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Abstract

This paper introduces a structural dynamic factor model (SDFM) for stock returns. In contrast to standard linear factor models, the new approach accounts for nonlinear effects of common factors when the distance-to-default is small. We develop a toolkit of econometric methods for the SDFM, based on indirect inference and Approximate Bayesian Computation (ABC) filtering and smoothing, which permit to estimate, filter and predict systematic risk. We apply the SDFM to measure the systematic risk of financial institutions and obtain their rating of default and speculative features.

Keywords: Systemically Important Financial Institutions (SIFI), Merton's model, Value of the Firm, indirect inference, rating, ABC filtering and smoothing.

1 Introduction

An extensive literature analyzes systematic risk from stock returns. The investigation usually proceeds from dynamic linear factor models, where the factors can feature conditional heteroscedasticity. Canonical models of systematic risk are derived from the Capital Asset Pricing Model (CAPM) and its various extensions, where the linear factors are returns on diversified portfolios of stocks (Markowitz, 1959; Sharpe, 1964; Lintner, 1965). These canonical models separate risk and required capital into two components. The systematic risk component measures the effects of the systematic factors, taking into account the sensitivities to the factors (or betas); the specific risk corresponds to the residual risk once the systematic component is taken into account.

The hypothesis of linear effect of the systematic factors has been questioned, in particular for factor values, which might create default of important financial institutions, the so-called systemically important financial institutions (SIFI). If a firm is close to default, the stock price integrates the cost of default-risk and can therefore co-move nonlinearly with the systematic factor.[1] This nonlinearity is consistent with financial theory, which views a stock as a call option written on the asset component of the firm's balance sheet with a strike equal to the debt component (Merton, 1974).[2] The call features of equity implies that stock returns can respond nonlinearly to changes in the underlying balance sheet. Nonlinear effects are likely to be more significant for financial institutions, which are exposed to inappropriate direct credit granting, but also to massive exposures to derivative products, such as mortgage-

[1]We prefer to use the terminology systematic instead of systemic for the common factors. Indeed, a shock on a common factor does not necessarily imply a risk of destruction of the financial or economic system (see e.g. Hansen (2012) for a discussion of the two notions).

[2]The extensive literature on the valuation of corporate debt and equity includes Black and Cox (1976), Chen (2010), Leland (1994), and Glasserman and Nouri (2012).

backed securities, collateralized debt obligations, or sovereign credit default swaps.

This paper contributes to the literature by including stochastic volatility and option features in the analysis of systematic risk. We develop a structural dynamic factor model (SDFM) with these features, which we apply to the stock returns of leading U.S. financial institutions and intermediaries. Specifically, we build two classes of systematic risk models of increasing accuracy.

First, we define a nonlinear multivariate Dynamic Factor Model (DFM) of stock returns that includes stochastic volatility, but excludes the option to default. Returns are linearly exposed to a common risk factor. The framework improves on the Sharpe-Lintner model by including stochastic volatilities. We include both (i) a stochastic volatility term for the common factor and (ii) stochastic volatility terms specific to each financial institution. Because the common factor and stochastic volatilities are latent, we develop an appropriate estimation approach based on indirect inference (Gouriéroux, Monfort and Renault, 1993; Smith, 1993) and Approximate Bayesian Computation (Calvet and Czellar, 2011, 2015a). This econometric toolkit allows us to infer the factor and volatility dynamics and measure the sensitivity of each stock to the factor. We apply this methodology to the returns on 10 leading U.S. financial institutions. We also investigate the dynamic link between the filtered factor and the submarket portfolio derived from the static approach.

Second, we develop a fullly fledged structural dynamic factor model (SDFM) that incorporates both stochastic volatility and default risk. We use the Value-of-the Firm model to quantify the effect of the distance-to-default on stock values. We approximate this nonlinear effect to get a link between the changes in the log excess asset/liability rates and the stock returns. Then, the analysis is extended to the multiasset framework. This leads to a model defined in two components. The linear DFM with stochastic volatilities is used to define the joint dynamics of the unobserved changes in the log asset/liability ratios. Then a nonlinear measurement equation relates the observed stock returns to these changes and includes the nonlinear effect of the distance-to-default. By design, the full-fledged SDFM nests the first model developed in this paper. The simpler model captures systematic risk in "normal" times, in which default risk is tiny, while the full-fledged SDFM is also able to capture systematic risk in turbulent times.

We estimate the SDFM on U.S. financial stocks and compare the results to the output of our simpler model without default option. We use our results to construct three ratings: a solvency rating, a rating based on the market price of default insurance, and a rating incorporating the speculative feature of the associated stock. We also provide the idiosyncratic and systematic Value-at-Risk measures.

Our structural approach to systematic risk is in the spirit of the rating models developed by KMV (see e.g. Crosbie and Bohn (2004)). We improve this earlier approach by expanding the factor dynamics and allowing for stochastic volatility. The paper is also related to the structural extension of multivariate GARCH models developed in Engle and Siriwardane (2015).

Section 2 presents the data. Section 3 constructs a model of systematic risk that includes stochastic volatilities in the factor and individual stocks. Section 4 develops the full-fledged SDFM, which also incorporates the option to default. Section 5 concludes. The ABC filtering and smoothing techniques are presented in Appendix A and smoothed hidden states in Appendix B.

2 Data and Summary Statistics

We consider the daily log returns between April 6, 2000, and July 31, 2015, of ten financial institutions: MetLife (MET), ING Group (ING), Goldman Sachs (GS), Lincoln National (LNC), HSBC Holdings (HSBC), JPMorgan Chase & Co. (JPM), Bank of America (BAC), Credit Suisse (CS), Wells Fargo (WFC), and Barclays (BCS). This set includes mainly risky institutions on the current list of SIFIs with total capital ratios set between 11.5% for ING or Wells Fargo to 13% for HSBC or JP Morgan, to be compared to the minimum ratio of 8%. This set includes two insurance companies: MetLife[3] and Lincoln National. The daily log returns are:

$$Y_{i,t} = \log(P_{i,t}/P_{i,t-1}),$$

where $P_{i,t}$ is the closing price of financial stock i at date t adjusted for dividends and splits, downloaded from Yahoo Finance. All returns are computed in U.S. dollars. The sample size of the log return series is $T = 3,853$ (trading days). Throughout the paper, we denote by $Y_t = (Y_{1,t}, \ldots, Y_{n,t})'$ the column vector of stock returns at date t, and by $Y_{1:t} = (Y_1', \ldots, Y_t')'$ the observations available up to date t.

The log return series are plotted in Figure 1. We immediately observe time-varying and persistent volatilities. Large volatilities arise to all series at the period of the recent financial crisis, i.e. 2008-2010, and also for several series in 2012.

Table 1 reports sample statistics of the return series. The mean returns are between -1% and 10% per year on this period, with large kurtosis revealing fat tails. These tail features are partly due to the crises, with negative daily returns up to 50% (for LNC and BCS), which corresponds to a decrease of 40% of the price. We also observe large log returns, up to a price increase of 60% for BCS. The skewnesses are negative, except for GS, JPM and WFC. This negative sign can be due to a crash going faster than the recovery.

[3]This company failed the stress test in 2012. As a consequence it solds its banking unit and its 70 billion mortgage servicing business, the latter one to JP Morgan.

Table 1: **Sample Statistics**

	MET	ING	GS	LNC	HSBC	JPM	BAC	CS	WFC	BCS
					A. Sample Moments					
Mean	0.00039	-0.00003	0.00023	0.00023	0.00011	0.00016	$5{\cdot}10^{-6}$	-0.00003	0.00037	0.00004
St.dev.	0.02758	0.03233	0.02442	0.03532	0.01743	0.02627	0.03125	0.02698	0.02528	0.03375
Skewn.	-0.31930	-0.81764	0.28792	-1.24505	-0.83182	0.26453	-0.34869	-0.36651	0.88155	-0.67903
Kurt.	20.98442	13.47540	12.25651	43.23296	15.34543	13.03115	25.40294	8.88474	27.09202	42.12672
Min.	-0.31156	-0.32229	-0.21022	-0.50891	-0.20852	-0.23228	-0.34206	-0.23460	-0.27210	-0.55549
Max.	0.24686	0.22626	0.23482	0.36235	0.13463	0.22392	0.30210	0.21056	0.28341	0.50756
AC(1)	-0.07837	-0.02256	-0.04989	-0.03458	-0.04682	-0.08004	-0.01004	-0.05813	-0.11286	-0.05603
Sq. AC(1)	0.31971	0.20103	0.23496	0.29467	0.19912	0.33998	0.32550	0.25721	0.21381	0.08837
					B. Covariance Matrix					
MET	0.00076	0.00051	0.00037	0.00068	0.00026	0.00044	0.00054	0.00040	0.00045	0.00050
ING	0.00051	0.00104	0.00041	0.00062	0.00036	0.00050	0.00057	0.00059	0.00046	0.00074
GS	0.00037	0.00042	0.00060	0.00048	0.00024	0.00045	0.00048	0.00038	0.00037	0.00042
LNC	0.00068	0.00062	0.00048	0.00125	0.00033	0.00056	0.00067	0.00051	0.00055	0.00065
HSBC	0.00026	0.00036	0.00024	0.00033	0.00030	0.00028	0.00031	0.00029	0.00025	0.00038
JPM	0.00044	0.00050	0.00045	0.00056	0.00028	0.00069	0.00062	0.00043	0.00050	0.00050
BAC	0.00054	0.00057	0.00048	0.00067	0.00031	0.00062	0.00098	0.00045	0.00064	0.00062
CS	0.00040	0.00059	0.00038	0.00051	0.00029	0.00043	0.00045	0.00073	0.00037	0.00054
WFC	0.00045	0.00046	0.00037	0.00055	0.00025	0.00050	0.00064	0.00037	0.00064	0.00049
BCS	0.00050	0.00074	0.00042	0.00065	0.00038	0.00050	0.00062	0.00054	0.00049	0.00114
					C. Eigenvalues					
Eigenv.	0.00524	0.00070	0.00054	0.00041	0.00032	0.00026	0.00023	0.00017	0.00013	0.00013
					D. Regression on Mean Log Returns					
Alpha	0.00025	-0.00020	0.00011	0.00004	0.00002	0.00002	-0.00017	-0.00017	0.00023	-0.00013
Beta	0.97036	1.15132	0.83225	1.24654	0.59355	0.98732	1.16722	0.93051	0.93459	1.18635

