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Frictional spatial equilibrium*

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Abstract

We study the properties of spatial equilibrium in an economy where locations have heterogeneous endowments and the labour market is subject to matching frictions. Both workers and firms make endogenous location decisions, which, in turn, determine the spatial distribution of unemployment, wage and firm density, as well as city population. We explain why diverse urban configurations may coexist in a country without any impediment to labour mobility, and in particular, why homogeneous workers, free to move at will, may be subject to spatial stickiness while welfare is not equalized across space. We also introduce a typology of cities based on the productivity of their local amenities, which describes the comovement of local economic outcomes and we show that the introduction of commercial real estate induces an asymmetry between urban decline and urban growth. Positive (negative) productivity shocks are more (less) likely to increase (decrease) population than rent, rent than wages, and wages than employment.

Keywords: amenities, search frictions, spatial equilibrium.

JEL Classification: J61, R12

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The competitive model of spatial equilibrium, pioneered by Rosen (1979) and Roback (1982), remains the cornerstone of theories of the location decision of firms and workers within a system of local labour markets. In this setting, equilibrium reflects a spatial compensation mechanism achieved through workers' mobility, which equalizes welfare across cities. In this paper, we develop a notion of equilibrium which is consistent with the striking and persistent disparities in key economic outcomes observed across local labor markets in developed countries.[1] Specifically, we show that heterogeneous local matching rates, reflecting the endogenous location decisions of workers and firms, are sufficient to induce an equilibrium mechanism where individuals do not move despite heterogenous welfare across cities.

We model a local labour market and introduce time consuming matching between firms and workers in a dynamic setting where both types of agents decide where to locate.[2] The competitive equilibrium is a special case of our framework, and classical results, depending on the impact of local amenities on firm productivity, are derived and extended. In contrast to the standard literature where the main mechanism involves a spatial trade-off between rents and wages, we investigate whether amenities might induce more firms (resp. workers) to locate not only because of the production (resp. wages) but also because of expectations over a larger workforce (resp. a larger pool of employers). Therefore, the model can explain the comovement of rents, unemployment and wages across cities, but also accounts for equilibrium population and firm densities.

The key mechanism underlying our model is the welfare consequence of the tradeoff between production, matching and congestion externalities in the location decision of firms and workers. Production externalities, stemming from local amenities, capture city-level indivisibilities not manipulable by agents, and independent from agents characteristics. Matching externalities are different in nature as they originate from the location decisions of workers and firms. Because these externalities allow agents to meet suitable partners faster, they may also improve communication flows within the economy and generate knowledge spillovers. Finally, congestion externalities act as a counterweight to the forces that lead to agglomeration. The respective strengths of these externalities determine unemployment, population size, number of firms, and therefore, market tightness.

In contrast to the frictionless literature, we show that the wage level, even when combined with the level of rent, is not a sufficient statistic to measure workers' welfare, because its negative correlation with unemployment risk breaks down when commercial and residential land markets are not

[1] See Moretti (2012) for an overview of the US.

[2] See Zenou (2009) for a number of relevant references.

fully segmented. Even though the model does not feature moving costs for workers, it induces spatial stickiness as positive productivity shocks are more likely to increase local earnings than local employment probability. For the same reason, and except under stringent assumptions, the equilibrium does not induce welfare equalization. This last conclusion underscores the key role played by informational frictions in keeping agents unequal but free to move. Finally, the model also sheds light on the determinants of a city structure (worker-intensive or firm-intensive). Compared to the dualism in the literature between productive and consumer cities, the trade-off between production, matching and congestion induces a far richer typology of cities where workers may be attracted to unproductive cities because of lower rents, or repulsed from very productive cities for the exact opposite reason. To sum up, we show that the diverse urban configurations observed in the data can result from the rational choices of agents (workers and firms) under technological constraints (production, matching and construction).

Following Roback (1982), several papers have proposed refinements of the frictionless theory of spatial equilibrium, featuring labour heterogeneity in skills and tastes, as well as adjustment through both wages and employment (for a discussion, see Moretti (2011)). However, the main indicator of labour market conditions in perfect competition models, expected wage, fails at accounting for equilibrium flows out of and into unemployment (Pissarides, 2000).[3] Arguably, one might even read the building of a theory of equilibrium unemployment as being motivated by earlier empirical work on the trade-offs between wages and unemployment risk in regional equilibrium (see, for example, Pissarides & McMaster (1990)). Contrary to Kline & Moretti (2013) and Wrede (2015), the two previous papers seeking to incorporate matching frictions into the classical spatial equilibrium framework, we truly consider the spatial foundations of local market tightness, which derives from the endogenous location decisions of both workers and firms, each of which having an impact on the level of rent faced by all other actors in the economy.

The structure of the paper is constructive. Section 1 describes some stylized facts on US metropolitan areas that may not be accounted for by previous models. Section 2 provides a frictional generalization of the homogeneous version of the Roback model. Then, section 3 studies how spatial equilibrium is shaped by production, matching and construction technologies and section 4 summarizes the findings into a typology of cities. Section 5 concludes.

[3] A recent paper by Behrens, Mion, Murata & Suedekum (2014), which provides an interesting study of the impact of various kinds of spatial frictions on spatial equilibrium, does so without mentioning unemployment.

1 Motivating facts

The main motivation for the model is the possible coexistence of heterogeneous levels of individual welfare across cities. However, besides anecdotal evidence or declarative data, such information is not easy to collect. We choose to illustrate this notion indirectly, by studying the interaction between the two economic components of welfare, earnings and employment probability, and their impact on city attractiveness. Arguably, the workings of spatial equilibrium in a system of cities may be observed through short-run population dynamics, which, as a first-order approximation, do not rely on structural changes. We provide below suggestive evidence that wage statistics alone do not convey enough information to forecast this kind of short-run urban evolutions.

