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Unemployment Insurance Take-up Rates in an Equilibrium Search Model *

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Abstract

From 1989 – 2012, on average 23% of those eligible for unemployment insurance (UI) benefits in the US did not collect them. In a search model with matching frictions, asymmetric information associated with the UI non-collectors implies an inefficiency in non-collector outcomes. This inefficiency is characterized along with the key features of collector vs. non-collector allocations. Specifically, the inefficiency implies that non-collectors transition to employment at a faster rate and a lower wage than the efficient levels. Quantitatively, the inefficiency amounts to 1.71% welfare loss in consumption equivalent terms for the average worker, with a 3.85% loss conditional on non-collection. With an endogenous take-up rate, the unemployment rate and average duration of unemployment respond significantly slower to changes in the UI benefit level, relative to the standard model with a 100% take-up rate.

Keywords: unemployment insurance, take-up, matching frictions, search, experience rating
JEL classification: E61, J32, J64, J65

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1 Introduction

The unemployed *not* collecting benefits they are eligible for may represent the most important issue in the U.S. unemployment insurance system. The existing literature on unemployment insurance (UI) has focused primarily on incentive problems, such as moral hazard, and recently on the effects of UI benefit extension programs (e.g. Nakajima (2012) or Hagedorn, Karahan, Manovskii, and Mitman (2013)). In the U.S., from 1989 – 2012, UI fraud and benefit extensions amounted to 2.4% and 13% of total benefit expenditures on average, respectively.[1] According to our analysis, the "unclaimed" benefits from eligible unemployed not collecting benefits are nearly double the *combined* expenditures on UI fraud and benefit extensions.

In addition, the empirical UI literature has found strong evidence that UI collectors and non-collectors have different unemployment outcomes. Most notably, UI non-collectors tend to have much shorter unemployment durations (see for example Katz and Meyer (1990) or Braun, Engelhardt, Griffy, and Rupert (2016)). Given that UI collectors and non-collectors have such heterogeneous outcomes, accounting for the take-up decision appears crucial to understanding the impact of UI benefits on equilibrium outcomes. Any change in UI benefits alters not only the behavior and outcomes of UI collectors, but it may also impact how many collect benefits and the behavior and outcomes of the non-collectors. The analysis in this paper shows this to be an important consideration. Our contribution includes a calculation of the fraction of eligible unemployed collecting UI (hereafter "take-up rate"), an equilibrium model incorporating the take-up decision, and an exploration of the implications of unclaimed benefits.

While many U.S. labor market statistics are tabulated and readily available from the Bureau of Labor Statistics (BLS), there exists little information regarding the UI take-up rate. Building on the methodology of Blank and Card (1991), an estimate of the UI take-up rate is calculated from 1989 – 2012. The calculation uses the March supplement of

[1] UI fraud includes issues such as concealed earnings, insufficient job search, job offer refusals, and quits, among others. See Fuller, Ravikumar, and Zhang (2015) for further details on the issue of UI fraud.

the Current Population Survey (CPS) along with detailed eligibility criteria by U.S. state. Over the $1989 - 2012$ time period, the take-up rate averaged 77%. Given these estimates of the take-up rate, the analysis then develops an equilibrium search model to explain the determinants and implications of this take-up rate.

The take-up decision is captured using a search model with matching frictions where risk-averse workers direct their search to the optimal wage and arrival rate combinations offered by risk-neutral firms.[2] Workers are heterogenous in their direct utility cost of collecting UI benefits. These costs, along with past UI collections, are private information for the worker. This informational friction combined with the UI financing scheme imply an inefficiency in equilibrium.

Specifically, we model the "experience rating" feature of UI taxes in the U.S., where firms finance UI benefits with the specific tax rate depending on their employees contributions to UI expenditures (see Feldstein (1976), Topel (1983), Albrecht and Vroman (1999), Wang and Williamson (2002), and Cahuc and Malherbet (2004), among others for analyses of experience rating). We demonstrate analytically how the informational friction implies that equilibrium is inefficient, taking as given the experience rating feature of the U.S. system. Moreover, we show that while UI collectors search efficiently, the inefficiency occurs because non-collector search behavior is distorted.

In the model, firms maximize profits by offering different wages depending on whether or not the worker prefers to collect UI benefits in the event of a future separation. They know the distribution of workers across UI collection costs, but do not observe whether or not the worker has collected in the past. In general, this is not problematic for the firm. They simply offer wages that maximize each type of worker's expected lifetime utility given the expected queue lengths associated with each wage. This would imply a natural "separating" equilibrium. The natural separation arises from the effects of different consumption levels with risk-averse workers. All else equal, UI collectors enjoy higher consumption while unemployed relative to a UI non-collector. Thus, UI collectors search for jobs offering rel-

[2]Rogerson, Shimer, and Wright (2005) offer an overview of a directed search environment and the related literature.

atively high wages, but longer unemployment durations (*i.e.* slower job arrival rates). In contrast, UI non-collectors prefer to search for relatively low wage jobs with shorter average unemployment durations.

In an experience rated system, however, this natural separation is distorted when UI collection costs are private information. With the UI tax accumulating only to those firms hiring a future UI collector, for an equivalent arrival rate, the UI non-collector has a higher wage than a collector. Thus, for some range of UI collection costs a UI collector may find it beneficial to collect benefits but search for the non-collector wage. Indeed, we show that this is true in equilibrium with private information.

We characterize how firms manage this incentive problem. In equilibrium, it implies that UI non-collector wages and arrival rates are distorted from the efficient levels. Relative to efficiency, non-collectors have a lower wage and shorter unemployment duration. This distortion makes the non-collector job (wage and queue length) less appealing to a UI collector.

To understand the magnitude of the inefficiency, we use our estimates of the UI take-up rate to calibrate the model to U.S. data and quantify the welfare consequences of the distortions. Given the observed take-up rate and level of experience rating, the non-collector distortion amounts to a welfare loss of 1.71% in consumption equivalent terms for the average worker. Conditional on being a non-collector, the welfare loss is much higher at 3.85%. The welfare loss is measured relative to the perfect information economy, which we also characterize analytically.

Interestingly, our analytical and quantitative results show the take-up rate is lower with perfect information relative to the economy with asymmetric information. Thus, to avoid paying the experience rated tax, firms prefer to dissuade some workers from collecting UI benefits by offering more appealing wages. When the idiosyncratic UI collection costs are not observable by the firm, however, their ability to provide an attractive alternative to collecting UI benefits is reduced; as a result, the take-up rate remains higher in the private information economy.

The analysis also focuses on how incorporating the UI take-up decision affects the impact of UI benefits on equilibrium outcomes. That is, how does an increase in UI benefits affect moments such as the unemployment rate and average duration of unemployment? We find that allowing for an endogenous take-up rate has important implications. Specifically, while an increase in UI benefits does imply an increase in both the unemployment rate and average duration of unemployment, these two moments respond slower relative to a standard search model with a fixed 100% take-up rate. This occurs in part because the average duration of unemployment actually decreases for non-collectors when UI benefits increase.

Indeed, the effect of UI benefits on the search behavior of UI collectors and non-collectors represents an important aspect of our analysis. As discussed above, UI collectors prefer longer durations and higher wages relative to non-collectors. Acemoglu and Shimer (1999, 2000) present a related finding: as UI benefits increase, workers prefer to search for higher wage jobs arriving less frequently.[3] They show how this feature may lead to UI benefits increasing productivity. We abstract from the productivity dimension, focusing instead on modeling the take-up decision and its implications. On this dimension, we explore an inefficiency in equilibrium outcomes not present in the models of Acemoglu and Shimer (1999, 2000).

The remainder of the paper proceeds as follows. In Section 2 we present information on the U.I. system in the U.S. and our procedure for estimating the take-up rate. Section 3 describes the model and equilibirum, and in Section 4 we analytically derive the key properties of equilibrium. Section 5 presents the calibration, policy experiments, and quantitative welfare results. Section 6 concludes.

2 Evidence on take-up rates

This section has two objectives: a description of the relevant features of the U.S. system and a description of our take-up rate estimation and exploration of its key features.

[3] Marimon and Zilibotti (1999) also examine the implications of UI benefits on productivity, focusing on how UI benefit differences may explain differences in U.S. and European labor market outcomes.

2.1 Unemployment Insurance System in the U.S.

Unemployment benefits in the U.S. are financed by a tax levied on firms, and this tax is "experienced rated". Firms pay a tax rate that is positively correlated with their contribution to *insured* unemployment in their particular U.S. state. For example, a firm that has never separated from a worker who collects benefits pays a lower tax rate than a firm that has frequent layoffs who collect benefits. Note, for the firm's tax rate, it does not matter how frequently they separate from workers, but how frequently they separate from workers who collect benefits.

In addition, each state has a maximum and minimum tax rate. This implies that the U.S. system has "partial" experience rating. A firm at the maximum tax rate will not see its tax rate increase in response to an increase in its contribution to UI expenditures. In this regard, firms at the maximum tax rate are subsidized. Similarly, a firm at the minimum tax rate may contribute to the UI fund while rarely sending an employee to insured unemployment. Indeed, this partial experience rating feature may have important impacts on labor market outcomes. For example, the work of Feldstein (1976) and Topel (1983) explore how this feature may influence firm layoff decisions. While we abstract here from the firm layoff decision, this previous work suggests important potential impacts from partial experience rating on this dimension.

2.2 Take-up rate estimates

While many statistics and data on the labor market are readily available for public use, there exists little information on take-up rates of unemployment insurance. There is data on the characteristics of the insured unemployed (those collecting benefits), as well as data on the ratio of insured unemployed to total unemployed (hereafter IUR). While this provides some characterization of the take-up rate, the IUR does not control for eligibility. That is, many of the unemployed are not eligible to collect benefits. To calculate the *take-up rate*, we first find the fraction of unemployed agents who are currently eligible to collect, and then take the ratio of insured unemployed to *eligible* unemployed.

We follow a method similar to Blank and Card (1991). Specifically, we start with IUR data, which refers to those collecting "Regular Program" benefits. These are the 26 weeks of benefits primarily financed by each state, and exclude any extended benefit programs financed by the state or federal government. We use the IUR series tabulated by the U.S. Department of Labor, which can be found at: http://workforcesecurity.doleta.gov/unemploy/chartbook.asp.

To determine the fraction of unemployed eligible for regular program benefits, we use data from the March Supplement of the CPS, along with the specific eligibility criteria of each state, for each year from 1989 – 2012.[4] The take-up rate is calculated as the ratio of the IUR to the Fraction of Unemployed Eligible. Figure 1 displays the IUR, our estimate of the take-up rate, and a decomposition of the different eligibility criteria.

Eligibility depends primarily on three factors, all of which are determined at the state level. First, as mentioned above, there is a fixed duration that an individual may collect benefits for. In the majority of states, regular program benefits have a potential duration of 26 weeks. In all of the years studied, Massachusetts and Washington have a maximum potential benefit duration of 30 weeks. Beginning in 2004, Montana has a maximum potential benefit duration of 28 weeks. Again, these potential durations refer to the *Regular Program* benefits, and thus exclude any extended benefit programs. Of course, being unemployed for longer than 26 weeks does not necessarily make an individual ineligible. The key issue is whether or not the individual exhausted their regular program benefits (*i.e.* already collected benefits for the maximum potential duration).

To control for this eligibility criteria, we use the information in the March CPS about whether an individual collected benefits in the previous year or not. If an individual is unemployed in March of a given year and has expired regular program benefits, then they have been unemployed for longer than 26 weeks (accounting for differences in Massachusetts, Washington, and Montana where applicable) and must have collected benefits in the previous

[4] Blank and Card (1991) and Anderson and Meyer (1997) provide estimates of the take-up rate prior to 1989.

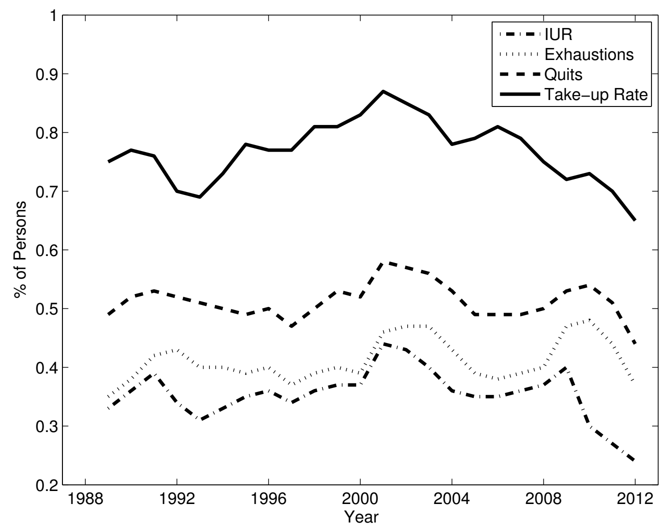


Figure 1: Take-up Rates by Eligibility Criteria Over Time

The bottom line labeled "IUR" is the ratio of insured unemployed to total unemployed. As the lines progress, unemployed individuals are eliminated from the denominator based on different eligibility criteria. Thus, the gap between lines illustrates roughly how many unemployed are ineligible for each criteria. A larger gap between lines indicates a larger number of unemployed ineligible for a certain criteria. "Exhaustions" removes to those ineligible because they exhausted their benefits and "Quits" removes those who are ineligible because they quit the job. The jump from the "Quits" line to the "Take-up Rate" line occurs when those unemployed who are ineligible because they do not meet the monetary requirements are removed. Thus, the "Take-up Rate" line plots the fraction of eligible unemployed collecting benefits.

year.[5] We consider such individuals as ineligible. In addition to the maximum length of benefits, many states also have a minimum waiting period, typically 1 week, and we control for this criteria where applicable.

In Figure 1 we plot the take-up rate along with a decomposition of the three eligibility criteria. The line labeled "IUR" is the ratio of insured unemployed to total unemployed. As the lines progress, we remove some unemployed individuals from the denominator (total unemployed) until we reach the number of unemployed eligible for benefits.

The line labeled "Exhaustions" removes from the total unemployed those who have exhausted their benefits. On average, over the period from $1989-2012$, 11% of those ineligible for benefits were deemed to have exhausted benefits.[6] As expected, this criteria has a cyclical

[5]In some states, certain individuals may have potential benefit durations less than 26 weeks, depending on their particular circumstances. The most common potential duration, however, is 26 weeks.

[6]Note, this fraction will not match with the BLS series calculating the fraction of collectors who exhaust

contribution, with more individuals exhausting benefits during periods of high unemployment. For example, in 2010, 31% of those ineligible were due to exhausted benefits.

The nature of the separation leading to the spell of unemployment represents the second element of eligibility criteria. Specifically, in most states, individuals who quit their previous job, or were fired for cause, are not eligible to collect benefits. In certain years, Georgia is an exception and does allow job leavers (quits) to collect benefits, but they face an increased waiting period before eligible. This criteria is intended to limit benefits to only those individuals who have lost their job through no fault of their own. In the CPS data, we can eliminate quits; however, we cannot determine whether or not the agent was fired for cause. We also use information on an individual's industry to focus only on covered employment. As in Blank and Card (1991), we eliminate postal workers, federal public administration workers, and ex-service persons, as this group is not eligible to collect UI benefits. In Figure 1, the line labeled "Quits" shows the contribution of this eligibility criteria. On average, 19% were ineligible because they quit their previous job.

