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# Ramsey-optimal Tax Reforms and Real Exchange Rate Dynamics

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**Abstract**

We solve for the Ramsey-optimal path for government debt, labor income taxes and capital income taxes for a small open economy with an endogenously-determined real exchange rate. Due to the endogenous exchange rate, the model must be solved using the 'primal problem': maximize the lifetime utility of the representative household subject to equilibrium conditions and the government budget constraint. The open economy constrains the government's setting of the capital income tax rate since physical capital cannot be dominated in rate of return by foreign assets. However, the endogenous real exchange rate loosens this constraint relative to a one good open economy model in which the real exchange rate is necessarily fixed.

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# 1 Introduction

Much is known in the macroeconomic literature about optimal taxation in closed economy macroeconomic models as evidence by the extensive literature that traces back to Lucas and Stokey (1983), Chamley (1986) and Judd (1985). Far less is known about Ramsey-optimal taxation in the context of open economy macroeconomic models.[1] In Correia (1996) and the subsequent literature on Ramsey taxation in open economy models, there is a homogeneous good, thereby fixing the real exchange rate at one. Our paper is the first to address Ramsey optimal taxation with distinct domestic and foreign goods, and so an endogenous real exchange rate.

The existent literature almost universally ends up solving the problem of the benevolent government by developing an implementability condition which incorporates the equilibrium conditions, thus reducing the government's problem to one of choosing an allocation: prices and tax rates are computed after the fact. This implementability condition approach cannot be applied here primarily because one cannot eliminate one price, the real exchange rate, and secondarily due to the presence of capital adjustment costs which would complicate the development of an implementability condition even for a closed economy model. Consequently, the government's primal problem must be solved directly: maximize the representative household's lifetime utility subject to the equilibrium conditions and the government's budget constraint. On the face of it, solving the primal problem seems straightforward, if tedious. However, this approach runs into a problem: the government's debt sequence implied by its budget constraint is inherently unstable. This instability issue is a known problem, and in those rare papers in the Ramsey literature that compute the path of government debt do so by solving for it backwards in time, starting from a terminal steady state, to avoid the accumulation of small numerical errors when debt is solved forwards.

In addition to studying Ramsey optimal taxation in an open economy environment, an important contribution of this paper is a procedure for computationally solving the government's primal problem. This procedure is best understood in steps. First, it is a straightforward matter to solve the model given a path for government debt. In essence, this step involves solving a (large) set of equations for the time paths of the model's variables, given initial and terminal conditions. Second, associated with any such solution is a value for lifetime utility of the representative household. The task, now, is to choose the path for government debt that *maximizes* households' lifetime utility. In our experience, gradient-based methods for finding such a path for government debt do not work very well. The third step

[1] By "Ramsey-optimal taxation" we mean work that solves for at least a set of tax rates as well as the path for government debt. The path for government debt is often implicit. This definition excludes those papers that hold fixed the level of government debt.

is to adopt a derivative-free optimization routine. We use an *evolutionary program* which is a directed random search procedure akin to a genetic algorithm or simulated annealing. In summary, the evolutionary program selects over candidate paths of government debt for the path that maximizes lifetime utility of the representative household; and, for each candidate debt path, we solve for the remaining variables given initial and terminal conditions.

If the procedure just described seems obvious, evidently it is only obvious in retrospect since, to the best of our knowledge, we are the first to apply this method. Furthermore, this procedure is applicable to a wide range of interesting Ramsey problems for which it simply is not possible to develop an implementability condition.

Applying this method to solve for optimal paths for tax rates and government debt in a small open economy model would be worthwhile even if we merely confirmed that results in the literature continue to hold when the real exchange rate is endogenous. In part, this is the case. A key finding in the literature is that capital income should initially be taxed at very high rates, but in the long run not at all. The intuition for this finding is as in Ramsey (1927): the government should tax most heavily those factors that are in inelastic supply. This intuition continues to hold in our small open economy. However, in setting the capital income tax rate, the government in the open economy environment faces a constraint that is absent from the closed economy model: The rate of return to capital, net of taxes and adjustment costs, must equal the return to international bonds. As a result, in the open economy, the government's choice of the capital income tax rate is initially lower than that chosen in a closed economy. The initial lower capital income tax revenue in the open economy leads to a higher level of government debt relative to the closed economy model. In addition, a real exchange rate appreciation lowers consumption tax revenue in the medium to long run. In addition, in the open economy model, provision of the public good increases relative to the initial steady state. All of these factors lead the government to set a higher labor income tax rate in the small open economy model.

In a one good small open economy model, the real exchange rate is necessarily fixed. In such an environment, the government is even more constrained in its initially setting of the capital income tax rate since the real exchange rate cannot adjust to alter the effective return to foreign assets – a mechanism that is clearly present in the two good open economy environment. In this case, the government finds it best not to increase the capital income tax rate, even in the very short term. This result is reminiscent of that found in Correia (1996). As a result, the long run level of government debt changes little from its initial value. Thus, the distinction between one and two good small open economy models – with the consequent implications for the determination of the real exchange rate – is an important one. In our opinions, the two good model, with its endogenous determination of the real exchange rate, better describes the environment faced by governments than either the one good open economy model or the closed economy model.

One may well wonder why the government does not simply apply very high tax rates early in its implementation of the Ramsey tax program in order to become a net creditor (drive its debt negative), financing its expenditures on public goods from its interest revenue. That is, why does the government not choose a value for its debt so that it replicates the Pareto optimal outcome with no taxes? The answer is that achieving such a level of government debt is, evidently, too costly. Indeed, analyses that follow the implementability condition approach do *not* predict zero labor income tax rates precisely because the Lagrange multiplier on the implementability constraint binds. This point is made more formally in Section 3.

Following the Ramsey literature, the government is restricted to linear tax schedules, although it is free to tax labor and capital income at different rates. The government is able to fully and credibly commit to the Ramsey optimal taxation program. Adopting a residence-based taxation scheme, income from foreign bonds are implicitly taxed by the foreign government, not the domestic government. Restricting attention to those papers in the literature that solve for the set of tax rates and government debt (even if only implicitly), there is a large literature analyzing closed economies, starting with the aforementioned Lucas and Stokey (1983), Chamley (1986) and Judd (1985). The open economy literature is considerably smaller, starting with Correia (1996). As stated earlier, to the best of our knowledge, no one has performed a full Ramsey optimal taxation analysis for a two good small open economy model. A practical advantage of analyzing a *small* open economy is that strategic interactions between governments can be ignored.

The bulk of Section 2 develops the two good small open economy model; towards the end of this section is a brief description of how the two good model can be reduced to either a closed economy model, or a one good small open economy model. Fiscal policy – the Ramsey problem – is presented in Section 3. Included is a description of our solution procedure. The model is calibrated in Section 4. The key results, in the form of time paths for macroeconomic variables, are discussed in Section 5. The implications of alternative settings for preference parameters and the trade parameters are presented in Section 6. Some final remarks are made in Section 7.

# 2 A Two Good, Small Open Economy Model

In the two good model, private consumption is a composition of domestic and foreign goods. Attention is focused on the home or domestic economy; an asterisk is used to denote values of foreign variables.

## Households

The typical domestic household starts period $t$ with three assets: $k_{t-1}$ units of domestic capital, $d_{t-1}$ units of domestic government debt, and $b_{t-1}$ units of internationally traded bonds. Capital income is taxed at the rate $\tau_t^k$, net of depreciation. The gross return to a unit of capital is, then, $R_t^k \equiv 1 + (1 - \tau_t^k)(r_t - \delta)$ where $r_t$ is the real rental rate for capital and $\delta$ the depreciation rate of capital. The gross return to a unit of government debt is $R_{t-1}^d$. Finally, international bonds pay off in terms of foreign output. In terms of *domestic* output, a unit of such bonds pays the gross return $e_t R_{t-1}^b$ where $e_t$ is the real exchange rate, expressed as the number of units of domestic output per unit of foreign output. Conceptually, the international bonds are state contingent. However, since the analysis focuses on perfect foresight equilibria, state contingent notation is suppressed in the interests of a cleaner presentation. The fact that the international bonds are state contingent means that the implicit assumption that only domestic households own domestic capital and domestic government is without loss of generality.

The representative domestic household receives utility from private consumption goods, $c_t$, government or public goods, $g_t$, and disutility from working, $h_t$. The household's problem is:

$$\max_{\{c_t, h_t, k_t, d_t, b_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U(c_t, g_t, h_t) \tag{1}$$

subject to the sequence of budget constraints,

$$\begin{aligned} &(1 + \tau_t^c)\, p_t c_t + k_t + \frac{\eta}{2}(k_t - k_{t-1})^2 + d_t + e_t b_t \\ &\quad \leq \left(1 - \tau_t^h\right) w_t h_t + R_t^k k_{t-1} + R_t^d d_{t-1} + e_t R_t^b b_{t-1}. \end{aligned} \tag{2}$$

The last two terms on the right-hand side of eq. (2) are the proceeds of previous period purchases of domestic and international bonds, respectively. The first two terms are payments to labor and capital: in addition to the capital income components discussed above, $w_t$ is the real wage which is taxed at the rate $\tau_t^h$.

In addition to bond purchases (the last two terms on the left-hand side of eq. (2)), the domestic household purchases private consumption goods, capital, and pays a capital adjustment cost. The price of a unit of consumption in terms of domestic output is $p_t$; its derivation is described shortly. Consumption purchases are taxed at the rate $\tau_t^c$.