Table 1 displays the estimation results of a panel regression of yearly population growth for US metropolitan areas in the years preceding the Great Recession, as a function of the lagged growth rates of mean real wage, unemployment rate and population (city fixed effects and a trend are also included). Several features stand out: first, there is no within-city raw correlation between wage dynamics and future urban growth (column 1), while this is not the case of unemployment risk: a higher

Table 1: Wage growth, unemployment growth, and urban dynamics

	Panel A: All MSAs (N=297)				Panel B: Small MSAs (N=192)			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
Hourly wage	0.001		0.020*	0.020*	0.002		0.012	0.010
	(0.013)		(0.012)	(0.012)	(0.014)		(0.014)	(0.014)
Unempl. rate		-0.036***	-0.037***	-0.037***		-0.028***	-0.029***	-0.029***
		(0.0031)	(0.0031)	(0.0031)		(0.0040)	(0.0041)	(0.0040)
Population				0.091***				0.099***
				(0.025)				(0.031)

	Panel C: Medium MSAs (N=80)				Panel D: Large MSAs (N=25)			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
Hourly wage	-0.003		0.070**	0.074**	-0.028		0.031	0.052
	(0.038)		(0.035)	(0.035)	(0.040)		(0.042)	(0.040)
Unempl. rate		-0.054***	-0.056***	-0.057***		-0.023***	-0.025***	-0.030***
		(0.0056)	(0.0057)	(0.0058)		(0.0066)	(0.0073)	(0.0070)
Population				0.063				0.31***
				(0.046)				(0.084)

Notes: (i) Ordinary-least-squares regression of the yearly workforce growth rate by metropolitan area from 2002 to 2006 as a function of the one-year-lagged growth rate of mean hourly wage (in 2000 dollars), of the one-year-lagged growth rate of the unemployment rate and of the one-year-lagged growth rate of the workforce; Standard errors are in parentheses and *, ** and *** respectively denote a 10%, 5% and 1% significance level; (iii) MSA fixed effects and a trend are included; (iv) Small MSAs have a maximum population of less than 200,000 workers over the period, while Large MSAs have a minimum population of more than 1,000,000 workers over the period; (v) Sources: *Local Area Unemployment Statistics* (Unemployment rate and workforce) and *Occupational Employment Statistics* (wages).

unemployment risk is strongly associated with lower population growth (column 2). Second, the positive impact of wage growth on city attractiveness may only sometimes be recovered when one takes unemployment risk into account (column 3). In addition, unemployment dynamics and wage dynamics are almost uncorrelated, as indicated by the stability of the coefficients of the unemployment rate, from columns 2 to 3. Even though one cannot fully rule out the possibility that these findings may reflect spurious correlations, this concern is mitigated by the last specification of each panel, which controls for lagged population growth, and where the results are virtually unaffected (column 4). Finally, these findings are observed all along the city size distribution, even though medium-sized cities stand out compared to small and large cities.

In the next sections, we build a model that aims to reproduce some of these features: in particular, the model equilibrium allows for a breakdown of the negative comovement between unemployment dynamics and wage dynamics and predicts that unemployment dynamics are a more powerful determinant of city attractiveness (through workers' welfare) than wage dynamics.

2 Framework

We consider the spatial search and matching equilibrium of a local labour market where homogeneous workers and firms meet, bargain over a wage and produce. Both workers and firms seek to maximize discounted total disposable revenue flow. The current section, which describes our theoretical model, is divided into three parts. We first describe the geographic component of the model. Second, we present the search and matching framework and describe how wages are formed. Finally, we combine these components to characterize the equilibrium distribution of agents.

2.1 Space

Space consists of a continuum of cities, which are heterogeneous in local amenities $\gamma \geq 0$. Amenities are local attributes that affect individual well-being. They may also impact firm production, as in Roback's (1982) model.

The world consists of an infinite population of workers, who live outside the city and enjoy a dynamic utility z/r. These workers are perfectly mobile, can move to the city at no cost and become unemployed there. This mobility process determines the equilibrium city population M. Similarly, there is an infinite supply of latent firms willing to post a vacancy in the city. The total number of firms in the city, either vacant or productive, is denoted N.

Workers and firms compete on the land market, which is assumed to be perfectly competitive. Workers consume one unit of land, while a job requires ψ unit(s) of land. The parameter ψ is a scaling factor which introduces a segmentation in the land market. Total city size denoted s emerges in equilibrium as workers and firms choose to locate in the city and determines the unit cost of the land q, such that market clearing rents verify $q = h(s)$ where $h(\cdot)$ is a land supply function.[4]

2.2 Search, bargaining and production

Workers and firms are brought together through a random and time consuming matching process. The matching technology is generically defined as a function of the market tightness θ, with classical properties: the worker arrival rate is $m(\theta)$ with $m'(\theta) < 0$, $\lim_{\theta\to 0} m(\theta) = \infty$ and $\lim_{\theta\to\infty} m(\theta) = 0$ and the matching elasticity is given by $\eta(\theta) := -\theta m'(\theta)/m(\theta) \in (0,1)$. Market tightness is an equilibrium outcome resulting from the number of workers M and the number of firms N in the economy. Unemployed workers meet a vacancy at rate $\theta m(\theta)$. There is no search on the job.[5] Finally, a match is destroyed at exogenous rate δ.

A firm in a location with γ amenities, when matched with a worker, benefits from a fixed irreversible technology to produce a unit of output valued at $p(\gamma)$, where $p = p(0)$ denotes the location-independent, firm-specific component of productivity, which is assumed to be high enough so that the participation constraint is always satisfied.[6] As for the dependency vis-à-vis amenities, it captures all indivisibilities (infrastructure, knowledge, ...) shared by firms within a location. This production function is continuously differentiable. The instant profit of the firm is the difference between $p(\gamma)$ and the sum of the operating costs: wage and rent. Rent is also paid when a vacancy is open, in addition to an advertising cost c. Therefore, we can define $s \equiv M + \psi N$.