Finally, there exist monetary eligibility requirements. These require an agent to have accumulated a sufficient amount of earnings in a specified "base-period," or worked a minimum number of weeks.[7] To estimate monetary eligibility, we use the earnings information contained in the March CPS, along with the state-level monetary eligibility requirements. Such monetary requirements vary significantly across states.

There are several different varieties of monetary eligibility. Perhaps the most standard is to require base period wages that exceed some multiple of the weekly benefit amount. The weekly benefit amount (WBA) is the benefit the worker would be entitled to, which is based on these previous earnings. For example, in 1989, Colorado required base period wages to exceed 40 times the WBA. Determining eligibility with this type of criteria also requires estimating the WBA for the individual; each state also has specific rules determining the

benefits, but is strongly correlated with it. What we measure are those individuals who exhaust benefits *and* are still unemployed as of March in that year. This fraction is necessarily below the fraction that exhaust benefits.

[7]The base-period differs across states. Many use a year, while others use two quarters. The base-period is used both to determine monetary eligibility and to calculate the specific benefit an individual is entitled to.

WBA given earnings. As another example, in 1989, California required base period earnings of at least $\$1,200$.

High Quarter Earnings (HQE) represents an important quantity for monetary eligibility in some states. This also represents a drawback to using the March CPS earnings information (Blank and Card (1991) also discuss these drawbacks). Since it only details earnings during the previous year, HQE cannot be determined. In some states, eligibility is based on earnings outside of the HQE. For example, in 1989, Georgia required base period earnings greater than 1.5 times the HQE. In such cases, we are unable to determine monetary eligibility.[8]

In Figure 1, moving from the "Quits" line to the take-up rate displays the contribution of monetary requirements to the number of unemployed deemed ineligible for benefits. On average, 71% of those deemed ineligible failed to satisfy their state's monetary eligibility requirements.

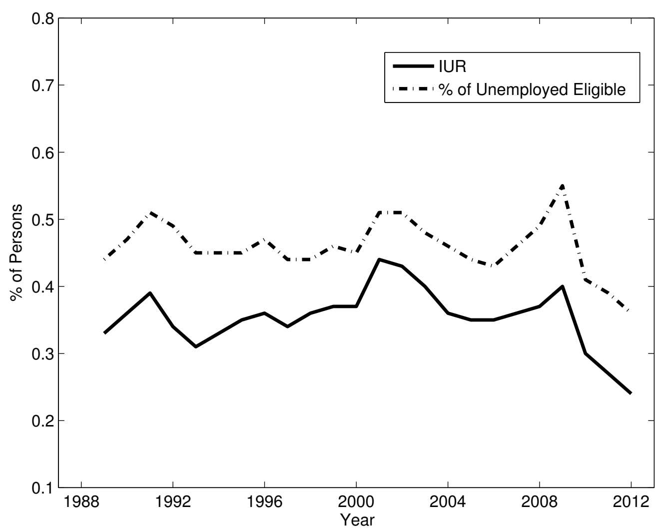


Figure 2: Insured Unemployed and Fraction of Unemployed Eligible

Figure 2 plots the IUR and the fraction of unemployed eligible for benefits. When the

[8]Using weeks worked represents one possible way to proxy for this type of eligibility. For example, in the case of Georgia above, we could require total weeks worked in the previous year to exceed 19.5 ($1.5*13$). This assumes constant earnings over the year, and simply requires the individual to have worked more than one quarter. We have implemented this alternative and it has a negligible impact on the fraction of unemployed eligible for benefits.

two lines move closer together, the take-up rate increases, and it decreases when the lines diverge. The Fraction of Unemployed Eligible for benefits displays a similar cyclical pattern to the IUR.

2.3 Reasons for Non-collection

The estimates above imply that from 1989 – 2012, on average 23% of those eligible for UI benefits did not collect them. One may ask what are the reasons for non-collection? Given the eligibility criteria discussed above, there clearly exist some costs to applying for UI benefits and thus verifying eligibility. The exact nature of these costs and the exact reason(s) for non-collection has not been determined or well-documented in the literature. Anderson and Meyer (1997) cite some survey results offering possible reasons for non-collection, but no particular reason dominates. Of course, an individual does not collect UI benefits if they believe the net benefit to doing so is negative. In our model below, we simply model this as a per-period utility cost of collecting UI benefits.[9]

3 Model

The economy consists of a unit-measure of infinitely-lived, risk-averse workers, and a large measure of risk-neutral firms. Time is continuous and goes on forever, and both agents and firms discount the future at rate $r > 0$. Workers have preferences over consumption, with flow utility given by $h(c)$, where c represents consumption. Firms are composed of a single job, either filled or vacant. Vacant firms are free to enter and pay a flow cost, $\gamma > 0$, to advertise a vacancy. Vacant firms produce no output. The flow output of a firm with a filled job is given by y. There are several components of the model to specify. We begin by describing the key features of how UI is modeled.

[9] Auray and Fuller (2016) explore possible micro-foundations for these utility costs associated with participating in the UI system.

3.1 Unemployment Insurance

As discussed in Section 2.3, we assume that applying for UI benefits and verifying eligibility imply a flow utility cost to workers who collect UI. Furthermore, we assume that this flow utility cost is additively separable and occurs each period the worker collects UI benefits.[10] This mirrors the feature of the U.S. system where workers must re-apply for benefits each week they are unemployed. Workers are heterogenous with respect to their costs of collecting UI benefits, which is denoted by ε. Let $F(\varepsilon)$ denote the distribution of workers over ε. If a worker collects UI benefits, they receive flow consumption b which only expires if the worker transitions to employment. If the worker decides not to collect UI benefits while unemployed, they receive flow consumption d, where $b > d$. Thus, each period of unemployment a UI-collector with collection cost ε receives flow utility of $h(b) - \varepsilon$ while a non-collector receives $h(d)$.[11]

Unemployment benefits are financed by lump-sum taxes levied on firms. These taxes are experienced rated in the following manner. If a firm separates from a worker who collects UI benefits, the firm pays a flow cost of τ. The value of τ determines the marginal cost to a firm of sending a worker to insured unemployment.

In addition, we assume that the worker's UI collection cost, ε, is private information, known only to the worker. Moreover, the firm does not observe whether or not the worker collected UI in the past. Since ε is permanent, knowledge of UI collection history would enable the firm to infer ε. The firm does know the distribution of ε, $F(\varepsilon)$.

Given this information structure, we have an environment with both moral hazard and adverse selection. Moral hazard arises as firms are unable to observe/control workers' actions regarding what jobs to search for. Adverse selection arises from the different types of workers', with types hidden from firms. The inefficiency we describe below hinges on the adverse

[10] Assuming a one-time, upfront cost of collecting UI benefits represents the alternative. Either assumption, per-period flow cost or one-time upfront flow cost deliver the same results, since we assume the cost is fixed for the worker's lifetime.

[11] We assume that UI benefits last forever. Since the flow cost of collecting UI benefits is constant each period the worker collects, the potential duration of UI benefits does not affect the take-up decision. If the worker finds collecting b in the current period beneficial given the costs, they will regardless of how long UI benefits last for.

selection problem. Indeed, the moral hazard problem is always present in directed search models. In our model, if workers' types were observable, the firm could reject applications from certain types. Although the moral hazard problem still exists, it does not affect the efficiency of equilibrium.[12]

Notice, we assume that all workers are UI eligible. We analyze the take-up decision, which applies only to those who are UI eligible. Indeed, while some unemployed are not eligible for UI benefits, adding this dimension to the model complicates the analysis, but does not provide any additional insights to the question at hand.[13]

3.2 Wages and Matching

We assume directed search. Firms post wages and workers direct their search to the wage maximizing their expected lifetime utility (see Moen (1997) or Acemoglu and Shimer (1999) for a similar formulation of the environment).

There exists a matching function, denoted $m(u, \mathrm{v})$, describing the number of matches formed between the v vacancies and u unemployed workers. We assume standard properties, *i.e.* m is continuous, strictly increasing, strictly concave (with respect to each of its arguments), and exhibits constant returns to scale. Furthermore, $m(0, \cdot) = m(\cdot, 0) = 0$ and $m(\infty, \cdot) = m(\cdot, \infty) = \infty$. Let $q = \frac{u}{\mathrm{v}}$ denote the "queue" length.

Given this matching technology, a vacancy is filled with Poisson arrival rate $m(\frac{u}{\mathrm{v}}, 1)$. Similarly, an unemployed worker finds a job according to a Poisson process with arrival rate $m(1, \frac{\mathrm{v}}{u})$. Let $\alpha_E(q) = m(\frac{u}{\mathrm{v}}, 1)$ and $\alpha_W(q) = m(1, \frac{\mathrm{v}}{u})$ denote the vacancy filling and job finding rates, respectively. Filled jobs receive negative idiosyncratic productivity shocks rendering the match unprofitable with a Poisson arrival rate λ.

[12] This type of moral hazard problem would matter for determining the optimal level of UI benefits, for example.

[13] See Auray and Fuller (2016) for an example where including both UI eligibles and ineligibles is important for the analysis.

3.3 Value Functions

We begin by describing the firm's and worker's value functions for a general wage function, w. After defining the equilibrium concept we then show that wages are a function of ε, and describe how the market naturally separates UI collectors and non-collectors.

3.3.1 Firms

Denote the value of a vacancy and the value of a matched firm by V and J, respectively. For any given w,

$$rV = -\gamma + \alpha_E[q(w)]\,[J - V] \tag{1}$$

According to Equation (1), the firm pays the flow cost γ to open the vacancy, and at rate $\alpha_E[q(w)]$ the firm fills the vacancy. For the value of a filled vacancy, J, denote the expected probability a worker collects UI benefits if separated (or the expected proportion of workers collecting) by p. Then,

$$rJ = y - w + \lambda\,[-p\tau + (V - J)] \tag{2}$$

That is, the firm earns flow profits $y - w$. At rate λ the job is destroyed, and whether or not the firm pays the experience rated tax, τ, depends on if the worker collects UI benefits or not. Since the firm's expects a worker collects with probability p, $p\tau$ is the expected flow cost of experience rated taxes, which the firm pays upon separation. Given free entry, $V = 0$, we have,

$$J = \frac{y - w_i}{r + \lambda} - \lambda p\tau \tag{3}$$

Plugging Equation (3) into Equation (1) under free entry and solving for w yields,

$$w = y - \frac{\gamma(r+\lambda)}{\alpha_E\Big(q(w)\Big)} - \lambda p \tau \tag{4}$$

Equation (4) represents the zero-profit curve (alternatively iso-profit curve), describing the relationship between w and q for the firm.

3.3.2 Workers

Unemployed workers can be in two possible states depending on whether or not they collect unemployment benefits. Denote unemployed collecting UI by $i = U$ and not-collecting by $i = N$. The worker decides which unemployment state to enter the instant a separation occurs, when the worker transitions from employment to unemployment.

Let $U(\varepsilon)$ denote the expected value of searching for a job with wage and expected queue length combination $\big(w, q(w)\big)$ for an unemployed worker collecting UI with cost of collecting ε. Similarly, let N denote the lifetime utility for the worker if not collecting UI, and E the lifetime utility of employment. Given this, the value functions are given by:

$$rU(\varepsilon) = h(b) - \varepsilon + \alpha_W(q(w))\left[E - U(\varepsilon)\right] \tag{5}$$

$$rN = h(d) + \alpha_W(q(w))\left[E - N\right] \tag{6}$$

$$rE = h(w) + \lambda\left(\max\{U(\varepsilon), N\} - E\right) \tag{7}$$

Equation (5) implies that an unemployed worker collecting benefits receives instantaneous flow utility $h(b)$ from unemployment compensation, and with arrival rate $\alpha_W(q(w))$ the worker matches with a firm and transitions to employment. Equation (6) has a similar interpretation for an unemployed worker not collecting. Finally, equation (7) states that an employed worker receives instantaneous flow utility $h(w)$ and with Poisson arrival rate λ, the job dissolves. If the job dissolves, the worker decides whether or not to collect unemployment benefits. Notice, since the costs of collecting are permanent, in the steady state, if a worker prefers to collect UI benefits once, he always prefers to.

It is useful to have closed form solutions for $U(\varepsilon)$ and N in the analysis below. Towards this end, using Equations (5) to (7) to solve for $U(\varepsilon)$ and N, respectively, gives,

$$rU(\varepsilon) = \left(\frac{1}{r+\lambda+\alpha_W\big(q(w)\big)}\right)\left((r+\lambda)(h(b)-\varepsilon)+\alpha_W\big(q(w)\big)h\big(w\big)\right) \tag{8}$$

$$rN = \left(\frac{1}{r+\lambda+\alpha_W\big(q(w)\big)}\right)\left((r+\lambda)h(d)+\alpha_W\big(q(w)\big)h(w)\right) \tag{9}$$

To ensure that worker's indifference curves in (q, w) space are strictly convex requires the following assumption:

Assumption 1 *The matching function satisfies:*

$$2\Big[\alpha'_W(q)\Big]^2\alpha_W(q) > \alpha''_W(q)\Big[\alpha_W(q)\Big]^2 \tag{10}$$

This assumption represents a sufficient condition for the worker's indifference curve (in (q, w) space) to be strictly convex. It is generally true for large enough q, and was satisfied in the relevant range for all of the numerical examples computed in Section 5. Another way to view this assumption is as a sufficient condition for the value functions $U(q)$ and $N(q)$ to be strictly concave in q.

3.3.3 Definition of Equilibrium

In this section we define equilibrium for the economy described above. Unemployed workers direct their search to the posted wages that maximize their expected lifetime utility. Firms determine the wage that maximizes profits, subject to a zero-profit condition in equilibrium. An allocation is defined as a set $\Big\{\mathbb{W}, Q, \mathbf{U}, \mathbf{E}, \mathbf{N}, \mathcal{T}\Big\}$. This consists of a set of wages, $\mathbb{W}$, a queue length associated with each wage $Q(w)$, indirect utilities, $\mathbf{E}, \mathbf{U}, \mathbf{N}$ for workers in each possible employment state, and a take-up decision rule, $\mathcal{T}$. An equilibrium is then defined as follows:

Definition 1 *An equilibrium is an allocation* $\left\{\mathbb{W}, Q, \mathbf{U}, \mathbf{E}, \mathbf{N}, \mathcal{T}\right\}$ *such that:*

1. ***Profit maximization****: for all* w *and all* ε*,*

$$y - w - \frac{\gamma(r+\lambda)}{\alpha_E[q(w)]} - \lambda p \tau \geq 0 \tag{11}$$

with equality if $w \in \mathbb{W}$*,*

2. ***Optimal job application****: for all* w *and all* ε*,*

$$\mathbf{U}(\varepsilon) \geq U(w, Q(w), \varepsilon) \tag{12}$$

$$\mathbf{N} \geq N(w, Q(w)) \tag{13}$$

for $Q(w) > 0$*, where*

$$\mathbf{U}(\varepsilon) = \sup_{w' \in \mathbb{W}} U(w', Q(w'), \varepsilon) \tag{14}$$

$$\mathbf{N} = \sup_{w' \in \mathbb{W}} N(w', Q(w')) \tag{15}$$

3. ***Optimal Take-up****: A worker* $i \in \mathcal{T}$ *if and only if* $\mathbf{N} \leq \mathbf{U}(\varepsilon)$

4. ***Consistency****: The firm's expected* p *in Equation (11) is consistent with* $\mathcal{T}$*.*

This represents a standard definition of equilibrium in a directed search environment, adjusted for the take-up decision. Profit maximization states that firms choose the w that maximizes profits, taking the expected queue length, q, and expected proportion of UI collectors, p as given. Free entry ensures that firms earn zero profits in equilibrium. Optimal application requires that unemployed workers direct their search to the wage offering the highest expected lifetime utility. Note, it is possible that search behavior remains different for UI collectors and non-collectors; equilibrium is a *set* of wages, and workers direct their search appropriately within that set. The Optimal Take-up condition specifies the set of workers that find it optimal to collect UI benefits upon separation. Finally, this optimal take-up condition must be consistent with the firm's expected p; if more or less workers collect UI benefits than the firm expects, either profit maximization or the zero profit condition are violated.

