Fiscal Rules as Bargaining Chips

F. PIGUILLEM\textsuperscript{1}

A. RIBONI\textsuperscript{2}

\textsuperscript{1} Einaudi Institute for Economics and Finance (EIEF). E-mail : facundo.piguillem@gmail.com

\textsuperscript{2} CREST, Polytechnique. E-mail : alessandro.riboni@gmail.com
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Facundo Piguillem†  Alessandro Riboni‡

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Abstract

Most fiscal rules can be overridden by consensus. We show that this does not make them ineffectual. Since fiscal rules determine the outside option in case of disagreement, the opposition uses them as “bargaining chips” to obtain spending concessions. This political bargain reduces the debt accumulation problem. We analyze various rules and show that when political polarization is high, a government shutdown maximizes the opposition’s bargaining power and leads to a sizeable debt reduction. When polarization is low, a balanced budget is preferable. Mandatory spending eliminates debt accumulation by removing political risk. However, it generates persistent static inefficiencies.

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† Einaudi Institute for Economics and Finance (EIEF)
‡ Ecole Polytechnique (France), CREST
1 Introduction

Can fiscal rules be effective even when they are not respected? Various explanations have been put forth to explain the steady growth in public debt in the past decades. Sources of friction such as polarization and political turnover have been shown to generate incentives to accumulate debt in ways reminiscent of hyperbolic discounting, and therefore fiscal rules are considered optimal.\(^1\) Recently, fiscal rules have spread worldwide, partly as a response to the fiscal legacy of the recent recession.\(^2\) Rules are usually embedded in statutory norms or constitutional laws. However, compliance is not guaranteed: ultimately, what matters is the political will to adhere to the rules. In fact, it is often possible for politicians to override the rules if there is consensus.\(^3\)

It is tempting to assume that when fiscal rules are not respected, they are not effective. We argue that, on the contrary, fiscal rules may improve outcomes even with the possibility of being overridden, if the fiscal rule is the default option when legislators disagree. The fact that the rules are the default option changes the incentive to override them in the first place. We show that their effectiveness may even approach the optimal outcome.

To this end, we consider a strategic model of debt similar to that of Alesina and Tabellini (1990) with two parties that alternate in power. There are two public goods financed by taxes and debt. The two parties differ in terms of the desired composition of spending: each party would like to allocate most (or all) of the budget to only one of the goods. As is well-known, political turnover and preference misalignment result in the overissuance of debt: if there is a positive probability of being turned out of office, the incumbent prefers spending according to her own preferences rather than transferring resources to the future.

We extend the model along two key dimensions. First, we assume that policies are the result of negotiations among elected policymakers. In the US, for instance, both the executive and legislative powers must agree.\(^4\) Second, we study the impact of a broad range of fiscal

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\(^1\)For a survey of political economy frictions generating over-accumulation of debt see Alesina and Pas-salacqua (2015). For the optimality of rules see Amador et al. (2006) and Halac and Yared (2014).

\(^2\)Fiscal rules usually take the form of quantitative constraints (e.g., budget balanced laws) or procedural rules by which fiscal policy is made (Drazen (2002), and Alesina and Perotti (1999)). In the 90s only a few countries (such as Germany, Japan and the US) had numerical limits on budgetary aggregates at the central level. By 2012 the number of countries with national or supranational fiscal rules had risen to 76 (Budina et al. (2012)).

\(^3\)Even in Germany, where fiscal rules have a long tradition, compliance with the golden rule, prohibiting borrowing in excess of investment, has been weak since its introduction. See Baumann and Kastrop (2007).

\(^4\)In addition, even if the executive and the legislative majority belong to the same party, institutional rules such as filibuster may endow the opposition with de-facto veto power.
rules. We model fiscal rules as determining the default values of key fiscal aggregates in case
the two parties do not find an agreement, rather than as hard constraints.

An example of a fiscal rule is a budget balance law that can be bypassed when both
parties agree.\footnote{In many countries, budget balance laws have escape clauses (such as wars, recessions, and natural disas-
ters) and can be overridden with a larger majority than is required to pass legislation.} If there is no consensus to override the rule, budget balance (zero deficit) requires that spending cannot be larger than the net tax revenue. A “government shutdown”
provision is another example, in which no tax revenue is spent if the budget is not approved.\footnote{For example, in the US, the Executive Branch must shut down agencies and programs that are funded
annually when the Congress creates a “funding gap”.}

Similarly, when spending is discretionary (e.g., national defence, foreign aid, education and
transportation) the outside option when legislators do not pass an appropriation bill is zero
spending (see \cite{Bowen et al. 2014}).

Since fiscal rules determine the outside option in case of disagreement, the opposition uses
the rules as “bargaining chips” to obtain a more favourable budget. American politics offers
several examples of fiscal rules being used as leverage. For example, in 2011, Republicans
used the threat that the government would be forced to default on its debt to influence
the Obama administration’s spending plans. More recently, the Democrats threatened a
government shutdown to force a withdrawal of Trump’s immigration proposals.\footnote{For instance, Rep. Luis Gutierrez (D-Ill.) stated: “I’m not saying we should shut down the government,
but if you want a budget with Democratic votes, then it’s got to have some Democratic priorities” (\textit{Washington
Post}, 9 October 2017). In 2017 President Trump also threatened a government shutdown to force Congress
to pass a bill to finance the border wall with Mexico.}

When a fiscal rule is present, the party in power offers concessions to the opposition
to avoid its application. As a result of this political bargain, budgets will be less skewed
towards the preferences of the incumbent government. The effect on debt accumulation
is not as straightforward. To garner the support of the opposition, the government must
spend more on the good that the opposition prefers. Because of this effect, fiscal rules could
lead to more spending (and more debt) compared to a situation with no fiscal rules. There
are, however, two other effects as well. First, political compromise raises the cost of debt
because the opposition agrees to increase debt only if she is sufficiently compensated. Second,
the incumbent realizes that when she is out of power the other party will also reach for a
compromise to override the rule. The expectation that the other party will partly share the
total resources increases the incumbent’s benefit of transferring resources to the future. We
show that the first effect is outweighed by the other effects. As a result, debt accumulation
is reduced when the parties negotiate in the shadow of fiscal rules.
We emphasize that the above result hinges on the assumption that the parties negotiate in every period. However, if compromise occurs in the current period but is not expected in the future (e.g., because the executive and legislative branches will likely be controlled by the same party in the next election), bargaining will be “occasional,” and the attenuating effect, due to the expectation of future bargaining, will be muted. In this case, we show that fiscal rules may lead to more debt.

When policies are negotiated, we obtain richer and more complex comparative statics compared to the canonical model with alternating policy dictators. For instance, we show that the standard intuition that political persistence leads to lower debt accumulation does not necessarily extend to a model with bargaining. This is because when power is more persistent, the opposition’s chances to take power decrease, making it less valuable for the opposition to transfer resources to the next period. Since it is cheaper to obtain the opposition’s consent to raise debt, the incumbent may find it optimal to accumulate more debt when her chance to remain in power increases.

This paper studies bargaining outcomes under a variety of fiscal rules that, in turn, imply different outside options. In Section 4, we consider fiscal rules that constrain the proportion of tax revenue (after interest payments) spent for public good provisions. If the parties do not reach consensus, total spending cannot exceed what the fiscal rule prescribes and the incumbent is free to choose how to allocate spending over the two public goods. We show that when the degree of political polarization is high (i.e., the two parties have opposite preferences), the most effective rule to reduce debt is to drastically reduce public spending in case of disagreement, e.g., a government shutdown. However, when polarization is moderate or low, the optimal budget rule calls for a balanced budget. The different results are due to the ways budget rules affect the value of the outside option in different environments. When there is high polarization, the opposition knows that in case of disagreement the incumbent will primarily make cut on goods valued by the opposition. Thus, a government shutdown is optimal because it postpones spending, making it possible for the opposition to consume in future periods if she is in power. Given that the opposition has high bargaining power, in equilibrium she is able to appropriate more resources in every period, decreasing the variability of the public good and, therefore, decreasing the incentive to overspend. On the other hand, when political polarization is low, the opposition knows that in case of disagreement the executive will allocate resources more evenly. Thus, a way for the opposition to increase its bargaining power is to keep total spending unchanged (i.e., budget balance), thereby securing a smooth consumption path for the opposition.
In Section 5, we consider a fiscal rule where the default option in case of disagreement is the previous period’s spending level: spending is mandatory. Unlike discretionary spending, mandatory spending (e.g., Social Security and Medicare) is not decided in periodic appropriation bills (see Bowen et al. (2014)). If there is no agreement to change it, spending will remain the same as in the previous period. Given that the outside option is a nominal quantity, it might be unsustainable. To ensure that the budget constraint always holds, we assume that when the spending level is not sustainable, there are equal proportional cuts (i.e., sequestration). The endogeneity of the default option creates a link between present and future political powers: when the incumbent increases current spending, she also secures future resources by improving her future default option. Indeed, we show that mandatory spending fully insures against political risk and eliminates the incentive to over-accumulate debt. The downside of mandatory spending is that the initial incumbent can appropriate a disproportionate amount of her favorite public good and this disproportion will be maintained over time. To summarize, the policy is dynamically optimal, but there are large static inefficiencies.

The paper is organized as follows. Section 2 reviews the literature. Section 3 presents the two-period model and studies debt accumulation with alternating dictators. In Section 4, we extend the model and suppose that policies are negotiated in the shadow of fiscal rules. Section 5 examines bargaining with mandatory spending. Section 6 briefly discusses some theoretical implications and the conclusion follows. Proofs are in Appendix A. In Appendix B, we study a model with an infinite horizon.