On the worker side, income has both a non-monetary component γ and a pecuniary wage component w (or b when unemployed). The tradeoff between wages and amenities is key in the location decision of agents in the standard literature. In our setting, the attractiveness of a local labour market also depends on the likelihood and the salience of unemployment, captured by unemployment benefits b and local job finding rate $m(\theta)$.

Wages are determined through a bargaining process, where workers are endowed with an exogeneous bargaining parameter β. With V_u, V_e, J_f and J_v the respective value functions of unem-

[4] Alternatively, one could think of land as a fixed factor, transformable into housing, which would be the real input, and the quantity of which would be endogenously determined.

[5] This is not crucial to the mechanisms we seek to highlight here.

[6] In the literature, it is generally assumed that $p(\gamma) = p + \sigma\gamma$ where $\sigma\gamma$ is the city-specific productivity component.

ployed workers, employed workers, filled vacancies and vacancies (for complete expressions, see Appendix A), and the surplus of a match as $S = V_e - V_u + J_f - J_v$, we consider that the optimal level of wages w solves $V_e - V_u = \beta S$. Note that a by-product of this wage determination process is that the match surplus can be rewritten as follows:

$$(r+\delta)S = p(\gamma) + \gamma - (1+\psi)q - rV_u, \tag{1}$$

which shows that whether higher amenities will increase or decrease the level of wealth created by a match depends on their relative impact on firm production and rent.

2.3 Equilibrium

We consider a *frictional spatial equilibrium* as a system of local labour markets where land markets clear, unemployed agents are indifferent between locations, profit opportunities are exhausted, and labour market aggregates are steady. This equilibrium is summarized by a set of endogenous variables $\left(q, w, u, M, N\right)$, given a vector of exogenous parameters $\left(b, r, \delta, z, c, \beta, \gamma, \psi\right)$ and functional forms $p(\cdot)$, $m(\cdot)$, $\theta(\cdot)$ and $h(\cdot)$. Matching, rent and wages can be described by equations 2 to 4:

$$\frac{r+\delta}{m(\theta)} = \frac{1-\beta}{c+\psi q}(p(\gamma) - b - \psi q) - \beta\theta \tag{2}$$

$$q = b + \gamma - z + (c+\psi q)\frac{\beta\theta}{1-\beta} \tag{3}$$

$$w = \beta(p(\gamma) - \psi q) + (1-\beta)\left(q + z - \gamma\right) \tag{4}$$

It is worth pointing out that equation 2 describes the job creation mechanisms, which is standard in the literature (Pissarides, 2000). Also, the unique market tightness is an increasing function of local productivity $p(\gamma)$ and a decreasing function of workers bargaining power β. Contrary to Kline & Moretti (2013), this equation is impacted by both local amenities γ and the amenity value of the outside world z, through the rent channel. Any change in the productivity of amenities causes movements along a downward vacancy unemployment locus. An increase in productivity, or a decrease in exogenous cost of employment b shift the job creation curve to the left. In other words, since $p(\gamma)$ captures the strength of a local labour market, this results stipulates that *matching rates* and *job creation* are higher in stronger labour market - higher productivity increases the number of jobs. However, the strength of this effect depends ultimately on the ratio between match creation and destruction rates.

Equation 3 shows that local rent q is an increasing function of market tightness θ and firm's land consumption ψ. This conclusion highlights the effect of congestion implied by competition between workers and firms. Equilibrium rents do not only reflect an equalization of quality of across cities, but also differences in unemployment rates, and local firm density. The additional layer is more likely to reflect the complex tradeoffs in the location choice of both individuals workers and firms. Let $\chi = 1-\beta-\psi\theta\beta$. For rent to be positive and the model to remain meaningful, χ must be positive. This will be the case when workers have low bargaining power, firms consume little land and market tightness is relatively small. Also, note that higher amenities can in theory have a negative impact on rent if they are extremely detrimental to market tightness.

As shown in Equation 4, wage can decomposed between two elements, which respectively capture the participation constraint of the worker and a non mobility compensation effect. The first part represents the share of the surplus obtained by workers and covers for their outside option. We refer to this component as the participation share as it ensures that the workers is compensated more than its reservation wage. The second is a bargaining power weighted cash flow $z+q-\gamma$ required for workers to obtain utility level z, which acts as a spatial equalization mechanism to ensure that employment is always more profitable than unemployment, ruling out strategic unemployment behaviour.

Finally, the remaining of the equilibrium is defined by the steady state motion law of unemployment (equation 5) and the market clearing equation for land (equation 6), which allow us to recover city population M and number of firms N given a formal definition of market tightness (equation 7).

$$u = \frac{\delta}{\theta m(\theta)+\delta} \tag{5}$$

$$s = h^{-1}(q) \tag{6}$$

$$\theta = \frac{N-(1-u)M}{uM} \tag{7}$$

3 Workings of the city

The model relies on three technological constraints that affect the location decision of workers and firms, and arise from *production*, *matching* and *housing supply* technologies. The composition of cities is subject to these different constraints, which summarize the forces that lead to the agglomeration or scattering of economic agents and activities in space.[7] In this section, we show that the impact

[7] Throughout the exercise, we maintain a few crucial assumptions: first, the firm-specific component of production is high enough to ensure participation when $\gamma = 0$; second, the elasticity of matching is positive and lower than one; and finally, rent is positive and increasing with city size.

of amenities on almost all city characteristics (rent, wage, unemployment and worker's welfare) may be derived from the single impact of amenities on production. We derive a functional definition of $p(\gamma)$ that guides the co-movement of amenities and model primitives, and use it to construct a typology of cities. The only feature that is not fully determined by the production function is the breakdown between residential and commercial real estate, which also depends on the relative sensitivity of matching and housing supply to amenities.

