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Unemployment, Borrowing Constraints and Stabilization Policies

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Abstract

In this paper, we develop a tractable incomplete-market model with unemployment and borrowing constraints. We analyze the effects of fiscal and unemployment insurance policies in reaction to a large economic downturn. Policy instruments are government spending and the unemployment replacement rate. First, our incomplete-market model magnifies fluctuations in the unemployment rate after various shocks, compared to a standard complete-market model. Second, in response to a large shock that replicates the effects of a crisis, we find that government spending should increase substantially and that the replacement rate should drop. The sign and size of government interventions are quite different in a model with complete markets. Unemployment and borrowing constraints happen to matter quite a lot in determining the aggregate effects of large shocks, the design of optimized policies in response to these shocks and the corresponding welfare gains/losses.

Keywords: Unemployment, Borrowing constraints, Incomplete markets, Unemployment Insurance, Public Spending.

JEL Class.: D52, E21, E62, J64, J65.

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1 Introduction

Unemployment is a major issue in most developed economies, especially in the event of large economic downturns. The dynamics of the labor market during the recent Great Recession was particularly important in determining the dynamics of aggregate variables. In addition, the interaction between unemployment, heterogeneity and the distribution of wealth and marginal propensities to consume are important to understand the effects of large shocks. Addressing these interactions is also crucial to study the transmission of fiscal and monetary policy measures implemented to mitigate the effects of crisis episodes, and to account for their aggregate effects. The interaction between heterogeneity and stabilization policies received increased attention in the recent literature (see Auclert (2017) or Kaplan, Moll, and Violante (2016) and references therein). However, only a few models with heterogeneous agents and incomplete-markets consider unemployment as the main source of income fluctuations.[1] We lay-out a simplified heterogeneous-agent model to investigate the role of unemployment and borrowing constraints in the diffusion of large macroeconomic shocks and in the design of optimized stabilization policies.

The model features three types of agents – employed workers, unemployed workers and firm owners – with potentially restricted access to two classes of assets: government bonds and capital.[2] Firm owners are not financially constrained, and workers have access to government bonds only. The steady-state distribution of wealth is non-degenerate and allows for rich savings dynamics. In particular, we show that, in equilibrium, employed workers are not constrained while unemployed workers are constrained. Further, the proportions of employed and unemployed workers are endogenous and determined by a search and matching environment, which introduces a tight connection between borrowing constraints and unemployment dynamics.

In this model, we first show that large macroeconomic shocks produce amplified fluctuations of labor market quantities compared to a standard family model in which households pool their resources to achieve an identical level of consumption. We trace the higher sensitivity of matches to the number of vacancies to what Ljungqvist and Sargent (2017) call the fundamental surplus. Indeed, in the incomplete-market model, unemployed workers receive a lower income and borrowing constraints induce consumption inequalities between employed and unemployed workers. Those contribute to raise the value of the match surplus and the steady-state real wage, compared to a complete-market model. The fundamental match surplus is thus lower, which makes matches more sensitive to the number of vacancies, and amplifies movements in labor market quantities. Borrowing constraints, unemployment and the resulting consumption inequality between employed and unemployed workers are crucial in shaping the fundamental surplus. Quantitatively speaking, the amplification effect is very large.

[1] Recent notable exceptions are Krusell, Mukoyama, and Şahin (2010), Ravn and Sterk (2016) or Challe (2017).

[2] The model is in the spirit of Challe, Matheron, Ragot, and Rubio-Ramirez (2017) and provides a rationale for a much larger impact of macroeconomic shocks on key labor market variables.

In addition, we show that unemployment dynamics in an incomplete-market model crucially matters for the sign and size of optimized government interventions to stabilize the economy. We consider a large rise in the vacancy posting cost to generate a crisis (see Auray, Eyquem, and Gomme (2017) for a discussion), and show that the optimized response of the government is to raise public spending and to lower the replacement rate (the opposite of unemployment benefits extension programs). The latter policy has very large positive effects on the labor market as it raises the value of a match and therefore dampens the rise in unemployment induced by the shock. Public spending policies have relatively less effects, at least quantitatively. In addition, the policy prescription regarding public spending is opposite in the family model: the government should lower public spending quite substantially. Whenever an optimized UI policy is considered, the replacement rate should be lowered in all cases, but much more in the family model than in the incomplete-market model. Movements in the replacement rate have much larger effects on the dynamics of the real wage and vacancies, with stronger propagation to the unemployment rate. So less action is required in the incomplete-market model. Finally, our analysis also shows that the welfare effects of a shock producing a large economic downturn are deeply affected by the presence of endogenous unemployment fluctuations combined with financial constraints.

The rest of the paper is organized as follows. The model is described and discussed in details in Section 2. Section 3 proposes a first insight into the model to grasp the main mechanism at work and characterize the dynamics of our model after large negative shocks without government intervention. Section 4 discusses the design of optimized spending and unemployment benefits policies in response to a large adverse shock and Section 5 compares these policies with optimized policies derived in a complete-market (family model). Section 6 offers some concluding remarks.

2 Model

The model features three types of households: employed workers, unemployed workers and firm owners. As will be clear, unemployed workers are financially constrained while employed workers pool their resources. Firm owners save, accumulate capital and rent it to the firms, and receive profits. The rest of the model is a standard search and matching framework with Nash bargained wages that will set the stage for the endogenous dynamics of the unemployment rate, that affects the composition of the household sector and the extent of liquidity-constrained households. Finally, we introduce a government sector that levies distortionary taxes on labor and capital income and a tax on consumption.

2.1 Households

The model follows Challe et al. (2017) for the heterogeneous-agent model structure but is somehow simplified. The economy is populated with a unit size continuum of households, a proportion $\omega \in [0, 1]$ of workers that can either be employed or not and a proportion $(1 - \omega)$ of firm owners that accumulate capital, rent it to the firm and receive profits from production. Resources are pooled among employed workers, which greatly simplifies the distribution of wealth, as each employed worker will be attributed the same individual level of wealth independently of its prior employment status. In addition, unemployed workers are either liquidity constrained (whenever they were previously unemployed) or liquidate their wealth (whenever they were employed in the previous period). A condition for the liquidity constraint to be binding in equilibrium is to consider that workers are relatively impatient compared to firm owners, *i.e.* their respective subjective discount factors satisfy $0 < \beta^w < \beta^f < 1$.

Workers. Household i that belongs to the category of workers maximizes the following welfare index

$$\mathrm{E}_t \left\{ \sum_{s=t}^{\infty} (\beta^w)^{s-t} \, u\left(c_s^i, h_s^i, g_s\right) \right\} \tag{1}$$

where β^w is the subjective discount factor, $c_{i,t} > 0$ is the consumption of household i, $h_{i,t}$ stands for the number of hours worked and g_t is the aggregate amount of public spending. The budget constraint of household i is:

$$a_t^i + (1 + \tau_{c,t}) \, c_t^i = (1 + r_{t-1}) \, a_{t-1}^i + (1 - \tau_{n,t}) \left(\varepsilon_t^i w_t h_t^i + \left(1 - \varepsilon_t^i\right) b_t h \right) - T_t^i, \;\; a_t^i > 0 \tag{2}$$

where a_t^i is the household's wealth and r_{t-1} the risk-free return on bonds between periods $t-1$ and t. Variable $\varepsilon_t^i = \{0, 1\}$ gives the employment status of the household: when $\varepsilon_t^i = 1$, the household is employed, works h_t^i hours paid at the hourly real wage w_t; when $\varepsilon_t^i = 0$, the household is unemployed and receives an unemployment insurance b_t and spends $\overline{h}$ hours searching – where h is the steady state level of hours worked. Labor income is taxed at the rate $\tau_{n,t}$ and consumption is taxed at $\tau_{c,t}$. Variable T_t^i is a lump-sum tax. Finally, we consider that the unemployment benefits are

$$b_t = b_t^r w \tag{3}$$

where $b_t^r < 1$ is the (potentially time-varying) replacement rate and w the steady-state real wage. The number of employed workers in the economy and the unemployment rate are tied by $n_t + u_t = 1$. The number of employed workers is further given by the following equation

$$n_t = (1 - s) \, n_{t-1} + m_t \text{ where } m_t = \psi u_t^{\mu} v_t^{1-\mu} \tag{4}$$

where s is the (exogenous) separation rate, m_t is the number of matches and v_t the number of vacancies. The job-finding rate f_t and the job-filling rate λ_t are respectively defined as

$$f_t = m_t / u_t \text{ and } \lambda_t = m_t / v_t \tag{5}$$

As a consequence, the number of workers employed at time t includes unseparated matched workers and newly matched workers $n_t = (1 - s) \, n_{t-1} + f_t u_t$, the number of recently unemployed workers is sn_{t-1} and the number of unemployed workers remaining unemployed is $(1-f_t)\,u_t$. We follow the approach of Challe et al. (2017) and consider that employed workers pool their resources together to achieve the same level of individual consumption and therefore next-period wealth.[3] The corresponding Euler equation on bonds writes

