour of first price auctions. In general the revenue comparison between the optimal first and second price auctions is undetermined. But if signals are not correlated, then only the 'shill bidding effect' matters: the first price auction with an optimal reserve price unambiguously outperforms the optimal equilibrium of the second price auction. We also derive a result that sheds some light on what drives this shill bidding effect: after having defined a partial order to capture the strength of the winner's curse, we show that the difference in revenue between the optimal second price auction with and without commitment, raises with the winner's curse.

This paper is organized as follows: Section 2 briefly reviews the literature related to shill bidding and commitment failure in mechanism design. In Section 3 we introduce the model and the notations. Section 4 briefly recalls the equilibrium derivation with the commitment ability and states its superiority relative to equilibria with a shill bidding activity. Section 5, the core of the paper, derives the whole set of equilibria of the second price auction where the seller can use shill bids in mixed strategy. Revenue comparison between first and second price auctions and a comparative statics result are presented in section 6. Extensions are gathered in Section 7: allocative externalities, a binding reservation value for the seller, entry fees, the english button auction and endogenous entry are considered.

2 A review of the literature

There are some previous works on shill bidding[4] in the auction literature, and more specifically on the profitability of a shill bidding activity from the seller. The first contributions consider a framework such as the english auction in the pure private value model where shill bidding is an opportunity for the seller to fix a reserve price depending on the whole history of the auction (e.g. on the identity and the time where potential buyers exit the auction). In Graham et al [8], the seller uses phantom bids against heterogenous bidders in a standard english auction in order to fix a non-constant reserve price which depends on the whole bid sequence. In MW's model, Lopomo [17] establishes that the english button-auction is optimal among a class of robust mechanisms: more precisely, "his" english auction involves the non-anonymous participation of the auctioneer which sets an optimal reserve price after having observed $n-1$ buyers' exits. Unlike Graham et al [8]'s private value model, this optimal history dependent reserve price policy in the english auction which depends on the entire set of exits is not implementable with a stricto sensu shill bidding activity. Shill bidding is anonymous whereas in Lopomo [17], the auctioneer's activity is transparent such that he could not fool the market. Recently, in a similar vein, Izmalkov [11] considers shill bidding from the seller as a way to implement the optimal mechanism of Myerson [21] with a standard english auction. In those papers shill bidding is not used to distort the perception of the value of the item as

[4] Closely related is the possibility for the seller to cancel the final allocation after all bids have been raised and both the winning bidder and the winning price have been set. This has been studied by Horstmann and LaCasse [10] under the terminology 'secret reserve price', whereas some works, as Vincent [24], uses this terminology for what we call shill bids. Contrary to 'secret reserve prices', shill bids is a way for the seller to manipulate directly the winning price.

in the recent eBay auction reported by [6].We are aware of only two papers that focus on this last aspect of shill bidding[5]: Vincent [24] and Chakraborty and Kosmopoulou [4].

Vincent [24] is the first attempt that formalizes that in a common value framework where the seller sets a random reservation price the expected value for a winner is the weighted sum of the expected value conditional on the second highest bid being another bidder and the expected value conditional on the second highest bid being the seller. This is the non-equality between those two expected values that creates the aforementioned gap (together joined with the ability to commit not to use shill bids) between the reserve price and the lowest possible active bid. Vincent[6] misses the issue that, in the event where the seller has a reservation value which is common knowledge, the standard equilibrium of the second price auction with the optimal reserve price is not implementable if the seller has the ability to submit strategically a secret reserve price.

Chakraborty and Kosmopoulou [4] analyse a very simplified framework: the english auction in a pure common value framework with two signals. On the other hand, there is a degree of generality that is absent here: they introduce a parameter $\overline{\rho}$ which corresponds to the probability[7] that the seller has the ability to submit a shill bid and their results mostly consist in deriving comparative statics on the surplus for the bidders, the seller and the auctioneer relative to $\overline{\rho}$. Our analysis only considers the two extreme cases $\overline{\rho} = 0$ or $\overline{\rho} = 1$ and also the possibility to submit numerous shill bids as suggested by [6].

On a more theoretical perspective, shill bidding has not been fully analyzed in the mechanism design literature. The first contribution on shill bidding is Yokoo et al [25]: they do not consider shill bidding from the mechanism designer's point of view but from the bidders who could use false-name bids by using multiple identifiers. They establish a sufficient condition on bidders' preferences to make the Vickrey-Clarke-Groves mechanism robust to shill bidding. Following [25] in multi-unit auctions, Ausubel [2] and Ausubel and Milgrom [3] investigate manipulations with multiple bidding identities. Nevertheless, though [2] contains an interdependent value model, the possi-

[5]Horstmann and LaCasse [10] show how the seller can communicate his private information with a secret reserve price. This signaling effect is specific to a dynamic framework where a seller with a good signal prefers to reauction the item later insofar the bidders will refine their assessment of their valuation from their new signals and the information contained in the fact that the seller has preferred to cancel the first auction's allocation.

[6]Vincent writes 'If the seller's use value s were common knowledge, then whether or not a reserve price was announced would make no difference in an auction - bidders would simply compute the seller's optimal [reserve price] and behave as if it were announced', [24] p 579

[7]It would have been perhaps more interesting to introduce a parameter that reflects the cost of shill bidding, which would have enabled the seller to use more than one shill bid.

bility to exit the auction very early under a false identifier to manipulate the bidders estimation of the items is absent.

Note that experimental works on auctions, as Kagel and Levin [14] who question the practical relevance of the linkage principle, omit the shill bidding issue from the seller as well as from the buyers. Katkar and Riley [15] have run on eBay a field experiment to test the benefit to use secret reserve price for Pokémon cards. An eBay's secret reserve price is equivalent to a shill bid if eBay's standard auction fits the second price auction model.[8] [15] show that for two identical items, it raises more revenue to impose a minimum bid (i.e. an announced reserve price) than to set an equivalent secret reserve price -a result that is consistent with our results.

Finally, this paper is also related to a growing strand of the mechanism design literature which relaxes the commitment ability of the designer. In McAfee and Vincent [18], the designer cannot commit never to attempt to resell the good if he fails to sell it. Zheng [26] analyzes a complementarity commitment failure: the designer cannot ban resale market. Vartiainen [23] considers auction design when parties cannot commit to any action in the mechanism.

3 The Model

We consider the General Symmetric Model introduced by MW, i.e. an auction in which n bidders compete for the possession of a single object and for which each bidder receives a uni-dimensional signal X_i such that $X_1, \cdots, X_n$ are affiliated.[9] Our subsequent notations match MW's. The actual value of the object for bidder i depends not solely on his own signal X_i but also on the entire vector of signal X: this value is denoted $V_i = u_i(X)$. The valuation function u_i is non negative (if a bidder does not acquire the object, his payoff is normalized to zero) and strictly non decreasing in all variables. Hence we will restrict ourselves to a non pure private value

[8] On the other hand, if the suitable model were the english auction, then a secret reserve price would be different than both [10]'s definition and what we calls shill bidding in the English auction. Indeed, a field experiment with shill bids would not be possible due to eBay's terms-of-service agreement.

[9] Our model is slightly simpler than MW's: we do not consider that the seller receives a signal that is affiliated with the bidders signals. An informed seller modifies considerably the analysis, e.g. the equilibrium concept should rely on Bayesian updating after that the seller announces the chosen mechanism (even if he can commit ex ante to some standard mechanism, e.g. a first price auction, he may not be able to commit to a given reserve price). If the private information of the seller is verifiable, then the first part of the so-called linkage principle suggests that the seller has interest to commit ex-ante to reveal this information. On the other hand, if the seller's information is both relevant to the buyers and the seller's reservation value and also not verifiable, then this corresponds to a lemon problem which lies outside the scope of our analysis. Jullien and Mariotti [13] have analysed such a signalling game, which we discuss later in section 7.6.

framework.[10] The model is symmetric: signals are distributed symmetrically (i.e. the related density function is exchangeable) and bidders' valuation is a symmetric function of the other bidders' signals. Let us define the function $v : \mathbb{R}^2 \to \mathbb{R}$ (respectively $w : \mathbb{R}^2 \to \mathbb{R}$) by $v(x,y) = E[V_1|X_1 = x, Y_1 = y]$ (respectively $w(x,y) = E[V_1|X_1 = x, Y_1 \leq y]$), where Y_1 denotes the first order statistic of the signals received by bidder 1's opponents. Due to the strict monotonicity of u_i, v and w are strictly increasing in both arguments. With an abuse of notation, we note $v(x) := v(x,x)$ and $w(x) := w(x,x)$. Thus $v(x) > w(x)$ for $x > \underline{x}$. Most of the analysis considers that the object is valueless for the seller[11] except proposition [4.2] which considers a framework, as in [24]'s note, where the seller is privately informed about his reservation value. Denote by $f^{(k:n)}$ the density of the k^{th} order statistic of $X_1, \cdots, X_n$ ($F^{(k:n)}$ the corresponding cumulative distribution function (CDF)) and $f^{(k:n)}_{-i,\theta}$ the density function of the k^{th} order statistic of $X_1, \cdots, X_{i-1}, X_{i+1}, \cdots, X_n$ conditional on $X_i = \theta$ ($F^{(k:n)}_{-i,\theta}$ the corresponding CDF). Due to symmetry, the index $-i$ is dropped in the following analysis. The lower bound (respectively upper bound) of the signals isdenoted by $\underline{x}$ (respectively $\overline{x}$).