3.1 Wages, unemployment and rents

We derive and discuss the impact of the production function on the equilibrium outcomes defined in section 2. Since local amenities affect labour market equilibrium indirectly through market tightness which is affected by the endogenous location decisions of firms and workers, we explicitly define the dependency using $\theta := \theta(\gamma)$ and $\chi(\gamma) := 1-\beta-\psi\beta\theta(\gamma)$. By solving a first-order ODE, we can characterize the threshold production functions that negate the impact of amenities on unemployment $(p^u(\cdot))$ and wages $(p^w(\cdot))$:

$$p^u(\gamma) = p + \psi\left(\frac{p+c-b}{c+\psi(b-z)}\right)\left(\frac{\theta(\gamma)\chi(0)}{\theta(0)\chi(\gamma)}\right)\gamma + (1-\beta)\left[p-b+\psi(b+\gamma-z)\right]\left(\frac{\theta(\gamma)-\theta(0)}{\chi(\gamma)\theta(0)}\right) \tag{8}$$

$$p^w(\gamma) = p + \psi(1-\beta)\left(\frac{1-\theta(\gamma)}{\chi(\gamma)}\right)\gamma - (1-\beta)\left(1-\beta-\psi\beta\right)\left[c+\psi(b-z)\right]\left(\frac{\theta(\gamma)-\theta(0)}{\chi(0)\chi(\gamma)}\right) \tag{9}$$

When $p(\gamma) > p^u(\gamma)$, higher amenities lead to a decrease in unemployment; while if $p(\gamma) > p^w(\gamma)$, they lead to a wage increase. These expressions feature firms' participation constraint as a constant, a linear component in γ, and an element that captures the effect of market tightness. Note that if $p(\gamma) = p^u(\gamma)$, equation 8 simplifies to $p^u(\gamma) = p + \psi\frac{p+c-b}{c+\psi(b-z)}\gamma$: the production function negating the impact of amenities on unemployment is linear in amenities. Note, also, that when $\psi = 0$, equations 8 and 9 simplify to:

$$\begin{aligned} p^u(\gamma) &= p + \frac{p-b}{\theta(0)}\left(\theta(\gamma)-\theta(0)\right) && (10)\\ p^w(\gamma) &= p - c\left(\theta(\gamma)-\theta(0)\right) && (11) \end{aligned}$$

Finally, we compute the market tightness function $\theta^q(\cdot)$ that negates the impact of amenities on rent, so that if $\theta(\gamma) < \theta^q(\gamma)$, higher amenities lead to a decrease in rent. This function verifies:

$$\theta^q(\gamma) = K[c+\psi(b+\gamma-z)] + \frac{1-\beta}{\beta\psi} \tag{12}$$

where $K < 0$ is an integration constant that depends on the model parameters. Rents may only decrease with amenities when the market tightness is very negatively impacted by amenities. In particular, one can see that $\theta^q(\gamma) \leq \theta^q(0)$, so that if $p(\gamma) > p^u(\gamma)$, rent cannot decrease with amenities. If $\psi = 0$, equation 12 becomes:

$$\theta^q(\gamma) = K - \frac{1-\beta}{c\beta}\gamma \tag{13}$$

The systematic study of equations 8 to 13 restricts the set of possible equilibria in a way that is summarized in proposition 1:

Proposition 1 *ROSEN-ROBACK WITH SEARCH FRICTIONS*

i) If higher amenities decrease unemployment, they increase wages; if they decrease wages, they increase unemployment.

ii) In some cases, higher amenities increase both unemployment and wages. These cases disappear if firms do not consume land.

iii) If higher amenities decrease rents, they decrease wages; if they increase wages, they increase rents.

iv) If local amenities have no impact on firm production, they cannot have a positive impact on the labor market.

Proof See Appendix B

Proposition 1 reformulates and extends classical results. As in Roback (1982), the rent gradient is almost always positive, whereas the impact of amenities on labour market outcomes depends on how productive local amenities are. In addition, local amenities always have at least some impact on the labour market or on the real estate market.

Points (i) and (ii) state that introducing search frictions and commercial real estate allows us to distinguish between the patterns of employment and wages. Unless amenities are very productive, higher amenities will attract more unemployed workers than new firms, hence a lower matching rate, or equivalently a higher unemployment rate. This result is more likely than the traditional compensation mechanism on wages because the bargaining mechanism between workers and firms attenuates the impact of negative productivity shocks. As shown in equation 1, match surplus is negatively correlated with rent. Therefore, if matching frictions are high because technology is not productive enough to attract a large number of new firms, match surplus will not be as low as expected because if firms consume land, local rents act as a stabilization device.

Point (iii) states that labour market dynamics and real estate dynamics are intertwined: negative real estate (resp., positive labour market) dynamics imply negative labour market dynamics (resp., positive real estate dynamics). Contrary to Roback (1982), the model allows for the possibility of a negative impact of amenities on rent, because city size is endogenous and unemployed workers will leave a city that has experienced a negative productivity shock. This phenomenon is all the more likely to be observed if firms also consume land, but it does not depend on this assumption.

Finally, point (iv) deals with the special case when there are no local effects on productivity. This assumption of constant returns in the production technology is interesting because it is often made in the search and matching literature, even if it is in contradiction with the well-documented impact of city size on productivity.[8] In this case, if firms do not consume land, amenities have no impact on unemployment nor on wages. This is the only case when such a null result happens. However, if firms do consume land, higher amenities may only translate into negative labor market outcomes: higher unemployment and lower wages.