$$E_t\left\{\beta^w\left(1+r_t\right)\frac{\left(1+\tau_{c,t}\right)}{\left(1+\tau_{c,t+1}\right)}\frac{\left(1-s\right)u_c\left(c^e_{t+1},h_{t+1},g_{t+1}\right)+su_c\left(c^u_{t+1},h,g_{t+1}\right)}{u_c\left(c^e_t,h_t,g_t\right)}\right\}=1 \tag{6}$$

where c^e_t and c^u_t denote the per capita consumption levels of employed and unemployed workers respectively. Given that $s>0$ and $u_c\left(c^e_{t+1},h_t,g_t\right)<u_c\left(c^u_{t+1},h,g_t\right)$, this equation shows the precautionary motive that arises due to the risk of unemployment. Employed workers face a potentially decreasing consumption dynamics driven by the risk of income loss, that pushes them to save to self-insure through savings. Further, the budget constraint of employed workers is

$$\left(1+\tau_{c,t}\right)c^e_t+\frac{a^e_t}{n_t}=\left(1-s\right)\left(1+r_{t-1}\right)\frac{a^e_{t-1}}{n_t}+\left(1-\tau_{n,t}\right)w_th_t-T^e_t \tag{7}$$

where a^e_{t-1} denotes the total asset held by employed workers in the previous period – of which only a fraction $(1-s)$ remains, the complement being liquidated by recently unemployed workers. In addition, we consider an intensive margin of labor supply: employed workers optimally choose the number of hours worked according to:

$$-\frac{u_h\left(c^e_t,h_t,g_t\right)}{u_c\left(c^e_t,h_t,g_t\right)}=\frac{\left(1-\tau_{n,t}\right)}{\left(1+\tau_{c,t}\right)}w_t \tag{8}$$

Unemployed workers are liquidity constrained and hold zero assets, *i.e.* $a^u_t=0$.[4] Their current-period per-capita consumption consists in their income (unemployment benefits) if they were already unemployed or their income plus the liquidation of their past wealth if they just became unemployed, which happens with probability s. Hence, the aggregate budget constraint for the unemployed workers writes:

$$\left(1+\tau_{c,t}\right)c^u_t=s\left(1+r_{t-1}\right)\frac{a^e_{t-1}}{u_t}+\left(1-\tau_{n,t}\right)b_th-T^u_t \tag{9}$$

Firm owners. Firm owners maximize a welfare index

$$\mathrm{E}_t\left\{\sum_{s=t}^{\infty}\left(\beta^f\right)^{s-t}u\left(c^f_s,g_s\right)\right\} \tag{10}$$

[3]A consequence is that the level of hours worked for these households is $h^e_t=h_t$.

[4]The condition for this to hold is for the Euler equation residual of recently unemployed:

$$E_t\left\{\beta^w\left(1+r_t\right)\frac{\left(1+\tau_{c,t}\right)}{\left(1+\tau_{c,t+1}\right)}\frac{f_tu_c\left(c^e_{t+1},h^e_{t+1},g_t\right)+\left(1-f_t\right)u_c\left(c^u_{t+1},h,g_t\right)}{u_c\left(c^u_t,h,g_t\right)}-1\right\}$$

to be negative, where c^u_t denotes the consumption of unemployed workers. It can be shown very easily that this inequality holds under very loose conditions.

subject to the following budget constraint

$$(1+\tau_{c,t})\,c_t^f + a_t^f + k_t = (1+r_{t-1})\,a_{t-1}^f + \left(1+(1-\tau_{k,t})\left(r_t^k-\delta\right)\right)k_{t-1} + \frac{\varphi_t}{1-\omega} - T_t^f \qquad (11)$$

In addition to the liquid asset (bonds), firm owners have access to an illiquid asset (capital) that is rented to the firm at a gross rate r_t^k, is subject to taxation $\tau_{k,t}$ and depreciates at rate δ. As owners of the firm, they also receive a profit φ_t. Firm owners potentially face borrowing constraints $a_t^f \geq 0$ and $k_t \geq 0$ but those will not be binding in equilibrium. The first-order conditions for firm owners are

$$E_t\left\{(1+r_t)\,\Delta_{t,t+1}\right\} = 1 \qquad (12)$$

$$E_t\left\{\left(1+(1-\tau_{k,t+1})\left(r_{t+1}^k-\delta\right)\right)\Delta_{t,t+1}\right\} = 1 \qquad (13)$$

where

$$\Delta_{t,t+1} = \beta^f\,\frac{(1+\tau_{c,t})}{(1+\tau_{c,t+1})}\,\frac{u_c\left(c_{t+1}^f\right)}{u_c\left(c_t^f\right)} \qquad (14)$$

which prices both assets.[5]

2.2 Production and wage determination

The firm posts vacancies v_t paying a unit vacancy cost κ_t, out of which a fraction ($\lambda_t = m_t/v_t$) will be filled and accrue the total number of employed workers. It also rents capital k_{t-1}^d to firm owners to produce goods with the following technology:

$$y_t = z_t\left(k_{t-1}^d\right)^{\alpha}(\omega n_t h_t)^{1-\alpha} \qquad (15)$$

Firms first order condition with respect to capital is simply:

$$r_t^k = \alpha y_t / k_{t-1}^d \qquad (16)$$

In addition, the marginal value of a filled position is:

$$J_t = (1-\alpha)\,y_t/n_t - \omega w_t h_t + E_t\left\{\Delta_{t,t+1}\,(1-s)\,J_{t+1}\right\} \qquad (17)$$

where $(1-\alpha)\,y_t/n_t - \omega w_t h_t$ is the net contribution of the marginal worker, his marginal product less his wage bill. The value of a position remaining vacant is

$$V_t = -\kappa_t + E_t\left\{\Delta_{t,t+1}\left((1-\lambda_t)\,V_t + \lambda_t J_{t+1}\right)\right\} \qquad (18)$$

[5]Notice that $\beta^w < \beta^f$ has to be adjusted in the steady state for the Euler equation on bonds for employed workers to be consistent with the Euler equation on bonds for firm owners.

where $\kappa_t = \kappa \phi_t$ with ϕ_t following an AR(1) process. As usual in the search and matching framework, we assume that the free entry condition $V_t = 0$ holds.[6] The profits made by the firm are

$$\varphi_t = y_t - \omega n_t w_t h_t - r_t^k k_{t-1}^d - \kappa_t v_t \tag{19}$$

and are transferred to firm owners.

The real wage is determined as the solution to a Nash bargaining problem that maximizes a geometric average of workers and firm job surpluses

$$w_t = \arg\max \left(V_t^e - V_t^u\right)^\theta J_t^{1-\theta} \tag{20}$$

where θ is the bargaining power of workers, V_t^e is the value of being employed and V_t^u the value of being unemployed, respectively given by

$$V_t^e = u_c\left(c_t^e, h_t, g_t\right) \frac{1-\tau_{n,t}}{1+\tau_{c,t}} w_t h_t + u_h\left(c_t^e, h_t, g_t\right) + \beta^w E_t\left\{(1-s) V_{t+1}^e + s V_{t+1}^u\right\} \tag{21}$$

$$V_t^u = u_c\left(c_t^u, h, g_t\right) \frac{1-\tau_{n,t}}{1+\tau_{c,t}} b_t + u_h\left(c_t^u, h, g_t\right) + \beta^w E_t\left\{(1-f_t) V_{t+1}^u + f_t V_{t+1}^e\right\} \tag{22}$$

The first equation states that the current value of being employed is the utility of the current net labor income minus the current disutility of working plus the discounted value associated with the future status. The second equation states that the value of being unemployed is the utility of the current unemployment benefit minus the disutility of searching plus the discounted value associated with the future status. Our assumptions, in particular the fact that $b_t^r < 1$ imply that $V_t^u < V_t^e$ always. The Nash bargaining solution to this problem implies

$$\omega(1-\theta)\left(V_t^e - V_t^u\right) = \frac{1-\tau_{n,t}}{1+\tau_{c,t}} \theta J_t \tag{23}$$

which, combining with the expressions of V_t^e, V_t^u and J_t yields:

$$w_t = \frac{\omega(1-\theta)\left(u_c\left(c_t^u, h, g_t\right) b_t + (1+\tau_{c,t}) u_h\left(c_t^u, h, g_t\right) / (1-\tau_{n,t})\right) + \theta\left((1-\alpha) y_t / n_t + E_t\left\{\Omega_{t+1}\right\}\right)}{\omega(1-\theta) u_c\left(c_t^e, h_t, g_t\right)(h_t - 1) + \theta \omega h_t} \tag{24}$$

with $\Omega_{t+1} = \left(\Delta_{t,t+1}(1-s) - \beta^w (1-s-f_t)\right) J_{t+1}$. This expression shows that the real wage is determined as an average between the current value received when a worker is unemployed, *i.e.* the lower bound of the bargaining set, and the product of the marginal worker, *i.e.* the upper bound of the bargaining set. These arguments are weighted by θ, the bargaining power of workers.