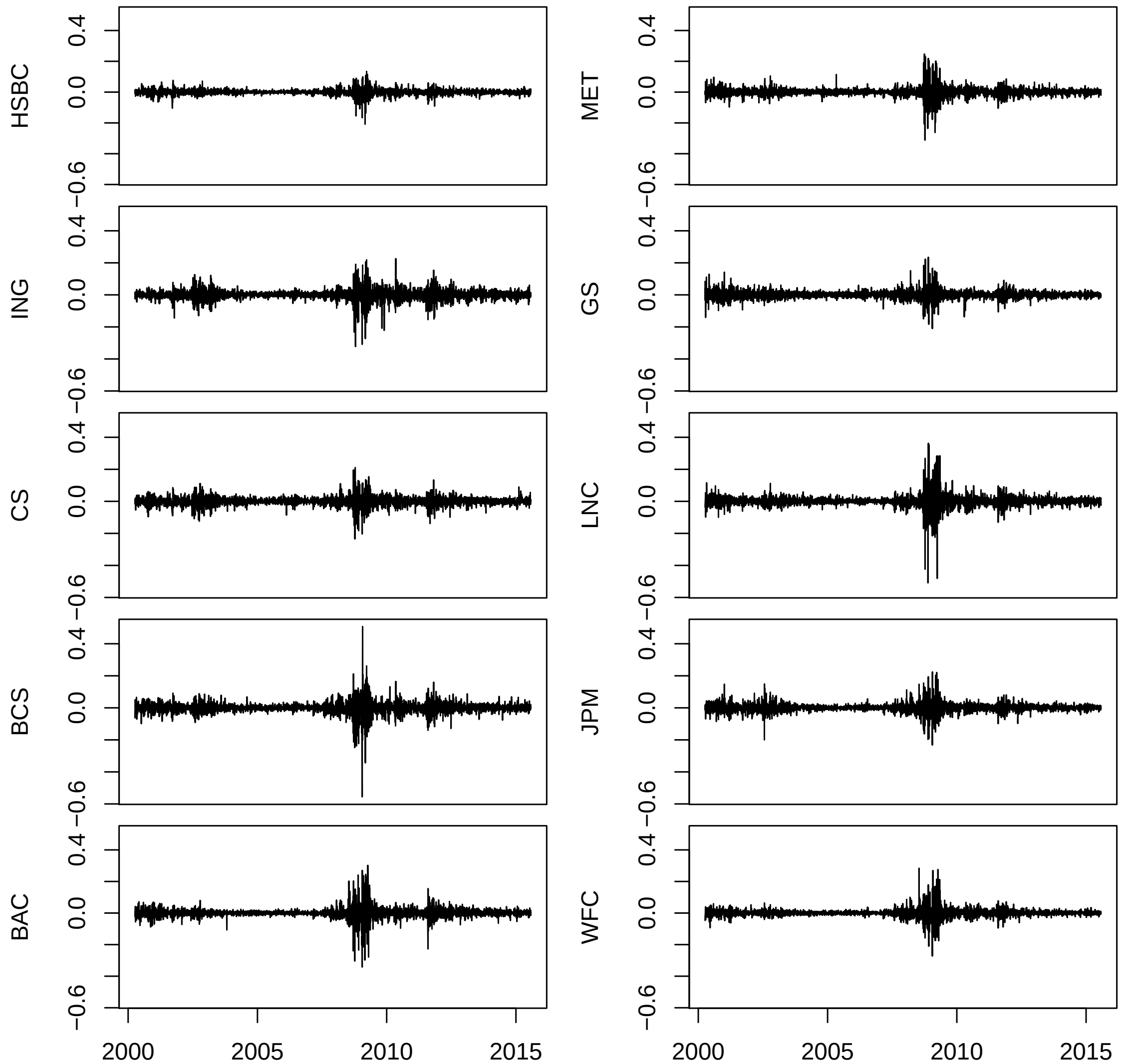

Figure 1: Log Return Series.

The dynamics of the series are summarized by the autocorrelations of the returns and squared returns, respectively. The first-order autocorrelations of the returns are very small (the largest one in absolute value is $6.7 \cdot 10^{-20}$ per month), as expected for an efficient market. At the opposite, the autocorrelations of the squared returns are larger (between 0.20 and 0.34, except for BCS), indicating the volatility persistence.

The realized daily volatility-covolatility matrix can be used for a static linear factor analysis. We provide the set of its eigenvalues in decreasing order. The largest eigenvalue is far above the next ones indicating the importance of the first linear factor. Then, the next eigenvalues decrease slowly from 0.0007 to 0.0001. The first eigenvector is $(0.30, 0.36, 0.25, 0.39, 0.18, 0.30, 0.36, 0.29, 0.29, 0.37)$. Its components are of a similar size and the associated portfolio is close to a submarket portfolio, where the submarket only includes these ten financial institutions. Finally, we provide the alphas and betas, when regressing the individual log returns on the logarithm of the submarket portfolio. The alphas are not significant showing that this standard consequence of the CAPM could be applied to the submarket considered as a whole.

Finally, let us compare the submarket portfolio and the market portfolio, i.e. the S&P500 index. Their returns are plotted in Figure 2. The slope is equal to 1.5, whereas the mean and standard errors of the submarket and market portfolios are 0.00014, 0.02247 and 0.00009, 0.01264, respectively. Thus, the submarket is not representative of the entire market. It features larger risk compensated by larger expected returns.

3 Dynamic Factor Model (DFM) With Stochastic Volatilities

This Section develops a multivariate model of stock returns that incorporates a systematic factor and stochastic volatilities.

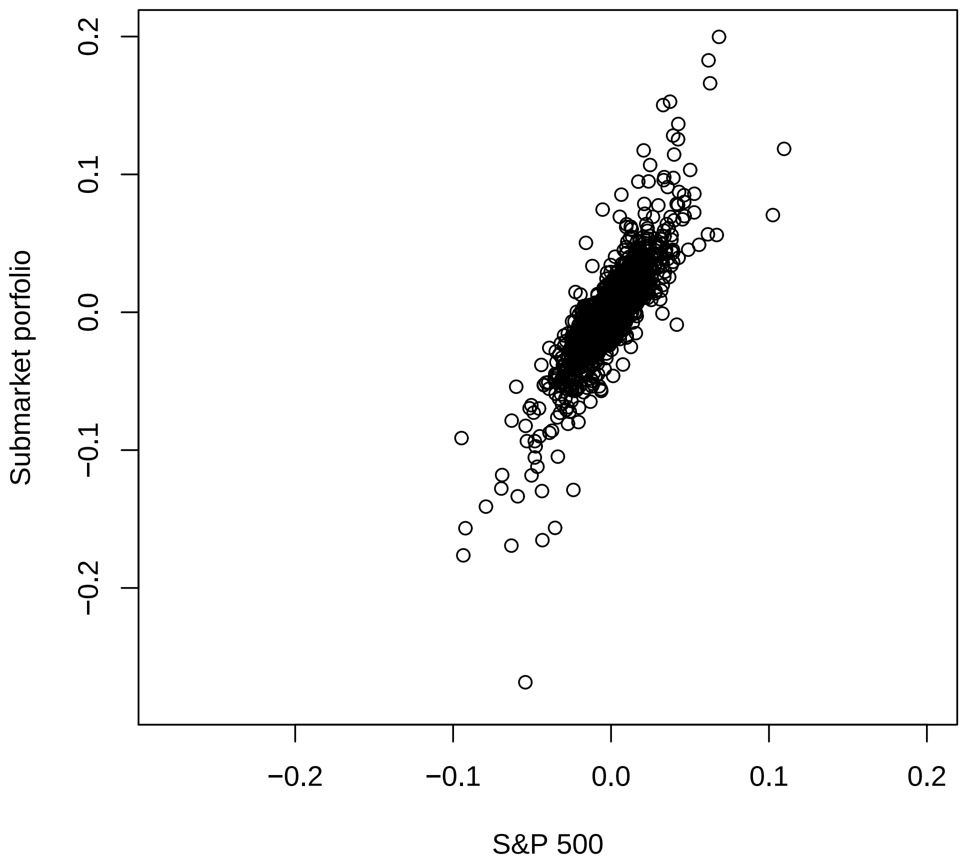

Figure 2: Submarket Portfolio Versus S&P500. The submarket portfolio log return is plotted versus the S&P500 log return. A simple linear regression provides an intercept of 10^{-5} and a slope of 1.5011.

