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Obfuscation Incentives of a Monopoly in a Dynamic Framework

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Abstract

Obfuscation is a well-known strategy developed by firms to lower the intensity of competition. We develop here a theoretical framework extending this result to a monopoly controlling consumer's search costs. In a dynamic structure where the future of the monopoly is today's competitor, as phrased by Coase's conjecture, obfuscating some products today turns clients into myopic consumers. This myopia generates higher profit through an optimal price discrimination. Introducing some correlation between the valuations of the goods, we are able to describe more complex but realistic equilibria. Interestingly, there exists a correlation value above which obfuscation is no longer profitable.

Keywords. Coase's conjecture, multiproduct-firm, monopoly, obfuscation, intertemporal price discrimination, sequential search model
JEL Classification. L12 D21 D42 D83 D92

Introduction

The sharp growth of the Internet has radically transformed the perception by consumers of a market. What used to be non relevant, because miles away is nowadays distant by a few clicks only. Cost of information reduced dramatically, allowing clients to compare many items easily and therefore increase the competition. In this context and to maintain their margins, many firms have developed new strategies.

One of the most frequent and studied one is to alter the accessibility of information for the consumers. Many websites obfuscate in different ways to reduce the comparability of their products with their competitors. Very often, this strategy takes the form of *add-on* or *personalization possibilities* that makes all comparisons useless.

Observing a very specific market, the *Online Exclusive Sales Market*, we noticed that all majors actors choose to obfuscate in a very specific and new way. Interestingly, even the first-mover obfuscated *before* the entry of its competitors, suggesting that this strategy is profitable even for a monopoly. This *Online Exclusive Sales Market* is made of websites selling products, mostly in the ready-to-wear industry, to their members. These websites act as platforms of a two-sided market, between producers and clients. A producer joins the platform to be able to destock rapidly his products while a client joins the platform to buy items at lower price. Each side of this market exerts a positive externality on the other: the more clients, the more the producers can sell; and the more producers, the more clients are interested in this website. The collaboration of the producer and the platform takes the form of *a sale*: for a limited duration (one week on average), many different articles of a particular brand (with a defined price and a limited stock) are for sale. Every day, new sales are opened and old ones are closed, whatever the remaining stock.

In this context, obfuscation takes the form of *not revealing the future products*. Indeed, even if the platform plans ahead sales approximatively two months in advance, consumers only discover the products at the opening of the appropriate sale. We would like in this paper to rationalize this behavior.

The base-model we develop is a two period game where one product is sold at each period. Consumers would like to buy *one and only one* of these two products. This extreme form of substitution is the standard one in the search literature and makes computations tractable. Therefore, they have to choose between the one available today and the one sold in the future, knowing that the first sale will be closed at that time. There exists search costs to come back. To maximize its profit the platform can control the search costs and has two possible strategies:

(i) Revelation: Disclose all the products at the first period, so the consumers know their valuations for the two goods. Clients can either buy now or wait the second period to discover the second period price and buy the second good.

(ii) Obfuscation: Hide the second period's good. Clients have to decide whether to buy now or to wait for the second period. If they do come back, they discover the second period's price and their valuation and can buy the second good.

We believe this pattern not to be restrictive to this context, but to have a high degree of generalization. In many markets, consumers are looking for only one quantity of a given good type, offers are of limited duration, the sellers know in advance the future goods he will sell but can choose not to reveal it, and there are search-costs for the consumers than can be influenced by the sellers. One could think of any market where firms are trying to generate impulsive purchases as another application of our theoretical model.

This idea that obfuscation helps generate higher profits has been developed for a long time now. The increasing use of Internet datasets allowed some empirical evaluations of obfuscation while many theoretical papers provided mechanisms through which obfuscation could be profitable. Ellison und Ellison (2009) evaluated empirically the influence of obfuscation on a given price search engine. They show that obfuscation helps the retailers to maintain their margin, even though the price elasticity is very high for the lowest quality goods.

Most of the theoretical literature focused on obfuscation in presence of competitors and showed that obfuscation helped the firms to lower the intensity of competition. Some models, *à la* Stahl (1989) are models of incomplete search. A proportion of consumers are without search costs which generates some randomization within the pricing strategy of the firms. In this context, obfuscation should be interpreted as the proportion of consumers with search costs. Thus, increasing obfuscation is directly related to diminishing the competition, as firms compete only on the consumers without search costs. Wilson (2010) modifies Stahl (1989) as obfuscation is, in his setup, a modification of the search cost, not the proportion of costless shoppers. Another way of softening competition is Ellison und Wolitzky (2012). With the crucial assumption of consumers convex search costs, the introduction of search costs by a firm increases the marginal cost of future searches and therefore reduces the competition. Other models are assuming there exists a differentiated sophistication between the consumers as in Ellison (2003). This heterogeneity in the consumers rationality can be exploited with, for example, *add-on*. In all this literature, obfuscation is related to a reduction of competition.

Some papers have tried to rationalize the use of obfuscation even for a monopoly. One way to do this is to use obfuscation as a reduction cost device. Shin (2005) or Taylor (2014) have exploited this idea that obfuscation can help a monopoly differentiate between high and low valuated consumers and therefore better allocate selling efforts. To our knowledge, only Petrikaite (2013) worked with a multiproduct firm selling multiple substitutable goods. In this context, obfuscation is used as a way to minimize the negative externalities between goods. A parallel approach of our modelisation can be found in the sequential search models. Very recently, Armstrong (2016) provided an extensive analysis of this literature, in a general setting where the sequence of inspections is endogenous.

We would like in our paper to extend these works in different ways. First, we develop a dynamic framework, while the previous models were fundamentally static. Instead of having multiple products at the same time, we only sell one at each period. The introduction of dynamics has many consequences, in terms of search behavior — at each period, the only available good is the one sold. There is no longer any possibility of coming back in the past to purchase the previous good, and thus, no possible recall — and price commitment — which was impossible by essence in a static framework. Second, we break the symmetry in many different ways, allowing to have two different price distributions, impatient consumers or introducing some correlation between the two goods. Without correlation, our results are more related to the standard search literature where draws are very often independent. This correlation is a departure from most of the search literature, but we think it is one in the good direction. Lastly, we provide another interpretation of the mechanism through which obfuscation is profitable.

Our work is also related to the monopoly intertemporal price discrimination's literature and especially Coase (1972)'s seminal paper. Many authors have explained how his conjecture applied and under which circumstances it could fail. Recently, Nava und Schiraldi (2016) provided another interpretation of Coase's conjecture and rephrased it as a market clearing condition. Using this new interpretation, the model we develop with revelation is Coasian, as the market is fully covered when time goes to infinity, but doesn't exhibit necessarily any zero-profit pattern.

When there is no correlation between the valuations, we find that obfuscation is always profitable. Economically speaking, the introduction of optimal search costs, joined with obfuscation enables the monopoly to turn rational and patient consumers into myopic ones. Indeed, all consumers have the same expectations of the future and therefore have the same option value to wait next period to buy. Controlling the search costs, the monopoly can reduce this option value to wait to zero, convincing the potential consumers to buy now if they like today's product. Because of this myopia, the monopoly can easily price discriminate as would any intertemporal monopoly do. The introduction of correlation between the two valuations softens this result as today's valuation is a signal on tomorrow's valuation. Because of this correlation, obfuscation cannot reduce totally the heterogeneity and the search costs don't reduce totally the option to wait. Correlation reduces the ability of the monopoly to introduce efficient search costs, and therefore its ability to turn consumers into myopic agents. Whenever correlation is sufficiently important, obfuscation is counterproductive and it would be better to reveal.

The rest of this paper is structured in the following way. In Section 1, we describe the basic assumptions and exhibit a toy model. We solve the model without correlation in Section 2. Then we allow for some extensions, including multi-periods and the introduction of correlation in Section 3. Our concluding remarks are in Section 4.

1 Model description

1.1 Description and timing

A mass 1 of consumers is willing to buy one and only one of two goods sold by a monopoly. They have a personal distinct valuation $v_i \in [0, 1]$ for each good, drawn from a distribution with cdf $F_i(v)$ and pdf $f_i(v)$. These valuations are private information, but the distributions are common knowledge. They have a time discount factor δ and there exists search costs s of coming back in the future to the monopoly.

The monopoly sells good i at period i and at price p_i. Therefore, it is impossible for the consumers to buy good 2 at period 1 or vice-versa. We also assume that the monopoly can control the search cost s paid by the consumers. This arguable assumption of possible perfect manipulation of search costs by the monopoly has been extensively used by previous works as Ellison und Wolitzky (2012) or Petrikaite (2013). We could also provide another interpretation of these search costs in our setting, if we assume that we have risk-adverse consumers and possible manipulation of expectations by the monopoly. Indeed, rewriting consumer's concave utility function as a mean-variance problem, as Levy und Markowitz (1979), we could reinterpret s as the risk-premium consumers have to pay whenever they are uncertain about their future valuation. If the monopoly could manipulate the variance of consumers belief without affecting other parameters of interest, our model would still applies. With revelation, there would be no risk premium, which is consistent with Lemma 1. Lastly, we extended our work to authorize exogenous search costs in Section 3.3 and find similar results. Therefore, we are going to assume in the rest of this paper that consumers are risk neutral and that the monopoly can manipulate s as it wishes.

The choice of $s \in [0, \infty)$ is costless. Finally, the monopoly can obfuscate or reveal to the consumers their own valuation v_2.

