Compulsory insurance and voluntary self-insurance: substitutes or complements? A matter of risk attitudes

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Summary:

Based on Ehrlich and Becker’s model (1972) on insurance and self-insurance substitutability, we study the effects of a compulsory partial insurance on self-insurance decisions of both risk-aversers and (mixed) risk-lovers. We show that when insurance is compulsory, risk-aversers adjust (by substituting) their self-insurance behavior to compensate for the level (too high or too low) of the compulsory coverage level. By contrast, even though they would refuse to invest in any voluntarily hedging scheme, (mixed) risk-lovers freely invest in self-insurance to complete a compulsory partial insurance coverage. Moreover, we prove that for a (mixed) risk-lover, an increase in the partial compulsory insurance coverage induces simultaneously a rise of the self-insurance marginal benefit and a decrease of its marginal cost. Therefore, while compulsory insurance and self-insurance are substitutes for risk-aversers, they are complements for (mixed) risk-lovers. This last result brings an unexpected justification for compulsory insurance policies.

Keywords: self-insurance; compulsory insurance; risk attitudes; risk-lovers

JEL-Classification: D11; D86 ; G22 ; K32 ; L51
Introduction:

Yet up to now, Insurance Theory has been based on the assumption of risk aversion. Several critical features justify the lack of concern for risk-lovers so far. Not only are the risk-lovers too few to deserve attention but were they more numerous than expected, risk loving issues would still appear as of casual interest since risk-lovers are not expected to buy insurance. Indeed, we know since Mossin (1968) that if a risk-averse decision-maker buys a comprehensive coverage when insurance price is actuarial, a risk-lover would refuse any positive insurance coverage based upon actuarial or more than actuarial prices: insuring a risk-lover would require subsidized rates getting insurers into an unsustainable (and somewhat unrealistic) loss situation. For all these reasons, Insurance Theory has been shaped by the assumption of risk aversion with no consideration for risk-loving matters.

This perspective is now being challenged by both practical and theoretical findings pointing out that some people are prone to be risk-lovers and that their behavior worth being studied. As suggested by Kahneman and Tversky (1979), according to the reflection effect, people could be risk-averse in the gain domain but risk-loving in the loss domain, which is insurance’s. Experimentally, the existence of risk-lovers is well documented. For example, the reflection effect is confirmed in Cohen et al. (1987) or Chakravarty and Roy (2009), while Noussair et al. (2014) highlight that about 15 percent of a large representative sample are risk-loving people. In their experimental work testing insurance demand in the Lab, Corcos, Pannequin and Montmarquette (2017) show that risk attitude - risk loving vs. risk aversion – is a key element of insurance behavior. First, their data show that risk-loving could be a genuine risk-attitude when it comes to insurance context. Moreover, Corcos and al. (2017) stress the fact that RLS’ insurance behavior could have much more in common with a strategic gambling behavior (RLS are more likely to buy insurance when no accident has occurred for a long time) than with a lack of interest for insurance.

On a theoretical point of view, several recent contributions also focused on risk loving behavior. Crainich, Eeckhoudt and Trannoy (2013) have showed that “combining good with good” is consistent with mixed risk loving (this property assesses that successive derivatives of the utility function are all positive). It follows that risk lovers are prudent (u’’’(w)>0) and share this behavior with risk averters. Jindapon (2013) focusing on self-protection stresses the fact that a risk lover may find optimal to invest in this type of hedging. Hence, theoretical risk-lovers’ insurance and self-insurance decisions deserve to be deepened.

Our paper addresses the risk lovers’ insurance behavior issue by extending the Ehrlich and Becker’s model (1972) on insurance and self-insurance to risk attitudes. However, since, in the real world, main insurance contracts (e.g. Health insurance, car insurance, household insurance and liability insurance) have to deal with compulsory insurance coming along with voluntary devices (complementary insurance, self-protection, and self-insurance), we specifically study the effect of a partial compulsory insurance on self-insurance decisions.