3.1 Specification

We specify the return dynamics by:

$$\begin{cases} Y_{i,t} = \alpha_i + \beta_i F_t + \sigma_{i,t}\varepsilon_{i,t} \, , \\ F_t = \gamma F_{t-1} + \eta_t u_t \, , \end{cases} \tag{3.1}$$

where F_t is a common unobservable linear factor, $\varepsilon_{i,t}$ and u_t are independent standard normals, $\sigma_{i,t}$ is the stochastic volatility specific to stock i, and η_t is the stochastic volatility of the factor. The stochastic volatilities, $\sigma_{i,t}$ and η_t, and the innovations, $\varepsilon_{i,t}$ and u_t, are mutually independent.

The stochastic volatilities follow autoregressive gamma (ARG) dynamics, as we now explain. A process ν_t is said to be ARG(a, b, c), if it is of the form $\nu_t \sim$ Gamma$(a + z_t, c)$, where $z_t \sim$ Poisson$(b\nu_{t-1}/c)$. The parameters a, b, c correspond, respectively, to a degree of freedom, a measure of serial dependence, and a scale parameter.[4] We accordingly assume that:

$$\eta_t^2 \sim \text{ARG}[\delta, \rho, (1-\rho)(1-\gamma^2)/\delta] \text{ and } \sigma_{i,t}^2 \sim \text{ARG}(\tilde{\delta}, \tilde{\rho}, \tilde{c}) \, ,$$

for every t.

Since a dynamic factor is defined up to a linear affine transformation, it is natural to consider the normalizing conditions:

$$\mathbb{E}(F_t) = 0 \quad \text{and} \quad \text{Var}(F_t) = 1.$$

Our dynamic model satisfies this normalization. The factor has a zero mean because

[4] ARG processes are exact time discretizations of Cox, Ingersoll and Ross (1985) processes. See Gouriéroux and Jasiak (2006) for further discussion.

it follows an autoregression with no intercept. The unit variance of the factor follows from the choice of the scale parameter in the ARG dynamics of η_t.

The model is specified by the vector of structural parameters:

$$\theta = (\alpha', \beta', \gamma, \delta, \rho, \tilde{\delta}, \tilde{\rho}, \tilde{c})'.$$

where $\alpha = (\alpha_1, \ldots, \alpha_n)'$ is the vector of expected stock returns, $\beta = (\beta_1, \ldots, \beta_n)'$ is the vector of sensitivities to the common linear factor, γ quantifies the persistence of the factor, δ and ρ determine the stochastic volatility of the factor, and $\tilde{\delta}$, $\tilde{\rho}$, and $\tilde{c}$ determine the idiosyncratic stochastic volatility of each stock. The model improves on the standard linear factor model[5] by allowing for stochastic volatilities.

The vector of state variables has $n + 2$ components:

$$x_t = (F_t, \eta_t, \sigma_{1,t}, \ldots, \sigma_{n,t}).$$

We get a nonlinear state-space model, to which the standard Kalman filter does not apply. In particular, the likelihood function corresponding to the observation of $Y_t = (Y_{1t}, \ldots, Y_{nt})'$, $t = 1, \ldots, T$, has an integral form, since the unobservable factor paths have to be integrated out. The likelihood involves an integral of dimension $(n + 2)T$, which increases with the number T of observation dates. To avoid the numerical optimization of the complicated likelihood function, we consider below a simple two-step estimation approach.

[5] See, e.g., Darolles and Gouriéroux (2015), Chapter 3.

3.2 Estimation Methodology

The discussion of the inference methodology proceeds in two steps. First, we develop a method-of-moments estimator for the vector of structural parameters θ and the latent common factor F_t, which would be consistent if the number of stocks n and the number of periods T were both large.[6] Second, we develop a consistent indirect inference estimator based on the auxiliary method of moments to adjust for the finite sample bias when only T is large.

3.2.1 Auxiliary Method of Moments Estimation

The method-of-moments (MM)[7] estimation of the structural parameters and the latent factor proceeds as follows:

(i) *Estimation of* α. Since $\mathbb{E}(Y_t) = \alpha$, we let

$$\hat{\alpha} = \frac{1}{T} \sum_{t=1}^{T} Y_t \, .$$

(ii) *Estimation of* F_t. Denote the demeaned row vector by $Z_t = Y_t - \hat{\alpha}$. Denote also $Z_{\cdot t} = n^{-1} \sum_{i=1}^{n} Z_{i,t}$ and its standardized version by:

$$\hat{F}_t = Z_{\cdot t} / \sqrt{T^{-1} \sum_{t=1}^{T} Z_{\cdot t}^2} \, ,$$

for every t.

[6]Gagliardini and Gouriéroux (2014) for the so-called granularity theory.

[7]The details of the computation of moments which uses properties of the ARG process are available upon request from the authors.

(iii) *Estimation of β and γ.* We let:

$$\hat{\beta}_i = \frac{1}{T}\sum_{t=1}^{T} Z_{i,t}\hat{F}_t \quad \text{and} \quad \hat{\gamma} = \frac{1}{T-1}\sum_{t=2}^{T} \hat{F}_t\hat{F}_{t-1}\,,$$

for every i.

(iv) *Estimation of δ and ρ.* We estimate $\eta_t u_t$ by $\widehat{\eta_t u_t} = \hat{F}_t - \hat{\gamma}\hat{F}_{t-1}$. We easily verify that $\eta_t u_t$ has population moments $\mathbb{E}[(\eta_t u_t)^4] = 3(1+\delta^{-1})(1-\gamma^2)^2$ and $\mathbb{E}[(\eta_t u_t)^2(\eta_{t-1}u_{t-1})^2] = (1+\rho/\delta)(1-\gamma^2)^2$. We therefore estimate the parameters ρ and δ by:

$$\hat{\delta} = \left[\frac{\sum_{t=2}^{T}\widehat{\eta_t u_t}^4}{3(T-1)(1-\hat{\gamma}^2)^2} - 1\right]^{-1} \quad \text{and} \quad \hat{\rho} = \hat{\delta}\left[\frac{\sum_{t=3}^{T}\widehat{\eta_t u_t}^2\widehat{\eta_{t-1}u_{t-1}}^2}{(T-2)\,(1-\hat{\gamma}^2)^2} - 1\right].$$

(v) *Estimation of the parameters $\tilde{\delta}$, $\tilde{\rho}$, and $\tilde{c}$.* A natural estimator of $\sigma_{i,t}\varepsilon_{i,t}$ is:

$$\widehat{\sigma_{i,t}\varepsilon_{i,t}} = Z_{i,t} - \hat{\beta}_i\hat{F}_t\,.$$

The process $\sigma_{i,t}\varepsilon_{i,t}$ satisfies $\mathbb{E}\left(\sum_{i=1}^{n}\sigma_{i,t}^2\varepsilon_{i,t}^2\right) = nA$, $\mathbb{E}\left(\sum_{i=1}^{n}\sigma_{i,t}^4\varepsilon_{i,t}^4\right) = 3nA^2(1+\tilde{\delta}^{-1})$, and $\mathbb{E}\left[\sum_{i=1}^{n}(\sigma_{i,t}\varepsilon_{i,t})^2(\sigma_{i,t-1}\varepsilon_{i,t-1})^2\right] = nA^2(1+\tilde{\rho}/\tilde{\delta})$, where $A = \tilde{c}\tilde{\delta}/(1-\tilde{\rho})$. We therefore let $\hat{A} = n^{-1}\sum_{i=1}^{n}\sum_{t=1}^{T}\widehat{\sigma_{i,t}\varepsilon_{i,t}}^2$,

$$\hat{\tilde{\delta}} = \left[\frac{\sum_{t=1}^{T}\sum_{i=1}^{n}\widehat{\sigma_{i,t}\varepsilon_{i,t}}^4}{3n\hat{A}^2(T-1)} - 1\right]^{-1},$$

$$\hat{\tilde{\rho}} = \hat{\tilde{\delta}}\left[\frac{\sum_{t=2}^{T}\sum_{i=1}^{n}\widehat{\sigma_{i,t}\varepsilon_{i,t}}^2\widehat{\sigma_{i,t-1}\varepsilon_{i,t-1}}^2}{n\hat{A}^2(T-1)} - 1\right], \quad \text{and} \quad \hat{\tilde{c}} = (1-\hat{\tilde{\rho}})\hat{A}/\hat{\tilde{\delta}}\,.$$

The resulting MM estimator:

$$\hat{\mu} = (\hat{\alpha}, \hat{\beta}, \hat{\gamma}, 1/\hat{\delta}, \hat{\rho}/\hat{\delta}, 1/\hat{\tilde{\delta}}, \hat{\tilde{\rho}}/\hat{\tilde{\delta}}, \hat{A})',$$

has closed-form expressions and is easy to compute and interpret. However, the filtered value $\hat{F}_t$ converges to F_t only if the number of stocks n goes to infinity. As a result, the method of moments is inconsistent when the number of stocks is fixed. Despite this shortcoming, the MM estimator can be useful in the context of indirect inference, as we now show.