The timing of the games with obfuscation or revelation are the following:

	Timing with Obfuscation	Timing with Revelation
1	• Monopoly chooses p_1 and s • Consumers find v_1, p_1 and s. • They form expectations v_2^e and p_2^e. • They choose whether they buy 1 and whether they come back.	• Monopoly chooses p_1 and s • Consumers find v_1, v_2, p_1 and s. • They form expectations p_2^e • They choose whether they buy 1 and whether they come back.
2	• Monopoly chooses a price p_2 • Consumers find v_2 and p_2 • They choose whether they buy 2	• Monopoly chooses a price p_2 • Consumers find p_2. • They choose whether they buy 2

We note p_2^e the *expected* price of the second good at the first period and v_2^e the expected valuation of the second good for each consumer. We call p_i^m the i-period monopoly price, i.e. $p_i^m \equiv \arg\max_x x(1 - F_i(x))$. Unless otherwise specified, as in Sections 3.4 and 3.5, the two valuations are drawn independently and there is no correlation between them.

In order to have existence and unicity of equilibrium prices, we make the following assumption:

Assumption 1 (Technical requirements)**.**

1-1 $x \mapsto \frac{1-F_i(x)}{f_i(x)}$ *is a decreasing function.*

1-2 $\forall p_1 \in [0,1], x \mapsto \int_x^1 \frac{F_1(p_1+\delta(v-x))-F_1(p_1)}{F_1(p_1)f_2(x)} f_2(v)\,\mathrm{d}v$ *is a decreasing function.*

The first part of this Assumption is the one usually done in the static literature, defining in an unambiguous way p_i^m. Nevertheless, the second part is necessary to ensure the concavity of the profits in our dynamic game. These technical constraints are satisfied for many distributions functions, including some Normal distributions when $F_1 = F_2$. Very often in the literature, very restrictive additional assumptions were made to ensure the concavity of the profit functions.

Before writing and solving the model, one could already notice that the choice of s is straightforward if the monopoly reveals, according to Lemma 1.

Lemma 1. *When v_2 is revealed, the optimal search cost is* 0*.*

Proof. When v_2 is revealed at the first period, all clients can compare $v_1 - p_1$ with $\delta(v_2 - p_2^e - s)$. Thus, for the consumer, the only thing that matters at the second period is the total price paid $p_2^e + s$. Therefore, the demands at the first and second period are function of p_1 and of $p_2^e + s$. Keeping this latter sum constant, it would always be profitable for the monopoly to lower s and to increase p_2^e. □

1.2 Toy model

We would like in this paragraph to provide a very intuitive and simple illustration of the economic forces in action in the comparaison of revelation with obfuscation.

Let's consider a mass 1 of consumers, willing to buy *one and one only* product. They have to choose between the good 1, sold at the first period, and the good 2, sold at the second period. Their valuations for each good can be $\frac{1}{3}$, $\frac{2}{3}$ or $\frac{3}{3}$. The probability for having any valuation is $\frac{1}{3}$. These valuations are independent and identically distributed. Figure 1 represents the demands of consumers under obfuscation and revelation. We also assume that there is no time discount factor: $\delta = 1$. We note v_1 and v_2, the valuations of a given client, p_2^e the expected price at the second period, p_1 the price of the first good and s the search cost of coming back.

1.2.1 Price Equilibrium Reveal

The monopoly could first reveal the information v_2 to its consumers. They know v_1 and v_2 at the beginning of the first period. Demands at the first and second periods are functions of p_1, p_2^e and s.

The full resolution of optimal prices is not straightforward. One should first notice that the prices at each period should always be $\frac{1}{3}$, $\frac{2}{3}$ or $\frac{3}{3}$to maximize profit. Then, there exists only $3^2 = 9$ possible price combinaisons. For each combinaison, we can compute the first and second period demands, and then check whether expectations of the second period prices are consistent. Finally for all consistent combinaisons, we can compare the total profit of the monopoly.

We find that two combinations of prices can yield the maximum profit: $(\frac{2}{3}, \frac{2}{3})$ or $(\frac{3}{3}, \frac{2}{3})$. In both cases, the profit of the monopoly is $\frac{16}{27}$.

1.2.2 Price Equilibrium Obfuscate

The second possible strategy for the monopoly is to obfuscate. Clients have now expectations about the utility they could get at the second period, and we need to determine the mass of consumers coming back. Then, we could compute the optimal second period price, and finally the first period price. Our resolution is by backward induction.

Search behavior The clients coming back at the second period are the ones such that the expected surplus at the second period is bigger than the surplus at the first period:

$$\begin{aligned}
&v_1 - p_1 < \mathbb{E}[v_2 - p_2^e | v_2 \geq p_2^e] - s \\
&\quad \Leftrightarrow v_1 < \tilde{v}_1 \\
&\text{with } \tilde{v}_1 = p_1 + \mathbb{E}[v_2 - p_2^e | v_2 \geq p_2^e] - s
\end{aligned}$$

There is a threshold, called $\tilde{v}_1$ below which clients consumers come back at the second period. If $v_1 \geq \tilde{v}_1$, clients buy the first period good. We can see on this last equation that $\tilde{v}_1$ does not depend of the price of the second period, but only on the *expected* price of the second period.

Optimal Second Period Price Clients coming back at the second period are the ones with $v_1 < \tilde{v}_1$. As v_1 and v_2 are independent, the distribution of v_2 conditionally on coming back at the second period is exactly the same as ex-ante. As the distribution of valuations is untouched, the monopoly should price at monopoly static price $p_2^m = \frac{2}{3}$ in the toy model.
Anticipating p_2, consumers expect to have $p_2^e = \frac{2}{3}$.

Mass of consumer searching We infer from this second period price the mass of consumer searching. Indeed, $\tilde{v}_1$ can be rewritten:

$$\begin{aligned}\tilde{v}_1 &= p_1 + \mathbb{E}[v_2 - 2/3 | v_2 > 2/3] - s \\ &= p_1 + \underbrace{\frac{1}{9} - s}_{\text{Option Value}}\end{aligned}$$

We can very clearly see the influence of search costs on the second period demand. Here, playing with s, the monopoly can change the *option value* to wait. This option value was an incentive to wait for the consumers, because they could always come back next period. We see that, if $s < \frac{1}{9}$, then consumers with $v_1 = p_1$ will *not* buy but rather wait. On the other hand, if $s = \frac{1}{9}$, consumers with $v_1 = p_1$ will buy now. Lastly, if $s > \frac{1}{9}$, the expected surplus of coming back at the second period is negative, and there is no second period demand.

Profits We can now compute the profits of the monopoly when it obfuscates, for each possible p_1 and for s. We restrict our search of the optimal search costs to $\{0, \frac{1}{9}\}$. Indeed, we don't want $s > \frac{1}{9}$ and with these two values, we generates all possible profits. With some simple algebra, we found that the optimal profit is reached for $p_1 = 3/3$ and $s = \frac{1}{9}$ with a profit equals to $\frac{17}{27}$.

1.2.3 Conclusions and generalization

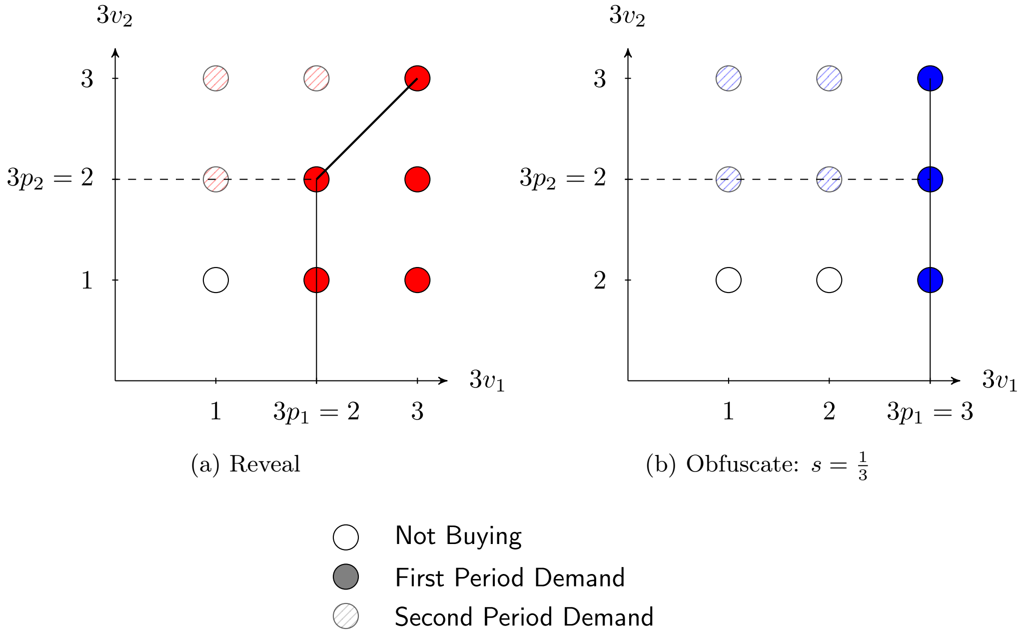
We notice that the profit is bigger in the obfuscation case than in the revelation case. To explain this result, we plotted in Figure 1 the first and second period demand associated with the equilibrium price couple (p_1, p_2). The filled points are the first period demand and the hatched ones, the second period demand.

With revelation, all consumers can choose which product to buy, given the first period price and the correct expectation they form about the second period price. Thus, when the monopoly reveals v_2, some clients with $v_1 \geq p_1$ choose not to buy because buying the second period good is more profitable. In our simple model, it corresponds to the clients with $(v_1 = 2/3, v_2 = 3/3)$. Revelation creates arbitrage possibilities.

With obfuscation and optimal search costs, there is no longer any option value. This absence of option value generates myopic consumers. Clients act today as if they were no futur and buy as long as $v_1 \geq p_1$. The monopoly prefers to have clients at the first period than to make them wait. Indeed, some of the waiting clients will not purchase at the second period.

Comparing these two modes, we understand how obfuscation and search costs interact. They transform patient consumers into myopic ones, which is profitable for the monopoly.