In a word, the effect of amenities on labour market equilibrium can be summarized by two different forces: intensive and extensive margins. When amenities are unproductive, the most important effect (extensive margin) comes from higher unemployment rates which correspond to a lower participation rate. Given unemployment rates, a spatial adjustment process (intensive margin) that affects wages takes place. The equilibrium effect of amenities on labour local market outcomes depends on the respective strengths of these two effects.

3.2 Spatial distribution of workers' welfare

One interesting by-product of the concept of frictional spatial equilibrium is to allow for a discussion about the impact of incomplete information on the spatial distribution of local economic well-being. We define workers' welfare as a weighted average between the respective utility levels of unemployed workers and employed workers:

$$\mathscr{W} = r\left[uV_u + (1-u)V_e\right] = ub + (1-u)w + \gamma - q \tag{14}$$

Local amenities impact workers' welfare both directly (consumption of the amenity) and indirectly, through their impact on employment, wages and rent. While workers' mobility leads to welfare equalization across locations in a frictionless setting, the existence of search frictions does not preclude

[8] See the discussion by Lutgen & Van der Linden (2015).

spatial heterogeneity in welfare as only utilities derived from a state of unemployment are equalized across locations. When the location decision of agents affects their probability to meet a suitable partner, welfare differences across cities can exist and persist over time without any adjustment cost at the individual level. In addition, while workers' welfare is generally unobserved and therefore proxied by other variables, and in particular by real consumption wage, it appears that wage dynamics do not always convey enough information to be used to infer welfare dynamics in a city. The findings are summarized in Proposition 2:

Proposition 2 — *SUFFICIENT STATISTICS FOR WORKERS' WELFARE*

i) If higher amenities decrease unemployment (rent), they increase (decrease) workers' welfare

ii) If firms consume land, workers' welfare dynamics cannot be inferred from wage dynamics

iii) If firms do not consume land and amenities have no impact on production, workers' welfare does not depend on amenities

iv) The only case where workers' welfare is equalized across locations regardless of the productivity of local amenities is when posting a vacancy is costless for the firms

Proof See Appendix C

From an empirical perspective, points (i) and (ii) show that when firms consume land, the only sufficient labour market statistic indicating a positive welfare outcome is a positive employment rate gradient.[9] On the real estate side, a decrease in rent is a non-ambiguous indication that the city is becoming a worse place to live. Wages, on the other hand, may be positively or negatively correlated with welfare. If firms do not consume land, the respective predictive powers of employment and wages become equivalent because of the mechanisms highlighted in proposition 1.

From a theoretical viewpoint, points (iii) and (iv) state that the process of welfare equalization predicted by the frictionless framework is only a special case. If firms do not consume land and amenities have no impact on production, there is a perfect capitalization of amenities such that they impact only the instant utility of workers and not the match surplus even in the presence of matching frictions. However, this result does not stand if firms consume land, because higher unproductive amenities attract more workers, hence lead to higher rents, which translates into lower welfare.

[9]This result may help explain the empirical evidence presented in Table 1. The relevance of the employment rate to analyze local economic well-being is also advocated by Amior & Manning (2015), in a very different framework.

More generally, point (iv) stipulates that when matching frictions are introduced, the classical result of a flat welfare gradient does not hold anymore.[10] As long as $p'(\gamma) \neq 0$, the mechanisms of wealth creation (through match surplus) and wealth distribution (through matching) cannot perfectly compensate each other. Let $p^S(\gamma)$ the threshold production function verifying that match surplus does not vary with amenities.

$$p^S(\gamma) = p + \psi\left(\frac{1-\beta+\beta\theta(\gamma)}{\chi(\gamma)}\right)\gamma + (1-\beta)\beta(1+\psi)\left[c+\psi(b-z)\right]\left(\frac{\theta(\gamma)-\theta(0)}{\chi(0)\chi(\gamma)}\right) \tag{15}$$

If $p(\gamma) = p^S(\gamma)$, it can be verified that higher amenities have a positive impact on workers' welfare if and only if they also have a positive impact on market tightness. Therefore, welfare will only be equalized across locations if workers' probability to appropriate some of the surplus is also equalized across locations.[11]

This result stresses the importance of incomplete information on the spatial distribution of workers' welfare. In a world with complete information, workers choose their location γ according to their expected lifetime utility $\mathscr{W}(\gamma)$, leading to a spatial equalization through real estate mechanisms. On the contrary, in the current setting, unemployed workers cannot afford to migrate to higher-welfare cities because of the general equilibrium effect of mobility on matching. A possible way of increasing the number of unemployed workers in high-welfare cities would be for workers to commit a type-dependent insurance contract in order to insure against unemployment risk in high-welfare cities, where local cost of living is higher.[12]

3.3 Firm density and city size

The previous results show the role of amenities on labour market outcomes, through their impact on market tightness, without separating the differential effect on firms and workers. In addition, they do not characterize the impact of amenities on the size of the economy both in terms of workers and firms. In this section, we study how amenities affect the vacancy creation behaviour of firms, and the location decision of workers. In doing so, we aim to provide a better understanding of how firms' and workers' behaviour shape not only the typology of cities but also the composition of cities.

Denoting by $f = N/M$ the firm density in the city, we can restate this object as a function of market tightness and unemployment rate $f \equiv 1-u(1-\theta)$. In addition, since unemployment rate

[10] Similarly, an egalitarian policy setting $b = b_{\max}$ such that $V_e = V_u$ leads to and $u = 0$ $\mathscr{W}(\gamma) = z$.

[11] Note that this result holds if $\psi = 0$ and $p^S(\gamma) = p + \frac{c\beta}{\theta(0)}\left(\theta(\gamma)-\theta(0)\right)$.