3.2.2 Indirect Inference Estimation

The indirect inference adjustment is based on a comparison of the auxiliary statistics computed from the data and those computed using simulated data generated from the structural model (3.1). More precisely, let us consider independent simulated pseudo-data samples: $Y_{1:T}^{(s)}(\theta) = \{Y_t^{(s)}(\theta)\}_{t=1}^{T}$, $s = 1, \ldots, S$, satisfying the data-generating process (3.1), where S is the number of replications. We use $S = 10$ throughout the paper. The auxiliary estimator based on simulated data is defined by:

$$\hat{\mu}_S(\theta) = S^{-1} \sum_{s=1}^{S} \hat{\mu}[Y_{1:T}^{(s)}(\theta)].$$

The indirect inference (II) estimator of θ is then:

$$\hat{\theta}_{II} = \arg\min_{\theta} [\hat{\mu} - \hat{\mu}_S(\theta)]' \Omega [\hat{\mu} - \hat{\mu}_S(\theta)],$$

where Ω is a symmetric positive-definite (SPD) weighting matrix. Since we use a just-identified auxiliary estimator, the II estimator does not depend on the SPD

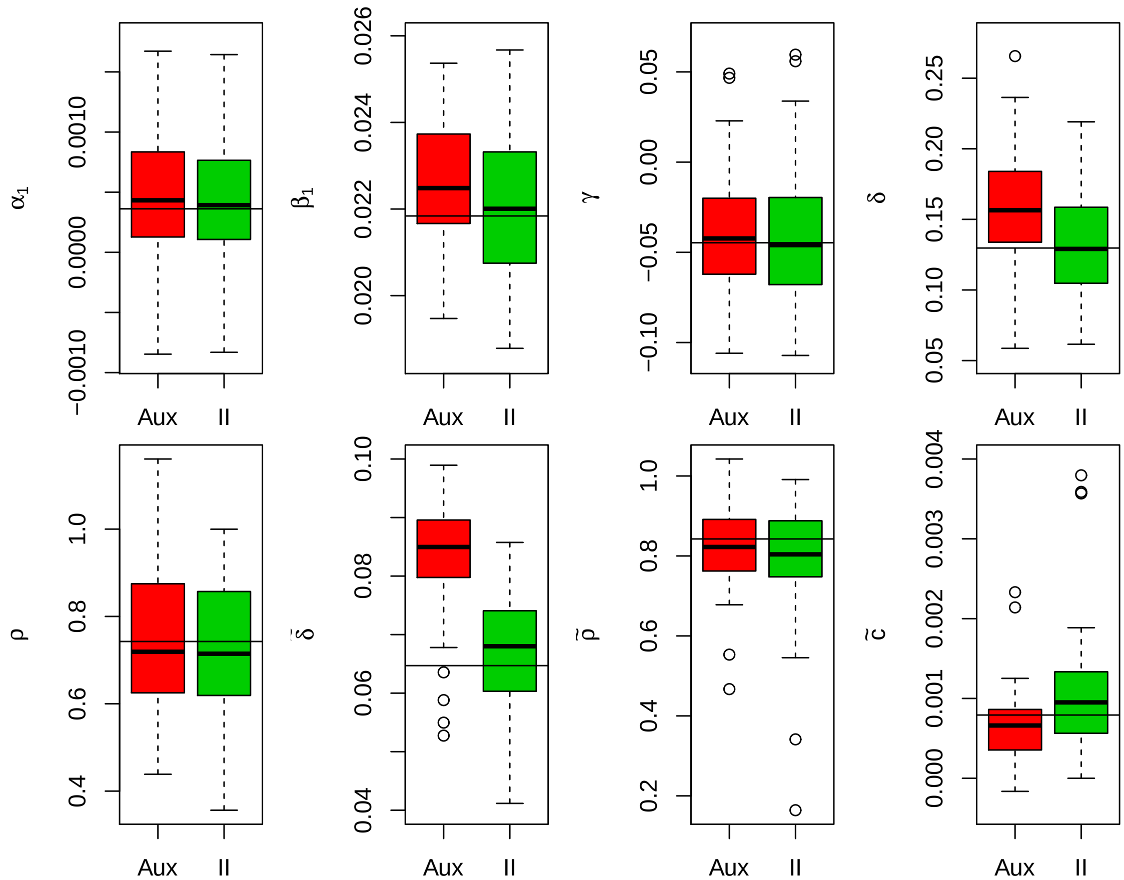

Figure 3: Method of Moments and Indirect Inference Estimates. This figure illustrates 100 MM and corresponding II estimates for $n = 10$ and sample size $T = 3,853$ as in the empirical dataset. In each panel the left boxplot is for MM and the right boxplot for II estimates. The true parameters are set to the empirical estimates and are reported with continuous lines.

weighting matrix Ω, which can be set equal to the identity matrix. The indirect inference estimator $\hat{\theta}_{II}$ consistently estimates θ as T goes to infinity.

3.3 Monte Carlo Results

Let us illustrate the properties of the first step method of moment (MM) estimation and of the II estimation by Monte Carlo study. We consider $n = 10$ stocks and a sample size $T = 3,853$ like in our empirical dataset. The true parameters are set to the empirical estimates obtained in the following subsection and reported in Table 2, second column.

We generate independently 100 datasets and for each of them we compute the MM estimate and the II estimate. We provide in Figure 3 the boxplots corresponding to the finite sample ($T = 3,853$) properties of both estimates. The true values of the parameters are set to the empirical estimates (see later in the second column of Table 2) and are reported with continuous lines. We only display α_1 and β_1 among the αs and βs. For the MM estimates, the boxes do not necessarily include the true values of the parameters, as a consequence of the nonconsistency. This arises for parameters characterizing the volatility dynamics, especially the δ and $\tilde{\delta}$ parameters. When the estimates are adjusted by indirect inference, we observe that the boxes now all include the true parameter values.

3.4 Empirical Results

Let us now apply the estimation approach to the real dataset. Table 2 reports the auxiliary MM estimate and the II estimate. The estimated betas can be directly compared with the estimated betas derived in the static analysis of Section 2. In Section 2, the submarket portfolio has not been standardized. Since the standard error of the return of the submarket portfolio is equal to 0.022, after standardization these betas are provided in the last column of Table 2. The betas estimated from the static model, the auxiliary moment method and the indirect inference estimates

Table 2: **Empirical Estimates**

	Aux	No option of default	With option of default	Static model
α_{MET}	0.000395	0.000362	0.000606	
α_{ING}	-0.000032	0.000059	0.000198	
α_{GS}	0.000232	0.000200	0.000596	
α_{LNC}	0.000227	0.000118	0.000358	
α_{HSBC}	0.000111	0.000163	-0.000122	
α_{JPM}	0.000164	0.000074	-0.000212	
α_{BAC}	0.000005	-0.000047	0.000440	
α_{CS}	-0.000028	0.000047	0.000159	
α_{WFC}	0.000366	0.000372	0.000694	
α_{BCS}	0.000043	-0.000053	-0.000950	
β_{MET}	0.021806*	0.021840*	0.026920	0.021348
β_{ING}	0.025872*	0.026084*	0.034525	0.025329
β_{GS}	0.018702*	0.018239*	0.020364*	0.018309
β_{LNC}	0.028012*	0.028496*	0.035498	0.027424
β_{HSBC}	0.013338*	0.012607*	0.016979	0.013058
β_{JPM}	0.022187*	0.022368*	0.036311	0.021721
β_{BAC}	0.026229*	0.026669*	0.034107	0.025679
β_{CS}	0.020910*	0.020599*	0.026767	0.020471
β_{WFC}	0.021002*	0.020969*	0.023378	0.020561
β_{BCS}	0.026659*	0.026857*	0.050174	0.026100
γ	-0.053297	-0.044679	-0.054902	
δ	0.162371*	0.129703*	0.161184*	
ρ	0.787698*	0.742651*	0.869338*	
$\tilde{\delta}$	0.077740*	0.064702*	0.077236*	
$\tilde{\rho}$	0.815843*	0.842556*	0.856746*	
$\tilde{c}$	0.000687	0.000792	0.000786	
ω			1.861301*	

Empirical estimates for daily log returns between April 6, 2000, and July 31, 2015, a sample size of 3,853. Each reported II estimate is the global minimum of the II objective function among 1,000 local minimizations with starting values chosen at random. In the Aux and No option of default columns, numbers with an asterix indicate significant parameters at the 5% level, where significance is measured by means of robust standard errors IQR/1.349 where IQR is the interquartile range over 100 Monte Carlo estimates.

are rather close to each other. Thus, the interest of the approach is essentially in the parameters characterizing the dynamics of factor and volatilities. These parameters are required for the prediction of future returns and in particular for the computation of the term structure of the Value-at-Risk and and their sensitivities with respect to shocks on either the systematic, or specific standardized innovations.

The estimated γ is small and negative. It has to be compared with the autoregressive coefficient corresponding to the return of the submarket portfolio derived by the static approach. This coefficient is -0.053, thus has the same order as the II estimates of γ.[8]

We observe a strong persistence, that is large ρ coefficients, for both systematic and specific volatilities. The estimated δ and $\tilde{\delta}$ coefficients provide information on the behaviour of the (marginal and conditional) distribution of volatilities in the tail and in the neighborhood of volatility zero.