3.3.4 Endogenous market segmentation

Showing that equilibrium involves endogenous separation on two dimensions represents the next step. Specifically, we show that markets endogenously separate (i) between UI collectors and non-collectors and (ii) along ε for UI collectors. Towards this end, consider the following result:

Lemma 1 *For any active equilibrium (i.e. $Q(w) > 0$), there exists a unique ε^* such that $\mathbf{U}(\varepsilon) \geq \mathbf{N}$, for all $\varepsilon \leq \varepsilon^*$ and $\mathbf{N} > \mathbf{U}(\varepsilon)$ for all $\varepsilon > \varepsilon^*$.*

Lemma 1 establishes a unique cut-off value for the costs of collecting UI benefits, denoted ε^*. If a worker's ε is below this cut-off, they prefer to collect UI benefits if separated. This is true of any equilibrium, which creates two "types" of workers the firm encounters. Moreover, which type of worker the firm encounters affects the profits earned; a worker with $\varepsilon \leq \varepsilon^*$ collects UI benefits if separated, implying the firm pays a higher flow cost upon separation, τ. Thus, the expected proportion of workers collecting benefits, p, is either $p = 1$ for $\varepsilon \leq \varepsilon^*$ or $p = 0$ for all $\varepsilon > \varepsilon^*$. Next we show that any equilibrium involves endogenous separation along this cut-off, ε^*, and also involves a wage function, $w(\varepsilon)$ for $\varepsilon \leq \varepsilon^*$.

Proposition 1 *Any equilibrium allocation, $\left\{\mathbb{W}, \mathbb{Q}, \mathbf{U}, \mathbf{E}, \mathbf{N}, \mathcal{T}\right\}$, involves a wage function, $w(\varepsilon)$, with $w_U(\varepsilon)$ for $\varepsilon \leq \varepsilon^*$ and w_N for $\varepsilon > \varepsilon^*$.*

Given this endogenous separation, firms post two different types of wages, $w_U(\varepsilon)$ targeted to UI collectors and w_N to non-collectors, which in turn implies potentially different queue lengths, which we denote by $q_U(\varepsilon)$ and q_N for UI collectors and non-collectors, respectively.

3.3.5 Labor market flows and stocks

Our description of equilibrium also requires the flow equations associated with the measures of workers in the different employment and unemployment states. Denote the number of unemployed workers collecting UI benefits for each ε by $n_U^u(\varepsilon)$, and the number of unemployed not collecting UI by n_N^u. Similarly, let $n_U^E(\varepsilon)$ denote the number of employed workers in state $i = U$ (*i.e.* will collect UI if separated) and n_N^E the number of employed workers in state $i = N$ (*i.e.* will not collect UI if separated).

To obtain a steady state equilibrium, for each ε the flows of workers into and out of employment must be equal. Since the market segments along ε, with $\varepsilon \leq \varepsilon^*$ collecting UI benefits and all others not, we can characterize these equilibrium flow equations as:

$$\lambda n_U^E(\varepsilon) = \alpha_W[q_U(\varepsilon)]n_U^u(\varepsilon) \tag{16}$$

$$f(\varepsilon) = n_U^E(\varepsilon) + n_U^u(\varepsilon) \tag{17}$$

Equation (16) states that for UI collectors, the flow of workers in and out of employment is equal and equation (17) ensures that the total measure of workers across the two employment states adds up to the population fraction, or $f(\varepsilon)$. Similarly, for $\varepsilon > \varepsilon^*$:

$$\lambda n_N^E = \alpha_W(q_N)n_N^u \tag{18}$$

$$1 - F(\varepsilon^*) = n_N^E + n_N^u \tag{19}$$

Given these flow equations, further denote the total number of employed (unemployed) with $\varepsilon \leq \varepsilon^*$ by $N_U^j = \displaystyle\int_0^{\varepsilon^*} n_U^j(\varepsilon)d\varepsilon, j = E, u$. Further let $N_N^j \equiv n_N^j, j = E, u$. Then, for $\varepsilon \leq \varepsilon^*$, equations (16) and (17) give

$$N_U^u = \int_0^{\varepsilon^*} \frac{f(\varepsilon)\lambda}{\lambda + \alpha_W[q_U(\varepsilon)]}d\varepsilon \tag{20}$$

Similarly we have:

$$N_N^u = \frac{[1 - F(\varepsilon)]\lambda}{\lambda + \alpha_W(q_N)} \tag{21}$$

Thus, the unemployment rate for this economy is given by $u = N_U^u + N_N^u$. The take-up rate is the fraction of eligible unemployed who collect UI benefits. Since we assume all workers remain eligible for UI benefits, this is given by:

$$\text{TUR} = \frac{N_U^u}{u} \tag{22}$$

4 Properties of Equilibrium

This section characterizes the key properties of equilibrium in the economy described above. We begin with the case of perfect information. Although the quantitative exercise uses the model with hidden UI collection costs, the perfect information economy provides a useful benchmark to help understand the inefficiency posed by unclaimed UI benefits.

4.1 Perfect Information

In the perfect information economy, we assume firms observe a worker's value of ε, and they can throw away applications at no cost. Thus, if a firm expects to hire a non-collector ($\varepsilon > \varepsilon^*$), but receives an application from a worker with $\varepsilon \leq \varepsilon^*$, they may disregard the application. This prevents UI collectors from applying to the wages posted for non-collectors and vice versa.

Proposition 2 *Assume that firms observe ε. If $\Big\{\mathbb{W}, Q, \mathbf{U}, \mathbf{E}, \mathbf{N}, \mathcal{T}\Big\}$ is an active equilibrium allocation, then any $w_U(\varepsilon) \in \mathbb{W}$, $w_N \in \mathbb{W}$, $q_U(\varepsilon) = Q\big(w_U(\varepsilon)\big)$, and $q_N = Q(w_N)$ must solve:*

$$\mathbf{U}(\varepsilon) = \max_{w_U(\varepsilon), q_U(\varepsilon)} \left(\frac{1}{r\big(r + \lambda + \alpha_W\big(q_U(\varepsilon)\big)\big)}\right)\Bigg((r+\lambda)(h(b) - \varepsilon) + \alpha_W\big(q_U(\varepsilon)\big)h\big(w_U(\varepsilon)\big)\Bigg) \tag{23}$$

$$s.t. \quad w_U(\varepsilon) = y - \frac{\gamma(r+\lambda)}{\alpha_E\Big(q(w_U(\varepsilon))\Big)} - \lambda\tau \tag{24}$$

$$\mathbf{N} = \max_{w_N, q_N} \left(\frac{1}{r(r + \lambda + \alpha_W(q_N))}\right)\Bigg((r+\lambda)h(d) + \alpha_W(q_N)h(w_N)\Bigg) \tag{25}$$

$$s.t. \quad w_N = y - \frac{\gamma(r+\lambda)}{\alpha_E\Big(q(w_N)\Big)} \tag{26}$$

Proposition 2 represents a standard formalization of equilibrium. Firms post equilibrium wages that maximize worker's utility, subject to the zero profit condition. In the case of perfect information, equilibrium is efficient. Figure 3 shows this equilibrium graphically. The equilibrium values of (q, w) occur at the point where the worker's indifference curve is tangent to the zero-profit curve in (q, w) space. This ensures both optimal application and profit-maximization. Since firm's hiring UI collectors pay an experience rated tax with certainty at separation (i.e. $p = 1$), their zero profit curve is shifted down by τ. Next we characterize some important features of the perfect information allocation, starting with how search behavior is affected by ε.

Lemma 2 *The job arrival rate $\alpha_W[q_U(\varepsilon)]$ is increasing in ε and the wage $w_U(\varepsilon)$ is decreasing in ε.*

Lemma 2 describes how the equilibrium allocations to UI collectors depend on the direct utility cost of collecting benefits. Intuitively, as ε increases, net benefit provided by UI is reduced which acts similarly to a decrease in UI benefits. Hence, the worker prefers to trade-off lower wages for a faster job arrival rate. This represents a similar result to the effect of increasing UI benefits characterized by Acemoglu and Shimer (1999), who show that an economy with higher UI benefits has longer durations and higher wages. Figure 4 displays the effect of ε on UI collector queue lengths and wages in equilibrium. As ε decreases, this "flattens" the worker's indifference curve, moving the point of tangency to the right. Note, this result also applies in the private information economy analyzed below, but we only state it once here.

Proposition 3 *In the perfect information equilibrium, the following is true:*

(i.) $\varepsilon^* < h(b) - h(d)$,

(ii.) $q_U^*(\varepsilon) > q_N^*, \forall \varepsilon \leq \varepsilon^*$

Proposition 3 illustrates some interesting features of the perfect information equilibrium. First, the value of ε^*, the point where the worker prefers to collect UI benefits for any $\varepsilon \leq \varepsilon^*$, has implications for the UI take-up rate. Recalling Equations (20) to (22), the take-up rate is generally increasing with ε^*. Below we compare the perfect information equilibrium to the case with asymmetric information and find that take-up rates are actually higher in the asymmetric information economy.

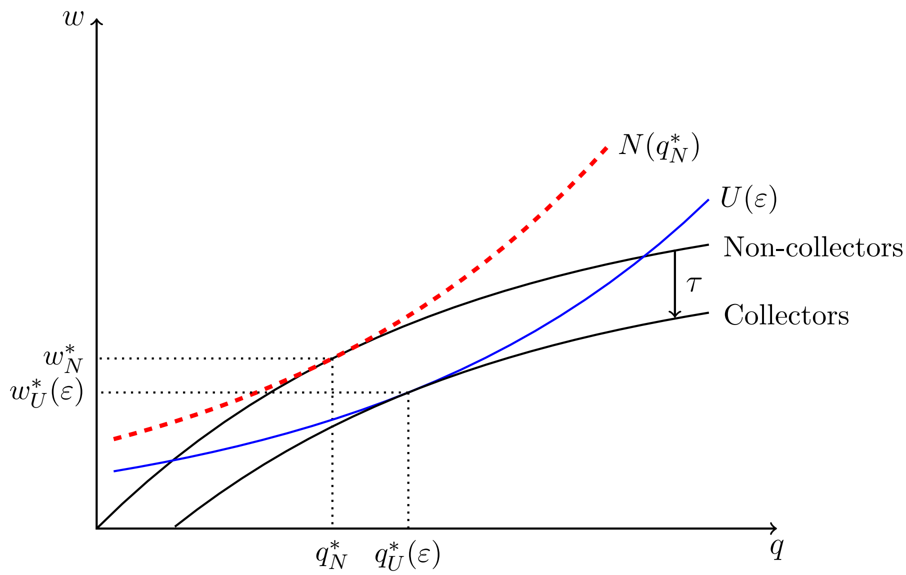


Figure 3: Determination of Equilibrium, Perfect Information

The graph shows the determination of $\big(q_U(\varepsilon), w_U(\varepsilon)\big), \varepsilon \leq \varepsilon^*$ and $\big(q_N^*, w_N^*\big)$ when there is perfect information. Wages offered by firms occur where the worker's indifference curve intersects the appropriate zero-profit curve. Since firms hiring UI collectors pay higher UI taxes, their zero-profit curve is shifted down by the tax, τ.

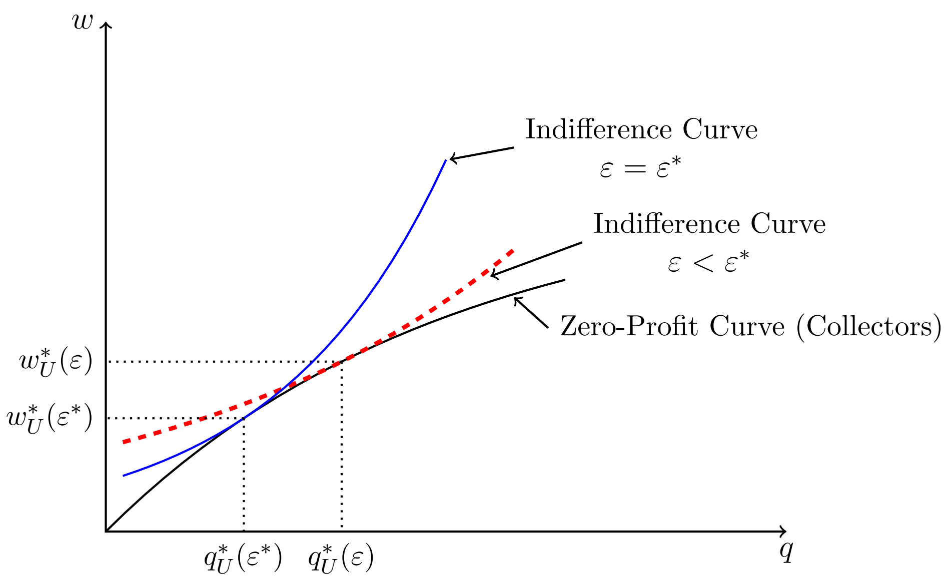


Figure 4: Effect of UI collection costs

The graph shows the determination of $\big(q_U(\varepsilon), w_U(\varepsilon)\big), \varepsilon \leq \varepsilon^*$. As ε decreases, the UI collector's indifference curve gets "flatter," as the net gain from UI benefits increases. This pushes the queue length and wage higher. These UI collectors are willing to wait longer for higher wage jobs.

Second, according to property (ii), UI collectors have longer unemployment durations, on average, relative to non-collectors. This follows from Lemma 2 (see Figure 4), where we show the queue length is inversely related to ε. Notice, a UI non-collector is equivalent (in flow utility) to a worker with $\varepsilon = h(b) - h(d)$. Since $\varepsilon^* < h(b) - h(d)$, all UI collectors lie below this point, and from Lemma 2, must have a higher queue length and longer average duration of unemployment. This is intuitive: UI non-collectors have lower flow utility in unemployment relative to a collector, and as a result they prefer shorter unemployment durations.

