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Abstract

We build measures of the demand for trade protection, and relate it to permanent productivity and transitory monetary policy shocks identified from the U.S. monthly and quarterly data. The demand for protection is counter-cyclical conditional on productivity shocks and pro-cyclical conditional on monetary policy shocks. We then lay out a two-country dynamic general equilibrium model with trade in intermediate and final goods, sticky prices, incomplete financial markets and endogenous monetary policy rules, and propose a repeated non-cooperative policy game that determines tariffs endogenously. These trade policies (i) are consistent with small but positive tariffs, as in the data, and (ii) fit empirical evidence about the cyclical pattern of the demand for trade protection under a wide range of plausible model calibrations. We then use the model to quantify the macroeconomic and welfare effects of a change in tariff setters' preferences that induces tariffs to rise in both countries.

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1 Introduction

Several key countries in the world trade have recently been shifting towards less cooperative trade policies. The shift is illustrated by the failure of multilateral negotiation rounds as well as by the increase in bilateral negotiations and agreements. Further, some countries like the US have clearly engaged in tariff wars. In this paper, we propose a dynamic general equilibrium model of non-cooperative trade policies that fits empirical evidence for the US about the demand for trade protection, and quantify macroeconomic and welfare effects of some of these non-cooperative policies.

A large literature in economics suggests that trade protection varies over the business cycle.[1] The most common view is that protectionism is counter-cyclical – a period of low growth and high unemployment will lead to greater calls for trade restrictions to protect domestic jobs. From a theoretical viewpoint, it is not always clear why trade restrictions should be increased during an economic downturn. If trade policy is aimed at domestic employment, an increase in tariffs during a recession would help domestic import competitors, but hurt exporters. It is not obvious that the balance of influence between import competition and exporters would shift in a clear pattern over the cycle. On the other hand, if trade restrictions are designed to improve the terms of trade, it is unclear why the pressure to exploit market power would be greater in periods of low economic activity. Empirically, the evidence on the cyclicality of trade policy is mixed however.[2]

Our paper first provides fresh evidence based on U.S. data on the cyclical behavior of the demand for trade protection and shows that the cyclical pattern conditional on different shocks may differ. We exploit Bown (2016)'s Global Anti-Dumping database. This is a recording of anti-dumping or trade disputes initiatives at the Dispute Settlement Body of the WTO, and these are often followed by the imposition of tariffs. With these data, we build measures of the demand for trade protection. We then relate it to productivity and monetary policy shocks identified from the U.S. data both at a monthly and quarterly frequency. We find robust evidence that the demand for protection falls after a positive permanent productivity shock, but rises after a expansionary monetary policy shock. The demand for protection in U.S. data is thus counter-cyclical conditional on productivity shocks and pro-cyclical conditional on monetary policy shocks.

Then, our paper builds a standard New Keynesian open economy macro model with sticky prices and exchange rate fluctuations with tariffs. The architecture of our open macro model when carefully calibrated to the U.S. economy leads to empirically consistent responses of the demand for trade protection – tariffs in the model – when trade policies are designed to improve a country's

[1] For instance see Bagwell and Staiger (1999), Bagwell and Staiger (2003), Irwin (2005), Bown and Crowley (2013b) and Knetter and Prusa (2003).

[2] Irwin (2005) finds that US anti-dumping filings are increasing in the US unemployment rate. Bown and Crowley (2013b) find that emerging market imposition of temporary trade barrier become increasingly counter-cyclical over the 1989-2010 period. Bown and Crowley (2013a) find strong evidence for counter-cyclical trade disputes among advanced economies prior to the Great Recession, but not afterwards. On the other and, Rose (2012) argues that there is little evidence for counter-cyclical trade policy in data over the 20th century.

terms of trade. More specifically, we consider trade policies to be designed as a repeated non-cooperative game along the lines of Bagwell and Staiger (2003). In the model of Bagwell and Staiger (2003), import tariffs are determined in a repeated game between two countries, and equilibrium sustainable protection satisfies an incentive constraint which balances the benefits of predatory tariffs against the future costs of a trade war. They show that a persistent fall in trade volume during a recession reduces the benefits of cooperation, and will lead to an increase in equilibrium tariffs. In a related paper, Bagwell and Staiger (1999) show that sustainable tariffs will increase during periods of high import growth, but tariffs will be lower, the greater is the volatility of imports.[3] The Bagwell and Staiger papers provide key insights into the forces determining trade protection, but we embed these forces into a fully-fledged dynamic general equilibrium with sticky domestic and export prices, trade in intermediate and final goods, flexible exchange rates, incomplete markets and monetary policy rules. The equilibrium tariff sequence represents a "sustainable equilibrium" in the sense of Chari and Kehoe (1990). Given a sequence of sustainable tariffs, there is a gain to one country from cheating, setting a high tariff so as to improve its terms of trade. But the (future) cost of cheating is a reversion to a total "trade war". The equilibrium sustainable tariff rates just balance the benefits and costs of cheating on the sustainable equilibrium itself. Our model can then address some of the key issues regarding the cyclicality of protectionism. The first question is why should protection be cyclical at all? Why would the temptation to impose tariffs vary over the business cycle?

First, our model offers an analysis of the incentives to raise tariffs unilaterally. We show that the presence of a well-documented terms-of-trade externality, by which the tariff setter can raise consumption and/or lower labor effort by raising its tariff, is key to build a system of endogenous and positive tariffs. In addition, as shown initially by Johnson (1953), we show that such a policy move is beggar-thy-neighbor, which could easily result in countries engaging in non-cooperative and aggressive trade policies.[4]

Second, we show that equilibrium sustainable tariffs, defined according to the repeated game described above, are time-varying, and move in response to monetary and productivity shocks. The essential intuition for this is that aggregate shocks have different impacts on the benefits and costs of cheating on the sustainable path of tariffs defined by the repeated game, and hence the equilibrium sustainable tariff rates themselves must respond to the shocks. In our baseline model and calibration, equilibrium sustainable tariffs will increase in response to positive monetary policy shocks, while tariffs will fall in response to productivity shocks. We show that there is a natural channel through which monetary policy shocks increase the gains from cheating on a sustainable

[3] Bown and Crowley (2014) construct an empirical test of the Bagwell and Staiger (1999) model. They find that, empirically, surges in imports tend to precipitate more trade restrictions, but increases in the volatility of imports tend to reduce average tariff rates.

[4] See Grossman (2016) for a recent overview. Further, as shown by Stockman and Dellas (1986) and later by Devereux and Min Lee (1999), this mechanism is only valid under imperfectly integrated (incomplete) financial markets. Under complete markets, risk-sharing eliminates any potential country-level benefit from raising tariffs unilaterally.

tariffs, while productivity shocks increase the consequences of cheating. As a result, depending on the preponderance of shocks, tariff rates may be pro-cyclical or counter-cyclical. This result is qualitatively robust but quantitatively sensitive to most key parameters of the model: the inverse of the Frisch elasticity of labor supply, the share of intermediate goods in total trade, and to the invoicing currency of exports (Local vs. Producer Currency Pricing). It is qualitatively sensitive to the value of the trade elasticity on final goods: when the latter becomes large enough, the cyclical pattern reverses because expenditure switching effects become larger than wealth effects, which alters the incentive to exploit the terms-of-trade externality.

Last, we engage in the simulation of a trade war, induced by a negative shock on the discount factors of tariff setters. The sequence and size of shocks are calibrated to reproduce the rise in tariffs observed recently in the US-China trade relations. When the shock is asymmetric, *i.e.* when only the domestic tariff setter becomes more impatient, we find that tariffs rise substantially in both countries. Indeed, the sustainable tariff equilibrium implies that the Foreign tariff setter should raise its tariffs in response to the change in the preferences of the domestic tariff setter. As such, endogenous retaliation and tariff wars are an equilibrium result of our model of endogenous demand for trade protection. Nevertheless, an asymmetric trade war still favors the domestic tariff setter, who is able to apply larger tariffs and produce a 0.07% welfare gain for domestic households, for a 64% increase in the domestic tariff. The Foreign economy experiences a 0.1% welfare loss and its sustainable tariff jumps by 48%. However, such an asymmetric policy move does not produce a major advantage for the domestic economy, as the real exchange rate appreciate by less than 1% and as the trade balance is barely affected. A symmetric shock on discount factors produces a doubling of tariff rates, producing welfare losses for both countries.

There has been an increasing interest in investigating the effects of trade restrictions in open economy macro models. Barattieri, Cacciatore, and Ghironi (2018) investigate empirically the impact of exogenous changes in tariffs in an SVAR framework, and show that they act as negative supply shocks, depressing GDP and raising inflation with little effects on the trade balance. They propose a small open economy model with firm entry and endogenous tradability that successfully rationalizes the empirical evidence. We adopt a mirror perspective, considering tariffs as endogenous and ask how governments react to economic conditions to determine trade policies over the business cycle. Another recent paper by Erceg, Prestipino, and Raffo (2018) studies the effects of trade policies in the form of import tariffs and export subsidies. They show that the macroeconomic effects of these policies critically depend on the response of the real exchange rate, and that in turn depends on the expectations about future policies and potential retaliation from trade partners. Finally, a recent paper by Furceri, Hannan, Ostry, and Rose (2019) examines the macroeconomic consequences of tariff shocks, and shows that these shocks are generally contractionary.

Focusing more closely on the endogenous determination of trade policies, we noted above that there is a large empirical literature investigating the link between trade restrictions and the economic cycle, and separately, the effect of real exchange rate undervaluation on trade policy (*e.g.* Oatley

(2010), Gunnar and Francois (2006), Bown and Crowley (2013a), among others). In a theoretical model Eaton and Grossman (1985) study optimal tariffs when international asset markets are incomplete and show that they can be used to partly compensate the lack of consumption insurance. Bergin and Corsetti (2008) also consider tariffs as policy instruments in addition to monetary policy but their focus is not specifically on tariffs, rather on the implications of monetary policy on the building of comparative advantages. Campolmi, Fadinger, and Forlati (2014) offer a detailed analysis of optimal non-cooperative policies with a large set of instruments, including tariffs. In a rich model with endogenous location of firms and an extensive margin of trade, they show that the terms-of-trade externality remains the dominating incentive to apply positive tariffs. As discussed above, Bagwell and Staiger (2003) propose a trade model featuring potential terms-of-trade manipulation by governments, and trade agreements as means to restrict this policy option. Our paper is complementary to theirs. Most importantly, we incorporate endogenous tariff formation within a standard open economy macro model, showing the importance of the types of shocks, and price stickiness for the equilibrium degree of trade protection.

The paper is structured as follows. Section 2 offers an empirical analysis based on SVARs to highlight the different pattern of the demand for trade protection to different shocks in the US. Section 3 develops a dynamic NK two-country model with sticky domestic and export prices, trade in intermediate and final goods, flexible exchange rates, incomplete markets and monetary policy rules. Tariffs are either exogenous or set according to a repeated non-cooperative policy game. Section 4 investigates the steady-state effects of varying tariffs exogenously to shed light on the incentives to raise tariffs. Section 5 offers an analysis of the dynamics of sustainable tariffs to both productivity and monetary policy shocks, and Section 6 simulates a trade war triggered by a reduction in the discount factor of tariff setters. Section 7 presents some conclusions.

2 Empirical evidence

We investigate the effects of macroeconomic shocks on the demand for trade protection. Following Barattieri, Cacciatore, and Ghironi (2018), our empirical analysis of this potential effect relies on Bown (2016)'s Global Anti-Dumping (GAD) database that collects anti-dumping or trade disputes initiatives at the Dispute Settlement Body of the WTO, that are likely to be followed by the imposition of tariffs. Our main objective is to uncover the responses of the demand for trade protection, approximated by our measure of trade disputes, to standard supply and monetary policy shocks. To do so, we estimate VAR models using monthly and quarterly U.S. data, identify structural productivity and monetary policy shocks, and track the response of our measure of demand for trade protection to these shocks.

