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Temporal disaggregation of short time series with structural breaks: Estimating quarterly data from yearly emerging economies data

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Temporal disaggregation of short time series with structural breaks: Estimating quarterly data from yearly emerging economies data

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Abstract

This article develops a methodology to compute up-to-date quarterly macroeconomic data for emerging countries by adapting a well known method of temporal disaggregation to time series with small sample size and instable relationships between them. By incorporating different procedures of structural break detection, the prediction of higher-frequency estimations of yearly official data can be improved. A methodology with a model selection procedure and disaggregation formulas is proposed. Its predictive performance is assessed by using empirical advanced countries data and simulated time series. An application to the Chinese national accounts allows the estimation of the cyclical components of the Chinese expenditure accounts and shows the Chinese economy to have second order moments more in line with emerging countries than advanced economies like the United States.

Keywords: Time series, macroeconomic forecasting, disaggregation, structural change, business cycles, emerging economies

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1 Introduction

In order to analyze the macroeconomic fluctuations of a particular economy, time series with a frequency higher than annual are preferable. They also have to meet conditions for them to be as less noisy as possible, such as being deflated to real values and seasonally adjusted. Unfortunately for both the academic community and the practitioners interested in studying the business cycles of important but opaque emerging countries such as China or Russia, these conditions are hardly met concerning fundamental aggregates such as the national accounts. Official free access time series are available from the World Bank, but their frequency is only annual. One must therefore rely on official datasets disclosed by national statistics administrations for higher frequency data. In the case of China, the National Bureau of Statistics (NBS) indeed publishes quarterly real growth rates of GDP, although their validity are very discussed in the literature (most recently in Perkins and Rawski (2008), Holz (2014) or Clark et al. (2017)), but only annual nominal values of the Chinese national accounts, and the most recently disclosed information concern the period from one to two years past the date of disclosure. If the annual data can be easily deflated by finding or approximating price indexes for each account, business cycle analyses still requires up-to-date quarterly or monthly series. Methods of disaggregation and extrapolation of quarterly data to monthly estimates have been discussed for decades, notably by relating the initial data to higher frequency indicators as introduced by Friedman (1962) and Chow and Lin (1971), or by exploiting the autocorrelation structure of the initial data without any additional information as in Stram and Wei (1986) and Wei and Stram (1990). While both methods are adaptable for disaggregating annual data, the first method which relies on related higher frequency series fits our case of interest considering the much lower abundance of the observable data. The principle is to incorporate fluctuations from higher order frequency indicators while retaining the annual aggregated level of the series to disaggregate, and the procedure is straightforward: an annual prediction model can be estimated by linking the annual accounts and the annual aggregations of the quarterly or monthly indicator, then the estimated coefficients are applied to the observed quarterly or monthly indicator to estimate a quarterly or monthly disaggregated national account data, and finally the annual discrepancies between the estimates and the original series is distributed to the estimated disaggregates. This method assumes a stable linear relationship between the national accounts series and the indicator series, what is unlikely to be true for emerging markets. For example, the available Chinese data period covering at the same time national accounts series and quarterly or monthly related indicators ranges from roughly 2000 to 2014, during which the economy has been affected by several economic and political changes (participation to the World Trade Organization in 2001, end of the exchange peg to the US dollar in 2005, worldwide financial crisis since 2008). As a consequence, the behaviour of the macroeconomic aggregates and their relationship may also change over the period, despite the shortness of the series, it is then useful to allow for structural change in the parameters of the linear relationship.

A most common test of a structural break in linear models is the one of Chow (1960) which consists in simply splitting the sample at a known date and assessing the significance of the difference in the parameter estimates. Hansen (1992a) provides a test with an endogenous or unknown date of structural break but is only valid if regressors are stationary. Since annual datasets are proven to hardly provides stationary series, especially for emerging markets, a test of parameter instability with non stationary regressors is more adapted, such as proposed by Hansen (1992b).

That being said, the validity of the predicting model resides in the stationarity of the error process, no matter how we decide to specify the structural changes. Although Friedman (1962) imposes the stationarity of the series, which can by attained by differencing the data (Fernandez (1981)), the residuals may already be stationary if the series are cointegrated, which would avoid unnecessary transformation of the data. A problem arises for short series such as data from the emerging markets, because the test hardly rejects the null of no cointegration when tested. Since conventional cointegration tests such as residual-based augmented Dickey-Fuller (ADF) tests lose significant power in the presence of a structural change, Gregory and Hansen (1996) consequently propose a test of the null of no cointegration against the alternative of cointegration with structural break, which we will use as a selection process of the predicting model for the interpolation of annual national accounts data by using related infra-annual indicators. Since this test is better used on long series, we prealably need to assess and adapt its use for time series of small or very small sample size.

However, our main objective is to predict quarterly estimates of annual aggregates, and testing for stationary residuals does not guarantee a small prediction error. We therefore confront the cointegration with structural break approach with a prediction focused selection process which minimizes an observable prediction error such as the root mean square error of the annual predicted aggregates.

The first section details the different methodologies we adopt for the disaggregation process. The second section illustrates the competing methodologies by disaggregating two components of the Chinese national accounts. The third section assesses the performance of the methodologies in order to deduce a final disaggregation process, by applying them to the US data where we observe the quarterly series, then to simulated data to study their performance in terms of power and prediction error. The final section gives an example of a business cycle study made possible by the disaggregation of the Chinese national accounts with a basic business cycle stylized facts study and its comparison to former results in the litterature for other countries.

2 Methodology

The method of disaggregation introduced by Friedman (1962) then Chow and Lin (1971) uses the contemporary linear relationship between a given time series and the temporal aggregate of a higher frequency indicator to predict the former, given the residuals of the linear relationship is stationary. In practice, the test of the stationarity of the residuals of the model linking the series of interest and its aggregated indicator is identical to a cointegration test. When studying emerging countries data, two problems arise: the available time series are small samples of data, and the linear relationship is likely to be unstable in the sense of its parameter being subjected to at least one structural change. In order to take into account time-varying parameters in short samples, we compare two competing methods to modelize and detect structural breaks : the test of cointegration with unknown structural break such as the one of Gregory and Hansen (1996) which minimizes the ADF-statistics of the unit root test of the residuals of the model, and a prodedure which minimize the prediction error of the annual series to disaggregate. In this section we detail a two-step methodogy for the interpolation of an annual data series using a related quarterly indicator, by in the first place quickly recall the case without a structural change derived initially by Chow and Lin (1971), and then expose the case with structural change.

2.1 The case without structural change

For the study we will define annual indexes $n \in (1, ..., N)$ and quarterly indexes $t \in (1, ..., 4N)$. Uppercase and lowercase letters respectively stand for annual and quarterly time series. Disaggregating time series with related series consists in 3 steps we will call regression, disaggregation and adjustment.
The regression step links the annual time series we are interested in disaggregating namely $Y = \begin{pmatrix} Y_1 & \dots & Y_N \end{pmatrix}'$ with dimension $N \times 1$, to a set of m quarterly indicators $x = \begin{pmatrix} x_1 & \dots & x_{4N} \end{pmatrix}'$ with dimension $4N \times m$, where $x_n = \begin{pmatrix} x_{1,n} & \dots & x_{m,n} \end{pmatrix}'$. The latter is annualized, yielding the annual indica-

tors X with dimension $N \times 1$ such that:

$$X_n = \sum_{i=-3}^{0} x_{4n+i} \quad \forall n$$

Following Chow and Lin (1971), the linear relationship linking Y and X without structural change, which we define as the model O, would be obtained by estimating the following model:

$$Y_n = \mu + \alpha X_n + U_n \tag{1}$$

where $U_n = \rho U_{n-1} + \varepsilon_n$, and $\alpha = \left(\alpha_1 \ldots \alpha_m\right)'$ are the coefficients associated with each component of X. If $U = \left(U_1 \ldots U_N\right)'$ is second-order stationary , i.e. $|\rho| < 1$, the linear relationship (1) will be retained for the following of the procedure. If second-order stationarity is rejected for U, or if a unit root in U cannot be rejected, the model will be estimated in first difference (see Fernandez (1981)), which we define as the model dO :

$$\Delta Y_n = \mu + \alpha \Delta X_n + V_n \tag{2}$$

where Δ is the first difference operator, such that $\Delta Y_n = (1 - L)Y_n$, $\forall n \in \{2, ..., N\}$.
Once the regression model linking Y and X has been selected in level or in first difference, preliminary quarterly estimations of Y, which we will note $\hat{y}$ with dimension $4N \times 1$ can be predicted during what we call the disaggregation step. If the regression step has been estimated in level, preliminary disaggregated data are computed using the following formula :

$$\hat{y}_t = \frac{\hat{\mu}}{4} + \hat{\alpha} x_t \quad \forall t \in \{1, ..., 4N\} \tag{3}$$

where $\hat{\mu}$ and $\hat{\alpha}$ are the estimators of respectively μ and α from (1).
If the regression step has been estimated in first difference and the intercept coefficient $\hat{\mu}$ is significant, it implies that there is a significant linear trend component in the relationship linking Y and X. The disaggregation formula is therefore (the proof is in Appendix A):

$$\hat{y}_t = \frac{Y_1}{4} + \hat{\mu}\frac{t - 2.5}{16} + \hat{\alpha}(x'_t - \frac{X'_1}{4}) \text{ for } t \in \{1, ..., 4N\} \tag{4}$$

By construction, the annual aggregation of $\hat{y}$ is strictly equal to the predicted value $\hat{Y}$ of the model estimated in the regression step, whatever the retained order of differenciation for the variables. Hence the preliminary estimated values do not sum up to the initial time series we want to disaggregate, i.e. :

$$\sum_{i=-3}^{0} \hat{y}_{4n+i} = \hat{Y}_n = Y_n + \hat{U}_n$$

where $\hat{Y} = \left(\hat{Y}_1 \quad \ldots \quad \hat{Y}_n\right)'$ are the predicted dependent values from the regression step and $\hat{U}$ are the annual discrepancies between Y and $\hat{Y}$, or by construction the residuals from the regression of model in level, or such that $\Delta \hat{U} = \hat{V}$ for model in first difference. $\hat{U}$ can therefore be distributed into quarterly residuals $\tilde{u} = \left(\tilde{u}_1 \quad \ldots \quad \tilde{u}_{4N}\right)'$ and added to the preliminary quarterly estimates $\hat{y}$. We consider a smooth autoregressive distribution , where the $\tilde{u}$ minimize the quarterly loss function and add up to the annnual discrepancies[1]:

$$\min_{u} \quad \sum_{t=1}^{4N} (u_t - u_{t-1})^2$$
$$s.t. \sum_{i=-3}^{0} u_{4n+i} = \hat{U}_n \quad \forall n \in \{1, ..., 4N\}$$

The final estimations of the infra-annual account series are then given by:

$$\tilde{y}_t = \frac{\hat{\mu}}{4} + \hat{\alpha} x_t + \tilde{u}_t \tag{5}$$

when the model is estimated in level,

$$\tilde{y}_t = \frac{Y_1}{4} + \hat{\mu}\frac{t-2.5}{16} + \hat{\alpha}(x'_t - \frac{X'_1}{4}) + \tilde{u}_t \tag{6}$$

when the model is estimated in first difference.

2.2 Taking into account parameter instability

It is known that stationarity test have low power against structural breaks (e.g. Gregory et al. (1996) for the test of the null of no cointegration). Taking into account parameter instability in the regression model can therefore avoid unneccessarily differenciating the variables. In this paper we consider only parameter instability in the value of the estimated coefficients in the regression model, and we only consider the case with one regressor ($m = 1$). Let us define n_b the annual date of structural break in the parameters, i the regime such that $i = 1$ when $n < n_b$ and $i = 2$ when $n \geq n_b$. Five types of structural break in the coefficient values are considered[2] and compete with the models without structural break O and dO:

- model C: a model in level with a shift in the intercept
$$Y_n = \mu_i + \alpha X_n + U_n \tag{7}$$
- model CS: a model in level with a shift in the intercept and the slope
$$Y_n = \mu_i + \alpha_i X_n + U_n \tag{8}$$
- model dC: a model in first difference with a shift in the intercept
$$\Delta Y_n = \mu_i + \alpha \Delta X_n + V_n \tag{9}$$
which implies a shift in the trend for the model in level
- model dCS: a model in first difference with a shift in the intercept and the slope
$$\Delta Y_n = \mu_i + \alpha_i \Delta X_n + V_n \tag{10}$$
which implies a shift in the trend and the slope for the model in level

We need a procedure to select at the same time the model $M \in \mathcal{M} = \{\text{O, C, CS, dO, dC, dCS}\}$ and the date of structural break n_b which yield the best prediction of disaggregated data. We compare two different approaches to select them: one based on a test of cointegration with structural break and another based on minimizing annual prediction errors.

[1] Initially proposed for the stationary case by Denton (1971), it is extended by Fernandez (1981) for a non stationary U, i.e. the model in first difference

[2] These are the models for which asymptotical properties of the statistics for testing cointegration with structural breaks have been derived by Gregory and Hansen (1996).

2.3 Selection by rejecting the absence of cointegration with an endogenous structural break

In an approach to consider unit root testing of the residuals as a model selecting criterion, a natural extension of the existing method, which partly consists in checking the absence of a unit root in the residuals of a model without structural break, is to consider a structural break in parameters in the cointegration model. The best model would therefore be the one which rejects the most the presence of a unit root in the residuals U for the models in level in (1), (7) and (8) or V for the models in first difference in (2), (9) and (10). Such a test of no cointegration with unknown structural break is characterized by Gregory and Hansen (1996).

In Gregory and Hansen (1996), the structural break date $\hat{n}_b$ is selected by minimizing the unit root test statistic of the residuals among models where $n_b \in \mathcal{N}_b = [\![0.15N, 0.85N]\!]$. Since we have short series, we consider augmented Dickey-Fuller statistics for each potentiel dates and model $ADF(n_b, M)$. For a model M considered, we note the resulting statistic testing the null hypothesis of no cointegration with endogenous structural break $ADF^*(M)$ such that

$$ADF^*(M) = \min_{n_b \in \mathcal{N}_b} ADF(n_b, M) \tag{11}$$

Gregory and Hansen (1996) compute asymptotical critical values for $ADF^*(M)$, which are reported in Table 1. They also assess the simulated performance of the test statistics and show that the ADF-type

M	1 %	2.5 %	5 %	10 %	97.5 %
C	-5.13	-4.83	-4.61	-4.34	-2.25
CS	-5.57	-5.19	-4.95	-4.68	-2.55

Table 1: Asymptotical critical values for the cointegration test of Gregory and Hansen (1996) with one regressor ($m = 1$), by model M

statistics are biased away from the null for small sample. More precisely, they find that for sample size $n = 100$ and using the asymptotical critical value at level 5% for models C and CS on 2 500 replications of data simulated under the null, the test statistic respectively rejects 8 and 5 percentage points too often the null hypothesis when the latter is true. For smaller sample size $n = 50$, the size distortion increases to respectively 12 and 8 percentage points.