[12] Given a target $\mathscr{W}(\gamma) = \underline{\mathscr{W}}$, it is possible to design a transfer scheme $(-t_e, t_u)$ such that $(1-u)t_e = ut_u$.

depends ultimately on market tightness $u := u(\theta)$, we can define matching conditions to depend only on firm density f. Given a matching technology $\widehat{m}(f) = m(\theta)$, we can solve the steady state unemployment as a function of firm density. Furthermore, bounds on the elasticity of the matching function, $\eta(\theta) \in (0,1)$ ensures that there is a strict positive bijection between f and θ, so that the relationship between amenities and market tightness described by equation 8 is still verified. Given a construction technology $h(\cdot)$, city size is the solution to equation 6, which allows then to back out population size M and number for firms N using prior knowledge of f. We define $f(\gamma) \equiv \widehat{m}^{-1}(m(\theta))$ the inverse firm density function and $s(\gamma) \equiv h^{-1}(q)$ the inverse land supply function. With $M \equiv \frac{s}{\psi f+1}$ and $N \equiv \frac{fs}{\psi f+1}$, we get:

$$M'(\gamma) > 0 \Longleftrightarrow f'(\gamma) < \frac{s'(\gamma)}{\psi M} \tag{16}$$

$$N'(\gamma) > 0 \Longleftrightarrow f'(\gamma) > -\frac{f}{M}s'(\gamma) \tag{17}$$

Equations 16 and 17 highlight the importance of the *cost effectiveness of matching* and allow us to write proposition 3:

Proposition 3 — *CITY ATTRACTIVENESS FOR FIRMS AND WORKERS*

i) While higher productive amenities always translate into an increase in city size, the breakdown of land between commercial and residential usage depends on how productive local amenities are and to what extent matching efficiency is capitalized into land prices.

ii) If the number of firms increases with amenities, so does the population of workers: the only possible exception to this rule takes place if amenities are so productive that they lead to a boom in commercial real estate at the expense of residential real estate.

iii) There is a possibility of a poverty trap whereby higher, unproductive amenities, combined with low construction costs, attract workers because of low cost of living conditions even though they also contribute to a derelict labour market.

Proof See Appendix D

Proposition 3 describes the mechanism of agglomeration in the city. Specifically, we show that although productive amenities are appealing to both workers and firms, the equilibrium city population depends on the comparative advantage of firms over workers, which is directly captured by the degree of supermodularity of the production function $p(\gamma)$. Generically, if the number of firms increases with amenities, so will the population of workers, unless the dynamics of commercial real estate take place at the expense of residential real estate. Finally, proposition 3 highlights the possibility of a poverty trap originating from the combination of unproductive amenities and low construction costs. Such a location is still attractive to workers because of the spatial compensation mechanism (low living cost vs failing labour market). Growing cities with flourishing labour markets, and slums with failing labour markets are extreme urban configurations, which are compatible with a spatial frictional equilibrium. Ultimately, these configurations can not be summarized using the duality between wages and rent, but require the introduction of matching technology and unemployment.

4 A typology of cities

To summarize the different mechanisms developed in the paper, Panel A in Table 2 presents a typology of cities based on the production technology, which corresponds to units where theoretical regularities can be established. Theoretically, different types of cities may coexist in a country as a result of spatial misallocation of workers across cities, and frictional job finding (Shimer, 2007).

In contrast to most urban economics, cities do not always offer positive agglomerations benefit in production, specifically for a range of production functions for which both labour and real estate markets are negatively impacted by higher amenities. In the lower type, *repulsive cities*, the combination of highly unproductive amenities and cost-ineffective matching yield lower populations of both workers and firms; in the less extreme type, *poor cities*, which is akin to a citywide poverty trap, unproductive amenities associated with affordable construction induce both the number of firms and the overall level of rent to decrease with larger amenities; however, the population of workers, who benefit from lower cost of living, increases. Interesting, those first two classes are not likely to be empirically relevant. The next class consists of *consumer cities*, where higher amenities translate into higher rents, lower wages and higher unemployment, and lower welfare. Quality of life is the major source in attractiveness of these locations. Both quality of life and wages attract workers into the fourth class of *divided cities*, where amenities are weakly productive and higher amenities translate into higher wages but not into lower unemployment, therefore benefiting both landlords and employed workers, but at the cost of unemployed agents. The welfare dynamics associated with these two classes of cities are unclear and depend on the structural parameters in the economy. The case where higher amenities would translate into higher rents and higher wages but not into higher workers' welfare could exist as as a result of workers' myopia: a focus on instant utility (amenities for

Table 2: Typology of cities: the role of commercial real estate

Panel A: Six city types when $\psi > 0$

<table>
<tr><th>Repulsive</th><th>Poor</th><th>Consumer</th><th>Divided</th><th>Productive</th><th>Edge</th></tr>
<tr><td>$M'(\gamma) < 0$</td><td colspan="4">$M'(\gamma) > 0$</td><td>$M'(\gamma) < 0$</td></tr>
<tr><td colspan="2">$q'(\gamma) < 0$</td><td colspan="4">$q'(\gamma) > 0$</td></tr>
<tr><td colspan="3">$w'(\gamma) < 0$</td><td colspan="3">$w'(\gamma) > 0$</td></tr>
<tr><td colspan="4">$u'(\gamma) > 0$</td><td colspan="2">$u'(\gamma) < 0$</td></tr>
<tr><td colspan="2">$\mathscr{W}'(\gamma) < 0$</td><td colspan="2">$\mathscr{W}'(\gamma) \lesseqgtr 0$</td><td colspan="2">$\mathscr{W}'(\gamma) > 0$</td></tr>
</table>