Figure 1: Demands of the Obfuscate and Reveal Cases



It can now choose two different prices at the first and the second period, as would a durable good monopoly do with myopic consumers. When the monopoly reveals, it cannot price discriminate in this way as there would be a mass of consumers willing to arbitrage between the two goods. One could finally note that without search costs, we no longer have the profitability of obfuscation alone.

We understand with this very simple example a set of general intuitions that we would like to extend. From a technical point of view, the resolution of the model with obfuscation will be done by backward induction, while the resolution of the model without obfuscation is more complex. Economically speaking, it is profitable for the monopoly to reduce the option value to 0 and therefore to have high s. The increase of profit is done through the transformation of the patient consumers into myopic ones and is captured through a price differentiation between the first and the second period good.

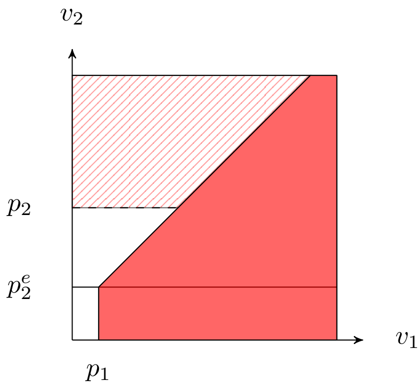
2 Resolution without correlation

In this section, we would like to extend our previous intuition into a more general framework. We explicit the driving forces of our model and establish a set of normative results between obfuscation and revelation.

2.1 Resolution with Revelation

To support our reasoning, we plotted on Figure 2 all the possible pair of valuations. Here, a point (x, y) represents a consumer whose valuations are: $v_1 = x$ and $v_2 = y$. As we revealed v_2, each consumer knows its location on the square $[0,1]^2$. He also forms some expectations about the second period price, p_2^e. Thus he can choose whether to wait or to buy at the first period.

Figure 2: Buying or waiting with Revelation



The diagonal line represents the indifference curve between buying now or waiting: $v_1 - p_1 = \delta(v_2 - p_2^e)$. All the clients above the diagonal prefer to wait and the clients below, in the colored area, prefer to buy now, if $v_1 \geq p_1$. We can therefore write the second period demand, as a function of p_2, p_2^e and p_1.

$$D_2^{\mathcal{R}}(p_1, p_2, p_2^e) = \int_{p_2}^{1} F_1(p_1 + \delta(v - p_2^e)) f_2(v)\, \mathrm{d}v$$

The FOC of the profit, joined with the consistency of the expectations yields the following equation

$$p_2 = \frac{1 - F_2(p_2)}{f(p_2)} + \underbrace{\int_{p_2}^{1} \frac{F_1(p_1 + \delta(v - p_2)) - F_1(p_1)}{F_1(p_1) f_2(p_2)} f_2(v)\, \mathrm{d}v}_{\geq 0}$$

There exists a unique price equilibrium satisfying this equation: the LHS is an increasing function of p_2, while the RHS is a decreasing function of p_2, because of Assumption 1.

Result 1. *At equilibrium, in the revelation model without correlation, p_2 is greater or equal to p_2^m.*

Proof. The last term of the RHS being positive, we conclude immediately that the second period price is greater or equal to p_2^m. □

We cannot provide any closed form solution for the second period price $p_2^{\mathcal{R}}$, but we can rewrite the monopoly total profit:

$$\begin{aligned}
\Pi^{\mathcal{R}}(p_1) =& p_1 \int_{p_1}^{1} F_2(p_2^{\mathcal{R}} + \frac{v-p_1}{\delta}) f_1(v)\,\mathrm{d}v + p_2^{\mathcal{R}} \int_{p_2^{\mathcal{R}}}^{1} F_1(p_1 + \delta(v - p_2^{\mathcal{R}})) f_2(v)\,\mathrm{d}v \\
\Pi^{\mathcal{R}}(p_1) =& (p_1 - c_1(p_1, p_2^{\mathcal{R}}))(1 - F_1(p_1)) + (p_2^{\mathcal{R}} - c_2(p_1, p_2^{\mathcal{R}}))(1 - F_2(p_2^{\mathcal{R}})) \\
& \text{with } c_1(p_1, p_2) = p_2 \int_{p_2}^{1} \frac{1 - F_1(p_1 + \delta(v - p_2))}{1 - F_1(p_1)} f_2(v)\,\mathrm{d}v \\
& \text{with } c_2(p_1, p_2) = p_1 \int_{p_1}^{1} \frac{1 - F_2(p_2 + \frac{v-p_1}{\delta})}{1 - F_2(p_2)} f_1(v)\,\mathrm{d}v
\end{aligned}$$

Where c_1 and c_2 can be seen as "costs". When one decreases the first period price, there is some substitution between, leading to a loss for the monopoly, with respect with two independent goods. The fact that the consumers perfectly anticipates the second period price allow them to arbitrage between the two goods. As $c_1(p_1, p_2)$ and $c_2(p_1, p_2)$ are positive functions, the two periods prices are expected to be strictly above their monopoly prices.

2.2 Resolution with Obfuscation

The monopoly's other strategy is to obfuscate v_2 and to introduce some search costs s to come back at the second period. Clients now longer know v_2. They must take expectations on the possible values to choose whether to buy now or to wait. Consumers will come back at the second period if the expected gains to search are bigger than the surplus of the first period. Mathematically, we can express it as:

$$\max(v_1 - p_1, 0) \leq \delta \int_{p_2^e}^{1} (v_2 - p_2^e) f_2(v_2)\,\mathrm{d}v_2 - \delta s$$

$$\Leftrightarrow \begin{cases} s \leq \int_{p_2^e}^{1} (v_2 - p_2^e) f_2(v_2)\,\mathrm{d}v_2 \\ v_1 < p_1 + \delta \left(\int_{p_2^e}^{1} (v_2 - p_2^e) f_2(v_2)\,\mathrm{d}v_2 - s \right) \end{cases}$$

We infer from this last equation the second period participation constraint:

$$\int_{p_2^e}^{1} (v_2 - p_2^e) f_2(v_2)\,\mathrm{d}v_2 \geq s \tag{PC}$$

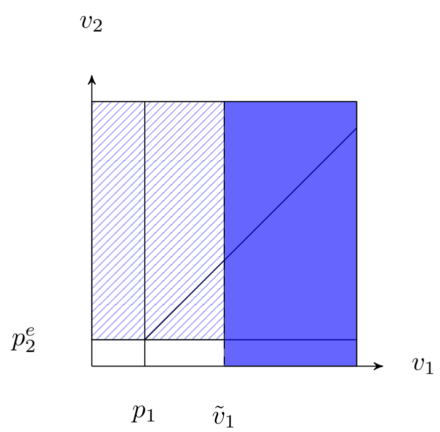
Indeed, the LHS corresponds to the maximum value of search costs compatible with the existence of a second period demand. If s is too high, there is no demand at the second period because consumers expect on average to have a negative surplus.

Clients coming back at the second period are the ones with a relatively low first period valuation : v_1 must be lower than a given threshold, $\tilde{v}_1$, given by the RHS of the following inequation.

$$v_1 < p_1 + \delta \left(\int_{p_2^e}^{1} (v_2 - p_2^e) f_2(v_2) \, \mathrm{d}v_2 - s \right) = \tilde{v}_1$$

Using PC, we immediately see that $\tilde{v}_1 > p_1$. Based on these equations, we can represent in Figure 3 the different demands. The filled grey area corresponds to the clients not coming back as $v_1 \geq \tilde{v}_1$ and buying at the first period. The hatched area corresponds to the second period demand.

Figure 3: Demands in the Obfuscation case



We can therefore write the second period profit of the monopoly in the obfuscation case as:

$$\Pi_2 = F_1(\tilde{v}_1) p_2 (1 - F_2(p_2))$$

We immediately conclude that the second period optimal price is p_2^m, as $\tilde{v}_1$ does not depends on p_2 but only p_2^e. At the second period, it is optimal for the monoply to price as a monopolist. Finally, the total profit without correlation can be rewritten:

$$\Pi^{\mathcal{O}}(p_1, s) = p_1(1 - F_1(\tilde{v}_1)) + p_2^m F_1(\tilde{v}_1)(1 - F_2(p_2^m))$$

And the monopoly has to choose the optimal values of s and p_1, with $p_2 = p_2^m$.

Result 2. *In the obfuscation case without correlation, it is optimal to bind the Participation Constraint (PC):*

$$s = \int_{p_2^m}^{1} (v_2 - p_2^m) f_2(v_2)\,\mathrm{d}v_2$$

Proof. A double deviation argument can prove this result. Suppose the optimal s to be strictly lower than the upper bound.
Then, it would be possible to increase p_1 and s such that $\tilde{v}_1$ remains constant. Keeping $p_2 = p_2^m$ and $\tilde{v}_1$ constant, the profit is increasing in p_1 and thus, our deviation would yields a strictly higher profit. □

We deduce from the Result 2 that: $\tilde{v}_1 = p_1$. The profit can be rewritten in a simpler way:

$$\begin{aligned}\Pi^{\mathcal{O}}(p_1, p_2) =& p_1(1 - F_1(p_1)) + p_2^m F_1(p_1)(1 - F_2(p_2^m)) \\ =& \left(p_1 - p_2^m(1 - F_2(p_2^m))\right)(1 - F_1(p_1)) + p_2^m\,(1 - F_2(p_2^m))\end{aligned}$$

Here, on average, each consumer coming back at the second period will yield on average $p_2^m(1 - F_2(p_2^m))$. Therefore, there is an incentive for the monopoly to increase its first period price above the monopoly price. The maximization with respect to p_1 gives the following equation, ensuring existence and uniqueness of the equilibrium:

$$p_1 = \frac{1 - F_1(p_1)}{f_1(p_1)} + p_2^m(1 - F_2(p_2^m))$$

Result 3. *The equilibrium prices in the obfuscation case are independent of δ.*

Proof. At equilibrium, $p_2 = p_2^m$, which is independent of δ. Also, the first order condition defining p_1 is independent of δ □

When Equation PC is binding, consumers are myopic. They do not take into account their future anymore as, on expectation, this future will yield a zero surplus.