The household's Euler equations are:

$$\frac{1-\tau_t^h}{1+\tau_t^c}\frac{w_t}{p_t}U_c(c_t,g_t,h_t)+U_h(c_t,g_t,h_t)=0 \tag{3}$$

$$[1+\eta(k_t-k_{t-1})]\frac{U_c(c_t,g_t,h_t)}{(1+\tau_t^c)\,p_t}=\beta\frac{U_c(c_{t+1},g_{t+1},h_{t+1})}{\left(1+\tau_{t+1}^c\right)p_{t+1}}\left[R_{t+1}^k+\eta(k_{t+1}-k_t)\right] \tag{4}$$

$$\frac{U_c(c_t,g_t,h_t)}{(1+\tau_t^c)\,p_t}=\beta\frac{U_c(c_{t+1},g_{t+1},h_{t+1})}{\left(1+\tau_{t+1}^c\right)p_{t+1}}R_t^d \tag{5}$$

$$\frac{U_c(c_t,g_t,h_t)}{(1+\tau_t^c)\,p_t}e_t=\beta\frac{U_c(c_{t+1},g_{t+1},h_{t+1})}{\left(1+\tau_{t+1}^c\right)p_{t+1}}R_t^b e_{t+1}. \tag{6}$$

A version of eq. (6) also holds for foreign households:

$$\frac{U_c^*(c_t^*,g_t^*,h_t^*)}{(1+\tau_t^{c*})\,p_t^*}=\beta\frac{U_c^*(c_{t+1}^*,g_{t+1}^*,h_{t+1}^*)}{\left(1+\tau_{t+1}^{c*}\right)p_{t+1}^*}R_t^b. \tag{7}$$

Following Chari, Kehoe, and McGrattan (2000), solve each of eqs. (6) and (7) for the common return on a bond, equate the two expressions, then iterate backwards in time to obtain

$$e_t=\vartheta\frac{U_c^*(c_t^*,g_t^*,h_t^*)}{U_c(c_t,g_t,h_t)}\frac{(1+\tau_t^c)\,p_t}{(1+\tau_t^{c*})\,p_t^*} \tag{8}$$

where

$$\vartheta=e_0\frac{U_c^*(c_0^*,g_0^*,h_0^*)}{U_c(c_0,g_0,h_0)}\frac{(1+\tau_0^c)\,p_0}{(1+\tau_0^{c*})\,p_0^*}. \tag{9}$$

An implication of eq. (8) is that the real exchange rate is determined by the price and consumption tax-adjusted ratio of marginal utilities of consumption between domestic and foreign households, and the arbitrary factor of proportionality, $\vartheta$.

The private consumption good is an aggregate of domestic, $c_{ht}$, and foreign, $c_{ft}$, goods,

$$c_t=\left[\varphi^{\frac{1}{\mu}}c_{ht}^{\frac{\mu-1}{\mu}}+(1-\varphi)^{\frac{1}{\mu}}c_{ft}^{\frac{\mu-1}{\mu}}\right]^{\frac{\mu}{\mu-1}} \tag{10}$$

where $\mu>0$ is the elasticity of substitution between domestic and foreign goods; $\varphi=1-(1-n)\gamma$; $n$ is the relative size of the domestic economy; and $\gamma$ is a measure of trade openness. Solving the relevant cost minimization problem yields the relative price for aggregated private domestic consumption,

$$p_t=\left[\varphi+(1-\varphi)e_t^{1-\mu}\right]^{\frac{1}{1-\mu}} \tag{11}$$

as well as demands for domestic and foreign goods,

$$c_{ht} = \varphi \left[\varphi + (1-\varphi)e_t^{1-\mu}\right]^{\frac{\mu}{1-\mu}} c_t \tag{12}$$

$$c_{ft} = (1-\varphi) \left[\varphi e_t^{\mu-1} + 1 - \varphi\right]^{\frac{\mu}{1-\mu}} c_t. \tag{13}$$

Slightly different expressions characterize the foreign economy.

## Firms

The representative firm has access to a neoclassical production function, $F$. The firm rents capital and hires labor on competitive factor markets to maximize period-by-period profits,

$$F(k_{t-1}, h_t) - w_t h_t - r_t k_{t-1}.$$

The demands for capital and labor are, then, governed by the usual first-order conditions:

$$r_t = F_k(k_{t-1}, h_t) \tag{14}$$

$$w_t = F_n(k_{t-1}, h_t). \tag{15}$$

## Government

The problem of a benevolent government planner is analyzed in Section 3. For the purpose of analyzing the competitive equilibrium, it is sufficient to note that the government finances the stream of public goods, $g_t$, by either issuing debt, $d_t$, or levying taxes to satisfy its budget constraint

$$d_t - R^d_{t-1} d_{t-1} = g_t - \tau^c_t p_t c_t - \tau^h_t w_t h_t - \tau^k_t \left(r_t - \delta\right) k_{t-1} \tag{16}$$

as well as the usual transversality condition concerning its debt. In eq. (16), it is understood that the quantities are expressed per capita.

## Competitive Equilibrium

The definition of a competitive equilibrium is standard: Given prices and government actions, households solve their utility maximization problems, firms solve their profit maximization problems, the government satisfies its budget constraint and 'no Ponzi scheme' condition, and markets clear.

As in de Paoli (2009), the small open economy is the limit case of a two country model.

The domestic goods market clearing condition is

$$y_t = \varphi p_t^{\mu} c_t + \gamma e_t^{\mu} c_t^* + k_t - (1-\delta)k_{t-1} + \frac{\eta}{2}\left(k_t - k_{t-1}\right)^2 + g_t. \tag{17}$$

## Model Variants

The Ramsey optimal taxation literature has considered two other classes of models that can be viewed as special cases of the two good, small open economy model. The bulk of the Ramsey optimal taxation literature has focused on closed economies. In this case, set foreign bond holdings to zero ($b_t = 0$). The Euler equation eq. (6) is no longer relevant. There are no foreign goods and so no consumption aggregator, and the relative price of the consumption good is unity ($p_t = 1$).

The open economy models analyzed in the literature have one good. In this case, since domestic and foreign goods are perfect substitutes, the real exchange rate ($e_t$) is one as is the relative price of consumption ($p_t$). There is no need to introduce the consumption aggregator, eq. (10). The goods market clearing condition now reads

$$y_t = c_t + nx_t + k_t - (1-\delta)k_{t-1} + \frac{\eta}{2}\left(k_t - k_{t-1}\right)^2 + g_t$$

where $nx_t$ is net exports. In turn, net exports are related to international bonds via

$$b_t - R_{t-1}^b b_{t-1} = nx_t.$$

For a more complete description of the closed and one good small open economy models, see Appendix A.

# 3 Fiscal Policy

The problem of the benevolent domestic government is to maximize the household's lifetime utility subject to the government's budget constraint as well as the equations characterizing the competitive equilibrium. As discussed in the Introduction, the typical approach in the Ramsey optimal taxation literature is to use the equilibrium conditions to eliminate all prices and taxes, reducing the government's problem to one of choosing an allocation. Unfortunately, this approach fails here owing to both the capital adjustment costs as well as the endogenously-determined real exchange rate.[2]

[2] Appendix A.1 illustrates the problem in developing an implementability condition with capital adjustment costs for the closed economy model. For the two good small open economy model, Appendix A.2 shows that the real exchange rate shows up, through the relative price of consumption, in the feasibility constraint.

To understand our approach, it is helpful to review the approach typically followed in the literature. To this end, consider the closed economy version of the model with no capital adjustment costs. The consumer's problem is

$$\mathcal{L} = \sum_{t=0}^{\infty} \left\{ \beta^t U(c_t, g_t, h_t) + \lambda_t \left[ (1-\tau_t^h) w_t h_t + R_t^k k_{t-1} + R_{t-1}^d d_{t-1} - (1+\tau_t^c) c_t - k_t - d_t \right] \right\}. \tag{18}$$

The associated first-order conditions are:

$$\beta^t U_c(c_t, g_t, h_t) = \lambda_t (1+\tau_t^c) \tag{19}$$

$$\beta^t U_h(c_t, g_t, h_t) + \lambda_t (1-\tau_t^h) w_t = 0 \tag{20}$$

$$\beta^t \lambda_t = \beta^{t+1} \lambda_{t+1} R_{t+1}^k \tag{21}$$

$$\beta^t \lambda_t = \beta^{t+1} \lambda_{t+1} R_t^d. \tag{22}$$

Focus on the second term in eq. (18). This term necessarily must equal zero (either the term in square brackets is zero, or the associated Lagrange multiplier is). Equations (21) and (22) imply that many of the terms in this sum are zero. Using eq. (19) to substitute out for $\lambda_t(1+\tau_t^c)$ and eq. (20) to substitute out for $\lambda_t(1-\tau_t^h)w_t$ yields the implementability condition,

$$\sum_{t=0}^{\infty} \beta^t \left[ U_c(c_t, g_t, h_t) c_t + U_h(c_t, g_t, h_t) h_t \right] = \left[ R_0^k k_{-1} + R_{-1}^d d_{-1} \right] \frac{U_c(c_0, g_0, h_0)}{1+\tau_0^c}. \tag{23}$$

The Ramsey allocation problem is now written

$$\begin{aligned} \mathcal{L} = & \sum_{t=0}^{\infty} \beta^t U(c_t, g_t, h_t) \\ & + \lambda \left\{ \sum_{t=0}^{\infty} \beta^t \left[ U_c(c_t, g_t, h_t) c_t + U_h(c_t, g_t, h_t) h_t \right] - \left[ R_{k0} k_{-1} + R_{-1}^d d_{-1} \right] \frac{U_c(c_0, g_0, h_0)}{1+\tau_0^c} \right\} \\ & + \sum_{t=0}^{\infty} \phi_t \left[ F(k_{t-1}, h_t) + (1-\delta) k_{t-1} - c_t - g_t - k_t \right]. \end{aligned} \tag{24}$$

The advantage of eq. (24) is that it involves only the allocation, $\{c_t, h_t, k_t, g_t\}_{t=0}^{\infty}$ – neither prices nor tax rates appear in the planner's objective. A minor nuisance of using eq. (24)

Given this last result, one might think that setting up the model with preferences defined 'directly' over home and foreign goods rather than over their composite might be a fruitful way to proceed. Appendix A.3 shows that this approach also fails, albeit because the planner is not constrained with regards to his choice of the quantity of foreign consumption goods.

is that one must solve for the value of the Lagrange multiplier on the implementability condition, $\lambda$. Typically, one starts with a guess for the value of $\lambda$, solves the model conditional on this value, then check whether the implementability condition is satisfied. If it is not, the value for $\lambda$ is updated until the implementability condition is satisfied.