The sample sizes we are interested in are much smaller, with $n = 50$ being a high upper bound when it comes to emerging country data. We therefore simulate data with sample size $n < 50$, using the following calibration under the null of no cointegration :

$$\begin{cases} Y_n = 1 + 2X_n + U_n, & U_n = U_{n-1} + \varepsilon_n, \quad \varepsilon_n \sim \text{NID}(0,1) \\ X_n = -1 + V_n, & V_n = V_{n-1} + \eta_n, \quad \eta_n \sim \text{NID}(0,2) \end{cases} \tag{12}$$

The chosen parameter values follow Gregory and Hansen (1996), and are nuisance parameters which do not matter under the null. Table 2 show the rejection frequencies under the null hypothesis, using the asymptotical critical values at the 5% level for 50 000 replications of data generated by (12). The test expectedly overrejects the null hypothesis as the sample size decreases, and size distortion can be very large for very small sample, up to 28 percentage points for both models C and CS. Using asymptotical critical values would therefore not be adapted in order to select the type of structural break model for our methodology, especially when the amplitude of size distortion is heterogenous between the models. To correct for size distortion, we compute size-adjusted critical values for each model.

n	model C	model CS
15	0.28	0.28
20	0.23	0.23
30	0.18	0.17
50	0.12	0.12

Table 2: Rejection frequencies at the 5% nominal level of significance using asymptotical critical values from Gregory and Hansen (1996) with one regressor, by model and by sample size n

50 000 replications of time series of size $n = 13, ..., 50, 100, 200, 500, 1000$ are simulated using the previous data generating process under the null hypothesis. We also use a response surface method à la MacKinnon (1991) where we fit a polynomial of $1/n$ by OLS for each q'th quantile of the simulated distribution of the test statistics and model m:

$$Crt(n, q, m) = \psi_\infty + \sum_{k=1}^{K} \psi_k n^{-k} + error$$

The order K of the polynomial is selected by minimizing the corrected Akaike information criterion (AICc hereafter). The polynomial functions of $1/n$ are mainly of order 3 or 4 for small quantiles, and of lesser order for higher quantiles. This differs with Gregory and Hansen (1996) for whom first order polynomials are fitted. We can explain it with the fact that they simulate series with $n = 50, 100, 150, 250$, for which the tests statistics are located in the flatter part of the function of $1/n$. As we obtain a function of n to estimate critical values of each quantile, we can compute an approximate distribution of the test statistic for each model and each sample size. Table 3 report the resulting size-adjusted critical values by model and for very small sample sizes ($n \leq 50$), as well as the asymptotic ones, which correspond to the estimated intercept of the polynomial fit. We naturally observe for every type of structural break model

		Level				
	model	0.01	0.025	0.05	0.1	0.975
$n = 15$	O	-5.13	-4.47	-3.97	-3.47	-0.37
	C	-7.52	-6.73	-6.16	-5.58	-2.38
	CS	-8.02	-7.18	-6.57	-5.96	-2.55
$n = 20$	O	-4.79	-4.25	-3.81	-3.37	-0.35
	C	-6.75	-6.17	-5.73	-5.25	-2.37
	CS	-7.19	-6.59	-6.11	-5.62	-2.56
$n = 30$	O	-4.48	-4.03	-3.67	-3.28	-0.34
	C	-6.16	-5.72	-5.35	-4.97	-2.35
	CS	-6.55	-6.08	-5.72	-5.3	-2.56
$n = 50$	O	-4.24	-3.85	-3.53	-3.18	-0.32
	C	-5.74	-5.38	-5.08	-4.73	-2.32
	CS	-6.11	-5.73	-5.42	-5.07	-2.55
$n = \infty$	O	-3.9	-3.59	-3.33	-3.04	-0.3
	C	-5.11	-4.83	-4.59	-4.32	-2.26
	CS	-5.41	-5.17	-4.94	-4.66	-2.54

Table 3: Approximate size-adjusted critical values for one regressor

that the critical values increase sharply for very small sample size, then converge to their asymptotic

value. Moreover, we succeed in replicating the ones obtained by Gregory and Hansen (1996). From the cumulative distribution of the test statistics we obtain for every sample size n, we can compute p-values which can be uses as a selection criterion for the choice of the model for the regression step of our methodology. More precisely, we can select the model which rejects the unit root in the residuals, i.e. the model which yields a p-value associated with the test statistic lower than a selected level. If several models rejects the unit root we choose the most parcimonious one, i.e. on priority order O, C, CS, dO, dC, or dCS. M^*_{test} denotes the model selected by cointegration test approach.
The performance of the size-adjusted critical values in terms of power is discussed in Section 4.2.

2.4 Selection by minimizing annual prediction errors

Choosing the model which rejects the most the unit root in the residuals puts in the model selection more weight into the absence of persistence in the residuals, which can be at the expense of the error variance. We therefore consider an alternative method which is more direct than testing for a unit root in the residuals by simply selecting the model and the structural break date which minimizes the error in the prediction of the annual aggregates, here measured by the root mean squared error. For a given model M, if $\hat{Y}(n_b, M)$ denotes the annual predictions of Y by the model M with a structural break in parameters occuring at n_b[3], we obtain for each model $M \in \mathcal{M}$ a prediction error criterion RMSE(M) such that:

$$\text{RMSE}(M) = \min_{n_b \in \mathcal{N}} \sqrt{\frac{1}{N} \sum_{n=1}^{N} \left(\hat{Y}_n(n_b, M) - Y_n \right)^2} \tag{13}$$

Then we would select the model such that :

$$M^*_{rmse} = \arg\min_{M \in \mathcal{M}} \text{RMSE}(M) \tag{14}$$

2.5 Disaggregation formulae with structural break in parameters

For every model M there is a disaggregation formula to apply to the observed quarterly indicator x using the estimated coefficients. In the case of the models with structural breaks, the quarterly predictions before adjustment $\hat{y}(n_b^*, M^*)$ are computed as follows (proofs are in Appendix B):

- model C: a model in level with a shift in the intercept

$$\hat{y}_t = \begin{cases} \frac{\hat{\mu}_1}{4} + \hat{\alpha} x_t & \text{for } t < 4n_b^* - 3 \\ \frac{\hat{\mu}_2}{4} + \hat{\alpha} x_t & \text{for } t \geq 4n_b^* - 3 \end{cases} \tag{15}$$

- model CS: a model in level with a shift in the intercept and the slope

$$\hat{y}_t = \begin{cases} \frac{\hat{\mu}_1}{4} + \hat{\alpha}_1 x_t & \text{for } t < 4n_b^* - 3 \\ \frac{\hat{\mu}_2}{4} + \hat{\alpha}_2 x_t & \text{for } t \geq 4n_b^* - 3 \end{cases} \tag{16}$$

- model dC: a model in first difference with a shift in the intercept

$$\hat{y}_t = \begin{cases} \frac{Y_1}{4} + \hat{\mu}_1 \frac{t-2.5}{16} + \hat{\alpha}(x_t - \frac{X_1}{4}) & \text{for } t < 4n_b^* - 3 \\ \frac{Y_1}{4} + \hat{\mu}_1 \frac{n_{s_b}-2}{4} + \hat{\mu}_2 \frac{t-4n_b^*+5.5}{16} + \hat{\alpha}(x_t - \frac{X_1}{4}) & \text{for } t \geq 4n_b^* - 3 \end{cases} \tag{17}$$

[3]The $\hat{Y}$ are direct estimates for models estimated in level, or such that $\hat{Y}_n = \widehat{\Delta Y}_n + \hat{Y}_{n-1}$ for $n > 1$ and $\hat{Y}_1 = Y_1$ for models estimated in first difference. The RMSE for all models have then the same magnitude by construction.

- model dCS: a model in first difference with a shift in the intercept and the slope

$$\hat{y}_t = \begin{cases} \frac{Y_1}{4} + \hat{\mu}_1 \frac{t-2.5}{16} + \hat{\alpha}_1 (x_t - \frac{X_1}{4}) & \text{for } t < 4n_b^* - 3 \\ \frac{Y_1}{4} + \hat{\mu}_1 \frac{n_{s_b}-2}{4} + \hat{\mu}_2 \frac{t-4n_{s_b}+5.5}{16} + \hat{\alpha}_1 \frac{X_{n_b-1}}{4} + \hat{\alpha}_2 (x_t - \frac{X_{n_b-1}}{4}) & \text{for } t \geq 4n_b^* - 3 \end{cases} \quad (18)$$

2.6 Layout of the new methodology

Our upgrade of the disaggregation method eventually involves a more sophisticated model selection step than the usual one. It comes down to the following procedure :

1. Fit the model explaining the observed series Y by its aggregated indicator X for every competing model $M \in \mathcal{M} = \{\text{O, C, CS, dO, dC, dCS}\}$. In the case of the structural break models, select the best break date following the test approach or RMSE approach. Then among all models, select the best model $M^* = M^*_{test}$ or M^*_{rmse} following the same approach. If M^* is a structural break model, it is associated with its selected break date $n_b^* = n^*_{b,test}$ or $n^*_{b,rmse}$.

2. Predict $\hat{y}(n_b^*, M^*)$ the disaggregated estimates of Y using the disaggregation formula associated with M^* on observed indicators x.

3. Compute $\tilde{u}$ the smooth autoregressive disaggregation of the annual error predictions $\hat{U} = \hat{Y} - Y$ using a smooth autoregressive distribution, then obtain the adjusted quarterly predictions $\tilde{y} = \hat{y} + \tilde{u}$.

3 Example: an application to the Chinese national accounts data

We illustrate the objective of the method by attempting to disaggregate annual data from China. Abeysinghe and Rajaguru (2004) disaggregate the Chinese GDP using two indicators (nominal M1 money supply and nominal total exports), but no application to the components of the national accounts has been done. Let us consider the time series of two expenditure accounts of the Chinese annual GDP data that are household consumption expenditures and net exports. The first series is an example of a stable series in terms of volatility, and the second one a more volatile one. Figure 1 shows the time series representation of both data series: the scatter points stand for observed annual national account, the bold lines for their associated quarterly indicator, that is to say national retail sales for consumption, and net exports of goods for net exports. The goal of the paper is to disagreggate the annual observations into quarterly observations using the quarterly indicator. In order to do so, we want to find the best model that links the annual observations of the national account data to the annual aggregate of their respective quarterly indicator, either in level or in first difference. Graphically, we want to find the best fit the scatter of representations in Figure 2.

3.1 Disaggregating the Chinese household personal expenditures

We fit the data with the previously discussed models. Table 4 reports the results for the models considered the using size-adjusted Gregory and Hansen (1996) test in the upper part, or minimizing the annual prediction errors in the lower part.

In the usual method, i.e. not considering structural breaks, estimating the model in level does not allow to reject the unit root in the residuals, hence the model in first difference dO is selected. When considering endogenous structural breaks, the presence of a unit root in the residuals is rejected at the 5%

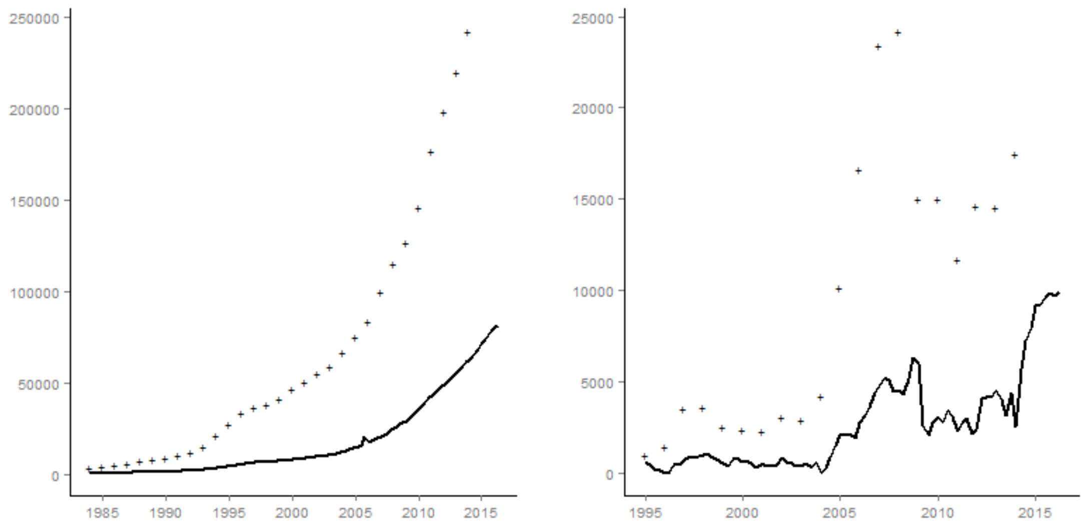

Figure 1: Chinese consumption personal expenditure and retail sales (left), net exports and net exports of goods (right)

	$\hat{n}_b$ selected by cointegration test					
	O	C	CS	dO	dC	dCS
$\hat{\mu}_1$	8384.16***	3560.39***	-828.3	1097.31 .	373.14	-149.41
	(5.78)	(4.14)	(-1.25)	(1.94)	(0.43)	(-0.14)
$\hat{\alpha}_1$	0.92***	0.87***	1.38***	0.79***	0.77***	1.42 **
	(60.81)	(95.47)	(41)	(18.14)	(15.77)	(3.32)
$\hat{\mu}_2 - \hat{\mu}_1$		13236.09***	21233.52***		1214.59	1311.53
		(9.77)	(18.78)		(1.09)	(0.76)
$\hat{\alpha}_2 - \hat{\alpha}_1$			-0.53***			-0.64
			(-15.42)			(-1.46)
RMSE(M)	5915.37	2816.6	1523.57	2827.01	1893.88	1678
$\hat{n}_b$	NA	1996	2002	NA	1992	2002
ADF*	-1.114	-3.558	-7.149	-5.725	-6.22	-7.047
p	0.883	0.591	0.003	0.001	0.01	0.003
	$\hat{n}_b$ selected by annual prediction error					
	O	C	CS	dO	dC	dCS
$\hat{\mu}_1$	8384.16***	2937.81 **	-828.3	1097.31 .	374.14	-144.02
	(5.78)	(3.32)	(-1.25)	(1.94)	(0.4)	(-0.14)
$\hat{\alpha}_1$	0.92***	0.87***	1.38***	0.79***	0.77***	1.41 **
	(60.81)	(99.41)	(41)	(18.14)	(16.14)	(3.39)
$\hat{\mu}_2 - \hat{\mu}_1$		13193.54***	21233.52***		1107.01	1058.01
		(9.92)	(18.78)		(0.96)	(0.55)
$\hat{\alpha}_2 - \hat{\alpha}_1$			-0.53***			-0.62
			(-15.42)			(-1.45)
RMSE(M)	5915.37	2783.5	1523.57	2827.01	1852.91	1674.9
$\hat{n}_b$	NA	1995	2002	NA	1991	2003

Table 4: Model fitting Chinese annual household consumption expenditures and aggregated retail sales (1984:2014), $n = 31$

level for the model in level with a change in the constant in the slope (CS), the model in first difference without structural break (dO), with a change in the constant (dC), and with a change in the constant