Departing from MW, we consider that the seller cannot commit not to use shill bids in the auction mechanism. Then the timing of the game is as follows:

- Step 0: each agent is privately informed about his signal.
- Step 1: the seller announces an auction mechanism which possibly contains a reserve price.
- Step 2: every agent, i.e. the bidders and possibly the seller, are free to submit any bids as they want.[12]
- Step 3: the object is allocated according to the auction mechanism, resale is banned and the seller cannot re-auction it.

[10] In the symmetric pure private value framework, the 'standard' equilibrium of the optimal auction, i.e. the second price with an optimal reserve price (Myerson [21]), remains an equilibrium with shill bidding. There is no gap and thus the inability to commit not to use shill bids is not an issue from an optimal design perspective. Nevertheless, shill bidding is still an issue in the pure private value framework insofar as the seller can not commit to a reserve price that is lower than the optimal reserve price. With endogenous entry, it could lower the revenue (see section 7).

[11] The analysis extends (as done in section 7) easily if the seller has a binding reservation value that is common knowledge: then the level of the seller's shill bid conveys no information on the value of the object for the bidders, a point which makes the analysis tractable.

[12] Except for the English auction to submit more than two bids is never strictly better than the best response with only one bid. More generally, it is a departure with the mechanism design literature which considers non-anonymous mechanisms where bidders can be identified and where consequently shill bidding is not an issue if the seller is able to commit to a given mechanism.

As been emphasized in the introduction, the timing is different than [24]'s: the seller chooses a mechanism after being informed. Our analysis considers mostly the second price auction with a reserve price. The analysis fits also with the english auction as emphasized in section 7. On the other hand, the analysis of the first price auction is not modified when the seller's reservation value is null and this format is especially considered in the specific case where signals are drawn independently. In the event, the first price and the second price auctions are equivalent under commitment.

In MW's analysis with commitment not to participate in the mechanism, the equilibrium bidding of such auctions ischaracterized by a function, denoted by $b(\cdot)$, mapping a buyer's own signal X_i into a bid $b_i = b(X_i)$. Hereafter, MW's equilibrium will be referred to as the symmetric equilibrium in pure strategy with commitment or more briefly as equilibrium without shill bids or equilibrium with commitment. On the other hand, if the seller cannot commit not to use shill bids, the auction is a Bayesian game between the seller and the buyers. The bidding strategy of the seller must be added in the equilibrium concept.

Definition 1 *In the first and second price auctions with shill bids, a strategy profile is a couple (b, G) where $b : \mathbb{R} \to \mathbb{R}^+$ is the function mapping a signal into a positive bid (the null bid is equivalent to non participation) and G represents the CDF according to which the seller sets her reserve price (possibly using a shill bid). A symmetric equilibrium in pure strategy with shill bids*[13] *is then a strategy profile (b, G) such that bidders and the seller follow their best response strategy.*

Throughout this work, we restrict ourselves to symmetric equilibria insofar bidders use the same strategy. If the randomization of the seller contains no atom then we denote by g the corresponding density. Both functions G and g will abusively be qualified as either the reserve price or shill bids strategy. Denote by $\underline{r}$ (respectively $\overline{r}$) the lowest (highest) possible shill bid, i.e. $\underline{r} = \max[x|G(x) = 0]$ ($\overline{r} = \min[x|G(x) = 1]$). Bids strictly below $\underline{r}$ are inactive insofar the probability to win is nul. Buyers who bid above this cut off point are called active buyers.

Note that the preceding equilibrium concept is not suitable for the english (button) auction where a bid strategy is not only a function of the bidder's signals but also of the whole history, i.e. the number of bidders who have quit and the levels at which they quit. More importantly, in the english auction, a bidder may find profitable to use shill bids whereas the second and first price auctions are immune to shill bids from the bidders. The examination

[13] Also shortly referred to as equilibrium with shill bids or equilibrium without commitment. Note that this definition applies from step 2 of the game and thus not to the whole game including the choice of the seller in step 1.

of the english auction is reported to Section 6. If not specified, the auction mechanism refers now to the second price auction.

4 Equilibria with commitment not to use shill bids

In this section, we recall the results without shill bids. The equilibrium of the second price auction without shill bidding is characterized by a gap between the reserve price and the lowest equilibrium bid: at equilibrium there will be no bids in a neighborhood of r [the reserve price] as originally noted by MW. More generally (e.g. also for the first price auction), for any bidder i and conditional on any signal X_i, the function mapping the expected value of the object to the highest opposing bid (the reserve price being included) is discontinuous at r. This distinctive feature of the equilibrium have been recently mentioned in the empirical auction literature as a way to test common value model against private value model when the reserve price is binding. Hendricks, Pinkse and Porter [9] and Athey and Haile [1] have considered this idea although no formal test has been yet developed.

Proposition 4.1 (Milgrom, Weber) *The equilibrium with commitment of the second price auction with a reserve price r is characterized by a threshold θ^*, such that $r = w(\theta^*)$, below which buyers do not participate (or equivalently raises a bid below the reserve price) and above which the equilibrium bid is given by:*

$$b_{SP}(x) = v(x), \quad if \quad x \geq \theta^*. \tag{1}$$

The threshold is qualified as the level of trade. Figure [1] illustrates the gap between the reserve price and the lowest bid of a participating bidder: with strict informational externalities and for $\theta^* > \underline{x}$, we have $r = w(\theta^*) < v(\theta^*) = b_{SP}(\theta^*)$. This gap implies that the seller would find strictly profitable to raise secretly the reserve price up to $v(\theta^*)$. If $v(\theta^*)$ is strictly inferior to the optimal reserve price of the equilibrium with commitment, then she would profitable to raise a secret reserve even strictly greater than $v(\theta^*)$. Then the equilibrium analysis with commitment is no more valid and the intuition is that the seller will not be able to set the optimal reserve price but rather that the equilibrium reserve prices will be too high at equilibrium. On the other hand, in the first price auction, when the object is valueless for the seller, raising the reserve price with a shill bid is always strictly dominated because it unambiguously raises a lower revenue. This remark implies that the seller is able to implement the optimal equilibrium of the first price auction with commitment without the ability to commit not to use shill bids. It is no longer true for the second price auction. Consequently, the ranking between those two formats that has been established by MW may be reversed without commitment.

Before the analysis of equilibria without commitment and anticipating the 'no-gap' lemma, the next result states that any (suitable) equilibrium without commitment is dominated by an equilibrium with commitment where a pure reserve price strategy is used. It illustrates a standard intuition: an agent is better off if he is able to commit. Indeed this result is slightly more general than our model and remains true under an informed seller with a reservation value. The proposition formalizes in a broader framework the gain of commitment. For this proposition (and only this), the seller may be privately informed by a multi-dimensional signal S and possibly have a reservation value which depends on his information but also on the whole set of signals, $u_0(S, X_1, \cdots, X_n)$. Denote by b_0 the seller shill bid.