For future reference, it is useful to have in hand the analytical results that go along with solving eq. (24). Defining

$$W(c_t, g_t, h_t) \equiv U(c_t, g_t, h_t) + \lambda \left[U_c(c_t, g_t, h_t)c_t + U_h(c_t, g_t, h_t)h_t\right]$$

the associated first-order conditions are

$$\beta^t W_c(c_t, g_t, h_t) = \phi_t \tag{25}$$

$$\beta^t W_g(c_t, g_t, h_t) = \phi_t \tag{26}$$

$$\beta^t W_h(c_t, g_t, h_t) + \phi_t F_h(k_{t-1}, h_t) = 0 \tag{27}$$

$$\phi_t = \phi_{t+1}\left[F_k(k_t, h_{t+1} + 1 - \delta\right]. \tag{28}$$

Equations (25) and (28) imply that in a steady state,

$$1 = \beta\left[F_k(k, h) + 1 - \delta\right].$$

The corresponding equation from the competitive equilibrium is

$$1 = \beta\left[1 + (1 - \tau^k)(F_k(k, h) - \delta)\right].$$

Comparing these two expressions delivers the familiar result that $\tau^k = 0$: in the long run, capital income should not be taxed.

Next eqs. (25) and (27) imply that in steady state,

$$U_h + \lambda\left[U_{hc}c + U_{hh}h + U_h\right] + F_h\left\{U_c + \lambda\left[U_{cc}c + U_c + U_{ch}h\right]\right\} = 0.$$

From the competitive equilibrium,

$$U_h + (1 - \tau^h)F_h U_c = 0.$$

There is, then, no presumption that the benevolent government planner should set the labor income tax rate to zero in the long run.

Finally, notice that the Ramsey allocation problem allows one to solve for only two of the three tax rates. In this case, attention is focused on the two factor income tax rates, although one could alternatively solve for one factor income tax rate and the consumption tax rate.

As mentioned earlier, the presence of capital adjustment costs – even in the closed economy model – makes it impossible to obtain an implementability condition like eq. (23).[3] Furthermore, even without capital adjustment costs, in the open economy version of the model, it simply is not possible to eliminate the real exchange rate, $e_t$, from the government's problem; see Appendix A.2. Previous open economy work has sidestepped this issue by assuming a single good that is produced in both the domestic and foreign economies, thus fixing the real exchange rate at unity.

Given these difficulties, optimal fiscal policy must be solved for 'directly', or by 'brute force.' A further problem relates to the stability of government debt. Much of the modern Ramsey taxation literature pretty much ignores debt. Those who are interested in the debt sequence solve for it backwards using the government budget constraint. The reason for solving backwards is that the government budget constraint implies that government debt dynamics are unstable, and so when solving forward for debt, small numerical errors lead to explosive debt sequences.

## Computation

Our approach is to start by specifying a sequence for government debt, $\{d_t\}_{t=0}^{\infty}$. Given this sequence for government debt, for the open economy model, the government's problem is to choose $\{c_t, g_t, h_t, k_t, \tau_{t+1}^k, \tau_t^h, p_t, e_t, R_t^d, w_r, r_t\}_{t=0}^{\infty}$ to maximize lifetime utility of the representative household, eq. (1), subject to the household's Euler equations, eqs. (3) to (5), the risk-sharing condition, eq. (8), the price equation, eq. (11), the firms' first-order conditions, eqs. (14) and (15), feasibility, eq. (17), and the government's budget constraint, eq. (16). As is typical in the Ramsey taxation literature, the government cannot change the initial capital income tax rate, otherwise the government would choose a confiscatory capital income tax rate which private agents are unable to avoid. As noted above, only two of the three tax rates can be determined; we chose the two factor income taxes.

Given the equations characterizing the solution to the government's problem, and given its debt sequence, we solve for the government's choice variables using an extended path algorithm, specifying an initial steady state and a 'no change' terminal conditions; for details, see Auray, Eyquem, and Gomme (2016).

We then choose the government debt sequence to maximize the lifetime utility of the representative domestic household. Given government debt for the first $J$ periods, the re-

[3] As shown in Appendix A.1, one can *almost* rewrite the government's problem in terms of choosing an allocation if one takes the path for the consumption tax as given.

mainder of the debt sequence is given by

$$d_t = \rho d_{t-1} + (1-\rho)\overline{d}. \tag{29}$$

where $\rho$ is an autoregressive parameter, and $\overline{d}$ is the long run value for government debt. The task, then, is to choose $(\{d_t\}_{t=1}^{J}, \rho, \overline{d})$ to maximize the household's lifetime utility. In our experience, applying gradient-based methods does not work very well for finding the debt sequence. Instead, we use an evolutionary program which is a stochastic optimization method in the same class as genetic algorithms and simulated annealing.

In general, an evolutionary program works as follow. Given an initial guess, the vector $x$, generate a population of candidate solutions around $x$ using Normally distributed random variables. Each candidate solution $x_i$ has an associated function evaluation $y_i$ (in the case at hand, the lifetime utility of the representative household). Sort the population of candidate solutions by their function evaluations, from best to worse. If $N$ is the total number of candidate solutions, generate new candidate solutions for the worst half of the population by copying a corresponding member from the best half of the population, with normally distributed noise:

$$x_i = x_{i-N/2}(1+\sigma\epsilon) + \sigma\eta, \quad i = N/2+1, \ldots, N$$

where $\epsilon$ and $\eta$ are vectors distributed $N(0,1)$ and $\sigma$ governs the 'noise'. This procedure of copying with noise is repeated many times. Notice that $\sigma$ determines how much new candidates differ from old ones. Over time, the value of $\sigma$ is decreased to search more locally. Since each candidate solution can be evaluated independently of the others, evolutionary programming lends itself quite well to parallel processing.

To start, we set $J = 1$ which means specifying the first value of the debt sequence, its long run value, and how quickly debt reaches its long run value. When the evolutionary program is done, we then set $J = 2$, using the previous solution for the debt sequence to give the first two values of the debt sequence, the long run value of government debt, and the autoregressive parameter. $J$ is successively increased to $J = 10$. As shown below, the government debt sequence settles into its long run steady state value after 2 periods, suggesting that our eventual choice of $J = 10$ is not influencing the solution.

# 4 Calibration

## Functional Forms

The utility function is of the constant relative risk aversion variety

$$U(c,g,h) = \frac{\left(C(c,g)(1-h)^{\chi}\right)^{1-\sigma}}{1-\sigma},$$

where $C(c,g)$ is an aggregator over private and public consumption goods, given by

$$C(c,g) = \left[(1-\kappa)c^{\frac{\psi-1}{\psi}} + \kappa g^{\frac{\psi-1}{\psi}}\right]^{\frac{\psi}{\psi-1}}.$$

It is understood that the consumption aggregator is Cobb-Douglas when $\psi = 1$, and the utility function is logarithmic when $\sigma = 1$.

Production is Cobb-Douglas:

$$y = F(k,h) = k^{\alpha}h^{1-\alpha}.$$

## Parameterization

The model is annual. The initial steady state is symmetric in the sense that foreign variables are assumed equal to the domestic variables. Hence the real exchange rate is $e = 1$ and net exports are zero in the initial steady state. These assumptions ensure that the initial steady state of the closed and open economies are the same.

Some parameters are set exogenously. The analysis of the tax reforms is somewhat more straightforward when utility is additively separable. Thus, the coefficient of relative risk aversion, $\sigma$, is set to one which implies logarithmic preferences, and the consumption aggregator is Cobb-Douglas ($\psi = 1$). Consequently, preferences are additively separable in private consumption, public consumption, and hours (or leisure). The capital adjustment cost, $\eta$, is set to 0.025 – a common value in the small open economy literature. The trade openness parameter, $\gamma$, is set to 0.3 on the basis that the world share of imports is 30%, and the elasticity of substitution between home and foreign goods, $\mu$, is set to 1.5 as in Backus, Kehoe, and Kydland (1992).

There remain nine parameters: $\kappa$, the utility weight on public versus private goods; $\chi$, the utility weight on leisure; $\beta$, the discount factor; $\alpha$, capital's share of income; $\delta$, the depreciation rate of capital; $\vartheta$, the constant in the risk-sharing condition (for the open economy versions of the model); and the three tax rates, $\tau^c$, $\tau^h$ and $\tau^k$. The corresponding nine targets are taken from observations for the U.S.:

1. U.S. time use surveys imply that people spend roughly 30% of their discretionary time working.

2. The real interest rate is set to 4%, a value commonly used in the macroeconomics literature.

3. Capital's share of income is 30%; see Gomme and Rupert (2007).

4. Depreciation of 7.5%; again, see Gomme and Rupert (2007).

5. Domestic consumption equals foreign consumption.

6. Government's share of output is 19.55%.

7. Observed effective average tax rates of 4.84% for consumption, 28.59% for labor income, and 37.10% for capital income. These rates are averages for the U.S. for 2005–07; see Auray, Eyquem, and Gomme (2017) for details.

The resulting parameter values are: $\kappa = 0.2334$, $\chi = 1.3281$, $\alpha = 0.3$, $\delta = 0.075$, $\vartheta = 2.7262$, $\beta = 0.9615$, $\tau^c = 0.0484$, $\tau^h = 0.2859$, $\tau^k = 0.3710$.