[6] It implies $E_t\left\{\Delta_{t,t+1} \lambda_t J_{t+1}\right\} = \kappa_t$.

2.3 Government, aggregation and equilibrium

The government purchases public goods g_t and provides unemployment insurance to the unemployed workers. It finances this stream of expenditure using labor and capital income taxes as well as the consumption tax and a lump-sum tax. The government also issues public debt – one-period bonds paying r_t between t and $t+1$ so that its budget constraint writes:

$$\begin{aligned} b_t^g = (1+r_{t-1})\, b_{t-1}^g + g_t \quad &+ \quad (1-\tau_{n,t})\,\omega u_t b_t h \\ &- \quad \left(\tau_{c,t} c_t + \tau_{n,t}\omega n_t w_t h_t + (1-\omega)\,\tau_{k,t}\left(r_t^k - \delta\right) k_{t-1}\right) - T_t \end{aligned} \quad (25)$$

where

$$c_t = \omega\,(n_t c_t^e + u_t c_t^u) + (1-\omega)\, c_t^f \quad (26)$$

The market clearing conditions on the bonds market, the market for capital and the market for goods and services are respectively

$$\begin{aligned} b_t^g &= \omega a_t^e + (1-\omega)\, a_t^f && (27) \\ k_t^d &= (1-\omega)\, k_t && (28) \\ y_t &= c_t + (1-\omega)\,(k_t - \delta k_{t-1}) + g_t + \kappa_t v_t && (29) \end{aligned}$$

Due to Walras law, the last equilibrium condition is redundant with the aggregation of the budget constraints (employed, unemployed workers and firm owners) consolidated with the government budget constraint. An equilibrium in this economy is defined as a situation where, for a given path of fiscal policy instruments $\mathcal{F}_t = \{g_t, \tau_{n,t}, \tau_{k,t}, \tau_{c,t}, b_t^r, T_t\}$: (i) for a given path of prices, households satisfy their optimality conditions and budget constraints, the firm optimizes and the government balances its budget, and (ii) for a given path of quantities, prices adjust so that markets clear.

3 First insight into the model

Before we run further experiments with this model, we contrast the dynamics implied by various shocks in the model with the effects of the same shocks within a standard complete-market model where households are all members of the same family and pool their resources together to achieve the same level of consumption, *i.e.* $c_t^e = c_t^u = c_t^f = c_t$. In addition, household-specific Euler equations and budget constraints are out of the picture and discount factors are now identical: $\beta^w = \beta^f$.

3.1 Set-up and calibration

Set-up. In this section, public spending and UI benefit policies are passive as $\{g_t, b_t^r\} = \{g, b^r\}$. In addition, we assume that the government keeps distortionary taxes constant $\tau_{n,t} = \tau_n$, $\tau_{k,t} = \tau_k$

and $\tau_{c,t} = \tau_c$, and uses lump-sum taxes to finance its policies. The latter are paid by firm owners only, and the government uses the following feedback rule to stabilize debt dynamics:[7]

$$T_t = d_T \left(\frac{b^g_{t-1}}{4y_{t-1}} - \frac{b^g}{4y} \right) \tag{30}$$

Calibration for production and the households. The model is quarterly. Following Gomme and Rupert (2007), the capital depreciation rate is 7% annually ($\delta = 0.018$) and the capital share is $\alpha = 0.3$. Following Challe et al. (2017) the proportion of workers is $\omega = 0.6$. In addition, because our model produces a non-degenerate distribution of wealth, we impose that the share of liquid (bond) wealth held by the 60% poorest households (workers in our case) is 5%. This pins down the distribution of bond holdings in the steady state and contributes to determine the individual levels of final good consumption. The implied level of consumption of employed workers is 39.8% larger than the level of consumption of unemployed workers. The discount factor of firm owners is $\beta^f = 0.99$, which pins down the risk-less interest rate at 4.1% annually. The discount factor of workers is adjusted for Equation (6) to hold. We assume that the utility function of firm owners is $u(c^f_t) = \log(c^f_t) + \gamma \log(g_t)$ and that the utility functions of workers are respectively $u(c^e_t, h_t) = \log(c^e_t) + \gamma \log(g_t) - \chi h_t^{1+\xi} / (1+\xi)$ for employed workers and $u(c^u_t, h_t) = \log(c^u_t) + \gamma \log(g_t) - \chi h^{1+\xi} / (1+\xi)$ for unemployed workers. Public spending are introduced in the utility function of agents to introduce meaningful policy trade-offs in case of shocks. We fix the Frisch elasticity of labor supply at $1/\xi = 0.5$ and the labor disutility parameter χ is adjusted to get $h = 1$ in the steady state.

Calibration for the government. We analyze the dynamics of European economies. We use European averages for distortionary tax rates and set $\tau_c = 0.15$, $\tau_n = 0.3$ and $\tau_k = 0.4$. The baseline replacement rate is relatively high, at $b^r = 0.6$ and public spending to GDP are set at $g/y = 0.27$. These numbers will give rise to a 101.39% debt-to-annual-GDP ratio. We set the lump-sum tax rule parameter at $\phi = 0.25$ to ensure the stability of debt dynamics. Finally, the government spending utility weight γ is calibrated so that, for our baseline calibration of g, the marginal utility of public spending equals the weighted average of marginal utilities of consumption, an application of "Samuelson's principle". It gives $\gamma = 0.5916$.

Calibration for the labor market. On the labor market, we also try to replicate key European statistics. We set the elasticity of matches with respect to vacancies at $\mu = 0.25$ and impose that the bargaining power of firms is equal to this elasticity $\theta = \mu$. The quarterly separation rate is $s = 0.025$, the job-finding rate is $f = 0.25$ and the job-filling rate is $\lambda = 0.8$. To hit those targets, we adjust the matching efficiency parameter $\psi = 0.5981$ and the vacancy posting costs $\kappa = 1.0857$.

[7] The following results are qualitatively robust to a feedback rule using the labor income tax rate instead of lump-sum taxes, see Appendix A.

Those numbers are reported in Table 1 below.

Table 1: Parameter values

Proportion of workers in population	$\omega = 0.6$
Discount factor (firm owners)	$\beta^f = 0.9900$
Discount factor (workers)	$\beta^w = 0.9802$
Share of liquid wealth held by the bottom 60%	$\omega a^e / \left(\omega a^e + (1-\omega)\, a^f\right) = 0.05$
Capital depreciation rate	$\delta = 0.018$
Elasticity of production to the capital stock	$\alpha = 0.3$
Frisch elasticity of labor supply	$1/\psi = 0.5$
Labor disutility parameter	χ adjusted to get $\overline{h} = 1$
Consumption tax rate	$\tau_c = 0.15$
Labor income tax rate	$\tau_n = 0.3$
Capital income tax rate	$\tau_k = 0.4$
Baseline public spending in GDP	$g/y = 0.27$
Utility weight of public spending	$\gamma = 0.5916$
Baseline replacement rate	$b^r = 0.6$
Separation rate	$s = 0.025$
Job-finding rate	$f = 0.25$
Job-filling rate	$\lambda = 0.8$
Elasticity of matches to unemployment	$\mu = 0.25$
Bargaining power of workers	$\theta = \mu$
Matching efficiency	$\psi = 0.5981$
Vacancy posting cost	$\kappa = 1.3135$

Up to slight differences, the calibration of the family model is pretty much the same. The absence of heterogeneity implies a higher discount factor for the workers, which, all else equal would raise the steady-state value functions of employed and unemployed workers (and thus the Nash bargained wage). However, the resource pooling also shuts down the possibility of consumption inequality between employed and unemployed workers. The contribution of workers to the match surplus is lowered, which results in a lower equilibrium real wage. The latter also hurts the income of unemployed workers, since the replacement rate applies to a smaller steady-state real wage. As a result, the value functions of workers (employed or not) are smaller in the family model compared to the baseline model. Hence, the family model produces a lower level of aggregate consumption and a lower level of labor income, which results in reduced fiscal receipts and lower sustainable debt in the steady state (20.94% of annual GDP in the family model against 101.39% in the incomplete-market model). These difference require an adjustment of the steady-state value of the vacancy posting cost κ (from 1.3135 in the baseline model to 4.38 in the family model) to hit the same targets on the labor market. Other variables and parameters are the same in both models. Table 2 below reports the steady state allocations produced by each model for the sake of clarity.