Proposition 4.2 *Consider an equilibrium of the second price auction with shill bids where potential buyers bid according to a strictly increasing mapping of their signal, $b(\cdot)$, and where the strategy of the seller is such that*[14]

$$E[V_1|X_1 = x, b_0 = b(x), Y_1 < x] < v(x), for\ any\ x \tag{2}$$

then this equilibrium is outperformed by an equilibrium without shill bids and a pure reserve price policy for some seller's types.[15]

Proof 4.1 *We show that the seller's type which uses the lowest reserve price $\underline{r}$ (with the shill bid activity) will be better off by committing not to use shill bids. Anticipating from the first 'no-gap' lemma (5.1), which applies in a very general framework, we have $\underline{r} = w(x^*)$, where x^* denotes the cut off signals such that buyers below x^* are not participating. At equilibrium without commitment, the revenue of the seller corresponds to an auction where bidders below signal x are not participating whereas bidders above bids according to b such that $b(x)$ is a weighted sum of $E[V_1|X_1 = x, b_0 = b(x), Y_1 < x]$, the expected value of the item conditional on winning after a tie with the seller and $v(x)$, the expected value of the item conditional on winning after a tie with another bidder. Due to our assumption in equation (2), this weighted sum is below $v(x)$. Therefore the seller could have raised a higher revenue by committing to the reserve price $w(x^*)$, which would have raised the same participant but with the commitment pushing them to bid more strictly aggressively, which strictly raises revenue in the event occurring with positive probability where there are two active bidders above $\underline{r}$.*

[14]The condition means that a tie with the seller is a bad news compared with a tie with another buyers.

[15]In a framework where the seller is uniformed, with a mixed reserve price policy, the result would have been immediate insofar as such any equilibrium without commitment can be duplicated with the corresponding reserve price policy. Such an argument is the standard manner in mechanism design to prove that commitment makes the seller better off.

In a framework where the seller's signals affect only his reservation value and not the valuation of the buyers (and where his signal is not correlated with the other buyer's signals), then the level of the shill bid conveys no information for the buyers and equation (2) is satisfied. We obtain the following corollary which includes [24]'s framework.

Corollary 4.3 *If the seller shill bidding strategy is independent of $X_1, \cdots, X_n$, which is satisfied if S is drawn independently of $X_1, \cdots, X_n$, then equilibria of the second price auction without commitment are outperformed by some equilibria with commitment and a pure reserve price strategy for some seller's types.*

This point may seem inconsistent with Vincent [24]'s note which exhibits a numeric example in a common value second price auction where the seller's ex ante expected revenue without commitment outperforms its revenue with the optimal second price auction where a pure reserve price policy is announced. This example relies on the fact that the seller chooses to commit after being privately informed. The timing of our model is slightly different: first the seller is privately informed, second he chooses to commit to be or not to be able to use shill bids.

Relative to the General Symmetric Model, we make an additional assumption that is not crucial but allows us simple characterization of the whole set of equilibria. Monotonicity assumptions on hazard rates are standard in the mechanism design literature to derive optimal mechanism. Here the assumption is not exclusively on the distribution of the signals but depends also on w the way the object is valued. But since w is defined up to a normalization, this assumption can be viewed as only depending on the distribution of signals. We assume that a given map, which corresponds to a 'quasi-revenue' as the discussion below emphasizes, is unimodal.

Assumption 4.1 (Unimodality of the quasi-revenue) *The map $x \to (F^{(2:n)}(x) - F^{(1:n)}(x)) \cdot w(x) + \int_x^\infty w(u) f^{(2:n)}(u)du$ is a unimodal (or quasi-concave) function.*

Denote by x^* the associated mode and $r^* = w(x^*)$ the corresponding reserve price.

Suppose that the potential buyers bid according to $u \to w(u)$, then the map corresponds to the revenue of the seller as a function of the level of trade x, i.e. equivalently the reserve price $w(x)$. The first term corresponds to the event where the second highest bid is the reserve price (w(x)) whereas the second term corresponds to the event where the second highest bid is a bid from a real buyer (bidding $w(u)$, if u is his private signal). To this extend, we qualify this map as the quasi-revenue. Then assumption 4.1 could be interpreted as follows: the quasi-revenue a unimodal function of the reserve price. In a pure private value framework where $w = v$, then the preceding assumption is equivalent with assuming that the expected revenue with commitment is an unimodal function of the reserve price. A marginal increase of the reserve price has two effect: first it reduces trade which reduce revenue and second it increase the revenue in the event when there is only one active bidder. The unimodality assumption, which is satisfied in all standard examples, states that the second marginal effect is dominant below r^* whereas the first one is dominant above r^*.

The following innocuous assumption states that the mode, x^*, of the quasi-revenue must be strictly higher than the lowest possible signal $\underline{x}$. This assumption is satisfied if for example the optimal equilibrium without shill bidding involves some restriction on trade. It aims is to avoid the case where the optima with and without shill bidding involves no binding reserve price and are thus equivalent.

Assumption 4.2 *The optimum of the quasi-revenue involves a restriction of the level of trade: $x^* > \underline{x}$.*

With those appropriate assumptions, r^* will have the nice property to separate equilibrium reserve price of the second price auction with and without commitment : with commitment, the optimal reserve price will be less than r^*, whereas without commitment in the second price auction the seller will submit shill bids that are higher than r^* with probability one. The following proposition concerns the first half of this statement.

Proposition 4.4 *The optimal reserve price policy of the second price auction with commitment is to use a reserve price denoted by r^{**}_{SP}, such that*[16]*:*

$$r^{**}_{SP} < r^*. \tag{3}$$

Denote by $x^{**} < x^*$ the corresponding cut off point for buyers' signals, such that $r^{**}_{SP} = w(x^{**})$.

Proof 4.2 *Note that the expression of the revenue as a function of $x = w^{-1}(r)$ the cut off point x corresponding to the reserve price r is $x \to (F^{(2:n)}(x) - F^{(1:n)}(x)) \cdot w(x) + \int_x^\infty b(u) f^{(2:n)}(u)du$, where $b(u) = v(u)$. Thus it equals to the quasi-revenue plus a strictly decreasing function because $b(u) > w(u)$. We conclude with assumption (4.1).*

The second half of the statement is proved by deriving the whole set of equilibria with shill bids.

[16] If signals are statistically independent then first and second price auctions are revenue-equivalent and $r^{**}_{SP} = r^{**}_{FP}$. With an additional unimodality assumption similar to (4.1), we obtain easily that $r^{**}_{SP} < r^{**}_{FP}$. The optimal second price auction raises a higher revenue and induces a higher participation than its first price counterpart.

5 Equilibria with Shill Bids

In this section the analysis focus on the second price auction with shill bids where the announced reserve price is initially set to zero in the first stage of the game.[17] We characterize the whole set of equilibria in proposition 5.2. This result leads us to give a key property of the equilibria: only shill bids above the mode of the quasi-revenue are sustainable at equilibrium. It means that the shill bidding activity reduces the level of trade. Moreover, we obtain that the union over possible equilibria of the support of the the shill bidding activity equals to $[r^*, \infty[$.

Proposition 5.1 *At any equilibrium with shill bids, the lowest possible shill bids is higher than the mode of the quasi-revenue:*

$$\underline{r} \geq r^*$$

Reciprocally, any bid above r^* is a possible shill bid of an equilibrium with shill bids.

This result is a straightforward corollary of proposition 5.2. The characterization of the set of equilibria proceeds in four lemmata which gives necessary conditions on candidate equilibria. The no-gap lemma establishes an initial condition. Added to the suitable differential equations resulting from the profit-maximizing behavior of buyers and the seller, it implies that, generically, an equilibrium is uniquely characterized by its lowest possible shill bid, i.e. $\underline{r}$. The fourth lemma states that only equilibria with $\underline{r} \geq r^*$ are possible whereas proposition 5.2 concludes by verifying that the remaining candidate equilibria are suitable equilibria.

The preceding section has established that no reserve price in pure strategy is sustainable by an equilibrium with shill bids and where the seller does not use shill bids, except the symmetric equilibrium where the seller submits a shill bid superior to $w(\overline{x})$ and where the object remains in the seller's hand with probability one. The following lemma states that with shill bids the equilibrium must have no gap between the lowest possible shill bid of the seller and the lowest possible bid of active bidders. Denote by $\underline{r}^*$, the cut off point for participation, i.e. $\underline{r}^*$ is defined such that $\underline{r} = w(\underline{r}^*)$.