2 Literature Review

This paper is closely related to the strategic-debt literature (e.g., Alesina and Tabellini (1990), Persson and Svensson (1989), Amador (2003), Debortoli and Nunes (2013), Aguiar and Amador (2011), and Chatterjee and Eyigungor (2016)). A well-known result in this literature is that political turnover and polarization generate a debt-bias. Politicians who are uncertain of re-election overspend in the current period and raise debt to tie the hands of future policy-makers. A recent contribution to the literature is Battaglini and Coate (2008), who assume that policies are made through legislative bargaining. According to their model,

\footnote{In Lizzeri (1999), voters favour candidates who propose a transfer of resources to the present because they fear that in the future, these resources will be offered to others. Bisin et al. (2015) study politicians’ incentives to accumulate debt when voters are time inconsistent.}
legislators in the minimum winning coalition do not fully internalize the tax burden of spending decisions and approve targeted transfers for their districts. In addition, fearing that they might not be included in future coalitions, legislators have the incentive to transfer resources from the future to the present, leading to over-accumulation of debt. Bouton et al. (2016) presented the first paper to model the joint determination of debt and entitlement programs (such as pensions and health care). Through entitlements, governments pre-commit a fraction of future resources to a particular use. Entitlements provide an additional instrument to constrain future governments, thus weakening the incentive to use debt. They assume that entitlements are decided by the incumbent and cannot be renegotiated by the future government. Instead, in Section 5 we assume that mandatory spending is decided through bargaining and can be renegotiated or scaled down in the future. In Bouton et al. (2016), current spending and the entitlement for the next period are represented as two separate choice variables. We assume instead that the entitlement for the next period coincides with the amount spent in the current period. In other words, mandatory spending programs run on autopilot unless legislators agree to change them.

The literature on fiscal rules has greatly expanded in recent years. Azzimonti et al. (2016) study the impact of a budget balance rule in the context of a calibrated version of Battaglini and Coate’s model. When there are shocks, fiscal rules impose a trade-off between the cost due to the lack of response to the shocks, and the benefit in terms of debt discipline. In contrast to our analysis Azzimonti et al. (2016) study budget balance rules that are strict and cannot be overridden. We study instead how the possibility of override leads to a political bargain. Moreover, we focus on a wider set of rules. In Bouton et al. (2016) constraints on debt would lead to increases in entitlements since debt and entitlement are strategic substitutes. Dovis and Kirpalani (2017) study fiscal rules in federal governments. When there are fiscal rules at the local level, a lax central government ends up revealing its type earlier (by choosing not to enforce the rules) relative to an environment without rules. Interestingly, because of this effect they show that fiscal rules might aggravate over-borrowing by local governments. Halac and Yared (2018) study fiscal rules in a world economy with integrated capital markets. They compare coordinated rules (which are chosen jointly by a group of countries) and uncoordinated rules (chosen independently by each country). In their model, rules affect countries not only by limiting their borrowing (and hence their flexibility to respond to shocks) but also by reducing interest rates, which has a redistributive

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effect across countries. [Halac and Yared (2017)] drop the assumption that fiscal rules can be perfectly enforced and study fiscal rules which are self-enforcing, so that complying with the rule is preferable to the punishment that follows a breach. In [Halac and Yared (2014, 2017, 2018)] politicians have an exogenous present-bias for spending. In our model the present-bias arises endogenously as a consequence of political turnover and interacts with the fiscal rule. Thus, fiscal rules have a direct effect reducing the temptation to accumulate debt that is not present in the aforementioned papers.

A key message of this paper is that political bargaining in the shadow of fiscal rules leads to lower debt accumulation. In other contexts, however, the need to reach a political compromise might lead to higher debt. For instance, in the “war of attrition” model by [Alesina and Drazen (1991)], a strong government that does not need to compromise with the opposition would find it easier to reduce the deficit, by making the opposition suffer a larger stabilization cost. Existing empirical evidence (e.g., [Milesi-Ferretti et al. (2002)]) points out that coalition governments are associated with higher debt. This evidence does not necessarily contrast with our results. First, this correlation might be explained by other variables, such as proportional representation. Second, this evidence often refers to coalition governments that are weak (with a small majority) while we study coalitions between the two main parties. Finally, in our model, there are instances (“occasional” bargaining) in which political compromise leads to higher debt.

Under mandatory spending (Section 5), the bargaining default depends on the previous policy outcome (the status quo). The political-economy literature dealing with an endogenous status quo has grown recently (among recent contributions, see [Riboni and Ruge-Murcia (2008), Diermeier and Fong (2011), Duggan and Kalandrakis (2012), Bowen et al. (2014), Bowen et al. (2017), and Dziuda and Loeper (2016)]) but generally abstracts from debt-accumulation issues. [Piguillem and Riboni (2015)] do not consider either debt or fiscal rules; they study a legislative bargaining model where legislators have a spending-bias, and show that when the status quo is endogenous, disagreement among legislators disciplines policy-making.

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10On fiscal rules in a monetary union, see [Chari and Kehoe (2007) and Beetsma and Uhlig (1999)]. Foarta (2018) analyzes fiscal rules in a banking union. [Alfaro and Kanczuk (2017)] and [Hatchondo et al. (2015)] study fiscal rules in models of debt and default. The political economy of default is studied in, among others, [Dovis et al. (2016)] and [Azzimonti and Quadrini (2016)].
3 The Model

We consider a two-period model: $t = 1, 2$. For tractability, we maintain this assumption in most of the paper. In Section 4.1 we solve an infinite horizon economy. We assume that the tax revenue is exogenous and equal to $\tau$ in both periods. There are two types of public goods. In the first period, public spending is financed by current taxes and debt. In the final period, any remaining debt must be paid.\[11\]

There are two parties that stochastically alternate in power and negotiate policies in the shadow of a fiscal rule. Fiscal rules constrain policy makers unless an agreement is reached between the two parties. The details of the rules will be discussed in the next sections. Bargaining in period 1 unfolds as follows (see Figure 1):

(i) At the beginning of period 1, one of the two parties is elected and becomes the incumbent government.

(ii) The incumbent makes a take-it-or-leave-it offer to the opposition, specifying spending levels and debt. The incumbent’s proposal does not need to satisfy the fiscal rule. The proposal, however, has to satisfy the government’s budget constraint.

(iii) As is standard in the legislative bargaining literature, the opposition accepts the proposal if and only if she will be at least as well-off as if she had rejected it.

(iv) In case the opposition rejects the proposal, the fiscal rule must apply. If there is more than one policy which satisfies the rule, the choice of policy is up to the incumbent.

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Footnote 11: The assumption that taxes are exogenous is not important for the main point of the paper, but greatly simplifies the analysis.
We denote the policy that is chosen in case of disagreement as the “no compromise policy”. Notice that if the “no compromise policy” is initially proposed, it is accepted. This implies that if the incumbent wants to abide by the rule, she does not need to make concessions to the opposition.

In period 2 the government at time 1 remains in power with a probability of \( q \in [0, 1] \). With a complementary probability of \( 1 - q \) the opposition at \( t = 1 \) becomes the government at \( t = 2 \). To simplify the exposition of the two-period model, the bargaining’s outcome in the final period is modeled in “reduced form” by assuming that available resources at \( t = 2 \) are exogenously split between the two parties (for details see Section 4). In Appendix B, where we solve the infinite horizon model, the spending composition is endogenously determined for all \( t \) following the bargaining protocol of Figure 1.

The two parties have different preferences over the desired composition of public spending: each party would like to allocate most (or all) of the budget to one of the two public goods. We use the following notation: \( g^I \) and \( g^O \) denote, respectively, the good that is favoured by the time-1 incumbent (denoted I) and the public good preferred by the time-1 opposition (denoted O). The per-period utilities of the incumbent and of the opposition are given by:

\[
\begin{align*}
  u_I(g^I, g^O) &= u(g^I) + \theta u(g^O) \\
  u_O(g^I, g^O) &= u(g^O) + \theta u(g^I)
\end{align*}
\]

where \( u(\cdot) \) is a CRRA utility function with a coefficient of relative risk aversion denoted by \( \sigma \in [0, 1] \). For \( g^j \), with \( j = I, O \), we have

\[
u(g^j) = \frac{(g^j)^{1-\sigma}}{1-\sigma}
\]

We denote by \( \beta \) the discount factor. The parameter \( \theta \in [0, 1] \) captures the political polarization: when \( \theta = 0 \), a party does not derive any utility from the public good favoured by the other party, implying maximum disagreement about the composition of spending. Disagreement disappears as \( \theta \to 1 \). The period 1 government’s budget constraint is:

\[
g^I_1 + g^O_1 + (1 + r)b_0 \leq b_1 + \tau
\]

where \( b_0 \) denotes the quantity of government debt inherited from the past, while \( b_1 \) denotes

\[12\]Similarly, the thrust of our analysis would be maintained in a model in which politicians also disagreed about the size of the government, as in Persson and Svensson (1989).
the quantity of debt at the end of period 1. The interest rate is \( r \). In the second period all debt must be paid, so new debt cannot be issued. The budget constraint is given by:

\[
g'_2 + g'_O + (1 + r)b_1 \leq \tau. \tag{5}
\]

We assume that \( b_1 \leq \tau/(1+r) \). This implies that it is always feasible to pay the outstanding debt. To avoid cluttered notation, we assume what follows:

**Assumption 1:** Let \( b_0 = 0, \beta = 1 \) and \( r = 0 \).

Under this assumption, the incentives to run debt only arise from political considerations.

**Proposition 1 (social optimum)** When policies are chosen by a social planner, debt is zero regardless of the planner’s weights for the two parties.

If the social planner is utilitarian (equal weights), it follows immediately that spending on both public goods will be the same: \( g'_I = g'_O = \tau/2 \) for \( t = 1, 2 \).