Panel B: Three city types when $\psi = 0$

<table>
<tr><th>Repulsive - Poor</th><th>Consumer - Divided</th><th>Productive - Edge</th></tr>
<tr><td>$M'(\gamma) < 0$</td><td colspan="2">$M'(\gamma) > 0$</td></tr>
<tr><td>$q'(\gamma) < 0$</td><td colspan="2">$q'(\gamma) > 0$</td></tr>
<tr><td colspan="2">$w'(\gamma) < 0$</td><td>$w'(\gamma) > 0$</td></tr>
<tr><td colspan="2">$u'(\gamma) > 0$</td><td>$u'(\gamma) < 0$</td></tr>
<tr><td>$\mathscr{W}'(\gamma) < 0$</td><td>$\mathscr{W}'(\gamma) \lesseqgtr 0$</td><td>$\mathscr{W}'(\gamma) > 0$</td></tr>
</table>

Notes: (i) The implications laid out in propositions 1 to 3 can be observed by reading Panel A from the top left corner to the bottom right corner. The only exception is the "edge cities", where the competition for land is so severe and firms are so productive that higher amenities come at the expense of residential real estate.

consumer cities, and both wages and amenities for illusive cities) at the expense of actual welfare. At the end of the productivity spectrum, *productive cities* correspond to the standard viewpoint of cities: amenities generate positive production externalities that benefit everyone. As a consequence, firms and workers earn more and both the population of workers, the number of firms, and firm density increase with higher amenities. However, there also is a possibility of more extreme *edge cities*, where the demand for commercial real estate is so high that it drives away some of the workers who cannot keep up with the level of rent, so that the population of workers actually decreases with higher amenities. This congestion externality does not prevent the city from expanding, from a land-use viewpoint.

Another way to read Table 2 is to discuss the urban dynamics following an local exogenous productivity shock affecting the production function $p(\cdot)$. Workers and firms react differently to booms and downturns. Workers directly benefit from local amenities, and this channel is not affected by productivity shocks. On the contrary, the decision to open a vacancy is more sensitive to local economic conditions. Therefore, the population of workers will not decrease, unless the shock is strongly negative. Similarly, at the other end of the productivity spectrum, an adverse shock will be very likely to be detrimental the probability of finding a job. Wage dynamics are in between because they are determined by the value of match surplus, which depends on firm productivity, but also on rent and amenities. These mechanisms, combined with the coexistence of commercial real estate and residential real estate, explain why a positive productivity shock is more likely to increase population than rent, and rent than wages and more likely to increase wages than reduce the unemployment rate, whereas everything is reversed for a negative productivity shock. Urban decline and urban growth are asymmetric. Such conclusion resonates with the study by Glaeser & Gyourko (2005), who use a perfect competition framework featuring durable housing to explain why cities tend to grow more quickly than they decline, why positive productivity shocks increase population more than they increase rents, and why negative productivity shocks decrease rents more than they decrease population.

Finally, Panel B of Table 2 shows that the typology is drastically simplified when firms land consumption is not considered ($\psi = 0$): as stated in Kline & Moretti (2013), such assumption blocks one channel of feedback from the housing market to the labour market. In addition, city size and population become synonymous. Under this scenario, and abstracting from repulsive cities or poverty traps, as well as from the question of workers' welfare, a frictional spatial equilibrium and the classical spatial equilibrium become observationally equivalent.

5 Conclusion

Following Roback (1982), this paper develops a framework which can be used to uncover two features that are generally implicit in a local labour market: local amenities and their impact of city productivity. The introduction of matching frictions and commercial real estate allows us to further discuss the dynamics of unemployment and local firm density. Among other findings, we show that unemployment and the level of earnings may not always be negatively correlated. A practical consequence is that any empirical work regarding the impact of city amenities on labour market outcomes should make use of statistical information on both dimensions separately.

Our findings suggest a number of venues for future research. We show that labour market policies targeting unemployed workers should make a better use of the spatial dimension of the labor market.

Notably, a national minimum wage policy is likely to induce more distortion and reduce mobility. As such developing a fully structural model with various sources of inefficiency would allow to test different policy margins. On an empirical side, testing the predictions of this equilibrium model would require very stable macroeconomic conditions. A venue for future research will therefore be to study the transitional dynamics that are likely to occur in the process of evolving local amenities or sectorial recompositions affecting local production technologies.

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A Value functions

$$rV_u = b + \gamma - q + \theta m(\theta)[V_e - V_u] \tag{18}$$

$$rV_e = w + \gamma - q + \delta[V_u - V_e] \tag{19}$$

$$rJ_v = -c - \psi q + m(\theta)[J_f - J_v] \tag{20}$$

$$rJ_f = p(\gamma) - \psi q - w + \delta[J_v - J_f] \tag{21}$$

B Proof of proposition 1

Points i) and ii) stipulate that if $\psi > 0$, only three cases are possible: $p^w(\gamma) < p^u(\gamma) < p(\gamma)$, $p^w(\gamma) < p(\gamma) < p^u(\gamma)$ or $p(\gamma) < p^u(\gamma) < p^w(\gamma)$.

- The market tightness function $\widehat{\theta}(\gamma) = \theta(0) - \frac{\psi\theta(0)\chi(0)\gamma}{(1-\beta)(c+\psi(b-z))+\psi\chi(0)\gamma}$ such that $p^u(\gamma) = p^w(\gamma)$ is bounded upwards by $\theta(0)$. Let $\widehat{\gamma}$ the root (possibly outside of the range of values taken by γ) to the equation $\theta(\widehat{\gamma}) = \widehat{\theta}(\widehat{\gamma})$.
- If $\gamma = 0$, we have $p^u(\gamma) = p^w(\gamma) = p$. In a right-neighborhood of 0, we know that $p^u(0^+) > p^w(0^+)$ because the lowest firm productivity p compatible with the participation constraint $S > 0$ for $\gamma = 0$ is higher than the value that would solve $p^u(0^+) = p^w(0^+)$.
- If $p(\gamma) > p^u(\gamma)$, then by definition, this means $\forall \gamma > 0, \theta(\gamma) > \theta(0)$ so that by continuity, $p^u(\gamma) > p^w(\gamma)$
- If $p(\gamma) < p^u(\gamma)$, then by definition, this means that $\forall \gamma > 0, \theta(\gamma) < \theta(0)$
  - If $\gamma < \widehat{\gamma}$, we have $p^u(\gamma) > p^w(\gamma)$ but $p(\gamma)$ may either be higher or lower than $p^w(\gamma)$.
  - If $\gamma > \widehat{\gamma}$, we have $p^u(\gamma) < p^w(\gamma)$