We now contrast the dynamics of the economy in the baseline model and in the family model after (i) a 5% negative productivity shock and (ii) a 25% positive shock on the cost of posting vacancies. The first one is to understand the dynamics of the model under the preferred business

Table 2: Steady state allocations

	Baseline model	Family model
Output	1.3729	1.3729
Capital	29.5758	29.5758
Investment	0.2127	0.2127
Employment	0.9091	0.9091
Hours worked	1.0000	1.0000
Aggregate consumption	0.7522	0.6651
Unemployed consumption	0.7205	0.6651
Employed consumption	1.0072	0.6651
Firm owners consumption	0.4087	0.6651
Real wage	1.6658	1.4416
Public spending	0.3707	0.3707
Debt to annual GDP ratio	1.0139	0.2094
Unemployment rate	0.0909	0.0909
Replacement rate	0.6000	0.6000
Job-finding rate	0.2500	0.2500
Job-filling rate	0.8000	0.8000
Real interest rate	0.0101	0.0101
Vacancies	0.0284	0.0284

cycle driving force of the literature. The second one is argued by Auray, Eyquem, and Gomme (2017) to be the main driving force behind the recent Great Recession. This interpretation is consistent both with results offered by Brinca, Chari, Kehoe, and McGrattan (2016) in their business cycle accounting approach to the Great Recession, and with the view that financial factors at the firm level are behind the significant rise in the cost of posting vacancies (or the cost of creating jobs).

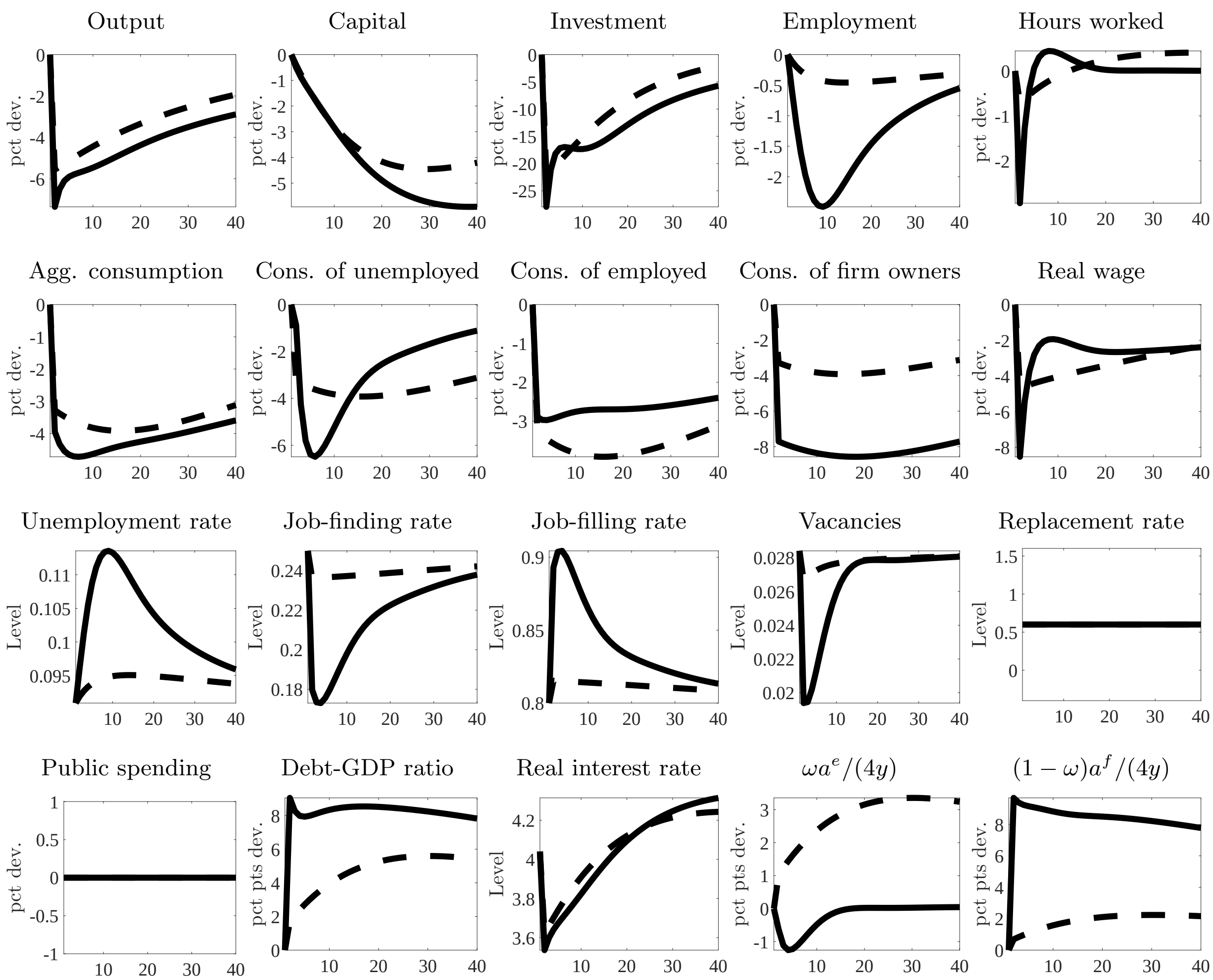
We start with the analysis of a negative productivity shock and report the Impulse Response Functions (IRFs hereafter) in Figure 1. The model is solved non-linearly under perfect foresight using a Newton-type algorithm.[8]

A negative productivity shock has expected consequences: output, consumption, employment, hours worked and investment all fall together. The marginal productivity of labor falls, which decreases the value of matches and results in both a lower bargained real wage and a lower number of vacancies posted. As a result the number of matches falls, consistently with the fall in employment and with the rise in unemployment. Tax bases shrink which makes public debt rise. As output is falling as well, the debt-to-GDP ratio surges.

Qualitatively, these dynamics are similar for both models. Quantitatively however, the two models display major differences. The rise in the unemployment rate is much larger in the

[8]The algorithm is a built-in routine of Dynare. It is an application of the Newton-Raphson algorithm that takes into consideration the special structure of the Jacobian matrix in dynamic models with forward-looking variables. The details of the algorithm are explained in Juillard (1996).

Figure 1: IRFs to a 5% negative productivity shock



Solid: baseline model. Dashed: Family model.

incomplete-market model. We attribute this result to the higher sensitivity of matches to vacancies and to the smaller "fundamental" surplus in our model. Indeed, as shown by Ljungqvist and Sargent (2017), what they call the fundamental surplus happens to drive the sensitivity of matches to vacancies in models with matching frictions: when the surplus is low, the sensitivity of matches to vacancies is large, which makes unemployment vary more in response to perturbations of the economy. Table 2 shows that the baseline and family models differ in the equilibrium level of the real wage – due to consumption inequality – but produce the same level of output. The fundamental surplus is defined as the difference between the productivity of the marginal worker minus his marginal cost, the real wage. The productivity of the marginal worker is basically the same under both models but the real wage is larger in our incomplete-market model, which makes the fundamental surplus smaller, and therefore matches more sensitive to vacancies.

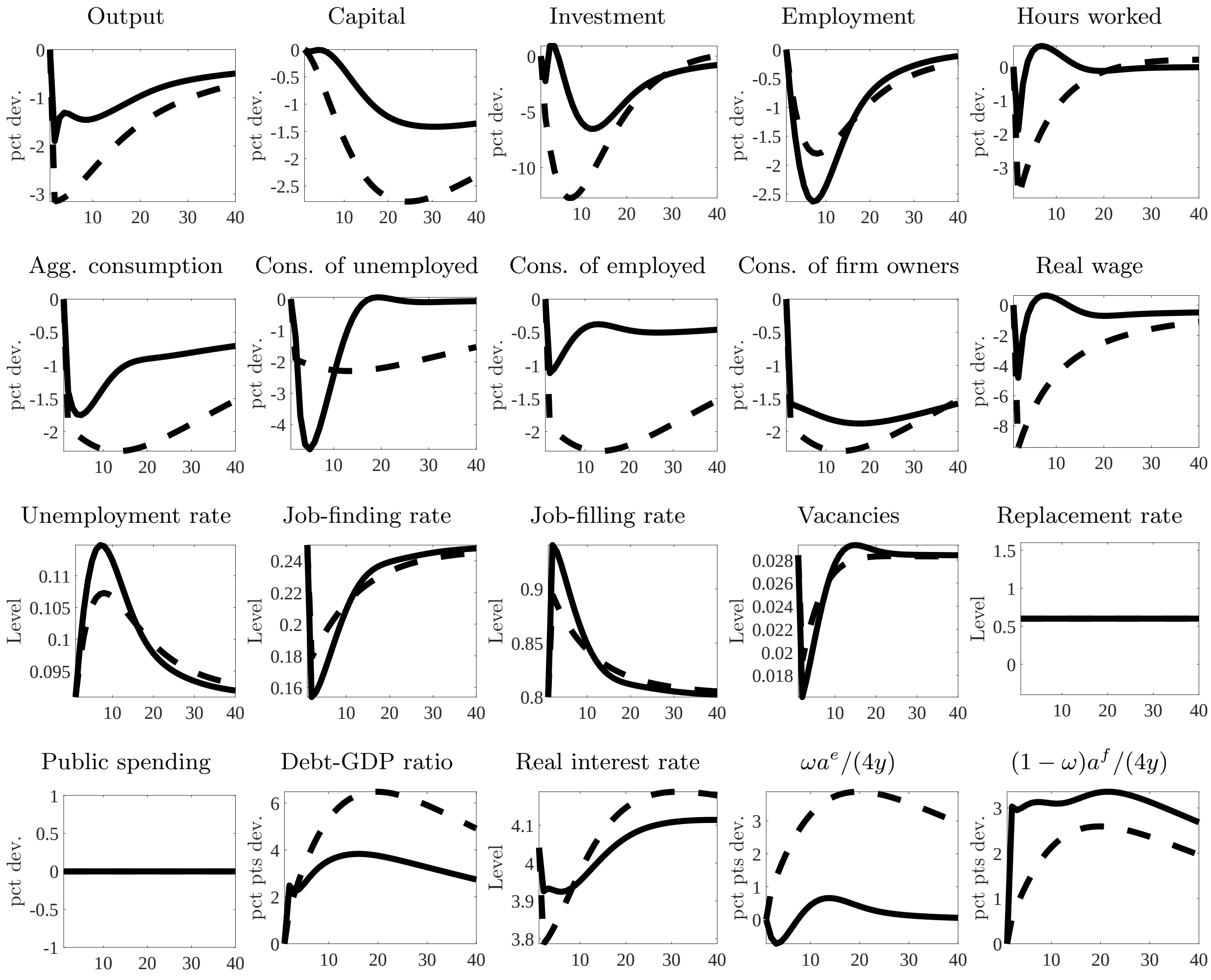
Further, the dynamics of inequalities contributes to magnify the response of the unemployment rate compared to the family model. Indeed, in the model with incomplete markets, unemployed workers face a much deeper fall in their consumption while employed workers face a lower fall in consumption. Since the marginal productivity of capital falls, the real interest rate on bonds falls too. This helps employed workers smooth their consumption by lowering their bond holdings.[9] This difference in consumption dynamics between unemployed and employed workers is, by definition, absent in the family model but shapes the dynamics of workers contribution to the match surplus. The latter falls more than in the family model, and pushes the number of matches further down.