### 3.1 Alternating Dictators

Before proceeding, it is instructive to compute the solution when the party in power is a policy dictator unconstrained by fiscal rules, the standard approach in the strategic-debt literature.\(^{13}\)

The problem of the (alternating) dictator can be easily solved backwards. At \( t = 2 \), from \( g'_2 + g'_O + (1 + r)b_1 \leq \tau \) and \( r = 0 \), the available resources are \( (\tau - b_1) \). The dictator spends \( (1 - \tilde{\theta})(\tau - b_1) \) for her preferred public good and \( \tilde{\theta}(\tau - b_1) \) for the other public good, where

\[
\tilde{\theta} = \frac{\theta^{\frac{1}{\sigma}}}{1 + \theta^{\frac{1}{\sigma}}}. \tag{6}
\]

Proceeding backwards, knowing that the budget constraints hold with equality and using the final period solutions, we obtain the following first-order condition for debt at \( t = 1 \):

\[
u'(\tau + b_1) = \frac{\Omega(\theta)}{\Theta(\theta)}u'(\tau - b_1) \tag{7}\]

\(^{13}\)Results of this section closely follow Propositions 1 and 2 in Tabellini and Alesina (1990).
where:

\[ \Theta(\theta) = u(1 - \tilde{\theta}) + \theta u(\tilde{\theta}) \quad \text{and} \quad \Omega(\theta) = [q + (1 - q)\theta]u(1 - \tilde{\theta}) + [\theta q + (1 - q)]u(\tilde{\theta}) \]

The left-hand side of (7) is the gain from issuing one more unit of debt; the right-hand side is the expected cost of increased debt in terms of the need to cut future expenditure. Note that the optimal allocation requires \( u'(\tau + b_1) = u'(\tau - b_1) \), and therefore \( b_1 = 0 \). It is clear that the deviation from the efficient allocation is given by the ratio \( \Omega(\theta)/\Theta(\theta) \). When \( \sigma \in [0, 1] \) the ratio \( \Omega(\theta)/\Theta(\theta) \leq 1 \). If the incumbent is always in power, \( \Omega(\theta)/\Theta(\theta) = 1 \) and there is no over-accumulation of debt. Suppose instead that in the first period, the incumbent believes that there is some probability of being turned out of office. If in the second period she happens to be out of power, the composition of public spending will be chosen according to the opposition’s preferences. As a result, the incumbent does not fully internalize the value of future resources. Any politician would overspend in the first period: they consume more when they have the chance of choosing their desired mix of public goods. The higher the probability of losing power, the larger the debt.

**Proposition 2 (alternating dictators)** Let \( q < 1 \) and \( \theta < 1 \). When the incumbent is a policy dictator, debt is strictly positive. Debt is decreasing in \( \theta \) and \( q \).

Since turnover is an intrinsic feature of every democracy, our focus is on analyzing how different fiscal rules reduce the incentive to over-accumulate debt.

### 4 Bargaining

When there is a fiscal rule, the incumbent has the incentive to reach a compromise with the opposition in order to avoid its application. The outcome of a political bargain crucially depends on the outside option in case of disagreement. For this reason, in this paper we consider different outside options. Each outside option is implied by a fiscal rule. In this Section we start by considering a set of fiscal rules that specify the amount of total available resources that can be spent in case the majority party’s proposal is rejected. In Section 5

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14 In addition to this effect, as shown in Tabellini and Alesina (1990), when \( \theta > 0 \) there is also an “insurance” effect which goes in the opposite direction. Because of the concavity in the utility function, the incumbent also has the incentive to lower the debt to smooth consumption over time. Following the strategic-debt literature, we assume parameters such that the “insurance” effect is dominated by the other effect: \( \sigma \in [0, 1] \).
we will consider instead a rule (mandatory spending) that specifies the composition of public good spending in case of disagreement.

Suppose that unless there is a political consensus, public good spending in period 1 must satisfy the following condition:

$$g^I_1 + g^O_1 \leq \alpha (\tau - rb_0)$$

This rule prescribes that in case of disagreement the total spending cannot exceed an exogenous proportion $\alpha \in [0, 1]$ of the net flow of income. When $r = 0$ and $b_0 = 0$, the rule becomes $g^I_1 + g^O_1 \leq \alpha \tau$. It is important to stress that rule (8) does not specify the composition of public spending. We assume that the way of meeting the fiscal rule’s requirements is at the discretion of the majority party. That is, in case of disagreement, the majority party acts as a dictator in choosing how to allocate spending, but with limited resources. With this assumption we capture the idea that when is necessary to implement spending cuts, the executive can exercise its discretionary power to implement cuts on items that she does not particularly value.

The assumption that $\alpha$ is smaller than 1 guarantees that the rule binds, so that both parties are willing to reach a compromise to bypass it. A fiscal rule specifying $\alpha = 1$ would correspond to a balanced budget rule: without the approval of the opposition, the incumbent cannot increase the debt. A fiscal rule with $\alpha = 0$ is equivalent to a rule that prescribes a government-shutdown in case the executive and the opposition do not agree. More generally, $\alpha = 0$ captures discretionary spending, that is, spending programs that must be approved each year and are ended if both parties cannot agree.

Since all debt must be paid in the final period, at $t = 2$ we cannot impose a rule equivalent to (8). To simplify the analysis, we model the outcome of the final-period bargaining in “reduced form”. We assume that both parties negotiate and the proportion $\gamma’/(1 + \gamma’)$ of the total resources (with $\gamma’ \in [0, 1]$) is spent for the public good favoured by the opposition while the remaining is spent for the public good favoured by the incumbent. The higher the value of $\gamma’$, the higher the opposition’s bargaining power. This assumption captures the repeated bargaining in the infinite horizon economy, where $\gamma’$ is endogenously determined (see Appendix B). As we will see, this variable is important and taking it as exogenous, only in the two period model, helps us show the intuition behind the main result. We will later

\[\text{For instance, a budget balance law, which would keep debt unchanged, would not be feasible in the last period. Hence, it would not be a credible rule.}\]

\[\text{This simplification is only needed to deal with bargaining in the final period. In the infinite-horizon model (Appendix B) we assume that a rule equivalent to (8) holds in all periods.}\]
choose $\gamma'$ to replicate the infinite horizon equilibrium.

From (5), when $r = 0$, available resources in the final period are given by $\tau - b_1$. Given the second period bargaining, and for any level of debt $b_1$, the spending composition is:

$$g_I^2(b_1) = \frac{\tau - b_1}{1 + \gamma'} \quad \text{and} \quad g_O^2(b_1) = \gamma' \frac{\tau - b_1}{1 + \gamma'} \quad (9)$$

To make the analysis more transparent, in what follows we assume that there is extreme political polarization: each party only cares about one public good. The case of $\theta = 0$ is the most interesting: this is when the debt-accumulation problem is more severe and, as shown below, rules greatly help in reducing debt. Later we show the results for an arbitrary $\theta$ in the infinite horizon case.

**Assumption 2:** Suppose $\theta = 0$.

Now we turn to the incumbent’s first-period problem. Using (4) and Assumptions 1 and 2, this problem can be written as:

$$\max_{\{b_1, g_O^1\}} \left\{ u(\tau + b_1 - g_O^1) + qu(g_I^2(b_1)) + (1 - q)u(g_O^2(b_1)) \right\}$$

subject to

$$u(g_O^1) + qu(g_O^2(b_1)) + (1 - q)u(g_I^2(b_1)) \geq \bar{U} \quad (10)$$

The constraint (10) is the acceptance constraint that is absent in the dictator’s problem. The default utility of the opposition in case of disagreement is given by

$$\bar{U} = u(\bar{g}_1^O) + qu(\bar{g}_2^O(\bar{b}_1)) + (1 - q)u(\bar{g}_2^I(\bar{b}_1)) \quad (11)$$

where $\bar{g}_1^O$ and $\bar{b}_1$ denote the spending and debt that the executive would set in case of disagreement. These values crucially depend on the fiscal rule in place. When $\theta = 0$, it is clear that upon disagreement the incumbent would choose $\bar{g}_1^O = 0$ (this will not be the case when $\theta > 0$) and therefore the evolution of debt is given by the budget rule, $\bar{b}_1 = -(1 - \alpha)\tau$. This implies that the total available resources in the second period would be $\tau - \bar{b}_1 = (2 - \alpha)\tau$.

Note that the incumbent and the opposition have opposite incentives. Besides disagreeing on the spending composition, they disagree on the dynamic allocation of resources. Since the incumbent is not sure to remain in power, she would like to transfer resources from tomorrow to today. Conversely, the opposition counts on the possibility of coming to power in the future.
Consequently, she would like to move to the next period with more resources. Borrowing terminology from the hyperbolic-discounting literature, the party in power is present-biased, while the opposition is future-biased.

Since (10) holds with equality, the acceptance constraint implicitly defines a function $g_1^O = G(b_1)$, which is increasing in debt:

$$G'(b_1) = \frac{1}{1 + \gamma'} \left[ qu'(g_2^O(b_1))\gamma' + (1 - q)u'(g_2^I(b_1)) \right]$$

(12)

When the agenda setter increases $b_1$ (increases debt) she will have to raise $g_1^O$ to keep the opposition indifferent. This is intuitive: the opposition wants to be compensated for going to the next period with a smaller amount of resources.

By replacing the solution $G(b_1)$ in the maximization problem we obtain the first order condition with respect to $b_1$, which equalizes the marginal benefit of an extra unit of debt with the marginal cost of cutting expenditure tomorrow.

$$(1 - G'(b_1))u'(\tau + b_1 - G(b_1)) = (1 - q)u'\left(\frac{\tau - b_1}{1 + \gamma'}\right)\frac{\gamma'}{1 + \gamma} + qu'\left(\frac{\tau - b_1}{1 + \gamma'}\right)\frac{1}{1 + \gamma}$$

(13)

We compare (13) with the policy dictator’s first-order condition when $\theta = 0$:

$$u'\left(\tau + b_1\right) = qu'\left(\tau - b_1\right)$$

(14)

The differences between the two first-order conditions are underbraced in equation (13).

1. One extra unit of debt does not translate into one more unit of consumption because additional $G'(b_1)$ units must be given to the opposition as compensation. Compared to a model with alternating dictators, this channel reduces the incentive to raise debt.

2. Consumption at $t = 1$ is $\tau + b_1 - G(b_1)$ instead of $\tau + b_1$, which increases the marginal utility of consumption. If the party in power wants to maintain a given level of current consumption in the first period, she must increase debt.

3. As long as $\gamma' > 0$, the incumbent realizes that increasing the debt not only reduces her consumption if she stays in power, but when she is out of power as well. The extra value of future resources reduces the incentive to accumulate debt.

4. In case power is maintained, future resources will partly be appropriated by the oppo-
sition, which increases the incentives to accumulate debt.

Because the four channels do not all go in the same direction, the overall effect of bargaining is ambiguous. Note that channels (1) and (2) are driven by negotiations at time $t = 1$, while (3) and (4) are driven by the expectation of future bargaining. This distinction will play a role later in this section (Proposition 4) when we assume that bargaining takes place occasionally rather than in every period.