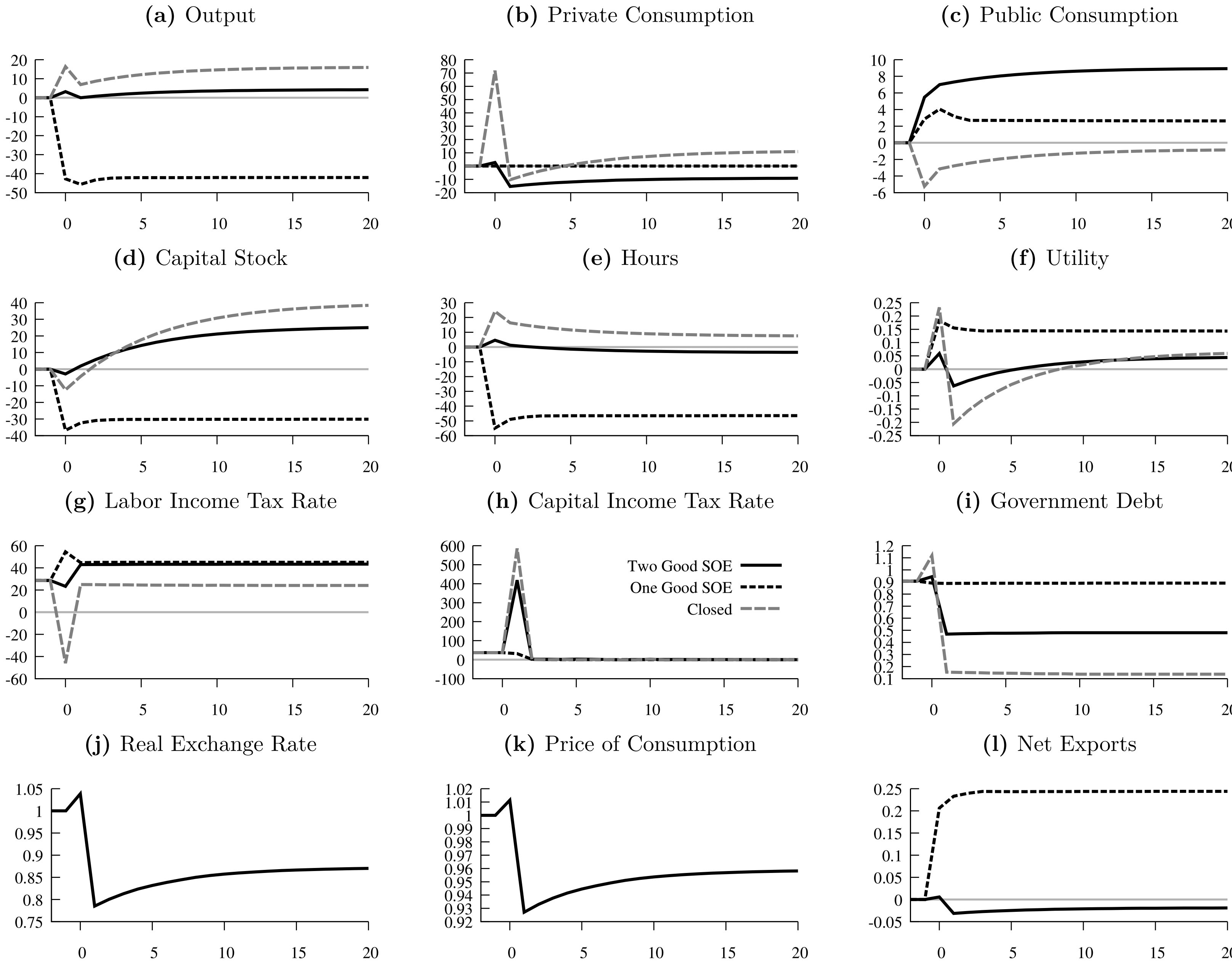
The steady state value for government debt is, finally, computed from the steady state version of the government budget constraint. The level of debt, or even the debt-output ratio, is not very consequential in the sense that one could add lump-sum transfers to the model in order to achieve any desired debt-output ratio.

# 5 Tax Reforms

At time 0, the government announces its policy in the form of time paths for public spending, income tax rates, and public debt: $\{g_t, \tau_t^h, \tau_{t+1}^k, d_t\}_{t=0}^{\infty}$. As is common in the Ramsey taxation literature, the government is not free to set the initial capital income tax rate, $\tau_0^k$. Results from the literature show that if the government could set $\tau_0^k$, it would set this tax rate sufficiently high to drive its debt negative enough to finance all of its current and future public spending from the interest income. The short run dynamics of these tax reforms are presented in Figure 1 while the long run steady states are summarized in Table 1.

To start, consider government policy, starting with the closed economy model, then the two good open economy model. The most dramatic effects are with respect to the capital income tax rate and government debt. In the closed economy model, the capital income tax rate rises from 37.1% to 586% in period 1, after which it immediately falls to around 0%.

**Figure 1:** Dynamic Paths For the Closed and Small Open Economy (SOE) Models



In period 0, the government actually *subsidizes* labor income ($\tau_0^h = -44\%$), presumably to forestall a drop in hours worked. This tax rate then rises to around its eventual long run value of 24%, roughly 4.4 percentage points below its initial level. Initially public spending drops, but then gradually recovers.

**Table 1:** Initial and Terminal Steady States

| | Initial | Closed | Two Good Small Open Economy | One Good Small Open Economy |
|---|---|---|---|---|
| $\tau^c$ | 0.0484 | 0.0484 | 0.0484 | 0.0484 |
| $\tau^h$ | 0.2859 | 0.2421 | 0.4341 | 0.4502 |
| $\tau^k$ | 0.3710 | 0.0000 | 0.0000 | 0.0000 |
| $y$ | 0.4177 | 0.4819 | 0.4327 | 0.2423 |
| $c/y$ | 0.6422 | 0.6194 | 0.5400 | 1.1070 |
| $k/y$ | 2.1646 | 2.5714 | 2.5714 | 2.6087 |
| $h$ | 0.3000 | 0.3215 | 0.2887 | 0.1606 |
| $g/y$ | 0.1955 | 0.1877 | 0.2854 | 0.2217 |
| $d/y$ | 2.1695 | 0.2816 | 1.0706 | 3.6752 |
| $e$ | 1.0000 | | 0.8741 | 1.0000 |
| $nx/y$ | 0.0000 | | −0.0427 | −0.5244 |
| $\omega$ | | 3.0600 | 3.4209 | 16.7012 |

For the two good small open economy model, the dynamics of the government policy variables are broadly similar to those in the closed economy model. While the initial hike in the capital income tax rate is still impressive ($\tau_1^k = 418\%$), it falls short of the 586% tax rate in the closed economy model. The fall in government debt is correspondingly more modest as seen in Figure 1(i). The three main factors leading to a higher labor income tax rate are: a relatively higher level of government debt, greater public spending (see Figure 1(c)), and lower consumption tax revenue due to the real exchange rate appreciation (see Figure 1(j)). In the long run, the labor income tax rate is 14.8 percentage points higher than its initial value.

It would seem that the differences in the responses of the capital income tax rates are at the heart of the differences between the two good small open economy and closed economy models. To understand these differences, it is useful to consider the "effective" returns to international bonds and capital:

$$\tilde{R}_t^b \equiv \frac{e_{t+1}R^b}{e_t} \tag{30}$$

$$\tilde{R}_{t+1}^k \equiv \frac{(1-\tau_{t+1}^k)(r_{t+1}-\delta)+q_{t+1}}{q_t}. \tag{31}$$

The effective return to capital includes the capital adjustment costs as summarized by Tobin's $q_t = 1 + \eta(k_t - k_{t-1})$ while the effective return on international bonds includes the real exchange rate movements. Apart from the initial period, there is an arbitrage condition that must hold: $\tilde{R}^k_{t+1} = \tilde{R}^b_t$. This arbitrage condition constrains the government in its setting of the capital income tax rate. To be sure, there are other margins of adjustment: in the effective return to capital, the changes in Tobin's $q$ serve to offset the fall in the return due to the capital income tax rate hike; for the effective return to international bonds, the exchange rate initially depreciates ($e_0$ rises), then appreciates ($e_1$ falls). The effects of this arbitrage condition are starkly seen in the one good small open economy in which the real exchange rate cannot change. In this case, $\tilde{R}^b_t = R^b$ and there is no variation in the effective return on international bonds. The government is now much more constrained in its setting of the capital income tax rate. Indeed, on impact, the government actually lowers this tax rate, from 37.1% to 31.4%, after which the capital income tax rate quickly goes to zero. This result is broadly consistent with that of Correia (1996). As a result, there is only a very modest decline in government debt, and the labor income tax rises, initially by 25.9 percentage points, and in the long run by 16.4 percentage points. In the two good small open economy model, the government is clearly less constrained than in the one good model, but more constrained than the closed economy model which explains why the behavior of the capital income tax rate in the two good model lies between those of the other two models.

The private sector macroeconomic effects are driven by the time paths of the two income tax rates. The sharp, transitory increase in the capital income tax rate in the closed economy model leads to a similarly sharp decline in capital accumulation, allowing a spike in private consumption. Given such a large increase in private consumption, the Euler equation governing the labor-leisure choice, Equation (3), would call for a similarly sharp decline in hours worked were it not for the negative labor income tax rate at time 0. The two good small open economy model sees a more modest (but still very large) increase in the capital income tax rate, and so a more modest fall in capital accumulation.

The initial real exchange rate depreciation raises the relative price of aggregate consumption, which serves to moderate the increase in private consumption in period 1. The subsequent appreciation starting at time 2 leads to a fall in the relative price of consumption. As discussed in Benigno and de Paoli (2010), this real exchange rate appreciation tends to raise the path for consumption and lower that of hours. There is no spike in the capital income tax rate in the one good small open economy model, although the capital stock nonetheless (permanently) declines. In this latter model, lower capital income taxation leads to virtually no change in the level of government debt; to finance it expenditures, the government must raise the labor income tax rate. In turn, this increase in the labor income tax rate reduces hours worked to such an extent that the marginal product of capital falls enough to actually reduce the return to capital, thus explaining the long term fall in the capital stock. Output is, naturally, quite a bit lower than in the initial steady state. Yet, with separable preferences, the international risk-sharing condition implies no change in private consumption. To accommodate unchanging private consumption with a substantial fall in output, net exports must rise as seen in Figure 1(l).

Once again, the results for the one good small open economy contrast with those of the other two models. In the closed economy model, zero capital income taxation and a lower labor income tax rate boost economic activity in the medium to long run; the capital stock is 40% higher, the labor input 7% higher, and output rises 16%. In the two good small open economy model, higher labor income taxation leads to a 4% decline in the labor input. Nonetheless, capital rises 25% in the long run, and output is 4.3% higher. The one good small open economy model predicts large declines in macroeconomic activity: 30% for the capital stock, 46% for hours, and 42% for output.