2.3 Comparisons of the cases

Due to the lack of closed form solutions in the general case, we impose here some restrictions allowing us to make additional analytical comparisons. We believe it is fair to have the following assumptions:

Assumption 2 (Equality of Distributions)**.** $F_1 = F_2 = F$

Assumption 3 (Infinite Patience)**.** $\delta = 1$

Result 4. *With Assumptions 2 and 3, without correlation, there exists a symmetric equilibrium in the revelation case such that:*

$$\begin{aligned} p_1 =& p_2 = p \\ 2pF(p)f(p) =& 1 - F^2(p) \end{aligned}$$

Yielding the following c_i functions and total profit :

$$\begin{aligned} c_1(p) &= \frac{p(1-F(p))}{2} \\ \Pi^{\mathcal{R}}(p) &= p(1-F^2(p) \end{aligned}$$

Proof. The full proof is in appendix, but the idea is to notice that, with these assumptions, the profit is a symmetric function. □

We can finally compare the profits with or without obfuscation:

$$\begin{aligned} \Pi^{\mathcal{R}}(p) &= p(1-F(p)) + pF(p)(1-F(p)) \\ \Pi^{\mathcal{O}}(p_1) &= p_1(1-F(p_1)) + F(p_1)\Pi^m \ , \forall p_1 \end{aligned}$$

Result 5. *With Assumption 2 and 3, without correlation, it is always profitable to obfuscate.*

Proof. The choice of the first period price p_1 is such that:

$$\Pi^{\mathcal{O}}(p_1) \geq \Pi^{\mathcal{O}}(p)$$

Thus, we have

$$\Pi^{\mathcal{O}}(p) - \Pi^{\mathcal{R}}(p) = F(p_1)(\Pi^m - p(1-F(p)) \geq 0$$

□

We have proven, that, whatever the distribution function, and with Assumptions 2 and 3, it was always profitable to obfuscate when there is no correlation between the valuations of the two goods.

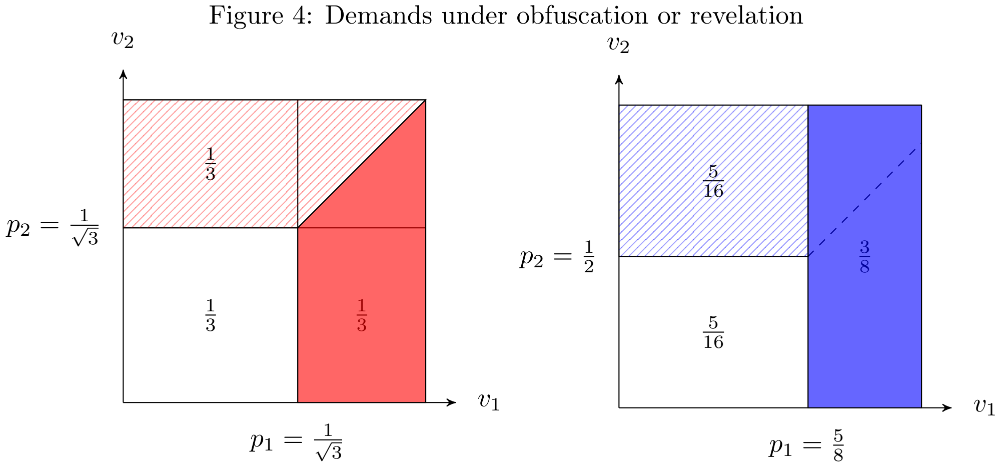
2.4 Application in the uniform case

Assumption 4 (Uniformity)**.** *We assume that $f_1 = \mathcal{U}_{[0,1]}$.*

We would like in this subsection to focus on the illustrative case of the uniform distribution, with Assumptions 2, 3 and 4.

In this context, demand functions are confounded with areas on Figures 3 and 2. Furthermore, we can compute the equilibria.
The first period demand is represented in the filled area, and the second period demand in the dashed area.

Figure 4: Demands under obfuscation or revelation



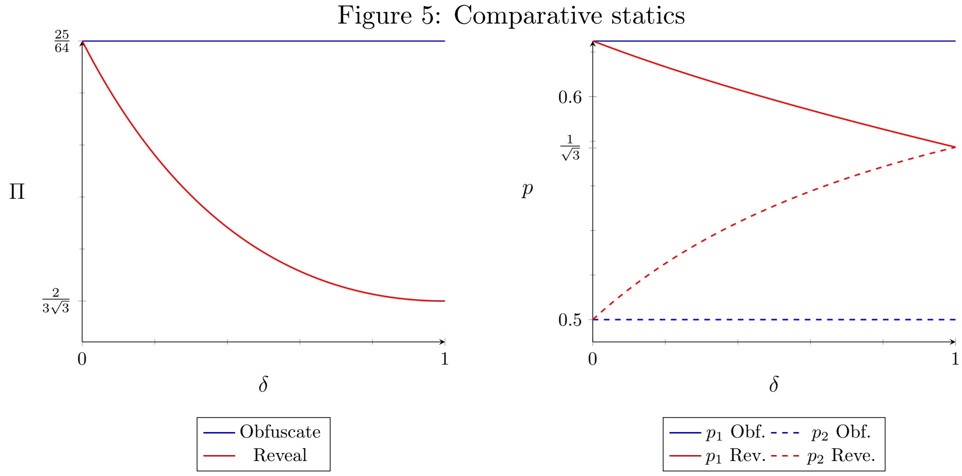
(a) Reveal (b) Obfuscate

When the clients are informed, it is optimal to choose the same price at the two periods : $p_1 = p_2 = \frac{1}{\sqrt{3}}$. This choice split in three equal parts the clients purchasing the first good, the second or none. It is not possible for the monopoly to price discriminate, because consumers would then have the possibility to arbitrage. Keeping total demand constant, if the two prices were not equal, more than half of the clients would buy the low price good.

With obfuscation, it is possible to price-discriminate. As the clients have no surplus, on average, of coming back at the second period, they act today as if there were no future. The ones with $v_1 \geq p_1$ have to buy now, while the ones with low valuations come back at the second period and are priced at the monopoly price $p_2 = \frac{1}{2}$. If the monopoly kept the same prices and revealed v_2, then many clients — all the clients in the filled blue area and above the dashed line — would substitute the first (and expensive) good with the second (and cheap) good.

The Figure 5 compares the profits and the prices of Obfuscation and of Revealing under Assumptions 2 and 4, allowing δ to vary from 0 to 1. As we already pointed out, there is no influence of δ on the equilibrium for the obfuscation. Indeed, s is such that there is no option value to wait, and thus, δ plays no role at all in this setup.
We see that the profit of the monopoly is always greater or equal when it obfuscate, compared to revealing. Only for $\delta = 0$, both profits are equal. We can see where this comes from, with the Figure 4. When δ converges to 0, the indifference curve between buying today and tomorrow goes steeper and steeper, and the two demands are very similar. From the point of view of the consumer, when the futur doesn't matter anymore ($\delta \to 0$), it is useless to know or not the next valuation.

Figure 5: Comparative statics



(a) Comparaison of Profits

(b) Comparaison of Prices

Regarding the prices, we find that the the prices are always more dispersed with obfuscation than without (the red lines are between the blue ones). Only for $\delta = 1$, are both prices equal in the Reveal cases. Otherwise, there is some price differentiation. Intuitively, we understand that a decrease in δ leads to a higher demands in the first good (the diagonal line in Figure 4b goes steeper), which is compensated by an increase in p_1 and a decrease in p_2. In the extreme case where $\delta = 0$, there is no arbitrage done by the clients anymore, which allows the monopoly to have the same prices as in the obfuscation case.

A natural interpretation of obfuscation is to turn clients into myopic consumers as they act *as if* $\delta = 0$, even if it is not the case. This result may have very large consequences in term of estimation of δ in environments where there is some search costs, as these search costs could lower the estimations of δ.

3 Extensions

We provided in the previous section a set of results in a simple model. We would like with this section to cover some natural extensions to prove the robustness of our results in more complex settings.

3.1 Multiple periods in the uniform case

The first extension of our model than comes in mind would be to allow for multiples periods. To ensure tractability, we rely in this extension on Assumptions 2, 3 and 4.

The monopoly is still facing a mass 1 of consumers willing to buy one in N goods. The monopoly sells good i at period i and has the choice between revealing the whole vector $(v_2, ...v_N)$ at the beginning of the first period or not. It can also choose a search costs s_i, paid by the consumers at the beginning of period i.

3.1.1 Revelation

Focusing once again on the symmetric equilibrium : $p_t = p$, the total profit of the monopoly can be written as: $\Pi^{\mathcal{R}}(p) = p(1 - F(p)^N)$. Some simple algebra yields : $p = (\frac{1}{N+1})^{\frac{1}{N}}$ and $\Pi^{\mathcal{R}} = \frac{N}{(N+1)^{1+\frac{1}{N}}}$

Under revelation, all periods are exactly the sames because of the arbitrage possibilities, and thus the price doesn't depend on t. The total number of periods tends to increase the equilibrium price and the profit of the monopoly converges to one. This result could first look like a counter-example to Coase's conjecture, but let's not forget that the monopoly is selling a *new* product at each period, as opposed to the usual definition of the durable good monopoly. Nevertheless, we rely on Nava und Schiraldi (2016), which rephrased Coase's conjecture in terms of market clearance. From this point of view, when time goes to infinity, the monopoly fulfills all the demand and clears the market.