We denote by $\gamma$ the first-period consumption ratio:

$$\gamma := \frac{g_O^1}{g_I^1}. \quad (15)$$

Solving the two-period model amounts to determining the equilibrium values of $\gamma$ and $b_1$ as a function of $\gamma'$. In Figure 2 we present an example to illustrate how debt and $\gamma$ vary with $\gamma'$. In drawing this figure, we assume $\sigma = 0.2$ and that the current fiscal rule in place is a government-shutdown, $\alpha = 0$. Results would be qualitatively unchanged for other values of $\alpha$. As a benchmark, we also plot the debt level in the alternating dictator model.

**Figure 2: Bargaining and Debt**

The first observation from Figure 2 is that debt is smaller than in the standard model with alternating dictators and no bargaining. Moreover, the budget is less skewed towards the preferences of the party in power. However, the spending composition is not egalitarian. Because the incumbent has agenda-setting power, she obtains a larger share of consumption than the opposition: in the configuration of parameters of Figure 2, $\gamma$ is about $1/2$.

In the alternating dictator’s model with extreme polarization, a higher $\sigma$ (relative risk aversion) induces the incumbent government to smooth consumption over time, leading to
lower debt. Similarly, a higher \( q \) (political persistence) reduces the incentives to tie the hands of the successor, also leading to smaller debt. Bargaining brings about new effects which may change these predictions. When studying the comparative statics in our model, we need to take into account how parameters change the opposition’s bargaining power. A higher \( \sigma \) lowers the bargaining position of the opposition, who would suffer more from the application of the fiscal rule. As a result, fewer goods must be given to the opposition in the current period, making debt less costly to run. Similarly, when \( q \) increases, the opposition is less likely to be in power in the next period, weakening the opposition’s incentives to transfer resources to the future. From (26), this decreases the opposition’s bargaining power and makes debt cheaper. In contrast to the standard intuition arising from the canonical model, increasing either \( \sigma \) or \( q \) may lead to more debt. Figure 3 shows an example with close to linear utility (low \( \sigma \)) where debt is hump-shaped in political persistence, measured by \( q \). Figure 3 also shows that more concavity (higher \( \sigma \)) increases debt when \( q \) is low.

Figure 3: Political Persistence and Debt

To obtain sharper predictions, after solving for the equilibrium \( \gamma \) as a function of \( \gamma' \), we compute the debt level when \( \gamma = \gamma' \). In Figure 2 we focus on the equilibrium corresponding to the intersection between the \( \gamma \) curve and the 45 degree line. This is a consistency requirement for the future expected bargaining and, as shown in Appendix B, the outcome in the infinite horizon economy. Intuitively, this arises because preferences are homothetic and the problem is symmetric. Thus the proportions of resources allocated to the incumbent and opposition remain constant over time. We obtain the following results.

Proposition 3 (bargaining) Let a fiscal rule, \( \alpha \in [0, 1] \), be given. Suppose Assumptions 1 and 2 hold. At time \( t = 1 \) the incumbent proposes a policy that does not satisfy the constraint specified by the fiscal rule and the proposal is accepted by the opposition. In equilibrium:
a) Debt is lower than debt with alternating dictators.

b) Debt is positive.

Proposition 3 states that bargaining reduces the accumulation of debt, but cannot eliminate it. The thrust of these results will be maintained in the infinite horizon model. Note that the constraint (8) does not hold on the equilibrium path. Nevertheless, fiscal rules influence debt accumulation. The existence of a fiscal rule induces the incumbent to reach a compromise with the opposition in order to bypass the rule. As stated in Proposition 3, political compromise leads to smaller debt.

The assumption that negotiations happen in both periods is important. Suppose, instead, that bargaining occurs only in the first period. For instance, imagine that the next government is expected to have a large majority. Then, fiscal rules will be overridden without the consent of the opposition. Proposition 4 states that the resulting debt might be higher.

**Proposition 4 (occasional bargaining)** If bargaining is occasional, debt may be larger than with alternating dictators.

The intuition behind this result is that when there is no bargaining in the final period, channel (3) is not operative. This decreases the cost of transferring fewer resources to the future. All in all, we conclude that in order for bargaining to reduce debt (Proposition 3) it must take place in both periods.

### 4.1 Infinite Horizon

We present the results from the infinite horizon model and show that results are in line with those in the two-period model. The full analysis is detailed in Appendix B. Given that the solution is stationary, we drop the time indexes from all variables. In what follows, we consider the whole range of the polarization parameter, i.e., \( \theta \in [0, 1] \). On the other hand, we assume that \( q = 1/2 \). Assuming equal probabilities makes the two parties completely symmetric, greatly simplifying the algebra. Finally, we assume \( \beta(1 + r) = 1 \), so that the social optimum is to keep the debt level constant over time. To summarize, the infinite horizon’s analysis is carried out under the following assumptions.

---

\(^{17}\) Whether or not political compromise is occasional may depend on whether it is driven by structural features of the institutional environment (e.g., filibustering), or the result of an indecisive election outcome, which is unlikely to occur again. The implications on debt accumulation would be different in the two cases.
Assumption (infinite horizon) Let \( \theta \in [0, 1] \), \( \beta \in [0, 1) \), \( q = 1/2 \), and \( \beta(1 + r) = 1 \).

Unless an agreement is reached between the two parties, total spending must satisfy the following rule:

\[
g^O + g^I \leq \alpha(\tau - rb) \tag{16}
\]

In each period, the budget constraint is

\[
g^I + g^O + (1 + r)b = \tau + b' \tag{17}
\]

Using a guess-and-verify approach, we compute the Markov strategies in the infinite horizon model in which debt is the payoff-relevant state variable. In Appendix B we show that the saving functions are linear in \( \tau - rb \), the tax revenue after interest payments on existing debt \( b \). In all periods, we find that the proportion of resources allocated to the incumbent’s preferred public good is given by

\[
g^I = \frac{\nu}{p}(\tau - rb) \tag{18}
\]

while total spending is given by

\[
g^I + g^O = \nu(\tau - rb) \tag{19}
\]

where \( \nu \geq 1 \) and \( p \geq 1 \) are endogenous parameters to be determined. When the opposition’s bargaining power is weak, the spending composition is more biased towards the preferences of the party in power. In the alternating dictator’s model with extreme polarization, \( p \) is one so that \( g^O \) is zero. Because per-period utility is homothetic, we find that \( \nu \) and \( p \) do not depend on the debt level and, thus, are constant over time. This greatly simplifies the analysis. In Appendix B, we write down the conditions that implicitly define these parameters. Such conditions depend, non trivially, on all of the model’s parameters, such as polarization, curvature in the utility function, and the fiscal rule. After replacing the solutions [18] and [19] into the government budget constraint we find that \( \nu \) determines the evolution of debt:

\[
b' = b + (\nu - 1)(\tau - rb) \tag{20}
\]

The closer \( \nu \) is to 1, the closer debt-dynamics is to the social optimum, which prescribes zero new debt: \( b' = b \). In Appendix B we show that there is a negative relation between the opposition bargaining power and debt accumulation: when the bargaining power of the
opposition increases, \( \nu \) decreases and the incentives to accumulate debt are reduced. After numerically computing the equilibrium value of \( \nu \), the debt dynamics in the infinite horizon solution are plotted in Figure 4. In the vertical axis, we plot future debt \((b')\) as a function of current debt \((b)\). We show debt dynamics in the alternating dictators case and under various fiscal rules (budget balance and government shutdown). For all \( \alpha \in [0, 1] \) debt grows over time. Since the interest payments on existing debt increase as well, from (19) we obtain that total spending decreases over time. We consider two cases, low polarization (left panel) and high polarization (right panel). In line with the intuition for the two-period model, Figure 4 shows that rules are not followed on the equilibrium path but they nevertheless affect debt-dynamics. The largest deviations from the optimum arise when polarization is high. When polarization is low, the debt problem is not severe in the first place and fiscal rules only slightly improve compared to the dictator’s problem. When polarization is maximal, bargaining has more bite and rules drastically reduce the debt accumulation problem. As shown in the Appendix (Figures 10 and 11), the effect of rules is stronger when utility is almost linear, because this raises the opposition’s bargaining power.

Next, for all levels of polarization, we compute the fiscal rule \( \alpha \) which minimizes debt accumulation and we denote it as “optimal”. Figure 5 plots the optimal \( \alpha \) as a function of \( \theta \) for an intermediate value of \( \sigma \). The value of \( \alpha \) that maximizes the opposition’s bargaining power and thus decreases debt is increasing in \( \theta \). The optimal \( \alpha \) is zero when polarization is extreme. Since upon disagreement the executive would cut as much as possible the opposition’s favorite goods, the way to increase the opposition’s bargaining power is to reduce spending as much as possible and move resources to the future. The opposition’s utility increases due to the fact that it could use these resources if her party becomes the new ex-
ecutive. This is why a government shutdown provision and discretionary spending programs are optimal when political preferences are highly polarized. Is this still true when \( \theta > 0 \)? When polarization is more moderate, the executive will allocate some resources to the goods that the opposition values. It follows that a government shutdown is no longer optimal. The concavity of the utility function plays an important role. A government shutdown increases bargaining power by increasing the availability of future resources, but at the cost of present consumption, which harms the opposition’s bargaining power. When polarization is moderate, the opposition also enjoys the spending allocated to the executive. Thus, moving some resources from the future to the present (raising \( \alpha \)) increases, by smoothing consumption over time, the expected utility of the opposition contingent on disagreement, and hence her bargaining power. The optimal \( \alpha \) converges to 1 as the polarization dissipates.

Before concluding this section, we investigate whether it would be desirable to “harden” the rules by eliminating the possibility of override. We focus, in particular, on balanced budget
laws.\textsuperscript{18} We compute welfare from 0 to infinity from an ex-ante perspective, before knowing the identity of the initial incumbent. The optimality of a strict balanced budget law is not obvious. On the one hand, it would eliminate the debt accumulation problem. On the other hand, when rules are “hard”, the incumbent would have no incentive to reach a compromise with the opposition. As a result, in each period the incumbent would choose her preferred spending mix, leading to excessive volatility in consumption. Figure 6, drawn assuming $q = 1/2$ and $\sigma = 1/2$, shows ex-ante welfare under the dictator’s model (no rules), when rules are flexible and when they are hard. Figure 6 shows that for all $\theta < 1$ a hard budget balance law is indeed suboptimal and allowing a supermajority override is ex-ante desirable. However, imposing rules (whether hard or flexible) is preferable to having no rules at all.