4.2 Hidden UI Collection Costs

In this section we examine the baseline case where ε is unobservable. That is, we now have both a moral hazard and an adverse selection problem. Specifically, since ε and previous UI collection status is unobservable by a firm, there is no way to prevent a current UI collector from searching for the non-collector wage. Why may a UI collector prefer this option? This occurs because of the experience rated tax. The tax, τ, implies that for a given queue length q, $w_U(\varepsilon) < w_N$ (follows from Equation (4)). Since under perfect information, non-collector jobs arrive faster than UI collector jobs, this introduces the possibility that a UI collector could search for a higher wage job that arrives faster, strictly dominating it. Define $\tilde{U}(\varepsilon)$ as the expected lifetime utility for a UI collector who deviates and searches for the non-collector job. This is given by,

$$\tilde{U}(\varepsilon) = \frac{1}{r(r + \lambda + \alpha_W(q_N))} \left[(r + \lambda)(h(b) - \varepsilon) + \alpha_W(q_N)h(w_N)\right] \tag{27}$$

In Equation (27), the worker receives the flow utility from collecting UI benefits, $h(b) - \varepsilon$, but searches for the (q_N, w_N) job, with employment at wage w_N arriving at rate $\alpha_W(q_N)$.

In order for the wage function (and associated queue lengths) $w_U(\varepsilon)$ and w_N to be viable in equilibrium, they must satisfy the constraint that:

$$U(\varepsilon) \geq \tilde{U}(\varepsilon), \text{ for all } \varepsilon \leq \varepsilon^*$$

If this constraint is violated, a UI collector prefers to search for the non-collector job, and q_N and firm profits are no longer consistent (since the firm opening a "non-collector" job will pay taxes when those workers separate and collect UI benefits). To maintain an equilibrium allocation, the non-collector allocation must be altered to satisfy this constraint.[14]

We now analyze the equilibrium with private information. First, consider the determination of equilibrium analogous to Proposition 2:

Proposition 4 *Assume that ε is unobservable. If $\Big\{\mathbb{W}, Q, \mathbf{U}, \mathbf{E}, \mathbf{N}, \mathcal{T}\Big\}$ is an active equilibrium allocation, then any $w_U(\varepsilon) \in \mathbb{W}$, $w_N \in \mathbb{W}$, $q_U(\varepsilon) = Q\big(w_U(\varepsilon)\big)$, and $q_N = Q(w_N)$ must solve:*

$$\mathbf{U}(\varepsilon) = \max_{w_U(\varepsilon), q_U(\varepsilon)} \left(\frac{1}{r(r+\lambda+\alpha_W\big(q_U(\varepsilon)\big))}\right)\left((r+\lambda)(h(b)-\varepsilon)+\alpha_W\big(q_U(\varepsilon)\big)h\big(w_U(\varepsilon)\big)\right) \tag{28}$$

$$s.t. \quad w_U(\varepsilon) = y - \frac{\gamma(r+\lambda)}{\alpha_E\Big(q(w_U(\varepsilon))\Big)} - \lambda\tau \tag{29}$$

$$\mathbf{N} = \max_{w_N, q_N} \left(\frac{1}{r(r+\lambda+\alpha_W(q_N))}\right)\Big((r+\lambda)h(d)+\alpha_W(q_N)h(w_N)\Big) \tag{30}$$

$$s.t. \quad w_N = y - \frac{\gamma(r+\lambda)}{\alpha_E\Big(q(w_N)\Big)} \; and, \tag{31}$$

$$U(\varepsilon) \geq \tilde{U}(\varepsilon), \; for\ all\, \varepsilon \leq \varepsilon^* \tag{32}$$

Notice, the formulation of equilibrium is similar to the baseline case with the exception of Equation (32), which represents the "incentive constraint" for firms in the non-collecting market. The addition of this incentive constraint, however, alters key aspects of equilibrium, including its efficiency. Denote by w_N^* and q_N^* the solution to the non-collector problem under perfect information given by Equations (25) and (26). Further, let $\tilde{w}_N$ and $\tilde{q}_N$ denote

[14]Potentially, one may consider altering the UI collector jobs, $(q_U(\varepsilon), w_U(\varepsilon))$ to ensure the constraint is satisfied; however, this remains infeasible. This is true because the firm posting $w_U(\varepsilon))$ has no control over a worker's utility when they deviate. That is, this firm does not control w_N (and thus q_N) and therefore can only alter the utility a worker receives from applying to their job. Since $w_U(\varepsilon)$ already maximizes a type ε worker's utility, no other possible $w_U(\varepsilon)$ can increase $U(\varepsilon)$ to satisfy the constraint. As a result, w_N must be altered.

the solution to the non-collector problem with private information given by Equations (30) to (32). The equilibrium under private information then has the following properties:

Proposition 5 *In the private information equilibrium, the following is true:*

(i). $\varepsilon^* = h(b) - h(d)$.

(ii). $\tilde{q}_N \neq q_N^*$ *and the equilibrium is inefficient.*

Compare (i) of Proposition 5 to (i) of Proposition 3: ε^* is higher with asymmetric information relative to perfect information. From Equations (20) to (22), a higher value of ε^* implies a higher take-up rate, all else equal. Thus, the asymmetric information economy has a higher take-up rate relative to perfect information. We explore this feature in more detail in Section 5.

Next, according to (ii) of Proposition 5, equilibrium is inefficient under private information. Why are non-collectors pushed away from the efficient wage and queue length achieved in the perfect information equilibrium? To understand this, define $\hat{\varepsilon}$ such that $\hat{\varepsilon} = h(b) - h(d)$. This is the value of ε where a UI collector and non-collector have the same flow utility in unemployment. Thus, for a UI collector with $\varepsilon = \hat{\varepsilon}$, $\tilde{U}(\varepsilon) = \mathbf{N}$; that is, this UI collector can always achieve the same level of utility as a non-collector. Since the efficient equilibrium under perfect information is such that $\mathbf{N}(q_N^*) > \mathbf{U}(\hat{\varepsilon})$, the constraint in Equation (32) is violated. Satisfying the constraint thus pushes the non-collector wage and queue length away from this efficient benchmark. The next result characterizes this distortion further.

Corollary 1 *For* $\tau > 0$*, there are two possible equilibrium values of* $\tilde{q}_N$ *satisfying* $\tilde{q}_N^L < q_N^* < \tilde{q}_N^H$*. Moreover,* $\tilde{q}_N^L < q_U(\varepsilon)$ *and* $w_N(\tilde{q}_N^L) < w_U(\varepsilon)$ *for all* $\varepsilon \leq \varepsilon^*$ *implying that non-collectors have a shorter unemployment duration and lower wage than UI collectors.*

Figure 5 shows the determination of equilibrium in the private information economy. Recall, an equilibrium has two essential features: optimal application and profit maximization. Thus, to satisfy the incentive constraint, in equilibrium non-collecting firms must offer a wage such that the UI collector indifference curve at $\varepsilon = \varepsilon^*$ intersects the non-collector zero-profit curve. At this wage and expected queue length, a UI collector is indifferent between searching for the non-collector vs. collector job. From Figure 5, it is clear that a strictly concave zero-profit function and strictly convex indifference curve imply that with private information, there exists two potential equilibrium values of $\tilde{w}_N$. This is true because the indifference curve described by $N\big(\tilde{q}_N(\tilde{w}_N)\big) = U(\varepsilon^*)$ intersects the non-collector zero-profit curve twice.

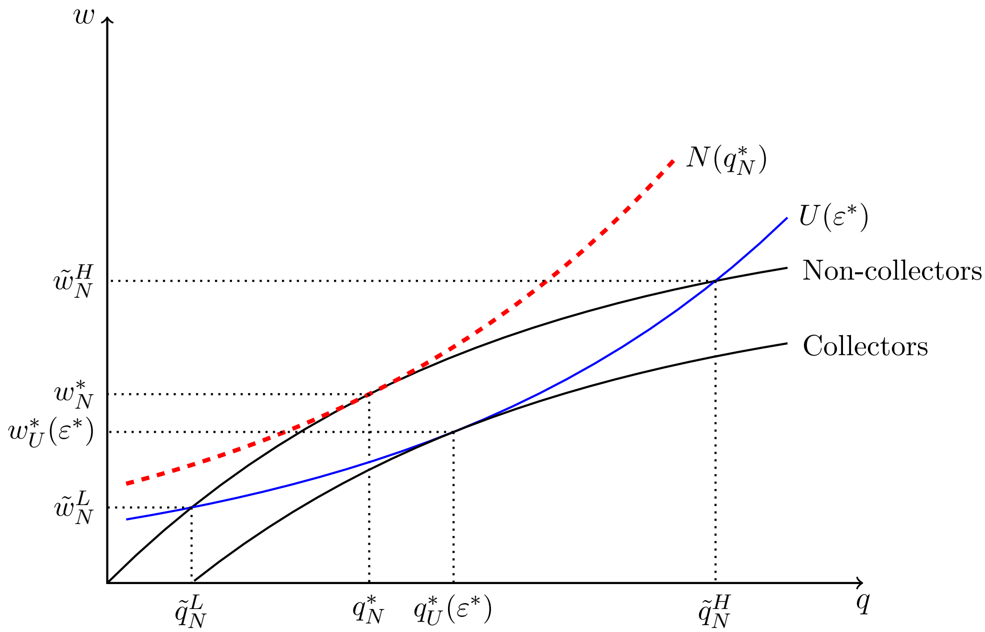


Figure 5: Determination of Equilibrium, Private Information

The graph shows the determination of $\big(q_U(\varepsilon), w_U(\varepsilon)\big), \varepsilon \leq \varepsilon^*$ and $\big(\tilde{q}_N, \tilde{w}_N\big)$ when ε is private information. Since a UI collector can search for $\big(q_N^*, w_N^*\big)$, and receives higher utility doing so, $\big(q_N^*, w_N^*\big)$ is not a feasible equilibrium for non-collectors. To solve this incentive problem, firms offer a $\big(\tilde{q}_N, \tilde{w}_N\big)$ where the UI collectors indifference curve intersects the non-collector zero-profit curve. Given a strictly convex indifference curve, there thus exist two possible equilibrium values, $\tilde{q}_N^L$ and $\tilde{q}_N^H$.

While indeed there exist two possible $\tilde{q}_N$'s, our empirical analysis below rules out $\tilde{q}_N^H$. Specifically, under $\tilde{q}_N^H$, the job-arrival rate for UI collectors exceeds that of non-collectors. This is contrary to the empirical evidence on the effects of UI benefits, all of which suggest a UI collector has a longer average duration of unemployment relative to a non-collector (for example see Katz and Meyer (1990) or Braun, Engelhardt, Griffy, and Rupert (2016)).[15] Having characterized the key properties of the equilibrium with endogenous take-up rates, we now turn towards quantifying these implications.

5 Quantitative analysis

In this section, we present a quantitative analysis of the aforementioned model and equilibrium. Our calibration focuses on the time period from $1989 - 2012$.

5.1 Calibration

The model described in Section 3 leaves the following parameters to be determined: r, b, d, λ, γ, $F(\varepsilon)$, τ , and functional forms for the matching function, m, and the utility function, h.

The time period is set to one month, so a per-annum risk-free interest rate of 4% implies $r = 0.0033$. The utility function is given by

$$h(c) = \frac{c^{1-\phi} - 1}{1 - \phi} \tag{33}$$

For the coefficient of relative risk aversion, ϕ, we use a value of 1.0, which falls within the range considered in Hansen and Imrohoroglu (1992) and the existing RBC literature.

The distribution $F(\varepsilon)$ is assumed to be exponential. Specifically, $f(\varepsilon) = \dfrac{1}{\mu_\varepsilon} \exp\left(-\dfrac{\varepsilon}{\mu_\varepsilon}\right)$, so that $F(\varepsilon) = 1 - \exp\left(-\dfrac{\varepsilon}{\mu_\varepsilon}\right)$. We normalize the value $\mu_\varepsilon = 1$.

For the matching function, m, we use the standard constant returns to scale form given by $m(u, \mathrm{v}) = u_j^\eta \mathrm{v}^{1-\eta}$.[16] As in Fredriksson and Holmund (2001), we use a value of 0.5 for η.

The job separation rate is set to match the average unemployment rate from $1989-2012$, which is 6.0%. This implies a value of $\lambda = 0.0157$. This value of λ is consistent with

[15] Braun, Engelhardt, Griffy, and Rupert (2016) find support for a directed search environment. Moreover, they also find evidence that UI benefits imply longer unemployment durations and higher wages.

[16] An equivalent alternative, used by others including Shimer (2005), is $m(u, \mathrm{v}) = m_0 u^\eta \mathrm{v}^{1-\eta}$ where u/v is normalized to 1, and m_0 is chosen to target the number of matches.

Shimer (2005), who finds a quarterly job separation rate of 0.035. The value of γ (vacancy creation costs) is set to match the observed average unemployment duration from $1989-2012$. According to the BLS tabulations from the CPS, the average unemployment duration from $1989-2012$ was 18.1 weeks, or 4.53 months, implying $\gamma = 22.14$. An alternative calibration strategy is to fix λ at the rate in Shimer (2005) and let the unemployment rate differ from the average in the data.

We parameterize the UI system setting $b = 0.444$ (the value of output, y is normalized to $y = 1$), consistent with an average replacement rate of 0.50. The replacement rate is calculated as b divided by the average wage for UI collectors, $\frac{b}{\bar{w_U}}$. The average wage for UI collectors is $\bar{w_U} = \int_0^{\varepsilon^*} w_U(\varepsilon)\phi_E(\varepsilon)d\varepsilon$, where $\phi_E(\varepsilon) = \frac{n_U^E(\varepsilon)}{N_U^E}$. For the minimum level of consumption (among UI non-collectors), we set d to match the observed take-up rate. Recall from Proposition 5, the equilibrium value of ε^*, a key determinant of the take-up rate (from Equation (22)), is determined by $h(b) - h(d)$. For the $1989 - 2012$ period, the take-up rate averaged 77%, requiring $d = 0.1661$.[17]

Finally, the value of τ is set to match data on experience rating in the U.S. system. Topel (1983) examines the specific experience rating system in a number of states finding an average marginal cost of a separation to a firm of approximately 80% of the implied UI expenditures. In the model, the average worker who collects UI contributes benefit expenditures equal to the UI benefit, b, times the average duration of unemployment (since benefits do not expire). We set τ to be 80% of this average benefit expenditure, implying $\tau = 1.8304$.

Table 1 lists the parameters and their values, and Table 2 presents the results from our calibration showing the key moments in the model and data.

5.2 Results

Figures 6(a) and 6(b) show the key properties of equilibrium established in Section 4. To begin, consider how equilibrium queue lengths and wages behave for UI collectors. As

[17] An alternative calibration strategy is to fix d at some level, and adjust μ_ε to hit the take-up rate. This does not affect the main results. We tried several permutations of these alternatives and obtained similar results.