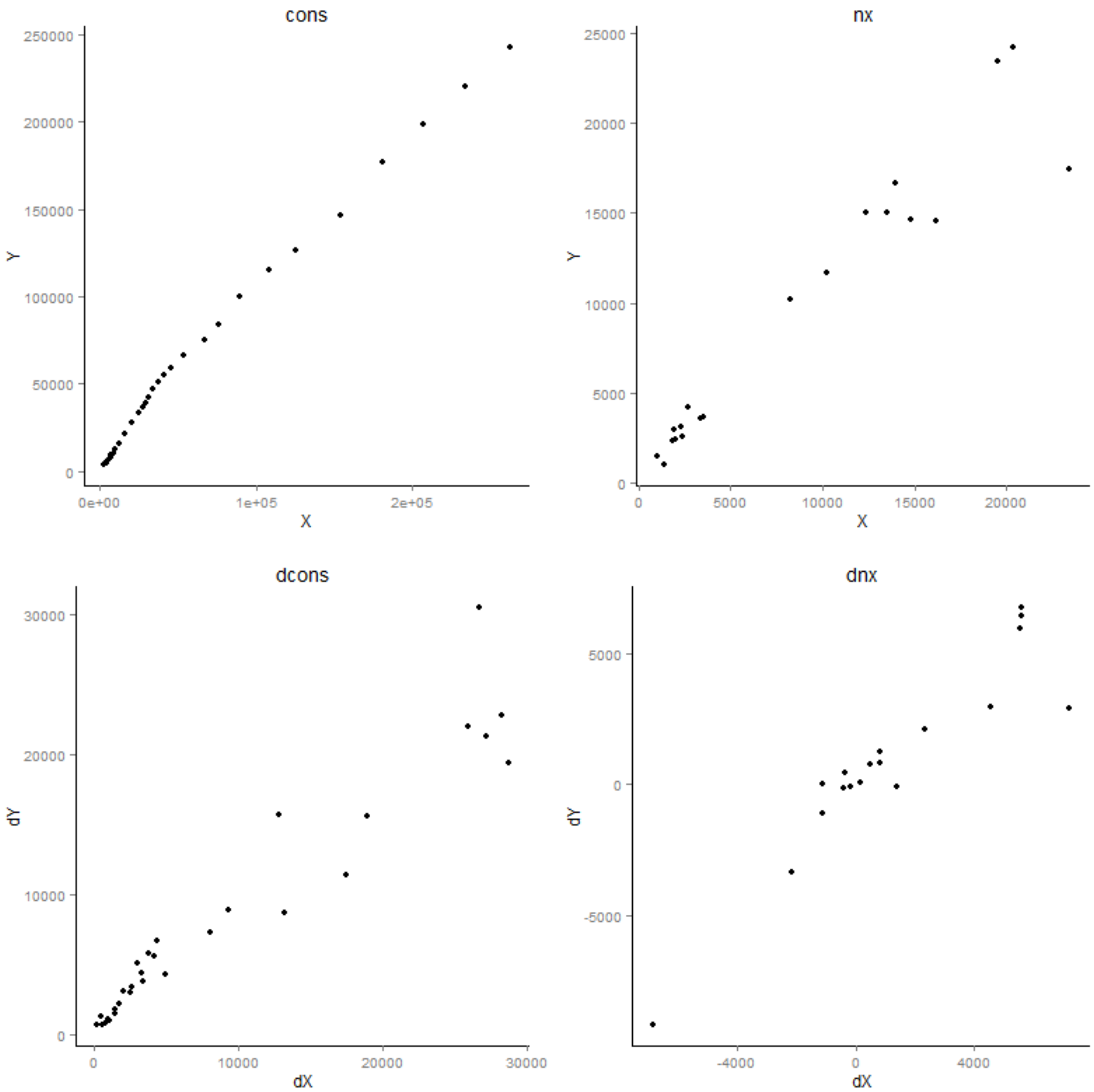

Figure 2: Scatter representation of the Chinese data, in level and in first difference

and slope (dCS). The most parcimonious of the three is model CS with a structural break occuring in 2002 and it does not differentiate the series. When only minimizing the annual prediction errors only, it is also the case of model CS with a structural break in 2002 which is selected. Therefore our two competing methods yield the same results in this case. Figure 3 represents the fit of the model using the usual method on the left, and our updated version on the right.
We can now predict the quarterly series of household consumption expenditures by applying the dissagregation formulae of the previous section, then adjusting the predicted series so their annual aggregates match the annual observations. Figure 4 shows the resulting predicted series which is also adjusted for seasonal variations. We observe that both method provide similar disaggregation over the whole considered period, but with a notable difference for the period 2002-2004 when we look at the cyclical components of the predictied series. Let us look at a more volatile and less trend driven series.

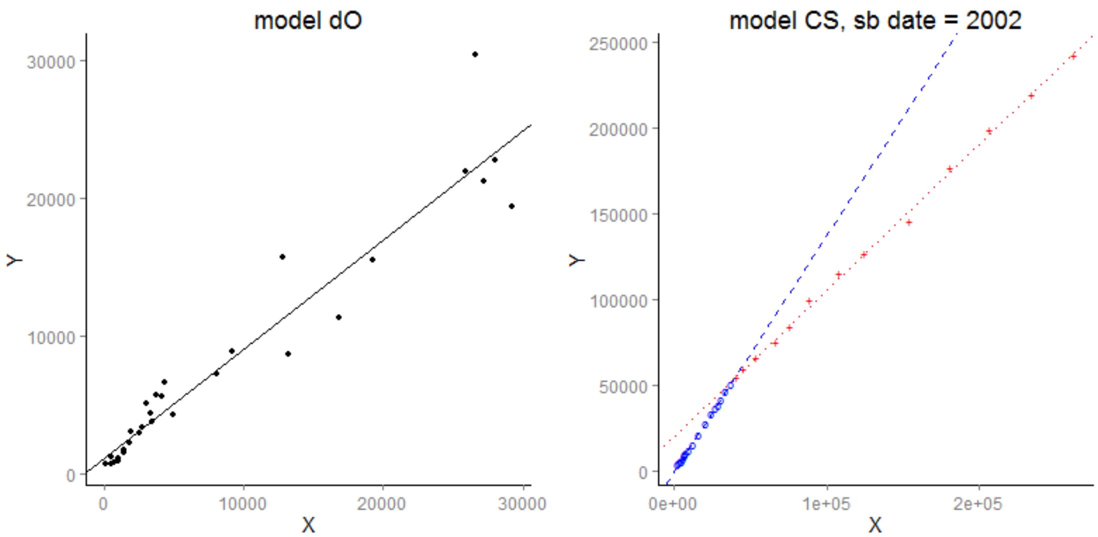

Figure 3: Fitted models for predicting the Chinese annual household consumption expenditures from the annualized retail sales

3.2 Disaggregating the Chinese net exports

We now apply the same procedures to disaggregate the Chinese net exports of goods and services using net exports of goods. Both time series are more volatile and seem to be less driven by a deterministic trend than the ones in the case of disaggregating annual consumption. Table 5 reports the results of the regression of each model.

When not considering structural breaks, both model in level and in first difference do not reject the unit root in the residuals. In this case we would select the one in first difference dO. The presence of the unit root in the residuals is rejected at the 5% level for the model in first difference with a change in the constant and the slope dCS at year 2010. However, the model yielding the lowest annual prediction errors is the model in level CS with a structural break at year 2011. Figure 5 represents the fitting of the models for each method. We now predict the disaggregated series of net exports of goods and services for each competing procedure. Figure 6 shows the predicted and seasonally adjusted disaggregated series. We observe from both representations in level and in first difference that the three methodologies yield fairly different quarterly series, notably for the period after 2008. In particular, not considering structural break yields more volatile quarterly predictions for the later period, and diverges significantly from the updated methods for the out-of-sample period (after 2015).

We have compared the updated methodology of disaggregation to the usual one by appling them to two time series of the Chinese national accounts whose profile are pretty different. These examples imply that particularly for volatile series the predicted disaggregated series are sensitive to the selected model and structural break date, even when this sensitivity is mitigated by the adjustment step. However, it is impossible at this stage to tell which method yields the best prediction because we don't observe the true values of the non aggregated series. In order to study how the methodology perfoms in disaggregating annual time series, we can look at how the competing procedures perform by using US data where the disaggregated data is known, and then simulated series.

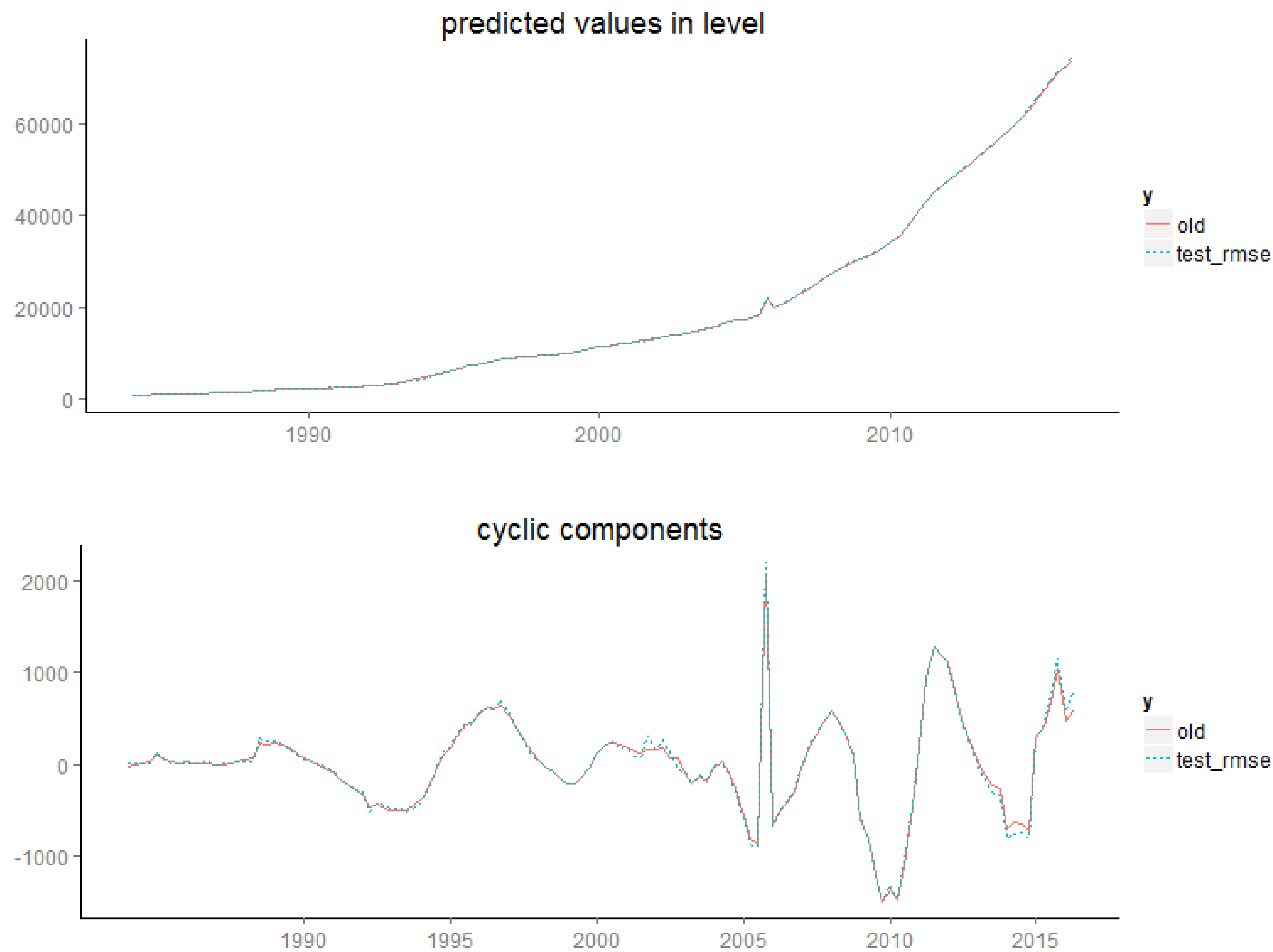

Figure 4: Predicted and seasonally adjusted Chinese quarterly household consumption expenditures by method

4 Performance of the method

Let us recall that the main objective of this methodology is to predict infra-annual estimations of an observed annual time series. Therefore we are more interested in the predictive ability of the entire method rather than the performance of the test that is used in the first step of it. The previous approximate critical values are used to select the type of structural break model between the candidates for the selection by co-integration test case.

Most emerging and advanced countries disclose their annual national account data of a particular year around the third quarter of the following year. It implies that quarterly indicators are disclosed between 3 and 7 quarters ahead of the annual publications of the national accounts. In order to assess if our method improves the prediction of the data, we look at its ability to predict the quarterly account series of the concomittant period to the indicator (in-sample prediction), as well as up to two years ahead of the last concomittant year (out-of-sample prediction). As a first application and empirical assessment of the method, we use US data on consumption expenditure from which we compare the quarterly disaggregation of annually aggregated real consumption personal expenditures using retail sales data as

	$\hat{n}_b$ selected by cointegration test					
	O	C	CS	dO	dC	dCS
$\hat{\mu}_1$	786.8	761.62	20.35	-248.18	143.4	-11.39
	(1.06)	(1.07)	(0.09)	(-0.73)	(0.43)	(-0.05)
$\hat{\alpha}_1$	0.99***	1.06***	1.17***	0.96***	0.99***	1.17***
	(15.23)	(14.11)	(46.14)	(10.06)	(11.81)	(18.68)
$\hat{\mu}_2 - \hat{\mu}_1$		-1968.47	9095.02***		-1626.11 *	-671.25
		(-1.65)	(8.54)		(-2.54)	(-1.53)
$\hat{\alpha}_2 - \hat{\alpha}_1$			-0.81***			-0.58***
			(-11.6)			(-5.15)
RMSE(M)	2002.41	1858.34	586.19	4507.4	1550.3	872.73
$\hat{n}_b$	NA	2009	2010	NA	2010	2010
ADF*	0.378	-3.678	-5.798	-1.896	-4.494	-8.596
p	0.996	0.589	0.08	0.602	0.272	0.001
	$\hat{n}_b$ selected by annual prediction error					
	O	C	CS	dO	dC	dCS
$\hat{\mu}_1$	786.8	635.4	23.95	-248.18	176.59	86.33
	(1.06)	(1.05)	(0.13)	(-0.73)	(0.67)	(0.47)
$\hat{\alpha}_1$	0.99***	1.09***	1.17***	0.96***	1.03***	1.15***
	(15.23)	(17.75)	(60.49)	(10.06)	(14.7)	(20.14)
$\hat{\mu}_2 - \hat{\mu}_1$		-3526.78 **	7823.46***		-2412.99***	-1370.62 *
		(-3.17)	(8.51)		(-4.18)	(-2.88)
$\hat{\alpha}_2 - \hat{\alpha}_1$			-0.76***			-0.47***
			(-13.24)			(-4.18)
RMSE(M)	2002.41	1586.85	459	4507.4	895.93	493.86
$\hat{n}_b$	NA	2011	2011	NA	2011	2011

Table 5: Model fitting Chinese net exports of goods and services and aggregated net exports of goods (1995:2014), $n = 20$

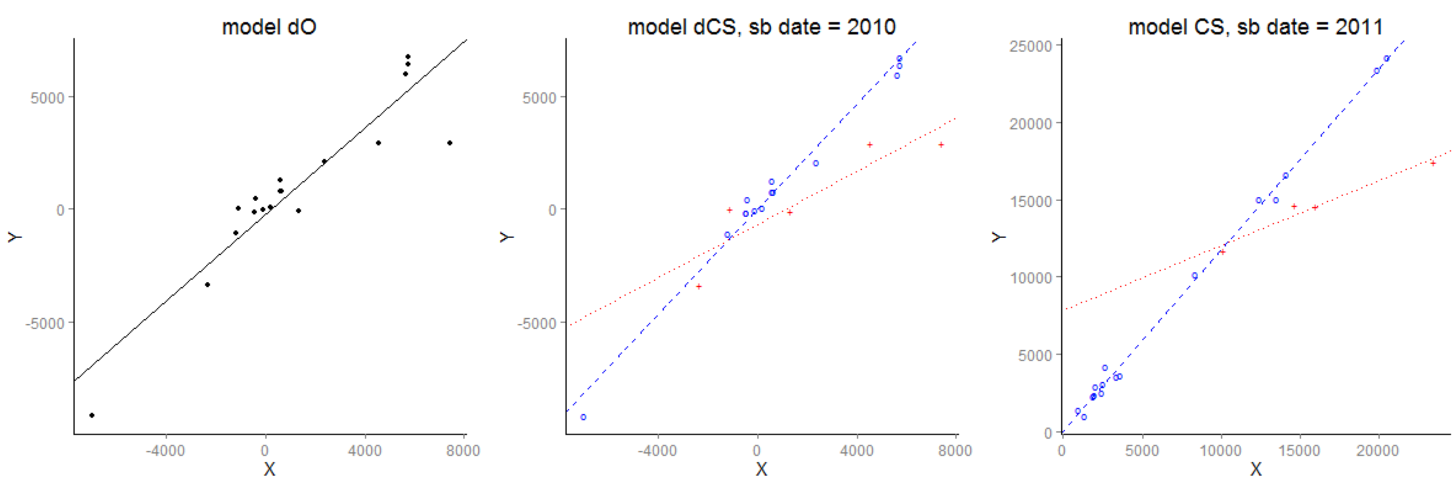

Figure 5: Fitted models for predicting the Chinese annual net exports of goods and services from the annualized net exports of goods

an indicator with observed quarterly data. After that we compute Monte Carlo simulations to analyze the predictive performance while controlling for the data generating processes.