2.1 Productivity shocks

The identification of productivity shocks follows Debortoli, Galì, and Gambetti (2019). According to this approach, a bi-variate VAR is estimated using the log-difference (growth rate) of labor productivity and the total number of hours worked. We use FRED data for the US and take the log level of GDP per capita (y_t) at the quarterly frequency and the index of industrial production at the monthly frequency. The log of total hours worked ($n_t = log(h_t \times e_t)$) is obtained by taking the quarterly or monthly series for the average weekly number of hours in the manufacturing sector (h_t) and total employment (e_t). Last, the growth rate of labor productivity obtains easily by combining both time series ($\Delta(y_t - n_t)$). In both the monthly and the quarterly set-up, the time range goes from the beginning of the year 1979 and ends at the end of the year 2015. The estimated VAR includes a constant and a trend:

$$X_t = \alpha_1 + \alpha_2 \times t + \alpha_3 (L) X_t + \alpha_4 \xi_t$$

where the vector of variables included in the VAR is thus $X_t = [\Delta(y_t - n_t)\ n_t]'$. Lag selection proceeds through Akaike information criterion. The structural shocks are identified using long-run restrictions *à la* Blanchard and Quah: demand shocks are assumed to be those that *do not* have permanent effects on the *level* of labor productivity. By contrast, the other shock is a productivity shock. Once the productivity shock has been identified, we estimate a bi-variate VAR including the trade dispute variable and the identified shock. In the quarterly specification, the trade dispute variable is simply the log of one plus the number of trade disputes initiated in the corresponding quarter. In the monthly specification, because of the presence of many null observations, we build a measure of the stock of trade disputes initiated over the last 6 months and the relevant variable is the log of one plus the stock of trade disputes over the last 6 months. The second estimated VAR also includes a constant and a trend:

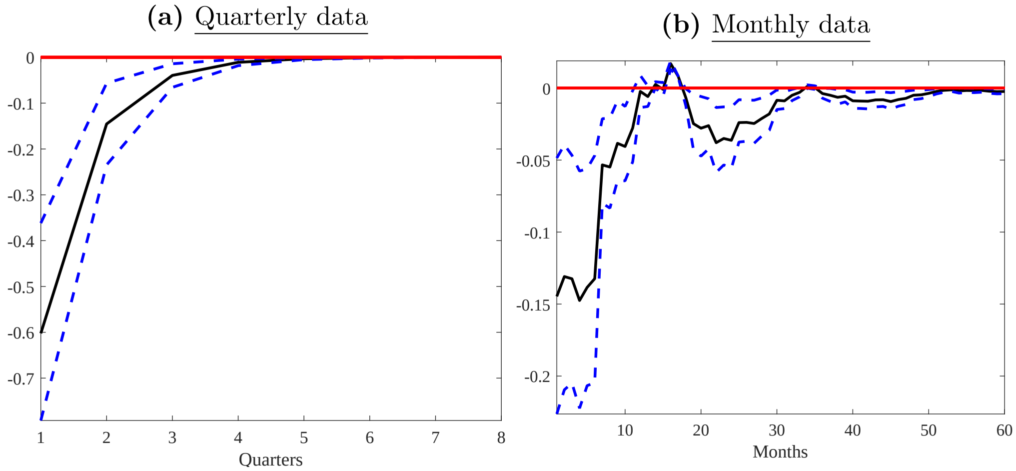
$$X_t = \alpha_1 + \alpha_2 \times t + \alpha_3 (L) X_t + \alpha_4 \xi_t$$

where the vector of variables included in the VAR is thus $X_t = [\log(1 + TD_t)\ Pdtyshock_t]'$. We report the effects of an innovation to the productivity shock in Figure 1 and show that a positive supply shock generates a substantial and statistically significant fall in trade disputes on impact and for roughly one year. Hence, conditional on productivity shocks driving the business cycle, we find evidence of counter-cyclical demand for protection, in line with the theoretical intuitions put forth by Bagwell and Staiger (1999).

2.2 Monetary policy shocks

We also investigate the response of trade protection to a monetary policy shock, identified using a Cholesky decomposition of the variance-covariance matrix of innovations, where the nominal interest rate is ranked last. The assumption implies that monetary policy shocks do not have any contemporaneous effects on all the variables included in the VAR. We consider the following variables and ranking: the log of one plus a measure of trade disputes, the log of GDP (or index of industrial production at a monthly frequency), a measure of oil-price inflation, the CPI inflation rate, a trade-weighted measure of the real exchange rate against all US trade partners and the

Figure 1: Responses of trade disputes to a permanent positive supply shock.



Note: Confidence bands respectively represent the 16^{th} and 84^{th} percentiles of IRFs. The trade dispute variable is $\log(1+TD_t)$ in the quarterly specification and $\log(1+\sum_{s=t-5}^{s=t} TD_s)$ in the monthly specification, where TD_t denotes the number of trade disputes initiated at time t.

nominal interest rate, measured by the Fed fund rate. All the data are taken from the FRED database except trade disputes, taken from the GAD database. The estimated VAR specification includes a constant and a trend:

$$X_t = \alpha_1 + \alpha_2 \times t + \alpha_3 (L) X_t + \alpha_4 \xi_t \tag{1}$$

where $X_t = [log(1 + TD_t) \ log(Y_t) \ 100 \times \pi_t \ 100 \times \pi_t^{oil} \ \log(RER_t) \ 100 \times i_t]'$, and lag selection proceeds through Akaike information criterion. Figure 2 reports the effects of a monetary policy shock identified through short-run restrictions using either the quarterly and monthly specification.

Figure 2 shows that an identified increase in the Fed fund rate lowers the level of output persistently and induces a slight fall in CPI inflation, consistently with previous results of the literature. The response of our measure of trade disputes is negative on impact and for roughly two quarters, before it overshoots and then returns to zero. Focusing on the impact and immediate response, we find that the demand for protection is pro-cyclical conditional on monetary policy shocks. This result goes against the idea that trade protection or the demand for trade protection is unconditionally counter-cyclical, but instead shows that the cyclical pattern critically depends on the nature of the macroeconomic shocks hitting the economy.

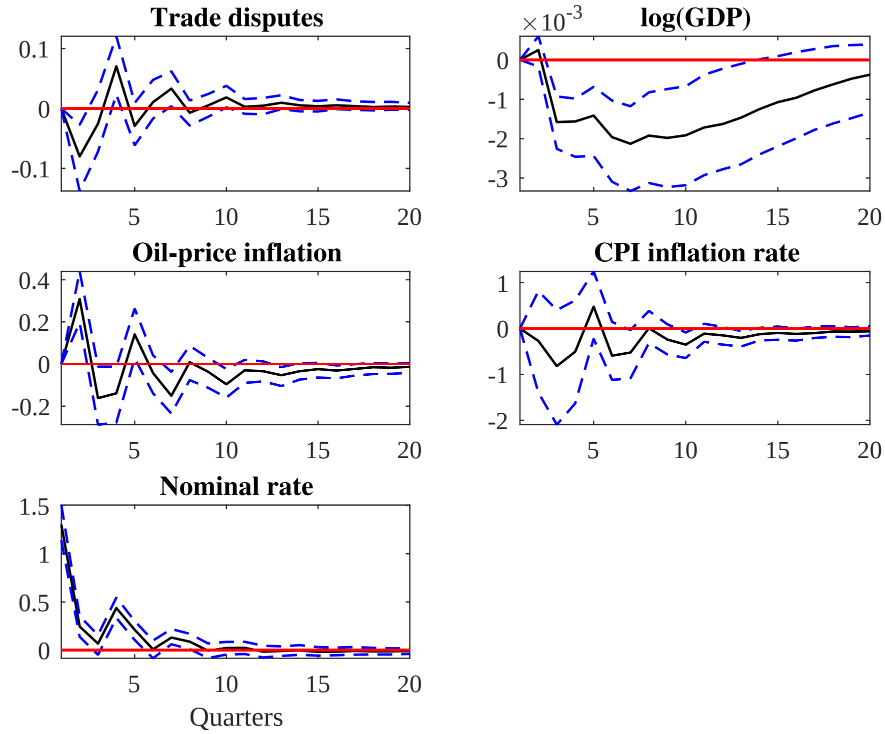
3 Model

3.1 Households

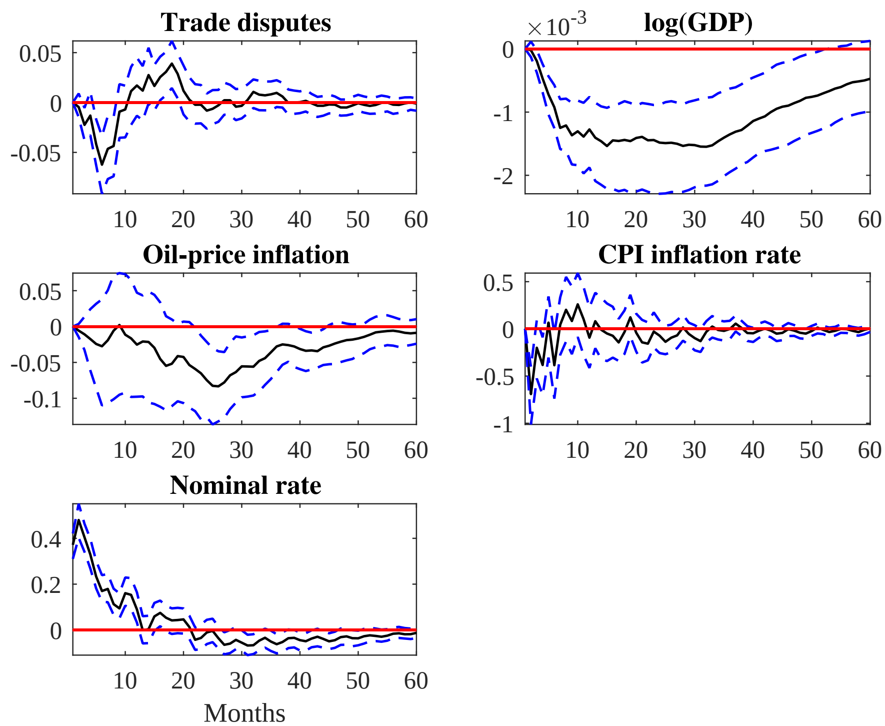
We consider a two-country open economy with a flexible exchange rate, Calvo pricing contracts, and incomplete financial markets. The Calvo pricing scheme applies to both domestic sales prices and export prices, so that local currency pricing (LCP) applies in the short run and producer

Figure 2: Responses to a restrictive monetary policy shock.

(a) Quarterly data



(b) Monthly data



Note: Confidence bands respectively represent the 16^{th} and 84^{th} percentiles of IRFs. The trade dispute variable is $\log(1+TD_t)$ in the quarterly specification and $\log(1+\sum_{s=t-5}^{s=t} TD_s)$ in the monthly specification, where TD_t denotes the number of trade disputes initiated at time t.

currency pricing (PCP) applies in the long run. We also introduce trade in intermediate goods, and consider monetary policy assuming that Central Banks commit to Taylor-type rules, subject to unexpected monetary policy shocks. The economy is made of two countries, Home and Foreign. The Home economy produces the final good h and the Foreign economy the final good f. Variables pertaining to the Foreign economy are denoted with an asterisk.

Home. The representative household of the Home economy maximizes a welfare index

$$E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} u\left(c_s, \ell_s\right) \right\}, 0 < \beta < 1 \tag{2}$$

subject to the following budget constraint

$$s_t b_{ft} + b_{ht} + p_{ht} c_{ht} + \left(1 + \tau_t\right) p_{ft} c_{ft} + p_t \Phi_t = s_t r_{t-1}^* b_{ft-1} + r_{t-1} b_{ht-1} + w_t \ell_t + \Pi_t - T_t \tag{3}$$

In Equation (2), β is the subjective discount factor, c_t is the level of aggregate consumption, a composite of domestic goods in quantity c_{ht} and Foreign goods in quantity c_{ft}, and ℓ_t is the level of hours worked. In Equation (3), b_{ft} is the amount of one-period international bonds denominated in the Foreign currency, that pay a gross return r_t^* between period $t-1$ and period t, and b_{ht} the amount of domestic bonds, that pay a gross return r_t. Variables p_{ht} and p_{ft} respectively denote the domestic prices of the Home and Foreign goods expressed in units of the domestic currency, p_t (defined below) is the domestic CPI. Imports of final goods are subject to tariffs at the rate τ_t where the proceeds are rebated to the household by the government. The nominal wage is w_t, $\Pi_t = \int_0^1 \Pi_t(i) di$ denotes the nominal profit paid by the monopolistic firms to the domestic household, and T_t is a lump-sum tax. Finally, $\Phi_t = \frac{\nu}{2}\left(\frac{s_t b_{ft}}{p_t} - \frac{s b_f}{p}\right)^2$ is a portfolio adjustment cost paid on international bonds. The domestic consumption basket has the following composition

$$c_t = \left((1-\omega)^{\frac{1}{\mu}} c_{ht}^{\frac{\mu-1}{\mu}} + \omega^{\frac{1}{\mu}} c_{ft}^{\frac{\mu-1}{\mu}} \right)^{\frac{\mu}{\mu-1}} \tag{4}$$

Home and Foreign goods are imperfectly substitutable with elasticity of substitution $\mu > 0$. In addition, households' preferences are biased towards the local good, as $\omega \in [0, 1/2]$ denotes the share of imported goods in the consumption bundle of the domestic households. Optimal good demands are derived maximizing c_t for a given total expenditure on goods $p_{ht} c_{ht} + \left(1 + \tau_t\right) p_{ft} c_{ft}$:[5]

$$c_{ht} = (1-\omega) \left(\frac{p_{ht}}{p_t}\right)^{-\mu} c_t \text{ and } c_{ft} = \omega \left(\frac{\left(1+\tau_t\right) p_{ft}}{p_t}\right)^{-\mu} c_t \tag{5}$$

where

$$p_t = \left((1-\omega) p_{ht}^{1-\mu} + \omega \left(\left(1+\tau_t\right) p_{ft}\right)^{1-\mu} \right)^{\frac{1}{1-\mu}} = p_{ht} \left(1 - \omega + \omega \left(\left(1+\tau_t\right) \Lambda_t^* q_t\right)^{1-\mu}\right)^{\frac{1}{1-\mu}} \tag{6}$$

[5]The payment of the portfolio adjustment cost on Foreign bonds gives rise to similar final good demand functions.

and $q_t = s_t p^*_{ft}/p_{ht}$ denotes the terms of trade, s_t is the nominal exchange rate and $\Lambda^*_t = p_{ft}/s_t p^*_{ft}$ captures the Foreign deviations from the law of one price. These conditions imply that

$$p_{ht}c_{ht} + (1+\tau_t)\, p_{ft}c_{ft} = p_t c_t \tag{7}$$

This two-country two-goods set-up with imperfect substitution across goods is known to give rise to a terms-of-trade externality, by which tariff setters can engineer an appreciation of their terms of trade by raising tariffs. The latter increases the relative price of the local good, the wealth of local households and produces a rise in their level of consumption. This externality is induced by the shape of consumption baskets, and would therefore apply in a pure exchange (endowment) economy. Its strength is thus tailored by the degree of home bias and by the extent of substitutability between both goods. For tariff setters, the larger the degree of home bias $(1-\omega)$, the larger the possibility to manipulate terms of trade, and the larger the gains in terms of the consumption of Foreign goods – and the lower the losses in terms of the consumption of Home goods. In addition, the lower substitutability between both goods, the larger market power tariff setters have over the price of their national good, and therefore the larger their ability to exploit the terms-of-trade externality. In the limiting case of a perfect substitutability between both goods $(\mu \to \infty)$, market power vanishes and so does the ability of tariff setters to manipulate terms of trade. The remaining first-order conditions imply:

$$\beta E_t \left\{ \frac{u_c(c_{t+1},\ell_{t+1})}{u_c(c_t,\ell_t)} \frac{r_{t+1}}{\pi_{t+1}} \right\} = 1 \tag{8}$$

$$\beta E_t \left\{ \frac{u_c(c_{t+1},\ell_{t+1})}{u_c(c_t,\ell_t)} \frac{r^*_t}{\pi_{t+1}} \frac{s_{t+1}}{s_t\left(1+\nu\left(d_t-d\right)\right)} \right\} = 1 \tag{9}$$

$$-\frac{u_\ell(c_t,\ell_t)}{u_c(c_t,\ell_t)} - \frac{w_t}{p_{ht}} \left(1-\omega+\omega\left(\left(1+\tau_t\right)\Lambda^*_t q_t\right)^{1-\mu}\right)^{\frac{1}{\mu-1}} = 0 \tag{10}$$

where $\pi_t = p_t/p_{t-1}$ is the gross CPI inflation rate and where $d_t = s_t b_{ft}/p_t$ denotes the real net foreign assets. The first two equations are standard Euler equations, which implies that a modified uncovered IUP condition holds in the model when both are combined. The third one is the labor supply equation. The latter makes apparent an additional channel through which the terms-of-trade externality operates. Indeed, while the real wage is paid in units of the Home final good, its purchasing power, that enters in the labor supply equation, is expressed in units of the consumption goods that incorporates both Home and Foreign final goods. Hence, an appreciation of the terms of trade raises the purchasing power of the labor wage, which allows households to consume more and supply less labor through the presence of the wealth effect on labor supply. Hence, applying positive tariffs will not only improve consumption, but also lower labor supply, therefore generating additional utility gains.