Table 1: Parameters

r	0.0033	Discount rate
ϕ	1.0	Coefficient of relative risk aversion
η	0.5	Elasticity of matching function
λ	0.0157	Job separation rate
b	0.444	Replacement rate, UI, non-binding
d	0.1661	Minimum consumption rate
γ	22.14	Vacancy cost
τ	1.8304	Experience rating parameter
μ_ε	1	Parameter of $F(y)$

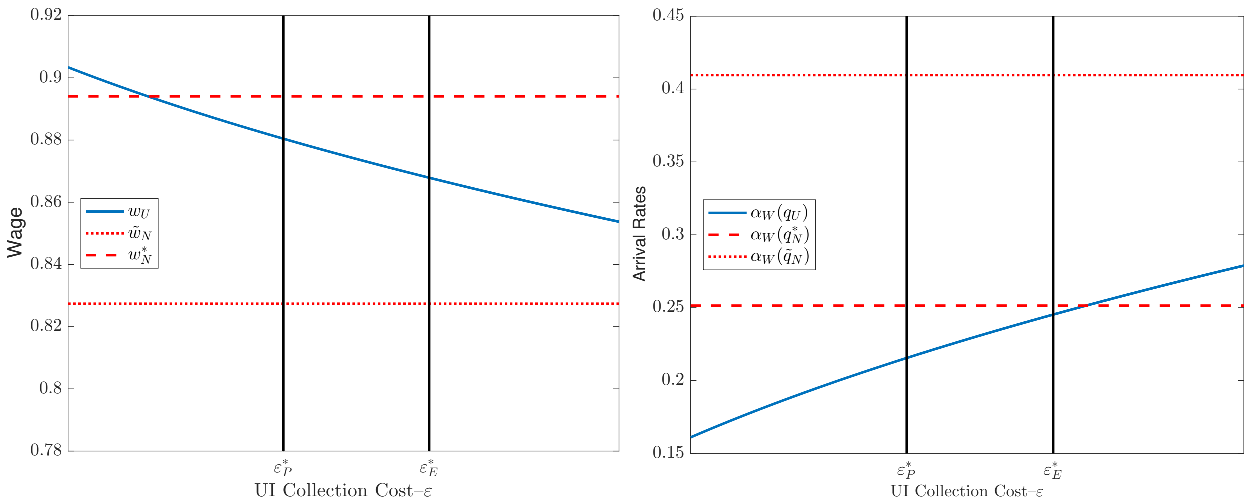
Table 2: Calibration Results

Moment	Model	Data
Unemployment rate	6.0%	6.0%
Unemployment duration	4.53	4.53
Take-up rate	0.77	0.77

in Lemma 2, wages are decreasing in ε while arrival rates are increasing in ε. Recall, as ε increases, the net gain from collecting UI benefits decreases so workers prefer to search for lower wage jobs that arrive faster.

Figures 6(a) and 6(b) also display the relationship between UI collector and non-collector wages and queue lengths. As indicated in Corollary 1, UI non-collectors have shorter unemployment durations than UI collectors (consistent with the empirical evidence in Katz and Meyer (1990) and Braun, Engelhardt, Griffy, and Rupert (2016)) and they have lower wages relative to UI collectors. This is consistent with the intuition above for the change in UI collector search behavior with ε. Recall, at $\varepsilon = \varepsilon^*$, $h(b) - \varepsilon^* = h(d)$. Since the net benefit of collecting is higher than $h(d)$ for all UI collectors, non-collectors prefer lower wage jobs arriving faster.

Proposition 5 also shows that non-collector wages and queue lengths are distorted from the efficient levels. Figures 6(a) and 6(b) display the effect of the information asymmetries



(a) Wages

(b) Arrival Rates

Figure 6: Wages and Arrival Rates Comparison

This Figure shows features of the equilibrium outcomes for UI collectors and non-collectors, under both perfect and private information. The left figure plots equilibrium wages for UI collectors under perfect and private information, as a function of ε. It also displays the non-collector wage, denoted $\tilde{w}_N$ for the private information equilibrium and w_N^* for the perfect information equilibrium. The right figure plots the respective arrival rates. The vertical lines occur at the cut-off value for ε, labeled ε_P^* and ε_E^* for the perfect information and private information economies, respectively.

on non-collector search behavior. In Figure 6(a), the non-collector wage under private information, $\tilde{w}_N$ remains below the efficient perfect information benchmark, w_N^*. With respect to queue lengths, Figure 6(b) displays that when ε is private information, job arrival rates for non-collectors are lower than the efficient benchmark under perfect information. These features have implications for several important moments in the model, which Table 3 displays and compares.

Consider first the effects of the inefficiency on the average duration of unemployment and the wage for non-collectors. According to Table 3, the inefficiency in the equilibrium with information asymmetries reduces non-collectors' average duration of unemployment from 3.98 months to 2.44 months. Thus, non-collectors move to employment over one and half months (6.14 weeks) *sooner* than what is optimal. Moreover, the non-collector wage is distorted below its efficient level by 8.1%; 0.894 under efficiency and 0.827 in the private information equilibrium.

Table 3: Efficient vs. Equilibrium (Inefficient)

Moment	Efficient	Inefficient
Average Duration, UI Collectors	5.42	5.15
Average Duration, UI Non-collectors	3.98	2.44
Average Duration (overall)	4.72	4.53
Average Wage, UI Collectors	0.89	0.89
Wage, UI Non-collectors	0.89	0.83
Unemployment rate	6.73%	6.00%
Take-up Rate	51%	77%

For UI collectors, the unemployment duration and wage vary with ε. The table reports the respective averages across ε. Specifically, $\bar{w_U} = \int_0^{\varepsilon^*} w_U(\varepsilon)\phi_E(\varepsilon)d\varepsilon$ and DUR $= \int_0^{\varepsilon^*} \frac{1}{\alpha_W[q_U(\varepsilon)]}\phi_U(\varepsilon)d\varepsilon$, where $\phi_i(\varepsilon) = \frac{n_U^i(\varepsilon)}{N_U^i}$.

The empirical literature on UI benefits has found that UI non-collectors have shorter unemployment durations relative to an equivalent collector. Our model also implies this feature, as does the existing search literature. In our model, however, part of the difference in unemployment durations between collectors and non-collectors is inefficient. Non-collectors are better off with a longer average duration of unemployment and higher wage than what occurs in equilibrium under private information.

Table 3 also shows a difference in the average unemployment duration and average wage for UI collectors between the two economies, with shorter durations and lower wages in the inefficient economy (0.892) relative to efficiency (0.894). This may appear surprising given that $w_U(\varepsilon)$ and $q_U(\varepsilon)$ solve the same problem under perfect and private information (i.e. Equations (23) to (24) and Equations (28) to (29) are the same problem). The averages differ, however, since the cut-off value of UI collection costs, ε^* differs between the two economies. Recall from Propositions 3 and 5, $\varepsilon_P^* < \varepsilon_E^*$, where ε_P^* and ε_E^* denote the cut-off values for the perfect information economy and the private information equilibrium, respectively. From Figures 6(a) and 6(b), wages increase and arrival rates decrease as ε decreases from ε^*. Since the efficient equilibrium has a lower cut-off, $\varepsilon_P^* < \varepsilon_E^*$, this implies a higher average wage and longer average duration of unemployment among the UI collectors.

As discussed in Section 4, this difference in ε^* also has implications for the UI take-up rate. Indeed, as displayed in Table 3, the take-up rate is lower in the efficient equilibrium, 51%, relative to the private information economy, 77%. This represents an interesting feature of the model. If firms can observe workers' UI collection costs, then they can effectively dissuade some fraction of workers from collecting UI benefits. Since hiring non-collectors lowers the experience rated tax firms face, the non-collector market is attractive to firms. Moreover, with no informational frictions, the market is able to provide some "natural" insurance to non-collectors in the form of shorter unemployment durations. By not having to pay the UI tax, these firms can also offer a much higher wage when no informational frictions exist. These features make the non-collector wage more appealing, reducing the UI take-up rate. That is, the informational asymmetries increase the UI take-up rate by reducing firms' options for non-collector wages; however, this higher take-up rate is inefficient.

5.3 Welfare

To measure the welfare cost of the inefficiency characterized above, we first look at non-collector welfare. Our welfare criterion is the standard consumption equivalent exercise. Letting H^* and $\tilde{H}$ denote expected lifetime utility in the efficient and inefficient allocations, respectively, then we characterize the percent consumption equivalent change $\triangle$ satisfying:

$$H^* = \tilde{H} + \frac{1}{r}h(1+\triangle)$$

Since we assume $h(c) = \log(c)$, this implies that

$$\triangle = \exp\left[r(H^* - \tilde{H})\right] - 1 \tag{34}$$

Table 4 presents the key welfare results. The first three rows indicate the welfare gain for non-collectors moving from the inefficient to efficient equilibrium. For example, an unemployed non-collector has expected lifetime utility of N^* in the efficient equilibrium and $\tilde{N}$ under the inefficient one. The % consumption equivalent welfare gains for N presented in Table 4 use N^* and $\tilde{N}$ in Equation (34). Similarly, the W_N row corresponds to the welfare gain for employed non-collectors. The"Avg. NC" row corresponds to the welfare gain, conditional on being a non-collector: $H = \dfrac{n_N N + n_N^E W_N}{n_N + n_N^E}$; i.e. the expected welfare for a non-collector. Finally, the last row labeled "Total Welfare" corresponds to the average welfare of all workers in the economy, or:

$$H = \int_0^{\varepsilon^*} \Big[n_u^E(\varepsilon) E_u(\varepsilon) + n_u^U(\varepsilon) U(\varepsilon) \Big] d\varepsilon + N_N^E E_N + N_N^U N \tag{35}$$

The middle column of Table 4 corresponds to the baseline parameterization with $\mu = 1.0$.

Table 4: % Consumption Equivalent Welfare Change

	$\mu = 0.5$	$\mu = 1.0$	$\mu = 1.5$
N	2.99	3.08	3.20
W_N	3.72	3.93	4.16
Avg. NC	3.65	3.85	4.08
Total Welfare	1.92	1.71	1.73

In the baseline parameterization, the welfare costs of the inefficiency are large: an average non-collector has a 3.85% welfare gain moving to the efficient economy, and overall the average worker has a 1.71% welfare gain moving to the efficient economy. Recall, for a given ε, $U(\varepsilon)$ and $E_U(\varepsilon)$ remain unchanged moving from the inefficient to the efficient economy. There are some welfare changes among UI collectors, however, as the value of ε^* and unemployment rate for UI collectors both change across economies.

The first and third columns of Table 4 presents some robustness checks with respect to the normalization of μ. Here we change the value of μ (the mean of the distribution of ε, $F(\varepsilon)$), and recalibrate λ, γ, b, d, and τ to maintain the initial moment targets. The largest and most relevant changes are to d. As μ decreases, d must increase in order to hit the target take-up rate, and vice versa. Appendix A discusses further details and additional robustness checks. In general, changing the value of μ has a relatively small effect on the welfare cost of the inefficiency.

5.4 Effects of UI Benefits

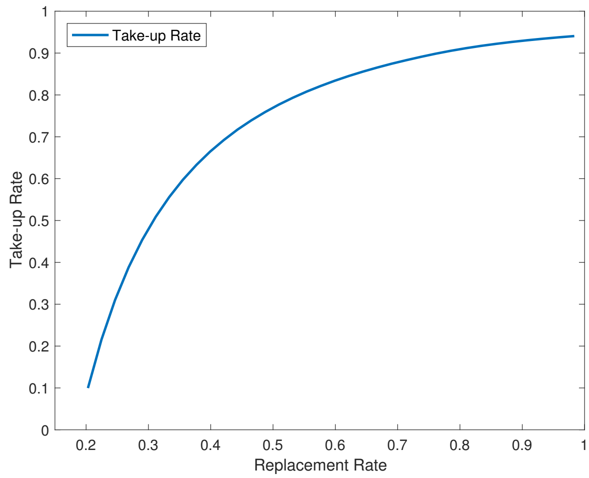
In this section we consider the equilibrium impact of changing the level of unemployment benefits.[18] The economy displays several interesting features when UI benefits increase. In the experiments below, we increase the UI replacement rate while setting τ to maintain the initial level of experience rating. That is, we maintain $\frac{\tau}{b * DUR_U} = 0.80$ as the replacement rate changes.

Figures 7(a) to 7(d) display the effects of an increase in b on the key equilibrium outcomes. First consider Figure 7(a). The take-up rate is increasing and concave in the UI replacement rate. Take-up increases from a low of 9.97% to a maximum of 94.06 as the replacement rate changes from 20% to 98%. Next, in Figure 7(b), the unemployment rate and average duration of unemployment are also increasing in the UI replacement rate. While indeed both moments increase with the replacement rate, they do so relatively slowly. A replacement rate of almost 100% is associated with an unemployment rate of 8.98%. Typically the unemployment rate and duration explode as the replacement rate approaches 100%. To understand the significance of these results, we compare them to a simple economy with a fixed 100% take-up rate.

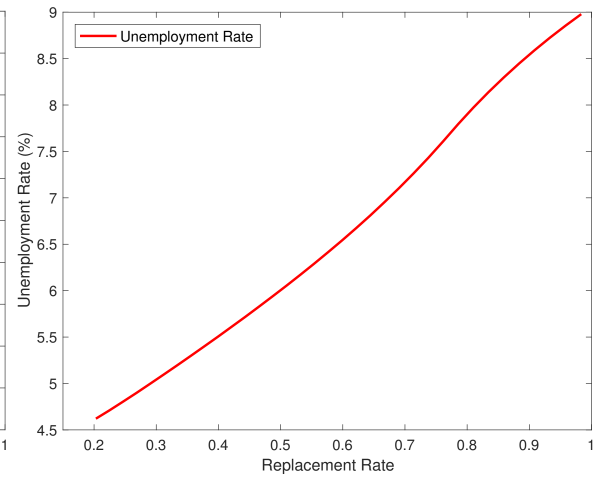
Consider a standard directed search model with no UI collection costs ($i.e.$ $\varepsilon = 0$ for all workers) and assume that all unemployed workers collect UI benefits. Thus, the take-up rate is fixed at 100%. We simulated the economy under different replacement rates and report the results in Figures 8(a) to 8(b). In these simulations, we maintain the same parametrization as the baseline case (with the exception that $\varepsilon = 0$).

Figure 8(a) and Figure 8(b) plot the response of the unemployment rate and average duration of unemployment, respectively. As one typically expects, as the replacement rate approaches 100%, both of the aforementioned moments begin to explode. At a replacement rate of 83.5% the unemployment rate is 19.32% and the average unemployment duration is 15.24 months. This is a significantly faster response relative to our baseline model, which is plotted in Figure 8(c) and Figure 8(d), respectively. Indeed, the endogenous take-up rate

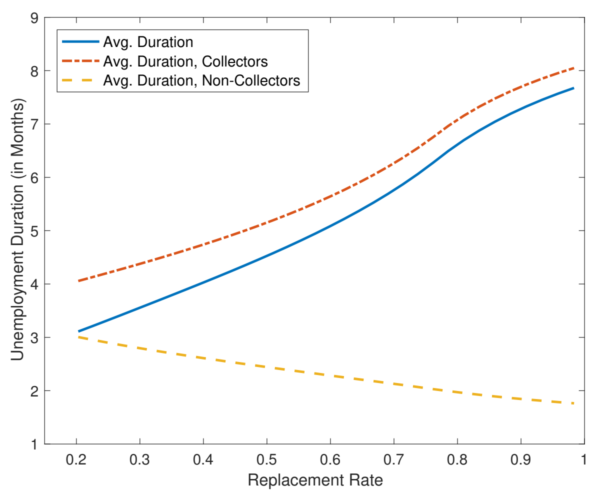
[18] Davidson and Woodbury (1998) and Wang and Williamson (2002) also examine unemployment insurance policies in models with take-up rates less than 1, but in these papers the take-up rate is exogenous.