To evaluate the efficacy of the tax reforms, compute the welfare benefit as the percentage of private consumption that can be taken from households (holding fixed both public consumption and hours worked) that leaves them with the same utility as the original steady state. That is, the welfare benefit is measured by the value of $\omega$ that satisfies

$$\sum_{t=0}^{\infty} \beta^t U((1-\omega)c_t, g_t, h_t) = \frac{U(c_0, g_0, h_t)}{1-\beta}. \tag{32}$$

For the closed economy model, the welfare benefit of the tax reform is just over 3% of consumption; for the two good small open economy model, 3.4%; and for the one good small open economy model, 16.7%. Clearly, the results for the latter model are driven by a combination of unchanged private consumption (arising from the international risk-sharing condition along with additively separable preferences), much lower hours worked (and so higher leisure), and a modest rise in public goods. The more modest, but still sizable, welfare implications for the closed and two good small open economy models chiefly reflect the smaller effects of the tax reforms on labor supply.

## Assessment

The stark contrast in the results across these economic models highlight the underlying economic mechanisms. The fixed real exchange rate in the one good small open economy model severely limits the government's ability to raise capital income tax revenue, an effect operating through a return arbitrage condition equating the effective return on international bonds with the effective return to capital. This consideration is entirely absent from the closed economy model, and the government chooses a very high capital income tax rate for one year (one model period). While the aforementioned arbitrage condition is present in the two good small open economy model, its effects are tempered by movements in the real exchange rate which allow the effective return on international bonds to respond to the tax reforms.

The scant existing literature on Ramsey-optimal tax reforms has focused on the one good small open economy model; see, for example, Correia (1996). As discussed earlier, an advantage of a fixed exchange rate is that one can cast the problem of the benevolent government planner in terms of feasibility constraints and an implementability condition, thus solving directly for the allocation. However, as shown in this section, fixing the real exchange rate is far from an innocuous assumption.

The results in this section lead to two conclusions. First, considering the international dimension of tax reforms is important: The arbitrage condition in the small open economy models constrains the government's choice for the capital income tax rate in the short term. Second, the real exchange rate dynamics that arise from the two good small open economy model – but not in the one good model – alleviates the effects of this arbitrage constraint. These real exchange rate movements also serve to counterbalance the effects of the rise in the labor income tax rate. As a result, the benchmark two good small open economy model's dynamics more closely resemble those of the closed economy model than the one good small open economy model.

# 6 Alternative Parameter Settings

Most of the parameters in the calibration are well pinned down. Chief among those that are not are: the coefficient of relative risk aversion, the elasticity of substitution between private and public goods, the degree of trade openness, and the elasticity of substitution between home and foreign goods. Implications for alternative values for these parameters are considered in this section.[4]

## Preference Parameters

The benchmark calibration set the coefficient of relative risk aversion, $\sigma$, and the elasticity of substitution between private and public consumption goods, $\psi$, equal to one. The advantage of these parameter values is that they imply that utility is additively separable between

[4] A more exhaustive set of figures with more parameter alternatives are contained in Appendix B.

private consumption, public consumption, and leisure. This separability implies that the cross partial derivatives of the utility function are zero which made it easier to work through the results in Section 5.

To start, consider the effects of a plausibly higher setting for risk aversion: $\sigma = 2$. As shown in Figure 2, the time paths for the government policy variables are little changed relative to the benchmark setting with logarithmic preferences. The paths of private consumption and hours worked are somewhat smoother reflecting the fact that higher risk aversion also implies a lower intertemporal elasticity of substitution. There are virtually no differences in the time paths of the real exchange rate or net exports. The welfare benefit of the tax reform program falls from 3.42% for the benchmark calibration to 2.34%.

There are larger differences in time paths when there is a lower elasticity of substitution between private and public consumption (specifically, $\psi = 1/2$). While the path for the public consumption good is quite similar to that observed for the benchmark calibration, that for private consumption is not, reflecting the lower substitutability between these two goods. To be able to afford this higher path for private consumption, households work more hours relative to the benchmark calibration. The one period spike in the capital income tax rate is, consequently, lower at a mere 295% compared to 418% for the benchmark case. The real exchange rate appreciates more than under the benchmark calibration, and net exports are correspondingly lower. The welfare benefit of the tax reform rises from 3.42% to 5.91%, chiefly due to the higher path for private consumption.

## Trade Parameters

The trade openness parameter, $\gamma$, was set to 0.3 on the basis that the world import share is around 30%. The U.S. is less open than the world as a whole: its import share in the early 2000s is closer to 15%. Here, $\gamma$ is set to 0.05 which corresponds to the U.S. import share circa 1970.

As shown in Figure 3, a less open economy naturally has time paths that look much more like a closed economy than the benchmark model. The one period spike in the capital income tax rate is now 533%, much closer to the closed economy's value of 586% than that observed for the benchmark small open economy model (418%). Due to the higher capital income tax rate, government debt falls more when the economy is less open. As in the closed economy model, labor income is initially subsidized, albeit at a lower rate ($\tau_0^h = -18\%$ rather than $-46\%$). At the start of the tax reform, there is a much stronger depreciation of the real exchange rate, followed by a more modest appreciation relative to the initial real exchange rate. The exchange rate dynamics can be understood through the arbitrage

**Figure 2:** Dynamic Paths For Alternative Preference Parameter Values

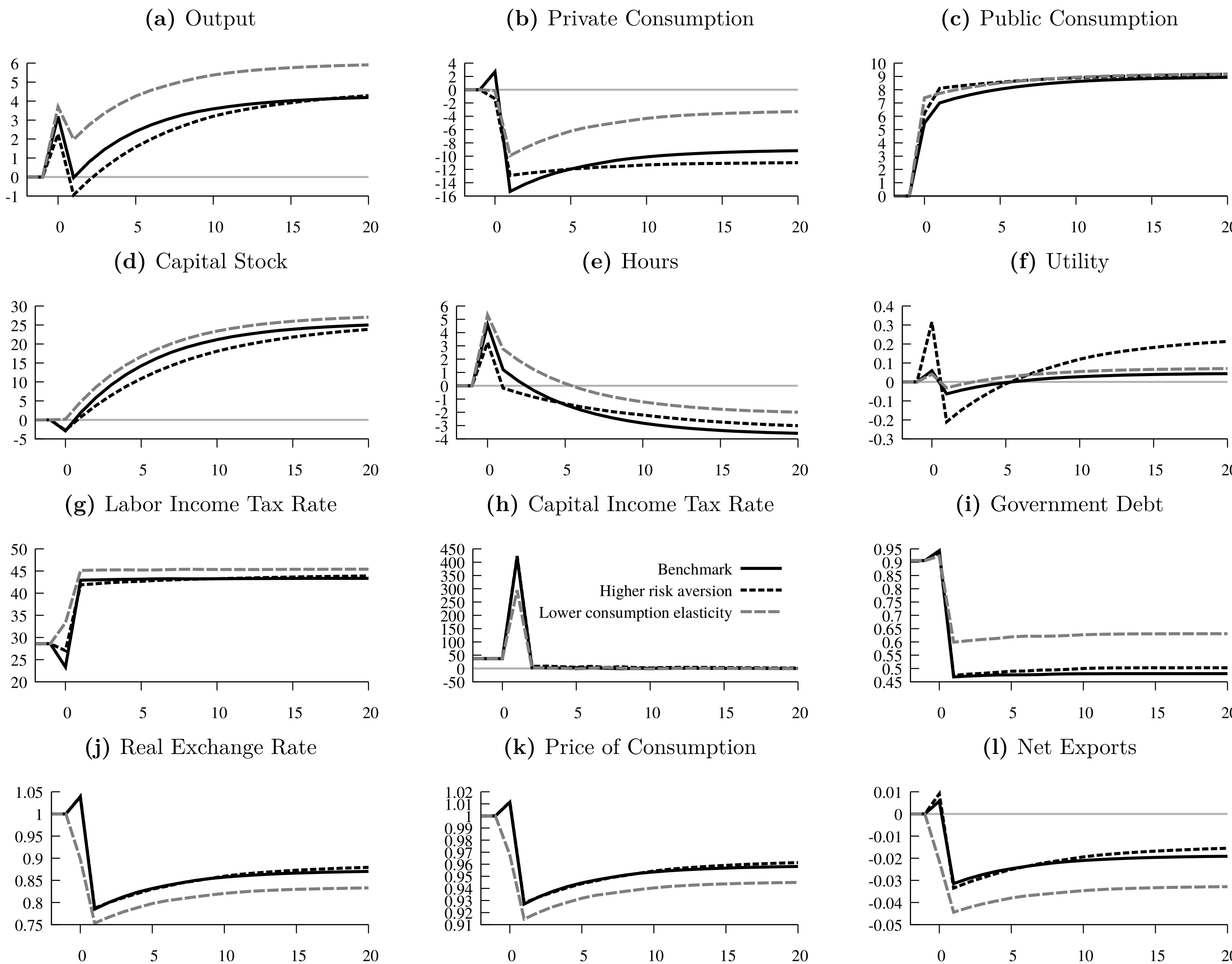
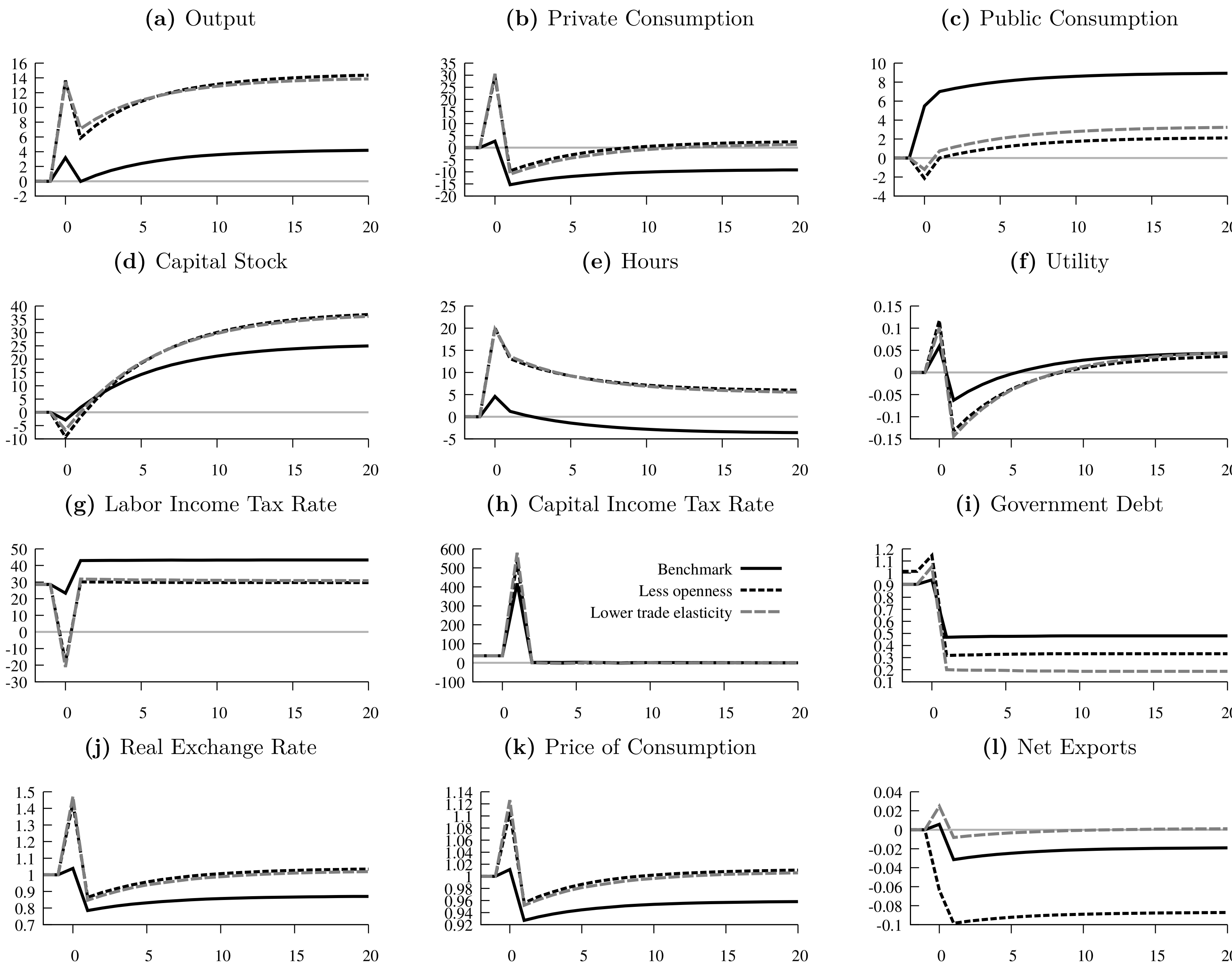