Foreign. Let us now turn to the description of the the Foreign representative household, which is almost perfectly symmetric. In particular, the labor supply equation arising from the optimization is

$$-\frac{u_\ell\left(c_t^*,\ell_t^*\right)}{u_c\left(c_t^*,\ell_t^*\right)}-\frac{w_t^*}{p_{ft}^*}\left(1-\omega+\omega\left(\left(1+\tau_t\right)\Lambda_t/q_t\right)^{1-\mu}\right)^{\frac{1}{\mu-1}}=0 \tag{11}$$

where $\Lambda_t = s_t p_{ht}^*/p_{ht}$ stands for the Home deviations from the law of one price. In addition, given that Foreign households have access to local bonds at no cost, the Foreign Euler equation on Foreign bonds yields

$$\beta E_t\left\{\frac{u_c\left(c_{t+1}^*,\ell_{t+1}^*\right)}{u_c\left(c_t^*,\ell_t^*\right)}\frac{r_t^*}{\pi_{t+1}^*}\right\}=1 \tag{12}$$

We further assume that the Foreign household does not have access to bonds issued in the Home economy, without loss of generality. Finally, the Foreign consumption basket is

$$c_t^*=\left((1-\omega)^{\frac{1}{\mu}}\,c_{ft}^{*\frac{\mu-1}{\mu}}+\omega^{\frac{1}{\mu}}c_{ht}^{*\frac{\mu-1}{\mu}}\right)^{\frac{\mu}{\mu-1}} \tag{13}$$

where $\omega^* \in [0,1/2]$ denotes the share of Home goods in the Foreign consumption bundle. Optimization yields

$$c_{ft}^*=(1-\omega)\left(\frac{p_{ft}^*}{p_t^*}\right)^{-\mu}c_t^*\text{ and }c_{ht}^*=\omega\left(\frac{\left(1+\tau_t^*\right)p_{ht}^*}{p_t^*}\right)^{-\mu}c_t^* \tag{14}$$

where τ_t^* is the tariff rate on imports imposed by the Foreign tariff setter. p_{ft}^* and p_{ht}^* are the prices of Foreign and Home goods, expressed in units of the Foreign currency and the corresponding Foreign CPI writes

$$p_t^*=\left((1-\omega)\,p_{ft}^{*1-\mu}+\omega\left(\left(1+\tau_t^*\right)p_{ht}^*\right)^{1-\mu}\right)^{\frac{1}{1-\mu}}=p_{ft}^*\left(1-\omega+\omega\left(\left(1+\tau_t^*\right)\Lambda_t/q_t\right)^{1-\mu}\right)^{\frac{1}{1-\mu}} \tag{15}$$

3.2 Firms

Production and marginal costs. In each country, a unit continuum of firms indexed in i produce varieties of each type of good using a combination of local labor ℓ and imported intermediate goods x:

$$y_t(i)=a_t x_t(i)^\alpha\,\ell_t(i)^{1-\alpha}\text{ and }y_t^*(i)=a_t^* x_t^*(i)^\alpha\,\ell_t^*(i)^{1-\alpha} \tag{16}$$

where $(\alpha,\alpha^*) \in [0,1/2]^2$ stand for the shares of intermediate imports in the Home and Foreign economy respectively. Variables a_t and a_t^* are exogenous measures of productivity, following AR(1) processes

$$\log a_t=\rho_a\log a_{t-1}+\xi_{at}\text{ and }\log a_t^*=\rho_{a^*}\log a_{t-1}^*+\xi_{at}^* \tag{17}$$

Before solving for optimal prices, cost minimization by firms in each country implies:

$$\alpha\varphi_t(i)\frac{y_t(i)}{x_t(i)}=\left(1+\tau_t\right)\Lambda_t^* q_t\text{ and }(1-\alpha)\,\varphi_t(i)\frac{y_t(i)}{\ell_t(i)}=\frac{w_t}{p_{ht}} \tag{18}$$

$$\alpha\varphi_t^*(i)\frac{y_t^*(i)}{x_t^*(i)}=\left(1+\tau_t^*\right)\Lambda_t/q_t\text{ and }(1-\alpha)\,\varphi_t^*(i)\frac{y_t^*(i)}{\ell_t^*(i)}=\frac{w_t^*}{p_{ft}^*} \tag{19}$$

where $\varphi_t(i)$ and $\varphi_t^*(i)$ are the real marginal production costs, implying

$$\varphi_t(i) = \varphi_t = \frac{((1+\tau_t)\Lambda_t^* q_t)^{\alpha}(w_t/p_{ht})^{1-\alpha}}{a_t\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \tag{20}$$

$$\varphi_t^*(i) = \varphi_t^* = \frac{((1+\tau_t^*)\Lambda_t/q_t)^{\alpha}\left(w_t^*/p_{ft}^*\right)^{1-\alpha}}{a_t^*\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \tag{21}$$

Pricing decisions. Prices are then set optimally subject to Calvo contracts whereby a randomly chosen proportion $1-\theta$ of firms are able to change their prices. As in Benigno (2004), re-setters chose two prices, one for the local market and one for the export market, maximizing the discounted sum of nominal profits in the event they are unable to reset in the future:

$$\max_{\widetilde{p}_{ht}(i),\widetilde{p}_{ht}^*(i)} E_t\left\{\sum_{s=t}^{\infty}(\theta\beta)^{s-t}\lambda_s\Pi_s(i)\right\} \text{ and } \max_{\widetilde{p}_{ft}^*(i),\widetilde{p}_{ft}(i)} E_t\left\{\sum_{s=t}^{\infty}(\theta\beta)^{s-t}\lambda_s^*\Pi_s^*(i)\right\} \tag{22}$$

where λ_t and λ_t^* are the Lagrange multipliers associated with the households budget constraints. Firms take into account the probability θ of being unable to reset the price in the future and compute profits and good demands accordingly. Profits write

$$\Pi_t(i) = (1-\tau_y)\left(\widetilde{p}_{ht}(i)\,y_{ht}(i) + s_t\widetilde{p}_{ht}^*(i)\,y_{ht}^*(i)\right) - p_{ht}\varphi_t\left(\underbrace{y_{ht}(i)+y_{ht}^*(i)}_{y_t(i)}\right) \tag{23}$$

$$\Pi_t^*(i) = \left(1-\tau_y^*\right)\left(\widetilde{p}_{ft}^*(i)\,y_{ft}^*(i) + s_t^{-1}\widetilde{p}_{ft}(i)\,y_{ft}(i)\right) - p_{ft}^*\varphi_t\left(\underbrace{y_{ft}^*(i)+y_{ft}(i)}_{y_t^*(i)}\right) \tag{24}$$

The demands for varieties depend on total demand, on the relative price of variety i and on the elasticities of substitution between varieties $\eta > 1$:

$$y_{ht}(i) = \left(\frac{\widetilde{p}_{ht}(i)}{p_{ht}}\right)^{-\eta} y_{ht} = \left(\frac{\widetilde{p}_{ht}(i)}{p_{ht}}\right)^{-\eta}(c_{ht}+\Phi_{ht}) \tag{25}$$

$$y_{ht}^*(i) = \left(\frac{\widetilde{p}_{ht}^*(i)}{p_{ht}^*}\right)^{-\eta} y_{ht}^* = \left(\frac{\widetilde{p}_{ht}^*(i)}{p_{ht}^*}\right)^{-\eta}(c_{ht}^*+x_t^*) \tag{26}$$

$$y_{ft}^*(i) = \left(\frac{\widetilde{p}_{ft}^*(i)}{p_{ft}^*}\right)^{-\eta^*} y_{ft}^* = \left(\frac{\widetilde{p}_{ft}^*(i)}{p_{ft}^*}\right)^{-\eta^*} c_{ft}^* \tag{27}$$

$$y_{ft}(i) = \left(\frac{\widetilde{p}_{ft}(i)}{p_{ft}}\right)^{-\eta^*} y_{ft} = \left(\frac{\widetilde{p}_{ft}(i)}{p_{ft}}\right)^{-\eta^*}(c_{ft}+\Phi_{ft}+x_t) \tag{28}$$

where x_t and x_t^* are the aggregate demands of intermediate goods. Finally, τ_y and τ_{y^*} are taxes introduced by the government to offset the steady-state distortions implied by monopolistic competition. Optimal pricing conditions are standard and reported in Appendix A. With a large number of firms and individual prices facing the same probability of being reset, the price levels are given by:

$$p_{ht} = \left(\theta p_{ht-1}^{1-\eta} + (1-\theta)\,\widetilde{p}_{ht}^{1-\eta}\right)^{\frac{1}{1-\eta}} \text{ and } p_{ht}^{*} = \left(\theta p_{ht-1}^{*1-\eta} + (1-\theta)\,\widetilde{p}_{ht}^{*1-\eta}\right)^{\frac{1}{1-\eta}} \quad (29)$$

$$p_{ft}^{*} = \left(\theta p_{ft-1}^{*1-\eta} + (1-\theta)\,\widetilde{p}_{ft}^{*1-\eta}\right)^{\frac{1}{1-\eta}} \text{ and } p_{ft} = \left(\theta p_{ft-1}^{1-\eta} + (1-\theta)\,\widetilde{p}_{ft}^{1-\eta}\right)^{\frac{1}{1-\eta}} \quad (30)$$

3.3 Governments and aggregation

We assume that the steady-state levels of taxes are set to correct mark-up distortions on varieties. As such, the tax rates are negative, $\tau_y = 1/(1-\eta)^{-1}$ and $\tau_{y^*} = 1/(1-\eta^*)^{-1}$, and the corresponding expenditure is financed using the proceeds from import taxes and a lump-sum tax on households. Government budget constraints are thus

$$T_t + \tau_t p_{ft} y_{ft} + \tau_y \left(p_{ht} y_{ht} + s_t p_{ht}^* y_{ht}^*\right) = 0 \text{ and } T_t^* + \tau_t^* p_{ht}^* y_{ht}^* + \tau_{y^*}\left(p_{ft}^* y_{ft}^* + s_t^{-1} p_{ft} y_{ft}\right) = 0 \quad (31)$$

The aggregate demands of imported intermediate goods x_t and x_t^* are

$$x_t = \alpha\left((1+\tau_t)\,\Lambda_t^* q_t\right)^{-1} \varphi_t y_t \text{ and } x_t^* = \alpha\left((1+\tau_t^*)\,\Lambda_t/q_t\right)^{-1} \varphi_t^* y_t^* \quad (32)$$

and market clearing conditions are:

$$y_t = y_{ht} + y_{ht}^* \quad (33)$$

$$y_t^* = y_{ft}^* + y_{ft} \quad (34)$$

where

$$y_{ht} = (1-\omega)\left(1-\omega+\omega\left((1+\tau_t)\,\Lambda_t^* q_t\right)^{1-\mu}\right)^{\frac{\mu}{1-\mu}} (c_t + \Phi_t) \quad (35)$$

$$y_{ht}^* = \omega\left((1-\omega)\left((1+\tau_t^*)\,\Lambda_t/q_t\right)^{\mu-1} + \omega\right)^{\frac{\mu}{1-\mu}} c_t^* + x_t^* \quad (36)$$

$$y_{ft}^* = (1-\omega)\left(1-\omega+\omega\left((1+\tau_t^*)\,\Lambda_t/q_t\right)^{1-\mu}\right)^{\frac{\mu}{1-\mu}} c_t^* \quad (37)$$

$$y_{ft} = \omega\left((1-\omega)\left((1+\tau_t)\,\Lambda_t^* q_t\right)^{\mu-1} + \omega\right)^{\frac{\mu}{1-\mu}} (c_t + \Phi_t) + x_t \quad (38)$$

As in many international macroeconomics models, we assume that local bonds are in zero net supply $b_{ht} = 0$. The dynamics of Home net foreign assets d_t obtain through the aggregation of all budget constraints in the Home economy:

$$d_t - \left(\frac{s_t}{s_{t-1}} \frac{r_{t-1}^*}{\pi_t}\right) d_{t-1} = \frac{p_{ht}}{p_t}\left(\Lambda_t y_{ht}^* - q_t \Lambda_t^* y_{ft}\right) \quad (39)$$

In addition, the clearing condition on Foreign bonds gives $b_{ft}^{*} + b_{ft} = 0$ which implies

$$q_t^r d_t^* + d_t = 0 \tag{40}$$

where, recall, $d_t = s_t b_{ft}/p_t$ and $d_t^* = b_{ft}^*/p_t^*$ are the real net foreign asset positions, and where $q_t^r = s_t p_t^*/p_t$ is the real exchange rate. We consider an environment of flexible exchange rates, where both Central Banks follow simple monetary policy rules targeting CPI inflation rates:

$$\log(r_t/r) = \rho_r \log(r_{t-1}/r) + (1-\rho_r)\, d_\pi \log(\pi_t/\pi) + \xi_{rt} \tag{41}$$

$$\log(r_t^*/r^*) = \rho_r \log\left(r_{t-1}^*/r^*\right) + (1-\rho_r)\, d_\pi \log(\pi_t^*/\pi^*) + \xi_{rt}^* \tag{42}$$

where ξ_{rt} and ξ_{rt}^* are monetary policy shocks following AR1 processes.