3.1.2 Obfuscation

We would like to know whether we could achieve higher profits with obfuscation. Let's first notice that we can always find some value for the search cost at period t such that consumers have no expected surplus of coming back. Indeed, the expected surplus of coming back at time $t+1$ can be written as:

$$ES_{t+1} = \int_{p_{t+1}}^{1} (v - p_{t+1})f(v)dv - s_{t+1} + \delta ES_{t+2}$$

With the final condition $ES_{N+1} = 0$, we can define a sequence $(s_t)_{t \in \{1,..,N\}}$ such that :

$$s_t = \int_{p_{t+1}}^{1} (v - p_{t+1})f_{t+1}(v)dv$$

This sequence of search costs ensures that, at any period t, consumers are willing to come back, but expect no surplus of the future. Thus, consumers are perfectly myopic and we can find the optimal prices by backward induction. At period t, the profit earned by the monopoly from period t to N can be written as :

$$\Pi^{\mathcal{O}}_{t \to N} = p_t(1 - F(p_t)) + F(p_t)\Pi^{\mathcal{O}}_{t+1 \to N}$$

Thus, we can define a recursive sequence of prices and Profit such that:

$$p_N = \tfrac{1}{2} \qquad p_t = \tfrac{1+\Pi^{\mathcal{O}}_{t+1 \to N}}{2}$$

And :

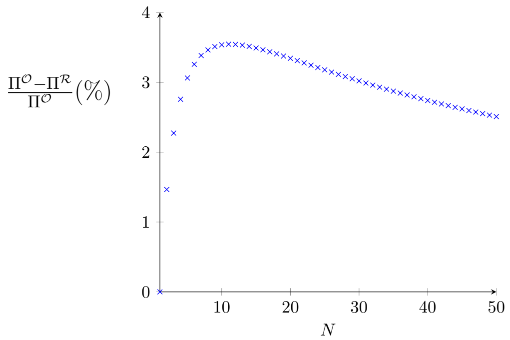
$$\Pi^{\mathcal{O}}_{N \to N} = \frac{1}{4} \qquad \Pi^{\mathcal{O}}_{t \to N} = (p_t)^2$$

For a given value of $t \in \{1, ..N\}$, the sequence of prices is decreasing. More interestingly, the total profit earned by the monopoly when there is N period is increasing in N and converges to 1.

3.1.3 Comparison of the cases

We would like now to compare the efficiency of these two strategies with the number of periods. We plot in Figure 6 the relative gain of Obfuscation with respect to Revelation as a function of the number of periods. At the end, when the number of periods is infinite, both modes are able to perfectly price discriminate and extract all the surplus of the consumer, so the difference should converge to zero. Obfuscation allows the monopoly to

Figure 6: Relative gain of Obfuscation with the number of periods



treat the consumer as if they were myopic leading to a more efficient price discrimination and a higher profit. This result remains true, even with multiple periods. Interestingly, there exists an optimal number of periods leading to a maximal gain with obfuscation compared to revelation.

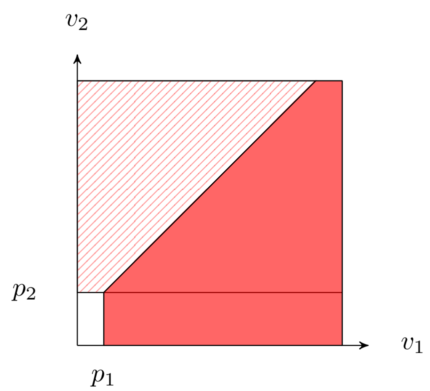
3.2 Commitment power

To better understand the role played by obfuscation itself, we would like to rule out any *commitment effect*. We introduce in this section a hypothetical scenario, where the monopoly could reveal v_2 and perfectly commit on a second period price p_2.

This possibility of price commitment with revelation changes the timing decisions : now the monopoly can simultaneously choose p_1 and p_2.

To support our reasoning, we replicate Figure 2, with some commitment on p_2 and p_1. Clients now longer have expectations, and they can perfectly choose whether to wait or

Figure 7: Revealing with Price Commitment



to buy at the first period.

Thus, we can write the total profit of the monopoly:

$$\Pi^{\mathcal{PCR}}(p_1, p_2) = p_1 \int_{p_1}^{1} F_2(p_2 + \frac{v - p_1}{\delta}) f_1(v)\,\mathrm{d}v + p_2 \int_{p_2}^{1} F_1(p_1 + \delta(v - p_2)) f_2(v)\,\mathrm{d}v$$

This profit is exactly the same as the one without price commitment,excepted that p_2 and p_1 are now longer tied by $p_2^{\mathcal{R}}(p_1)$.We don't have any closed form solutions in the general case, but with Assumptions 3 and 2, we can once again display the same symmetric equilibrium as without price commitment. Indeed the two first order conditions in the symmetric equilibrium are identical, they satisfy the optimality of the second period price without commitment. This allows us to extent Result 5, even with price commitment.

3.3 Exogeneity of search costs

Let's assume in this section that the search costs $s > 0$ are fixed and cannot be modified by the monopoly. Therefore, its only possible strategic variables are the prices p_1, p_2 and the possibility to obfuscate v_2. We also take $\delta = 1$,i.e. Assumption 3.

3.3.1 Revealing

Without commitment The setting with revelation of v_2 becomes slightly more complex. With the same notations as before, clients willing to come back at the second period are the ones such that :

$$v_2 - p_2^e - s \geq v_1 - p_1$$

And the first second period profit is simply :

$$\Pi_2 = p_2 \int_{\max(p_2^e + s, p_2)}^{1} F_1(p_1 + v - p_2^e - s) f_2(v_2)\,\mathrm{d}v_2$$

As, at equilibrium, we must have $p_2 = p_2^e$, we have $p_2^e + s > p_2$. Thus, the demand doesn't depend on the second period price, and there is no equilibrium price ! More simply, consumers expect to pay $s + p_2^e$ at the second period. Thus, they only come at the second period if v_2 is strictly greater than $p_2^e + s$. But once they paid s to come back, the monopoly would have an incentive to increase its price, and there is no price p_2 satisfying simultaneously the consistency of the beliefs and the optimality conditions. To circumvent this issue, we assume here that the monopoly could commit on a second period price.

With commitment The total profit is written as :

$$\Pi^{\mathcal{R}} = p_1 \int_{p_1}^{1} F_2(p_2 + s + v_1 - p_1) f_1(v_1)\,\mathrm{d}v_1 + p_2 \int_{p_2+s}^{1} F_1(p_1 + v_2 - p_2 - s) f_2(v_2)\,\mathrm{d}v_2$$

As previously observed when s was endogeneous, an increase of s is prejudicial to the monopoly. Thus, one can state that $\Pi^{\mathcal{R}}$ is a decreasing function of s.

3.3.2 Obfuscating

The introduction of an exogeneous s doesn't modifiy the second period problem for the monopoly. Clients coming back at the second period should be priced at the monopoly price.

Rewriting the participation constraint PC, we notice that there exists a maximal search cost above which no client would come back :

$$s_{\max} = \int_{p_2^m}^{1} (v_2 - p_2^m) f_2(v_2)\,\mathrm{d}v_2 \tag{1}$$

For $s > s_{\max}$, there is no second period demand, and our model collapses to a single good monopoly. For $s \leq s_{\max}$, the profit of the monopoly can be rewritten as :

$$\begin{aligned}\Pi^{\mathcal{O}}(p_1) =& p_1(1 - F_1(p_1 + \delta(s_{\max} - s)) + p_2^m F_1(p_1 + \delta(s_{\max} - s))(1 - F_2(p_2^m)) \\ =& F_1(p_1 + \delta(s_{\max} - s))(\Pi_2^m - p_1) + p_1\end{aligned}$$

Once again, we can observe that, under Assumption 2, the profit under obfuscation is an increasing function of s, as long as it is below $s_{\max}$.

3.3.3 Comparisons of the cases

Using our previous results, we have the following result.

Result 6. *There exists a range of search costs for which it is profitable to obfuscate.*

Proof. As previously stated, $\Pi^{\mathcal{O}}$ is an increasing function of s, and $\Pi^{\mathcal{R}}$ a decreasing one. For $s < s_{\max}$, both profit functions are continuous, and $\Pi^{\mathcal{R}}(s = 0) < \Pi^{\mathcal{R}}(s = s_{\max})$. According to the intermediate value theorem, there exists a range $(\underline{s}, s_{\max}]$, such that the monopoly is strictly better off with obfuscation. □

3.4 Positive correlation

Let's work now with an arbitrary correlation between the valuations v_1 and v_2. We work with the following modelization:

$$\begin{cases} v_2 = v_1 \text{ with probability } \mu \\ v_2 \sim F_2 \text{ with probability } 1-\mu \end{cases}$$

We note that, in this context, the probability to have a valuation v_2 above a given threshold is given by

$$\mathbb{P}[v_2 \leq y] = \mu \mathbb{1}\{v_1 \leq y\} + (1-\mu)F_2(y)$$

The introduction of correlation modifies the repartition of valuation within the square $[0,1]^2$. We have now a mass μ on the diagonal, while the remaining mass of consumers $(1-\mu)$ is distributed in the valuations (v_1, v_2) with $v_1 \neq v_2$. μ is not the correlation between the two variables v_1 and v_2, but an increasing function of it. For simplicity, we are going to abusively name μ the correlation between v_1 and v_2.
As the resolution of the model with correlation is non trivial, we choose to work with $\delta = 1$, i.e. Assumption 3 and with equality of distributions, i.e. Assumption 2. Nevertheless, for clarity, we keep the notations f_1 and f_2 at least at the beginning of the resolution.
We make the addition assumption that the monopoly is not excluding anyone from the second period with search costs.