Using the II parameter estimates, we simulate series of the same size as the original dataset. The related sample statistics are reported in Table 3. The comparison of Tables 1 and 3 shows a reasonable goodness of fit for the different characteristics of historical distribution, including the measures of state dependence between returns, and of their dynamic features by means of first-order autocorrelations of returns and squared returns.

[8] The autoregressive coefficient for the return of the S&P 500 is -0.084.

Table 3: **Sample Statistics of Simulated Series**

	MET	ING	GS	LNC	HSBC	JPM	BAC	CS	WFC	BCS
					A. Sample Moments					
Mean	0.00070	0.00037	0.00105	0.00029	0.00039	0.00044	$-8\cdot10^{-6}$	$-2\cdot10^{-5}$	0.00056	0.00030
St.dev.	0.02653	0.02931	0.02688	0.03260	0.01967	0.02718	0.03115	0.02680	0.02648	0.03060
Skewn.	-0.37902	-0.65573	-0.04512	-0.12469	-0.22193	-0.37643	-0.76003	-0.75318	-0.79803	-0.81506
Kurt.	14.18529	14.94072	21.18191	17.27276	15.01790	15.93163	15.70523	12.95755	18.91468	18.59897
Min.	-0.22352	-0.27242	-0.24653	-0.29785	-0.16614	-0.25429	-0.27710	-0.21414	-0.30902	-0.34973
Max.	0.16933	0.20106	0.32663	0.30655	0.16804	0.22008	0.20753	0.18743	0.25635	0.22679
AC(1)	-0.09970	-0.07468	-0.03627	-0.06953	-0.07767	-0.06166	-0.06460	-0.06251	-0.01950	-0.05747
Sq. AC(1)	0.23862	0.25440	0.20127	0.25253	0.32141	0.26033	0.27648	0.32238	0.22400	0.27619
					B. Covariance Matrix					
MET	0.00070	0.00053	0.00037	0.00059	0.00026	0.00046	0.00055	0.00042	0.00042	0.00054
ING	0.00053	0.00089	0.00044	0.00070	0.00032	0.00056	0.00067	0.00050	0.00051	0.00065
GS	0.00037	0.00044	0.00072	0.00049	0.00022	0.00038	0.00045	0.00035	0.00035	0.00046
LNC	0.00059	0.00070	0.00049	0.00106	0.00034	0.00062	0.00074	0.00055	0.00056	0.00072
HSBC	0.00026	0.00032	0.00022	0.00034	0.00039	0.00027	0.00032	0.00024	0.00025	0.00032
JPM	0.00046	0.00056	0.00038	0.00062	0.00027	0.00074	0.00057	0.00043	0.00044	0.00056
BAC	0.00055	0.00067	0.00045	0.00074	0.00032	0.00057	0.00097	0.00051	0.00052	0.00067
CS	0.00042	0.00050	0.00035	0.00055	0.00024	0.00043	0.00051	0.00072	0.00039	0.00051
WFC	0.00042	0.00051	0.00035	0.00056	0.00025	0.00044	0.00052	0.00039	0.00070	0.00052
BCS	0.00054	0.00065	0.00046	0.00072	0.00032	0.00056	0.00067	0.00051	0.00052	0.00094
					C. Eigenvalues					
Eigenv.	0.00523	0.00041	0.00033	0.00030	0.00028	0.00028	0.00027	0.00026	0.00024	0.00022
					D. Regression on Mean Log Returns					
Alpha	0.00031	-0.00009	0.00071	-0.00022	0.00015	0.00004	-0.00049	-0.00040	0.00018	-0.00017
Beta	0.96396	1.14046	0.84320	1.26499	0.58183	0.99771	1.18975	0.92071	0.92748	1.16992

4 Structural Model With Default Risk and Stochastic Volatilities

4.1 Theoretical Model

4.1.1 Value of the Firm Model

The structural model or Value-of-the-Firm model has been initially introduced by Black and Scholes (1973) and Merton (1974). This is a two-period model based on a description of the balance sheet of the firm at the second period. Since the structural model is now extended to a dynamic framework, we denote by $(t,\, t+1)$ the period of interest. The balance sheet of the firm at date $t+1$ is summarized by the value of assets, A_{t+1}, and the liability, L_{t+1}. The econometrician does not observe these components of the balance sheet.[9]

The value of the firm at date $t+1$ for the shareholders is $(A_{t+1} - L_{t+1})^+ = \max(A_{t+1} - L_{t+1}, 0)$, taking into account their limited liability. Thus the stock can be considered as a European call option written on the underlying asset value with strike the level of liability. In the absence of arbitrage, the firm's equity is worth:

$$P_t = e^{-r_t}\, \mathbb{E}^{\mathbb{Q}} \left[(A_{t+1} - L_{t+1})^+ \mid \mathcal{I}_t \right] \,, \tag{4.1}$$

where r_t the riskfree rate for period $(t, t+1)$, $\mathcal{I}_t$ is the information available to investors at date t, and $\mathbb{Q}$ is a risk-adjusted measure. Formula (4.1) can be applied to the total number of shares, or be standardized per share, as long as the number

[9] When A and L are latent, we do not have to be more precise on the way they are defined. Typically, the asset component is marked-to-market, whereas the debt is generally book valued. Similarly, the debts can have different maturities. The L component has to be interpreted as the part of the debt to be reimbursed at date $t+1$. Similarly, A may be the part of assets liquid at date $t+1$.

of shares is independent of time. The stock price is observed by the econometrician and can be used to impute the components of the balance sheet.

In order to facilitate the imputation of assets and liabilities, we assume that Merton (1974)'s hypotheses hold. That is, the information set of the investor at date t consists of: (i) the value of the assets, A_t, (ii) the present value of the debt, $L_{t|t+1} = e^{-r_t} L_{t+1}$, and (iii) the interest rate r_t. Under the risk-adjusted measure, the assets follow the geometric Brownian motion:

$$dA_t = A_t(r_t\, dt + \omega\, dz_t)\,,$$

where ω is a constant volatility term and z_t is a standard Brownian motion under $\mathbb{Q}$. The price of equity is then:

$$P_t = A_t\Phi(d_{1,t}) - L_{t|t+1}\Phi(d_{2,t})\,, \tag{4.2}$$

where $d_{1,t} = [\log\left(A_t/L_{t|t+1}\right) + \omega^2/2]/\omega$ and $d_{2,t} = d_{1,t} - \omega$.

In the modelling above, it is important to distinguish two time frequencies. The discrete dates $t = 1, 2, \ldots$ are the times at which the debt is due. They define the maturity dates of the call options. The underlying continuous time model specifies the evolution of the assets within the period and allows us to obtain the Black-Scholes expression (4.2).

4.1.2 Stock Returns and Financial Leverage

Financial leverage drives the dynamics of stock returns, as we now illustrate. Let

$$eal_t = \log\left(\frac{A_t}{L_{t|t+1}} - 1\right).$$

denote the log of the excess asset equity-to-liability ratio, which is a measure of the financial leverage. By (4.2), the price of the stock satisfies $\log(P_t) = \log(L_{t|t+1}) + h\left(eal_t; \omega\right)$, where

$$h(eal, \omega) \equiv \log\left\{(1 + e^{eal})\Phi[\omega^{-1}\log(1 + e^{eal}) + \omega/2] - \Phi[\omega^{-1}\log(1 + e^{eal}) - \omega/2]\right\}.$$

The log return on the stock between $t - 1$ and t,

$$Y_t = \log(L_{t|t+1}/L_{t-1|t}) + h\left(eal_t; \omega\right) - h\left(eal_{t-1}; \omega\right) ,$$

is therefore a nonlinear function of eal_t and eal_{t-1}.

We henceforth focus on companies for which the value of assets are much more volatile than the value of liabilities between two consecutive dates $t - 1$ and t. This condition seems plausible for financial institutions at the daily frequency. The log returns can then be rewritten as:

$$Y_t \approx h\left(eal_t; \omega\right) - h\left(eal_{t-1}; \omega\right) , \tag{4.3}$$

in every t.

If the firm is far from default (large eal_t), the log return satisfies

$$Y_t \approx a_t, \tag{4.4}$$

where $a_t = eal_t - eal_{t-1}$ Therefore, the relative change in firm value is equal to the relative change in the excess asset-to-liability ratio. In particular, if a_t is stationary under the physical measure $\mathbb{P}$, the stock return is also stationary under $\mathbb{P}$.

However, in the general case, formula (4.3) implies that

$$Y_t \approx \frac{\partial h}{\partial eal}\left(eal_{t-1};\omega\right)\, a_t\,. \tag{4.5}$$

This shows that the stock return depends not only on the change in financial leverage, a_t, but also on the level of financial leverage itself, and this leverage effect will increase with the proximity to default. The stock return amplifies changes in financial leverage in the proximity of default. This is the *volatility leverage connection* in the terminology of Engle and Siriwardane (2015). If a_t is stationary, the stock return will become nonstationary. This nonstationary feature has to be taken into account in the analysis. In our application, this leverage effect can be significant, while staying limited, since we consider financial institutions, which have not experimented default during all the period, and thus are still at some distance to default.