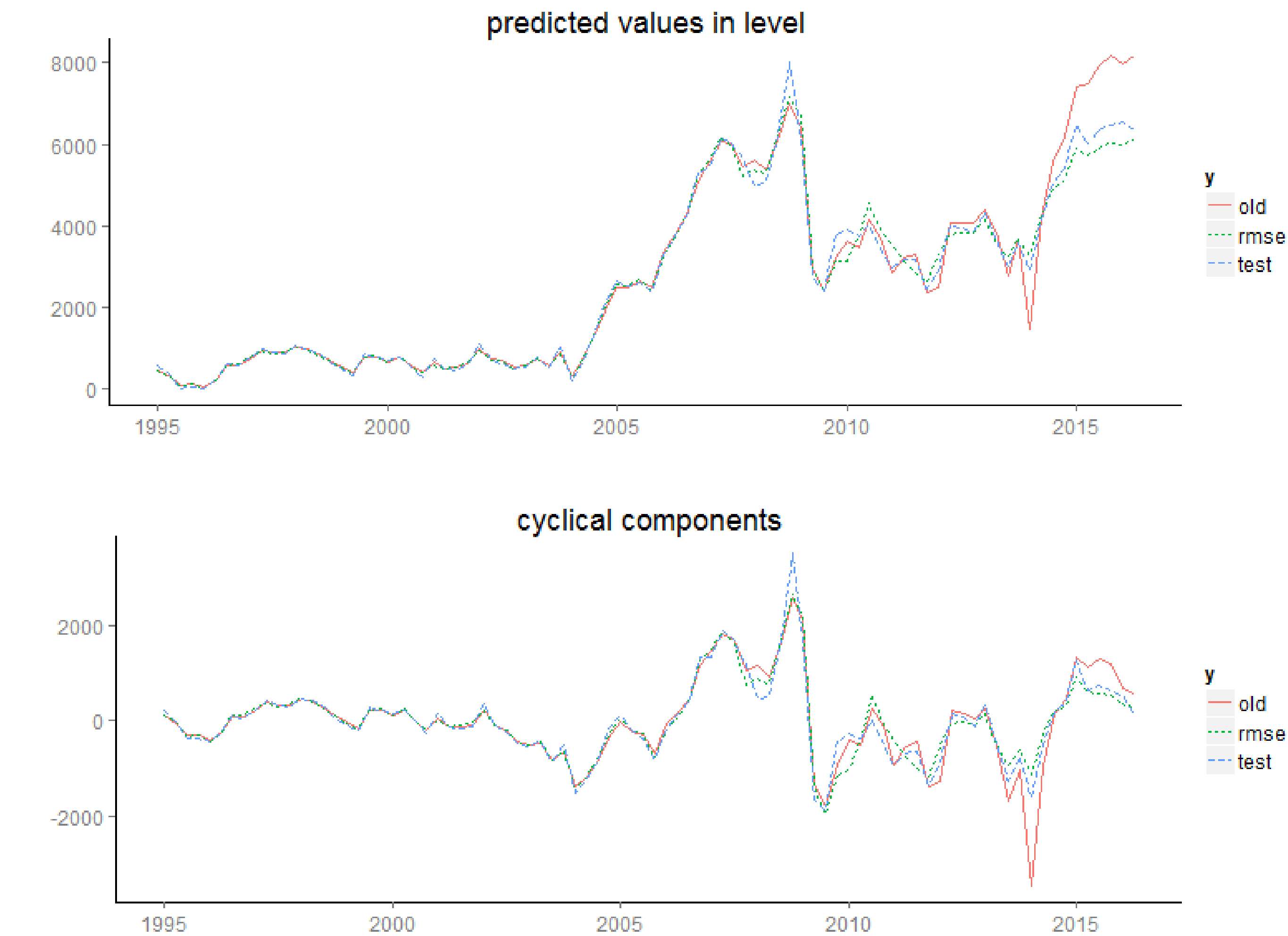

Figure 6: Predicted Chinese quarterly net exports of goods and services by method

4.1 Application and empirical performance on US data

In terms of predictive power, we can expect that taking into account a structural break in the regression step improves the fit of the annual data and reduces in-sample annual prediction error, especially since it introduces a non-linearity in a very small sample. However, the adjustment step corrects the annual prediction error at the quarterly frequency, which mechanically reduces in-sample quarterly prediction error. In the end, it is not trivial if a better annual fit improves the quarterly predictive ability. We can even expect that the adjustment step produces less distortion at the quarterly frequency when no structural break is taken into account because it will be uniform over longer intervals of the data, thus induces less arbitrary distribution of the error among the quarterly estimates.
For this exercise, we aggregate quarterly data into annual series, and we use a quarterly indicator to predict the original quarterly series.

4.1.1 The Data

As before, we try to predict quarterly series of national accounts representing national consumption expenditures and net exports of goods and services. For the first case, the national account data we are interested in are the time series of real personal consumption expenditure of the US economy (*rpce*), for which quarterly frequency of seasonal adjusted data are available for the period from 1959 to 2016. The quarterly indicator we use is seasonal adjusted quarterly series of real retail sales of the US economy (*rretails*), available from 1992Q1 to 2016Q2. For the case of the trade data, quarterly series of net exports of goods and services for the US (*nxgs*) are available from 1947Q1 to 2016Q2, and we use the quarterly series of net exports of goods (*nxg*) as an indicator, which is available from 1989Q1 to 2016Q2. After aggregating the quarterly national accounts data into annual series, we apply the same methodology as for the Chinese data.

4.1.2 Predicting the US personal consumption expenditures

Table 6 reports the results from the regression of annual *rpce* by annual *rretails*, for each type of model considered in the previous section. If no structural break is considered as in the usual method, the unit root in the residuals is not rejected either in the model in level or in first difference, therefore the model in first difference dO is selected. If we consider a structural break in the parameters, there is also no model for which the unit root in the residual is rejected. In this case we will also select the model in first difference with no structural break dO. However, the annual prediction errors are minimal for model in difference with a change in the constant and the slope dCS occuring in 1997.

	$\hat{n}_b$ selected by cointegration test					
	O	C	CS	dO	dC	dCS
$\hat{\mu}_1$	-1360.06***	-419.88	-652.13***	211.8***	143.99***	355.77 .
	(-4.61)	(-1.33)	(-4.14)	(8.52)	(4.51)	(1.85)
$\hat{\alpha}_1$	2.85***	2.47***	2.56***	1.06***	1.06***	-0.58
	(32.37)	(21.71)	(47.7)	(7.43)	(8.68)	(-0.39)
$\hat{\mu}_2 - \hat{\mu}_1$		803.46***	3080.49***		92.89 *	-120.33
		(4.18)	(4.52)		(2.86)	(-0.62)
$\hat{\alpha}_2 - \hat{\alpha}_1$			-0.55 **			1.65
			(-3.24)			(1.12)
RMSE(M)	333.5	243.64	129.89	251.21	116.81	112.15
$\hat{n}_b$	NA	2006	2008	NA	1999	1999
ADF*	-2.93	-4.124	-4.287	-2.284	-3.111	-3.179
p	0.186	0.384	0.433	0.419	0.822	0.866
	$\hat{n}_b$ selected by annual prediction error					
	O	C	CS	dO	dC	dCS
$\hat{\mu}_1$	-1360.06***	-494.63 *	-652.13***	211.8***	115.74 **	199.29
	(-4.61)	(-2.72)	(-4.14)	(8.52)	(3.16)	(0.78)
$\hat{\alpha}_1$	2.85***	2.5***	2.56***	1.06***	1.07***	0.46
	(32.37)	(40.53)	(47.7)	(7.43)	(9.07)	(0.25)
$\hat{\mu}_2 - \hat{\mu}_1$		892.78***	3080.49***		115.27 **	31.39
		(8.06)	(4.52)		(3.18)	(0.12)
$\hat{\alpha}_2 - \hat{\alpha}_1$			-0.55 **			0.61
			(-3.24)			(0.33)
RMSE(M)	333.5	161.81	129.89	251.21	108.48	108.45
$\hat{n}_b$	NA	2008	2008	NA	1997	1997

Table 6: Model fitting US annual household consumption expenditures and aggregated retail sales (1992:2014), $n = 23$

With the selected models, we dissagregate *rpce* into quarterly estimates which we compare with the observed quarterly values. Figure 7 represents the predicted values and its cyclical components. We observed that models selected by each method (*old*, *test*, *rmse*) yield similar quarterly predictions. Compared to the actual observations of *rpce* (*obs*), the predictions have a similar shape but there are discrepancies in the values, which are noticeable especially when we compare the cyclical components.

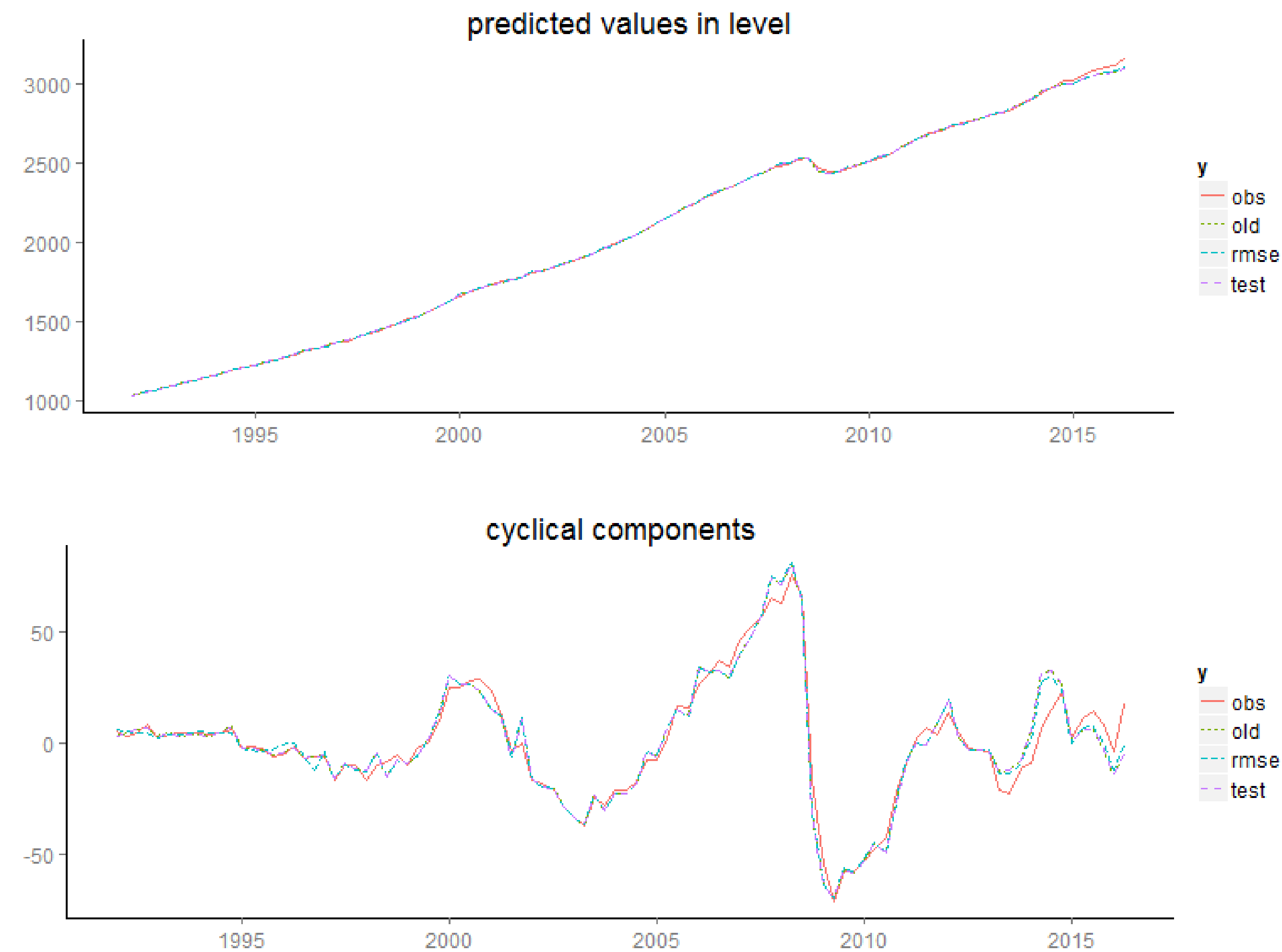

Figure 7: Observed and predicted US quarterly household consumption expenditures by method

For this application we can also compute the quarterly prediction error by comparing the estimates by method $\tilde{y}$ to the observations y, then assess the actual gain in accuracy by using our new methodology. Each method me has its quarterly prediction error:

$$qerr_{me} = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(\tilde{y}_{t,me} - y_t)^2}$$

Not considering structural changes (*old*) implies a quarterly root mean square error of 11.09 (1.73% of observed standard deviation). For this particular case, considering structural changes but selecting the type and date with the test-based approach (*test*) does not actually detect a structural change, therefore there is no gain in accuracy. However, selecting the type and date by minimizing the annual prediction error (*rmse*) implies a quarterly error of 9.63 (1.51% of observed standard deviation), therefore a 13.1% gain in accuracy (but only 0.22 % of observed standard deviation).

4.1.3 Predicting the US net exports of goods and services

Table 7 reports the results from the regression of annualized *nxgs* by annualized *nxg*, for the same models as previously. The unit root in residuals is never rejected at the 5% level whatever the type of model, therefore the model in first difference without structural change dO is selected in the usual method. When considering structural changes, we could select the model in first difference with a shift in 2006 in the constant dC by being more laxist since the unit root it rejected at the 10% level, what we do for a matter of comparison. If the structural change model and date is selected by minimizing the annual prediction error, the model in first difference with a shift in 1997 in the constant and the slope dCS is the best one.

	$\hat{n}_b$ selected by cointegration test					
	O	C	CS	dO	dC	dCS
(Intercept)	45.01	65.35 **	63.63***	9.11 *	-2.52	-7.21
	(1.49)	(3.86)	(4.3)	(2.45)	(-0.64)	(-1.26)
X	0.91***	0.99***	1***	1.03***	0.99***	0.91***
	(17.73)	(32.14)	(36.88)	(27.75)	(34.48)	(12.43)
d		113.92***	-143.69		22.04***	26.68 **
		(6.51)	(-1.68)		(4.02)	(3.92)
dX			-0.36 *			0.09
			(-2.85)			(1.13)
rmse.a	49.53	26.5	22.34	63.87	14.41	14.52
sb date	NA	2011	2009	NA	2006	2006
inf ADF	-1.198	-2.152	-4.108	-2.169	-5.373	-5.018
pval	0.867	0.99	0.518	0.476	0.09	0.222
	$\hat{n}_b$ selected by annual prediction error					
	O	C	CS	dO	dC	dCS
(Intercept)	45.01	64 **	63.63***	9.11 *	-2.2	-6.36
	(1.49)	(3.81)	(4.3)	(2.45)	(-0.6)	(-1.19)
X	0.91***	1***	1***	1.03***	0.98***	0.91***
	(17.73)	(32.19)	(36.88)	(27.75)	(35.46)	(13.01)
d		106.4***	-143.69		23.58***	27.6***
		(6.56)	(-1.68)		(4.4)	(4.21)
dX			-0.36 *			0.08
			(-2.85)			(1.06)
rmse.a	49.53	26.35	22.34	63.87	13.32	12.76
sb date	NA	2010	2009	NA	2007	2007

Table 7: Model fitting US net exports of goods and services and aggregated net exports of goods (1995:2014)

Figure 8 represents the quarterly observations and estimates of *nxgs* in level and the cyclical components. As in the case of US consumption, the three methods yield fairly similar estimates. We can however notice the estimates using the old method to be closer to the observed values for the out-of-sample period (2015-).