3.4.1 Revealing

We don't expect much to change in the revealing case. Indeed, as clients already know their two valuations at the beginning of the first period, this correlation does not modify the expected second period valuation and there is no *informational gain* form the point of view of the consumer.
For the monopoly not much changes too. The distribution of the valuations on (v_1, v_2) is now different, as there is a mass of consumers for which both goods are identical and are therefore perfectly arbitrating between the two goods. Thus, the incentive to have equal prices should be reinforced.

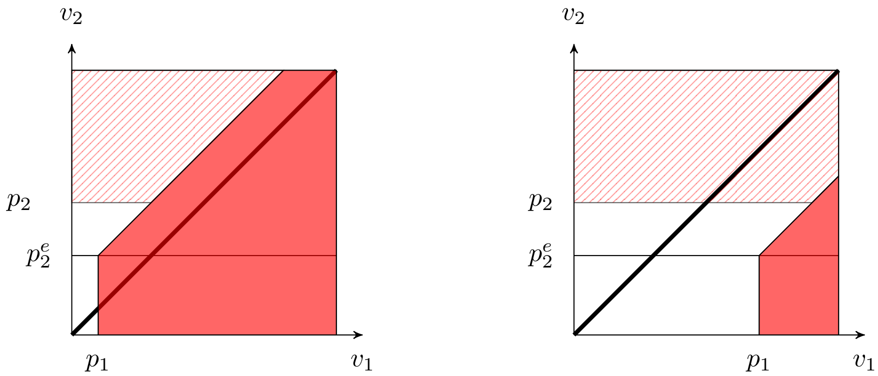
We represented in Figure 8 the different demands, depending on p_1, p_2^e and p_2. We see two different regimes in the demands for the first and the second good, depending on p_1, p_2. The mass of clients on the diagonal can switch from one good to the other.

With Figure 8, we can write the second period demand :

$$D_2^{\mathcal{C}}(p_1, p_2, p_2^e) = (1-\mu) \int_{p_2}^{1} F_1(p_1 + v - p_2^e) f_2(v) \, \mathrm{d}v + \mu \mathbb{1}_{p_1 > p_2^e}(1 - F_1(p_2))$$

The FOC condition of the second period profit, joined with the equality of distribution

Figure 8: Revelation with $\mu \neq 0$



(a) $p_1 \leq p_2^e$

(b) $p_1 \geq p_2^e$

and the consistency of the expectations yields the following equation

$$p_2 f(p_2) = 1 - F(p_2) + \frac{(1-\mu) \int_{p_2}^{1} (F(p_1 + v - p_2) - F(p_1)) f(v) \, \mathrm{d}v}{(1-\mu)F(p_1) + \mu \mathbb{1}_{p_1 > p_2}}$$

Some simple considerations of this last equation show that there is always existence of an equilibrium price. We cannot provide any closed form solution for the second period price p_2^C, but we can rewrite the monopoly total profit:

$$\Pi^{\mathcal{R}}(p_1) = (1-\mu)\Pi^{\mathcal{PER}}_{\mu=0}(p_1, p_2^C) + \mu(1 - F_1(\min(p_1, p_2^C))) \min(p_1, p_2^C)$$

Without any additional assumption, we are unable to provide a closed form solution. If we assume that $p_1 \geq p_2$, as it was the case without correlation, we can rewrite the system of equations as

$$p_2 f(p_2) = 1 - F(p_2) + \frac{1}{1 + \frac{\mu}{(1-\mu)F(p_1)}} \int_{p_2}^{1} \frac{(F(p_1 + v - p_2) - F(p_1))}{F(p_1)} f(v) \, \mathrm{d}v$$
$$\Pi^{\mathcal{PER}}(p_1) = (1-\mu)\Pi^{\mathcal{PER}}_{\mu=0} + \mu(1 - F(p_2^C))p_2^C$$

3.4.2 Obfuscating

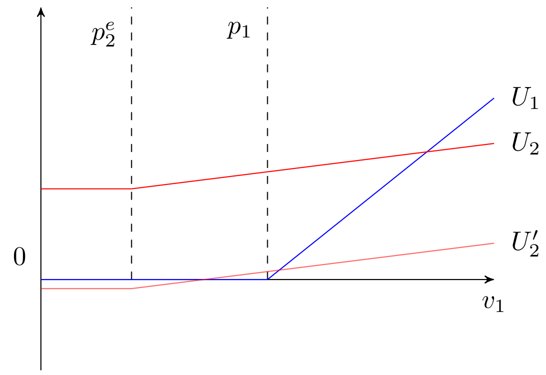
We first need to determine the clients willing to come back at the second period. To do so, we compare the utility at the first and at the second period. These utilities are given by:

$$U_1 = \begin{cases} 0 \text{ if } v_1 \leq p_1 \\ v_1 - p_1 \text{ otherwise} \end{cases}$$

$$U_2 = \begin{cases} (1-\mu) \int_{p_2^e}^{1} (v_2 - p_2^e) f_2(v_2) \, \mathrm{d}v_2 - s \text{ if } v_1 \leq p_2^e \\ (1-\mu) \int_{p_2^e}^{1} (v_2 - p_2^e) f_2(v_2) \, \mathrm{d}v_2 - s + \mu(v_1 - p_2^e) \text{ otherwise} \end{cases}$$

To compare these functions, we plotted them in Figure 9 for different values of s, with $s < s'$.

Figure 9: Utilities for different values of s



From this graph, we can see the first period demand, characterized by $U_1 \geq \max(0, U_2)$, and the mass of consumers coming back at the second period. We notice that there exists a maximum value of s above which we would exclude some consumers from coming back at the second period. For example, for s', all clients with a negative expected utility at the second period don't come back. We can conclude from this limitation the following result.

Result 7. *When there is correlation, the monopoly chooses the maximum possible search cost :*

$$s_{\max} = (1-\mu) \int_{p_2^e}^{1} (v_2 - p_2^e) f_2(v_2) \, \mathrm{d}v_2$$

Proof. Let's assume that $s < s_{\max}$. Then, it is possible to increase p_1 and s, such that the first period demand and the second period demand remains unchanged. This double deviation would yield a strictly higher profit. □

Result 7 is very easy to understand. It simply reflects an extension of our previous results. When there is correlation, we don't want to exclude any second period demand, but we want to have a very low reservation valuation. Thus, it is optimal to have s as big as possible without excluding anyone from the market.

Using Figure 9, we write the demand functions as:

$$\begin{aligned} D_1 =& 1 - F_1\left(\frac{p_1 - \mu p_2^e}{1-\mu}\right) = 1 - F_1(\bar{x}) \\ D_2 =& (1-\mu)(1 - F_2(p_2))F_1(\bar{x}) + \mu\left(F_1(\bar{x}) - F_1(p_2)\right) \end{aligned}$$

We can write the first order condition of the second period price, assuming that $F_1 = F_2$:

$$\begin{aligned} p_2 f(p_2)\left((1-\mu)F(\bar{x}) + \mu\right) =& (1-\mu)(1-F(p_2))F(\bar{x}) + \mu\left(F(\bar{x}) - 1 + 1 - F(p_2)\right) \\ p_2 f(p_2) =& (1 - F(p_2)) - \frac{1}{1 + \frac{1}{\mu(1-F(\bar{x}))}} \end{aligned}$$

This equation allows us to ensure the existence and the uniqueness of p_2 and to provide some information on p_2 at equilibrium. We find that the second period price is below the monopoly price, and that the correlation between the two valuations tends to decrease this equilibrium price. An interesting point is to notice that p_2 is no longer independent of p_1 which prevent us from having a closed form solution for the equilibrium prices.
We can still write the profit of the monopoly:

$$\Pi = p_1(1 - F(\bar{x})) + p_2 F(\bar{x})(1-\mu)(1 - F(p_2)) + \mu p_2(F(\bar{x}) - F(p_2))$$

3.4.3 Comparaisons of the results in the Uniform Case

As it remains difficult to have a closed form solution, and to provide more intuitive results, we make the additional Assumption 4 of uniformity: $F_1 = F_2 = F = \mathcal{U}_{[0,1]}$. We can now provide some numerical comparisons of the obfuscation with the revelation cases.

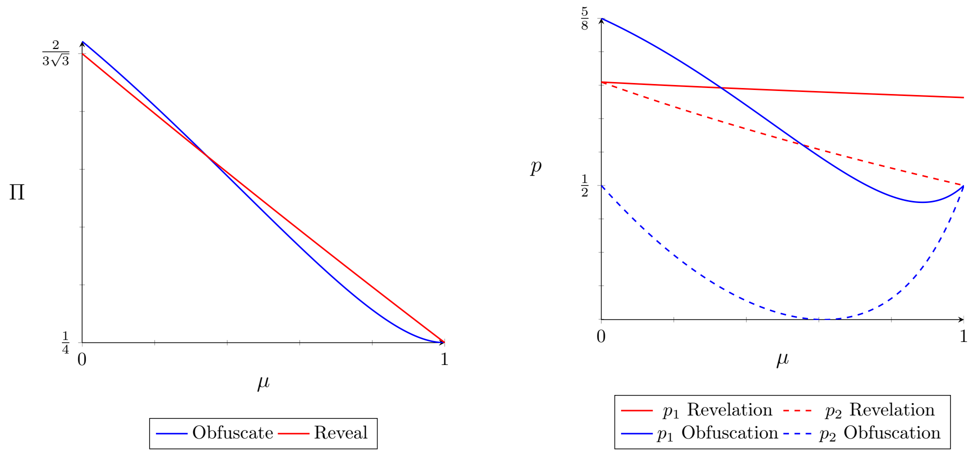
In the Figure 10, we represented the profit and the prices of the monopoly in the revelation and obfuscation cases. The first observation regarding the profit is that the correlation decreases the profit in both cases. Indeed, the correlation between v_1 and v_2 tends to lower $\max(v_1, v_2)$ and thus, clients have a lower ex-ante valuation for this two period game. We also observe Result 8.