Lemma 5.1 (The No-gap Lemma) *If the equilibrium contains some trade, i.e. $\underline{r}^* < \overline{x}$, then the equilibrium bid of a buyer with type $\underline{r}^*$ equals to the lowest possible shill bid $\underline{r}$: $b(\underline{r}^*) = w(\underline{r}^*)$.*

[17] The results immediately extend to the general case where the auction contains an initial announced reserve price. In this case, we should possibly truncate the set of equilibria that we derived such that the lowest possible shill bid must be greater than the announced reserve price.

Proof 5.1 *Suppose on the contrary that $b(\underline{r}^*) > \underline{r}$, then the seller can raise its revenue, provided that $\underline{r}^* < \overline{x}$, by secretly raising strictly its reserve price and staying below $b(\underline{r}^*)$. It does not change the probability of selling the object whereas it strictly raises its price in the case where the shill bid corresponds to the second order statistic of bids, an event which occurs with a strictly positive probability.*

In this very strong form, the no-gap lemma relies strictly on the assumption that the shill bidding activity involves no cost. It is no longer the case if there are, as in eBay, positive final value fees when the item is sold whereas no fee is charged when the item is not sold. This point is discussed in section 7 and only weaker versions of the no-gap lemma will emerge. In particular, there are some equilibria with trade where the seller commit to a reserve price and does not find profitable to raise shill bids. Nevertheless, such fees do not alter the general insight of the paper which is the inability to use low reserve prices and not to raise shill bids without commitment.

The second no-gap lemma states that $b(\cdot)$ must be continuous above $\underline{r}$.

Lemma 5.2 (The Second No-gap Lemma) *The map $b(\cdot)$ is continuous on $[G^{-1}(\underline{r}), \overline{x}]$*

Proof 5.2 *Suppose on the contrary that there is a point x such that $b(x^-) < b(x^+)$ where $b(x^-)$ (respectively $b(x^+)$) denotes the left (right) limit at x which are well defined since $b(\cdot)$ is monotone. Two events may happen depending on the fact that shill bids may occur in the left neighborhood of x. First, no shill bids occurs in the left neighborhood of x, then locally we have $b(x) = v(x)$ and $b(x^-) = v(x)$, moreover, as will be proved independently in lemma 5.3, $b(x^+) \in [w(x), v(x)]$. Then $b(x^-) = b(x^+)$ since $b(\cdot)$ is monotone and this raises a contradiction. Second, shill bids are used in the left neighborhood. Then, similarly to the proof of the first 'no-gap' lemma, a reserve price of $b(x^+)$ raises unambiguously a strictly higher revenue than a reserve price about $b(x^-)$, which raises a contradiction with the seller using a best response strategy. Thus we have proved the second no-gap lemma.*

Since $b(\cdot)$ is monotone, by Lebesgue's Theorem, it is differentiable almost everywhere. Indeed, anticipating on the analysis of the seller's optimization program, $b(\cdot)$ is a solution almost everywhere of equation (8) and the initial condition $b(\underline{r}^*) = w(\underline{r}^*)$, then is uniquely characterized and differentiable on the whole range of active bidders and also strictly increasing. Denote by b' the first derivative of b.

In the last section, we have proved that any pure reserve price strategy below $w(\overline{x})$ can not be sustained by an equilibrium without commitment. More generally, we can prove that any reserve price strategy with an atom (in the range $]w(\underline{x}), w(\overline{x})[$) cannot be part of an equilibrium without commitment because it will induce a discontinuity in the bid function. Such a discontinuity is not possible as the second no-gap lemma have established. From now on, we consider that G contains no atom.[18]

Our aim is now to characterize the set of equilibria. In particular, we derive the set of possible reserve price strategy, which support equals to $[r^*, \infty]$.

From standard arguments in auction theory, we know than the equilibrium function b is necessary non-decreasing. Similarly, it can be shown here that no equilibria with pooling exists. Thus without loss of generality, we can look for strictly increasing solutions. Assuming that its opponents bid according to a (common) strategy $\beta(\cdot)$ which is a monotonically strictly increasing and differentiable function of its type and that the seller shill bids according to g, buyer's 1 maximization problem given that he has type x is:

$$\max_{b\geq 0}\left(\int_{\underline{r}}^{b}\left[\left(w(x,\beta^{-1}(u))-u\right)\cdot F_x^{(1:n-1)}(\beta^{-1}(u))+\int_{\beta^{-1}(u)}^{\beta^{-1}(b)}\left(v(x,s)-\beta(s)\right)f_x^{(1:n-1)}(s)ds\right]g(u)du\right) \tag{4}$$

The first term in the integral corresponds to the event where the highest competing bid is from the seller whereas the the second term represents the payoff when the highest competing bid is from a 'real' buyer.

Let $g^* = g \circ b$, G^* its corresponding CDF and $[\underline{r}^*, \overline{r}^*]$ the corresponding convex hull of its support. The seller's randomization over bids corresponds to a randomization over types and then to convert the type into a shill bid according to a bid equilibrium b: the density g^* corresponds exactly to this density of types. Then, the first order condition implies:

$$b(x) = \alpha(x)\cdot w(x) + (1-\alpha(x))\cdot v(x) \tag{5}$$

where

$$\alpha(x) = \frac{g^*(x)F_x^{(1:n-1)}(x)}{g^*(x)F_x^{(1:n-1)}(x) + G^*(x)f_x^{(1:n-1)}(x)} \tag{6}$$

$b(x)$ is a weighted sum of $w(x)$ and $v(x)$. If bidder 1's type is x and the maximum opposing bid is also $b(x)$, then the expected value for the object for bidder 1 is the sum of $w(x)$ weighted with the probability that the highest bid is a shill bid of the seller and $v(x)$ weighted with the probability that the highest bid is from one of his opponents. If $x > b^{-1}(\overline{r}^*)$, the probability to compete with the seller is nul and this expression reduces to the standard expression $b(x) = v(x)$.

Remark 5.1 *If G is atomless then, if $\overline{r} < \overline{x}$, we have the condition $b(x) \to_{x\to\overline{r}, x<\overline{r}} v(\overline{r})$ which implies that $\alpha(x) \longrightarrow_{x\to\overline{r}} 0$. As a consequence, $g(x) \longrightarrow_{x\to\overline{r}} 0$.*

[18] Equilibria with an atom at $w(\underline{x})$ have not yet been ruled out. But from lemma 5.4, such equilibria are ruled out if $r^* > w(\underline{x})$ as assumed by assumption 4.2

$\alpha(x) \in [0,1]$, then the analysis above implies the following lemma.

Lemma 5.3 *$w(x) \leq b(x) \leq v(x)$, if $x \geq \underline{r}$. As a corollary, $b'(\underline{r}^*) \geq w'(\underline{r}^*)$.*

This lemma states that in the interval where the seller uses shill bids, the bidding function lies between two bounds: the lower bound $w(x)$ which corresponds to the bidder expected value conditional on his signal being x and the tie-bidder being the seller and the upper-bound which corresponds to the bidder expected value conditional on his signal being x and the tie-bidder being one of his opponent bidder. For signals below $\underline{r}^*$, we haven't made any restriction on equilibrium bids provided that they are below $\underline{r}$. For signals above $\overline{r}^*$, equilibrium bids are such that $b(x) = v(x)$.

We now turn to the seller's equilibrium condition. Denote by $U_S^b(x)$ the seller's expected revenue if the buyers bid according to b and if he submits a shill bid corresponding to the bid of a buyer with a type x, i.e. corresponding to a reserve price of $b(x)$. The seller's expected revenue is:

$$U_S^b(x) = (F^{(2:n)}(x) - F^{(1:n)}(x)) \cdot b(x) + \int_x^{\overline{x}} b(s) f^{(2:n)}(s) ds \tag{7}$$

This expression should be put in parallel with the one of the quasi-revenue introduced in assumption 4.1. The same comments applies. The similarity between those expression will be used in next lemma.

If the equilibrium strategy is g, then the seller is indifferent between any bid b such that $g(b) > 0$. Differentiating this expression with respect to x we obtain the following differential equation for the bid function in the corresponding range $[\underline{r}^*, \overline{r}^*]$ over signals:

$$(F^{(2:n)}(x) - F^{(1:n)}(x)) \cdot b'(x) - f^{(1:n)}(x) \cdot b(x) = 0 \tag{8}$$

Suppose that $\underline{r}$ -the lowest possible shill bids under strategy g- is given, then b is uniquely defined through first-order linear differential equation (8).