4.2 The Structural Dynamic Factor Model (SDFM)

We incorporate the effect of the distance-to-default[10] by specifying the dynamics of stock returns under the physical measure $\mathbb{P}$ as follows:

State Equations. Changes in the excess asset-to-liability ratio satisfy

$$\begin{cases} a_{i,t} = \alpha_i + \beta_i F_t + \sigma_{i,t}\varepsilon_{i,t}\,,\; i = 1,\ldots,n\,, \\ F_t = \gamma F_{t-1} + \eta_t u_t\,, \end{cases} \tag{4.6}$$

with the same dynamic assumptions as in (3.1). Compared to (3.1), this dynamic model includes new state variables that are the $a_{i,t}$. As usual, it will be interesting to

[10] We use distance-to-default in its general meaning and not with the specific definition by KMV [see Crosbie and Bohn (2004)].

reconstitute these variables, that are the implied financial leverages, from the stock return data only.

Measurement Equations. Stock returns satisfy

$$Y_{i,t} = h\left(eal_{i,t}; \omega_i\right) - h\left(eal_{i,t-1}; \omega_i\right)\ ,\ i = 1, \ldots, n\,. \tag{4.7}$$

The model includes additional parameters, that are the intraperiod volatilities ω_i appearing in the measurement equations. It is known from the theoretical model that such a parameter ω_i can be difficult to identify if the firm is very far from default.[11] For this reason, we assume $\omega_i = \omega$, independent of the firm, in the illustration below.

The measurement equations involve the levels of financial leverages: $eal_{i,t} = \log(A_{i,t}/L - 1)$, which will have to be reconstructed from their changes $a_{i,t} = eal_{i,t} - eal_{i,t-1}$. This demands the introduction of initial conditions on $\log(A_{i,t}/L - 1)$. Since the institutions under analysis are not on default during the considered period, we can select initial values sufficiently high to avoid default at the initial date. The results are weakly dependent on this initial choice, which corresponds to the extreme situation of stationarity.

The discrete time dynamic model (4.6) for the asset liability ratios, written under the historical distribution $\mathbb{P}$, is compatible with Merton's pricing equations derived under the risk-neutral probability $\mathbb{Q}$. Indeed, a discrete time framework implies an incomplete market framework, that is an infinity of discrete time risk-neutral distribution compatible with a given historical distribution. In our framework the risk-neutral log-normal distribution for $a_{i,t}$ given $a_{i,t-1}$ is compatible with the historical distribution for $a_{i,t}$ given $a_{i,t-1}$ by choosing the appropriate stochastic discount

[11] See equation (4.4), where the effect of ω_i disappears.

factor. If the riskfree interest rate is set to zero, the stochastic discount factor is just the ratio of the risk-neutral and historical transition densities. In our specification, the investors have the information on the balance sheets $eal_{i,t}$ and on the intraperiod volatility (not only on the returns), but they do not have the information on the underlying factors $F_t, \sigma_{i,t}, \eta_t$. In contrast, the econometrician has only the information on the stock returns.

The first state equation is written on the log excess asset liabilities. Therefore the excess asset liability ratio is always positive. This feature is compatible with the most recent financial stability regulation. The very risky institutions have to be treated by the supervisors before reaching default; this is the so-called *resolution regime*[12].

4.3 Comparison with the Literature

A dynamic structural model has recently been introduced in Engle and Siriwardane (2015). Their full recursive model is deduced from expansions of the risk-neutral dynamics and structural pricing formulas underlying a stochastic volatility model written on the process of asset value. This expansion provides a simplified dynamics for the equity returns, but its validity can be questioned, when distance-to-default is small and nonlinear effects potentially important. For instance, the expansion neglects the effect of time-to-maturity, the effect of the drift and of the volatility shocks to focus on the link between equity and asset volatilities. Moreover, the expansion is performed in continuous time, and the impact of this expansion for discrete time data, that is the time aggregation effect, is not really taken into account. Another difference compared to our model is the ARCH specification introduced for

[12]This resolution regime is specific to financial institutions. In this respect, our model is not appropriate for other types of corporates.

the estimated econometric model, which is not in line with the underlying structural model, where the volatilities were stochastic with their own shocks. Finally, the set of observable variables is not the same, since the authors assumed observable by the econometrician both the equity and asset values, whereas we preferred to assume observable the equity values only. This is more in line with the Moody's-KMV practice. Moreover, our approach solves the question of the different frequencies in which these data are available and avoid the use of proxies for the monthly asset values[13].

4.4 Estimation Results

As mentioned above, for identification reasons, we assume that $\omega_i = \omega$ are constant for all $i = 1, \ldots, n$[14]. We follow an approach similar to the approach in Section 3.2. We first consider a set of moment restrictions to calibrate the parameters and the underlying factors. This step can lead to inconsistent estimators. This lack of consistency is adjusted for by indirect inference in the second step. In the first step, in addition to our $(2n+6)$ auxiliary statistics in $\hat{\mu}$ of Section 3.2.2, we need an additional statistic to identify ω. For this purpose, we use the mean standard deviation of individual returns:

$$\tilde{\omega} = \frac{1}{n} \sum_{i=1}^{n} \sqrt{\frac{1}{T-1} \sum_{t=1}^{T} \left(\exp(Y_{i,t}) - \overline{\exp(Y_i)} \right)^2}. \tag{4.8}$$

[13] In Engle and Siriwardane (2015) the daily book values of the debt are obtained by an automatic exponential average with smoothing parameter of 0.01 from monthly book values provided by Bloomberg

[14] We have checked that the objective function of the estimation problem was poorly informative on the individual heterogeneity of the intraperiod volatilities.

Clearly, this auxiliary moment $\tilde{\omega}$ will not be a consistent estimator of ω. Indeed, $\tilde{\omega}$ has the interpretation of an interperiod volatility averaged on the banks and differ from the intraperiod volatility. However, this lack of consistency will be solved by indirect inference. Thus, we use the augmented auxiliary vector $\hat{\mu}^{OD} = (\hat{\mu}', \tilde{\omega})'$ and estimate the structural parameter $\theta^{OD} = (\theta', \omega)'$ by indirect inference as described in Section 3.2.2.

The estimation results are reported in Table 2, column 4. From this table we can compare the estimates for the models with and without option of default. The estimates of the beta coefficients are significantly different for Barclays and JP Morgan. We also observe more persistence in the stochastic volatility η_t and a value of the intraperiod volatility indicates the need to adjust for the option of default.

To assess the linearity of the common factor F_t we provide the pseudo R^2 measure:

$$R_i^2 = \frac{\sum_{t=1}^{T} (\beta_i\, F_t)^2}{\sum_{t=1}^{T} \left[(\beta_i\, F_t)^2 + (\sigma_{i,t}\, \varepsilon_t)^2\right]}\,. \tag{4.9}$$

Since the $\{F_t\}_{t=1}^{T}$ are not observed, we need a particle smoother to provide estimated values. Using the II parameter estimates, we impute a sequence of hidden states $x_{1:T} = (x_1, \ldots, x_T)$, $x_t = (F_t, \eta_t, \sigma_{1,t}, \ldots, \sigma_{n,t})$ over the sample period. To estimate the distribution of $x_t|Y_{1:t}$ at a given date t, we use an Approximate Bayesian Computation (ABC) smoother (see Appendix A for the ABC smoothing)[15]. We replace $\beta_i\, F_t$ in the numerator (4.9) by $\hat{\beta}_i$ times the median of the smoothed $\{\tilde{F}_t^{(m)}\}_{m=1}^{M}$ and $\sigma_{i,t}\, \varepsilon_t$ is replaced by the median of $\{Y_{i,t} - \hat{\alpha}_i - \hat{\beta}_i\, \tilde{F}_t^{m)}\}_{m=1}^{M}$. We report the R^2 measures in Table 4 for both estimated models DFM (second column) and SDFM (third column). As a matter of comparison, we measure the same proportions when

[15] A standard particle smoothing is not applicable in the SDFM model since the distributions of $x_t|x_{t-1}$ and of $y_t|(x_t, y_1, \ldots, y_{t-1})$ are not available in closed form.

Table 4: R^2 **Measures for Various Models**

	Static model	DFM	SDFM
MET	0.62513	0.25282	0.29712
ING	0.64071	0.25170	0.30877
GS	0.58649	0.22973	0.24579
LNC	0.62914	0.25735	0.30117
HSBC	0.58584	0.22771	0.28922
JPM	0.71357	0.26583	0.36402
BAC	0.70451	0.26740	0.31741
CS	0.60075	0.24063	0.29478
WFC	0.69037	0.25688	0.27342
BCS	0.62410	0.24433	0.36895

using a static factor model and calculate the R^2s associated with regressions of the returns on the submarket portfolio. We report these R^2 measures in the first column of Table 4. The R^2 measures show that the effect of F_t is overestimated by the static linear model (around 60%) and this effect is drastically reduced when using a DFM (around 25%) or a SDFM (around 30%).

To complete the comparison between the DFM and SDFM models, we report in Appendix B smoothed median F_t obtained via SDFM plotted against the smoothed median F_t obtained via DFM. We also report in Appendix B the smoothed 90% confidence bands for all the hidden states in x_t using the SDFM.