These larger fluctuations in unemployment and employment in the baseline model also induce larger movements in GDP and consumption. The same 5% persistent drop in productivity lowers GDP by roughly 5% in the family model while it falls by more than 6% in the baseline model. Employment falls by more than 2% against only 0.5% in the family model, and unemployment reaches almost 11.5%, roughly 2.5 pp above steady-state, instead of "only" 9.5% in the family model (0.5 pp above the steady state). Incomplete markets, borrowing constraints and the resulting consumption inequalities are thus key to the dynamics of the real wage and thus to the dynamics of unemployment, with feedback effects on the dynamics of unemployment and aggregate macroeconomic variables.

Is this amplification effect general or specific to productivity shocks? To answer this question, we now contrast the responses of the baseline and family models after a vacancy posting cost shock in Figure 2. The way this shock affects the economy is quite different, at least in the core mechanism. Through the free entry condition, the vacancy posting shock produces a mix of a

[9] If the real interest rate remained constant, the precautionary motive attached with the higher probability of receiving a low income due to unemployment would dominate: the savings of employed workers would rise and their consumption fall more, amplifying the recession.

Figure 2: IRFs to a 25% positive vacancy posting cost shock



Solid: baseline model. Dashed: Family model.

rise in the marginal value of a match for the firm J_t and a rise in the job-filling rate, resulting in an important drop in the number of vacancies posted. The fall in vacancies results in less matches, less employment, more unemployment, and less output. In turn, the decrease in output triggers a fall in consumption due to the diminishing real wage, and a drop in investment. Tax bases shrink, pushing the debt level and the debt-GDP ratio to rise. However, contrary to the case of a productivity shock, the fall in output is somehow attenuated by the reduction in total vacancy costs. Indeed, the fall in vacancies is large and more than proportional to the rising unit cost, so total vacancy costs $\kappa_t v_t$ fall. This helps smooth the dynamics of output, as total vacancy costs are paid in units of output and appear in the goods market clearing condition.

As in the previous case, the shock produces larger fluctuations in employment, unemployment and magnifies the response of vacancies compared to the family model. However, precisely because vacancies fall much more in the baseline model, the magnification effect applies to the extensive margin of quantities on the labor market (employment) only. It does not spread to GDP, consumption and then to other real variables such as debt-to-GDP or the real interest rate.

Finally, Appendix A reports the very same impulse responses when the government uses distortionary labor taxation to ensure long-run fiscal solvency instead of a lump-sum tax rule. Our results are unaffected from a qualitative perspective.

4 Optimized public spending and UI policies

In this section, the baseline economy is hit by a large vacancy posting shock. The baseline calibration is unchanged. Public spending and UI benefit policies $\{g_t, b_t^r\}$ are potentially conducted based on simple rules by which government spending and the replacement rate respond to both the lagged deviations of output from its steady state value, and the lagged deviations of the unemployment rate:

$$g_t = g + d_{gy}\left(y_{t-1} - y\right) + d_{gu}\left(u_{t-1} - u\right) \tag{31}$$

$$b_t^r = b^r + d_{ru}\left(u_{t-1} - u\right) + d_{ry}\left(y_{t-1} - y\right) \tag{32}$$

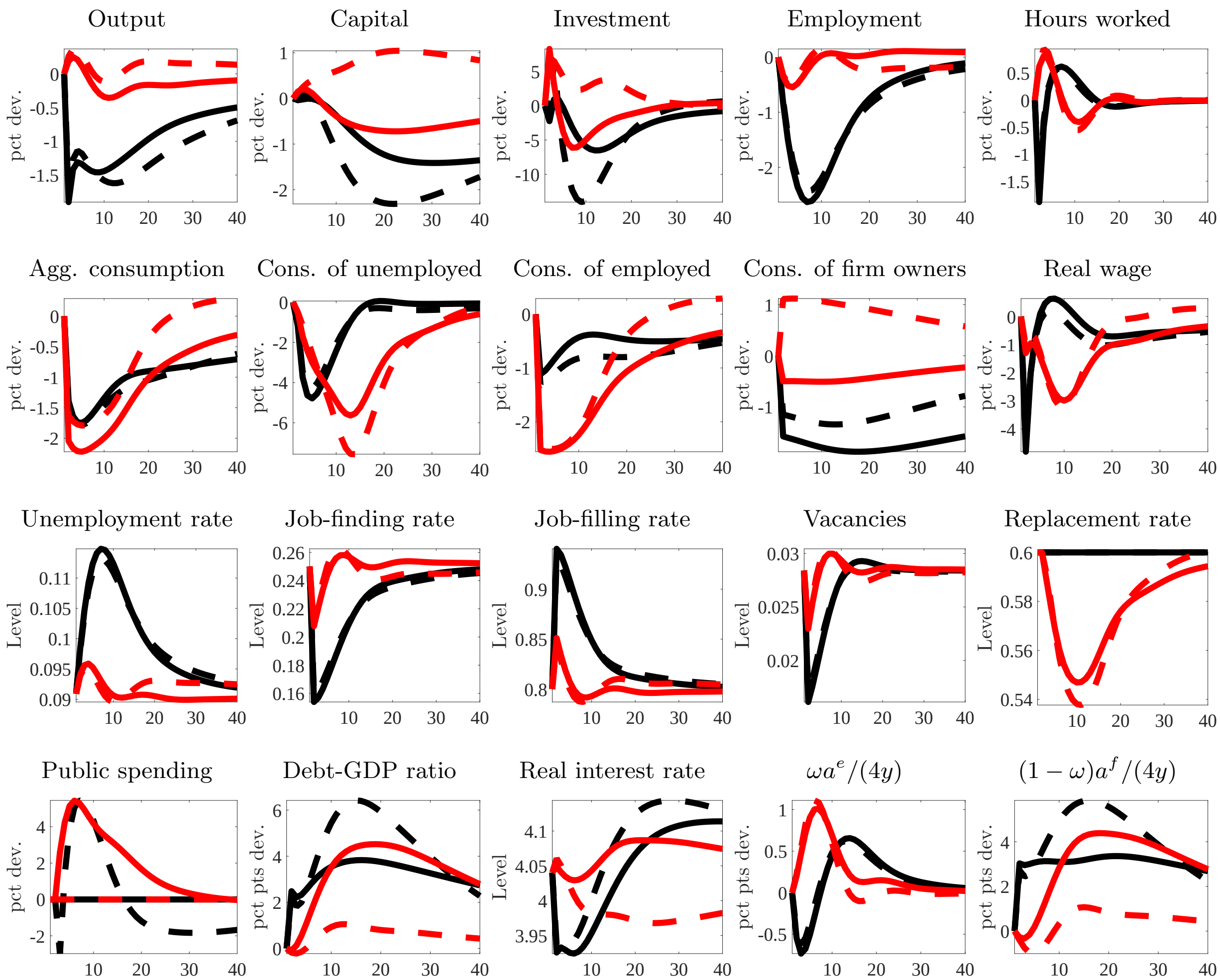
We consider four cases: (i) an equilibrium with no policy response, (ii) an optimized public spending rule along with a constant replacement rate (iii) an optimized replacement rule with constant public spending, and (iv) an equilibrium with optimized spending and replacement rules. Optimized rules are obtained by selecting (a subset or all of) the elasticities d_{gy}, d_{gu}, d_{ru} and d_{ry} so as to minimize the welfare losses from the shock. The welfare metric η is the Hicksian consumption equivalent that solves:

$$E_0 \sum_{t=0}^{\infty} \widetilde{\beta}^t \left(\sum_i \varpi_t^i u\left(c_t^i, h_t^i, g_t\right) - \sum_i \varpi^i u\left(c^i\left(1+\eta\right), h^i, g\right) \right) = 0 \tag{33}$$

where $\widetilde{\beta} = \omega\beta^w + (1-\omega)\,\beta^f$ and where ϖ_t^i represent population weights, *i.e.* $\varpi_t^e = \omega n_t$, $\varpi_t^u = \omega u_t$ and $\varpi_t^f = 1 - \omega$.[10]

Figure 3 shows the responses produced by our 4 different policy set-ups. The solid black line represents the equilibrium with passive government policies and is isomorphic to the dynamics reported in Figure 2, so we do not discuss this case any further.