5 Mandatory Spending

We now consider another rule that is widespread: we assume that the two public goods constitute mandatory spending governed by enacted laws rather than annual appropriations.\textsuperscript{19} Unless the law is changed, the previous year’s spending levels are maintained. In other terms, the status quo (the previously chosen policy) is the endogenous default option.\textsuperscript{20} Mandatory spending introduces a key dynamic link across periods: policy makers internalize the fact that current spending changes the default option in case of disagreement in the next period. In contrast, spending on discretionary programs is governed by periodic appropriation bills and the default level of spending is exogenous and equal to zero ($\alpha = 0$).

The prevalent view is that mandatory spending aggravates the debt accumulation problem by putting expenditures on autopilot. Surprisingly, we show that mandatory spending actually eliminates debt.

Consider a two period model and, again without loss of generality, assume extreme polarization, $\theta = 0$. At the beginning of $t = 1$ a party is recognized as incumbent. As before, we denote by $g^I$ and $g^O$ the public goods preferred, respectively, by the incumbent and the opposition. The outside option in case of disagreement specifies how spending should be

\textsuperscript{18}This is the most interesting case. For a government shutdown, committing to zero spending in all periods is obviously suboptimal.

\textsuperscript{19}Examples of mandatory spending include entitlement programs (e.g., Social Security, Medicaid, and Medicare), which in the US account for a large fraction of total federal spending (about 60 per cent, in the 2014 fiscal year). See Levit et al. (2015).

\textsuperscript{20}In various Latin American countries (e.g., Argentina and Uruguay) in case the budget is not approved, the budget of the previous year remains in effect in the interim (Stapenhurst et al. (2008)). A similar reversion to the status quo occurs in case of deadlock in the negotiations over the EU’s multi-annual financial framework.
allocated. The default option at time $t = 1, 2$ for good $g_j$ (with $j = I, O$) is denoted by $\bar{g}_j$.

The timing is as follows. At the beginning of the first period, the incumbent makes a take-it-or-leave-it offer to the opposition. The default policy in case of disagreement is exogenously given and equal to $(\bar{g}_I^0, \bar{g}_O^0)$ which is the (non-modeled) spending at time 0, with $g_I^0 + g_O^0 \leq \tau$. At time $t = 2$, the time-1 incumbent remains in power with probability $q$. With complementarity probability, the opposition goes to power. The party in power at time 2 makes a take-it-or-leave-it offer. The default policy in the second period is endogenous and is linked to $(g_I^1, g_O^1)$, the implemented policy in the first period. Define the following ratio:

$$\chi := \frac{\tau - b_1}{g_I^1 + g_O^1} = \frac{\text{Resources available}}{\text{Total spending compromised}}$$  \hspace{1cm} (21)

When the status quo is endogenous, there is the possibility that continuing to the next period with the same level of spending is not feasible. We distinguish two cases: when the default spending at $t = 2$ is feasible ($\chi \geq 1$) and when it is not feasible ($\chi < 1$). In the first case, the outside option in case of disagreement is assumed to exactly coincide with the policy implemented at $t = 1$:

$$\bar{g}_I^2 = g_I^1 \quad \text{and} \quad \bar{g}_O^2 = g_O^1$$ \hspace{1cm} (22)

Second, suppose that upon disagreement the default spending at $t = 2$ is not feasible. In this case we assume that spending is proportionally downsized by the factor $\chi$. Therefore, the new default spending in case of disagreement becomes:

$$\bar{g}_I^2 = \chi g_I^1 \quad \text{and} \quad \bar{g}_O^2 = \chi g_O^1$$ \hspace{1cm} (23)

The model can be solved backwards. The equilibrium outcome at $t = 2$ is straightforward to determine. If in the final period the default is feasible, the incumbent will optimally leave the opposition’s consumption unchanged and consume the remaining part of the budget. Conversely, when the default is not feasible ($\chi < 1$), the status quo will have to be downsized proportionally to render it feasible. As a result, the party in power in the second period has no discretion in choosing the budget composition. If the incumbent stays in power, she will get $\tau - b_1 - \chi g_O^1$ in the second period. If she is turned out of office, she will obtain $\chi g_I^1$. It is key to notice that the two expressions are identical. In fact,

$$\chi g_I^1 = \chi (\tau + b_1 - g_O^1) = \frac{\tau - b_1}{\tau + b_1} (\tau + b_1 - g_O^1) = \tau - b_1 - \chi g_O^1$$

\footnote{It is important that spending is downsized proportionally. If instead the incumbent government is able to selectively cut spending, results might be partially affected.}
In other words, political risk disappears: spending composition in \( t = 2 \) does not depend anymore on the identity of the incumbent but is linked in a deterministic way to previous spending. We summarize the above discussion in the following Lemma.

**Lemma 1:** When \( \chi \leq 1 \) there is no political risk at \( t = 2 \): consumption levels both in and out of power at time \( t = 2 \) are identical.

We now move backwards. In the first period the incumbent has the incentive to choose levels of spending such that all future resources are compromised. Because there is no variance of consumption in the second period, the solution of the first-period bargaining is to equalize today’s and tomorrow’s marginal utilities by choosing zero debt, which in turn implies \( \chi = 1 \). Spending is the same in the two periods. In this framework, if all spending is mandatory, there is no over-accumulation of debt and the economy is dynamically efficient.

**Proposition 5 (mandatory spending)** For any initial status quo spending \((\bar{g}_I^0, \bar{g}_O^0)\), the incumbent at \( t = 1 \) chooses zero debt.

To understand the intuition of this result, it is important to bear in mind what creates the problem in the first place: the fluctuation of power. If the allocation of power were constant over time, there would be no over-accumulation of debt. This happens, for instance, when \( q = 1 \). However, as power changes hands randomly, whoever is in power temporarily tries to obtain as many resources as possible, not only from the present but the future as well. The endogenous status quo allows politicians to smooth political power over time. When they are in power, they know that by increasing spending today, they are also increasing the default spending that they will obtain in case they are out of power. In other words
they are also increasing their future political power and stake in future resources. Hence, the
two parties accurately internalize the costs of increasing debt. The drawback of mandatory
spending is that the first period incumbent is able to appropriate a disproportionate amount
of her favorite public good. Because mandatory spending introduces path dependence, the
disproportion is maintained over time.

The amount of spending that the incumbent is able to secure depends on the initial
default, which defines the opposition’s bargaining power. In Figure 7 we show debt levels
and spending at \( t = 1 \) as a function of the default option in the first period, which we assume
to be the same for both goods. Dynamic efficiency (zero debt) is achieved for all initial
status quo policies. Static efficiency (equal spending on both goods) is obtained only when
the initial status quo is \( \bar{g}_0^O = \bar{g}_0^I = \frac{\tau}{2} \).

6 Discussion

In this paper, we have taken as given the nature of public spending, i.e., whether it is manda-
tory or discretionary. Alternatively, we could assume that policymakers have the possibility
to establish (once-and-for-all) whether new spending should be considered as mandatory or
discretionary. If the incumbent had this choice, she would make new spending mandatory
and the opposition would accept it. The reason for this result is that mandatory spending is
Pareto efficient, while discretionary spending generates allocations that are inside the Pareto
frontier. The incumbent would satisfy the opposition’s acceptance constraint and appropri-
ate all the additional efficiency gains. This discussion suggests that discretionary spending
should not be observed in equilibrium. However, to properly analyze this question one should
enrich the model in dimensions that allow discretionary spending to be preferred. For in-
stance, when there are aggregate shocks or varying parties’ tastes, mandatory spending is
not necessarily Pareto efficient (Riboni and Ruge-Murcia (2008), Dziuda and Loeper (2016)).
Since the focus of this paper is to understand the mechanisms by which fiscal rules affect
debt accumulation, we leave this analysis for future research.

Before concluding, we briefly discuss how our results contribute to the debate on the im-
lications of mandatory programs. Various authors (Riboni and Ruge-Murcia (2008), Bowen
et al. (2014), Bowen et al. (2017)) have stressed that mandatory programs provide insur-
ance against power fluctuations because with such programs the party in power cannot fully
undo past policies. The downside, according to many commentators and politicians, is that
by running on autopilot, mandatory programs make government debt uncontrollable. Our
model shows that by reducing political risk, mandatory spending also reduces the incentive to accumulate debt in the first place. This suggests that the observed joint rise of US debt and mandatory programs is non causal and could be driven by other factors.\textsuperscript{22}

7 Conclusion

In recent years, the number of countries adopting fiscal rules has continued to increase. Since most fiscal rules can be overridden by consensus, the effectiveness of rules is widely debated. We show that the possibility of override does not make fiscal rules irrelevant. Since fiscal rules determine the outside option in case of disagreement, the opposition uses fiscal rules as “bargaining chips”. In exchange for the opposition’s consent to raise debt and bypass the rule, the party in power offers spending concessions to the other party. This political bargain has two main implications. First, debt becomes more costly to accumulate, because the opposition will only agree to bypass the fiscal rule in exchange for more spending on her preferred public goods. Second, the expectation of future compromise increases the benefit of transferring resources to the future. All in all, we show that these two channels reduce the incentives for inefficient debt accumulation. In addition, since budgets are less skewed towards the incumbent’s preferences, we find that the possibility of override improves welfare by insuring against power fluctuations. Finally, we study mandatory spending programs and show that they remove political risk and eliminate the incentives to accumulate debt.

In the wake of the recent US budget crisis, which led to a gap in budget funding and a near default on the nation’s debt, various commentators have questioned the usefulness of rules such as the government shutdown or the debt ceiling. According to many, these rules create unneeded uncertainty and can potentially lead to worse fiscal outcomes. In this paper, we have argued that there are also reasons to hold a more favorable view. In a highly polarized political system, we have shown that these rules push conflicting parties to reach compromises, leading to lower debt. These results are obtained in a model which abstracts from bargaining inefficiencies and delays. We leave these extensions to future research.