**Figure 3:** Dynamic Paths



condition equating the effective returns on international bonds and capital (see eqs. (30) and (31)). The larger increase in the capital income tax rate in period 1 leads to a similarly larger change in the effective return to capital. Effective return equality then requires a more substantial appreciation in the real exchange rate between dates 0 and 1. Further, given that the economy is far less open, the effects of these real exchange rate movements on the domestic macroeconomy are less onerous. The welfare benefits of the tax reform are now 1.53% of consumption, smaller than both the benchmark model (3.42%) and the closed economy model (3.06%). That the welfare benefits are smaller than the closed economy model likely reflects the fact that the model more closely resembles a closed economy except with the additional constraint on the time 1 capital income tax rate associated with the effective rate of return equality condition.

Finally, the elasticity between domestic and foreign goods in the private consumption aggregator, eq. (10), is $\mu = 1.5$ in the benchmark model, a value used by Backus, Kehoe, and Kydland (1992) and much of the subsequent international finance literature with distinct home and foreign goods. While there is much recent controversy in the empirical literature concerning this elasticity, de Paoli (2009) shows that for international risk sharing what matters is whether the size of this elasticity relative to the Cobb-Douglas case ($\mu = 1$). To this end, the trade elasticity is set even smaller: $\mu = 0.8$. In many ways, this setting for the trade elasticity makes the economy behave like a relatively closed economy – that is, the time paths for macroeconomic variables look quite similar to the previous case in which trade openness, $\gamma$ was set to 0.05. The intuition is that by lowering the trade elasticity, the domestic and foreign sides of the small open economy operate more independently since there is less latitude to substitute between domestic and foreign goods. Compared to the less-openness case, there is a larger one period hike in the capital income tax rate (580%, nearly as high as for the closed economy model) and so a correspondingly lower long run level for government debt. Whereas the less-openness case saw a fall in net exports, there is almost no change when the trade elasticity is lower. The welfare benefit of tax reform is similarly smaller than the benchmark model: 2.07% rather than 3.42%.

## Summary

Relative to the benchmark small open economy model, the alternative preference parameter values considered in this section lead to only modest changes in the time paths of macroeconomic variables. There are far larger differences regarding the alternative values for the trade parameters. Reducing the substitutability between domestic and foreign goods, governed by the parameter $\mu$, has much the same effect as simply making the economy less open (the parameter $\gamma$). The welfare benefits of tax reform are, nonetheless, substantial.

# 7 Conclusion

Two important results arise from the economic analysis above. First, opening an economy to trade imposes an additional constraint on the benevolent government planner in the Ramsey optimal taxation problem: In choosing the tax rate on capital income, the rate of return on capital, net of taxes and adjustment costs, cannot be dominated by the return on available international assets. This consideration is, obviously, absent from closed economy models. Second, real exchange rate movements alter the real return on foreign asset received by domestic households. When the real exchange rate is fixed, there is essentially no scope for the government to lower its debt level through capital income taxation. In a two good small open economy model, real exchange rate movements restore the role of the capital income tax rate to (optimally) reduce the level of government debt. In this way, the results of the two good small open economy model more closely resemble those of a closed economy than the one good small open economy model. In our opinion, the two good small open economy better reflects the economic environment faced by governments, and so the tradeoffs in optimally choosing tax rates and debt.

Solving the two good small open economy model for the optimal set of taxes as well as for the path for government debt presented some technical difficulties. In particular, the endogenous real exchange rate precludes applying the usual procedure of solving for the allocation via an implementability condition. We solved the government's primal problem: maximize lifetime utility of the representative household subject to the equilibrium conditions and the government budget constraint. A further problem relates to the inherent instability of government debt implied by the sequence of government budget constraints. We developed a procedure that addresses all of these issues. This solution procedure is an independent contribution of this paper and opens up the set of economic environments for which optimal taxation issues can be studied.

# A Ramsey Impossibility Conditions

## A.1 Closed Economy with Capital Adjustment Costs

The Lagrangian for the typical household is

$$\begin{aligned}\mathcal{L} = &\sum_{t=0}^{\infty}\Big\{\beta^t U(c_t, g_t, h_t) \\ &+ \lambda_t \Big[(1-\tau_t^h)w_t h_t + R_t^k k_{t-1} + R_{t-1}^d d_{t-1} - (1+\tau_t^c)c_t - k_t - d_t - \frac{\eta}{2}(k_t - k_{t-1})^2\Big]\Big\}.\end{aligned} \tag{33}$$

The associated first-order conditions are:

$$c_t: \quad \beta^t U_c(c_t, g_t, h_t) = \lambda_t(1+\tau_t^c) \tag{34}$$

$$h_t: \quad \beta^t U_h(c_t, g_t, h_t) + \lambda_t(1-\tau_t^h w_t = 0 \tag{35}$$

$$k_t: \quad [1+\eta(k_t - k_{t-1})]\,\lambda_t = \lambda_{t+1}\left[R_{t+1}^k + \eta(k_{t+1} - k_t)\right] \tag{36}$$

$$d_t: \quad \lambda_t = \lambda_{t+1} R_t^d. \tag{37}$$

Substitute the first-order conditions into the second term in eq. (33):

$$\begin{aligned}&\sum_{t=0}^{\infty}\lambda_t \left[(1-\tau_t^h)w_t h_t + R_t^k k_{t-1} + R_{t-1}^d d_{t-1} - (1+\tau_t^c)c_t - k_t - d_t - \frac{\eta}{2}(k_t - k_{t-1})^2\right] \\ &= -\sum_{t=0}^{\infty}\beta^t\left[U_h(c_t, g_t, h_t)h_t + U_c(c_t, g_t, h_t)c_t\right] + R_0^k k_{-1} + R_{-1}^d d_{-1} \\ &\quad - \sum_{t=0}^{\infty}\beta^t \frac{U_c(c_t, g_t, h_t)}{1+\tau_t^c}\left[\eta(k_t - k_{t-1})(k_t - k_{t-1}) - \frac{\eta}{2}(k_t - k_{t-1})^2\right] \\ &= -\sum_{t=0}^{\infty}\beta^t\left[U_h(c_t, g_t, h_t)h_t + U_c(c_t, g_t, h_t)c_t\right] + R_0^k k_{-1} + R_{-1}^d d_{-1} \\ &\quad - \sum_{t=0}^{\infty}\beta^t \frac{U_c(c_t, g_t, h_t)}{1+\tau_t^c}\left[\frac{\eta}{2}(k_t - k_{t-1})^2\right].\end{aligned}$$

## A.2 Two Good Small Open Economy

The Lagrange-an for the household:

$$\begin{aligned}\mathcal{L} = \sum_{t=0}^{\infty} \big\{ & \beta^t U(c_t, g_t, h_t) \\ & + \lambda_t \big[ (1-\tau_t^h) w_t h_t + R_t^k k_{t-1} + R_{t-1}^d d_{t-1} + e_t R_{t-1}^b b_{t-1} \\ & \quad - (1+\tau_t^c) p_t c_t - k_t - d_t - b_t \big] \big\}.\end{aligned} \tag{38}$$