1 and a partial coverage if insurance price is more than actuarial.
These mandatory provisions are usually partial and come along with voluntary devices: complementary insurance, self-protection, and self-insurance.²

The risk-aversion-related results are in line with the economic intuition: risk-avers adjust their self-insurance behavior to compensate for the level (too high or too low) of coverage offered by a compulsory insurance. However, surprisingly, when insurance is not mandatory, risk-lovers³ choose not to invest in any hedging scheme, whereas they may freely invest in self-insurance to complete a compulsory partial insurance coverage. While compulsory insurance and self-insurance are substitutes for risk-avers, they are proved to be complements for (mixed) risk-lovers.

Our article is organized as follows. In Section 1, we present the interactions between compulsory insurance and self-insurance decisions for risk-averse individuals. Section 2 is devoted to (mixed) risk-lovers and shows how a partial compulsory insurance leads risk-lovers to invest freely in self-insurance. The last section addresses the political implications of our results.

1. The theoretical framework of Compulsory insurance and self-insurance

We consider an individual endowed with an initial wealth $W_0$ and facing a probability $q$ to lose a share $x_0$ of this wealth. The individual is subject to a compulsory insurance which scheme imposes her to pay an insurance premium $P = p\bar{I}$ in exchange for a compensation $\bar{I} > 0$ in case of accident. The unit price of insurance is denoted by $p$ and is assumed to be at least actuarial ($p \geq q$).⁴

Also, to reduce her risk-exposure, this individual can use a self-insurance technology with diminishing returns. The actual loss $x$ is therefore a function of the investment $a$ in self-insurance (SI): $x = x(a) = x_0 - SI(a)$, where $SI(a)$ represents the sheltered wealth, with $SI'(a) > 0$, $SI''(a) < 0$, and $x(0) = x_0$. We assume the marginal return of the first unit of self-insurance to be strictly greater than that of insurance in any case ($p \geq q$): $SI'(0) > 1/q$.⁵

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² Since Ehrlich and Becker (1972), self-insurance relates to the expenses reducing the size of the loss in case of accident (example: security seat belt) while self-protection is an investment intended to reduce the loss probability.

³ Assuming that they are prudent ($u'''' > 0$).

⁴ It would be irrational for an insurer to offer a less-than-actuarial price, or incurring losses. However, all results related to risk-avers also hold when $p < q$. But for the sake of consistency with the study of the risk-lovers behavior, we focused on this specific case.

⁵ Without this assumption, prevention would be meaningless (with $p=q$): if the marginal return of the 1st unit of money invested in SI is less than the marginal (constant) return of insurance ($1/q$), prevention would never be profitable.
The final wealth is therefore a function of the level of both self-insurance investment \( SI(a) \) and compulsory insurance \( I \). Depending on the state of nature, the final wealth can be written as follows:

\[
W_1 = W_0 - pI - a, \text{ in the absence of accident;}
\]

\[
W_2 = W_0 - pI - a - (x_0 - SI(a)) + I, \text{ if an accident occurs.}
\]

A rational decision maker aims at maximizing her expected utility according to her self-insurance choice \( SI(a) \). If the individual is risk-averse (RA), preferences are characterized by \( U(W) \), a strictly concave utility function \((U'(W) > 0, U''(W) < 0)\). The utility function is a strictly convex utility function \((U'(W) > 0, U''(W) > 0)\) if the individual is risk lover.