\textsuperscript{22}For instance, Bouton et al. (2016) find that an increase of polarization may explain the rise of both entitlements and debt.
A Proofs

Proof of Proposition 1: Suppose that \( \beta(1+r) = 1 \) and \( b_0 = 0 \). We denote the two parties by I and O. Let \( \lambda \) and \( 1-\lambda \) denote the planner’s weights for party I and O, respectively. Define

\[
\rho_I := \lambda + (1-\lambda) \theta \quad \text{and} \quad \rho_O := \lambda \theta + (1-\lambda)
\]

The planner’s problem can be written as

\[
\max_{\{b_1, g_O^1, g_O^2\}} \left\{ \rho_I u(\tau + b_1 - g_O^1) + \rho_O u(g_O^2) + \beta [\rho_I u(\tau - b_1(1+r) - g_O^2) + \rho_O u(g_O^2)] \right\}
\]

Taking the derivative with respect to \( b_1 \), it immediately follows that regardless of \( \lambda \in [0,1] \) the first-order-condition holds when \( b_1 = 0 \). □

Proof of Proposition 2: We solve the alternating-dictator model by backward induction. Under Assumption 1, in the second period the available resources are \( (\tau - b_1) \). The time-2 dictator spends \( (1-\tilde{\theta})(\tau - b_1) \) for her favorite public good and \( \tilde{\theta}(\tau - b_1) \) for the other public good, where \( \tilde{\theta} \) is given by (6). The first period problem can be written as

\[
\max_{\{b_1, g_I^1\}} \left\{ u(\tau + b_1 - g_I^1) + \theta u(g_I^1) + [q + (1-q)\theta]u((1-\tilde{\theta})(\tau - b_1)) + [\theta q + (1-q)]u(\tilde{\theta}(\tau - b_1)) \right\}
\]

Notice that the choice between \( g_I^1 \) and \( g_O^1 \) is a static decision, so it can be solved independently from the dynamic decision. It is straightforward to show that the static problem generates an analogous allocation to period 2: \( g_I^1 = \tilde{\theta}(\tau + b_1) \) and \( g_O^1 = (1-\theta)(\tau + b_1) \). Replacing these expressions in the previous problem and taking derivatives, we obtain the first-order condition (7), where

\[
\frac{\Omega(\theta)}{\Theta(\theta)} = q + \frac{(1-q)(\theta + \theta^{1-\sigma})}{1 + \theta^{\frac{1}{2}}}
\]

Define

\[
\Lambda := \frac{\Omega(\theta)}{\Theta(\theta)}
\]

From (7), when \( \Lambda \) is larger, debt is lower. We therefore study how parameters affect \( \Lambda \).
First, we show that when \( q \) is higher, debt is smaller:

\[
\frac{\partial \Lambda}{\partial q} = 1 - \frac{(\theta + \theta^{\frac{1-\sigma}{\sigma}})}{1 + \theta^{\frac{1}{\sigma}}} > 0
\]

In fact,

\[
(\theta + \theta^{\frac{1-\sigma}{\sigma}}) < 1 + \theta^{\frac{1}{\sigma}}
\]

can be written as \( \theta > \theta^{\frac{1}{\sigma}} \). The inequality holds since \( \theta < 1 \) and \( \sigma \leq 1 \).

We now study the effect of polarization on debt. We compute

\[
\frac{\partial \Lambda}{\partial \theta} = (1 - q) \left( 1 \right) \left( 1 + \theta^{\frac{1-2\sigma}{\sigma}} \right) - \frac{1 - \sigma}{\sigma} \frac{\theta^{\frac{1}{\sigma}} \cdot \left( 1 + \theta^{\frac{1}{\sigma}} \right) - \frac{1}{\sigma} \theta^{\frac{1}{\sigma}} \cdot \left( \theta + \theta^{\frac{1-\sigma}{\sigma}} \right)}{(1 + \theta^{\frac{1}{\sigma}})^2}
\]

This derivative is positive when \( \theta < 1 \) and \( \sigma \leq 1 \). In fact, we rewrite it as

\[
\frac{\partial \Lambda}{\partial \theta} = (1 - q) \left[ 1 - \frac{1 - \sigma}{\sigma} + \frac{1 - \sigma}{\sigma} \left( \theta^{\frac{1-2\sigma}{\sigma}} - \theta^{\frac{1}{\sigma}} \right) \right]
\]

This expression is strictly positive since the first two terms in the square brackets add up to a positive value and the last term is also positive. \( \square \)

**Proof of Proposition 3**: Let \( \gamma \) be the equilibrium ratio of initial consumptions: \( \gamma := \frac{g^O}{g^I} \).

Reorganizing the first order condition (13):

\[
u'(g^1_I) = \frac{1}{1 + \gamma} \left[ qu'(g^2_I) + (1 - q)u'(g^2_O) \gamma \right] - G'(b_1)u'(g^1_I)
\]

\[
u'(g^1_I) = \frac{1}{1 + \gamma} \left[ qu'(g^2_I) + (1 - q)u'(g^2_O) \gamma \right] + \frac{1}{1 + \gamma} \left[ qu'(g^2_O) \gamma' + (1 - q)u'(g^2_O) \right] u'(g^1_I)
\]

Knowing that (4) and (5) are satisfied with equality, we write the first-order condition as

\[
\frac{u'(\frac{\sigma + b}{1 + \gamma})}{u'(\frac{\sigma - b}{1 + \gamma})} = \frac{1}{1 + \gamma} \left[ q + (1 - q) \frac{u'(g^2_O)}{u'(g^2_I)} \gamma' + q \frac{u'(g^2_O)}{u'(g^2_I)} u'(g^1_I) \gamma' + (1 - q) \frac{u'(g^1_I)}{u'(g^1_I)} \right]
\]

(24)

First, we show that debt is reduced. To do this, we impose the consistency requirement \( \gamma = \gamma' \). With this restriction, it is easy to see that \( \frac{u'(g^2_O)}{u'(g^2_I)} = u'(\gamma) = u'(\gamma) = \frac{u'(g^2_O)}{u'(g^2_I)} \). Then,
\[
\frac{u'(\tau + b_1)}{u'(\tau - b_1)} = \frac{1}{1 + \gamma} \left[ q + (1 - q)u'(\gamma)\gamma + q\gamma + (1 - q)u'(\gamma^{-1}) \right]
\]
\[
\frac{u'(\tau + b_1)}{u'(\tau - b_1)} = q + (1 - q) \frac{[u'(\gamma)\gamma + u'(\gamma^{-1})]}{1 + \gamma}
\]

From the last equation it is straightforward that debt grows at a smaller rate than in the dictator’s problem. The last term in the right-hand side of the above equation ensures it. Loosely speaking, it is analogous to increasing the discount factor from \(q\) in the dictator’s problem to \(q + (1 - q)\frac{[u'(\gamma)\gamma + u'(\gamma^{-1})]}{1 + \gamma}\) in the bargaining problem. Since \(\gamma > 0\) simple algebra confirms point (a) of Proposition 3.

To prove that debt is still positive, we only need to show that \(\frac{[u'(\gamma)\gamma + u'(\gamma^{-1})]}{1 + \gamma} \leq 1\), or:

\[
\gamma^{1-\sigma} + \gamma^\sigma \leq 1 + \gamma
\]

This is true for all \(\gamma > 0\) and \(\sigma \leq 1\). To see this, consider the function \(f(\gamma) = \gamma^{1-\sigma} + \gamma^\sigma - 1 - \gamma\). We want to show that \(f(\gamma) \leq 0\) for all \(\gamma\). Rewriting the function as \(f(\gamma) = (1 - \gamma^{1-\sigma})(\gamma^\sigma - 1)\) it is clear that \(f(\gamma)\) is negative and it equals zero only when \(\gamma = 1\). □

Proof of Proposition 4: First, suppose that bargaining takes place in the first period but not in the second period: \(\gamma' = 0\). In this case the first order condition (13) is given by

\[
(1 - G'(b_1))u'(\tau + b_1 - G(b_1)) = qu'(\tau - b_1)
\]

and

\[
G'(b_1) = (1 - q) \frac{u'(g_1^I)}{u'(g_1^O)}
\]

Following similar steps as in the previous proof, we obtain

\[
\left( \frac{g_1^I}{g_1^O} \right)^\sigma = q + (1 - q)\gamma^\sigma
\]

According to the last equation, the incumbent’s consumption grows at a smaller rate than the dictator’s consumption. Does this imply that debt grows at a smaller rate? Using the budget constraints (4) and (5), we write (27) as

\[
\frac{1}{(\tau + b_1)^\sigma} = q + (1 - q)\gamma^\sigma \frac{1}{(\tau - b_1)^\sigma}
\]
In addition, note that $\gamma$ is a continuous function of $b_1$, which is implicitly defined by the acceptance constraint

$$u(\gamma \left( \frac{\tau + b_1}{1 + \gamma} \right)) + u(\tau - b_1)(1 - q) = u((2 - \alpha)\tau)(1 - q)$$

To argue that debt might be higher when bargaining occurs only in the first period, we show that there are parameters such that

$$\frac{q + (1 - q)\gamma^\sigma}{(1 + \gamma)^\sigma} < q$$

(30)

For instance, when $\sigma$ approaches 1, (30) holds when $q > 1/2$. □

Proof of Proposition 5

Step 1: Suppose that in equilibrium $\chi \leq 1$. We show that zero debt is optimal.