The associated first-order conditions are:

$$c_t: \quad \beta^t U_c(c_t, g_t, h_t) = \lambda_t (1+\tau_t^c) p_t \tag{39}$$

$$h_t: \quad \beta^t U_h(c_t, g_t, h_t) + \lambda_t (1-\tau_t^h w_t = 0 \tag{40}$$

$$k_t: \quad \lambda_t = \lambda_{t+1} R_{t+1}^k \tag{41}$$

$$d_t: \quad \lambda_t = \lambda_{t+1} R_t^d \tag{42}$$

$$b_t: \quad e_t \lambda_t = \lambda_{t+1} R_t^b e_{t+1}. \tag{43}$$

Substitute the first-order conditions into the second sum in eq. (38):

$$\begin{aligned} & \sum_{t=0}^{\infty} \lambda_t \left[ (1-\tau_t^h) w_t h_t + R_t^k k_{t-1} + R_{t-1}^d d_{t-1} + e_t R_{t-1}^b b_{t-1} - (1+\tau_t^c) p_t c_t - k_t - d_t - b_t \right] \\ & = -\sum_{t=0}^{\infty} \beta^t \left[ U_h(c_t, g_t, h_t) h_t + U_c(c_t, g_t, h_t) c_t \right] + \frac{U_c(c_0, g_0, h_0)}{p_t (1+\tau_t^c)} \left[ R_0^k k_{-1} + R_{-1}^d d_{-1} + R_{-1}^b b_{-1} e_0 \right] \\ & = 0. \end{aligned}$$

In this case, the implementability condition is

$$\sum_{t=0}^{\infty} \beta^t \left[ U_h(c_t, g_t, h_t) h_t + U_c(c_t, g_t, h_t) c_t \right] = \frac{U_c(c_0, g_0, h_0)}{p_t (1+\tau_t^c)} \left[ R_0^k k_{-1} + R_{-1}^d d_{-1} + R_{-1}^b b_{-1} e_0 \right]. \tag{44}$$

The government's problem is:

$$
\begin{aligned}
\mathcal{L} = &\sum_{t=0}^{\infty} \beta^t U(c_t, g_t, h_t) \\
&+ \lambda \left\{ \sum_{t=0}^{\infty} \beta^t \left[ U_h(c_t, g_t, h_t) h_t + U_c(c_t, g_t, h_t) c_t \right] \right. \\
&\qquad \left. - \frac{U_c(c_0, g_0, h_0)}{p_t(1+\tau_t^c)} \left[ R_0^k k_{-1} + R_{-1}^d d_{-1} + R_{-1}^b b_{-1} e_0 \right] \right\} \\
&+ \phi_t \left[ F(k_{t-1}, h_t) - \varphi p_t^\mu - \gamma e_t^\mu c_t^* - k_t + (1-\delta) k_{t-1} - g_t \right].
\end{aligned} \tag{45}
$$

The problem in this case lies not with the implementability condition, eq. (44), but rather the feasibility constraint in the government's problem where $e_t$ and $p_t$ appear.

## A.3 Two Good Small Open Economy, No Consumption Aggregator

Perhaps the problem with the exchange rate and relative price of the consumption good appearing in the government's feasibility constraint is an artifact of using an aggregator over home and foreign goods. To investigate this possibility, preferences are now defined over home and foreign goods directly.

The Lagrangian for the household:

$$
\begin{aligned}
\mathcal{L} = \sum_{t=0}^{\infty} \Big\{ &\beta^t U(c_{ht}, c_{ft}, g_t, h_t) \\
&+ \lambda_t \Big[ (1-\tau_t^h) w_t h_t + R_t^k k_{t-1} + R_{t-1}^d d_{t-1} + e_t R_{t-1}^b b_{t-1} \\
&\qquad - (1+\tau_t^c) \left[ c_{ht} + e_t c_{ft} \right] - k_t - d_t - b_t \Big] \Big\}.
\end{aligned} \tag{46}
$$

The associated first-order conditions are:

$$
\begin{aligned}
c_{ht}: &\quad \beta^t U_1(c_{ht}, c_{ft}, g_t, h_t) = \lambda_t (1+\tau_t^c) && (47) \\
c_{ft}: &\quad \beta^t U_2(c_{ht}, c_{ft}, g_t, h_t) = \lambda_t (1+\tau_t^c) e_t && (48) \\
h_t: &\quad \beta^t U_4(c_t, g_t, h_t) + \lambda_t (1-\tau_t^h w_t = 0 && (49) \\
k_t: &\quad \lambda_t = \lambda_{t+1} R_{t+1}^k && (50) \\
d_t: &\quad \lambda_t = \lambda_{t+1} R_t^d && (51) \\
b_t: &\quad e_t \lambda_t = \lambda_{t+1} R_t^b e_{t+1}. && (52)
\end{aligned}
$$

Following steps as above, the implementability condition reads:

$$\sum_{t=0}^{\infty} \beta^t \left[ U_4(c_{ht}, c_{ft}, g_t, h_t)h_t + U_1(c_{ht}, c_{ft}, g_t, h_t)c_{ht} + U_2(c_{ht}, c_{ft}, g_t, h_t)c_{ft} \right] \\ = \frac{U_c(c_0, g_0, h_0)}{p_t(1+\tau_t^c)} \left[ R_0^k k_{-1} + R_{-1}^d d_{-1} + R_{-1}^b b_{-1} e_0 \right]. \tag{53}$$

Again, no particular problem here.

To obtain the feasibility constraint, combine the household's budget constraint with the government's budget constraint, here written as

$$d_t + \tau_t^c (c_{ht} + e_t c_{ft}) + \tau_t^h w_t h_t + \tau_t^k (r_t - \delta) k_{t-1} = g_t + R_{t-1}^d d_{t-1}. \tag{54}$$

Combining these budget constraints gives

$$c_{ht} + e_t c_{ft} + k_t + e_t b_t + g_t = y_t + (1-\delta)k_{t-1} + e_t R_{t-1}^b b_{t-1}. \tag{55}$$

Domestic net exports are $c_{ht}^* - e_t c_{ft}$. Thus,

$$e_t b_t - e_t R_{t-1}^b b_{t-1} = c_{ht}^* - e_t c_{ft}. \tag{56}$$

Substituting into eq. (55),

$$c_{ht} + c_{ht}^* + k_t + g_t = y_t + (1-\delta)k_{t-1}. \tag{57}$$

The problem that arises in this case is that in the Ramsey problem, the government's choice of foreign consumption goods, $c_{ft}$, is unconstrained. The government would, then, try to set $c_{ft}$ as large as possible, which is clearly not feasible with the world feasibility constraint (which the domestic government does not take into account).

# B Additional Figures

This appendix includes additional figures to complement those reported in section 6. Figure 4 reports model solutions for both higher ($\psi = 2$) and lower ($\psi = 1/2$) elasticities of substitution between private and public goods. Figure 5 presents paths for the two good small open economy with both lower trade openness ($\gamma = 0.05$) and more trade openness ($\gamma = 0.8$). Finally, Figure 6 facilitates comparison of the lower trade openness case with the closed economy model.