Numerically, the old method provides estimations with a quarterly root mean square error of 2.08 (4% of observed standard deviation), whereas selecting a structural break model and date with a test-based approach has a quarterly RMSE of 2.25 (5% of observed standard deviation), and by minimizing the annual prediction a quarterly RMSE of 2.73 (5.2% of observed standard deviation). Considering a structural break for predicting the US net exports therefore implies a loss of prediction accuracy of more than 20% (but only 1.2 percentage points relatively to observed standard deviation).

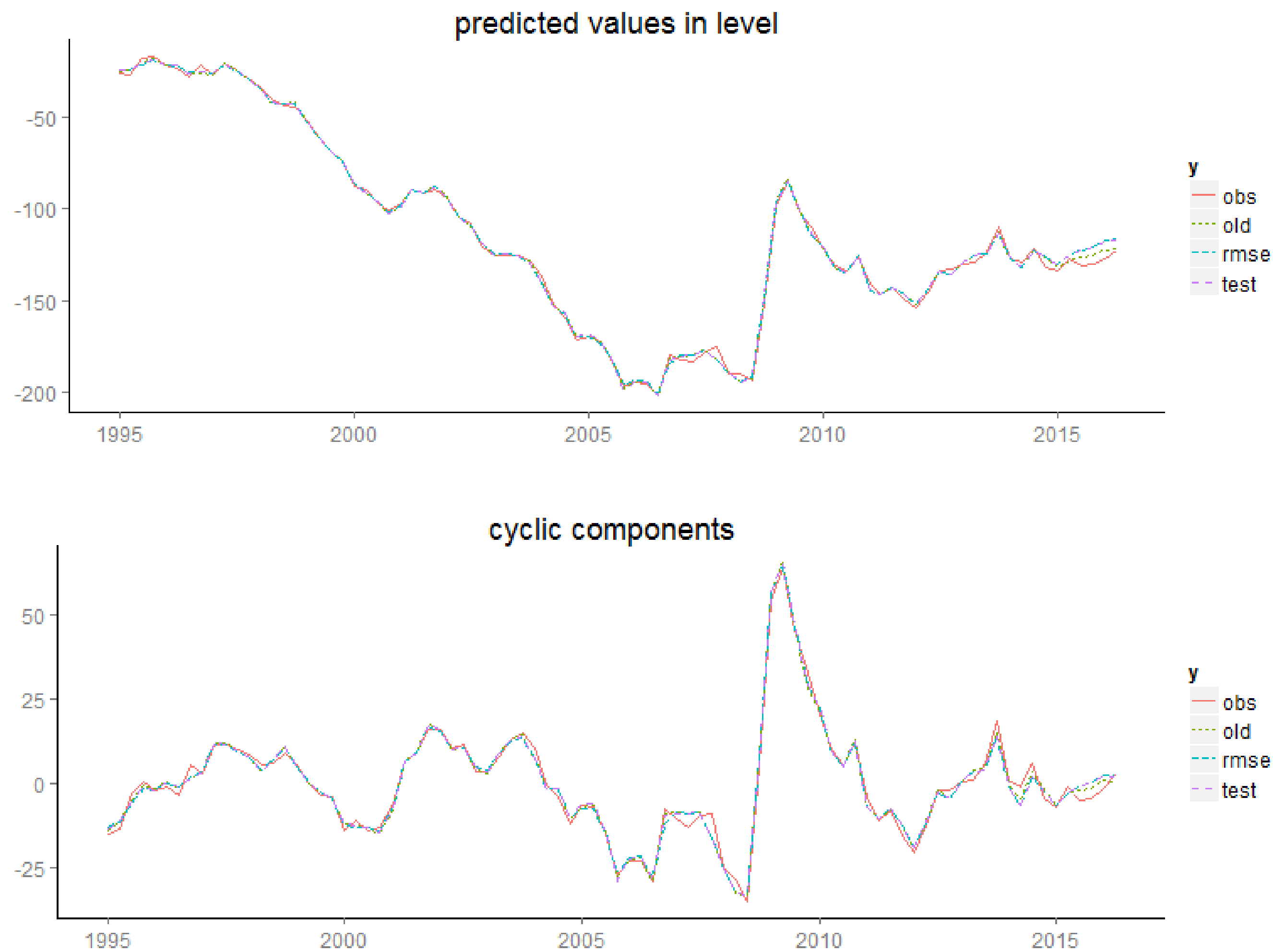

Figure 8: Observed and predicted US net exports of goods and services by method

The example on some US time series hints that considering a structural change in the prediction model may or may not improve accuracy. We can indeed expect that considering a structural change when there is not would not improve or even worsen the estimates. We now assess the prediction performance of simulated series for which we can control for the data generating process, especially to consider the cases with or without structural changes.

4.2 Monte Carlo simulations

In the first section we disaggregated the Chinese annual data into quarterly series, but nothing could assess the quality of the quarterly predictions. In the second second we used observed quarterly official national account data from the United States and looked if the disaggregation method could correctly reproduce the data. However the US data prove to be less adapted to our modelization than the Chinese data. In order to obtain a more precise assessment of the ability of our method to predict a quarterly series by disaggregating its annual aggregation using a related series, we simulate small and independent quarterly series and a related quarterly indicator, for which we will be able to control for the type of structural break model which generates them.

4.3 Calibration

For each replication of the simulated series, we first simulate the indicator x as a random walk:

$$x_t = 2 + x_{t-1} + \eta_t \qquad \eta_t \sim \mathcal{N}(0, 10)$$

In a typical case of temporal disaggregation, the related series x used as a predictor must be a good indicator of the series Y we want to disaggregate. As a consequence most of the variance of the disaggregated values y must be explained by the variance of x. Moreover, we want to distinguish the performance of our method under the presence or the absence of a structural change in the parameters, so we choose parameter values which takes values that match the Chinese national accounts data but also imply a sharp structural change. For each model, replications of a couple of quarterly series $\{y, x\}$ are simulated, following the subsequent generating processes:

$$u_t = \rho u_{t-1} + \varepsilon_t \qquad \varepsilon_t \sim \mathcal{N}(0, 1)$$

For the models in level:

(O) $y_t = 8 + 0.9x_t + u_t$

$$\text{(C)} \begin{cases} y_t = 3 + 0.9x_t + u_t & t < 4n_{sb} - 3 \\ y_t = 16 + 0.9x_t + u_t & t \geq 4n_{sb} - 3 \end{cases}$$

$$\text{(CS)} \begin{cases} y_t = -1 + 1.4x_t + v_t & t < 4n_{sb} - 3 \\ y_t = 19 + 0.9x_t + v_t & t \geq 4n_{sb} - 3 \end{cases}$$

For the models in first difference,

$$v_t = v_{t-1} + u_t$$

and consistently with the data the change in the constant coefficient is of much smaller amplitude since it implies a change of trend in the model in level:

(dO) $y_t = 1 + \frac{t-2.5}{16} + 0.8x_t + v_t$

$$\text{(dC)} \begin{cases} y_t = 1 + 0.3\frac{t-2.5}{16} + 0.8x_t + v_t & t < 4n_{sb} - 3 \\ y_t = 1 + 0.3\frac{t-2.5}{16} + 3.3\frac{t-4n_{sb}+5.5}{16} + 0.8x_t + v_t & t \geq 4n_{sb} - 3 \end{cases}$$

$$\text{(dCS)} \begin{cases} y_t = 1 + 0.3\frac{t-2.5}{16} + 1.4x_t + v_t & t < 4n_{sb} - 3 \\ y_t = 1 + 0.3\frac{t-2.5}{16} + 3.3\frac{t-4n_{sb}+5.5}{16} + 1.4\frac{X_{n_{sb}-1}}{4} + 0.8(x_t - \frac{X_{n_{sb}-1}}{4}) + v_t & t \geq 4n_{sb} - 3 \end{cases}$$

We consider $\rho = 0$ and $\rho = 0.5$. For each replication of quarterly series, we can aggregate to obtain annual series of sample size $N_{total} = N_{in-sample} + 2N_{out-of-sample}$, such that $N_{in-sample} = N \in \{15, 20, 25, 30, 50\}$ and $N_{out-of-sample} = 4$. The date of structural break n_{sb} is random and, if $n \in \{-3, -2, ..., N, N+1, ..., N+4\}$, is drawn in from uniform distribution $\mathcal{U}[\![0.15N, 0.85N]\!]$. In the end we have 10 000 replications of quarterly series $\{y, x\}$ of size T=92,112,132,152,232.

4.4 Performance of the methodology

The main objective of the methodology is to better predict quarterly series than the usual method that doesn't consider neither structural breaks nor short series. We consider three alternatives in selecting the regression model:

- the usual method (*old*): select the model in level without structural change O if the unit root in the residuals is rejected, the model in first difference without structural change dO if not
- the test-approach (*test*): select the most parcimonious model with or without a structural change which rejects the unit root in the residuals, or the model in first difference without structural change dO when none of the models rejects the unit root
- the annual error approach (*rmse*): select the model which minimizes the annual prediction error in level.

Firstly, the test-approach allow us to assess the power of the cointegration test under our calibration. Secondly we compare the predictive performance of each method, especially the accuracy gain of considering a structural change in the parameters.

4.4.1 Performance of the test of cointegration with endogenous structural change

Selecting a model which rejects the presence of a unit root in the residuals can help improving the prediction of the disaggregated estimates. Indeed, the less persistence there is in the residuals, the lesser the adjusting step of the methodology will affect the predictions. However, using the p-values as a selection criterion requires the corresponding test to have good power for the small samples we are considering. Let us look at the ability of the test to reject the unit root when there is none in our calibration. Table 8 report the probability to reject the unit root in the residuals at the 5% nominal level when there is no unit root in the residuals and the regression model is the true model, for sample sizes 15, 20, 25, 30 and 50 (the complete tables of the regression of each model against each true model are reported in Appendix C).

	$\rho = 0$					$\rho = 0.5$				
N	15	20	25	30	50	15	20	25	30	50
O	0.589	0.826	0.94	0.985	0.995	0.351	0.572	0.775	0.899	0.994
C	0.174	0.309	0.537	0.865	1	0.095	0.13	0.307	0.725	1
CS	0.263	0.38	0.572	0.796	1	0.188	0.238	0.363	0.606	0.996
dO	0.203	0.417	0.649	0.83	0.99	0.116	0.233	0.406	0.608	0.973
dC	0.092	0.167	0.268	0.406	0.921	0.064	0.101	0.149	0.237	0.7
dCS	0.093	0.171	0.284	0.444	0.941	0.057	0.088	0.138	0.212	0.696

Table 8: Power of the test: probability to reject the unit root in the residuals at the 5% nominal level when there is no unit root in the residuals and the regression model is the true model

We expectedly see overall that power declines when the autocorrelation ρ increases. More specifically for the models estimated in level with $\rho = 0$, the test without structural break rejects the unit root in the residuals when there is none with 59% probability for the smallest sample size $N = 15$, then with a more reasonable 83% for $N = 20$, and at least 94% for $N = 25$ and higher. For the models with structural breaks, the probability is only higher than 50% from $N \geq 25$ and higher than 80% from $N \geq 30$ which show at low power for the test in level in very small samples but reasonable power in less small samples, considering our calibrations. When the models are estimated in first difference, we see that the model without structural break has low power (20% probability at $N = 15$, increasing to a reasonable 65% at $N = 25$, 84% at $N = 30$, and higher than 99% for N higher than 50), whereas the models with structural break (dC) and (dCS) have even lower power (probablity lower than 50% for $N \leq 40$, and higher than 90% for $N \geq 50$). In general we can say that, considering our calibrations, the test has reasonable power for small sample sizes when there is no structural change, whereas the power is low for small sample size when there is no structural breaks. We can therefore expect the method relying on a test of cointegration with endogenous structural break not to improve very much the prediction of the disaggregated estimates for very small samples $N \leq 30$ for the true models in level and for small samples $N \leq 50$ for the true models in first difference.

4.4.2 Predictive performance

Table 9 reports the ratio of the quarterly prediction error by using the method *test* or *rmse* on the quarterly prediction error by using the old method.

	$\rho = 0$									
	test-approach					rmse-approach				
N	15	20	25	30	50	15	20	25	30	50
O	1.002	1	1	1	1	1.164	1.101	1.041	1.029	1.002
C	1.168	0.992	0.901	0.827	0.916	0.495	0.592	0.669	0.727	0.848
CS	1.07	0.92	0.855	0.828	0.892	0.868	0.872	0.873	0.822	0.807
dO	1.049	1.047	1.034	1.038	1.039	1.421	1.307	1.254	1.243	1.142
dC	1.02	1.001	0.984	0.975	0.998	0.937	0.845	0.794	0.77	0.733
dCS	1.04	1.043	1.049	1.057	1.09	1.035	0.947	0.909	0.894	0.816
	$\rho = 0.5$									
	test-approach					rmse-approach				
N	15	20	25	30	50	15	20	25	30	50
O	1.007	1.002	1	1	1	1.26	1.108	1.091	1.055	1.009
C	1.229	1.087	1.035	0.929	0.984	0.554	0.625	0.688	0.733	0.838
CS	1.118	1.025	0.952	0.904	0.911	0.866	0.902	0.854	0.837	0.81
dO	1.029	1.037	1.03	1.03	1.035	1.384	1.299	1.255	1.24	1.188
dC	1.025	1.026	1.029	1.029	1.043	1.243	1.152	1.105	1.092	1.019
dCS	1.037	1.034	1.033	1.032	1.046	1.25	1.178	1.129	1.122	1.048

Table 9: Quarterly prediction error by method with structural breaks relative to the prediction error of the method without structural breaks

For $\rho = 0$, when the true model is not subject to a structural change (models O and dO), considering a structural change and selecting the type and date by a cointegration with endogenous break test approach does not improve the prediction (the gain in accuracy is almost null). The predictions are in fact almost identical, because the procedure would select the models without any structural change. When there is a structural change for the model in level (C and CS), there is a slight loss of accuracy for $N = 15$ but a gain in accuracy of at least 10% for model C and $N \geq 25$ and for model CS and $N > 20$. The gain in accuracy is almost inexistant or slightly negative for the cases in first difference for our calibration, mainly because the true change in parameters is of much smaller magnitude in the regression models.
However, selecting the type and date of structural change by minimizing the annual prediction error significantly improves the predictions when there is indeed a structural change in the true model. For the models in level, the gain in accuracy is from 15% to 50% for model C, and from 13% to 20% for model CS, and respectively for the models in frst difference from 6% to 26% and from -4% to 18%. The drawback is that when there is no structural change in the true model, considering a structural

change in the regression implies a loss of accuracy which becomes very significant for the model in first difference (from -14% to -42% for model dO against from 0% to -16% for model O).
For $\rho = 0.5$, all predictions lose accuracy, and there is only a gain in accuracy for the true models in level.

In conclusion to our simulated studies, updating the disaggregation method by considering a parameter instability of the linear model has different implications depending on the procedure of the selection of the type and date of structural change. When there is a structural change, considering one and selecting its type and date with the rmse-based approach improves significantly the accuracy of the prediction, much more than selecting them with the test-based approach. However when there is no structural change, the rmse-based approach implies a loss in accuracy while the test-based approach doesn't change anything, so it is better not to consider any structural change. A mixed procedure which would conciliate all advantages of our method is the following:

- if the test-based approach rejects the unit root in any model with a structural change, one should use the rmse-based approach to find a better model with structural change
- if the test-based approach does not reject the unit root for a model with a structural change, one should not consider any structural change in the model, especially when the sample size is very small.