4 Steady-state analysis with exogenous tariffs

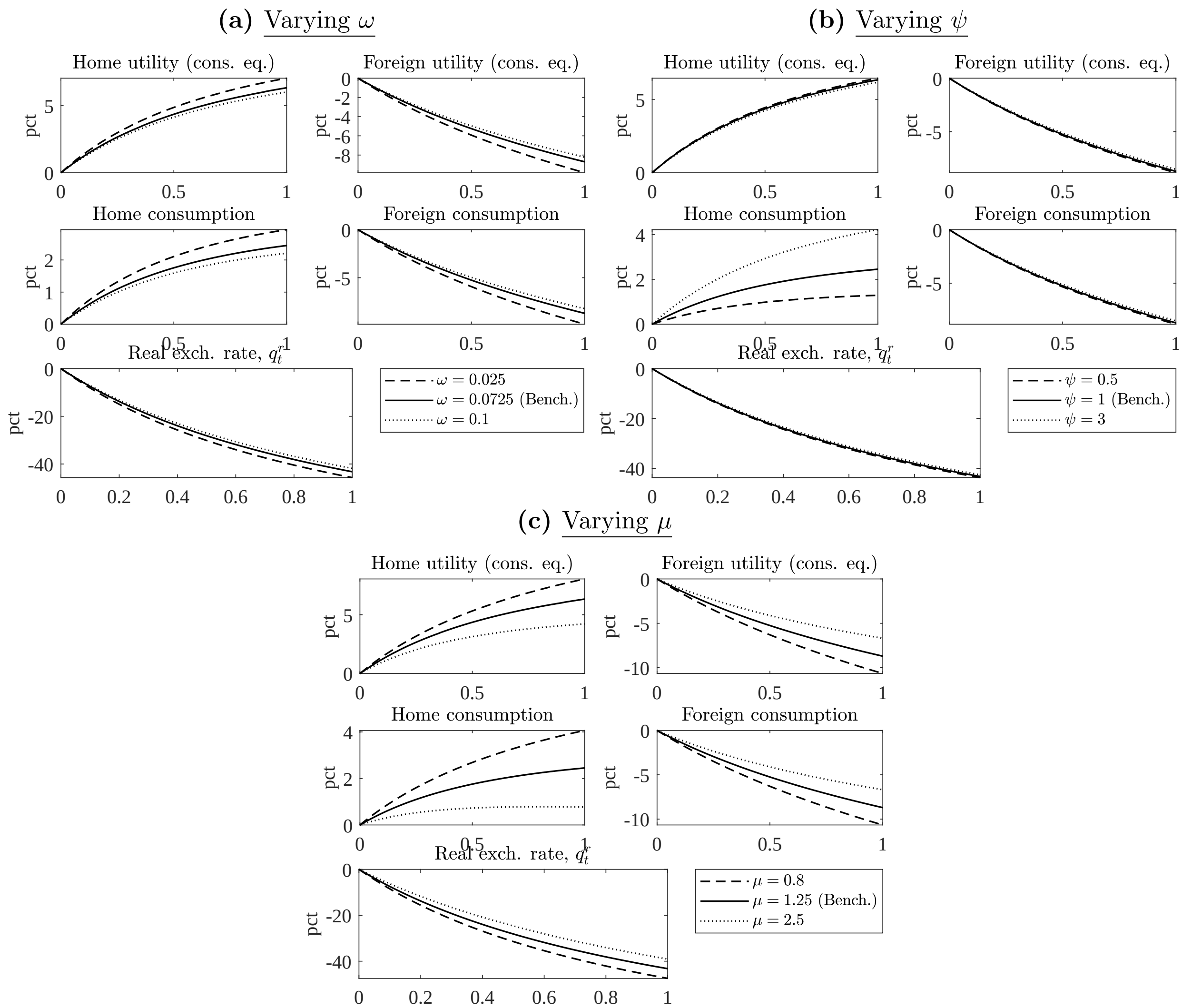
Let us first analyze the steady-state effects of applying tariffs unilaterally and exogenously. Starting from a situation of zero tariffs $\tau = \tau^* = 0$, we solve the steady state for a continuum of positive values in the Home economy $0 < \tau < 1$ while keeping the Foreign tariff at zero $\tau^* = 0$. We track the steady-state changes in utility – Hicksian consumption equivalents, aggregate consumption and the real exchange rate for the various steady-state levels of Home tariffs. We also contrast the effects resulting from the benchmark calibration with those obtained for alternative parameter values to shed light on how each of them affects the incentives to levy tariffs from a non-cooperative perspective.

The benchmark calibration is the following. The model is quarterly and the discount factor of households is $\beta = 0.99$. We specialize the utility function to a log specification

$$u(c_t, \ell_t) = \log c_t - \chi \ell_t^{1+\psi}/(1+\psi) \tag{43}$$

where ψ is the inverse of the Frisch elasticity of labor supply. We set $1/\psi = 1$ and consider alternative values in the steady-state exercise. Further, we assume $\chi = 1$, so as to let the level of hours worked vary in the steady state. Total trade openness is assumed to be 30%, as in U.S. data, which implies that total imports represent 15% of GDP. Therefore, we assume $\omega + \alpha = 0.15$, and set $\omega = 0.0725$ in the benchmark case, which also implies $\alpha = 0.0725$. In words, 50% of the trade flows between countries are flows of final goods while the remaining 50% are flows of intermediate goods. These parameters will also be subject to a sensitivity analysis to shed light on the role of the composition of trade on the incentives to set tariffs. In the benchmark case, the trade elasticity is $\mu = 1.25$. Again, alternative values are considered given the importance of this parameter. On the production side, we assume $\eta = \eta^* = 6$ and impose a negative tax that removes steady-state distortions induced by monopolistic competition: $\tau_y = 1/(1-\eta)$ and $\tau_{y^*} = 1/(1-\eta^*)$. Correcting the local monopolistic distortions makes the terms-of-trade externality the only motive to set tariffs non-cooperatively in our model.[6] Other parameters of the model affect its dynamic properties and not the steady state, and will be discussed in a dedicated section below. Figure 3 reports the steady-state results in each country for various levels of the Home tariff τ. We track the consumption equivalent (Hicksian) steady-state variation in utility, expressed in percents, and the percentage variation of Home and Foreign consumption, as well as the percentage variation of the real exchange rate. Each panel offers alternative values for key parameters. Figure 3 displays a set of striking results about the incentives faced by policymakers to impose tariffs in the steady-state.

Figure 3: Steady-state variables as a function of the Home tariff (τ), % change from a zero-tariff steady state.



Note: The x-axis is the Home tariff τ, keeping the Foreign tariff at zero, $\tau^* = 0$. Parameter ω is the steady-state share of final imports in output, $\alpha = 0.15 - \omega$ the steady-state share of intermediate imports in output, ψ the inverse of the Frisch elasticity of labor supply and μ the price elasticity of trade flows of final goods. The price elasticity of trade flows of intermediate goods is implicitly set to unity due to the functional form chosen for the production function.

[6] See Campolmi, Fadinger, and Forlati (2014) for a more general discussion of optimal policy with many instruments, including tariffs.

Figure 3 shows that raising the Home tariff while the Foreign tariff remains null always improves the Home terms of trade, *i.e.* the real exchange rate q falls. In addition, raising the steady-state Home tariff increases the utility of domestic households and lowers the utility of Foreign households. Tariffs are thus *beggar-thy-neighbor*, which is at the heart of the policy game we present in the next section: each policymaker has strong incentive to apply higher tariffs than their counterpart to raise the utility of local households at the expense of households in the other country. These results are known since Johnson (1953), and hold under any of the trade structures considered, and under all parameter values. The terms-of-trade externality remains at the heart of most papers focusing on non-cooperative trade policies. In addition, the Home utility gains from raising the Home tariff are smaller than the utility losses imposed on Foreign utility, so that unilateral tariffs, while attractive from a country-level perspective, are welfare damaging in terms of world-wide utility. Here too, the result holds true for alternative trade structures and for any combination of parameters investigated in Figure 3. Again, this result is not particularly novel and constitutes one of the most common arguments in favor of trade agreements (See Grossman (2016) for a recent survey).

How do changes in parameters shape the incentive to raise tariffs unilaterally? Figure 3 shows that the potential utility gains from raising tariffs unilaterally are smaller when trade is more intensive in final goods (ω large implying α small), larger when the labor supply elasticity $1/\psi$ is low (ψ large) as well as when the trade elasticity μ is low. With a larger share of trade in final goods ω, panel (a) of Figure 3 shows that unilateral tariffs generate smaller utility gains. When trade is more intensive in final goods, tariff setters have less room to manipulate terms of trade, which increases consumption less, producing smaller welfare gains. Hence, the presence and size of trade in intermediate goods magnifies the unilateral incentives to levy tariffs, as long as some trade in final goods remains. Further, as shown in panel (b) of Figure 3, when the labor supply elasticity is smaller (ψ is larger), the output response to changes in the terms of trade is dampened and the relative price of the Home good rises by less, leading aggregate consumption to rise more, and producing larger consumption utility gains from tariffs. However, given that hours worked are less responsive to changes in tariffs, they fall by less when ψ increases. Overall, since changes in hours worked are given a larger weight in utility as ψ increases, the larger changes in consumption are not reflected in larger changes in total utility gains or losses because the smaller welfare gains from hours worked almost perfectly compensate for the larger welfare gains from consumption.[7] Finally, the effects of the trade elasticity on final goods are depicted in panel (c) of Figure 3. As μ increases, the strength of the terms-of-trade externality decreases as the ability of tariff setters to manipulate terms of trade falls because their market power over local goods shrinks. Therefore, the utility gains of tariffs are larger whenever the elasticity of substitution between Home and Foreign goods μ is lower.

[7] This effect on utility would be absent in a pure exchange economy or with an exogenous amount of labor supply.

5 Sustainable tariffs

5.1 Policy game and endogenous tariffs

We now consider that tariffs are determined endogenously based on the following policy game. Tariff setters can choose between being on the sustainable tariff path, and cheating on this path of tariffs. If the Home (resp. Foreign) country cheats but not the Foreign (Home) country, the Home (Foreign) country gets to apply higher tariffs and manipulate its terms of trade with some short term gains in their objective function. However, cheating by one of the two countries triggers a retaliation of the other country, in which case the equilibrium switches to the punishment equilibrium where both countries apply a maximum tariff τ^H forever, a situation labeled below as the worst Nash equilibrium.

Let us denote variables under the worst Nash equilibrium where $\tau_t = \tau_t^* = \tau^H$ by a 'N' superscript, the variables under the equilibrium where the Home (Foreign) tariff setter cheats by a 'HC' ('FC') superscript, and the equilibrium under sustainable tariffs with a 'S' superscript, in which case $\tau_t = \tilde{\tau}_t$ and $\tau_t^* = \tilde{\tau}_t^*$. The value functions for Home and Foreign when tariffs are on the sustainable path are:

$$v_t^S = u\left(c\left(\tilde{\tau}_t, \tilde{\tau}_t^*\right), \ell\left(\tilde{\tau}_t, \tilde{\tau}_t^*\right)\right) + \beta^g E_t\left\{v_{t+1}^S\right\} \text{ and } v_t^{S*} = u\left(c^*\left(\tilde{\tau}_t, \tilde{\tau}_t^*\right), \ell^*\left(\tilde{\tau}_t, \tilde{\tau}_t^*\right)\right) + \beta^{g*} E_t\left\{v_{t+1}^{S*}\right\} \quad (44)$$

where β^g and β^{g*} respectively denote the Home and Foreign discount factors of tariff setters. We choose to consider a different discount factor for tariff setters because tariffs are in the hands of the government administration and, in regard of political economy considerations, the latter make decisions at shorter horizons than households, therefore implying $\beta^g < \beta$.[8] The value of cheating for the Home and Foreign countries are respectively:

$$v_t^{HC} = u\left(c\left(\tau^H, \tilde{\tau}_t^*\right), \ell\left(\tau^H, \tilde{\tau}_t^*\right)\right) + \beta^g E_t\left\{v_{t+1}^N\right\} \text{ and } v_t^{FC*} = u\left(c^*\left(\tilde{\tau}_t, \tau^H\right), \ell^*\left(\tilde{\tau}_t, \tau^H\right)\right) + \beta^{g*} E_t\left\{v_{t+1}^{N*}\right\} \quad (45)$$

while the Home and Foreign value functions associated with the Nash equilibrium write:

$$v_t^N = u\left(c\left(\tau^H, \tau^H\right), \ell\left(\tau^H, \tau^H\right)\right) + \beta^g E_t\left\{v_{t+1}^N\right\} \text{ and } v_t^{N*} = u\left(c^*\left(\tau^H, \tau^H\right), \ell^*\left(\tau^H, \tau^H\right)\right) + \beta^{g*} E_t\left\{v_{t+1}^{N*}\right\} \quad (46)$$

The conditions that determine sustainable tariffs in both countries are then

$$E_t\left\{v_{t+1}^S\right\} = E_t\left\{v_{t+1}^{HC}\right\} \text{ and } E_t\left\{v_{t+1}^{S*}\right\} = E_t\left\{v_{t+1}^{FC*}\right\} \quad (47)$$

These conditions determine both the levels and the variations of sustainable tariffs.