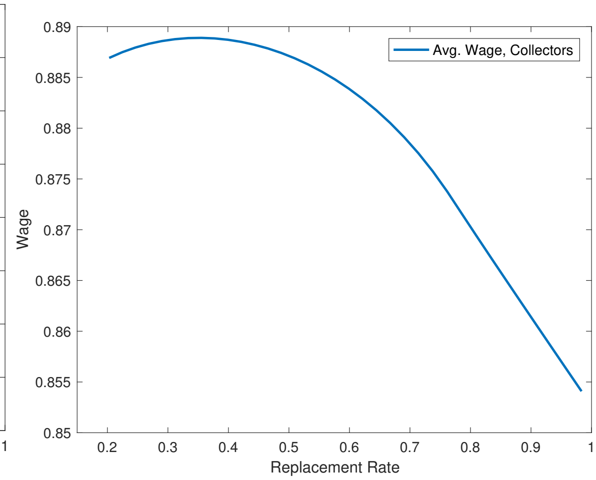
(a) Take-up Rate



(b) Unemployment Rate



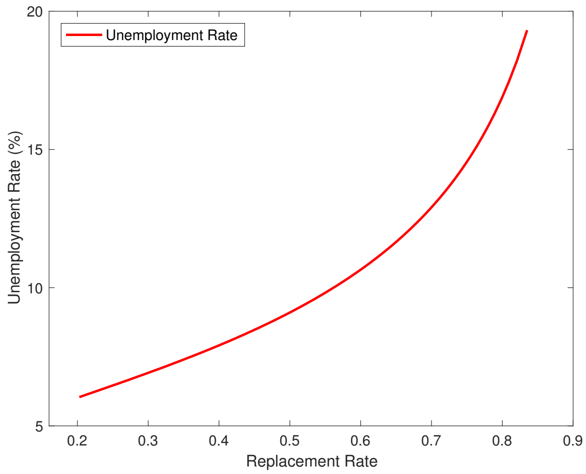
(c) Unemployment Durations



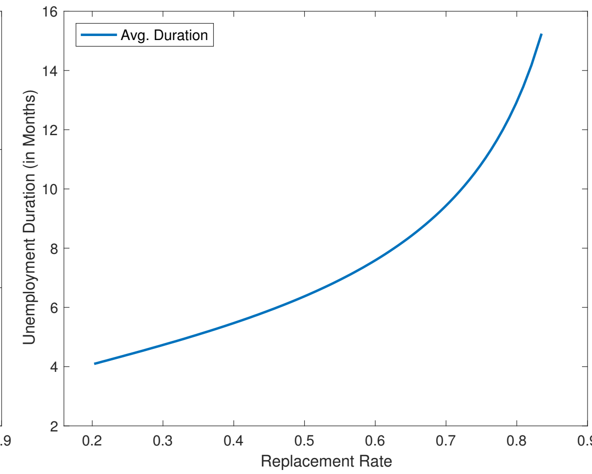
(d) Wages

Figure 7: Effects of UI Benefits

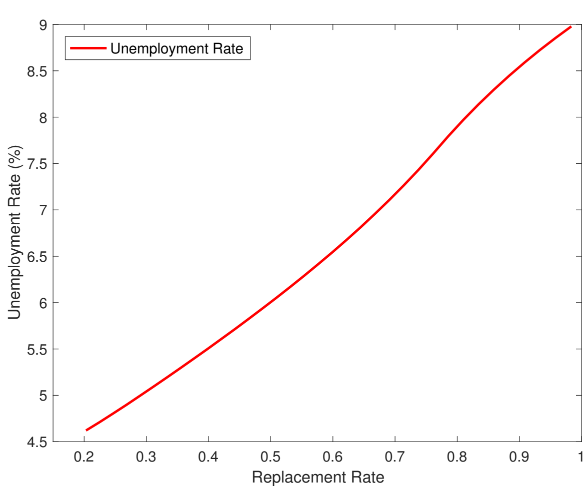
This figure plots the effects of UI benefits on equilibrium outcomes. The top two graphs plot the Take-up and Unemployment rates, respectively. The bottom-left graph plots the effect of b on unemployment durations. It plots the overall average unemployment duration, as well as for collectors and non-collectors separately. Similarly, the bottom-right graph plots wages for collectors and non-collectors. In all figures, the horizontal axis corresponds to the average replacement ratio for that particular b, or $\frac{b}{\bar{w_U}}$, where $\bar{w_U} = \int_0^{\varepsilon^*} w_U(\varepsilon)\phi_E(\varepsilon)d\varepsilon$ is the average wage for UI collectors.



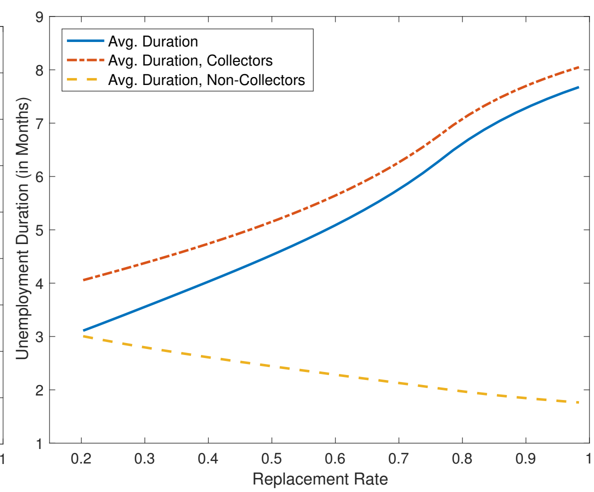
(a) Unemployment Rate



(b) Unemployment Durations



(c) Unemployment Rate: Baseline Model



(d) Unemployment Durations: Baseline Model

Figure 8: Effects of UI Benefits: 100% Take-up Model

This figure plots the effects of UI benefits on equilibrium outcomes for a standard model with 100% take-up. The top two graphs plot the responses of the Unemployment Rate and Average Unemployment Duration for the 100% take-up rate economy, respectively. For comparison, the bottom-left graph plots the effect of b on the Unemployment Rate in the baseline economy, and the bottom-right graph plots Unemployment Durations in the baseline economy. In the top two figures, the horizontal axis corresponds to the average replacement ratio for that particular b, or $\frac{b}{\bar{w_U}}$. The bottom two have the average replacement rate, which is $\frac{b}{\bar{w_U}}$, where $\bar{w_U} = \int_0^{\varepsilon^*} w_U(\varepsilon)\phi_E(\varepsilon)d\varepsilon$ is the average wage for UI collectors.

represents the key difference between the two models.

The difference in responses of the unemployment rate and average duration of unemployment to UI benefits derives in part from the following. As UI benefits increase, the average duration of unemployment for UI collectors indeed increases; however, it *decreases* for UI non-collectors. This occurs because of the informational frictions that bind the non-collector wage to a UI collector's utility at $\varepsilon = \varepsilon^*$. In the efficient equilibrium with perfect information, the non-collector queue length is unaffected by the level of UI benefits. In addition, the take-up rate increases at a decreasing rate with UI benefits. Put together, the average duration of unemployment (economy-wide), which is the average between collectors and non-collectors), eventually levels off similarly to the take-up rate.

6 Conclusion

We estimate the UI take-up rate for the U.S. economy from $1989 - 2012$. An equilibrium directed search model is developed to explain this empirical fact and explore its implications for the provision of UI benefits. Modeling the take-up decision leads to an informational friction that creates an inefficiency in non-collector outcomes.

Consistent with empirical studies on the effects of UI benefits, the model predicts that UI collectors have longer unemployment durations than non-collectors. Part of this difference is inefficient, as non-collectors transition to employment faster than the optimal rate. After calibrating the model, we explore several counterfactual policy experiments. We find that the inefficiency imposed by the presence of non-collectors amounts to a welfare cost of 1.71%. Finally, we also show that incorporating the take-up decision matters when examining the effects of UI benefits on equilibrium outcomes. The analysis indicates that the unemployment rate and average duration of unemployment respond slower to changes in UI benefits than the standard search model with a fixed 100% take-up rate.

It is also interesting to note that our model may have important implications for the efficiency of experience rated taxes. Many have previously studied the effects of experience rating on the labor market. Examples include Feldstein (1976), Topel (1983), Albrecht and

Vroman (1999), Wang and Williamson (2002), and Cahuc and Malherbet (2004), among many others. We do not explore the overall effects of experience rating in this paper, but rather take that feature as given and examine the implications of unclaimed UI benefits. Our results do indicate that a full exploration of experience rating, accounting for the unclaimed benefits aspect, represents an interesting direction for future research. Indeed, current work in progress explores the full implications of experience rating allowing for both endogenous separations and unclaimed UI benefits.

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A Robustness Checks

This section performs robustness checks with respect to the normalization of μ in the calibration procedure. As μ changes, d must change to maintain a take-up rate of 77% in the baseline private information economy. Of course the other calibrated parameters also adjust to hit their respective targets, but the changes to d represent those with the largest implications for the results. Table 5 presents the results of this robustness exercise. The third column, labeled TUR presents the take-up rate in the perfect information economy; the take-up rate in the private information economy by design remains at 77%.

Table 5: Changing μ and d

μ	d	TUR	$\tilde{w}_N$	w_N^*	$\frac{1}{\alpha_W(\tilde{q}_N)}$	$\frac{1}{\alpha_W(q_N^*)}$	Tot. wel.	N wel.	E_N	Avg. NC
0.50	0.27	0.29	0.86	0.92	2.49	4.32	1.92	2.99	3.72	3.65
0.60	0.25	0.37	0.85	0.91	2.47	4.23	1.84	3.01	3.76	3.69
0.70	0.23	0.42	0.84	0.91	2.45	4.15	1.80	3.04	3.82	3.75
0.80	0.20	0.47	0.84	0.90	2.46	4.09	1.74	3.03	3.83	3.76
0.90	0.19	0.49	0.83	0.90	2.45	4.03	1.73	3.05	3.88	3.80
1.00	**0.17**	**0.51**	**0.83**	**0.89**	**2.44**	**3.98**	**1.72**	**3.08**	**3.93**	**3.86**
1.10	0.15	0.53	0.82	0.89	2.43	3.92	1.71	3.10	3.98	3.90
1.20	0.14	0.54	0.82	0.89	2.42	3.88	1.72	3.13	4.02	3.94
1.30	0.12	0.56	0.81	0.88	2.42	3.84	1.72	3.15	4.07	3.98
1.40	0.11	0.57	0.81	0.88	2.41	3.79	1.72	3.17	4.11	4.02
1.50	0.10	0.58	0.80	0.87	2.40	3.75	1.73	3.20	4.17	4.08

The table presents from left to right: the mean parameter of $F(\varepsilon)$, non-collector flow utility (d) necessary to maintain a 77% UI take-up rate in the private info economy, the take-up rate in the perfect info economy (TUR), non-collector wage in private information economy ($tildew_N$), non-collector wage in the perfect info economy, w_N^*, average duration for non-collectors in each economy, the total welfare gain, the welfare gain for unemployed non-collectors, and the average welfare gain conditional on being a non-collector. The row in bold ($\mu = 1.0$) corresponds to the baseline parametrization.

In general, changing the value of μ has relatively small effects on the welfare cost of private information. The largest change is the take-up rate in the perfect information economy, which goes to a low of 29% for a relatively low value of μ. The welfare gain for non-collectors is also increasing with the value of μ, although the changes are relatively small.

B Proofs

The following Lemma is used in the Proof of Lemma 1.

Lemma 3 *The value function, $U(\varepsilon)$ is strictly decreasing in ε; i.e.* $\dfrac{\partial U(\varepsilon)}{\partial \varepsilon} < 0$.

Proof: This follows immediately from Equation (8) and the envelope theorem. ■

Proof of Lemma 1:

Proof: To prove Lemma 1, define the function $\Gamma(\varepsilon) = U(\varepsilon) - N$. Let $w \in \mathbb{W}$ and $q = Q(w)$ be an equilibrium wage and associated queue length, with $q > 0$. Consider first $\Gamma(0)$. Since $b > d$, we have:

$$\mathbf{N} = \sup_{w'} \left(\frac{1}{r(r+\lambda+\alpha_W(q(w')))} \right) \left((r+\lambda)h(d) + \alpha_W\big(q(w')\big)h(w') \right) <$$
$$\sup_{w'} \left(\frac{1}{r(r+\lambda+\alpha_W(q(w')))} \right) \left((r+\lambda)h(b) + \alpha_W\big(q(w')\big)h(w') \right) = \mathbf{U}(0)$$

which implies that $\Gamma(0) > 0$. Next consider $\lim_{\varepsilon\to\infty} \Gamma(\varepsilon)$. Notice, from Lemma 3, $U(\varepsilon)$ is decreasing in ε. Specifically, $\lim_{\varepsilon\to\infty} U(\varepsilon) = -\infty$; therefore, there exists some $\underline{\varepsilon}$ such that $\Gamma(\underline{\varepsilon}) < 0$. From Lemma 3, $\Gamma(\varepsilon)$ is strictly decreasing (and continuous); therefore, there exists a unique ε^* such that $U(\varepsilon^*) = N$. ■

The following Lemma is useful in the proof of Proposition 1.

Lemma 4 *Any equilibrium $w \in \mathbb{W}$ and $q(w) = Q(w)$ must satisfy: $B(w) + C(q) = 0$ for all $\varepsilon \leq \varepsilon^*$, where*

$$B(w) = \frac{\alpha_W\big[q(w)\big]h'(w)}{r\Theta} \tag{36}$$
$$C(q) = \frac{\alpha'_W(q)q'(w)(r+\lambda)\big[h(w) - (h(b)-\varepsilon)\big]}{r\Theta^2} \tag{37}$$

and $\Theta = (r+\lambda+\alpha_W[q(w)])$.