4.5 Asset/Liability Ratio and Default Risk Premia

Then, the estimated model can be used to impute the smoothed values of the state variables $a_{i,t}$ and the smoothed values of the asset/liability ratios $A_{i,t}/L$, that are the implied financial leverages. For each institution i and date t, denote by $\hat{a}_{i,t}$ the median of the smoothed $\{a_{i,t}^{(m)}\}_{m=1}^{M}$ using $M = 100$ paths and $N = 10,000$ particles in the ABC smoothing algorithm (see Appendix A). The estimated asset/liability ratios $\widehat{A_{i,t}/L}$ are then recovered from $\hat{a}_{i,t}$, by using the initial condition $eal_{i,0} = 1$, $\forall i$. We can then compute measures of distance-to-default for the different banks and analyze how they evolved over time. Such measures are the excess asset/liability ratio:

$$EAL_{i,t} = \widehat{A_{i,t}/L} - 1\,, \tag{4.10}$$

and the risk premia of the insurance against default:

$$DD_{i,t} = h(eal_{i,t}; \hat{\omega}) - eal_{i,t}\,, \tag{4.11}$$

where $eal_{i,t} = \log(EAL_{i,t})$ and $\hat{\omega}$ denotes the indirect inference estimator of the intraperiod volatility. Compared to the asset/liability ratios, this measure takes into account the uncertainty on the ratio as evaluated by the market. $DD_{i,t}$ captures the magnitude of the nonlinear impact of the leverage on stock returns.

The change in the risk premia is defined by:

$$\Delta DD_{i,t} = h(eal_{i,t}; \hat{\omega}) - h(eal_{i,t}; \hat{\omega}) - \hat{a}_{i,t}\,. \tag{4.12}$$

The $EAL_{i,t}$ values are reported in Figure 4. For comparison, we also report in Figure 5 the EAL measures obtained by using the DFM of Section 3. Red lines indicate the $EAL_{i,t} = \exp(\sum_{k=1}^{t} Y_{ik})$ obtained with the DFM, while black ones indicate the $EAL_{i,t}$ measures from the SDFM. The evolution of the implied asset/liability ratios are very different according to the financial institutions. We observe for several institutions a regular increase of the ratios after the financial crisis, likely due to the more severe demand of required capital (see e.g. GS, LNC, WFC), but several institutions (HSBC, BCS, ING, JPM) still have rather small implied excess asset/liability

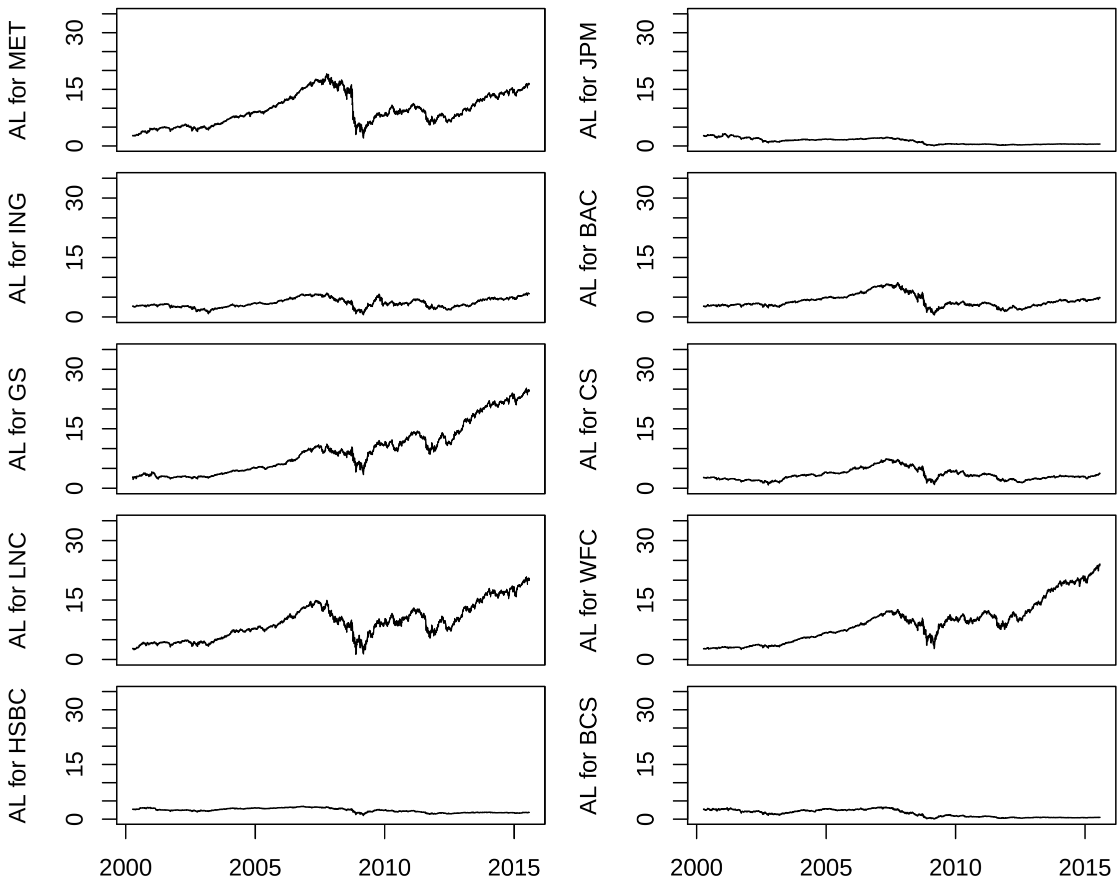

Figure 4: Implied Asset/Liability Ratios.

ratios. The patterns for MET is expected for an insurance company, with on average more protection before the crisis. So the need of liquidity during the financial crisis is partly fulfilled by their reserves, which are reconstructed later on at a rather high level.

The comparison of DFM and SDFM measures of financial leverages show significant differences. This misleading effect can be either positive, or negative, according to the institution.

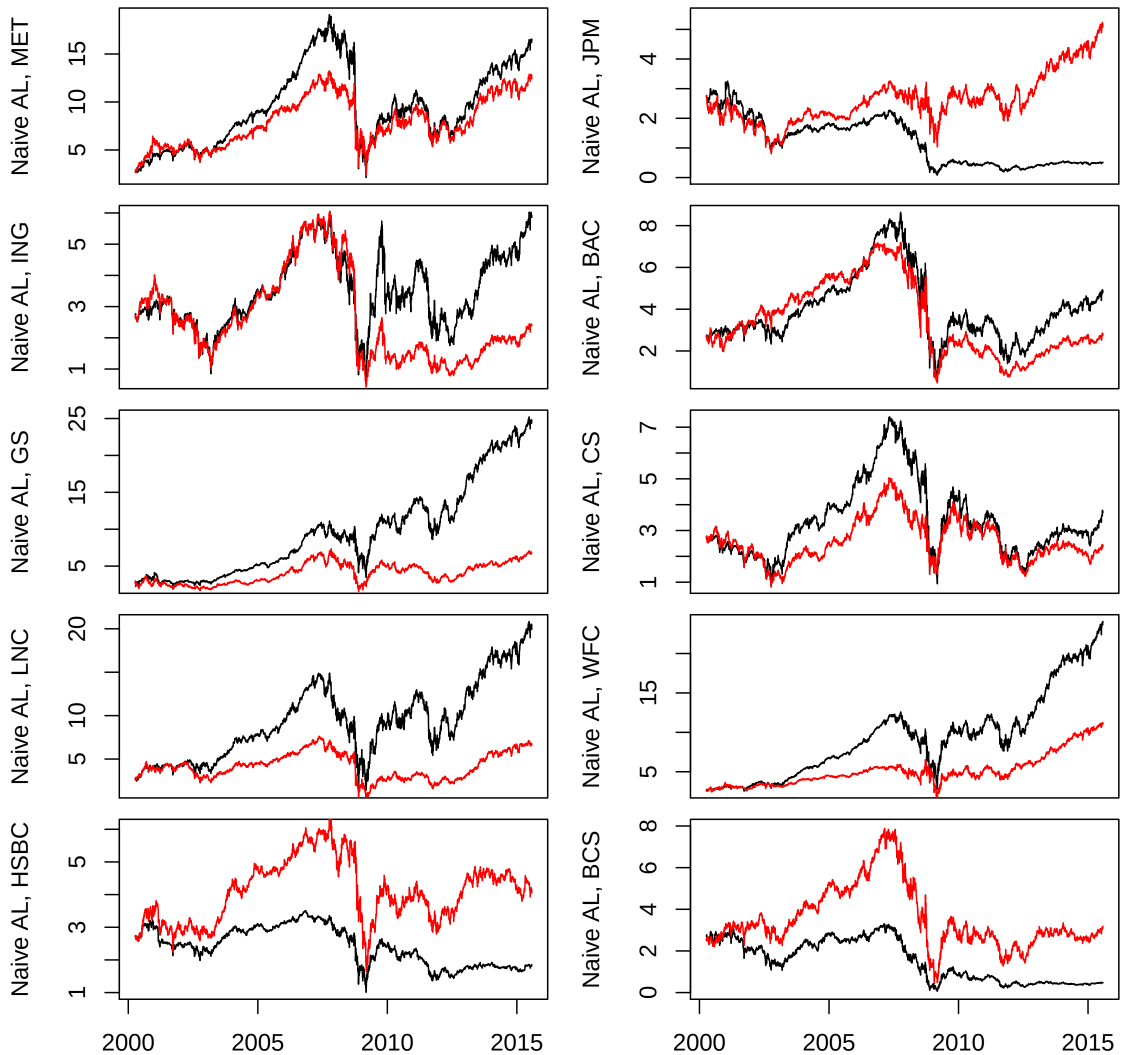

Figure 5: DFM Versus SDFM Asset/Liability Ratios. Red lines indicate the $EAL_{i,t} = \exp(\sum_{k=1}^{t} Y_{ik})$ from the DFM, while black ones the $EAL_{i,t}$ measures obtained by the SDFM.

