Estimation of Functionals of High-Dimensional Parameters: Bias Reduction and Concentration

Vladimir Koltchinskii

School of Mathematics, Georgia Institute of Technology

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Summary:

The main focus of this course will be on a circle of problems related to estimation of real valued functionals of parameters of high-dimensional statistical models. In such problems, it is of interest to estimate one-dimensional features of a high-dimensional parameter that are often represented by nonlinear functionals of certain degree of smoothness defined on the parameter space. The functionals of interest could be estimated with faster convergence rates than the whole parameter (sometimes, even with parametric rates). Potential examples include, for instance, such problems as estimation of linear functionals of principal components (that are nonlinear functionals of unknown covariance) in high-dimensional PCA. The goal is to discuss several mathematical methods that provide a way to develop estimators of smooth functionals of high-dimensional parameters with parametric $\sqrt{n}$ convergence rates and to prove their asymptotic efficiency.

The following topics will be covered (at least, to some extent):
- preliminaries in high-dimensional probability and analysis (concentration inequalities, comparison inequalities, smoothness of operator functions, etc);
- non-asymptotic bounds and concentration inequalities for sample covariance in high-dimensional and dimension-free frameworks;
- some approaches to concentration inequalities for smooth functionals of statistical estimators;
- a recent approach to bias reduction in functional estimation based on an approximate solution of integral equations with respect to Markov kernels on the parameter space;
- a coupling method (based on representations of Markov chains as superpositions of independent “random homotopies”) that allows one to obtain bounds on the bias of the resulting estimators;
- minimax lower bounds in functional estimation based on van Trees inequality and on Nemirovski's method.
Ces cours sont proposés aux étudiants de 3ème année de l'ENSAE, de l'ENSAI, ouverts aux étudiants de M2 ou inscrits en thèse. Une inscription préalable est demandée impérativement pour tous les étudiants de l'ENSAE, de l'ENSAI, ou extérieurs, à Lyza RACON : lyza.racon@ensae.fr ou par téléphone au 0170266926 afin de pouvoir être admis dans les locaux de l'ENSAE et pouvoir être joints en cas de nécessité par les organisateurs du cours.

**Literature:**