Proof: For $w \in \mathbb{W}$ and $q(w) = Q(w)$ to be equilibrium wages and queue lengths, they must satisfy Equation (12); that is, they must be optimal for a UI collector. Thus, there does not exist another w' such that the worker has higher lifetime expected utility. Suppose instead that $B(w) + C(q) \neq 0$, and take the case where $B(w) + C(q) > 0$. This is without loss of generality, as the $B(w) + C(q) < 0$ follows by reversing the direction of the proof. Now, suppose we increase w by any small amount. Let $q'(w)$ denote the associated increase in q required to remain on the firm's zero-profit curve in Equation (24) (or Equation (29)). The increase in w increases an unemployed worker's utility by $\frac{\partial U(\varepsilon)}{\partial w} = \frac{r\alpha_W(q)h'(w)}{r(r+\lambda+\alpha_W(q))} = B(w)$. When q increases by $q'(w)$, utility changes by:

$$\begin{aligned}
\frac{\partial U(\varepsilon)}{\partial q}q'(w) &= q'(w)\Big[\frac{\alpha'_W(q)h(w)\big(r(r+\lambda+\alpha_W(q))\big) - \big((r+\lambda)(h(b)-\varepsilon) + \alpha_W(q)\big)r\alpha'_W(q)}{r^2(r+\lambda+\alpha_W(q))^2}\Big] \\
&= q'\Big[\frac{r(r+\lambda)\alpha'_W(q)h(w) - r(r+\lambda)\alpha'_W(q)(h(b)-\varepsilon)}{r^2(r+\lambda+\alpha_W(q))^2}\Big] \\
&= \frac{\alpha'_W(q)q'(w)(r+\lambda)\big[h(w) - (h(b)-\varepsilon)\big]}{r(r+\lambda+\alpha_W(q))^2} = C(q)
\end{aligned}$$

Thus, the worker's utility changes by $B(w)+C(q)$. Recall, $B(w)+C(q) > 0$ so this increases $U(\varepsilon)$. Since the changes maintained profit maximization, this is a contradiction to $w \in \mathbb{W}$ and $q(w) = Q(w)$ as an equilibrium wage and queue length. ■

For the remaining proofs, it is useful to work with the worker's indifference curve. This is derived using Equation (8). Specifically, for any level of utility $\bar{U}$, an unemployed UI collector's indifference curve is given by:

$$\mathcal{W}(q) = h^{-1}\Big\{\frac{1}{\alpha_W(q)}\Big[r\bar{U}(r+\lambda+\alpha_W(q)) - (r+\lambda)\big[h(b)-\varepsilon\big]\Big]\Big\} \tag{38}$$

To ease the notation, define $T(q)$ as:

$$T(q) = \frac{1}{\alpha_W(q)}\Big[r\bar{U}(r+\lambda+\alpha_W(q)) - (r+\lambda)\big[h(b)-\varepsilon\big]\Big] \tag{39}$$

Given this, we have

$$\frac{\partial \mathcal{W}(q)}{\partial q} = \frac{T'(q)}{h'\Big(h^{-1}\big(T(q)\big)\Big)} \tag{40}$$

where

$$T'(q) = \frac{-\alpha'_W(q)}{[\alpha_W(q)]^2}\Big(r\bar{U} - [h(b) - \varepsilon]\Big) \tag{41}$$

Note that in equilibrium, since we restrict attention to $q(\varepsilon)$ such that $w(q) \geq \max\{h(b) - \varepsilon, h(d)\}$ (depending on whether or not the worker collects UI), $\bar{U} \geq h(b) - \varepsilon$; as a result, since $\alpha'_W(q) < 0$, $T'(q) > 0$. That is, the worker's indifference curve is strictly increasing in (q, w) space. Related, define the zero profit function defined in Equation (24) and/or Equation (26) as:

$$\mathcal{P}(q) = y - \frac{\gamma(r+\lambda)}{q_U(\varepsilon)\alpha_W\big(q_U(\varepsilon)\big)} - \lambda\tau \tag{42}$$

Viewed in this way, the problem of determining the optimal q becomes one of finding the indifference curve tangent to the firm's zero profit curve. The next Lemma shows that $\mathcal{P}(q)$ is strictly increasing and strictly concave.

Lemma 5 *The wage defined in Equation (42) (and Equation (29)) is such that $\mathcal{P}'(q) > 0$ and $\mathcal{P}''(q) < 0$.*

Proof: First, recall that our matching function is assumed to be such that $\alpha'_E(q) > 0$ and $\alpha''_E(q) < 0$. Differentiating Equation (42) with respect to q gives,

$$\mathcal{P}'(q) = \frac{\gamma(r+\lambda)\alpha'_E(q)}{\big[\alpha_E(q)\big]^2}$$

which is > 0 given the properties of $\alpha_E(q)$. Differentiating again with respect to q yields,

$$\mathcal{P}''(q) = \frac{\gamma(r+\lambda)\alpha''_E(q)\big[\alpha_E(q)\big]^2 - 2\gamma(r+\lambda)\alpha'_E(q)\alpha_E(q)}{\big[\alpha_E(q)\big]^2}$$

which is < 0 since $\alpha''_E(q) < 0$ and $\alpha'_E(q) > 0$. ■

Proof of Proposition 1:

We begin by showing that there must exist a wage function $w(\varepsilon)$ for UI collectors, and then show there exists a distinct wage for non-collectors, w_N. Suppose that there exists only one equilibrium wage, $\hat{w}$ that all employed workers receive if matched with a firm. Denote the expected queue length associated with this wage by $q(\hat{w})$. Next, consider any $\varepsilon_1 \leq \varepsilon^*$, where ε^* is the unique cut-off value given by Lemma 1. By definition of equilibrium, $\hat{w}$ must satisfy $U(\varepsilon_1) = \sup_{w'} U(\varepsilon_1, w')$. Then consider any $\varepsilon_2 < \varepsilon_1$. Notice that $B(w)$ from Equation (36) does not depend on ε and $C(q)$ from Equation (37) is decreasing in ε. Thus, $B(\hat{w}; \varepsilon_1) = B(\hat{w}; \varepsilon_2)$ and $C(q(\hat{w}); \varepsilon_1) > C(q(\hat{w}); \varepsilon_2)$. From Lemma 4, $\hat{w}$ and $q(\hat{w})$ satisfy$B(\hat{w}; \varepsilon_1) + C(q(\hat{w}); \varepsilon_1) = 0 > B(\hat{w}; \varepsilon_2) + C(q(\hat{w}); \varepsilon_2)$. This implies, however, that the marginal cost of decreasing $\hat{w}$ exceeds the gain (of decreasing q along the zero-profit curve), increasing utility for a worker with $\varepsilon = \varepsilon_2$, a contradiction to $\hat{w}$ being an equilibrium wage. Therefore, any equilibrium wage must be a function of ε for $\varepsilon \leq \varepsilon^*$. Call this function $w_U(\varepsilon)$.

The final step in the proof is to verify a distinct wage for non-collectors. The alternative is that all non-collectors receive the wage $w_U(\varepsilon^*)$. Notice, then, at this wage, some workers prefer to collect UI benefits (those with $\varepsilon = \varepsilon^*$ while the rest ($\varepsilon > \varepsilon^*$) do not collect. Thus, for a firm opening this job, the expected probability a separated worker collects is $0 < p < 1$. Suppose this is true. We take two cases separately: Case 1: Observable ε and Case 2: Unobservable ε. In Case 1: Since profits are higher for a firm with $p = 0$ (relative to $p > 0$), for any wage, a firm can offer $w_N = w_U(\varepsilon^*) + \mu$, for some $\mu > 0$, for only workers with $\varepsilon > \varepsilon^*$. Although those workers with $\varepsilon = \varepsilon^*$ would also like to apply, the perfect observability of ε prevents this. Since this clearly raises utility for those with $\varepsilon > \varepsilon^*$, all non-collectors prefer to apply for this job, violating optimal application of $w_U(\varepsilon^*)$ for all $\varepsilon \geq \varepsilon^*$.

Finally, consider Case 2: unobservable ε. In this case, the previous argument does not hold, since the firm cannot prevent any worker from applying to a particular job (wage). Again, suppose that all $\varepsilon \geq \varepsilon^*$ search for the $w_U(\varepsilon^*)$ job, at the expected queue length $q_U(\varepsilon^*)$. Notice that $p > 0$ for this job since both some collectors and non-collectors apply, but $p = 1$ for all $\varepsilon < \varepsilon^*$ as only collectors apply to these jobs. We will show that in this case there exits a $\delta > 0$ such that $w_U(\varepsilon^*; 1 > p > 0) > w_U(\varepsilon^* - \delta; p = 1)$ and $q_U(\varepsilon^*; 1 > p > 0) < q_U(\varepsilon^* - \delta; p = 1)$. This would imply a neighborhood around ε^* where optimal application is violated, since for those $\varepsilon \in [\varepsilon^* - \delta, \varepsilon^*)$, the $w_U(\varepsilon^*; 1 > p > 0)$ job would pay a higher wage and arrive faster, strictly dominating the $w_U(\varepsilon^* - \delta; p = 1)$ job. To show this, we use the fact that optimal application and profit maximization imply equilibrium wages and queue lengths must be such that the worker's indifference curve is tangent to the zero profit curve (in (q, w) space). That is, they must satisfy,

$$\mathcal{P}'(q_U^*(\varepsilon)) = \frac{\partial \mathcal{W}}{\partial q} = \frac{T'(q_U^*(\varepsilon))}{h'\Big(h^{-1}\big(q_U(\varepsilon)\big)\Big)} \tag{43}$$

Now, begin with the claim that $w_U(\varepsilon^*; 1 > p > 0) > w_U(\varepsilon^* - \delta; p = 1)$. We first show that $w_U(\varepsilon^*; 1 > p > 0) > w_U(\varepsilon^*; p = 1)$ and then a standard continuity argument delivers the desired result. Suppose instead that $w_U(\varepsilon^*; 1 > p > 0) \leq w_U(\varepsilon^*; p = 1)$. Given the zero profit curves for each p, profit maximization implies that $q_U(p < 1) < q_U(p = 1)$. Then, Assumption 1 implies $T''(q) > 0$ so that $T'(q_U(p = 1)) > T'(q_U(p < 1)$. Moreover, strict convexity of the utility function $h(\cdot)$ implies that $\frac{1}{h'(w_U(p=1))} \geq \frac{1}{h'(w_U(p<1))}$. Combining these inequalities along with $P''(q) < 0$ and Equation (43) we have:

$$\begin{aligned}
\frac{T'(q_U(p=1))}{h'(w_U(p=1))} &> \frac{T'(q_U(p<1))}{h'(w_U(p<1))} \\
\Rightarrow P'(q_U(p=1)) &> P'(p<1) \\
\Rightarrow q_U(p<1) &> q_U(p=1)
\end{aligned}$$

which is a contradiction to the argument above that $q_U(p < 1) < q_U(p = 1)$. Thus, $w_U(\varepsilon^*; 1 > p > 0) > w_U(\varepsilon^*; p = 1)$.

Now, given this, we can then show that $q_U(p < 1) < q_U(p = 1)$. Suppose instead $q_U(p < 1) \geq q_U(p = 1)$. Then, from the properties of $T(\cdot)$ discussed above, $T'(q_U(p < 1)) \geq T'(q_U(p = 1)$. Since $w_U(\varepsilon^*; 1 > p > 0) > w_U(\varepsilon^*; p = 1)$, the strict concavity of $h(\cdot)$ implies $\frac{1}{h'(w_U(p<1))} \geq \frac{1}{h'(w_U(p=1))}$. Combining these inequalities again with Equation (43) yields:

$$\frac{T'(q_U(p<1))}{h'(w_U(p<1))} > \frac{T'(q_U(p=1))}{h'(w_U(p=1))}$$
$$\Rightarrow P'(q_U(p<1)) > P'(p=1)$$
$$\Rightarrow q_U(p=1) > q_U(p<1)$$

a contradiction. Thus, $q_U(p<1) < q_U(p=1)$. Combining these results, if all non-collectors search for the wage $w_U(\varepsilon^*)$, then this wage is such that $w_U(\varepsilon^*; 1>p>0) > w_U(\varepsilon^*; p=1)$ and $q_U(\varepsilon^*; p<1) < q_U(\varepsilon^*; p=1)$. Then, since the policy functions, $w_U(\varepsilon)$ and $q_U(\varepsilon)$ are continuous functions of ε (for a given p), there exists a $\delta > 0$ such that $w_U(\varepsilon^*; 1>p>0) > w_U(\varepsilon^* - \delta; p=1)$ and $q_U(\varepsilon^*; p<1) < q_U(\varepsilon^* - \delta; p=1)$. Notice, however, that this implies the $w_U(\varepsilon^*)$ job has a higher wage and lower job arrival rate than the jobs offered to UI collectors with $\varepsilon \in [\varepsilon^* - \delta, \varepsilon^*)$. As a result, those workers with $\varepsilon \in [\varepsilon^* - \delta, \varepsilon^*)$ have higher utility from searching for the $w_U(\varepsilon^*)$ job, violating optimal application. Therefore, there must exist a distinct wage w_N for $\varepsilon > \varepsilon^*$. ■

We now turn towards the proof of Proposition 2. Towards this end, the following Lemma is used:

Lemma 6 *The function defined by* $G(q) = \dfrac{\alpha_W(q)}{r+\lambda+\alpha_W(q)}$ *is such that* $G'(q) < 0$.

Proof: Differentiating with respect to q yields,

$$G'(q) = \frac{\alpha'_W(q)(r+\lambda)}{\big(r+\lambda+\alpha_W(q)\big)^2} < 0$$

where the inequality follows from the properties of the matching function that imply $\alpha'_W(q) < 0$. ■

Proof of Proposition 2:

We show that any equilibrium must satisfy the optimization problems in Equations (23) to (26). Suppose that $\left\{\mathbb{W}, Q, \mathbf{U}, \mathbf{E}, \mathbf{N}, \mathcal{T}\right\}$ is an equilibrium with $w_U^* \in \mathbb{W}$ and $q_U^*(w_U^*) = Q(w_U^*) > 0$, for all ε. Furthermore, suppose there exists another w'_U and $q'_U(w'_U)$ that achieves a higher value of the objective function in Equation (23) (it is without loss of generality that we use w_U, q_U, as the case for q_N, w_N and the objective in Equation (25) follows analogously). That is, suppose that for any $\varepsilon \leq \varepsilon^*$, q'_U and w'_U satisfy

$$\left(\frac{1}{r+\lambda+\alpha_W\big(q'_U(\varepsilon)\big)}\right)\left((r+\lambda)(h(b)-\varepsilon)+\alpha_W\big(q'_U(\varepsilon)\big)h\big(w'_U(\varepsilon)\big)\right) > \mathbf{U}(\varepsilon) \tag{44}$$

Then, by definition of $Q(w)$ in equilibrium, for the wage w'_U it must satisfy optimal application, which implies

$$\left(\frac{1}{r+\lambda+\alpha_W\big(Q(w'_U)\big)}\right)\left((r+\lambda)(h(b)-\varepsilon)+\alpha_W\big(Q(w'_U)\big)h\big(w'_U(\varepsilon)\big)\right) \leq \mathbf{U}(\varepsilon) \tag{45}$$

Combining Equations (44) and (45) implies that

$$\frac{\alpha_W\big(Q(w'_U)\big)}{r+\lambda+\alpha_W\big(Q(w'_U)\big)} < \frac{\alpha_W\big(q'_U(w'_U)\big)}{r+\lambda+\alpha_W\big(q'_U(w'_U)\big)}$$

From Lemma 6, this inequality implies that $q'_U(w'_U) < Q(w'_U)$. Now, profit maximization in equilibrium implies that w^*_U and $Q(w^*_U)$ satisfy Equation (24) with equality. Furthermore, this profit maximization implies that any other w_U cannot earn greater than this. Combining $q'_U(w'_U) < Q(w'_U)$, the fact that $\alpha_E(q)$ is increasing in q, along with the profit maximization just discussed implies that

$$y - w'_U - \frac{\gamma(r+\lambda)}{\alpha_E(q'_U)} - \lambda\tau < y - w'_U - \frac{\gamma(r+\lambda)}{\alpha_E(Q(w'_U))} - \lambda\tau \leq 0$$

Thus, q'_U, w'_U do not satisfy profit maximization, and thus are not feasible in equilibrium. Therefore, any equilibrium allocation must satisfy Equations (23) to (26). ■

Proof of Lemma 2:

Proof: Consider ε_1 and ε_2 such that $\varepsilon_2 > \varepsilon_1$. Denote $q^*_U(\varepsilon)$ the optimal choice of queue length (defined by Equation (23) or Equation (28)) for a given ε. Since $\alpha_W(q)$ is strictly decreasing in q, we need to show that $q^*_U(\varepsilon_1) > q^*_U(\varepsilon_2)$. Now, suppose instead that $q^*_U(\varepsilon_2) \geq q^*_U(\varepsilon_1)$. Notice, for a given q, $T'(q)$ is increasing in ε; therefore, $T'(q;\varepsilon_2) > T'(q;\varepsilon_1)$. Since by assumption, $q^*_U(\varepsilon_2) \geq q^*_U(\varepsilon_1)$ and $T(q)$ is increasing in q, we also have that $T'(q_U^*(\varepsilon_2)) > T'(q_U^*(\varepsilon_1))$. Moreover, since $\mathcal{P}(q)$ is increasing in q, we also have that $w_U(q_U(\varepsilon_2)) \geq w_U(q_U(\varepsilon_1))$. Thus, the strictly concave utility function implies $\frac{1}{h'\Big(h^{-1}\big(q_U(\varepsilon_2)\big)\Big)} \geq \frac{1}{h'\Big(h^{-1}\big(q_U(\varepsilon_1)\big)\Big)}$, implying that $\frac{T'(q_U^*(\varepsilon_2))}{h'\Big(h^{-1}\big(q_U(\varepsilon_2)\big)\Big)} > \frac{T'(q_U^*(\varepsilon_1))}{h'\Big(h^{-1}\big(q_U(\varepsilon_1)\big)\Big)}$.