The policyholder’s problem is the following:

\[
\max_a EU(a) = (1 - q)U(W_0 - pI - a) + qU(W_0 - pI - a - x_0 + SI(a) + I)
\]  

(1.1)

The following first order condition characterizes an interior solution:

\[
\frac{\partial EU}{\partial a} = -(1 - q)U'(W_0 - pI - a) - [1 - SI'(a)]qU'(W_0 - pI - a - x_0 + SI(a) + I) = 0
\]

This condition can be rewritten to make both the marginal cost and the marginal benefit of self-insurance apparent:

\[
\frac{\partial EU}{\partial a} = -\{(1 - q)U'(W_1) + qU'(W_2)\} + \{SI'(a)qU'(W_2)\} = 0
\]  

(1.2)

2. The risk averters’ behavior

Since the second order condition is checked for a risk averter with \( U''(.) < 0 \) and \( SI''(a) < 0 \), the optimal level of self-insurance expenditure under a compulsory insurance equalsizes the marginal return of self-insurance to the following expression: \(^6\)

\[
SI'(a) = \frac{(1 - q)U'(W_1)}{qU'(W_2)} + 1
\]  

(1.3)

The risk averter’s demand for self-insurance depends on whether the compulsory insurance coverage leads to a shortage or an excess of insurance—i.e. whether the compulsory level is below or above the optimal voluntary insurance level. Characterizing the resulting choice of prevention requires, therefore, to provide the optimal self-insurance decision under a

\(^6\) \( \frac{\partial^2 EU}{\partial a^2} = (1 - q)U''(W_0 - pI - a) + SI''(a)qU'(W_0 - pI - a - x_0 + SI(a) + I) + [1 - SI'(a)]^2qU''(W_0 - pI - a - x + SI(a) + I) < 0 \)
voluntary insurance context. This choice implies the maximization of the expression (1.1) as a function of both \( a \) and \( I \). Then, still making marginal cost and marginal benefit apparent, we obtain a 1st order condition for optimal insurance:

\[
\frac{\partial EU}{\partial I} = -p\{(1 - q)U'(W_1) + qU'(W_2)\} + \{qU'(W_2)\} = 0
\] (1.4)

Combining conditions (1.2) and (1.4), suggests that at the optimum of the voluntary insurance program, the marginal returns of insurance and self-insurance are equal:

\[
SI'(a^*) = \frac{1}{p}
\] (1.5)

Comparing equations (1.3) and (1.5) makes it possible to characterize the optimal prevention behavior under a compulsory insurance as a function of the difference between the level of compulsory insurance and the optimal voluntary insurance levels \((\bar{I} - I^*)\):

- When \( I = I^* \), we get:
  \[
  SI'(a) = \frac{(1 - q)U'(W_1)}{qU'(W_2)} + 1 = SI'(a^*) = \frac{1}{p}
  \] (1.6)

  Then the prevention levels in the compulsory and voluntary insurance schemes are the same (a=a*);

- When \( I < I^* \), the compulsory insurance leads to an insurance rationing\(^{\text{7}}\) and the ratio of marginal utilities given in expression (1.6) decreases. With respect to the optimal situation, \( W_1 \) increases and \( W_2 \) diminishes; as a consequence: \( SI'(a) < \frac{1}{p} = SI'(a^*) \). Therefore a shortage in insurance provides an increase in the self-insurance investment: \( a > a^* \).

- When the compulsory insurance level exceeds the optimal voluntary insurance level \((\bar{I} > I^*)\), a symmetric argument results in a reduction of the prevention level: \( a < a^* \).

These findings are summarized in the following proposition:

\textbf{Proposition 1}: Whatever the insurance context (either voluntary or compulsory) RAs keep substituting \( I \) and SI: if the compulsory level of insurance leads to a shortage (resp. an excess) in insurance, the RA increases (resp. decreases) her SI investment.

\(^{\text{7}}\) It is worth noting that a complementary insurance hedging, if available, would eliminate this shortage. The policyholder could combine a voluntary complementary hedging \( (I_c) \) with \( \bar{I} \), so as to achieve the optimum \((I^* = \bar{I} + I_c)\). However, we kept this case since it is entirely plausible that a complementary insurance hedging cannot be organized (adverse selection, moral hazard, narrow market, regulation...). Moreover, even if provided, it would not be necessarily available for everyone.
It is also possible to characterize the effect of a change in the unit price $p$ (with $p \geq q$). For this purpose, let us rewrite the condition (1.3) and develop the arguments of marginal utilities:

$$SI'(a) = \frac{(1 - q)U'(W_0 - pI - a)}{qU'(W_0 - pI - a - x_0 + SI(a) + I)} + 1 \quad (1.7)$$