5 Business cycle stylized facts

Now that we have studied the performance of our considered methods of temporal disaggregation we apply them to the Chinese national accounts so we can undertake for example a business cycle stylized facts analysis à la Backus et al. (1992), which quantitatively assesses the relevance of various dynamic and stochastic general equilibrium (DSGE) business cycles models in the case of the United States from 1954Q1 to 1989Q4 by computing second order moments. In order to compute comparable relative volatiliy and correlation tables between the components of the Chinese national accounts from 1998Q1 to 2016Q2, we first construct quarterly estimates of the Chinese capital formation and governement consumption expenditures. Then by using the former estimates of consumption and net exports, we can finally compute quarterly estimates we can compare to the official data.

5.1 Estimating the Chinese quarterly GDP

In order to estimate the quarterly national accounts data for the Chinese we apply the mixed methodology we just mentionned. For the Chinese consumption, the previous sections show the test-based approach detecting a structural break in several models with a structural change (CS, dC and dCS) so we use the rmse-based approach to disaggregate it, i.e. selecting the model CS with a structural break occuring in 2002. Concerning the Chinese net exports, the test-based approach selected model dCS, so we use the rmse-based approach which selects the model CS with a break in 2011. Now we apply the same procedure to the Chinese fixed capital formation (a measure of national investment) and government expenditures (the regression results are in Appendix D). For the investment (cf), models dO, dO and dCS are respectively selected by methods *old*, *test* and *rmse*, and for the governement expenditures (g) models dO, dO and CS. Since the test-based approach never detects a structural change, we do not consider any of them and use the usual method, which selects the model in first difference without structural change dO.
Now that we have every expenditure component of GDP, we can aggregate them to obtain our quarterly

estimates of nominal GDP which we compare to the official GDP data.

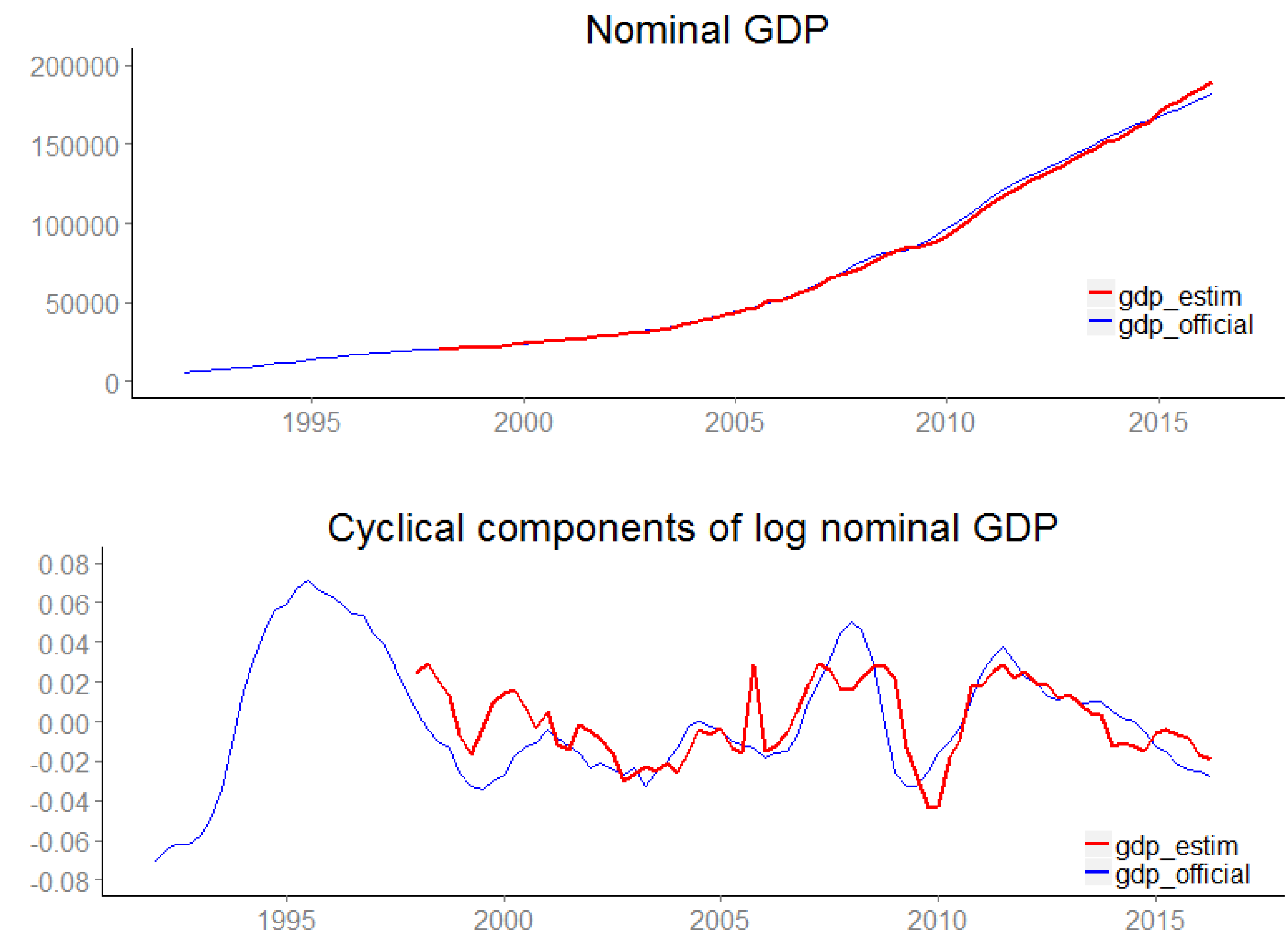

Figure 9: Estimated Chinese quarterly nominal GDP

Figure 9 represents both estimated and official nominal GDP after seasonal adjustment, in level and in their cyclical components. In terms of level magnitude our estimates are similar to the official data, which means that our methodology doesn't alter the consistency of the annual national account data. Looking at the cyclical components of log nominal GDP, we reproduce the general fluctuations of the official data, especially troughs around 2003 and before 2010, peaks around 2008 and 2012, and a recession phase since 2012. Our estimates however show more volatile cyclical components. We are now able to compute the moments of second order of the cyclical components of the Chinese national accounts.

5.2 Second order moments of the cyclical components

Business cycle stylized facts are computed from the cyclical components of the logarithmic transformation of seasonally adjusted and deflated time series, except for net exports which are considered as the non logarithmic share of GDP. Considering what have been done in the previous sections, we still have to deflate the series in a way that the components of the GDP are consistent with the GDP data in real values. The traditional way is to apply the GDP deflator on the nominal data in levels. An implied GDP deflator can be retrieved from the official data by dividing the quarterly nominal GDP by the quarterly real GDP. The latter is constructed by applying the quarterly year-over-year real growth rates (available for 1992Q1-2016Q2) on the quarterly real GDP in level (available for 2011Q1-2016Q2). However, we showed that the official GDP quarterly data are not very suitable for our quarterly estimates of the national accounts data. If we want to be agnostic about the use of the official data, we had better not to apply the implied GDP deflator to our estimates. Therefore, we consider two alternatives as GDP quarterly data and deflator. One case where we use the official GDP quarterly data and implied deflator, and another case where we use our GDP estimates instead of the official data and the official quarterly Consumer Price Index (CPI)[4] as a deflator.

Having disaggregated the nominal annual Chinese national accounts into nominal quarterly estimates, we deflate the series with the implicit deflator of the Chinese GDP or the CPI, then we seasonally adjust the resulting real series by using the X-13ARIMA-SEATS program and finally recover the cyclical components by Hodrick-Prescott filtering. Figure 10 represents the cyclical components of the selected disaggregated national accounts, compared to the cyclical components of the Chinese GDP for the two cases of GDP estimates previously mentionned.
Table 10 reports the moments of second order of the cyclical components of the Chinese national accounts, also when considering the two alternatives measures of GDP values and deflator. We consider only the period 2000Q1:2014Q4 in order to mitigate the effects of the estimation in the border period dates from the HP filtering and in the most recent dates which are subject to regular revisions. By looking at the standard deviations, we observe that after deflating the series our estimates of the quarterly GDP have almost twice as volatile cyclical components as the official data. This difference is also translated into the relative volatilities between GDP and the expenditure components. When we consider the official GDP data, Chinese consumption is 2.17 times as volatile as the GDP, which imply no consumption smoothing at the national level. It is also little persistent (autocorrelation of 0.15) and lowly procyclical (contemporary correlation of 0.19 less for the correlation to delayed GDP). Investment is 3 times as volatile as GDP, highly persistent and contracyclical ($\rho(\mathrm{cf}_t, \mathrm{gdp}t) = -0.32$). Governement consumption expenditure are even more volatile (4.89 times as volatile as GDP) and lowly procyclical ($\rho(\mathrm{g}_t, \mathrm{gdp}_t) = 0.16$), while net exports are 1.34 times as volatile as GDP and procyclical ($\rho(\mathrm{nx}/\mathrm{gdp}_t, \mathrm{gdp}_t) = -0.36$).
As a matter of comparison, we proceed to compute the stylized facts of the US business cycles for the same period in Table 11. Also, Aguiar and Gopinath (2007) calculate them for various emerging markets for the later decades of the last century. The second order moments computed when considering the official GDP data takes values very far from what we usually find for other countries, where in general investment is procyclical and net exports countercyclical. Also in developed countries, relative volatility of consumption is much smaller and implies consumption smoothing (relative volatilty of 0.86 for the US in the same period). However, our computed relative volatility of consumption is of the same magnitude as for some emerging countries such as Brazil in 1991Q1:2002Q1 (2.01), Ecuador in 1980Q1:2002Q2 (2.39) or Slovakia in 1993Q1:2003Q1 (2.04).

Our quarterly estimates have cyclical components which all have a low contemporaneous correlation to the cyclical components to GDP. Since GDP is the aggregate of the national accounts, it indicates that the cyclical components of the official GDP data are inadequate for our estimated fluctuation data. Therefore using our own estimates of GDP data, computed as the sum of the quarterly estimates of the national accounts data, as well as the CPI as a aggregate price deflator, allows us to perform a

[4]Computed as the quarterly average of the official monthly CPI.

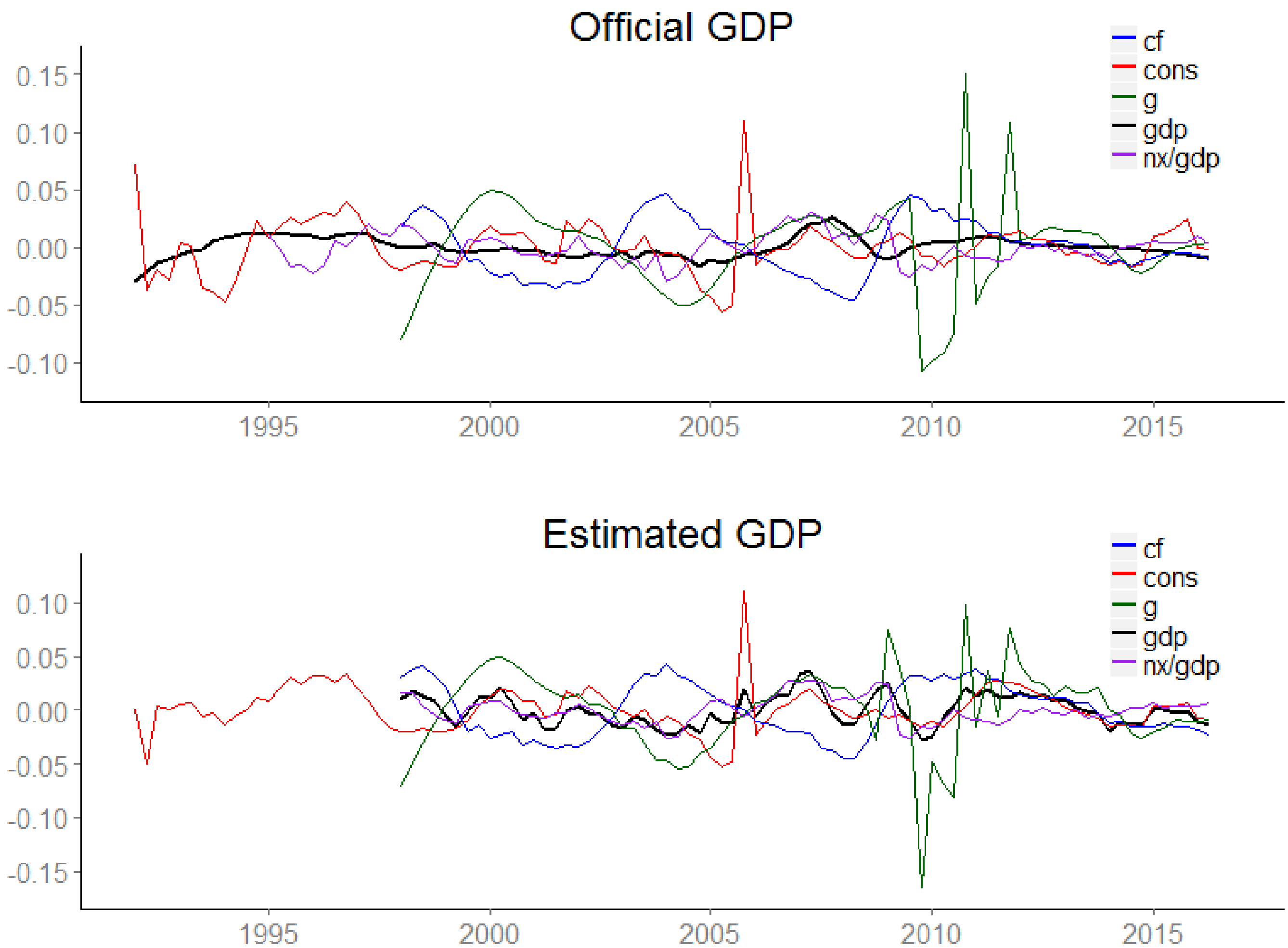

Figure 10: Cycle components of the estimated Chinese quarterly national accounts, by GDP estimates and deflator

more reliable analysis of the business cycle stylized facts. Indeed, when we consider our estimates of the quarterly GDP, contemporaneous correlation of consumption, governement expenditures and share of net exports to GDP are significantly higher, with respective values of 0.54, 0.6 and 0.55. The correlation of investment to GDP however drops to -0.1, which can indicate either an inadequacy of the CPI as a deflator for investment or the investment of fixed asset as an indicator of the fluctuations, but we can note that all components of GDP retain the same sign of cocyclicality as previously. Consistently with the higher volatility of our GDP estimates, the relative volatilities of the expenditure accounts to GDP drop to 1.46 for consumption, 1.64 for investment, 2.7 for governement expenditures. The lower but still high relative volatility of consumption which imply an absence of consumption smoothing at the national level is more in line with mid-range emerging markets (in terms of consumption smoothing) such as Argentina in 1993Q1:2002Q4 (1.38), Israel in 1980Q1:2003Q1 (1.6) or South Africa in 1980Q1:2003Q1 (1.61). It is however still almost twice the relative volatility to GDP of the US in the same period.