Coupled with assumption (4.1), which states that $(F^{(2:n)}(x) - F^{(1:n)}(x)) \cdot w'(x) - f^{(1:n)}(x) \cdot w(x) \geq (>) 0$ if and only if $x \leq (<) r^*$, the first-order condition (8) rules out reserve prices that are below $\underline{r}^*$. The following lemma puts the first restriction on the strategy g which was unconstrained up to this point:

Lemma 5.4 *A necessary condition on $\underline{r}$ is: $\underline{r} \geq r^*$*

Proof 5.3 *$x \to (F^{(2:n)}(x) - F^{(1:n)}(x)) \cdot w(x) + \int_x^{\infty} w(u) f^{(2:n)}(u) du$ is strictly increasing for $x < r^*$ due to assumption 4.1. Then in $\underline{r}$, we have:*

$$(F^{(2:n)}(\underline{r}) - F^{(1:n)}(\underline{r})) \cdot w'(\underline{r}) - f^{(1:n)}(\underline{r}) \cdot w(\underline{r}) > (F^{(2:n)}(\underline{r}) - F^{(1:n)}(\underline{r})) \cdot b'(\underline{r}) - f^{(1:n)}(\underline{r}) \cdot b(\underline{r}) \tag{9}$$

where $b(\underline{r}) = w(\underline{r})$ We conclude that $b'(\underline{r}) < w'(\underline{r})$ which raises a contradiction with lemma 5.3

This lemma formalizes one of the key insight of this paper: low reserve prices are not sustainable at equilibrium. It is not sufficient that $x \to (F^{(2:n)}(x) - F^{(1:n)}(x)) \cdot w(x) + \int_x^\infty w(u) f^{(2:n)}(u) du$ is locally decreasing at $\underline{r}$ to guarantee the existence of such an equilibrium. There may be solutions of the first-order differential equation (8) with the initial condition $b(\underline{r}) = w(\underline{r})$ which are hitting the lower bound before reaching the upper bound and are therefore not suitable solutions. Indeed, due to the unimodality assumption, $x \to (F^{(2:n)}(x) - F^{(1:n)}(x)) \cdot w(x) + \int_x^\infty w(u) f^{(2:n)}(u) du$ is decreasing for $r > r^*$ and solution of (8) can not hit again the lower bound w and are therefore suitable solutions.

Now define the set of possible candidates for $\overline{r}^*$ the highest possible type picked by the seller to shill bids.

Definition 2 [19]

Let $\Theta(r) = \{x | b(x) = v(x) \ \ and \ \ b(s) \leq v(s) \ if \ s \leq x\}$, where b is the solution of the differential equation (8) with the initial condition $b(\underline{r}) = w(\underline{r})$. Then if $\Theta(r)$ is not empty, denote by $\theta(r)$ an element of $\Theta(r)$. Otherwise we set $\theta(r) = \overline{x}$.

Finally, we can characterize the whole set of equilibria with shill bidding. For a given reserve price, if we restrict ourselves to the strategy of active bidders (i.e. $x \geq \theta^* = w^{-1}(\underline{r})$) then the set of equilibria with commitment is a singleton. On the contrary, there is a continuum of equilibria with shill bids. A typical equilibrium is depicted in Figure [2].

Proposition 5.2 *The set of equilibria is parameterized by $\underline{r} \geq r^*$ and $\overline{r} = \theta(\underline{r})$. The bidding strategy b of the active buyers is uniquely defined by equation (8) and the initial condition $b(\underline{r}^*) = w(\underline{r})$. Non active buyers are free to bid provided*[20] *that $U_S^b(x) \leq U_S^b(\underline{r}^*)$. Then, the shill bidding strategy g of the seller is uniquely defined by equation (5) and the normalization $\int_{\underline{r}}^{\overline{r}} g = 1$.*

Proof 5.4 *The preceding analysis has established that those are the necessary form of any equilibrium without commitment and that agents bid according to their best response strategy provided that $b(\cdot)$ is strictly increasing, which is immediately true from equation (8). The remaining point to check is that those equilibria are well defined: more precisely it remains to show that the shill bidding strategy g is a density function. From equation (5), we obtain that:*

[19] The definition is a bit intricate and is reduced to $\Theta(r) = \min_{x \geq \underline{r}^*} \{x | b(x) = v(x)\}$ in generic cases where $\Theta(r)$ is a singleton. A necessary condition for $\Theta(r)$ to contain more than one item is that at the first point where b hits the upper bound v we must have $b'(x) = v'(x)$ and $b''(x) \leq v''(x)$, i.e. b and v must be tangent with b staying below v.

[20] This condition is always satisfied if non active buyers bid zero.

$$G(x) = \exp\left(-\int_x^{\theta(\underline{r})} \frac{v(u)-b(u)}{b(u)-w(u)} \cdot \frac{f_u^{(1:n-1)}(u)}{F_u^{(1:n-1)}(u)} \cdot du\right), \quad x > \underline{r} \qquad (10)$$

$G(\theta(\underline{r})) = 1$. We can also check that $lim_{x \to \underline{r}} G(x) = 0$ noting that the order of the integrand $\frac{v(u)-b(u)}{b(u)-w(u)} \cdot \frac{f_u^{(1:n-1)}(u)}{F_u^{(1:n-1)}(u)}$ is $O(\frac{1}{(x-\underline{r})})$. Finally, $g \geq 0$ because b lies between the two bounds w and v what has been obtained by the restriction $\underline{r} \geq r^*$ (b can therefore not hit the lower bound w again) and the suitable choice of $\overline{r} = \Theta(\underline{r})$ (which ensures that b remains under the upper bound v). Thus we have proved that the function g corresponds effectively to a mixed reserve price strategy.

As predicted by section 4's analysis, pure strategy equilibria are only those such that the reserve price is above $w(\overline{x})$ and no trade takes place. Note that any selection $r \to \theta(r)$ is monotonically non-decreasing. Otherwise, those solutions of (8) would cross which is not possible. Then due to the unimodality assumption the equilibria can be ranked relative to seller's revenue: increasing $\underline{r}$ decreases seller's revenue and the particular equilibrium where $\underline{r} = r^*$ can be qualified as the seller's preferred equilibrium. We also obtain that it is always optimal to announce any reserve price in the interval $[0, r^*]$. Otherwise the truncation will eliminate the best equilibria from the seller's point of view.

Remark 5.2 *The equilibria with commitment are qualified as robust*[21] *because the equilibrium strategy of a potential buyer is still a best response if the highest bid of his opponents and the corresponding identity were disclosed. Without commitment, the equilibrium strategy of a potential buyer is still a best response if only the highest bid of his opponents were disclosed but not his identity: if he learns that it is the seller, then the winner could possibly regret his bid.*

Remark 5.3 *Note that if non-active buyers bid according to the mapping w, which could be rationalized by a suitable trembling of the seller, then among the set of equilibria only the equilibrium with $\underline{r} = r^*$ survives. In that manner, the seller's most preferred equilibrium can be viewed as the single remaining equilibrium after a suitable refinement.*

Let us illustrate our results in the special case where signals are statistically independent. In this case, closed form solution can be easily derived and reserve price supports are geometrically characterized. In such a case,

[21] This is a weaker robustness property than the ex-post Nash Equilibrium which applies for the english button auction with commitment where the equilibrium strategy of a potential buyer is still a best response if the bids of all his opponents were disclosed.

$(F^{(2:n)}(x) - F^{(1:n)}(x))) = n(1-F(x)) \cdot F^{n-1}(x)$ and $f^{(1:n)}(x) = n \cdot F^{n-1}(x)$. Then assumption (4.1) is equivalent to $x \rightarrow (1-F(x)) \cdot w(x)$ being unimodal and r^* equals to its mode. Equation (8) reduces to $(1-F(x)) \cdot b(x)$ being constant. For any $r \geq r^*$, $\theta := \theta(r)$ is uniquely defined by $(1-F(r)) \cdot w(r) = (1-F(\theta)) \cdot v(\theta)$ where $\theta \geq r$ (which is easily geometrically characterized in Figure [3]). Then the closed form of b as a function of $\underline{r}$ the minimum of the shill bids is:

$$b(x) = \frac{1-F(\underline{r})}{1-F(x)} \cdot w(\underline{r}), for \quad \underline{r} \leq x \leq \theta(\underline{r}) \tag{11}$$

$$b(x) = v(x), for \quad x > \theta(\underline{r}) \tag{12}$$

Finally, the CDF of seller's shill bids is given by:

$$G(x) = \exp\left(-\int_x^{\theta(\underline{r})} \frac{v(u)-b(u)}{b(u)-w(u)} \cdot (n-1)\frac{f(u)}{F(u)} \cdot du\right) \tag{13}$$

Figure [3] depicts the set of equilibria in the case of statistically independent signals on a finite support. The dotted interval $[\underline{r}, \overline{r}]$ represents a possible support for the shill bidding activity. The dotted interval $[\underline{r}^{opt}, \overline{r}^{opt}]$ represents the particular candidate where $\underline{r}^{opt} = r^*$ which corresponds to the seller's preferred equilibrium. At any equilibrium, shill bidding activity is above r^*. On the other hand, the optimal reserve price with commitment is r^{**}, which is strictly lower that r^*.