Point iii) The threshold market tightness for a negative rent gradient, $\theta^q(\gamma) = \theta(0) - \frac{\chi(0)\gamma}{\beta[c+\psi(b-z)]}$, can be shown to be always lower than $\widehat{\theta}(\gamma)$.

- If $\theta(\gamma) < \theta^q(\gamma)$, then $p(\gamma) < p^u(\gamma) < p^w(\gamma)$.
- If $p(\gamma) > p^w(\gamma)$, then $\theta(\gamma) > \widehat{\theta}(\gamma)$

Point iv) stipulates that if $\psi = 0$, only two cases are possible: $p^w(\gamma) < p^u(\gamma) < p(\gamma)$ and $p(\gamma) < p^u(\gamma) < p^w(\gamma$.

- From the definition of $p^u(\gamma)$ and equations 10 and 11, we have $p(\gamma) > p^u(\gamma)$ if and only if $p^u(\gamma) > p^w(\gamma)$

C Proof of proposition 2

Point i) Stipulates that $u'(\gamma)<0 \Longrightarrow \mathscr{W}'(\gamma)>0$ and $q'(\gamma)<0 \Longrightarrow \mathscr{W}'(\gamma)<0$

- We can show that if $p(\gamma)=p^u(\gamma)$, then:

$$\mathscr{W}'(\gamma)>0 \Longleftrightarrow \theta'(\gamma)>\theta'_{\mathscr{W}} \equiv -\frac{\psi\theta(\gamma)\chi(\gamma)}{(1-\beta)[c+\psi(b+\gamma-z)]} \tag{22}$$

- By definition of $p^u(\gamma)$, if $p(\gamma)=p^u(\gamma)$ then $\theta'(\gamma)=0>\theta'_{\mathscr{W}}$.
- From equation 3, we can show that $q'(\gamma)=0 \Longleftrightarrow \theta'(\gamma)=-\frac{\chi(\gamma)}{\beta[c+\psi(b+\gamma-z)]}<\theta'_{\mathscr{W}}$

Point ii) Stipulates that we may observe $w'(\gamma)\lesseqgtr 0$ and $\mathscr{W}'(\gamma)\lesseqgtr 0$. It can be proven by constructing parametric examples corresponding to the four situations

Point iii) Stipulates that if $\psi=0$, then $p'(\gamma)=0 \Longrightarrow \mathscr{W}'(\gamma)=0$.

- Assume $p(\gamma)=p$ and $\psi=0$. Then, we can show that $\mathscr{W}'(\gamma)$ can be factored into $\theta'(\gamma)$.
- Since $p^u(\gamma)=p$ in this case, this means that $\theta'(\gamma)=0$ so that $\mathscr{W}'(\gamma)=0$

Point iv) Stipulates that if $\psi=0$, then $\forall p(\gamma), c=0 \Longrightarrow \mathscr{W}'(\gamma)=0$

- With $\psi=0$, it is possible to show that $\mathscr{W}'(\gamma)$ can be factored into c, so that $\lim_{c\to 0}\mathscr{W}'(\gamma)=0$.

D Proof of proposition 3

The existence of a bijection between θ and f stems from the property of the matching function whereby $-\theta m'(\theta)/m(\theta)\in(0,1)$. Indeed, we have $f'(\gamma)=\theta'(\gamma)u^2\left(\delta+m(\theta)+(1-\theta)\theta m'(\theta)\right)/\delta$, which yields either $f'(\gamma)>\theta'(\gamma)u^2\left(1-\theta^2 m'(\theta)/\delta\right)>0$ or $f'(\gamma)<\theta'(\gamma)u^2\left(1-\theta^2 m'(\theta)/\delta\right)<0$. From Equations 16 and 17, we can distinguish the following cases:

- If $s'(\gamma)<0$ and $\frac{f'(\gamma)}{s'(\gamma)}<\frac{1}{\psi M}$, then $M'(\gamma)<0$ and $N'(\gamma)<0$
- If $s'(\gamma)<0$ and $\frac{f'(\gamma)}{s'(\gamma)}>\frac{1}{\psi M}$, then $M'(\gamma)>0$ and $N'(\gamma)<0$
- If $s'(\gamma)>0$ and $\frac{f'(\gamma)}{s'(\gamma)}<-\frac{f}{M}$, then $M'(\gamma)>0$ and $N'(\gamma)<0$
- If $s'(\gamma)>0$ and $-\frac{f}{M}<\frac{f'(\gamma)}{s'(\gamma)}<0$, then $M'(\gamma)>0$, $N'(\gamma)>0$ and $f'(\gamma)<0$
- If $s'(\gamma)>0$ and $0<\frac{f'(\gamma)}{s'(\gamma)}<\frac{1}{\psi M}$, then $M'(\gamma)>0$, $N'(\gamma)>0$ and $f'(\gamma)>0$
- If $s'(\gamma)>0$ and $\frac{f'(\gamma)}{s'(\gamma)}>\frac{1}{\psi M}$, then $M'(\gamma)<0$ and $N'(\gamma)>0$

Point i) derives from the observation all cases. Point ii) derives from the observation of the last case. Point iii) derives from the observation of the second case.