The property of substitutability between insurance and prevention results from equation (1.7). Indeed, under the DARA (Decreasing Absolute Risk Aversion) assumption, when the price of insurance increases, the denominator of the right-hand side of expression (1.7) rises relatively more than the numerator does. Therefore, the ratio decreases and according to equality (1.7), so does $SI'(a)$. Finally, $a$ increases with the price. Symmetrically, when the price of insurance lessens, $a$ decreases.

Proposition 2: When insurance is compulsory, an increase (respectively decrease) in the unit price $(p \geq q)$ induces the risk-averse individual to increase (respectively decrease) his investment in self-insurance.

3. The (mixed) risk-lovers’ behavior

We turn next to the case of a risk-lover who can voluntarily invest in self-insurance while the partial insurance coverage is mandatory. The only difference with the previous case lies in the utility function $U(W)$ which now reports a risk-loving behavior $(U''(W > 0))$. Once $p \geq q$, a risk-lover has no incentive to invest in a voluntary insurance activity. We show that, providing that a risk-lover does not voluntarily invest in insurance, were he committed to do so, he would reconsider his prevention behavior and would invest more in self-insurance. In that sense, a complementarity between both types of hedging arises. We present both a graphical reasoning and a formal proof.

Figure 1 illustrates the consequences of a partial insurance obligation on the risk-lover’s demand for self-insurance. In the plane $(W_1, W_2)$, the indifference curve of a risk-loving agent is concave with respect to the origin. Its slope, evaluated at point $(W_1, W_2)$ is $-\frac{(1-q)U'(W_1)}{qU'(W_2)}$. Therefore, the slope of any indifference curve at the intersection with the 45° line is equal to $-\frac{(1-q)}{q}$, which is also the slope of the actuarial insurance price line.
We start from an initial situation (point D whose coordinates are \((W_0, W_0-x_0)\)). When the unit price of insurance is at least actuarial \((p \geq q)\), a compulsory insurance scheme would cause a shift along a segment whose slope is \(\frac{1-p}{p}\), from initial point D to point D’.

The self-insurance technology is graphically represented by the wealth’s transformation curve DB whose slope is equal to \((1-SI'(a))\), as investing one more unit of money in self-insurance generates a marginal increase in the loss wealth amounting to \(SI'(a)\).

In figure 1, the indifference curve \(V(q, D)\) drawn through the initial situation D dominates all hedging opportunities: all insurance (or self-insurance) opportunities located on the curve DD’ (or DB respectively) worsen the risk-lover situation.

Therefore the shift from point D to point D’ induced by the compulsory insurance is illustrated by the shift to indifference curve \(V(q, D’)\), which reduces the well-being of the risk lover. As \(SI'(0)>1/q\), there is a value of “a” below which the marginal return of self-insurance is greater

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\(8\) At point D, the level of self-insurance is zero \((a = 0)\) and the maximum possible loss is therefore \(x(0) = x_0\).
than that of insurance. This, combined with a partial compulsory insurance coverage, makes the self-insurance investment profitable: foreseeing the welfare consequences of the shift from D to \( D' \), resulting from the compulsory insurance, the risk lover will exploit self-insurance opportunities.

As the insurance coverage has no impact on the self-insurance technology, the decision maker will implement it from \( D' \). In figure 1, starting from point \( D' \), the curve \( D'B' \) exhibits strictly the same opportunities as the initial wealth’s transformation curve \( DB \). Then, it is straightforward that a positive investment will be desired by the risk lover: along the “self-insurance curve” \( D'B' \), the shift from point \( D' \) to point \( F \) enhances her welfare since a higher indifference curve is reached, \( V(q, F) \) instead of \( V(q, D') \).