[8] See Grossman and Huyck (1988) or more recently Alfaro and Kanczuk (2017) for a justification of this assumption in the context of sovereign debt, or Acemoglu, Golosov, and Tsyvinski (2011) in the context of Ramsey optimal policies with self-interested politicians.

5.2 Solution and calibration

With endogenous tariffs, the equilibrium under sustainable tariffs in which $\tilde{\tau}_t$ and $\tilde{\tau}_t^*$ are determined by Equation (47) depends on three other sets of equilibrium conditions: those of the (worst) Nash equilibrium, those associated with the equilibrium in which the Home tariff setter cheats, and those of the equilibrium in which the Foreign tariff setter cheats, because the determination of sustainable tariffs depends on the value functions arising from these three equilibria. A solution of the model thus implies solving the joint system formed by the equations of the model under the four equilibria, by the value functions given by Eqs. (44)-(46) and by the two equilibrium conditions in Equation (47). Indeed, shocks affect differently key variables under the four equilibria (Home cheats, Foreign cheats and worst Nash), all of which are required to determine the sustainable tariffs in each country. Our focus will however be on how key variables evolve in the sustainable equilibrium, other equilibria acting as threat points that do not actually describe how the economy behaves.

The system is solved under rational expectations using perturbation methods up to a first order using Dynare.[9] Part of our calibration has already been exposed in the previous section. We now justify the values of new parameters and parameters that affect the dynamics of the model. We set the maximum feasible tariff rate at sixty-two percent, so that $\tau_H = 0.62$. This is the average tariff rate estimated by Ossa (2014) that would apply in a full scale world "tariff war", and hence represents the appropriate limit for the Nash equilibrium tariff rate within our model. As explained above, we assume that tariff setters face lower discount factors. We adjust them to produce a benchmark level of tariffs of 4.65%, which is the world average reported by the World Bank between 1988 and 2017. It implies setting $\beta^g = \beta^{g*} = 0.7076$. In line with Schmitt-Grohe and Uribe (2003), we assume that the portfolio adjustment cost parameter is $\nu = 0.0007$. On the production side, we consider a $\theta = 0.75$ probability of being forced to keep the price constant both for domestic and export prices. As such, Local Currency Pricing (LCP) is the benchmark pricing scheme for exports, and Producer Currency Pricing (PCP) is investigated as an alternative.[10] Parameters of the Taylor rules are $\rho_r = 0.9$ and $d_\pi = 1.5$. In line with our empirical approach, we look at the macroeconomic effects of quasi-permanent productivity shocks and impose $\rho_a = \rho_{a^*} = 0.9999$. Finally, we set the standard deviations of productivity and monetary policy shocks to $\sigma(\xi_a) = \sigma(\xi_a^*) = 0.01$ and $\sigma(\xi_r) = \sigma(\xi_r^*) = 0.0025$. Table 1 summarizes our parameter values, including those presented in the previous section.

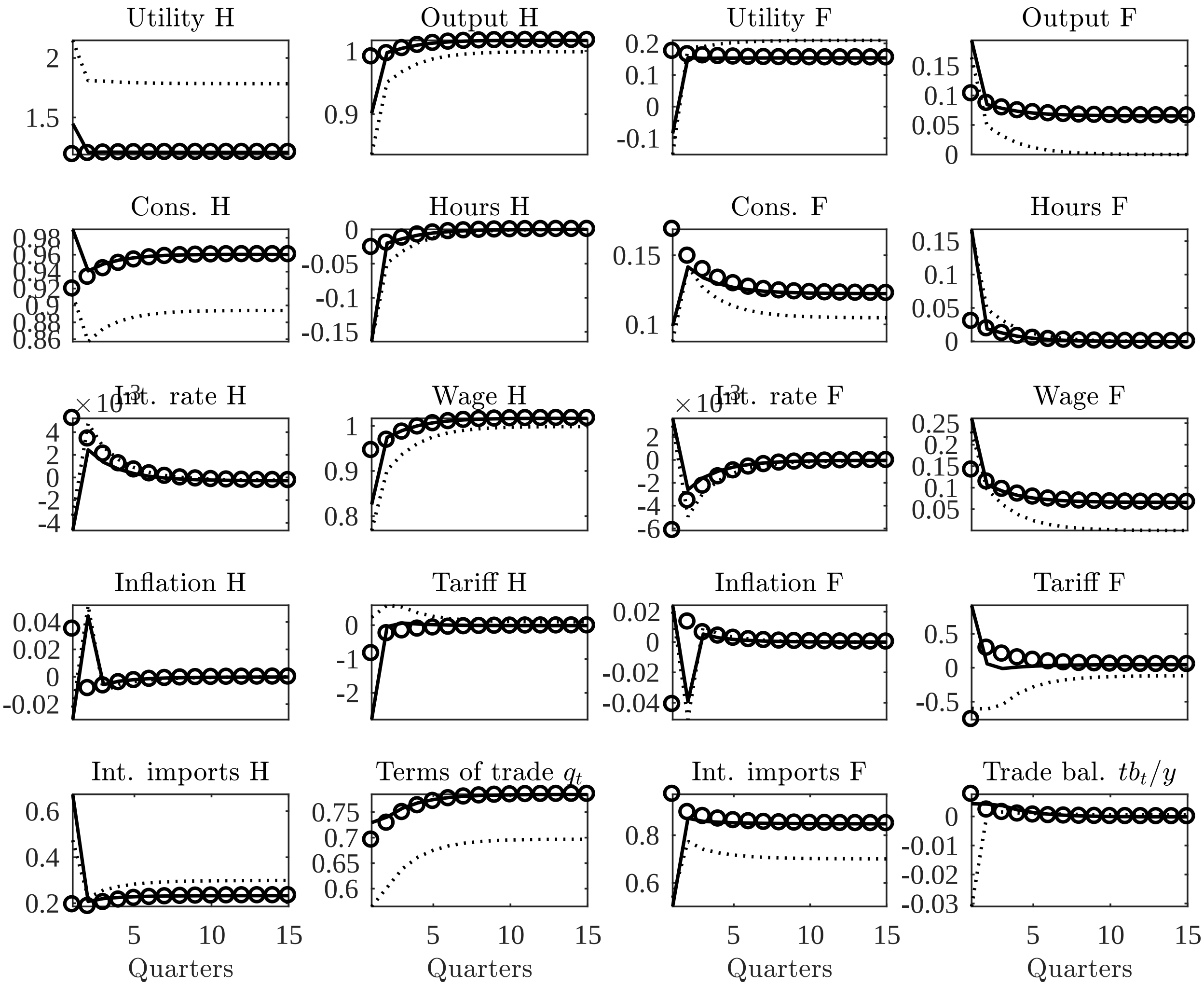
Figures 4 and 5 report the dynamics of sustainable tariffs respectively to a quasi-permanent productivity shock and to a restrictive monetary policy shock under alternative calibrations.

Our model of sustainable tariffs replicates the empirical evidence presented in Section 2, according to which trade protection falls after a permanent productivity shock and after a restrictive monetary

[9] See Adjemian, Bastani, Juillard, Karamé, Mihoubi, Perendia, Pfeifer, Ratto, and Villemot (2011).

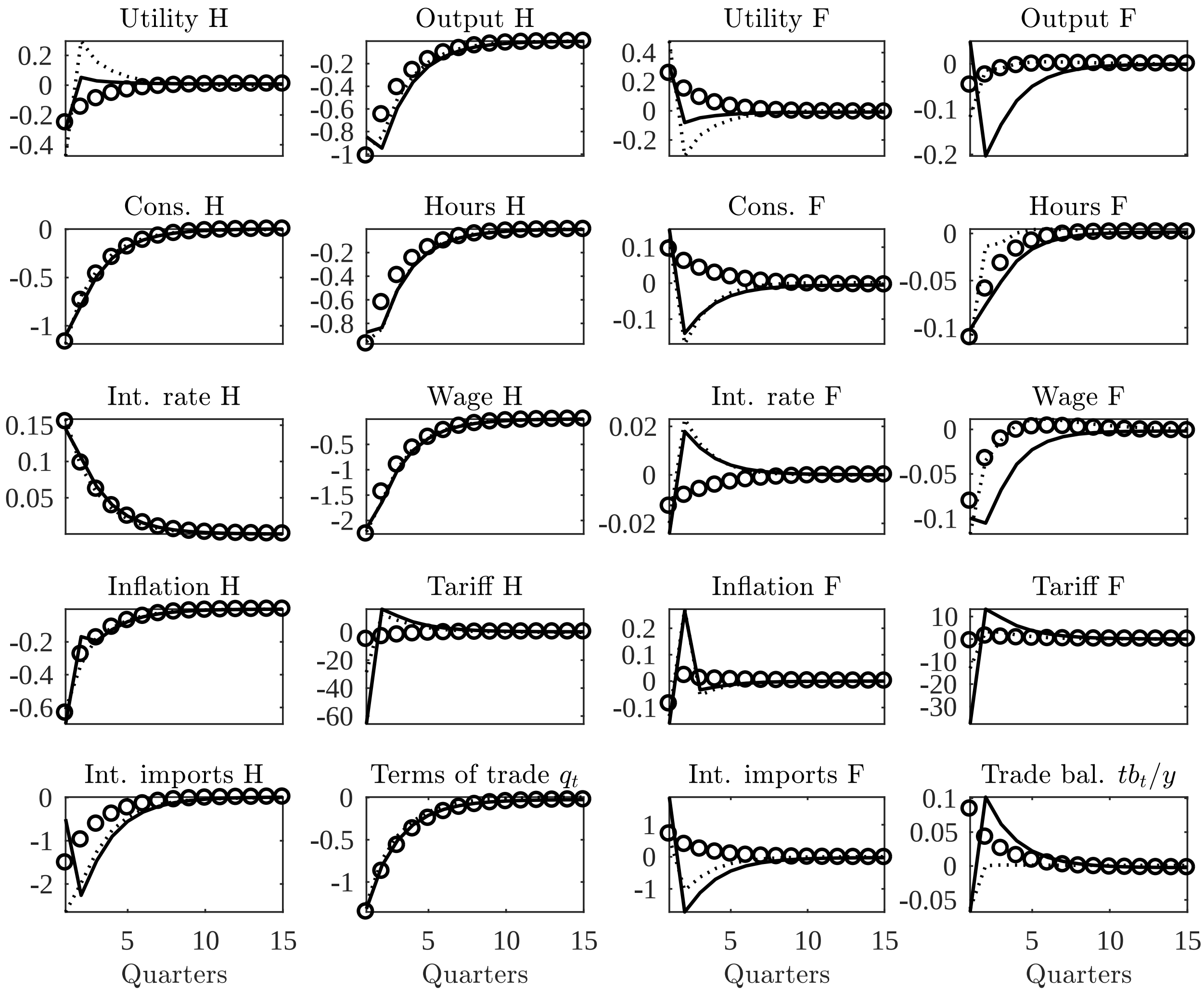
[10] In this case, the pricing conditions are modified to impose the law of one price on export prices, which is equivalent to setting the Calvo parameter to zero on export prices, *i.e.* $\theta^x = 0$.

Figure 4: IRFs to a quasi-permanent positive Home productivity shock on the path of sustainable tariffs, percent deviations from the steady state.



Solid: Benchmark calibration. Circled: Producer Currency Pricing ($\theta^x = 0$). Dashed: No trade in intermediate goods ($\alpha \approx 0$ and $\omega = 0.15$).

Figure 5: IRFs to a restrictive Home monetary policy shock on the path of sustainable tariffs, percent deviations from the steady state.



Solid: Benchmark calibration. Circled: Producer Currency Pricing ($\theta^x = 0$). Dashed: No trade in intermediate goods ($\alpha \approx 0$ and $\omega = 0.15$).

Table 1: Parameter values

Discount factor	$\beta = 0.99$
Frisch elasticity of labor supply	$1/\psi = 1$
Labor disutility parameter	$\chi = 1$
Portfolio adjustment cost	$\nu = 0.0007$
Total trade openness	$2(\omega + \alpha) = 0.3$
Share of final imports in GDP	$\omega = 0.075$
Share of intermediate imports in GDP	$\alpha = 0.075$
Trade elasticity for final goods	$\mu = 1.25$
Trade elasticity for intermediate goods	implicitly $= 1$
Elasticity of substitution btw varieties	$\eta = \eta^* = 6$
Steady-state tax rate on production	$\tau_y = \tau_{y^*} = (1-\eta)^{-1}$
Maximum feasible tariff	$\tau_H = 0.62$
Tariff setters discount factors	$\beta^g = \beta^{g*} = 0.7076$
Steady-state sustainable tariff	$\widetilde{\tau} = \widetilde{\tau}^* = 0.0465$
Calvo parameter (domestic prices)	$\theta = 0.75$
Calvo parameter (export prices)	$\theta^x = 0.75$
Taylor rule persistence parameter	$\rho_r = 0.9$
Taylor rule inflation parameter	$d_\pi = 1.5$
Persistence of productivity shocks	$\rho_a = 0.9999$
Standard deviation of productivity shocks	$\sigma(\xi_a) = 0.01$
Standard deviation of monetary policy shocks	$\sigma(\xi_r) = 0.0025$

policy shock. To gain insight on why this is the case, consider how productivity and restrictive monetary policy shocks affects the incentives to cheat, and focus on the Home economy.