6 Revenue Comparison - The 'Shill Bidding Effect'

In the standard independent private-values environment, the well-known revenue equivalence theorem establishes that the first price and the second price auction raise the same revenue for any given reserve price. This result has first been established by Myerson [21], but it is more general and applies in MW's General Symmetric Model provided that signals are statistically independent (Theorem 3.5 in Milgrom [19]). This equivalence result holds without shill bids. Since the first price auction's equilibrium is immune to shill bidding whereas the second price auction's performance is deteriorated by the shill bidding activity as a corollary of proposition 5.2, the first price auction raises a strictly higher revenue than the second price auction in the framework with independent signals and with shill bids. Moreover, the level of trade is unambiguously reduced with shill bidding in the second price auction compared to the first price auction with an optimal reserve price. The following proposition gathers those insights.

Proposition 6.1 (The shill bidding effect) *Under the additional assumption of independence of signals and without commitment, the optimal first* *price auction strictly dominates the whole set of equilibria of any second price auction. More precisely, we have the following inequalities:*

$$r^{**} = r^{**}_{SP} = r^{**}_{FP} < r^* = \underline{r}_{opt} \tag{14}$$

where r^{**}_{FP} equals to the optimal reserve price of the first price auction (with or without commitment) and $\underline{r}_{opt}$ is the minimum shill bid of the seller in her most preferred equilibrium of the second price auction without commitment.

The proposition is silent about the extent of the difference in term of revenue of the first price and the second price auction without commitment. In particular is there any factor which drives this difference? The next proposition sheds some light on this difference and more generally on the difference in term of revenue of the optimal auction with commitment and the optimal equilibrium of the second price auction without commitment. This difference equals exactly to the difference between the first price and the second price auction without commitment in the framework with statistically independent signals. The response is that the winner's curse increases this difference. Let us firstly define precisely what we call an increase of the winner's curse.[22]

Definition 3 *Consider two environments* $1, 2$ *with the same distribution of signals and the same function $w = w_1 = w_2$ but two different functions v denoted by v_1 and v_2, then environment 1 is said to suffer from a greater winner's curse than environment 2 if and only if $v_1(x) > v_2(x)$, for $x > \underline{x}$.*

This definition imposes a very partial ordering on different environments insofar as the distribution of types must be the same. The point that the functions w_i are the same is somehow a normalization in order to make the level of welfare comparable in the two different environments: suppose that you have two environment (w_1, v_1) and (w_2, v_2), then to make the suitable comparison payoffs should be renormalized via the suitable strictly increasing function. Then the two environments with the couple (w, v) as (w_1, v_1) and $(w_1, w_1 \circ w_2^{-1} \circ v_2)$ must be considered to apply the preceding definition.

Proposition 6.2 *The difference in utility between the revenue of the second price auction with an optimal reserve price (with commitment) and the revenue of the seller's preferred equilibrium of the second price auction without commitment is strictly increasing with the winner's curse.*

[22] We do not pretend to give a definitive definition of the winner's curse, which has been controversial and a source of some debate. We do not stand in line with the tradition which considers the winner's curse as a judgmental failure. In this strand, Compte [5] gives a different definition of the winner's curse: for a given signal and a buyer, [5] defines the winner's curse as the difference between the expected value of the item conditional on his signal and the expected value conditional on his signal and the fact that he wins.

The proof is relegated in the appendix. It relies in particular on the fact that $\theta_1(r) > \theta_2(r)$. For the same distribution of signals, the similarity of w and r in both environment implies that the function b are equal until it reaches the bound $\min_i v_i$. Because $v_2 < v_1$, the bound v_2 is the first to be reached. This point is easily seen in figure [3] where an increase of the winner's curse implies an increase of $(1-F(x))\cdot v(x)$ everything else staying unchanged, which pushes $\theta(r)$ on the right.

This proposition states that the loss of revenue due to the non-commitment ability, i.e. the 'shill bidding effect', is increasing with the winner's curse, i.e. the importance of the shift of the expected value with the other's signal. Then in the general case without commitment, two effects are at work to compare the first price and the second price auction: the 'linkage principle' highlighted by MW which relies on the correlation between types, but another strength is at work which could be called the 'shill bidding effect'. Whereas the 'Linkage principle' was a benefit of formats which conveys more information, it is not a surprise that those formats are exactly those that are vulnerable to shill bidding: the incentive to shill bid increases with the ability of the format to convey information. The main insight is that the channels of the linkage principle and the shill bid effect do not coincide exactly and that finally the global effect remains undetermined.

Example 6.1 *Let $n = 2$. Each buyer i's signal x_i is drawn independently from the interval $[0,1]$ with density $f(x_i) = 1$. Let the value be defined by $u_i(x_i, x_{-i}) = \alpha \cdot x_i + (1-\alpha)x_{-i}$ where $\alpha \in [\frac{1}{2}, 1)$. Note that the winner's curse is decreasing with α relative to definition 3*[23]*. Then $w(x) = \frac{1+\alpha}{2} \cdot x$ and $v(x) = x$.*

The optimal reserve price with commitment r^{**} equals to $\frac{2\alpha}{3\alpha+1}$, whereas the lowest reserve price without commitment is $r^* = \frac{1}{2}$ and the optimal equilibrium for the seller corresponds to the one with the support $[\frac{1}{2}, \frac{2+\sqrt{2(1-\alpha)}}{4}]$. It illustrates also a point that has been previously mentioned: $\theta(r^*)$ is increasing with α, i.e. the seller uses higher shill bids when the winner's curse is stronger because the incentive to use shill bids that convey a better information is then stronger.

In Table 1, we compare the revenue of the optimal equilibrium with commitment with the seller's preferred equilibrium without commitment varying the degree of the winner's curse which is captured by the parameter α. We can check that the difference in the revenues with and without shill bids is decreasing with α as predicted by proposition 6.2[24]*. The last line of the table corresponds to the gain in percentage of the commitment ability, which also corresponds in this framework to the gain of the first price auction over the*

[23] The renormalization consists to apply the suitable homothetic transformation with the factor $\frac{2}{1+\alpha}$.

[24] More precisely the prediction of 6.2 applies after the suitable normalization.

α	1/2	2/3	3/4	1
r^{**}	0.4	0.44	0.46	0.5
shill bidding	0.333	0.363	0.378	0.42
commitment	0.360	0.377	0.386	0.42
%	8.0	3.9	2.1	0

Table 1: Revenue with and without commitment varying the degree of the winner's curse

second price auction when the commitment ability fails.

Example 6.2 *Effect of the variation on the number of buyers. Consider n potential buyers with signals drawn independently. Let the value be defined by $u_i(x_i, x_{-i}) = \alpha \cdot x_i + (1-\alpha)\frac{\sum_{j\neq i} x_j}{n-1}$ where $\alpha \in [\frac{1}{2}, 1)$. Then $w(x)$ is independent of n, whereas $v(x)$ is a decreasing function of n for any x. In this example, a drop in the number of bidders unambiguously raises the winner's curse and applying proposition 6.2, it raises the 'shill bidding effect'.*

7 Extensions

7.1 Negative allocative externalities

Jehiel and Moldovanu [12] consider a single-unit auction with two potential buyers such that a loser suffers from a negative externality: a non-purchaser prefers that the good remains unsold. Then the valuation for the good is not properly defined and depends on whether the best competing offer is the seller (by means of the reserve price or a shill bid in our framework) or the other buyer. Then with a reserve price r, the function mapping the expected 'valuation' of the object to the highest opposing bid is discontinuous at r and the same gap is present.