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Moreover, as shown in figure 1, it may exist an optimal positive investment in self-insurance for a risk lover: at point \( F \), the curve \( D'B' \) is tangent to the RL’s indifference curve. Such a case is plausible since SI’(a) is decreasing in “a” and SI’(0)>1/q. Indeed, the slope of \( D'B' \) increases from \((1-SI’(0))\textless \frac{(1-q)}{q}\) and if it increases faster than RL’s indifferences curves, such a tangency point is possible. Therefore, figure 1 shows that, combining self-insurance opportunities with a compulsory partial insurance, induces may induce a risk lover to voluntarily invest in prevention, whereas this was not the case without any insurance enforcement. Obviously, figure 1 remains an example and many other scenarios could arise such as full insurance or over insurance.

The graphical effect - a partial compulsory insurance encourages the risk-lover to invest more in prevention – comes from the policyholder’s program under compulsory insurance. It leads to equation (1.2) where the marginal cost and the marginal benefit of self-insurance\(^9\) can be identified and their behavior, after an increase in the level of compulsory insurance, studied:

\[
\frac{\partial EU}{\partial a} = -\{(1 - q) U'(W_0 - p\bar{I} - a) + qU'(W_0 - p\bar{I} - a - x_0 + SI(a) + \bar{I}) \} + \{SI'(a)qU'(W_0 - p\bar{I} - a - x_0 + SI(a) + \bar{I}) \} = 0
\] (1.5)

As both \( U''(W) \) and (1-p) are positive, the marginal benefit \( B_m = SI'(a)qU'(W_2) \) increases as the compulsory insurance level.

The impact of an increase in the compulsory insurance on the marginal cost \( C_m = (1-q) U'(W1) \) + qU'(W2) \) is more ambiguous: an increase in the insurance coverage reduces the wealth in state 1 but it increases that in state 2. Deriving the marginal cost with respect to \( \bar{I} \) provides equation (1.9): 

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\(^9\) This condition is supposed to characterize a maximum, which requires additional assumptions about the behavior of self-insurance. Graphically (see Figure 1), we obtain a maximum if the DB curve is more concave than the indifference curves of the risk-loving agent. This is far from unrealistic since it only needs the marginal return of the 1st unit of prevention to be high enough and to decrease thereafter.
\[
\frac{\partial C_m}{\partial I} = -p(1 - q)u''(W_0 - p\bar{I} - a) + (1 - p)q u''(W_0 - p\bar{I} - a - x_0 + SI(a) + I)
\]

(1.6)

When \( p \geq q \) and when the compulsory insurance is partial \((W_1 > W_2)\), this expression is unambiguously negative if \( u''(W) > 0 \). As shown by Crainich, Eeckhoudt, and Trannoy (2013), this assumption on the third derivative of the utility characterizes a precautionary behavior and is fully justified in the case of (mixed) risk-lovers. The marginal cost of self-insurance diminishes with the degree of the compulsory coverage.\(^{10}\) The following proposition is resulting:

**Proposition 3**: Whatever the initial level of self-insurance, an increase in the (partial) coverage of the compulsory insurance enhances the marginal benefit of self-insurance and reduces its marginal cost. In this case, the risk-lover is prone to invest more in prevention.

Returning to our main issue – compulsory insurance and secondary prevention – the lessons from Proposition 3 are straightforward. Starting from a situation where the risk-lover is already investing in prevention, the introduction of compulsory insurance leads her to increase her level of self-insurance.