Official GDP with implied GDP deflator

x	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(\text{gdp})}$	$\rho_1(x)$	$\rho(x_t, \text{gdp}_{t+k})$, where k= -4	-3	-2	-1	0	1	2	3	4
gdp	0.87	1	0.9	0.28	0.52	0.74	0.9	1	0.9	0.74	0.52	0.28
cons	2.17	2.5	0.15	0.02	0.08	0.06	0.1	0.19	0.18	0.2	0.13	0.06
cf	2.55	2.94	0.94	-0.21	-0.32	-0.4	-0.39	-0.32	-0.24	-0.14	-0.08	-0.03
g	4.24	4.89	0.32	0.34	0.3	0.2	0.16	0.16	0.19	0.19	0.14	0.08
nx/gdp	1.34	...	0.72	0.26	0.34	0.33	0.32	0.29	0.36	0.44	0.47	0.43

Estimated GDP with the official CPI as deflator

x	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(\text{gdp})}$	$\rho_1(x)$	$\rho(x_t, \text{gdp}_{t+k})$, where k= -4	-3	-2	-1	0	1	2	3	4
gdp	1.53	1	0.69	-0.1	0.13	0.4	0.69	1	0.69	0.4	0.13	-0.1
cons	2.24	1.46	0.21	0.05	0.31	0.31	0.35	0.54	0.23	0.13	0.05	-0.06
cf	2.51	1.64	0.95	-0.36	-0.26	-0.2	-0.15	-0.1	-0.12	-0.08	-0.01	0.1
g	4.13	2.7	0.41	0.12	0.24	0.54	0.58	0.6	0.35	0.13	-0.05	-0.16
nx/gdp	1.3	...	0.76	0.12	0.03	0.13	0.38	0.55	0.56	0.42	0.15	-0.06

Note: Statistics are calculated on HP-filtered season-adjusted deflated data. Except for net exports/output, all series are in logarithms.
gdp : Gross domestic product, *cons*: household personal consumption expenditures, cf: fixed capital formation (private investment), g: government consumption expenditures, nx: net exports of goods and services

Table 10: Second order moments of the business cycles of the Chinese economy 2000Q1:2014Q4

x	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(\text{gdp})}$	$\rho_1(x)$	$\rho(x_t, \text{gdp}_{t+k})$, where k= -4	-3	-2	-1	0	1	2	3	4
gdp	1.29	1	0.89	0.31	0.51	0.72	0.89	1	0.89	0.72	0.51	0.31
cons	1.11	0.86	0.87	0.32	0.5	0.69	0.86	0.93	0.84	0.63	0.39	0.15
cf	7.5	5.83	0.91	0.22	0.44	0.68	0.86	0.94	0.86	0.73	0.57	0.42
g	1.3	1.01	0.9	-0.29	-0.38	-0.46	-0.48	-0.48	-0.53	-0.58	-0.59	-0.52
nx/gdp	0.49	...	0.79	-0.03	-0.26	-0.51	-0.71	-0.76	-0.72	-0.56	-0.39	-0.26

Note: Statistics are calculated on HP-filtered season-adjusted deflated data. Except for net exports/output, all series are in logarithms.

Table 11: Second order moments of the business cycles for the United States 2000Q1:2014Q4

Conclusion

We have developped a methodology to construct quarterly data for emerging economies with an application to the Chinese economy based on a mixed procedure of endogenous structural break testing and prediction error minimizing. We did not take any instablility in the volatility into account, which we think is reasonable for the small samples we considered. Simulations confirmed that our choice of non linear parametrization improved the accuracy of the quarterly disaggregation of time series which behave like recent data from emerging markets such as China. It allowed us to compute the so-called stylized facts of its quarterly business cycle. We showed that the Chinese business cycle fluctuates in a different way from such an advanced economy as the US, but have similarities with other emerging countries such as exacerbated volatilies of the expenditure accounts relative to GDP and an absence of consumption smoothing at the national level. In addition we also find low persistences of the cyclical components of all national account components, as well as a much lower cyclicality to GDP. We can now apply this methodology to other emerging economies which are at a similar stage of development of their national statistical apparatus, or to future emerging economies. Some direct extensions of our methodology are for example to consider multiples indicators and structural breaks, while considering the limits due to the small sample size of the time series we are interested in. Also, a more thorough parametrization of the alternative hypothesis will provide a better assessment of the power the test procedure.

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A Estimating the trend component in the case of models estimated in first difference

When estimating a model in first difference, a siginificant intercept is translated into a trend component for the annual estimates in level, thus also for the dissagregated estimates. Estimating model (dO) yields predicted first difference values :

$$\Delta\hat{Y} = \begin{pmatrix} E(N-1) & \Delta X \end{pmatrix} \begin{pmatrix} \hat{\mu} \\ \hat{\alpha} \end{pmatrix}$$

which imply predicted level values that are defined up to an initial value $\hat{Y}_1$. We set the latter to the initial observed annual value Y_1, therefore :

$$\begin{cases} \hat{Y}_1 & = & Y_1 \\ \hat{Y}_n & = & \hat{Y}_{n-1} + \hat{\mu} + (X_n - X_{n-1})'\hat{\alpha} \quad \text{for} \quad n = 2, ..., N \end{cases}$$

or equivalently,

$$\begin{cases} \hat{Y}_1 & = & Y_1 \\ \hat{Y}_n & = & Y_1 + (n-1)\hat{\mu} + (X_n - X_1)'\hat{\alpha} \quad \text{for} \quad n = 2, ..., N \end{cases} \tag{19}$$

which we write matricially

$$\hat{Y} = \begin{pmatrix} E(N) & T(N) & X \end{pmatrix} \begin{pmatrix} Y_1 - \hat{\mu} - X_1'\hat{\alpha} \\ \hat{\mu} \\ \hat{\alpha} \end{pmatrix}$$

where $T(N) = \begin{pmatrix} 1 & \dots & N \end{pmatrix}'$. The presence of an annual trend component implies also that the predicted values at the disaggregated level also have a trend component:

$$\hat{y}_t \quad = \quad \hat{y}_{t-1} + (g(t) - g(t-1))\,\hat{\mu} + (x_t - x_{t-1})'\hat{\alpha} \tag{20}$$

where $g(t)$ is a quarterly trend component, i.e for a constant ν

$$g(t) = g(t-1) + \nu \;\text{ for }\; t \in \{2, ...fN\}$$

therefore

$$g(t) = g(1) + (t-1)\nu \;\text{ for }\; t \in \{2, ...fN\} \tag{21}$$

By backward induction we can rewrite (20) up to an initial value $\hat{y}_1$.

$$\hat{y}_t \quad = \quad \hat{y}_1 + (g(t) - g(1))\,\hat{\mu} + (x_t - x_1)'\hat{\alpha}$$

therefore

$$\hat{y}_t \quad = \quad \hat{y}_1 + (t-1)\nu\hat{\mu} + (x_t - x_1)'\hat{\alpha} \tag{22}$$

The first difference of the annual aggregation must sum up to the first difference of the annual estimates, i.e. in the case of a disaggregation to a higher frequency f:

$$\sum_{t=f(n-1)+1}^{fn} \hat{y}_t - \sum_{t=f(n-2)+1}^{f(n-1)} \hat{y}_t = \hat{Y}_n - \hat{Y}_{n-1} \;\text{ for }\; n \in \{2, ..., N\}$$

It implies for $n=2$:

$$\begin{aligned}\hat{\mu}\nu\left(\sum_{t=f+1}^{2f}(t-1)-\sum_{t=1}^{f}(t-1)\right)+\left(\sum_{t=f+1}^{2f}x_t-\sum_{t=1}^{f}x_t\right)'\hat{\alpha} &= \hat{Y}_2-\hat{Y}_1\\ \hat{\mu}\nu\left(\sum_{t=f}^{2f-1}t-\sum_{t=0}^{f-1}t\right)+(X_2-X_1)'\hat{\alpha} &= Y_1+\hat{\mu}+\hat{\alpha}(X_2-X_1)-Y_1\end{aligned}$$

therefore

$$\nu=\left(\sum_{t=f}^{2f-1}t-\sum_{t=0}^{f-1}t\right)^{-1}=\left(\sum_{t=0}^{2f-1}t-2\sum_{t=0}^{f-1}t\right)^{-1}=\left(\frac{(2f-1)2f}{2}-2\frac{(f-1)f}{2}\right)^{-1}=f^{-2}$$

The higher frequency estimates become:

$$\hat{y}_t = \hat{y}_1+\hat{\mu}\frac{t-1}{f^2}+(x_t-x_1)'\hat{\alpha} \tag{23}$$

Now we want the first annual aggregation to equal the first annual observation Y_1, therefore:

$$\begin{aligned}\sum_{t=1}^{f}\hat{y}_1+\hat{\mu}\sum_{t=1}^{f}\frac{t-1}{f^2}+\hat{\sum}_{t=1}^{f}(x_t-x_1)'\hat{\alpha} &= Y_1\\ f(\hat{y}_1-x_1'\hat{\alpha})-\frac{f-1}{2f}\hat{\mu}+X_1'\hat{\alpha} &= Y_1\\ \hat{y}_1 &= \frac{1}{f}\left(Y_1-X_1'\hat{\alpha}-\frac{f-1}{2f}\hat{\mu}\right)+x_1'\hat{\alpha}\end{aligned}$$

The quarterly estimates $\hat{y}_t$ when the model is estimated in first difference are therefore :

$$\hat{y}_t=\frac{Y_1}{f}+\hat{\mu}\frac{t-0.5(f+1)}{f^2}+\left(x_t-\frac{X_1}{f}\right)'\hat{\alpha}\quad\forall t\in\{1,...,4N\} \tag{24}$$

B Structural break variables and disaggregation formulas

B.1 Model in level with a change in the constant (C)

$$\hat{Y}_n=\hat{\mu}+X_n'\hat{\alpha}+\hat{\mu}_1\mathbb{1}_{n\geq n_{sb}}$$

$$\Leftarrow\quad \hat{y}_t=\frac{\hat{\mu}}{f}+x_t'\hat{\alpha}+\frac{\hat{\mu}_1}{f}\mathbb{1}_{t\geq f(n_{sb}-1)+1}$$

<u>Proof:</u>

It can be easy shown that $\forall n,\ \sum_{i=-f+1}^{0}\hat{y}_{fn+i}=\hat{Y}_n$

B.2 Model in level with a change in the constant and the slope coefficient (CS)

$$\hat{Y}_n = \hat{\mu} + X_n'\hat{\alpha} + (\hat{\mu}_1 + X_n'\hat{\alpha}_1)\mathbb{1}_{n\geq n_{sb}}$$

$$\Leftarrow \quad \hat{y}_t = \frac{\hat{\mu}}{f} + x_t'\hat{\alpha} + (\frac{\hat{\mu}_1}{f} + x_t'\hat{\alpha}_1)\mathbb{1}_{t\geq f(n_{sb}-1)+1}$$

Proof:

It can be shown again that $\forall n, \sum_{i=-f+1}^{0} \hat{y}_{fn+i} = \hat{Y}_n$

B.3 Model in first difference with a change in the constant (dC)

$$\Delta\hat{Y}_n = \hat{\mu} + \Delta X_n'\hat{\alpha} + \hat{\mu}_1\mathbb{1}_{n\geq n_{sb}} \text{ for } n \in \{2, .., N\}$$

$$\Leftarrow \quad \hat{y}_t = \frac{Y_1}{f} + \hat{\mu}\frac{t-0.5(f+1)}{f^2} + \left(x_t - \frac{X_1}{f}\right)'\hat{\alpha} + \hat{\mu}_1\frac{t-f(n_{sb}-1.5)-0.5}{f^2}\mathbb{1}_{t\geq f(n_{sb}-1)+1} \text{ for } t \in \{2, .., 4N\}$$

Proof:

The quarterly estimates in level have an implicit quarterly trend like in the case in first difference without a structural break but that changes at the structural break, so have the following form

$$\hat{y}_t = \frac{Y_1}{f} + \hat{\mu}\frac{t-0.5(f+1)}{f^2} + \left(x_t - \frac{X_1}{f}\right)'\hat{\alpha} + \hat{\mu}_1 h(t, n_{sb})$$

where

$$h(t, n_{sb}) = \begin{cases} 0 & \text{for } t < f(n_{sb}-1)+1 \\ g(t) & \text{for } t \geq f(n_{sb}-1)+1 \end{cases} = g(t)\mathbb{1}_{t\geq f(n_{sb}-1)+1}$$

with

$$\begin{aligned} g(t) &= g(t-1) + \nu \text{ for } t \in \{f(n_{sb}-1)+1, ..., fN\} \\ &= g(f(n_{sb}-1)+1) + (t - f(n_{sb}-1) - 1)\nu \end{aligned} \quad \text{and} \quad \sum_{t=f(n-1)+1}^{fn} g(t) = n - n_{sb} + 1 \text{ for } n \in \{n_{sb}, ..., N\}$$

It can be shown again that $\nu = f^{-2}$. At the structural break date $n = n_{sb}$:

$$\begin{aligned} \sum_{t=f(n_{sb}-1)+1}^{fn_{sb}} g(t) &= 1 \\ \sum_{t=f(n_{sb}-1)+1}^{fn_{sb}} \left(g(f(n_{sb}-1)+1) + \frac{t-f(n_{sb}-1)-1}{f^2}\right) &= 1 \\ fg(f(n_{sb}-1)+1) + f^{-2}\left(\sum_{t=1}^{fn_{sb}} t - \sum_{t=1}^{f(n_{sb}-1)} t - f^2(n_{sb}-1) - f\right) &= 1 \\ fg(f(n_{sb}-1)+1) + f^{-2}\left(\frac{fn_{sb}(fn_{sb}+1)}{2} - \frac{f(n_{sb}-1)(f(n_{sb}-1)+1)}{2} - f^2(n_{sb}-1) - f\right) &= 1 \\ fg(f(n_{sb}-1)+1) + f^{-2}\left(0.5f^2 - 0.5f\right) &= 1 \\ fg(f(n_{sb}-1)+1) + 0.5 - \frac{0.5}{f} &= 1 \end{aligned}$$

Hence

$$g(f(n_{sb}-1)+1) = \frac{0.5f+0.5}{f^2}$$

Thereforee

$$h(t,n_{sb}) = \frac{t-f(n_{sb}-1.5)-0.5}{f^2}\mathbb{1}_{t\geq f(n_{sb}-1)+1} \tag{25}$$

B.4 Model in first difference with a change in the constant and the slope coefficient (dCS)

$$\Delta\hat{Y}_n = \hat{\mu} + \Delta X_n'\hat{\alpha} + (\hat{\mu}_1 + \Delta X_n'\hat{\alpha}_1)\,\mathbb{1}_{n\geq n_{sb}} \text{ for } n \in \{2,..,N\}$$

$$\Leftarrow \hat{y}_t = \frac{Y_1}{f} + \hat{\mu}\frac{t-0.5(f+1)}{f^2} + \left(x_t - \frac{X_1}{f}\right)'\hat{\alpha} + \left(\hat{\mu}_1\frac{t-f(n_{sb}-1.5)-0.5}{f^2} + (x_t - \frac{X_{n_{sb}-1}}{f})'\hat{\alpha}_1\right)\mathbb{1}_{t\geq f(n_{sb}-1)+1}$$
for $t \in \{2,..,fN\}$

<u>Proof:</u>

$$\sum_{t=f(n-1)+1}^{fn}\hat{y}_t - \sum_{t=f(n-2)+1}^{f(n-1)}\hat{y}_t = \hat{Y}_n - \hat{Y}_{n-1} \text{ for } n \in \{2,...,N\}$$

For $n > n_{sb}$:

$$\begin{aligned}
\sum_{t=f(n_{sb}-1)+1}^{fn_{sb}}\hat{y}_t - \sum_{t=f(n_{sb}-2)+1}^{f(n_{sb}-1)}\hat{y}_t &= \Delta\hat{Y}_{n_{sb}} \\
\hat{\mu} + \Delta X_n'\hat{\alpha} + \hat{\mu}_1 + \left(\Delta X_n + \sum_{t=f(n_{sb}-1)+1}^{fn_{sb}} h(t,n_{sb}) - \sum_{t=f(n_{sb}-2)+1}^{f(n_{sb}-1)} h(t,n_{sb})\right)'\hat{\alpha}_1 &= \hat{\mu} + \Delta X_n'\hat{\alpha} + \hat{\mu}_1 + \Delta X_n'\hat{\alpha}_1
\end{aligned}$$