Now, by definition of being an optimal solution, $q_U^*(\varepsilon_1)$ satisfies,

$$\mathcal{P}'(q_U^*(\varepsilon_1)) = \frac{\partial \mathcal{W}}{\partial q} = \frac{T'(q_U^*(\varepsilon_1))}{h'\Big(h^{-1}\big(q_U(\varepsilon_1)\big)\Big)}$$

Since $\mathcal{P}''(q) < 0$, $\mathcal{P}'(q_U^*(\varepsilon_2)) \leq \mathcal{P}'(q_U^*(\varepsilon_1))$, which combined with the results above implies,

$$\frac{T'(q_U^*(\varepsilon_2))}{h'\Big(h^{-1}\big(q_U(\varepsilon_2)\big)\Big)} > \frac{T'(q_U^*(\varepsilon_1))}{h'\Big(h^{-1}\big(q_U(\varepsilon_1)\big)\Big)} = \mathcal{P}'(q_U^*(\varepsilon_1)) \geq \mathcal{P}'(q_U^*(\varepsilon_2))$$

which is a contradiction to $q_U^*(\varepsilon_2)$ being an optimal solution to Equation (23). Therefore, $q_U^*(\varepsilon_2) < q_U^*(\varepsilon_1)$. ■

Lemma 2 describes how the equilibrium allocations to UI collectors depend on the direct utility cost of collecting benefits. Intuitively, as ε increases, net benefit provided by UI is reduced which acts similarly to a decrease in UI benefits. Hence, the worker prefers to trade-off lower wages for a faster job arrival rate.

Proof of Proposition 3:

Proof: First consider (i) $\varepsilon^* < h(b) - h(d)$. Define $\hat{\varepsilon} = h(b) - h(d)$. Suppose instead that $\varepsilon^* \geq \hat{\varepsilon}$. Notice that if $\varepsilon = \hat{\varepsilon}$, collectors and non-collectors have the same flow utility. Moreover, the set of feasible wages for a firm hiring non-collectors, given any q, includes as a subset the wages available to a firm hiring collectors. This implies that by definition of $\mathbf{N}$, $\mathbf{N} \geq \mathbf{U}(\hat{\varepsilon})$. Then, if $\varepsilon^* > \hat{\varepsilon}$,

$$\mathbf{N} \geq \mathbf{U}(\hat{\varepsilon}) > \mathbf{U}(\varepsilon^*)$$

where the last inequality comes from Lemma 3. This is a contradiction to the definition of ε^* where $\mathbf{U}(\varepsilon^*) = \mathbf{N}$. Then, what if $\varepsilon^* = \hat{\varepsilon}$? Consider the determination of (q, w) as the tangency point of the worker's indifference curve and the zero-profit curve. This implies that the equilibrium q must satisfy $\mathcal{P}'(q) = \mathcal{W}'(q)$, or:

$$\frac{\gamma(r+\lambda)\alpha_E'(q)}{\left[\alpha_E(q)\right]^2} = \frac{-\alpha_W'(q)}{\left[\alpha_W(q)\right]^2}\left[r\bar{U} - H(c)\right] \tag{46}$$

where $H(c) = h(b) - \varepsilon$ for a UI collector and $H(c) = h(d)$ for a non-collector. Notice, for $\varepsilon = \hat{\varepsilon}$, Equation (46) is the same for UI collector's and non-collectors. Since $\mathcal{P}(q)$ and $\mathcal{W}(q)$ are strictly concave and convex, respectively, this has a unique solution. Therefore, $q_U(\hat{\varepsilon}) = q_U(\varepsilon^*) = q_N$. Since $p = 1$ for $\varepsilon = \varepsilon^*$, the wage for a UI collector, from Equation (24), is such that $w_U(\varepsilon^*) < w_N$. Given the same flow utility, however, $q_U(\varepsilon^*) = q_N$ and $w_U(\varepsilon^*) < w_N$ imply that $\mathbf{U}(\varepsilon^*) < \mathbf{N}$, a contradiction to the definition of ε^*. Thus, $\varepsilon^* < \hat{\varepsilon}$.

To show property (ii), $q_U(\varepsilon) > q_N$, for all $\varepsilon \le \varepsilon^*$, we can start with the fact argued above that from Equation (46), $q_U(\hat{\varepsilon}) = q_N$. Combining this with $\varepsilon^* < \hat{\varepsilon}$ and Lemma 2 yields the desired result. ■

The following two Lemmas are needed in the proof of Proposition 5:

Lemma 7 *When $\tau = 0$, $\mathbf{U}(\hat{\varepsilon}; \tau = 0) = \mathbf{N}(q_N^*)$, where q_N^* is defined as the solution to Equations (25) to (26), $\mathbf{U}(\varepsilon)$ is given by Equations (28) to (29), and $\hat{\varepsilon} = h(b) - h(d)$.*

Proof: When $\tau = 0$, from Equations (26) and (29), the zero-profit curves for collector and non-collector firms coincide. Moreover, at $\hat{\varepsilon} = h(b) - h(d)$, a UI collector and non-collector have identical flow utility, and thus identical indifference curves in (q, w) space. As a result, the utility maximizing (q, w) combination must also coincide, implying that $\mathbf{U}(\hat{\varepsilon}; \tau = 0) = \mathbf{N}(q_N^*)$. ■

Proof of Proposition 4:

Proof: The proof follows the same logic as the proof of Proposition 2 above, and is thus omitted here.. The only difference is the additional constraint in Equation (32) that is required for optimal application. In the proof of Proposition 5 we show that this constraint must bind at $\varepsilon = \varepsilon^*$; i.e. the constraint must be imposed. ■

The following Lemma is also used in the proof of Proposition 5:

Lemma 8 *The value function $U(\varepsilon; \tau)$ is decreasing in τ.*

Proof: Differentiating $U(\varepsilon)$ in Equation (8) with respect to τ (using the Envelope Theorem) gives:

$$\frac{\partial U}{\partial \tau} = \left(\frac{\alpha_W\big(q_U(\varepsilon)\big)}{r+\lambda+\alpha_W\big(q_U(\varepsilon)\big)}\right)\left(h'\big(w_U(\varepsilon)\big)\right)\left(\frac{\partial w_U}{\partial \tau}\right)$$

The first two terms in parenthesis are positive, and from Equation (29) (or Equation (24)), $\dfrac{\partial w_U}{\partial \tau} < 0$. ■

Proof of Proposition 5:

Proof: To show property (i), denote ε^* as the unique cut-off value such that $\mathbf{U}(\varepsilon) \geq \mathbf{N}$ for all $\varepsilon \leq \varepsilon^*$, with equality at $\varepsilon = \varepsilon^*$. This simply represents the unique crossing point of U and N identified in Lemma 1. Further let $\hat{\varepsilon} = h(b) - h(d)$. Property (i) says $\varepsilon^* = \hat{\varepsilon}$. Suppose instead this is not true. Then there are two possibilities: Case 1: $\varepsilon^* < \hat{\varepsilon}$. In this case, notice that since $h(b) - \hat{\varepsilon} = h(d)$, it must be that $\tilde{U}(\hat{\varepsilon}) = \mathbf{N}(\tilde{q}_N)$, for any equilibrium $\tilde{q}_N$. Moreover, by definition, $\mathbf{U}(\varepsilon^*) = \mathbf{N}(\tilde{q}_N)$. Using this along with Lemma 3, given $\hat{\varepsilon} > \varepsilon^*$:

$$\tilde{U}(\varepsilon^*) = \mathbf{N}(\tilde{q}_N) = \mathbf{U}(\varepsilon^*) > \mathbf{U}(\hat{\varepsilon})$$

a contradiction to equilibrium conditions, as the constraint in Equation (32) is violated. The other possibility then is Case 2: $\varepsilon^* > \hat{\varepsilon}$. In this case, using the definitions of $\hat{\varepsilon}$ and ε^*, the fact that $\tilde{U}(\varepsilon)$ (given by Equation (27)) is decreasing in ε, and the constraint in Equation (32) implies:

$$\mathbf{N}(\tilde{q}_N) = \mathbf{U}(\varepsilon^*) \geq \tilde{U}(\varepsilon^*) > \tilde{U}(\hat{\varepsilon}) = \mathbf{N}(\tilde{q}_N)$$

a contradiction. Therefore, $\varepsilon^* = \hat{\varepsilon} = h(b) - h(d)$.

Next, to show property (ii), that $\tilde{q_N} \neq q_N^*$, suppose instead that $\tilde{q}_N = q_N^*$. It is sufficient to show that at q_N^* the constraint is violated at $\varepsilon = \varepsilon^* = h(b) - h(d)$. That is, $\tilde{U}(\varepsilon^*) > U(\varepsilon^*)$. Towards this end, notice that by definition of $\tilde{U}(\varepsilon)$ and ε^*, $\tilde{U}(\varepsilon^*) = \mathbf{N}(q_N^*)$. Moreover, from Lemma 8 $\mathbf{U}(\varepsilon^*; \tau > 0) < \mathbf{U}(\varepsilon^*; \tau = 0)$, and from Lemma 7 $\mathbf{U}(\varepsilon^*; \tau = 0) = \mathbf{N}(q_N^*)$.

Combining these relationships implies:

$$\mathbf{U}(\varepsilon^*;\tau>0)<\mathbf{U}(\varepsilon^*;\tau=0)=\mathbf{N}(q_N^*)=\tilde{U}(\varepsilon^*)$$

a contradiction to the equilibrium conditions. Thus, $\tilde{q}_N \neq q_N^*$ in the private information equilibrium. ■

Proof of Corollary 1:

Proof: For notation, denote the firm's zero profit curve from Equation (42) for workers collecting UI (with τ paid at separation) by $\mathcal{P}_U(q)$ and for a non-collector as $\mathcal{P}_N(q)$. Note, given $\tau>0$, Equation (42) implies that given any q, $\mathcal{P}_N(q)>\mathcal{P}_U(q)$.

In equilibrium, the "incentive" constraint imposed by Equation (32) implies that a non-collector and a UI collector with $\varepsilon=\varepsilon^*$ must be on the same indifference curve. That is, $\mathcal{W}(\tilde{q}_N)=\mathcal{W}(q_U^*(\varepsilon^*))$. In addition, equilibrium requires the zero profit curve to intersect the indifference curve at the equilibrium q. For UI collectors this is a tangency, while for non-collectors (under private information) it is an intersection, which we show happens twice. To show this, we start by showing that at $q=q_N^*$, $\mathcal{W}(q)-\mathcal{P}_N(q)<0$ and crosses zero twice, once with $\tilde{q}_N^L<q_N^*$ and once with $\tilde{q}_N^H>q_N^*$.

From Lemmas 7 and 8, $\mathbf{N}(q_N^*)=\mathbf{U}(\varepsilon^*;\tau=0)>\mathbf{U}(\varepsilon^*;\tau>0)=\mathbf{N}(\tilde{q}_N)$. As a result, $\mathcal{P}_N(q_N^*)=\mathcal{W}(q_N^*;\mathbf{N}(q_N^*))>\mathcal{W}(\tilde{q}_N;\mathbf{N}(\tilde{q}_N))=\mathcal{W}(q_U^*(\varepsilon^*);\mathbf{U}(\varepsilon^*))=\mathcal{P}_U(q_U^*(\varepsilon^*))$. Thus, at q_N^*, $\mathcal{W}(q_N^*)-\mathcal{P}_N(q_N^*)<0$. Now, consider $\mathcal{W}(q)-\mathcal{P}_N(q)$ as q decreases. Towards this end, given the properties of the matching function, notice that $\lim\limits_{q\to 0}\alpha_W(q)=\infty$, $\lim\limits_{q\to\infty}\alpha_W(q)=0$, $\lim\limits_{q\to 0}\alpha_E(q)=0$, and $\lim\limits_{q\to\infty}\alpha_E(q)=\infty$.

$$\lim_{q\to 0}\mathcal{W}(q)=U^*-(r+\lambda)h(d) \tag{47}$$

$$\lim_{q\to\infty}\mathcal{W}(q)=h^{-1}\big[\infty\big]=\infty \tag{48}$$

$$\lim_{q\to 0}\mathcal{P}(q)=-\infty \tag{49}$$

$$\lim_{q\to\infty}\mathcal{P}(q)=y-\chi_i\lambda\tau \tag{50}$$

where recall $\chi_i, i = U, N$ is an indicator variable with $\chi_U = 1$ and $\chi_N = 0$. Using Equations (47) to (50) implies that $\lim_{q \to 0} \mathcal{W}(q) - \mathcal{P}_N(q) > 0$. Thus, it starts negative at $q = q_N^*$ and is eventually positive. Since $\mathcal{W}(q)$ is strictly convex (and strictly increasing) and $\mathcal{P}(q)$ is strictly concave (and strictly increasing), this crossing only happens once. As a result, there exists an equilibrium $\tilde{q}_N^L < q_N^*$. We similarly show that there exists an equilibrium $\tilde{q}_N^H > q_N^*$. Specifically, Equations (47) to (50) imply that $\lim_{q \to \infty} \mathcal{W}(q) - \mathcal{P}_N(q) > 0$, which combined with the strict convexity of $\mathcal{W}(q)$ and strict concavity of $\mathcal{P}(q)$ yields a unique crossing above q_N^*.

■