Jehiel and Moldovanu [12] establish that, with commitment, the first price and the second price auctions are equivalent. This is a consequence of the revenue equivalence theorem which relies on the symmetry and the independence of types. The same analysis could be pursued in their framework and the seller is better off with the first price auction. In a welfare perspective, the second price auction with shill bidding may be desirable if the seller's reservation value is high due to the tension between the profit maximizing incentives for a low reserve price and the welfare maximizing perspective which prefers that the object remains in the seller hand if negative externalities on non-buyers are high.

7.2 A binding reservation value

A surprising insight of the literature on auctions with externalities is that it may be optimal for the seller to fix a reserve price that is lower than her reservation value. In Jehiel and Moldovanu [12], such a low reserve price pushes buyers to bid more aggressively because they are fearing that the item goes in their competitor's hand instead of staying in the seller's hand. Such an insight fail if the seller cannot commit not to use shill bids.

Suppose that the seller's valuation for the object is Π_S which is common knowledge. The preceding analysis of the expected profit of the seller should be modified by adding the term $F^{(1:n)}(x) \cdot \Pi_S$ to $U_S^b(x)$ the previous expected revenue. Then for the second price auction as well as for the first price auction, the lowest equilibrium reserve price should be greater than Π_S. Thus this new constraint on the reserve price affects both type of auctions. Thus in the general case, the 'shill bidding effect' may damage both auction formats. But as our analysis emphasizes the whole set of constraints is always more restricting for the second price than the first price auction. Note that if the reservation value is greater than $w(\overline{x})$, then symmetric equilibrium without commitment involves no trade. Thus bilateral negociations should be unambiguously preferred to auctions.

7.3 Entry fees

As suggested by MW's Theorem 19, in order to raise her revenue, the seller should better use entry fees and a nul reserve price rather than only a reserve price either for the first price or for the second price auction. In the first price auction such a policy is still feasible with shill bids. On the other hand, with the second price auction, such a policy cannot be used to raise the revenue because of the binding constraint on the feasible reserve price policy. Consequently, the possibility to use entry fees increases the discrepancy between the first price and the second price auction.

7.4 The English (Button) Auction

Our previous analysis restricts to the first and second price auctions where shill bids are only valuable for the seller who is interested to submit one shill bid. In the english auction, incentives to raise shill bids are much stronger: the seller may be willing to submit any number of shill bids[25], similarly real potential buyers may be willing to use multiple identities to quit early the auction in order to convey an information making the object less valuable. If potential buyers are not able to submit shill bids and the number

[25] Note that Chakraborty and Kosmopoulou [4]'s analysis considers at most one shill bid from the seller, which does not fit with [6]'s story where her analysis bring her to 'a list of 33 Internet names that repeatedly bid on one another's offerings' and that is suspected to have formed a ring.

of potential buyers are exactly known[26] to be n, then shill bidding from the seller will not affect the english auction in the way it conveys information: there is a symmetric equilibrium where each buyer interprets the $(n-1)$ first departures from the auction as being departures from real bidders and the remaining bidders as being shill bids from the seller.

Similarly, if the seller is not able to submit shill bids and the number of potential buyer is known, then the shill bidding activity from the bidders does not affect[27] the english auction: there is a symmetric equilibrium where each buyer interprets that, until it remains n active bidders, departures from the auction are departures from shill bids and that among the remaining bidders no real bidder has quitted.

On the other hand, if both the seller and the bidders can submit freely any shill bids as they want, the english auction is no more immune to shill bidding. The result is a kind of babbling equilibrium where an infinite number of shill bids invades the english auction until the highest possible reserve price at equilibrium for the seller. After this highest reserve price is reached (which corresponds to the ones that has been derived previously for the second price auction), all identities of the seller quit the auction which then corresponds to a standard english auction without shill bidding. Before the switch, the only information that makes sense is that it remains other active bidders or not. Otherwise, if the strategy of a shill bid can either inflate or deflate the expected value of real bidders, then it will be used respectively by either the seller or some buyers. After the switch, the analysis corresponds to the standard english auction (which may then outperform the second price auction if signals are strictly correlated). We can doubt that this sharp switch in the shill bidding activity has any practical appeal insofar it corresponds to the english button and not the 'real-life' english auction formats. Nevertheless, it gives the prediction that low bids' exits, before the switch, do not convey any positive information to remaining bidders.

To summarize, with shill bidding, it may be suitable to consider that, before this cut off point that has been derived previously as $\theta(\underline{r})$ the highest possible shill bid, the english auction is equivalent to the second price auction.

7.5 Endogenous Entry of Bidders

There is a couple of papers, e.g. Engelbrecht-Wiggans [7] and Levin and Smith [16], in the auction literature that endogenize the number of bidders participating in the mechanism (due to the costly activity to get informed

[26] It is the reason why [4] introduces uncertainty in the total number of potential buyers.

[27] We just argue that there is an equilibrium without shill bids. However, there exist equilibria where all bidders submit an infinite number of shill bids, which remains an equilibrium and makes the english auction equivalent to the second price auction. However, those kind of babbling effect are not robust if shill bidding has an infinitesimal cost contrary the equilibria where shill bidding technology is available to the seller and the buyers.

about their valuations for example). Those papers express a severe critic of the traditional optimal design insights as the discrepancy between the seller and the social planner objectives. Nevertheless, those papers rely on a strong commitment ability of the designer: the seller must be able to commit to an auction mechanism with a given reserve price (and where she does not participate with shill bids) before the agents decide to incur the costs for participation. Even if she can commit in advance to the selling mechanism, she may not be able to commit not to use shill bids. Then the results of the literature with endogenous entry are modified even in the private value framework.

Levin and Smith [16] highlight that MW's ranking between the second and price auction is still valid with endogenous entry. We argue that it may not be longer true even in private value due to the 'shill bidding effect': with a first price auction, the commitment of a nul reserve price is credible whereas in the second price or english auction the seller will deviate with a positive reserve price, a point that will be anticipated at equilibrium in the entry process. Indeed in the symmetric independent private value framework, for the second price or English auction with shill bidding, Myerson's reserve price is still the optimum with endogenous entry whereas for the first price auction, the optimum is attained when the seller charges neither entry fees nor reservation price.

7.6 Auctioneer's fees

So far, we have considered that the seller is the auctioneer and that he consequently receives the entire share of the winning price. In practice, however, it is generally not exactly the case: either because the seller should pay a tax on the final price or because the auction is run by an intermediary. For example, on eBay, according to the range of the winning price, the seller must pay a final value fee: between 0.01 \$ and 25.00\$, the fee represents 5.25% of the winning price, then this fee decreases to 1.50%. However, if the item is not sold and thus remains "officially" in the seller's hand, then no fee is charged. Thus our argument that for a given announced reserve price $r = w(x^*)$ the seller will find it profitable to raise a shill bid at $b(x^*) = v(x^*)$ is no more valid. Denote by τ the fee on the final price, such that the seller receives only $(1-\tau)\cdot p$. Then the impact in seller's surplus of such a deviation would equal to:

$$(F^{(2:n)}(x^*) - F^{(1:n)}(x^*)) \cdot (v(x^*) - w(x^*)) \cdot (1-\tau) - F^{(1:n)}(x^*) \cdot \tau \cdot w(x^*) \quad (15)$$

The first term corresponds to the gain of such a shill bid by raising the price paid by the buyer from $w(x^*)$ to $v(x^*)$ in the event where only one buyer is participating. The second term corresponds to the fee paid by the seller when he bought the item by means of the shill bid: it reflects the financial cost of the shill bidding activity due to the fee. If expression (15) is negative, the profitability to raise shill bids is no more guaranteed. However, the fact that this term is negative is not sufficient to guarantee that not to use shill bids is an equilibrium for this announced reserve price. Shill bids above $v(x^*)$ may be profitable, especially if the seller assigns a high valuation for the item or equivalently if the initial reserve price r is too low. On the whole, an increase of τ makes shill bids less attractive and is thus a way for the seller not to use shill bids.

eBay's fees are mostly of two kinds: first an insertion fee which is an increasing function of the reserve price and second a final value fee which represents the percentage of the winning price that eBay gets. As an example, under the actual fees in US, a seller who chooses the reserve price 30$ and obtains a winning price of 35$ is charged typically 2.8$: 1.2$ is charged as an insertion fee and about 1.6$ as a final value fee. This current paper has a strong policy recommendation for auctions' houses as eBay from a welfare perspective: in order to limit the shill bidding activity, they should charge more as a final fee than as an insertion fee. Whereas we have argued that an increase in the final value fee reduces shill bidding, insertion fee are also not neutral beside shill bidding. A steep insertion fee function is an incentive for the seller to choose a null announced reserve price and to use shill bids as an effective reserve price, a point that is also true in a pure private framework. Thus there will be a double dividend in putting a null insertion fee for any announced reserve price and to raise instead the final value fee. But although eBay officially prohibits shill bidding, it is not clear whether the auctioneer really suffers from this activity, as formerly emphasized by [4] and is thus inclined to make a switch in its fees' policy.