Focus on productivity shocks first. As shown in Figure 8 in Appendix B, a positive productivity shock in the Home economy raises output substantially and permanently in both countries, which raises the output and consumption losses from cheating and applying tariffs. It raises the cost of cheating for the Home economy and therefore pushes the tariff setter to lower tariffs on impact (see Figure 4). Figure 8 in Appendix B also shows that a Home productivity shock raises Foreign output and consumption through enhanced good demands from the Home economy, and through the impact of the real exchange depreciation on Foreign inflation, which triggers a fall of the Foreign nominal rate. The Home supply shock thus acts as a positive demand shock in the Foreign economy, which raises the incentive for the Foreign tariff setter to widen tariffs on impact. Indeed, as output and consumption increase more in the Home economy than in the Foreign economy, the Home economy would bear most of the losses from switching to the worst Nash equilibrium and the Foreign economy would gain in the short run. To remain on the sustainable path of tariffs, the Foreign economy must be allowed – through the incentive compatibility constraint – to raise its tariff temporarily (see Figure 4 again).

Quantitatively speaking, Figure 4 shows that changes in tariffs are small in both countries (1.5 or 2% of 4.65%, which means that tariffs jump/fall from 4.65% to 4.56/4.74% at most). This is consistent with the fact that the IRFs to a permanent productivity shock in the Home economy are overall very similar on the path of sustainable tariffs (Figure 4) and under exogenous tariffs (Figure 8 in Appendix B). Indeed, a positive Home productivity shock raises Home output, real wages and consumption, lowers hours worked and the relative price of Home goods. Despite the "unfavorable" dynamics of the real exchange rate, intermediate imports rise, driven by the rise in Home output and the corresponding demand for intermediate goods. The transmission to the Foreign economy goes through a rise of Foreign exports, driven by the higher demand for Foreign goods – Home consumption and Home output rise, which raises the demand for Foreign final and intermediate goods, respectively – and through the imported deflation, which drives inflation down and induces the Foreign Central Bank to lower its nominal rate.

Let us now shift the focus to restrictive monetary policy shocks in the Home economy. Again, in this case, the sustainable tariff in the Home economy falls, in line with empirical evidence shown in Section 2. Since the shock implies that current output and consumption are lower than future output, cheating on the sustainable path would bring short-run gains and generate important future output and consumption losses, at a time where these two variables are at a higher level. This raises the future costs of cheating compared to the current gains, leading the Home tariff to fall (see Figure 5). Similar dynamics are at work in the Foreign economy, pushing the tariff down. A restrictive monetary policy shock in the Home economy also depresses the Foreign economy: the fall in aggregate demand at Home lowers the Home demand for Foreign final and intermediate goods through trade linkages. The drop in Foreign output and consumption is lower than in the Home economy because the deflationary pressures in the Foreign economy are partly accommodated by the Central Bank by lowering the nominal rate on impact. In addition, the appreciation of Home terms of trade partly alleviates the fall in the demand for Foreign goods.

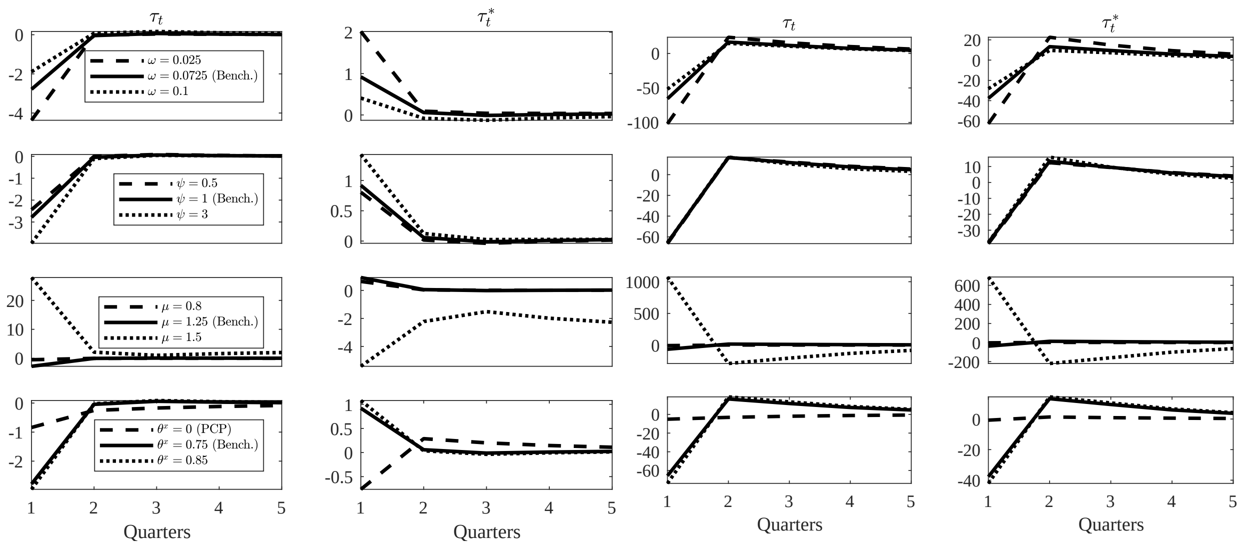
From a quantitative perspective, the size of tariff changes conditional on monetary policy shocks is much larger than conditional on productivity shocks: Figure 5 shows that the Home tariff falls by almost 60% and the Foreign tariff by more than 30% after a restrictive shock in the Home economy. This is especially true for the benchmark case with LCP, changes under PCP being much smaller. This additional result highlights the critical importance of nominal rigidities and pricing decisions in shaping the incentives to apply tariffs in the sustainable equilibrium conditional on monetary shocks. Nevertheless, this should not be too surprising given that nominal rigidities in general and LCP in particular govern the sensitivity of changes in terms of trade to changes in macroeconomic conditions in general, and to changes in tariffs in particular.

Our model of sustainable tariffs and endogenous trade protection thus produces conditional tariff dynamics that are consistent with empirical evidence presented in Section 2. Tariffs are pro-cyclical conditional on monetary policy shocks and counter-cyclical conditional on permanent productivity shocks. Focusing on tariffs only now, we report in Figure 6 their responses to the two shocks for various values of key parameters, namely the trade elasticity (μ), and the Frisch elasticity of labor supply $1/\psi$.

From the first line of Figure 6 we learn about the effects of the structure of trade flows on the dynamics of sustainable tariffs. When the share of final goods in trade flows ω is smaller, changes in

Figure 6: Responses of sustainable tariffs, in percentage deviation.

(a) Positive Home productivity shock **(b)** Restrictive Home MP shock



Note: Parameter ω is the steady-state share of final imports in output, $\alpha = 0.15 - \omega$ the steady-state share of intermediate imports in output, ψ the inverse of the Frisch elasticity of labor supply and μ the price elasticity of trade flows of final goods. $1 - \theta^x$ is the Calvo probability of adjusting export prices, $\theta^x = 0$ implying PCP. The price elasticity of trade flows of intermediate goods is implicitly set to unity due to the functional form chosen for the production function.

tariffs have less effects on macroeconomic variables because we have assumed a lower trade elasticity on intermediate goods trade flows than on final goods trade flows in the benchmark calibration. Hence, tariff setters have to apply larger changes in tariffs to achieve the same macroeconomic outcomes. Qualitatively however, our main result is robust to alternative trade structures.

Looking at the second line of Figure 6 shows that the inverse of the Frisch elasticity ψ on labor supply has a limited impact on our results, both qualitatively and quantitatively. While the relative size of wealth and substitution effects are crucial in shaping the sign and size of the terms-of-trade externality, considering a relatively wide range of different values for ψ does not alter much the dynamics of sustainable tariffs.

On the contrary, alternative values of the trade elasticity on final goods critically affect our results, as shown by the third line of Figure 6. Indeed, when final-good trade flows are more sensitive to changes in relative export prices, that is when μ is larger, the responses of sustainable tariffs are reversed compared to the benchmark case and much larger: the Home (Foreign) tariff increases (falls) after a positive productivity shock in the Home economy, and tariffs rise after a restrictive monetary policy shock in the Home economy. This opposite qualitative pattern in sustainable tariffs is the mere reflection of the fact that a larger trade elasticity on final goods magnifies expenditure switching effects compared to wealth effects and lowers the ability of tariff setters to exploit of the terms-of-trade externality. It lowers the gains from raising tariffs and raises the future losses from doing so, leading to reversed dynamics of tariffs on the sustainable path. The quantitative aspect of our results with a larger trade elasticity μ also has to do with the steady-state *levels* of tariffs in this case: since incentives to manipulate the terms of trade are lower with a larger μ, the gains from doing so are smaller, which entails *lower* steady-state levels of tariffs, increasing the size of percentage deviations from the steady-state value.

Finally, the last line of Figure 6 shows that our results are quantitatively but not qualitatively sensitive to the assumption made about export pricing decisions for the Home tariff. When export prices are sticky, *i.e.* with LCP, variations of the nominal exchange rate affect the relative prices of intermediate and final goods less in the short run. As such, whenever tariff setters want to alter the dynamics of the real exchange rate or terms of trade, they need to apply larger changes to their instruments – tariffs in the present case – compared to the case of PCP. In addition, the sign of the response of the Foreign tariff after shocks in the Home economy is reversed under PCP compared to the benchmark LCP case. Quantitatively, the fall in the Home tariff is roughly three and a half times larger under LCP (-3%) than under PCP (-0.84%) after a positive Home productivity shock and more than 14 times larger under LCP (-74%) than under PCP (-5.1%) after a restrictive monetary policy shock. Assumptions about pricing decisions thus matter critically for the size of tariff fluctuations and for the response of the tariffs of partners to local shocks, given their influence on the dynamics of the real exchange rate, inflation rates and endogenous interest rates.

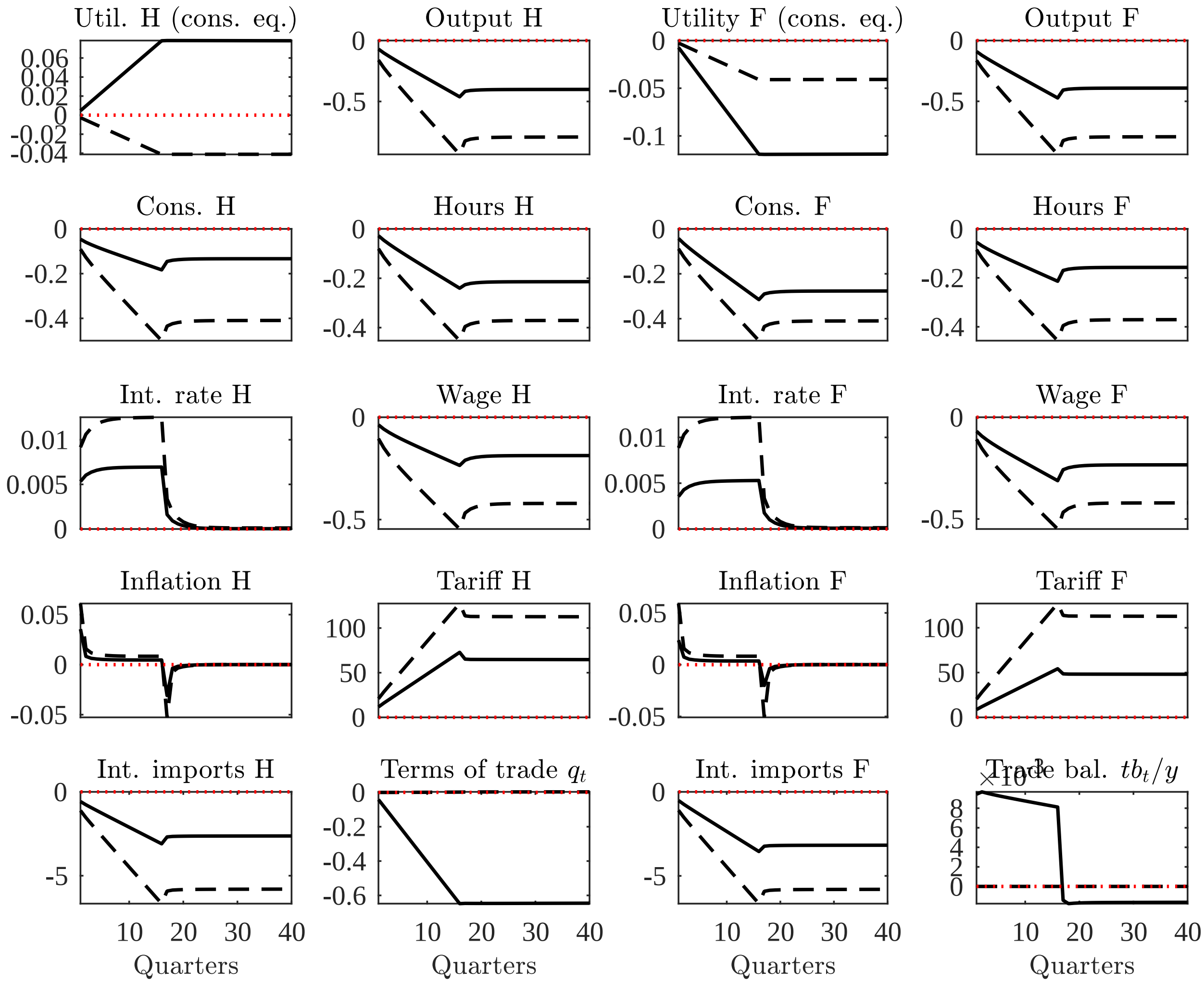
6 Simulating the effects of a trade war

Over the last few years, some countries around the world among which the U.S. have been raising tariffs substantially. For instance, according to Bown's calculations of the U.S.-China trade war, the average level of U.S. tariffs against China has risen from 3.1% in 2017 to 26.6% at the end of 2019.[11] Symmetrically, China has been raising its average level of tariffs against imports from the U.S. from 8% in January 2018 to 25.9% at the end of 2019. On average, these tariff hikes represent a 370% increase in tariffs. However, given that China represents 13.5% of total U.S. trade in goods and assuming that tariffs with respect to other countries and from other countries remain constant, the above increase in tariffs translates into a 100% increase in the aggregate level of tariffs. Through the lens of our model, for given preferences and technical parameters of the private sector, such a steady rise in sustainable tariffs can only be the product of a change in tariff setters' preferences, which in our case are entirely encapsulated in their discount factor. A rise in tariff setters impatience – a reduction of their discount factor β^g – should raise tariffs because the current utility gains are given a larger weight in their value function in comparison to the future losses induced by the worst Nash equilibrium in case they cheat. Figure 7 reports the effects of a succession over 16 quarters (4 years) of quasi-permanent small, negative (-0.25%) shocks on tariff setters' discount factors. Shocks are unexpected and we consider two alternative cases: (i) asymmetric shocks (affecting the Home tariff setter), or (ii) symmetric shocks (affecting both tariff setters). The value of the shock is computed so as to produce an approximate 100% rise in both

[11] See https://www.piie.com/blogs/trade-and-investment-policy-watch/us-china-trade-war-guns-august.