By contrast, because the early self-insurance units are more productive than the unit price of insurance \((SI'(0) > 1/p)\), there is a compulsory coverage threshold at which a risk-lover who does not invest in prevention when insurance is voluntary, invests in self-insurance. For proof, just evaluate equation (1.8) when both self-insurance and insurance are nil \((a = 0 \text{ and } I = 0)\). In this case, the marginal cost of the first self-insurance units are above their marginal benefit:

\[
\{(1 - q)U'(W_0) + qU'(W_0 - x)\} > \{SI'(0)qU'(W_0 - x)\}
\]

By Proposition 3, we know that the introduction of compulsory insurance reduces the marginal cost (left term) and increases the marginal benefit (right term). At the other extreme, when the compulsory insurance is comprehensive \((\bar{I} = x(0))\) and always for \( a = 0 \), the comparison of the marginal cost and benefit leads to the following inequality:

\[
U'(W_0 - px) < SI'(0)qU'(W_0 - px)
\]

\[
\{U'(W_0 - px(0))\} < \{-x'(0)qU'(W_0 - px(0))\}
\]

Then, the marginal benefit of the 1\(^{st}\) unit of self-insurance is positive, as it was assumed that \(SI'(0) > 1/p\). Therefore, given the monotonous variations - and in opposite direction - of \(C_m\) and \(B_m\), there necessarily exists a coverage rate, between full risk retention and full coverage, at

\(^{10}\) By symmetry, if there is over-insurance, the marginal cost of prevention is increasing and the total effect of an increase in the insurance coverage on the marginal perception of prevention is not clear.
which the use of self-insurance becomes profitable to the risk-loving agent. These results are summarized in the following proposition:

**Proposition 4:**
Imposing a partial compulsory insurance to a risk-loving individual induces himself:
- Either to increase an initially pre-existing self-insurance investment;
- Or to invest in an initially non-existent self-insurance activity, provided that the mandatory coverage ratio exceeds a certain threshold.

4. **Conclusion:**

Whether or not insurance is compulsory, for risk-aversers, insurance and self-insurance are substitutes. Dealing with secondary prevention (self-insurance), public policies account for this substitutability property, since the generosity of the public (or/and compulsory) insurance coverage could be responsible for a decrease in self-insurance investments and correlativelly an increase in the risk exposure. Deductibles in health insurance system are therefore supposed to provide a significance counterbalance to such a mechanism: a partial insurance coverage is expected to give some incentives for both types of prevention, self-insurance and self-protection and therefore mitigate the crowding out effect of insurance on protection activities.

In a compulsory insurance context, this mechanism (the substitutability property and its regulation policy) no longer holds as our contribution highlights a complementary property for risk-lovers between compulsory insurance and self-insurance. Indeed, although a risk-lover invests neither in insurance\(^{11}\) nor in self-insurance activities (or at most partially) when insurance is voluntary, we show that a partial compulsory insurance induces him to supplement this coverage by voluntarily investing in self-insurance.

When insurance is compulsory, the coexistence of risk-lovers and risk-aversers may have therefore surprising implications for public policies. For the population as the whole, depending on the intensity of the effects and on the proportion of risk-lovers versus risk-aversers, a partial compulsory insurance coverage could result in a global increase in self-insurance. To illustrate this, suppose a competitive market for each voluntary hedging scheme and two groups of homogeneous individuals in the population: risk-aversers and risk-lovers. At equilibrium, risk-aversers optimally combine both types of risk hedging while risk-lovers invest, at most, in self-insurance depending on its return. Now, suppose a compulsory insurance coverage settled precisely at the optimum of the risk-aversers. While the self-insurance investment remains the same for this group, risk-lovers’ investment is enhanced. The global effect is therefore unambiguous: compulsory rather than voluntary insurance

\(^{11}\) When the unit price is at least actuarial \( p \geq q \).
results in a global increase of self-insurance investments. In this regard, our results mitigate the standard statement that insurance and self-insurance are substitutes.

Finally, our analysis brings an unexpected justification for compulsory insurance schemes. If such a regulation enables to overcome asymmetric information, as mentioned by Akerlof (1970) for example, its efficiency is generally challenged by its negative incentive effects on prevention. Our contribution mitigates this point of view and underlines that the nature of these incentive effects is risk-attitude-dependent. Moreover, a compulsory insurance scheme could, under some circumstances, increase the global level of self-insurance.

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