Result 8. *With the previous assumptions, there exists a correlation value above which it is not profitable anymore to obfuscate.*

This result could be very likely generalized for any distribution. The economic intuition behind this result is straightforward. Correlation is more harmful to the obfuscation case because it also set a maximal value to the possible search costs. When correlation increases, search costs are less and less efficient to lower the option value to wait. Therefore, one cannot any longer turn consumers into myopic consumers. And we know that, whith to small search costs, it is no longer optimal to obfuscate.

The pattern of the prices in obfuscation and revelation are opposed. Under obfuscation, the prices tends to converge toward the monopoly prices. Under revelation, the second

Figure 10: Comparaison of Obfuscation and Revelation



(a) Profit

(b) Prices

period price decreases to the monopoly price, while the first period price decreases to a strictly higher level. We find than $p_1 \geq p_2$, but the first period demand goes to 0 as μ increases.

3.5 Negative correlation

Let's now complete our modelisation with a negative correlation. We have to slightly change our modelization to have now:

$$\begin{cases} v_2 = 1 - v_1 \text{ with probability } \mu \\ v_2 \sim F_2 \text{ with probability } 1 - \mu \end{cases}$$

To provide an easier economic intuition, we assume that the valuations are uniformly distributed, i.e. Assumption 4.

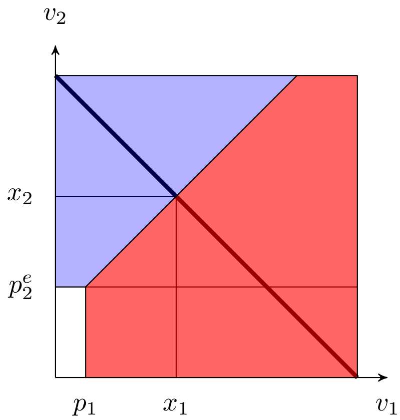
3.5.1 Revealing

Let's analyze what is going on when we have a negative correlation between v_1 and v_2.

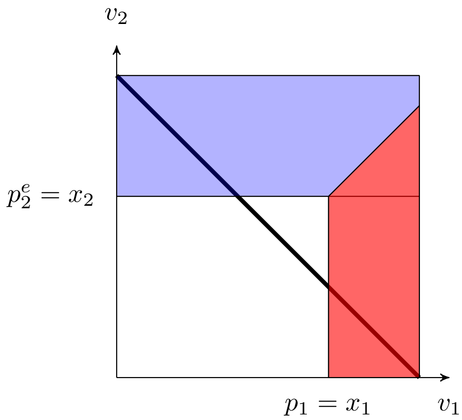
We represented in Figure 11 the different demands, depending on p_1, p_2^e. We see two different regimes in the demands for the first and the second good, depending on p_1, p_2^e. Indeed, the diagonal with a mass μ can now either intersect the demand for no good (in blank) or not. Let's call $x_1 = \max(p_1, \frac{1}{2} + \frac{p_1 - p_2^e}{2})$ and $x_2 = \max(p_2, \frac{1}{2} + \frac{p_1 - p_2^e}{2})$

We can now write the total profit of the monopoly in each of these regimes as the following. In the uniform case, under the assumption that $1 - p_1 \geq p_2$, we can rewrite the

Figure 11: Negative correlation



(a) $x_i > p_i$



(b) $x_i = p_i$

demand at the second period and the optimal second price as:

$$D_2(p_1, p_2, p_2^e) = (1 - p_2) - (1 - \mu)\frac{(1 - p_1 + p_2^e - p_2)^2}{2}$$

$$p_2(p_1) = \frac{1}{2}\frac{1 - \frac{1-\mu}{2}(1 - p_1)^2}{1 - \frac{1-\mu}{2}(1 - p_1)}$$

Reimplementing this functional form in the total profit, we can provide some closed form solutions for the total profit as a function of μ.

3.5.2 Obfuscating

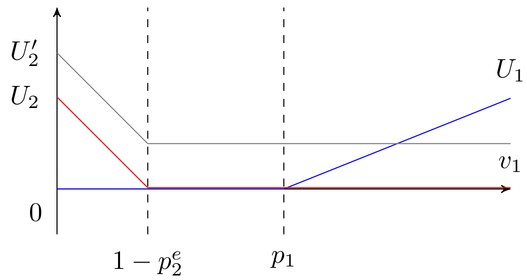
We assume the monopoly chooses not to reveal the second period valuation, and commits on an optimal search cost s consumers will have to pay at the beginning of the second period. We first have to compare the utilities of the consumers at the first and at the second period :

$$U_1 = \begin{cases} 0 \text{ if } v_1 \leq p_1 \\ v_1 - p_1 \text{ otherwise} \end{cases}$$

$$U_2 = \begin{cases} (1 - \mu) \int_{p_2^e}^{1} (v_2 - p_2^e) f_2(v_2)\, \mathrm{d}v_2 - s \text{ if } v_1 \geq 1 - p_2^e \\ (1 - \mu) \int_{p_2^e}^{1} (v_2 - p_2^e) f_2(v_2)\, \mathrm{d}v_2 - s + \mu(1 - v_1 - p_2^e) \text{ otherwise} \end{cases}$$

These utilities are very similar to the ones we previously had with a positive correlation, except that now, U_2 is negatively linked to v_1. We can easily deduce from the Figure 12 the following result.

Figure 12: Utilities with negative correlation for different values of s



Result 9. *In presence of negative correlation, there exists an optimal search costs $s^\star$ such that all clients with $v_1 > p_1$ purchase the first period good, and all clients with $v_1 \leq p_1$ come back at the second period.*

Proof. This result is equivalent to say that, in figure 12, U_2 are the optimal utilities left for the consumers, from the point of view of the firm.
Indeed, if, search costs are too high, some consumers won't purchase or come back at the second period: there are values of v_1 such that U_1 is null and U_2 is strictly negative. In this case, it would be profitable to decrease these search costs to attract more clients at the second period, without any effect on the first period demand.
If search costs are too small, some consumers have a strictly positive utility purchasing any of the goods (U_2'). In this case, there exists a double deviation yielding stricly higher profits: one could simultaneously increase p_1 (if $1-p_2^e \leq p_1$) of p_2^e (if $1-p_2^e \geq p_1$) and s such that the first and second periods demands remains the same. □

We make the computations under uniformity, with either $\min(p_1, 1-p_2) = p_1$ or $\min(p_1, 1-p_2) = 1-p_2$. We can prove that it is impossible to have a solution with $p_1 \leq 1-p_2$. Thus, we can rewrite the demands :

$$D_1(p_1) = 1 - F_1(p_1) = (1-p_1)$$
$$D_2(p_1, p_2) = F_1(p_1)(1-\mu)(1-F_2(p_2)) + \mu F_1(1-p_2) = (1-p_2)\left((1-\mu)p_1 + \mu\right)$$

We deduce from the second period demand, that the second period price remains the monopoly price, whatever the negative correlation. We have the following equilibrium prices and profit in the obfuscation case :

$$p_2 = \frac{1}{2}$$
$$p_1 = \frac{1}{2} + \frac{1-\mu}{8}$$
$$\Pi_{\mathcal{O}} = \frac{\mu}{4} + \left(\frac{1}{2} + \frac{1-\mu}{8}\right)^2$$

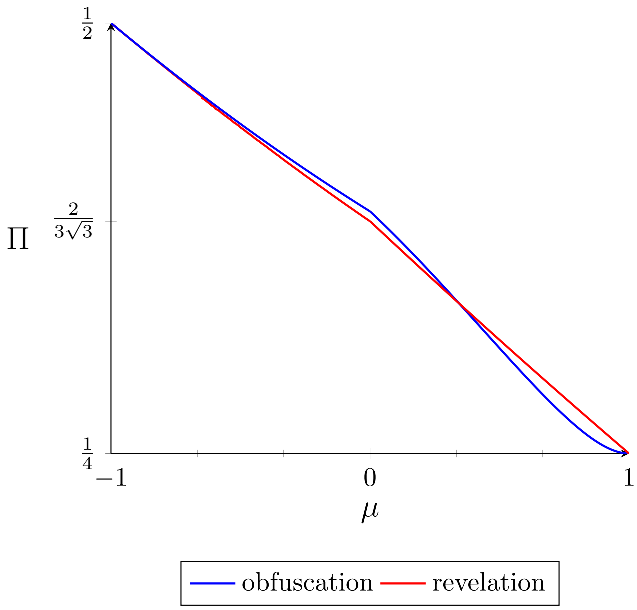
3.5.3 Comparaison of the modes under Uniformity

Using our results from Section 3.4 and 3.5, we can plot Figure 13, of profits under obfuscation and revelation for any correlation, positive or negative.

We don't observe the same inversion of optimal strategy below a given threshold as we had for positive correlation. Obfuscation remains the best strategy for all negative correlations. This is due to the fact that high valuated consumers at the first period are low valuated consumers at the second. Thus, it is easier for the monopoly to convince first period consumers with a high valuation to buy now and can reduce its search costs. In the same time, low valuated consumers at the first period have higher expectations of their future surplus. Thus, participation constraints are even easier to satisfy. In this context, it makes perfectly sense to always have obfuscation more profitable than revelation for negative correlations.

We still observe that the difference between the two strategies converges towards 0, as the correlation converges to -1. Indeed, with a perfect negative signal, it is pointless to obfuscate as consumers know their future valuation with perfect precision.