Home and Foreign tariffs after 16 quarters under the sequence of symmetric shocks, our reference simulation.

Figure 7: Simulation of a trade war, in percentage deviation.



Solid: Sequence of asymmetric 0.25% shocks on the discount factor of the Home tariff setter (β^g). Dashed: Sequence of symmetric 0.25% shocks on the discount factor of both Home and Foreign tariff setters (β^g and β^{g*}). Utility panels report the Hicksian consumption equivalent, in percents. Positive numbers signal utility gains.

Let us start by looking at the effects of a sequence of asymmetric discount factor shocks. The sequence leads the Home tariff to rise progressively over 16 quarters and settle 64% above its steady-state value, at $\tau = 7.66\%$. The consequences of an asymmetric tariff shock are the same as those depicted in Figure 10 in Appendix B: final and intermediate Home imports fall, leading output, hours worked and real wages to fall. Contrary to what is reported in Figure 10 in Appendix B, aggregate consumption falls, because the Foreign tariff setter retaliates and raises the Foreign tariff rate by almost 50%. The rise in imported intermediate goods drives CPI inflation up which in turn pushes the Home Central Bank to raise its nominal rate, further fueling the recession by depressing private consumption. As intended, the shock appreciates the real exchange rate which, by the labor supply effects of the terms-of-trade externality, generates welfare gains: hours worked fall more than consumption does. Quantitatively speaking, the sequence of asymmetric shocks lowers output by 0.4%, consumption by 0.14% and hours worked by 0.22%, producing a modest 0.0783% Hicksian equivalent consumption gain, a small welfare gain.[12]

The real exchange rate appreciates marginally, falling by 0.64%. The chief reason is that, on the sustainable path of tariffs, even though the preferences of the Foreign tariff setter remain unchanged, the latter ends up raising its tariffs by 48%, almost as much as in the Home economy. Our model thus predicts that a unilateral change in the preferences of the Home tariff setter can generate an endogenous positive and large response of the tariff set by the Foreign tariff setter, an endogenous trade war. As a matter of fact, the sequence of asymmetric shocks produces very similar dynamics for the Foreign economy, that are marginally compensated by the real exchange rate dynamics. Foreign output falls by 0.4% as well, consumption by 0.28% and hours worked by 0.16%. Since consumption falls more than in the Home economy and hours worked fall less, the shock produces a 0.11% welfare loss for the Foreign economy. Given that the sequence of shocks produces relatively minor asymmetries among both countries, it should not be surprising that the effects on the Home trade balance are positive but negligible, as it generates a trade surplus of 0.008% of GDP.

In the case of a sequence of symmetric shocks, tariffs rise by 113%, and settle around 9.9%. The effects of a joint rise in tariffs are perfectly symmetric: output falls by 0.8% in both countries, consumption by 0.42% and hours worked by 0.38%. In addition, intermediate imports jointly drop by 5.8%, inflation jumps more than under the sequence of asymmetric shocks and thus triggers a larger reaction of Central Banks, that further aggravate the recession. Overall, in the scenario of a sequence of symmetric discount factor shocks, both countries experience welfare losses of 0.04%. Notice that consumption falls by 0.42%. Given the level of personal consumption expenditure in the U.S. in 2018, this drop amounts to approximately 57 billion dollars, a number that is remarkably close to that reported by Fajgelbaum, Goldberg, Kennedy, and Khandelwal (2019).

7 Conclusion

We have analyzed the cyclical pattern of trade protection both empirically on U.S. data, and theoretically within in a two-country model with sticky domestic and export prices, incomplete markets, trade in intermediate and final goods, and two types of shocks: permanent productivity and monetary policy shocks.

First, the demand for trade protection in the data has been found to be counter-cyclical conditional on productivity shocks and pro-cyclical conditional on monetary policy shocks.

[12]The Hicksian equivalent consumption gain γ is defined as:

$$\sum_{t}^{\infty} \beta^{s-t} u\left(c_t, \ell_t\right) = \sum_{t}^{\infty} \beta^{s-t} u\left(c\left(1+\gamma/100\right), \ell\right)$$

Second, we offered a model that was able to rationalize this finding. In the model, tariff setters were willing to exploit the terms-of-trade externality by raising tariffs unilaterally. When tariffs were set according to a dynamic tariff game, we found that local tariffs rose in response to local monetary policy shocks, and fell in response to local permanent productivity shocks. This pattern was consistent with the empirical analysis conducted using SVARs on U.S. monthly and quarterly data, and robust to almost all parameters of our model, to the exception of the trade elasticity, that needed to remain relatively small for the empirical evidence to be replicated in the model.

Finally, we engaged in the quantification of the effects of a trade war triggered by a fall in the degree of patience of tariff setters, tariffs being set endogenously. When shocks were asymmetric, tariffs rose more at Home than abroad, producing small welfare gains at Home and welfare losses abroad. When shocks were symmetric, both countries experienced welfare losses.

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A Pricing conditions and inflation dynamics

The optimal pricing conditions are:

$$\widetilde{p}_{ht}(i) = \frac{\eta}{(1-\tau_y)(\eta-1)} \frac{E_t\left\{\sum_{s=t}^{\infty}(\theta\beta)^{s-t}\lambda_s p_{hs}\varphi_s y_{hs}(i)\right\}}{E_t\left\{\sum_{s=t}^{\infty}(\theta\beta)^{s-t}\lambda_s y_{hs}(i)\right\}} \tag{48}$$

$$\widetilde{p}^*_{ht}(i) = \frac{\eta}{(1-\tau_y)(\eta-1)} \frac{E_t\left\{\sum_{s=t}^{\infty}(\theta\beta)^{s-t}\lambda_s p_{hs}\varphi_s y^*_{hs}(i)\right\}}{E_t\left\{\sum_{s=t}^{\infty}(\theta\beta)^{s-t}\lambda_s s_s y^*_{hs}(i)\right\}} \tag{49}$$

$$\widetilde{p}^*_{ft}(i) = \frac{\eta}{(1-\tau_y)(\eta-1)} \frac{E_t\left\{\sum_{s=t}^{\infty}(\theta\beta)^{s-t}\lambda^*_s p^*_{fs}\varphi^*_s y^*_{fs}(i)\right\}}{E_t\left\{\sum_{s=t}^{\infty}(\theta\beta)^{s-t}\lambda^*_s y^*_{fs}(i)\right\}} \tag{50}$$

$$\widetilde{p}_{ft}(i) = \frac{\eta}{(1-\tau_y)(\eta-1)} \frac{E_t\left\{\sum_{s=t}^{\infty}(\theta\beta)^{s-t}\lambda^*_s p^*_{fs}\varphi^*_s y_{fs}(i)\right\}}{E_t\left\{\sum_{s=t}^{\infty}(\theta\beta)^{s-t}\lambda^*_s s_s^{-1} y_{fs}(i)\right\}} \tag{51}$$

Using the expressions for the dynamics of aggregate price levels

$$p_{ht} = \left(\theta p_{ht-1}^{1-\eta} + (1-\theta)\widetilde{p}_{ht}^{1-\eta}\right)^{\frac{1}{1-\eta}} \text{ and } p^*_{ht} = \left(\theta p_{ht-1}^{*1-\eta} + (1-\theta)\widetilde{p}_{ht}^{*1-\eta}\right)^{\frac{1}{1-\eta}} \tag{52}$$

$$p^*_{ft} = \left(\theta p_{ft-1}^{*1-\eta} + (1-\theta)\widetilde{p}_{ft}^{*1-\eta}\right)^{\frac{1}{1-\eta}} \text{ and } p_{ft} = \left(\theta p_{ft-1}^{1-\eta} + (1-\theta)\widetilde{p}_{ft}^{1-\eta}\right)^{\frac{1}{1-\eta}} \tag{53}$$

we get the following dynamics for inflation rates

$$\pi_{ht} = \left(\theta^{-1} - \theta^{-1}(1-\theta)\left(\frac{\eta}{(1-\tau_y)(\eta-1)}\frac{F_{ht}}{G_{ht}}\right)^{1-\eta}\right)^{\frac{1}{\eta-1}} \tag{54}$$

$$\pi^*_{ht} = \left(\theta^{-1} - \theta^{-1}(1-\theta)\left(\frac{\eta}{(1-\tau_y)(\eta-1)}\frac{F^*_{ht}}{G^*_{ht}}\right)^{1-\eta}\right)^{\frac{1}{\eta-1}} \tag{55}$$

$$\pi^*_{ft} = \left(\theta^{-1} - \theta^{-1}(1-\theta)\left(\frac{\eta}{(1-\tau_y)(\eta-1)}\frac{F^*_{ft}}{G^*_{ft}}\right)^{1-\eta}\right)^{\frac{1}{\eta-1}} \tag{56}$$

$$\pi_{ft} = \left(\theta^{-1} - \theta^{-1}(1-\theta)\left(\frac{\eta}{(1-\tau_y)(\eta-1)}\frac{F_{ft}}{G_{ft}}\right)^{1-\eta}\right)^{\frac{1}{\eta-1}} \tag{57}$$

where

$$
\begin{aligned}
F_{ht} &= \theta\beta E_t \left\{\pi_{ht+1}F_{ht+1}\right\} + \lambda_t\varphi_t y_{ht} \text{ and } G_{ht} = \theta\beta E_t \left\{G_{ht+1}\right\} + \lambda_t y_{ht} && (58)\\
F^*_{ht} &= \theta\beta E_t \left\{\frac{s_{t+1}}{s_t}\pi^*_{ht+1}F^*_{ht+1}\right\} + \lambda_t\varphi_t y^*_{ht}/\Lambda_t \text{ and } G^*_{ht} = \theta\beta E_t \left\{\frac{s_{t+1}}{s_t}G^*_{ht+1}\right\} + \lambda_t y^*_{ht} && (59)\\
F^*_{ft} &= \theta\beta E_t \left\{\pi^*_{ft+1}F^*_{ft+1}\right\} + \lambda^*_t\varphi^*_t y^*_{ft} \text{ and } G^*_{ft} = \theta\beta E_t \left\{G^*_{ft+1}\right\} + \lambda^*_t y^*_{ft} && (60)\\
F_{ft} &= \theta\beta E_t \left\{\frac{s_t}{s_{t+1}}\pi_{ft+1}F_{ft+1}\right\} + \lambda^*_t\varphi^*_t y_{ft}/\Lambda^*_t \text{ and } G_{ft} = \theta\beta E_t \left\{\frac{s_t}{s_{t+1}}G_{ft+1}\right\} + \lambda^*_t y_{ft} && (61)
\end{aligned}
$$

In these equations, $\Lambda_t = s_t p^*_{ht}/p_{ht}$ and $\Lambda^*_t = p_{ft}/s_t p^*_{ft}$ stand for the Home and Foreign deviations from the law of one price. Notice that, under flexible export prices, the Foreign price of the Home good and the Home price of the Foreign goods are respectively

$$\widetilde{p}^*_{ht} = s_t^{-1}\widetilde{p}_{ht} \text{ and } \widetilde{p}_{ft} = s_t\widetilde{p}^*_{ft} \tag{62}$$

which implies that the law of one price holds. Taking the dynamic version of the LOP deviations further implies

$$\frac{\Lambda_t}{\Lambda_{t-1}} = \frac{s_t}{s_{t-1}}\frac{\pi^*_{ht}}{\pi_{ht}} \text{ and } \frac{\Lambda^*_t}{\Lambda^*_{t-1}} = \frac{s_{t-1}}{s_t}\frac{\pi_{ft}}{\pi^*_{ft}} \tag{63}$$