Figure 3: IRFs to a 25% positive vacancy posting cost shock with active policies



Solid black: no policy. Dashed black: Public spending. Dashed red: Replacement rate. Solid red: Public spending and replacement rate

[10]Recall that we have imposed $h_t^u = h$ and $h_t^f = 0$. In addition, our assumptions imply that $h_t^e = h_t$.

With an active spending policy rule (dashed black line) but a constant replacement rate, the optimized policy is to increase public spending quite substantially (after a fall in the first period). The size of the intervention is quite large since spending increase by more than 5% but the effects are rather limited. Investment and aggregate consumption are slightly crowded out (in net terms), output is slightly stabilized with a marginal stabilization of employment and unemployment dynamics. Most of the crowding out effect bears on employed workers while unemployed workers and firm owners experience a slightly smaller fall in their consumption levels.

The unemployment benefit policy has much more impact on the dynamics of the model after a vacancy posting shock. The optimized response of the replacement rate should be negative, starting from 60% initially and progressively lowered to roughly 54%. This policy has basically two effects: it lowers V^u since the direct contribution of b_t falls, and raises the contribution of workers to the match surplus since the net value of being employed jumps. The match surplus increases and leads to a stabilized dynamics of the real wage. Therefore the number of matches, as well as the value of the marginal worker J_t, both increase. The rise in unemployment is much smaller. In addition, the stabilized dynamics of the real wage pushes the number of hours worked up, and the total contribution of employment to output increases through the intensive margin. A side effect of this policy is also to raise investment by quite a large amount, which further contributes to raise output. Indeed, Figure 8 in Appendix B shows that a reduction in the replacement rate and the corresponding cut in unemployment both lower public deficits so much that public debt falls, which raises the real interest rate and pushes firm owners to accumulate capital – they seek to boost firm profits and their consumption level – with a positive effect on output.

Finally, a jointly optimized policy combines the stabilizing properties of both policies. The replacement rate should fall less (55% instead of 54%) and public spending increase more (almost 7% instead of 5%) than when a single policy instrument is used. The benefits of the replacement rate policy in terms of labor market dynamics are preserved while the public spending policy introduces a crowding-out effect on consumption and investment, leading to a smoother path for output. Overall, our results suggest that the replacement rate policy is the most effective stabilization instrument compared to public spending, and that its beneficial effects go through a larger stabilization of the labor market and the positive spillovers attached to the reduction in unemployment.

Table 3: Hicksian consumption equivalents - incomplete-market model, in percents

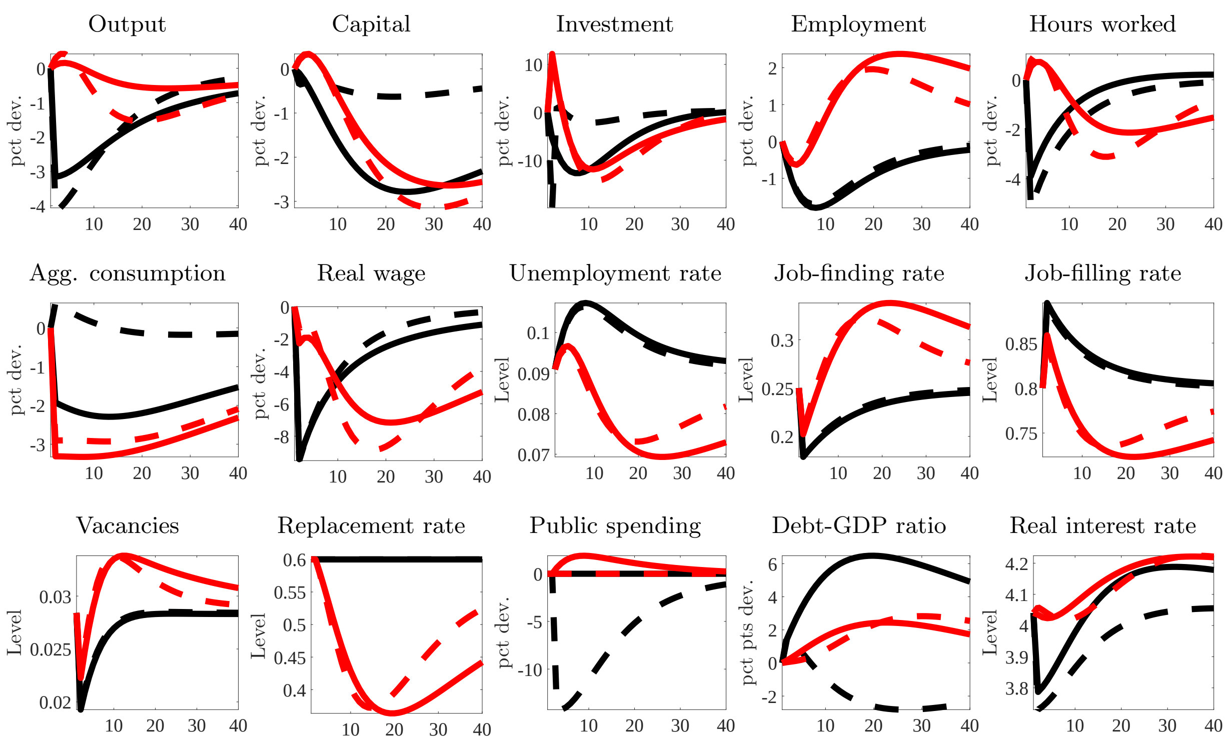
Quarters	Passive	optimized g	optimized b^r	Joint
4	-1.1680	-0.9133	-1.1974	-0.3674
8	-1.5422	-0.0862	-1.1361	0.2748
32	-1.2988	-0.8835	-0.5021	0.1797
100	-1.0105	-0.9540	-0.1393	0.0023
∞	-0.8570	-0.7841	-0.1054	0.0000

In terms of aggregate welfare, Table 3 shows that the welfare losses from the shock with passive policies are substantial, with a maximum loss at 8 quarters that reaches 1.5422% of consumption equivalent. Table 3 also depicts the relatively marginal stabilizing properties of government spending policies alone. The welfare numbers implied by the replacement policy rule are quite interesting: the fall in the replacement rate induces welfare losses in the short-run, that can be traced to the large fall in the consumption of unemployed workers, against relatively large long-term gains that relate to the stabilized dynamics of unemployment. Last but not least, the welfare numbers attached to the joint stabilization policy show that there are complementarities between both policy instruments, as welfare losses from the shock are very limited in the short run in this case, and positive in the medium run and zero in the long run.

5 Comparison with the family model

We now perform the same analysis but within the family model. Figure 4 reports the IRFs produced by the family model under the four cases considered in the previous section.

Figure 4: IRFs to a 25% positive vacancy posting cost shock in the family model



Solid black: no policy. Dashed black: Public spending. Dashed red: Replacement rate. Solid red: Public spending and replacement rate

First, when policy variables remain constant, remember that the family model broadly predicts a larger recession (due to the smoother dynamics of vacancies) but less movements in labor-market variables. Second, when the only policy instrument is the level of public spending, the optimized policy implies a *fall* rather than a rise in public spending. The chief reason is the incomplete-market model predicts a fall in unemployment after a spending shock while the family model predicts a rise. In addition, households are Ricardian in the family model, which implies that public spending directly crowd out private consumption: a rise in private consumption requires a fall in public spending in the family model. The transmission mechanism of movements in public spending is thus radically different in the family model, and drives an opposite policy prescription when public spending is the only policy instrument (see Figure 7 in Appendix B).