This policy recommendation should be compared with the optimal broker's mechanism proposed by Jullien and Mariotti [13] with a menu of transaction fees contingent on the reserve price. In their model, which is reminiscent of the lemon problem, the seller is privately informed about the quality of the object which corresponds to his valuation and also enters additively in the buyers' valuations. If the seller runs himself the auction, he has an incentive to raise a high reserve price (running the risk of no trade) in order to make believe that the object is of high quality and this leads to an equilibrium with fewer trade compared to the situation where the seller's type would be common knowledge (this is the lemon effect). On the other hand, if the seller faces the broker's optimal mechanism with a menu of insertion fees contingent on the reserve price, then the lemon effect is reduced.

We conclude that eBay fee's policy may be a trade-off between the 'lemon problem' that calls for insertion fee that are contingent on the reserve price and the shill bidding issue that calls for a final value fee.

A Proof of proposition 6.2

Denote by $U_i^{commit}(r)$ (respectively $U_i^{non-com}(r)$) the revenue of the second price auction with a reserve price of r (with a upper bound of r for the support of the reserve price sustainable by the mixed strategy equilibrium) in environment i with commitment (without commitment). Assume that environment 1 suffers from a greater winner's curse that environment 2, i.e $v_1 > v_2$.

In order to prove, for $r > r^*$, that

$$U_1^{commit}(r_1^{**}) - U_1^{non-com}(\theta_1(r)) > U_2^{commit}(r_2^{**}) - U_2^{non-com}(\theta_2(r)) \quad (16)$$

the left-hand (right-hand) term representing the gain of commitment in environment 1 (resp 2), it is sufficient to prove

$$U_1^{commit}(r_2^{**}) - U_1^{non-com}(\theta_2(r)) > U_2^{commit}(r_2^{**}) - U_2^{non-com}(\theta_2(r)) \quad (17)$$

since:

- $U_1^{commit}(r_2^{**}) < U_1^{commit}(r_1^{**})$ (from the definition of r_1^{**})
- $\theta_2(r) < \theta_1(r)$ and $r \to U_1^{non-com}(r)$ is decreasing in r for $r > r^*$. $U_1^{non-com}(r)$ can be viewed as the sum of two decreasing function in the range $[r^*, \overline{r}]$: $x \to (F^{(2:n)}(x) - F^{(1:n)}(x)) \cdot w(x) + \int_x^\infty w(u) f^{(2:n)}(u)du$ (assumption 4.1) and $\int_x^\infty (v(u) - w(u)) f^{(2:n)}(u)du$

Finally, equation (17) is equivalent to:

$$\int_{r_2^{**}}^{\theta_2(r)} (v_1(u) - v_2(u)) f^{(2:n)}(u)du > 0 \quad (18)$$

which is true because $v_1 > v_2$ and $\theta_2(r) > r_2^{**}$

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Figure 1 : Equilibrium with commitment not to use shill bids

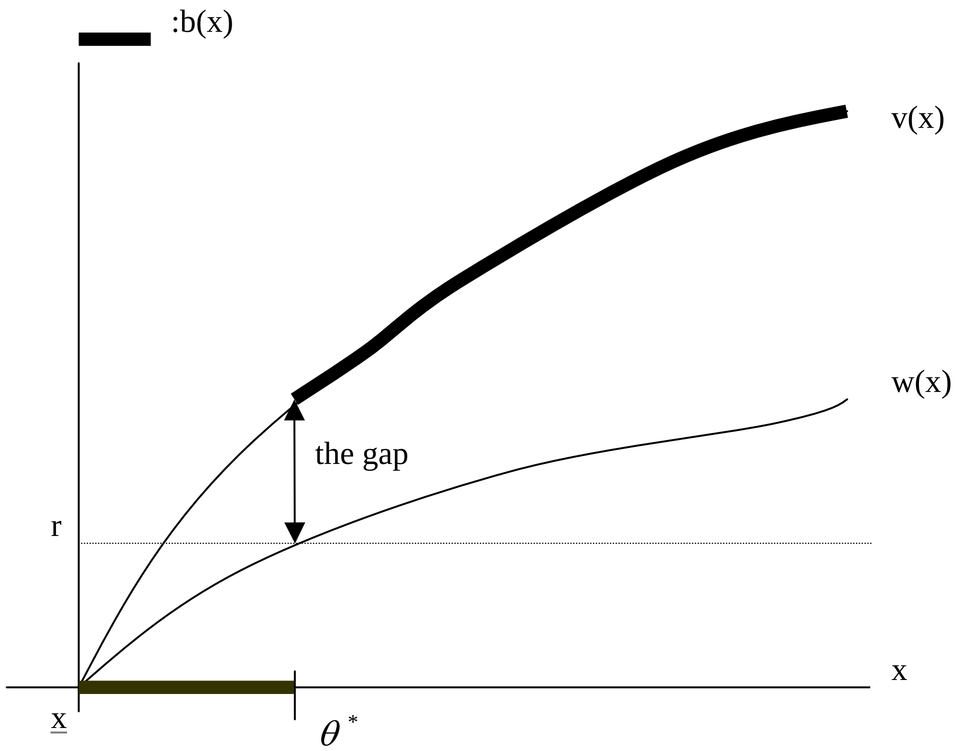

Figure 2 : Equilibrium with shill bidding

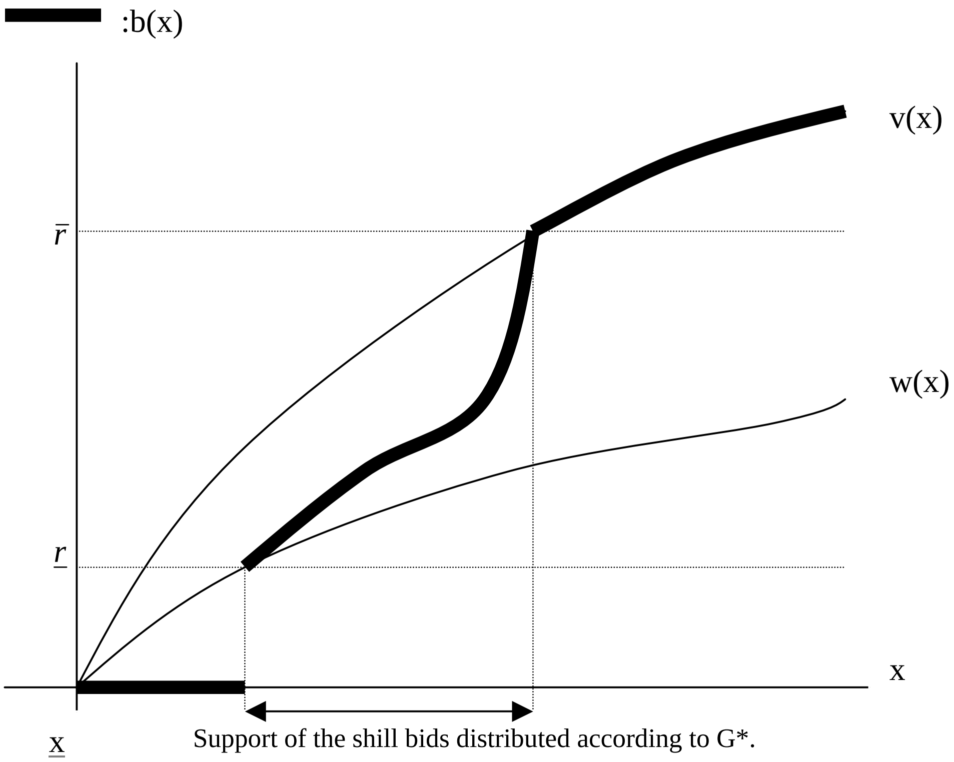

Figure 3 : Geometrical Resolution of the set of equilibria