Figure 13: Profits of obfuscation and revelation



3.6 Similar search costs at the two periods

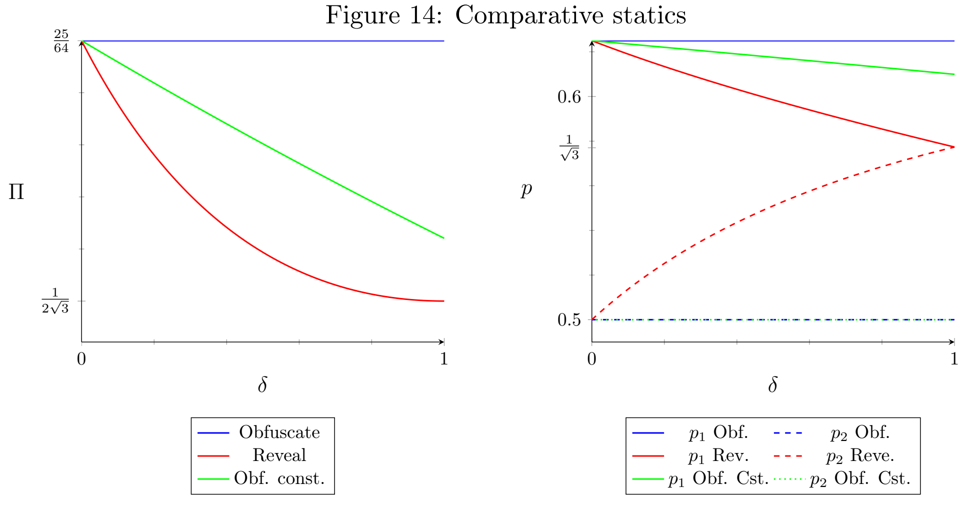
One may argue it is unrealistic to have high search costs at the second period and no search costs at the first one. Even though this modelization is a standard one in the search literature, we could doubt that the *Online Exclusive Sales Market* industry can so easily manipulate the search costs and make them vary from one period to another one.

If we think of our model as a succession of sales, $\{\dots, A, B, C, \dots\}$, search costs of sale B cannot be simultaneously high in the pair $\{A, B\}$ and low in the pair $\{B, C\}$ as they should be.
To circumvent this issue, we want to add two additional constraints : search costs have to identical at the two periods and the total expected surplus of the consumers must be positive to ensure the existence of our market.
When there is no correlation, we can combine these constraints in the following inequation:

$$(1 - F_1(\tilde{v}_1))(\int_{\tilde{v}_1}^{1}(v_1 - p_1)f_1(v_1)\,\mathrm{d}v_1) + F_1(\tilde{v}_1)(\int_{p_2}^{1}(v_2 - p_2)f_2(v_2)\,\mathrm{d}v_2 - s) \geq s \qquad \text{(PC')}$$

With the same notations as previously. Of course, in the revelation case, this additional constraint is not binding as optimal search costs were already null. In the obfuscation case, this is no longer true and we can have, depending on F_1 and F_2, the PC' constraint to be binding.
As this additional constraint can be computationally difficult to manage, we provide here only numerical computations in the uniform case. In this context, we find that this constraint was not satisfy, and the monopoly has to limit its search costs to a lower level. This also impact the first period price.

Figure 14: Comparative statics



(a) Comparaison of Profits

(b) Comparaison of Prices

We plotted on Figure 14 the profit and prices of this *constrained obfuscation case* in green. We see that it is still profitable to obfuscate when there are the same search costs in the uniform case. We are also quite confident in claiming that there should exists a range of parameter values $[0, \hat{\mu}]$ such that, even with these additional constraints, it is still profitable to obfuscate when there is correlation. This range will be included in the previous one as we restricted the efficiency of obfuscation.

3.7 Choice of first period product

In this section, we would like to provide a rationale for the optimal choice of product at the first or second period, when there is no correlation between the two valuations and with infinitively patient consumers ($\delta = 1$).

We work in the following restricted setting. Let's assume we have two distributions F_a and F_b, such that F_a and F_b yield the same monopoly profit Π^m. The monopoly has to choose whether it should sell product a or b at period 1, and the other at period 2. Without loss of generality, we impose that $p_a^\star < p_b^\star$.

We have the following result.

Result 10. *Under Revelation, with price commitment, the monopoly is indifferent between a and b at the first period. Under Obfuscation, it is better off with the product with the higher variance of valuations*

Proof. The economic intuition is quite straightforward:

- With revelation: Consumers already know their valuations, and arbitrage between the two good. The profit can be written as:

$$\Pi^{\mathcal{R}}(p_1, p_2) = p_1 \int_{p_1}^{1} F_2(p_2 + v - p_1) f_1(v)\,\mathrm{d}v + p_2 \int_{p_2}^{1} F_1(p_1 + v - p_2) f_2(v)\,\mathrm{d}v$$

  If this function admits an optimal vector of prices $(p_1^\star, p_2^\star)$, then inverting F_1 and F_2, the vector $(p_2^\star, p_1^\star)$ would be optimal too. Thus, the monopoly is indifferent between good a or good b at the first period.

- With obfuscation: The profit of the monopoly writes:

$$\begin{aligned}\Pi^{\mathcal{O}}(p_1, p_2) =& p_1(1 - F_1(p_1)) + F_1(p_1)\Pi^m \\ =& \Pi^m + p_1(1 - F_1(p_1)) - \Pi^m(1 - F_1(p_1))\end{aligned}$$

  And the monopoly has to choose between : $(F_1 = F_a, F_2 = F_b)$ and $(F_1 = F_b, F_2 = F_a)$. As $p_a^\star < p_b^\star$ and $p_a^\star(1 - F_a(p_a^\star)) = p_b^\star(1 - F_b(p_b^\star))$, we have $(1 - F_a(p_a^\star)) > (1 - F_b(p_b^\star))$. Thus, distribution F_a is more penalized by the last Π^m term than distribution F_b. It follows that the monopoly will prefer distribution F_b at the first period.

  Economically speaking, we understand it is better to have at the first period the good with a lot of variance in its valuation. Indeed, as we price above the static monopoly price, we can increase the first period price without loosing too much demand.

□

4 Conclusions

We have been able to develop in this work a theoretical model explaining how a monopoly could benefit from obfuscation. Compared to the previous literature, obfuscation is not a cost reduction device, but a choice allowing the firm to better price discriminate. Not revealing their own valuation to the consumers and imposing the optimal search costs turns consumers into myopic clients. They behave *as if* they didn't take into account the future when they are in fact infinitively patient. We could connect this idea with one of the marketing goals of the *Exclusive Sale* industry, which is to generate impulsive behavior, and not carefully considered purchases. Using the concept of *fun shopping*, this industry is in fact preventing consumers from considering the future products that may appears in the following weeks.

Other mechanisms could explain the use of obfuscation in this example, using for example stocking costs or fear of cannibalization between distribution channels of the producers. Still, we are confident in the relevance of this analysis in this market and others. Some assumptions in our modelization could probably be soften, like our extreme form of substitutability. Having a more flexible setting would be untractable, but shouldn't change dramatically our results. Another interesting further work would be to make sure this obfuscation remains stable in presence of competition.

Price discrimination on the Internet became in the last years a major concern of many public or regulatory institutions, including the European Commission. Our work could enlighten one mechanism through which firms manage to generate intertemporal price discrimination.

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A Proofs

A.1 Proof of Symmetric Equilibrium

Proof. With the two assumptions, we can notice that : $c_1(p_1, p_2) = c_2(p_2, p_1)$. It is then striking that the profit is a symmetric function: $\forall(x, y), \Pi^{\mathcal{PCR}}(x, y) = \Pi^{\mathcal{PCR}}(y, x)$. We suggest then to write the profit in PCR model as a function of p_1 and $h = p_2 - p_1$.

$$\begin{aligned}
\Pi^{\mathcal{PCR}} =& p_1 \int_{p_1}^{1} F(p_2 + v - p_1) f(v) \,\mathrm{d}v + p_2 \int_{p_2}^{1} F(p_1 + v - p_2) f(v) \,\mathrm{d}v \\
=& p_1 \int_{p_1}^{1} F(v + h) f(v) \,\mathrm{d}v + (p_1 + h) \int_{p_1+h}^{1} F(v - h) f(v) \,\mathrm{d}v
\end{aligned}$$

And the derivative of the profit are:

$$\begin{aligned}
\frac{\partial \Pi^{\mathcal{PCR}}}{\partial p_1} =& \int_{p_1}^{1} F(v + h) f(v) \,\mathrm{d}v + \int_{p_1+h}^{1} F(v - h) f(v) \,\mathrm{d}v \\
& - (p_1 + h) F(p_1) f(p_1 + h) - p_1 F(p_1 + h) f(p_1) \\
\frac{\partial \Pi^{\mathcal{PCR}}}{\partial h} =& p_1 \int_{p_1}^{1} f(v + h) f(v) \,\mathrm{d}v - (p_1 + h) \int_{p_1+h}^{1} f(v - h) f(v) \,\mathrm{d}v \\
& + \int_{p_1+h}^{1} F(v - h) f(v) \,\mathrm{d}v - (p_1 + h) F(p_1) f(p_1 + h)
\end{aligned}$$

With $h = 0$, both FOC conditions becomes identical and are simply:

$$p_1 F(p_1) f(p_1) = \int_{p_1}^{1} F(v) f(v) \,\mathrm{d}v = \frac{1 - F^2(p_1)}{2}$$

So, the couple (p, h) where p is defined by $2pF(p)f(p) = 1 - F^2(p)$, and $h = 0$ is a solution. Lastly, we can be sure there exists such a p by monotonicity of the LHS and RHS of this equation.

$\square$