Whenever the replacement rate is allowed to be changed by the government, the policy prescription is qualitatively similar that of the incomplete-market model: the government should lower the replacement rate. Quantitatively however, the optimized size of the drop is much larger in the family model than it was in the incomplete-market model: the replacement rate should fall below 40%, more than 20 pp below its steady-state value. The chief objective is the same than in the incomplete-market model, that is increase the difference between the value of being employed and the value of being unemployed, and thereby stabilize vacancies and limit the rise in unemployment. The fall in the replacement rate is so much larger once again because (i) the replacement rate is the only wedge between value functions of employed vs. unemployed – while consumption inequalities were an additional source in the incomplete-market model – and (ii) because matches are less sensitive to vacancies in the family model, due to a larger steady-state fundamental surplus. In addition, for the positive spillover on debt dynamics, private investment and output to materialize, the fall in the replacement rate must be much larger. Finally, when both instruments are used together, the prescription for the replacement rate remains broadly unchanged while the prescription for public spending is reversed. Preventing the unemployment rate to rise is much more easily achieved using large swings in the replacement rate than changes in public spending in the family model, as it was the case in the incomplete-market model.

Table 4: Hicksian consumption equivalents - Family model, in percents

Quarters	Passive	optimized g	optimized b^r	Joint
4	1.6439	−0.1508	−3.2335	−2.9772
8	0.9730	−1.6873	−2.8445	−2.7039
32	−0.9606	−2.0030	−0.6232	−0.9336
100	−1.0734	−1.0658	−0.7389	−0.6027
∞	−0.7439	−0.6972	−0.5221	−0.4191

Table 4 shows that the order of magnitude of lifetime welfare losses produced by the family model are roughly similar when policies are passive, or when public spending policies alone are considered. The family model produces lower short-run welfare losses though. However, lifetime welfare losses remain relatively large in the family model when active UI policies are conducted while the equivalent losses almost vanished in the incomplete-market model. In addition, the short-run welfare losses produced by the family model are particularly large, as much as 3%

of steady-state consumption, and remain as large as 0.5% of steady-state consumption from a lifetime perspective. So considering a realistic set-up for the joint dynamics of unemployment, savings and consumption inequalities after large shocks matters for the design of optimized stabilization policies, but also for the size of the corresponding aggregate welfare effects.

6 Conclusion

We develop a simplified heterogeneous agent model with incomplete market, unemployment insurance and borrowing constraints. We show that this simplified framework is relevant to study the effects of fiscal and unemployment insurance policies in reaction to a large economic downturn. The interaction between unemployment and borrowing constraints are crucial in determining the aggregate effects of large shocks, the design of optimized policies in response to these shocks and the corresponding welfare gains/losses. Finally we also compare our incomplete-market model with a family model to understand the key assumptions of our results.

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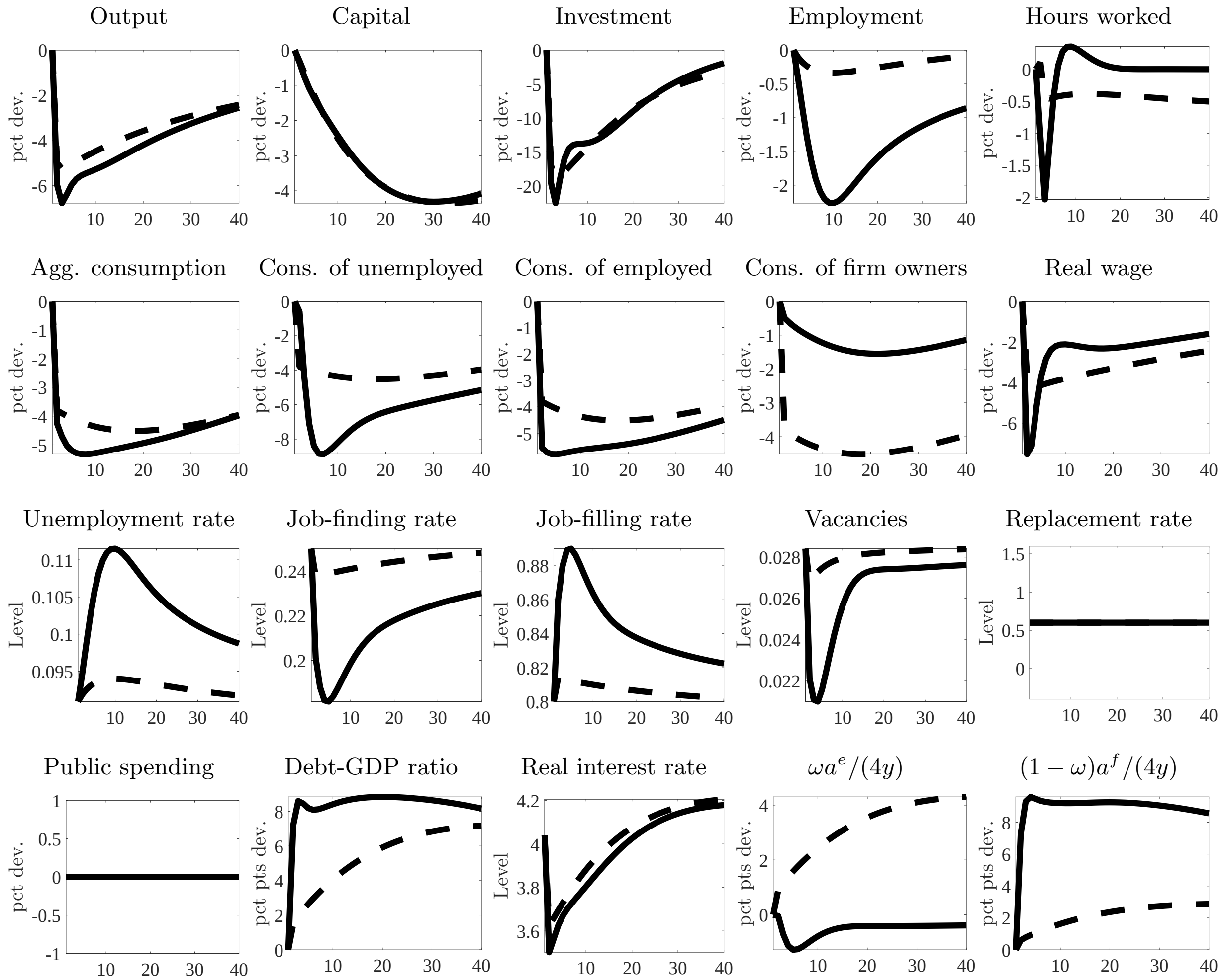
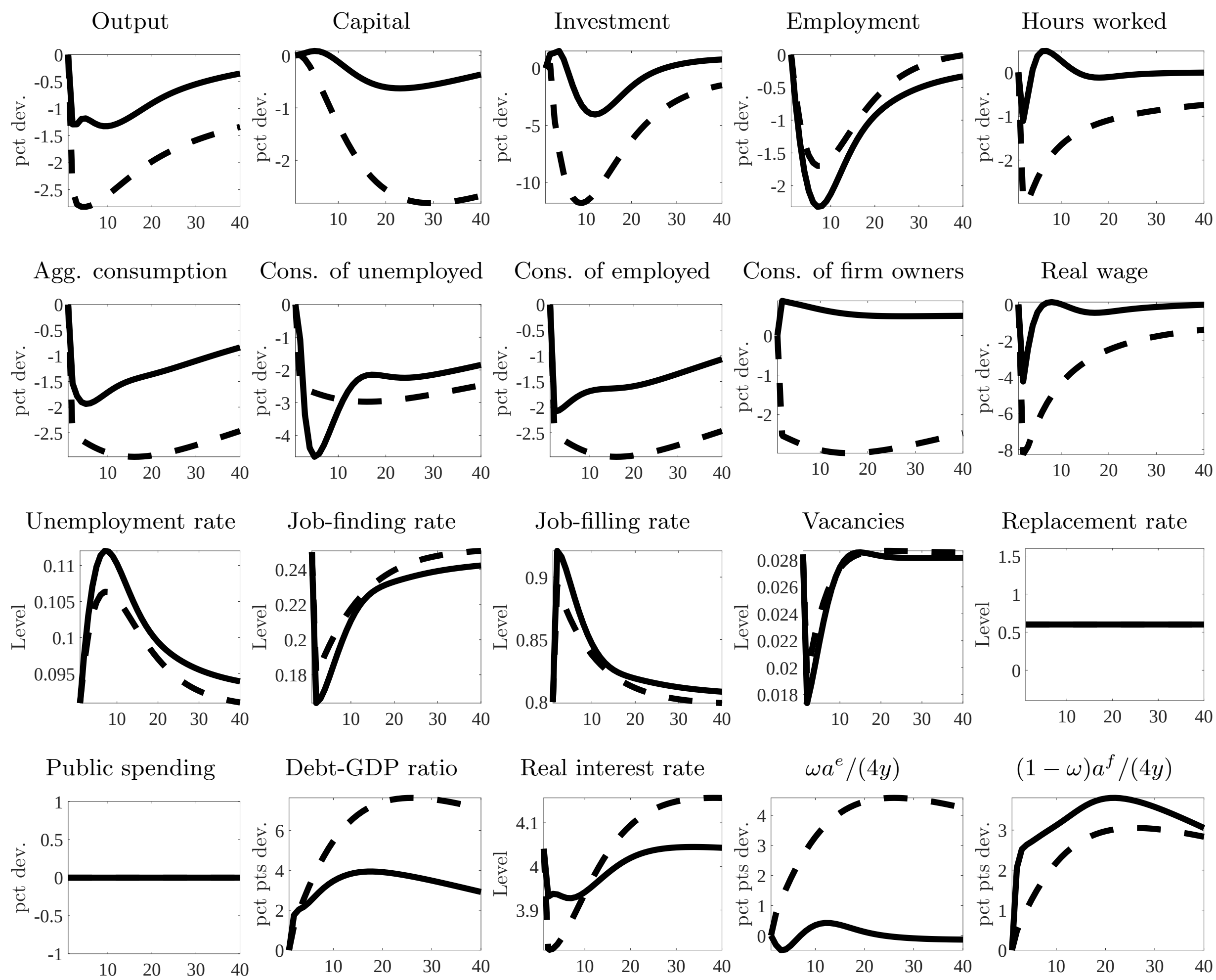
A Impulse responses to shocks with distortionary labor taxation

Figure 5: IRFs to a 5% negative productivity shock



Solid: baseline model. Dashed: Family model.