The initial default $(g_{01}, g_{01}^O)$ is given. Suppose that in equilibrium $\chi \leq 1$. We will verify this claim in Step 2. Given Lemma 1, there is no political risk in the second period. The agenda setter’s problem is therefore given by:

$$\max_{\{b_1, g_{01}, g_{11}^I\}} u(\tau + b_1 - g_{01}^I) + u(\chi g_{11}^I)$$

subject to

$$u(g_{01}^I) + u(\chi g_{11}^I) \geq u(g_{01}^O) + q u(g_{01}^O) + (1 - q) u(2\tau - 2g_{01}^I - g_{01}^O)$$

$$\chi = \min \left\{ \frac{\tau - b_1}{\tau + b_1}, 1 \right\}$$

and

$$g_{11}^I + g_{01}^O = \tau + b_1$$

Let $\lambda$ be the multiplier associated to the acceptance constraint. Differentiating with respect to $g_{11}^O$ we obtain

$$u'(\tau + b_1 - g_{01}^O) + \chi u'(\chi g_{11}^I) = \lambda [u'(g_{01}^O) + \chi u'(\chi g_{11}^O)]$$

$$u'(g_{11}^I)(1 + \chi^{-\sigma}) = \lambda [u'(g_{11}^O)(1 + \chi^{1-\sigma})]$$

which gives us
\[ \lambda = \frac{u'(\tau + b_1 - g_1^O)}{u'(g_1^O)} \] (31)

Next, we take the derivative with respect to \( b_1 \) and obtain

\[
u'(\tau + b_1 - g_1^O) + u'(\chi(\tau + b_1 - g_1^O)) \left[ \frac{-2\tau}{(\tau + b_1)^2} (\tau + b_1 - g_1^O) + \frac{(\tau - b_1)}{\tau + b_1} \right] - \lambda \left[ u'(\chi g_1^O) g_1^O \frac{2\tau}{(\tau + b_1)^2} \right] = 0 \] (32)

After inserting (31) we verify that when \( b_1 = 0 \), the above condition is satisfied. In fact,

\[
u'(\tau - g_1^O) + u'(\tau - g_1^O) \left[ \frac{-2}{\tau} (\tau - g_1^O) + 1 \right] - \frac{2u'(\tau - g_1^O)}{\tau} g_1^O = 0 \] (33)

or

\[
u'(\tau - g_1^O) = u'(\tau - g_1^O) \] (34)

Zero debt is therefore optimal. Note that \( g_1^O \) solves the acceptance constraint with equality, for given initial default option \((g_0^O, g_0^I)\). That is,

\[ 2u(g_1^O) = u(g_0^O) + qu(g_0^O) + (1-q)u(2\tau - 2g_0^I - g_0^O) \] (35)

**Step 2**: We show that \( \chi > 1 \) is not optimal.

When \( \chi > 1 \) (i.e., \( \tau + b_1 < \tau - b_1 \)) uncertainty in the second period is maintained. The agenda setter’s problem is written as follows:

\[
\max_{\{b_1, g_1^O, g_1^I\}} u(\tau + b_1 - g_1^O) + qu(\tau - b_1 - g_0^O) + (1-q)u(\tau + b_1 - g_1^O)
\]

subject to

\[ u(g_1^O) + qu(g_1^O) + (1-q)u(g_1^O - 2b_1) \geq u(g_0^O) + qu(g_0^O) + (1-q)u(2\tau - 2g_0^I - g_0^O) \]

We take the derivatives with respect to \( g_1^O \):

\[
-u'(\tau + b_1 - g_1^O)(2-q) - qu'(\tau - b_1 - g_1^O) \\
+ \lambda \left[ u'(g_1^O)(1 + q) + u'(g_1^O - 2b_1)(1 - q) \right] = 0
\] (36)
We take the derivatives with respect to $b_1$:

$$u'(\tau + b_1 - g^O_1)(2 - q) - qu'(\tau - b_1 - g^O_1) - \lambda [(1 - q)u'(g^O_1 - 2b_1)2] = 0$$

(37)

We evaluate the first-order condition (36) at $b_1 = 0$:

$$\lambda = \frac{u'(\tau - g^O_1)(1 - q)}{2u'(g^O_1)}$$

(38)

The first-order condition (37) evaluated at $b_1 = 0$ does not hold. In fact,

$$u'(\tau - g^O_1)(2 - q) - qu'(\tau - g^O_1) - \frac{u'(\tau - g^O_1)(1 - q)}{2u'(g^O_1)} [2(1 - q)u'(g^O_1))] > 0$$

(39)

Therefore, we have verified that the initial agenda setter makes sure that all resources are compromised: $\chi \leq 1$. □

B Infinite horizon

In this section we extend the results from Section 3 to an infinite horizon economy. To that end we assume that $q = 1/2$. We maintain all the assumptions and notation from the previous sections. The utility is given by (1). We will solve the model for all $\theta \in [0, 1]$. Since the two parties are symmetric, nothing depends on the identity of the party in power and we only distinguish parties by whether they are in power or not. To summarize:

Assumption: (infinite horizon) Let $\beta(1 + r) = 1$, $q = 1/2$ and $\theta \in [0, 1]$.

We write the problem recursively, with $b$ as state variable. Let $V_I(b)$ and $V_O(b)$ denote, respectively, the value functions of the incumbent and opposition. In order to simplify notation, define

$$W(b) := V_I(b) + V_O(b)$$

In the infinite horizon the party in power solves:

$$V_I(b) = \max_{\{g^I, g^O, b'\}} \{u_I(g^I, g^O) + \frac{\beta}{2} W(b')\}$$
\[ s.t. \tau - (1 + r)b + b' - g^I - g^O \geq 0 \quad (BC) \]
\[ u_O(g^I, g^O, \tau) + \frac{\beta}{2} W(b') \geq m(b) \quad (AC) \]
\[ b \leq b' \leq \bar{b} \]
\[ V_O(b) = u_O(g^*_I(b), g^*_O(b), \tau^*) + \frac{\beta}{2} W(b^*(b)) \]

The constraint (BC) is the government’s budget constraint. (AC) is the acceptance constraint: the opposition’s utility must be greater than \(m(b)\). The expression \(m(b)\) is the value of disagreement, which depends on the fiscal rule in place (more on this later). Throughout we assume that the acceptance constraint will be binding. We assume that \(b'\) must be smaller than the natural debt limit: \(\bar{b} = \tau/r\). This implies that it is always feasible to pay the outstanding debt. For \(b\) we can assume any negative number, including \(-\infty\). Since \(\beta(1+r) = 1\), it is straightforward to show that the social planner keeps the debt level constant over time.

### B.1 Static Problem

The political problem can be split in two sub-problems: a static problem to decide the spending allocation and a dynamic one to decide \(b'\). Let \(\mu\) the multiplier in the acceptance constraint. From the first-order conditions with respect to \(g^O\) and \(g^I\), it follows that

\[ g^O = \left( \frac{\theta + \mu}{1 + \theta \mu} \right)^{\frac{1}{\sigma}} g^I; \quad \text{and} \quad g^O + g^I = \left[ 1 + \left( \frac{\theta + \mu}{1 + \theta \mu} \right)^{\frac{1}{\sigma}} \right] g^I = pg^I \quad (40) \]

One can write the incumbent and opposition utilities, respectively, as:

\[ u_I(g^I, g^O) = \left[ 1 + \theta \left( \frac{\theta + \mu}{1 + \theta \mu} \right)^{\frac{1}{\sigma}} \right] \frac{(g^I)^{1-\sigma}}{1-\sigma} = \phi_I u(g^I) \quad (41) \]
\[ u_O(g^I, g^O) = \left[ \theta + \left( \frac{\theta + \mu}{1 + \theta \mu} \right)^{\frac{1}{\sigma}} \right] \frac{(g^I)^{1-\sigma}}{1-\sigma} = \phi_O u(g^I) \quad (42) \]

We can write

\[ p = 1 + (\phi_O - \theta)^{\frac{1}{1-\sigma}} \quad (43) \]
Instead of expressing $\phi_O$ and $\phi_O$ as a function of $\mu$, using the acceptance constraint, which is assumed binding, one can write:

$$\phi_O = \frac{m(b) - \frac{\beta}{2} W(b')}{u(g^I)}; \quad \text{and} \quad \phi_I = 1 + \theta(\phi_O - \theta) \quad (44)$$

Using (41) and (42)

$$W(b) = (\phi_O + \phi_I)u(g^I) + \beta W(b') \quad (45)$$

We rewrite the incumbent’s problem as:

$$V_I(b) = \max_{g^I, b'} \{\phi_I u(g^I) + \frac{\beta}{2} W(b')\}$$

s.t.  

$$\tau - (1 + r)b + b' - [1 + (\phi_O - \theta)\frac{1}{1-\sigma}g^I] g^I \geq 0$$

$$b \leq b' \leq \bar{b}$$

where $\phi_O$, which is given by (44), reflects the opposition’s bargaining power.

### B.2 Fiscal Rules

Before proceeding we need to specify how the budget rule affects outside value in case of disagreement, $m(b)$. As in Section 4, a budget rule is summarized by the parameter $\alpha \in [0, 1]$. Given $(\tau - rb)$, $\alpha$ determines total spending available for the two public goods. That is, the fiscal rule requires

$$g^O + g^I \leq \alpha(\tau - rb) \quad (46)$$

Inequality (46) must hold unless the two parties reach a consensus. If there is no consensus, the incumbent is free to choose the spending mix satisfying (46). Since upon disagreement the incumbent chooses the allocation, the optimal allocation $g^I$ and $g^O$ is the same as before with $\mu = 0$, i.e., $g^O = \theta^\frac{1}{2} g^I$ and $g^O + g^I = \left(1 + \theta^\frac{1}{2}\right) g^I = p_s g^I$. Therefore, we can write

$$m(b) = \phi_s u(g^I) + \frac{\beta}{2} W(b^s) \quad (47)$$

where

$$\phi_s^O := \theta + \theta^{\frac{1}{\sigma}} \quad (48)$$
and

\[ b^s := b + (\alpha - 1) (\tau - rb) \]  (49)

Equation (47) is the value of the outside option for the opposition and plays a fundamental role on the following derivations.

**B.3 Equilibrium characterization.**

To find the optimal budget rule we need to further characterize the equilibrium. To this end we use a guess and verify approach. We guess a solution and we find that it satisfies all the optimality conditions. The key feature that we exploit is that with the proposed rule the government spending grows at a constant rate, rendering the continuation value function proportional to the flow utility, \( W(b) = au(g^I) \) for some constant \( a \). We guess and verify that the ratio \( g^O / g^I \) is constant for all \( t \). In an environment with homothetic preferences the value function is also homothetic and therefore the opposition’s bargaining power can be represented by a constant independent of debt. We guess that spending is linear in \( (\tau - rb) \):

\[ g^I = \nu \tau - rb \]
\[ g^O = (p - 1) \nu \tau - rb \]
\[ W(b) = au \left( \frac{\tau - rb}{p} \right) \]

where \( a, p \) and \( \nu \) are constant to be determined.