Finally, CPI inflation rates are defined as

$$
\begin{aligned}
\pi_t &= \pi_{ht}\frac{\left(1-\omega+\omega\left(\left(1+\tau_t\right)\Lambda^*_t q_t\right)^{1-\mu}\right)^{\frac{1}{1-\mu}}}{\left(1-\omega+\omega\left(\left(1+\tau_{t-1}\right)\Lambda^*_{t-1}q_{t-1}\right)^{1-\mu}\right)^{\frac{1}{1-\mu}}} && (64)\\
\pi^*_t &= \pi^*_{ft}\frac{\left(1-\omega+\omega\left(\left(1+\tau^*_t\right)\Lambda_t/q_t\right)^{1-\mu}\right)^{\frac{1}{1-\mu}}}{\left(1-\omega+\omega\left(\left(1+\tau^*_{t-1}\right)\Lambda_{t-1}/q_{t-1}\right)^{1-\mu}\right)^{\frac{1}{1-\mu}}} && (65)
\end{aligned}
$$

where terms of trade dynamics are given by

$$\frac{q_t}{q_{t-1}} = \frac{s_t}{s_{t-1}}\frac{\pi^*_{ft}}{\pi_{ht}} \tag{66}$$

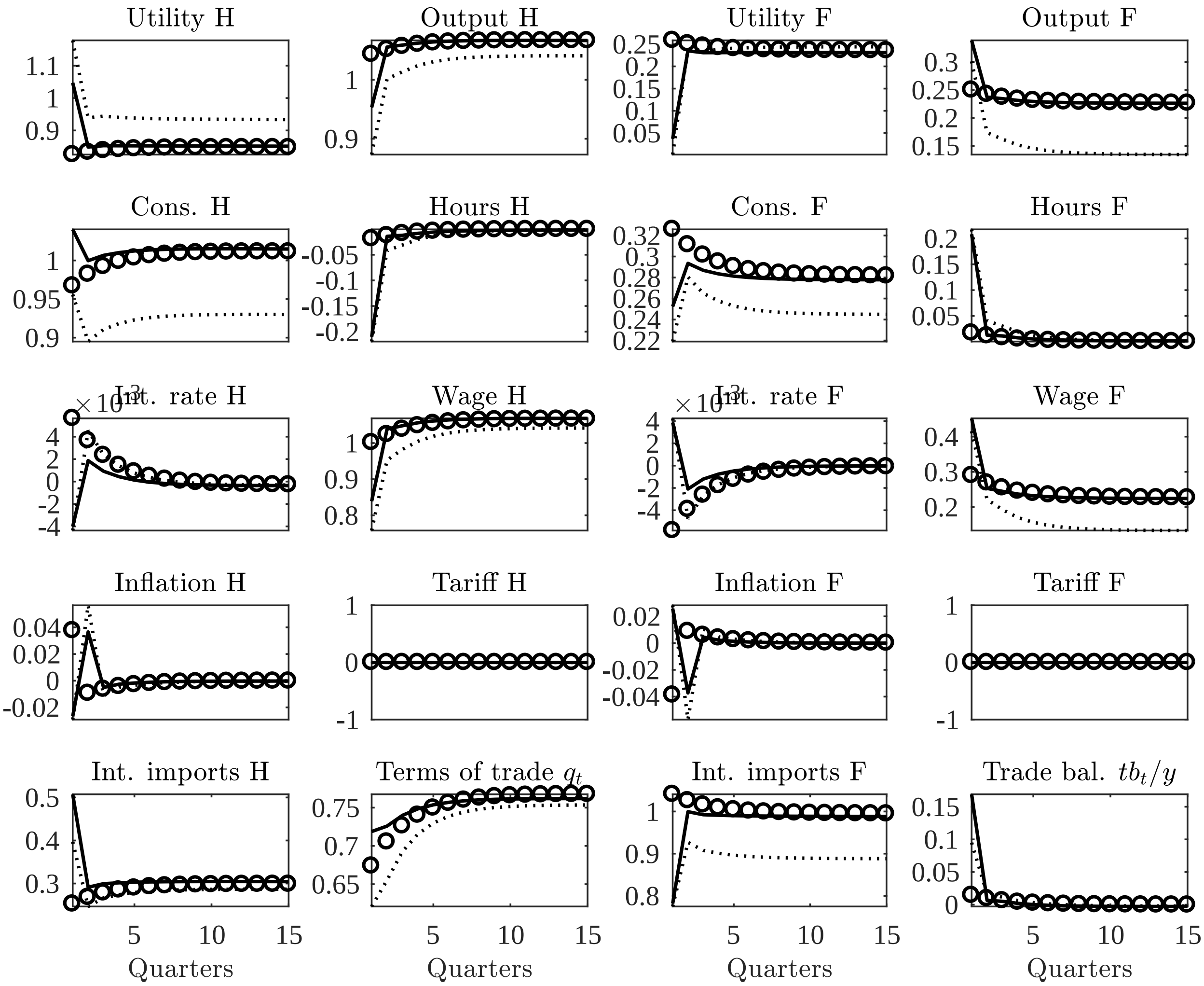
and the real exchange rate by

$$q^r_t = \frac{s_t p^*_t}{p_t} = q_t\left(\frac{1-\omega+\omega\left(\left(1+\tau^*_t\right)\Lambda_t/q_t\right)^{1-\mu}}{1-\omega+\omega\left(\left(1+\tau_t\right)\Lambda^*_t q_t\right)^{1-\mu}}\right)^{\frac{1}{1-\mu}} \tag{67}$$

B Impulse responses with exogenous tariffs

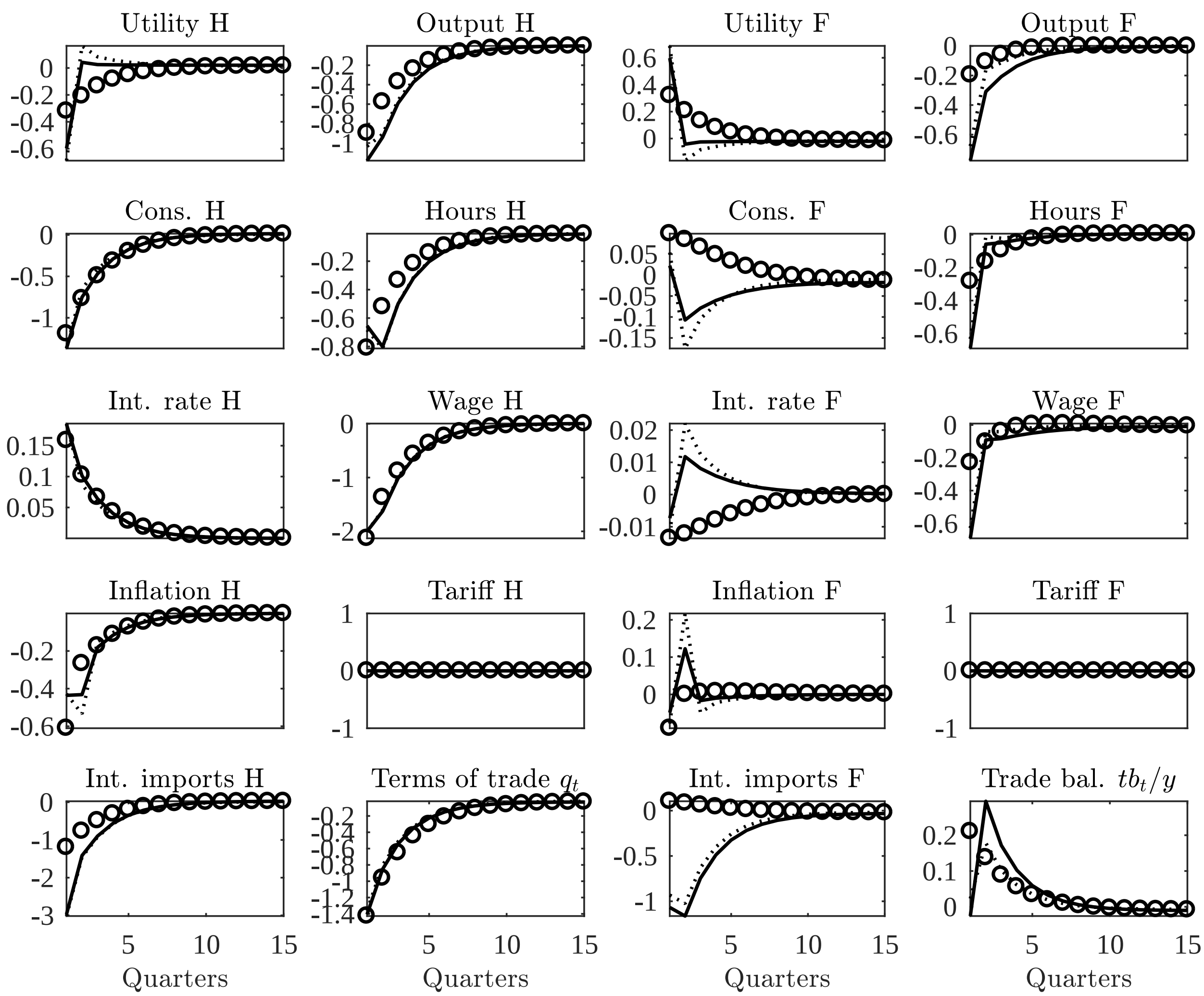
Figure 8, 9 and 10 report the impulse response functions respectively to a quasi-permanent Home productivity shock, a monetary policy shock and a quasi-permanent tariff shock.

Figure 8: IRFs to a quasi-permanent positive Home productivity shock with exogenous tariffs, percent deviations from the steady state.



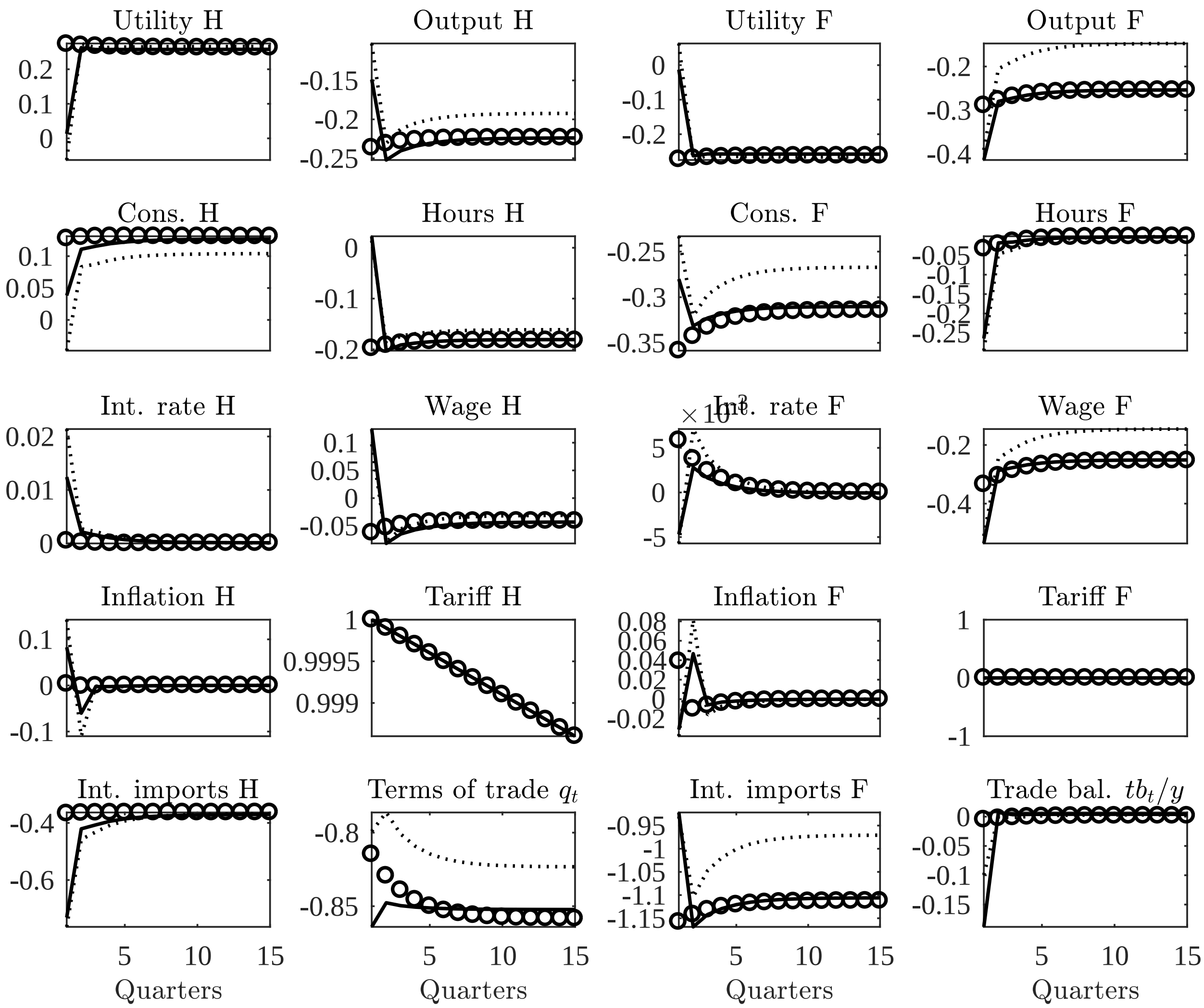
Solid: Benchmark calibration. Circled: Producer Currency Pricing ($\theta^x = 0$). Dashed: No trade in intermediate goods ($\alpha \approx 0$ and $\omega = 0.3$).

Figure 9: IRFs to a restrictive Home monetary policy shock with exogenous tariffs, percent deviations from the steady state.



Solid: Benchmark calibration. Circled: Producer Currency Pricing ($\theta^x = 0$). Dashed: No trade in intermediate goods ($\alpha \approx 0$ and $\omega = 0.3$).

Figure 10: IRFs to a quasi-permanent positive Home tariff shock, percent deviations from the steady state.



Solid: Benchmark calibration. Circled: Producer Currency Pricing ($\theta^x = 0$). Dashed: No trade in intermediate goods ($\alpha \approx 0$ and $\omega = 0.3$).