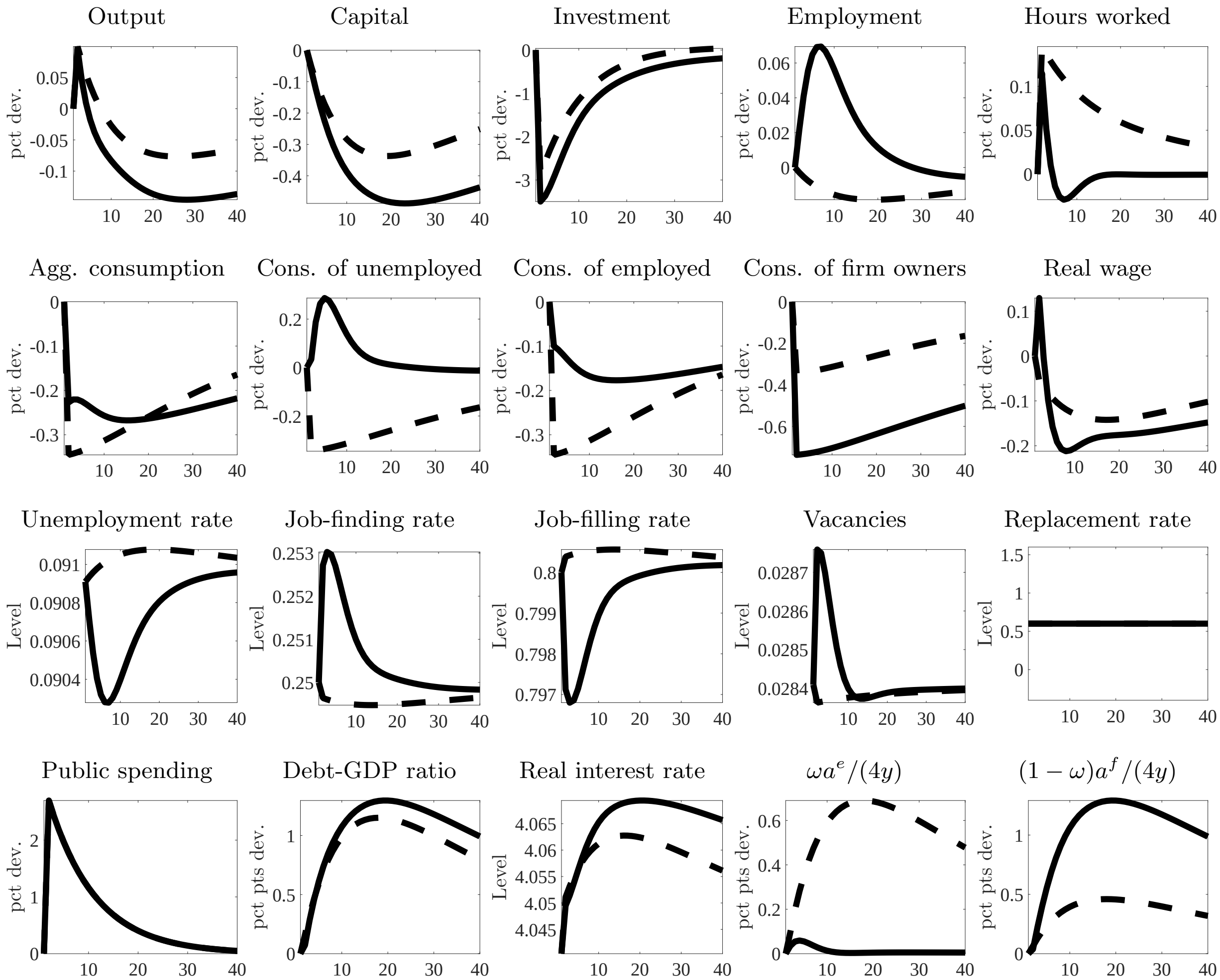
Figure 6: IRFs to a 25% positive vacancy posting cost shock



Solid: baseline model. Dashed: Family model.

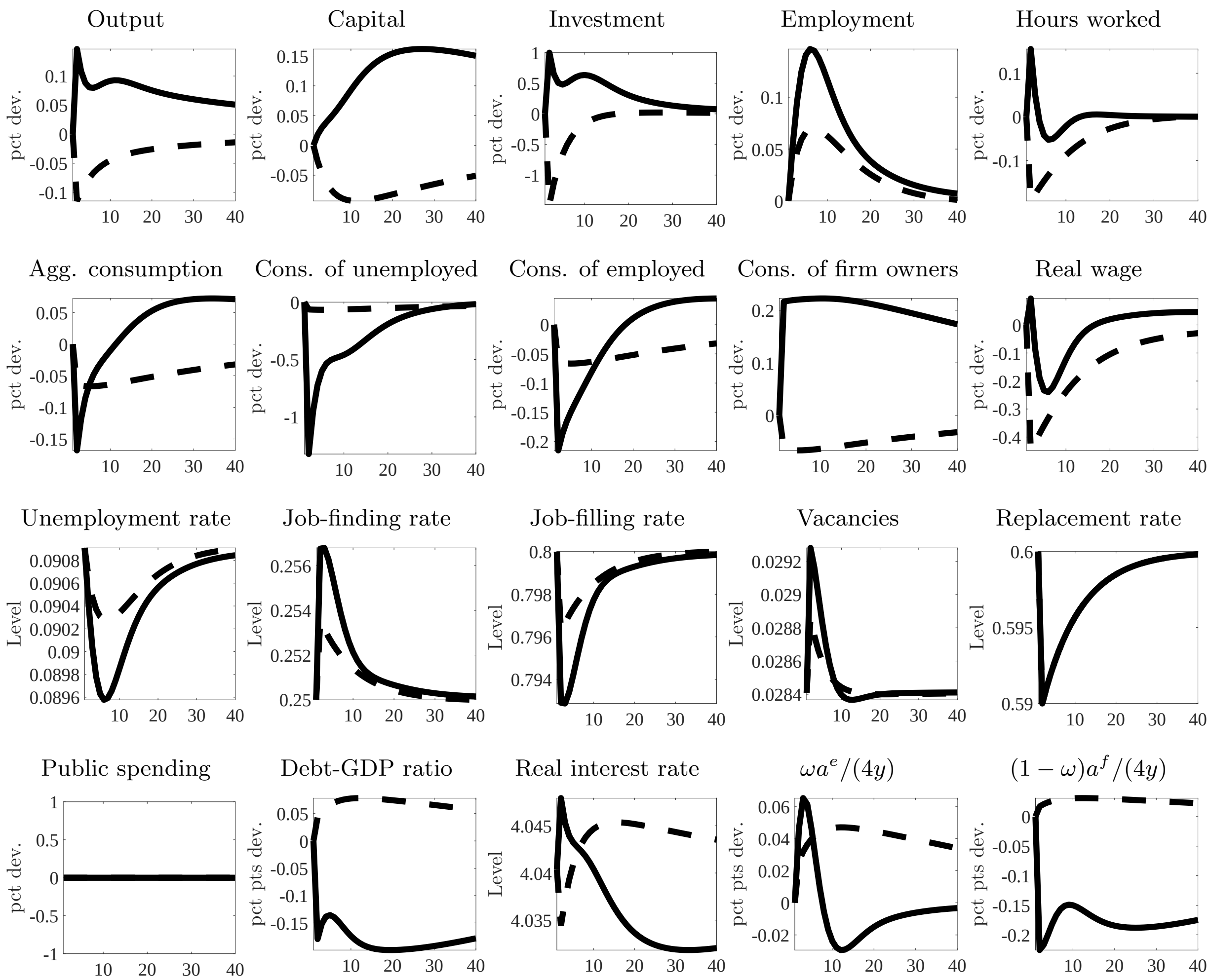
B Impulse responses to policy instruments

Figure 7: IRFs to a 1% positive public spending shock



Solid: baseline model. Dashed: Family model.

Figure 8: IRFs to a 1% negative replacement rate shock



Solid: baseline model. Dashed: Family model.