Note that under this guess:

\[ b' = b + (\nu - 1) (\tau - rb) \]

The lower \( \nu \), the smaller the incentive to accumulate debt. Using the guess:

\[ au \left( \frac{\tau - rb}{p} \right) = (\phi_O + \phi_I)u \left( \frac{\nu \tau - rb}{p} \right) + \beta au \left( \frac{\tau - rb}{p} \right) \left( 1 - r(\nu - 1) \right)^{1-\sigma} \]

Therefore,

\[ a = \frac{(\phi_O + \phi_I)\nu^{1-\sigma}}{1 - \beta \left( 1 - r(\nu - 1) \right)^{1-\sigma}} \]  (50)

Given \( a \), the derivatives of the value function are:

\[ W'(b) = \frac{-ra}{p} u' \left( \frac{\tau - rb}{p} \right) \]
\[ W'(b') = -\frac{ra'}{p} u' \left( \frac{\tau - rb}{p} \right) (1 - r (\nu - 1))^{-\sigma} \]

The Euler equation is given by

\[ u'(g^I) = -\frac{\beta}{2(1 + \theta)} [1 + (\phi_O - \theta)^{1-\sigma}] W'(b') \]

Using our guess:

\[ u'(g^I) = \frac{\beta}{2(1 + \theta)} [1 + (\phi_O - \theta)^{1-\sigma}] \frac{ra'}{p} u' \left( \frac{\tau + rb}{p} \right) (1 - r (\nu - 1))^{1-\sigma} \]

Then,

\[ \nu^{-\sigma} = \frac{(1 - \beta)a}{2p(1 + \theta)} [1 + (\phi_O - \theta)^{1-\sigma}] (1 - r (\nu - 1))^{-\sigma} \]

Define

\[ \rho := \frac{(1 - \beta)a}{2p(1 + \theta)} [1 + (\phi_O - \theta)^{1-\sigma}] \]

We obtain:

\[ \nu = \frac{1 + r}{\rho^{1-\sigma} + r} \quad (52) \]

Thus, for a given \( \phi_O \), using (50) and the definition of \( \nu \) we have a fixed point in \( \rho \). It is easy to show that there is a solution for all \( \phi_O \in [\theta, 1 + \theta] \) and, using \( a \), that in an equilibrium with \( \phi_O < 1 + \theta \), \( \nu > 1 \) which implies \( \rho < 1 \).

Now, we solve for \( \phi_O \) which depends on the budget rule. Recall that:

\[ \phi_O = \frac{\phi^s_O u(g^I) + \beta/2[W(b^s) - W(b)]}{u(g^I)} \]

where \( \phi^s_O \) and \( b^s \) are given by (48) and (49). Thus,

\[ \phi_O = \alpha^{1-\sigma} \phi^s_O + \frac{a\beta}{2u(g^I)} [u((\tau + rb^s)/p) - u((\tau + r b)/p)] \]

\[ \phi_O = \alpha^{1-\sigma} \phi^s_O + \frac{\beta a}{2u(g^I)} [u((1 - r (\alpha - 1))(\tau + rb)/p) - u((1 - r (\nu - 1))(\tau + rb)/p)] \]

\[ \phi_O = \alpha^{1-\sigma} \phi^s_O + \frac{\beta a}{2} [(1 - r (\alpha - 1))^{1-\sigma} - (1 - r (\nu - 1))^{1-\sigma}] \quad (53) \]
Proposition 6 For any budget rule $\alpha$ a Markov-perfect equilibrium is fully characterized by the factor $\nu$ and bargaining power $\phi_O$ that simultaneously solve equations (52) and (53).

The remaining parameter to be determined is $p$, which can be computed using (43).

Before studying how fiscal rules affect $\nu$ it is instructive to compute the dictator’s problem. The solution to it is a special case of the above with $\mu(b) = 0$ for all $b$. That generates:

$$x^p_g = \frac{g_{t+1}}{g_t} = 1 \quad \text{(Planner’s)}$$

$$x^D_g = \frac{g_{t+1}}{g_t} = \left[\frac{(1 - \beta)(1 + \theta)(1 + \theta^{1-\sigma})}{(1 - \tilde{\beta})\frac{\nu^{1-\sigma}}{2(1 + \theta^{1-\sigma})}}\right]^{\frac{1}{\sigma}} < 1, \quad \text{(Dictator)}$$

where $\tilde{\beta} = \beta (1 - r (\nu - 1))^{1-\sigma}$. With bargaining

$$x_g = \frac{g_{t+1}}{g_t} = \left[\frac{(1 - \beta)(\phi_O + \phi_I)(1 + (\phi_O - \theta)^{1-\sigma})}{(1 - \tilde{\beta})\frac{\nu^{1-\sigma}}{2(1 + \theta)(1 + (\phi_O - \theta)^{1-\sigma})}}\right]^{\frac{1}{\sigma}} \leq 1 \quad \text{(Bargaining)}$$

Notice that $x^D_g \to 1$ as $\theta \to 1$. As polarization vanishes, the dictator’s effective discount factor converges to the optimal. On the other extreme, when $\theta = 0$, polarization attains its maximum and $\rho$ is at its minimum. Comparing the constant multiplying the value function in the planner’s solution with $a$ for the dictator’s problem, which is $\frac{(1 + \theta)(1 + \theta^{1-\sigma})\nu^{1-\sigma}}{1 - \tilde{\beta}}$, one can see that the main difference stems from the internalization of the other’s party future preferences. The planner weighs both kinds of constituents equally, which is represented by the term $2(1 + \theta)$ multiplying the value function, while the dictator only values her own constituents, hence the term $(1 + \theta)(1 + \theta^{1-\sigma})$. The smaller $\theta$, the larger the difference between the problems, and therefore the larger the over accumulation of debt.

When there is bargaining the incumbent is forced to internalize some of the opposition’s preferences. Hence the term $(\phi_O + \phi_I)$ in (50). The larger the bargaining power of the opposition, $\phi_O$, the larger $\rho$ and the closer the solution is to the optimal. For instance, if $\phi_O = 1 + \theta$, then $\phi_I = 1 + \theta$, and therefore the planner’s solution is implemented. Of course, whether this is possible will depend on the values of $\alpha$ and $\phi_O^*$. What must be clear from the above is that in order to reduce debt accumulation the goal is to maximize $\phi_O$.
B.4 The optimal budget rule.

What is the optimal $\alpha$? Recall that the purpose of the optimal budget rule is to maximize $\rho$, but for that it is necessary to maximize $\varphi_O$. Using equation (53), taking the first order condition (and keeping in mind that $a$, $\nu$ and $p$ are functions of $\varphi_O$):

$$\frac{\partial \varphi_O}{\partial \alpha} = (1 - \sigma)\alpha^{-\sigma} \phi^s_O + \frac{\partial a(\alpha)}{\partial \varphi_O} \frac{\partial \varphi_O}{\partial \alpha} \frac{\beta}{2} \left[ (1 - r(\alpha - 1))^{1-\sigma} - (1 - r(\nu - 1))^{1-\sigma} \right]$$

$$- r(1 - \sigma)\frac{\beta a}{2} (1 - r(\alpha - 1))^{-\sigma} + r(1 - \sigma)\frac{\beta a}{2} (1 - r(\nu - 1))^{-\sigma} \frac{\partial \nu}{\partial \varphi_O} \frac{\partial \varphi_O}{\partial \alpha}$$

Since at the optimum $\frac{\partial \varphi_O}{\partial \alpha} = 0$, there is an analogous to the envelope theorem: only the direct effect matters; indirect effects are negligible. Rearranging the equation we obtain:

$$\alpha^{-\sigma} \phi^s_O = r\frac{\beta a(\alpha)}{2} (1 - r(\alpha - 1))^{-\sigma}$$

$$\alpha = \left( \frac{2\phi^s_O}{(1 - \beta) a(\alpha)} \right)^{\frac{1}{\sigma}} \left( 1 - (1 - \beta)\alpha \right)$$

As a result:

$$\alpha = \frac{1}{\beta} \left( \frac{2\phi^s_O}{(1 - \beta) a(\alpha)} \right)^{\frac{1}{\sigma}}$$

Revising $a$ in the above

$$\alpha = \frac{\left( \frac{2(1 - \tilde{\beta})}{(1 - \beta)} \frac{\phi^s_O}{(\varphi_O + \varphi_P)\nu^{1-\sigma}} \right)^{\frac{1}{\sigma}}}{\beta + (1 - \beta) \frac{2(1 - \tilde{\beta})}{(1 - \beta)} \frac{\phi^s_O}{(\varphi_O + \varphi_P)\nu^{1-\sigma}}}$$

where $\tilde{\beta} = \beta (1 - r(\nu - 1))^{1-\sigma}$. Using the equilibrium $\phi^s_O$ with full discretion:

$$\alpha = \frac{\left( \frac{2(1 - \tilde{\beta})}{(1 - \beta)} \frac{\phi^s_O}{(1 + \theta)(1 - \theta + \phi_O)\nu^{1-\sigma}} \right)^{\frac{1}{\sigma}}}{\beta + (1 - \beta) \frac{2(1 - \tilde{\beta})}{(1 - \beta)} \frac{\phi^s_O}{(1 + \theta)(1 - \theta + \phi_O)\nu^{1-\sigma}}}$$

Notice that $\alpha \to 0$ as $\theta \to 0$. If the degree of polarization is too high, it is optimal to have a government shutdown as the default option in case of disagreement.
As discussed in the main text, reducing $\alpha$ has the following two effects. 1) In case of disagreement it reduces current spending and shifts resources from today to future periods. It provides very low consumption in the current period, implying little intertemporal consumption smoothing (a negative insurance effect). 2) A lower $\alpha$ implies that the incumbent has fewer resources to use for his favorite public good. Therefore, a lower $\alpha$ strategically affects the incumbent’s choice. When polarization is high, the negative insurance effect is negligible because the incumbent does not spend on the public good favoured by the opposition, regardless of $\alpha$. On the other hand, the opposition can strategically affect the incumbent’s proposed spending mix. This is why the optimal budget rule implies $\alpha = 0$. As polarization decreases, the negative insurance effect is more important and the optimal budget rule is a higher level of $\alpha$. When there is no polarization, $\alpha = 1$ is optimal.

In Figure 8 and 9 we see that when $\sigma$, the inverse of the intertemporal elasticity of substitution, increases, the optimal $\alpha$ is closer to one. Also notice that when $\sigma$ increases, it is equivalent to a reduction of polarization: a more concave utility decreases the present-bias of the incumbent and the future-bias of the opposition.
References