Therefore

$$\sum_{t=f(n_{sb}-1)+1}^{fn_{sb}} h(t,n_{sb}) = \sum_{t=f(n_{sb}-2)+1}^{f(n_{sb}-1)} h(t,n_{sb}) \tag{26}$$

which means that for $n > n_{sb}$ the annual sum of the adjustment to x_t must be equal to the preceding annual sum. At the break location $n = n_{sb}$:

$$\begin{aligned}
\sum_{t=f(n_{sb}-1)+1}^{fn_{sb}}\hat{y}_t - \sum_{t=f(n_{sb}-2)+1}^{f(n_{sb}-1)}\hat{y}_t &= \Delta\hat{Y}_{n_{sb}} \\
\hat{\mu} + \Delta X_{n_{sb}}'\hat{\alpha} + \hat{\mu}_1 + \left(X_{n_{sb}} - \sum_{t=f(n_{sb}-2)+1}^{f(n_{sb}-1)} h(t,n_{sb})\right)'\hat{\alpha}_1 &= \hat{\mu} + \Delta X_{n_{sb}}'\hat{\alpha} + \hat{\mu}_1 + \Delta X_{n_{sb}}'\hat{\alpha}_1
\end{aligned}$$

Therefore

$$\sum_{t=f(n_{sb}-2)+1}^{f(n_{sb}-1)} h(t,n_{sb}) = X_{n_{sb}-1} \tag{27}$$

which means that for $n > n_{sb}$ the annual sum of the adjustment to x_t must be equal to the annual sum of the indicator at the year preceding the structural break location. A simple condition satisfying (26) and (27) would be :

$$h(t, n_{sb}) = \frac{X_{n_{sb}-1}}{f} \quad \text{for} \quad t \in \{1, ..., fN\} \tag{28}$$

C Power of the cointegration test

	$\rho = 0$					$\rho = 0.5$				
N	15	20	25	30	50	15	20	25	30	50
O	0.589	0.826	0.94	0.985	0.995	0.351	0.572	0.775	0.899	0.994
C	0.017	0.01	0.007	0.005	0.006	0.019	0.013	0.01	0.007	0.009
CS	0.04	0.039	0.038	0.041	0.049	0.039	0.04	0.04	0.047	0.057
dO	0.039	0.041	0.04	0.042	0.051	0.051	0.058	0.053	0.055	0.066
dC	0.02	0.016	0.018	0.016	0.017	0.03	0.028	0.025	0.023	0.021
dCS	0.031	0.031	0.03	0.029	0.032	0.035	0.036	0.035	0.033	0.037

Table 12: Power of the test by true model, for regression model (O)

	$\rho = 0$					$\rho = 0.5$				
N	15	20	25	30	50	15	20	25	30	50
O	0.307	0.521	0.731	0.871	0.997	0.175	0.294	0.449	0.613	0.969
C	0.174	0.309	0.537	0.865	1	0.095	0.13	0.307	0.725	1
CS	0.145	0.174	0.279	0.512	0.615	0.108	0.132	0.247	0.482	0.612
dO	0.04	0.037	0.032	0.035	0.037	0.041	0.04	0.041	0.041	0.052
dC	0.02	0.013	0.014	0.01	0.012	0.024	0.022	0.018	0.019	0.017
dCS	0.033	0.036	0.036	0.037	0.045	0.032	0.027	0.029	0.032	0.036

Table 13: Power of the test by true model, for regression model (C)

	$\rho = 0$					$\rho = 0.5$				
N	15	20	25	30	50	15	20	25	30	50
O	0.304	0.492	0.691	0.837	0.997	0.181	0.276	0.42	0.559	0.954
C	0.247	0.349	0.541	0.797	1	0.167	0.207	0.344	0.637	1
CS	0.263	0.38	0.572	0.796	1	0.188	0.238	0.363	0.606	0.996
dO	0.048	0.046	0.04	0.044	0.047	0.047	0.045	0.043	0.047	0.063
dC	0.032	0.034	0.035	0.037	0.054	0.032	0.034	0.036	0.031	0.042
dCS	0.04	0.045	0.042	0.048	0.076	0.036	0.032	0.034	0.036	0.052

Table 14: Power of the test by true model, for regression model (CS)

	$\rho = 0$					$\rho = 0.5$				
N	15	20	25	30	50	15	20	25	30	50
O	0.916	0.994	0.998	0.999	1	0.812	0.98	0.997	0.998	1
C	0.342	0.944	0.998	1	1	0.377	0.922	0.996	1	1
CS	0.314	0.793	0.934	0.973	0.986	0.347	0.813	0.951	0.983	0.992
dO	0.203	0.417	0.649	0.83	0.99	0.116	0.233	0.406	0.608	0.973
dC	0.084	0.136	0.193	0.282	0.673	0.082	0.157	0.244	0.389	0.852
dCS	0.199	0.356	0.532	0.698	0.953	0.121	0.239	0.403	0.586	0.962

Table 15: Power of the test by true model, for regression model (dO)

	$\rho = 0$					$\rho = 0.5$				
N	15	20	25	30	50	15	20	25	30	50
O	0.534	0.878	0.984	0.998	0.999	0.368	0.728	0.938	0.992	0.999
C	0.183	0.257	0.59	0.917	1	0.178	0.283	0.6	0.896	1
CS	0.219	0.349	0.592	0.855	0.997	0.218	0.346	0.609	0.871	0.999
dO	0.096	0.161	0.264	0.415	0.915	0.068	0.093	0.148	0.239	0.695
dC	0.092	0.167	0.268	0.406	0.921	0.064	0.101	0.149	0.237	0.7
dCS	0.11	0.207	0.322	0.489	0.934	0.071	0.115	0.177	0.265	0.76

Table 16: Power of the test by true model, for regression model (dC)

	$\rho = 0$					$\rho = 0.5$				
N	15	20	25	30	50	15	20	25	30	50
O	0.503	0.856	0.978	0.999	1	0.347	0.691	0.917	0.99	1
C	0.262	0.324	0.491	0.781	1	0.234	0.322	0.516	0.793	1
CS	0.276	0.437	0.639	0.856	1	0.266	0.415	0.635	0.858	1
dO	0.082	0.131	0.221	0.346	0.869	0.057	0.069	0.119	0.179	0.595
dC	0.07	0.116	0.189	0.299	0.845	0.049	0.073	0.107	0.172	0.578
dCS	0.093	0.171	0.284	0.444	0.941	0.057	0.088	0.138	0.212	0.696

Table 17: Power of the test by true model, for regression model (dCS)

D other accounts

	$\hat{n}_b$ selected by cointegration test					
	O	C	CS	dO	dC	dCS
(Intercept)	30280.78***	23040.57***	14954.08 *	5381.46 *	2785.17 **	2445.51
	(6.69)	(5.66)	(2.92)	(2.67)	(3.07)	(1.03)
X	0.57***	0.53***	0.81***	0.36***	0.55***	0.54 *
	(27.71)	(25.75)	(7.99)	(7)	(17.31)	(2.54)
d		22044.7 **	45969.64***		-1.6e+04***	17254.98 **
		(3.4)	(5.59)		(-8.1)	(3.3)
dX			-0.33 **			-0.43 .
			(-3.16)			(-1.87)
tt						
rmse.a	12098.19	8957.37	6508.15	9671.23	2227.32	4434.53
sb date	NA	2004	2007	NA	2012	2007
inf ADF	-2.834	-2.225	-5.536	-0.757	-3.365	-5.9
pval	0.222	0.983	0.133	0.941	0.745	0.097
	$\hat{n}_b$ selected by annual prediction error					
	O	C	CS	dO	dC	dCS
(Intercept)	30280.78***	28853.27***	19375.24***	5381.46 *	2785.17 **	2744.09 *
	(6.69)	(8.55)	(12.04)	(2.67)	(3.07)	(2.94)
X	0.57***	0.5***	0.71***	0.36***	0.55***	0.55***
	(27.71)	(19.37)	(48.16)	(7)	(17.31)	(16.87)
d		28415.16 **	95131.22***		-1.6e+04***	-2158.64
		(3.65)	(9.43)		(-8.1)	(-0.09)
dX			-0.36***			-0.21
			(-12.52)			(-0.61)
tt						
rmse.a	12098.19	8654.74	3204.09	9671.23	2227.32	2137.75
sb date	NA	2007	2011	NA	2012	2012

Table 18: Model fitting Chinese fixed capital formation and investment in fixed assets (1998:2014)

	$\hat{n}_b$ selected by cointegration test					
	O	C	CS	dO	dC	dCS
(Intercept)	12795.56***	11905.18***	6217.29***	2220.6 **	2126.24 *	2155.74 *
	(9.95)	(13.86)	(5.3)	(3)	(2.88)	(2.84)
X	0.49***	0.43***	0.79***	0.26***	0.24 **	0.24 **
	(27.22)	(24.65)	(16.61)	(4.35)	(3.85)	(3.66)
d		7705.16***	11933.97 **		1533.95	-3127.54
		(4.63)	(4.13)		(1.14)	(-0.36)
dX			-0.36***			0.33
			(-6.74)			(0.55)
tt						
rmse.a	3320.34	2087.89	1555.62	4175.96	3178.21	3244.73
sb date	NA	2006	2009	NA	2012	2012
inf ADF	-1.967	-3.511	-4.463	-3.336	-3.772	-4.114
pval	0.569	0.658	0.402	0.115	0.579	0.54

	$\hat{n}_b$ selected by annual prediction error					
	O	C	CS	dO	dC	dCS
(Intercept)	12795.56***	9810.97***	7324.51***	2220.6 **	1608.16 *	427.83
	(9.95)	(9.67)	(10.62)	(3)	(2.41)	(0.41)
X	0.49***	0.63***	0.73***	0.26***	0.14 .	0.68 *
	(27.22)	(19.87)	(30.35)	(4.35)	(1.97)	(2.86)
d		-1.6e+04***	-1217.42		3073.09 *	5138.21 *
		(-4.88)	(-0.39)		(2.6)	(2.47)
dX			-0.2***			-0.58 *
			(-5.93)			(-2.28)
tt						
rmse.a	3320.34	2019.59	1049.56	4175.96	1786.49	1515.19
sb date	NA	2010	2010	NA	2006	2009

Table 19: Model fitting Chinese government consumption expenditures and government expenditures (1998:2014)

E Cyclical components of the Chinese indicators

x	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(\text{gdp})}$	$\rho_1(x)$	$\rho(x_t, \text{gdp}_{t+k})$, where k= -5	-4	-3	-2	-1	0	1	2	3	4	5
gdp	0.8	1	0.9	0.12	0.28	0.52	0.74	0.9	1	0.9	0.74	0.52	0.28	0.12
cons	2.76	3.44	0.32	-0.18	-0.28	-0.23	-0.24	-0.17	-0.05	0.01	0.12	0.24	0.36	0.5
cf	3.95	4.93	0.75	-0.12	-0.23	-0.32	-0.31	-0.23	-0.14	-0.01	0.08	0.11	0.2	0.25
g	9.17	11.45	0	-0.01	0.04	0.05	0.02	0.01	0	0.03	0.03	0	-0.03	-0.07
nx/gdp	1.23	1.54	0.71	0.11	0.23	0.27	0.23	0.23	0.21	0.27	0.37	0.4	0.35	0.26

Note: Statistics are calculated on HP-filtered season-adjusted deflated data. Except for net exports/output, all series are in logarithms.

Table 20: CHN indicators 1998Q1:2016Q2

F Cyclical components of other series for the Chinese economy

				$\rho(x_t, \text{gdp}_{t+k})$, where k=										
x	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(\text{gdp})}$	$\rho_1(x)$	-5	-4	-3	-2	-1	0	1	2	3	4	5
gdp	1.53	1	0.69	-0.1	0.13	0.4	0.69	1	0.69	0.4	0.13	-0.1		
cons	2.24	1.46	0.21	0.05	0.31	0.31	0.35	0.54	0.23	0.13	0.05	-0.06		
cf	2.51	1.64	0.95	-0.36	-0.26	-0.2	-0.15	-0.1	-0.12	-0.08	-0.01	0.1		
g	4.13	2.7	0.41	0.12	0.24	0.54	0.58	0.6	0.35	0.13	-0.05	-0.16		
nx/gdp	1.3	...	0.76	0.12	0.03	0.13	0.38	0.55	0.56	0.42	0.15	-0.06		

Note: Statistics are calculated on HP-filtered season-adjusted deflated data. Except for net exports/output, all series are in logarithms.

Table 21: CHN cycles with estimated gdp, deflated by CPI 1998Q1:2016Q2

				$\rho(x_t, \text{gdp}_{t+k})$, where k=										
x	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(\text{gdp})}$	$\rho_1(x)$	-5	-4	-3	-2	-1	0	1	2	3	4	5
gdp	1.38	1	0.55	-0.06	-0.27	-0.13	0.17	0.55	1	0.55	0.17	-0.13	-0.27	-0.06
cons	2.04	1.48	0.2	-0.11	-0.13	0.13	0.2	0.23	0.54	0.17	-0.1	-0.17	-0.16	-0.04
cf	2.43	1.77	0.94	-0.22	-0.22	-0.15	-0.09	-0.08	-0.11	-0.22	-0.27	-0.26	-0.17	0
g	4.08	2.97	0.39	-0.02	-0.09	-0.01	0.3	0.37	0.4	0.23	0.06	-0.03	-0.09	-0.05
nx/gdp	1.22	0.89	0.75	0.17	0.05	-0.05	-0.02	0.22	0.54	0.6	0.5	0.24	0.03	0

Note: Statistics are calculated on HP-filtered season-adjusted deflated data. Except for net exports/output, all series are in logarithms.

Table 22: CHN cycles with estimated gdp, deflated by official implied gdp.defl 1998Q1:2016Q2

				$\rho(x_t, \text{gdp}_{t+k})$, where k=										
x	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(\text{gdp})}$	$\rho_1(x)$	-5	-4	-3	-2	-1	0	1	2	3	4	5
gdp	2.52	1	0.84	-0.43	-0.18	0.2	0.56	0.84	1	0.84	0.56	0.2	-0.18	-0.43
cons	3.04	1.21	0.62	-0.42	-0.18	0.21	0.5	0.7	0.82	0.66	0.47	0.2	-0.1	-0.3
cf	3.77	1.5	0.91	-0.38	-0.17	0.12	0.4	0.59	0.64	0.55	0.36	0.13	-0.09	-0.27
g	4.27	1.7	0.27	-0.09	0.03	0.13	0.34	0.41	0.42	0.29	0.14	-0.04	-0.21	-0.25
nx/gdp	1.22	0.49	0.75	0.2	0.13	0.02	-0.04	-0.03	0.06	0.1	0.11	0.04	-0.05	-0.08

Note: Statistics are calculated on HP-filtered season-adjusted deflated data. Except for net exports/output, all series are in logarithms.

Table 23: CHN cycles with estimated gdp, deflated by reconstructed implied gdp.defl 1998Q1:2016Q2

G US business cycle correlations (1947:2016)

				$\rho(x_t, \text{gdp}_{t+k})$, where k=										
x	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(\text{gdp})}$	$\rho_1(x)$	-5	-4	-3	-2	-1	0	1	2	3	4	5
gdp	1.62	1	0.85	-0.07	0.12	0.35	0.62	0.85	1	0.85	0.62	0.35	0.12	-0.07
cons	1.21	0.75	0.79	-0.11	0.02	0.18	0.42	0.64	0.79	0.75	0.61	0.43	0.25	0.07
cf	7.66	4.72	0.8	-0.26	-0.09	0.15	0.42	0.66	0.83	0.74	0.56	0.32	0.11	-0.03
g	3.16	1.95	0.89	0.3	0.35	0.33	0.28	0.23	0.15	0.04	-0.04	-0.08	-0.08	-0.05
nx/gdp	0.43	0.26	0.77	0.23	0.15	0.05	-0.11	-0.26	-0.34	-0.38	-0.35	-0.29	-0.23	-0.2

Note: Statistics are calculated on HP-filtered season-adjusted deflated data. Except for net exports/output, all series are in logarithms.

Table 24: US 1947Q1:2016Q2