**Figure 4:** Dynamic Paths For Alternative Elasticities of Substitution Between Private and Public Goods

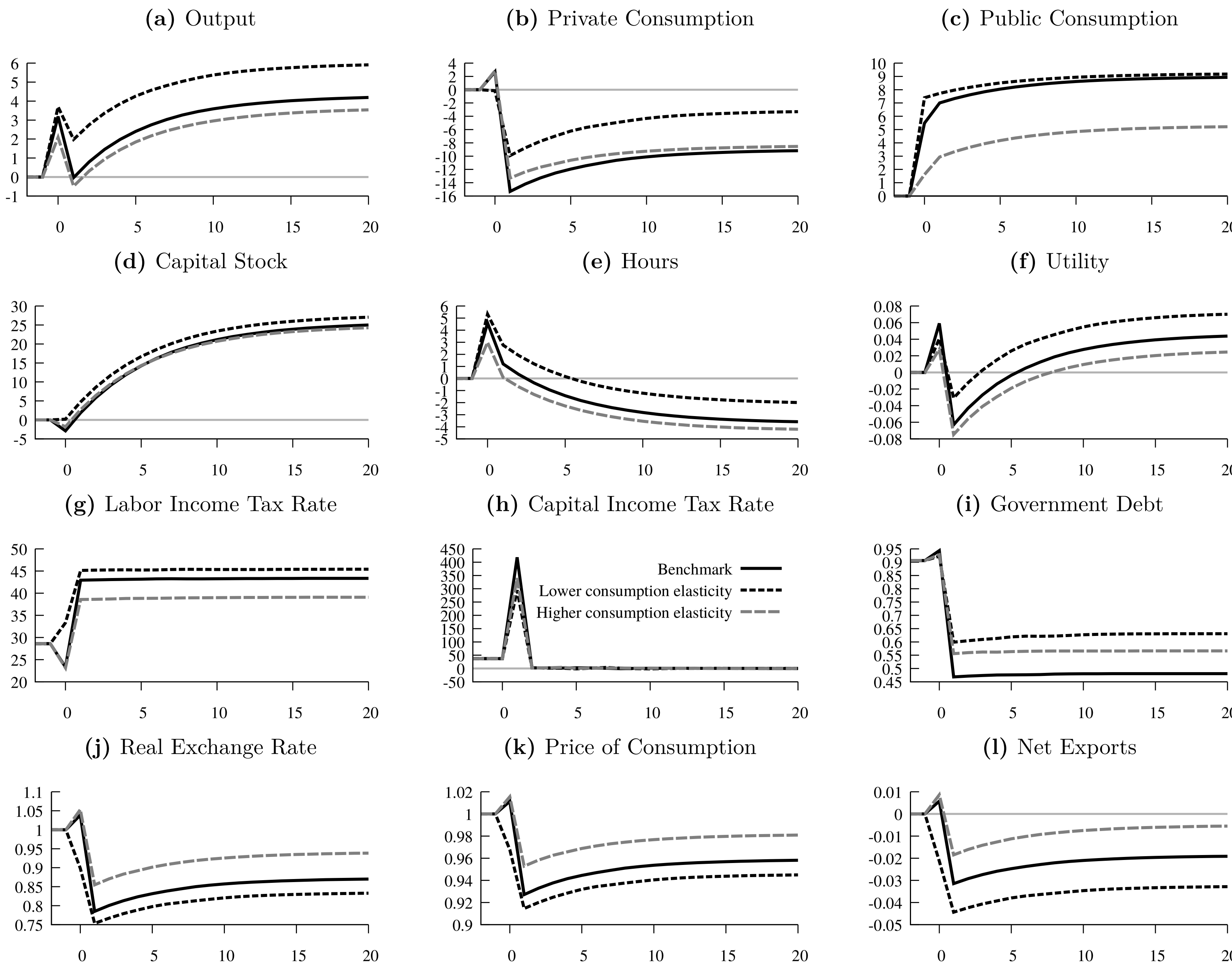


**Figure 5:** Dynamic Paths For Alternatives Settings for Trade Openness

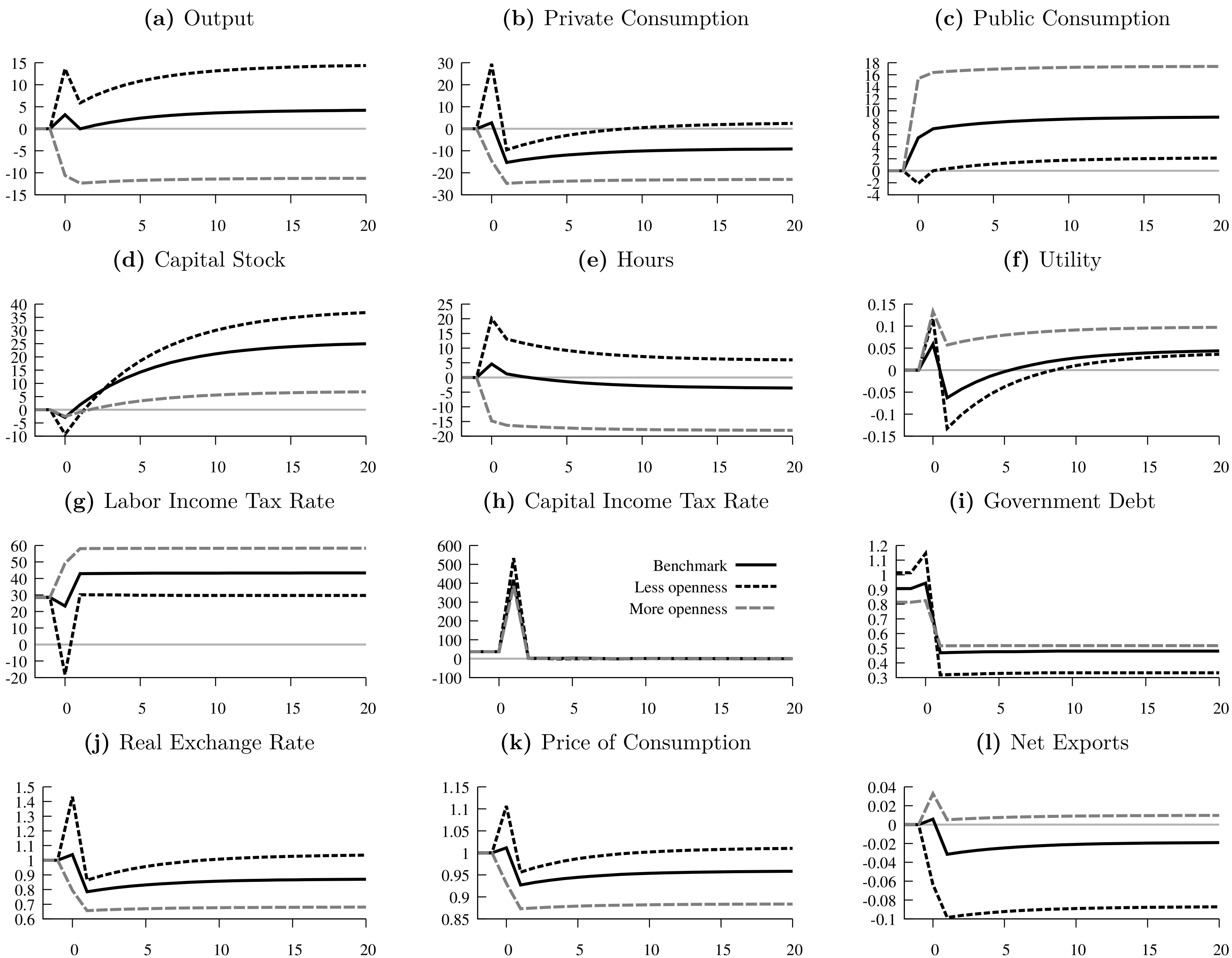
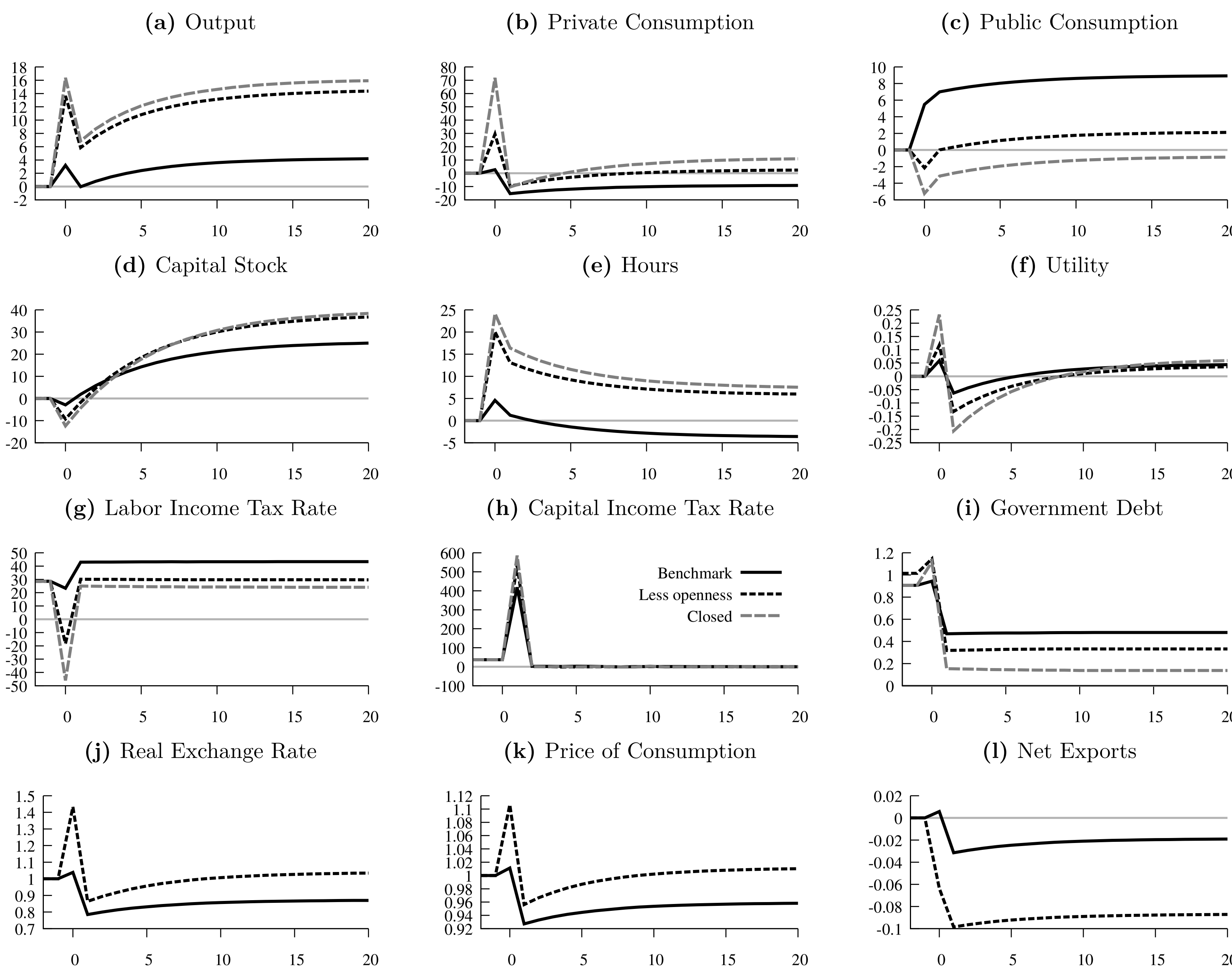


**Figure 6:** Dynamic Paths: Less Trade Openness Versus a Closed Economy



# References

Auray, Stéphane, Aurélien Eyquem, and Paul Gomme (2016). “A Tale of Tax Policies in Open Economies”. *International Economic Review* 57 (4), pp. 1299–1333.

Auray, Stéphane, Aurélien Eyquem, and Paul Gomme (2017). “Debt Hangover in the Aftermath of the Great Recession”. Working Paper 17-001. Concordia University.

Backus, David K., Patrick J. Kehoe, and Finn E. Kydland (1992). “International Real Business Cycles”. *The Journal of Political Economy* 100 (4), pp. 745–775.

Benigno, Gianluca and Bicanca de Paoli (2010). “On the International Dimension of Fiscal Policy”. *Journal of Money, Credit and Banking* 42 (8), pp. 1523–1542.

Chamley, Christophe (1986). “Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives”. *Econometrica* 54 (3), pp. 607–622.

Chari, V. V., Patrick J. Kehoe, and Ellen R. McGrattan (2000). “Sticky Price Models of the Business Cycle: Can the Contract Multiplier Solve the Persistence Problem?” *Econometrica* 68 (5), pp. 1151–1179.

Correia, Isabel H. (1996). “Dynamic Optimal Taxation in Small Open Economies”. *Journal of Economic Dynamics and Control* 20 (4), pp. 691–708.

de Paoli, Bicanca (2009). “Monetary Policy and Welfare in a Small Open Economy”. *Journal of International Economics* 77 (1), pp. 11–22.

Gomme, Paul and Peter Rupert (2007). “Theory, Measurement, and Calibration of Macroeconomic Models”. *Journal of Monetary Economics* 54 (2), pp. 460–497.

Judd, Kenneth L. (1985). “Redistributive Taxation in a Simple Perfect Foresight Model”. *Journal of Public Economics* 28, pp. 59–83.

Lucas Jr., Robert E. and Nancy L. Stokey (1983). “Optimal Fiscal and Monetary Policy in an Economy without Capital”. *Journal of Monetary Economics* 12 (1), pp. 55–93.

Ramsey, Frank P. (1927). “A Contribution to the Theory of Taxation”. *The Economic Journal* 37 (145), pp. 47–61.