To understand these differences, more information is provided by the series of distance-to-default DD. The average risk premia of the insurance over the ten firms are reported in the top panel of Figure 6. Note the similarity between the average

DD_t series and the VIX volatility index (bottom panel), which is a measure of implied volatility on the S&P500 option, usually referred as the fear index. We do not have a complete similarity, especially since the level of the average risk premia is significantly higher after the financial crisis. However, these almost similar patterns do not account for the heterogeneity among the financial institutions.

The risk premia of the insurance for each financial institution are reported in Figure 7. These results show that the jump in the average DD in Figure 6 (top panel) was mainly due to two institutions: JP Morgan and Barclays. The jump in DD for JP Morgan is likely due to its 2008 acquisitions of Bear Sterns (for 1.5 billion dollars) and Washington Mutual (for 1.9 billion dollars), both on bankruptcy. Similarly, the jump in Barclays is likely due to its 2008 acquisition of Lehman Brothers (for 1.7 billion dollars) after its bankruptcy. The other bumps on risk premia, smaller though, correspond to the European sovereign debt crisis. As expected, this effect is significant for Barclay's, which has supported a loss of about 2 billions of dollars by speculating on the Greek and Spanish debts by means of derivatives.

Finally, Figure 8 reports the changes in insurance risk premia. We cannot reasonably interpret these results obtained by analyzing stock returns, without having in mind derivatives written on the default of these firms and, in particular, the Credit Default Swaps (CDS). These products are often used for arbitrage and, hence, for speculation. In a neighborhood of the default, we expect to have more volatility on these insurance products and this increase in volatility should appear in the evolution of $(\Delta DD_{i,t})^2$. This behaviour indeed appears in the series of Figure 8. When the implied asset/liability ratio decreases, the variability in $\Delta DD_{i,t}$ increases. This negative correlation between A/L and $(\Delta DD_{i,t})^2$ translates the financial leverage effect discussed by Black (1976) and Hasanhodzic and Lo (2011). As suggested by Black, the stylized negative correlation between return and volatility is likely an indirect

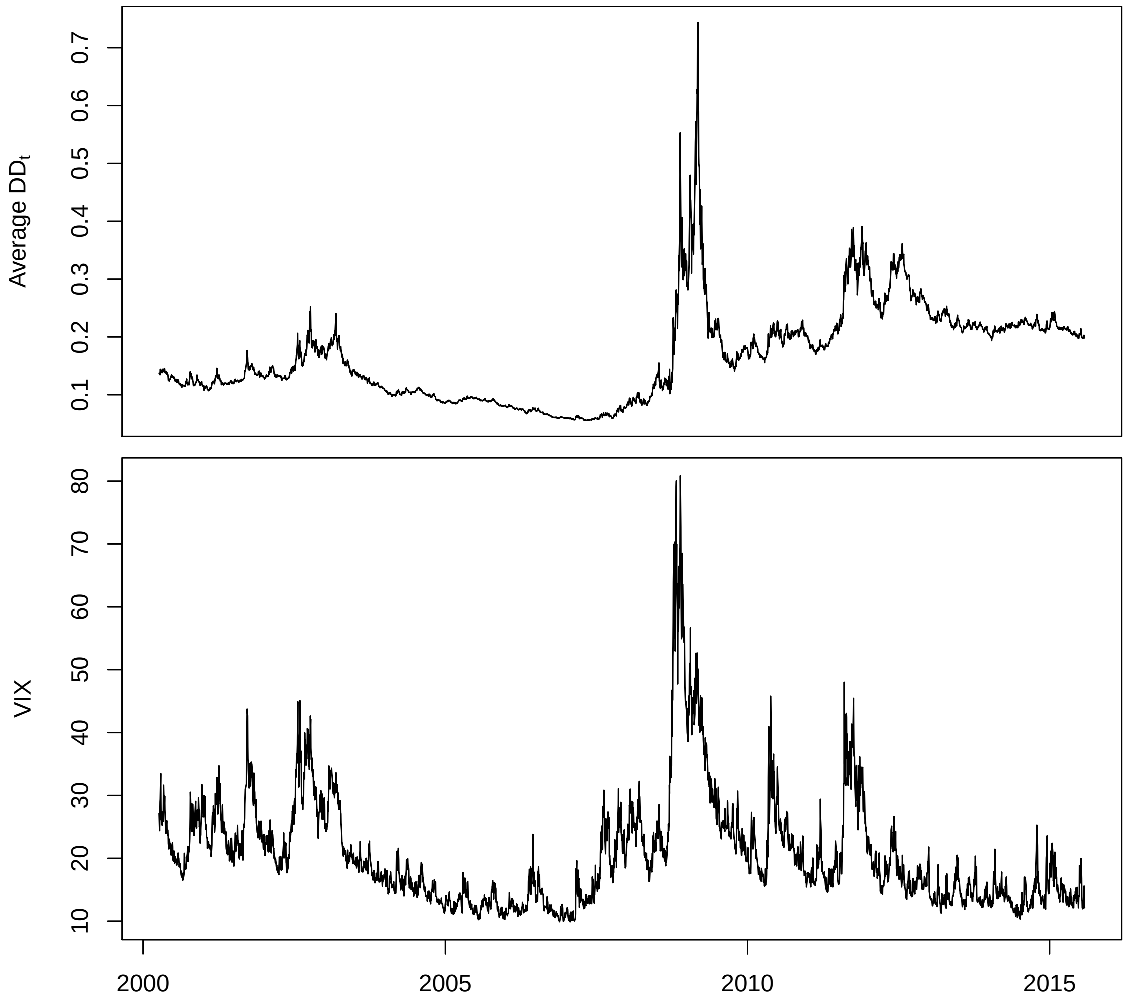

Figure 6: Average Risk Premia of the Insurance and VIX.

effect by means of the balance sheet. This is this effect of A/L, which is noted in our framework.

Table 5: **Rankings**

	Solvency		Cost of Default Ins.		Speculative Assets	
A. Rankings before the crisis (Aug 2006-July 2007)						
MET	10	(15.979)	10	(0.010)	10	($1.75\,10^{-8}$)
ING	4	(5.490)	4	(0.053)	3	($6.73\,10^{-7}$)
GS	7	(9.481)	7	(0.024)	6	($2.05\,10^{-7}$)
LNC	9	(13.428)	9	(0.013)	8	($4.26\,10^{-8}$)
HSBC	3	(3.326)	3	(0.106)	4	($6.10\,10^{-7}$)
JPM	1	(2.099)	1	(0.188)	1	($5.99\,10^{-6}$)
BAC	6	(7.661)	6	(0.032)	7	($1.80\,10^{-7}$)
CS	5	(6.440)	5	(0.043)	5	($3.31\,10^{-7}$)
WFC	8	(11.075)	8	(0.018)	9	($3.84\,10^{-8}$)
BCS	2	(3.051)	2	(0.119)	2	($4.71\,10^{-6}$)
B. Rankings during the crisis (Aug 2008-July 2009)						
MET	9	(7.097)	8	(0.050)	8	($6.36\,10^{-5}$)
ING	4	(2.365)	4	(0.203)	4	($8.85\,10^{-4}$)
GS	10	(7.367)	10	(0.038)	10	($1.89\,10^{-5}$)
LNC	7	(5.471)	7	(0.070)	5	($3.26\,10^{-4}$)
HSBC	3	(1.951)	3	(0.216)	7	($1.65\,10^{-4}$)
JPM	1	(0.459)	1	(0.972)	2	($5.07\,10^{-3}$)
BAC	5	(2.619)	5	(0.198)	3	($9.19\,10^{-4}$)
CS	6	(2.996)	6	(0.150)	6	($3.09\,10^{-4}$)
WFC	8	(7.025)	9	(0.042)	9	($3.07\,10^{-5}$)
BCS	2	(0.606)	2	(0.843)	1	($9.35\,10^{-3}$)
C. Rankings after the crisis (Aug 2010-July 2011)						
MET	7	(9.760)	7	(0.022)	8	($3.14\,10^{-7}$)
ING	6	(3.804)	6	(0.090)	3	($9.39\,10^{-6}$)
GS	10	(12.907)	10	(0.014)	10	($9.71\,10^{-8}$)
LNC	9	(11.169)	9	(0.018)	7	($4.41\,10^{-7}$)
HSBC	3	(2.119)	3	(0.187)	5	($5.40\,10^{-6}$)
JPM	1	(0.449)	1	(0.832)	2	($1.55\,10^{-4}$)
BAC	4	(3.178)	4	(0.114)	4	($8.58\,10^{-6}$)
CS	5	(3.336)	5	(0.106)	6	($4.97\,10^{-6}$)
WFC	8	(10.872)	8	(0.019)	9	($1.85\,10^{-7}$)
BCS	2	(0.688)	2	(0.594)	1	($2.08\,10^{-4}$)

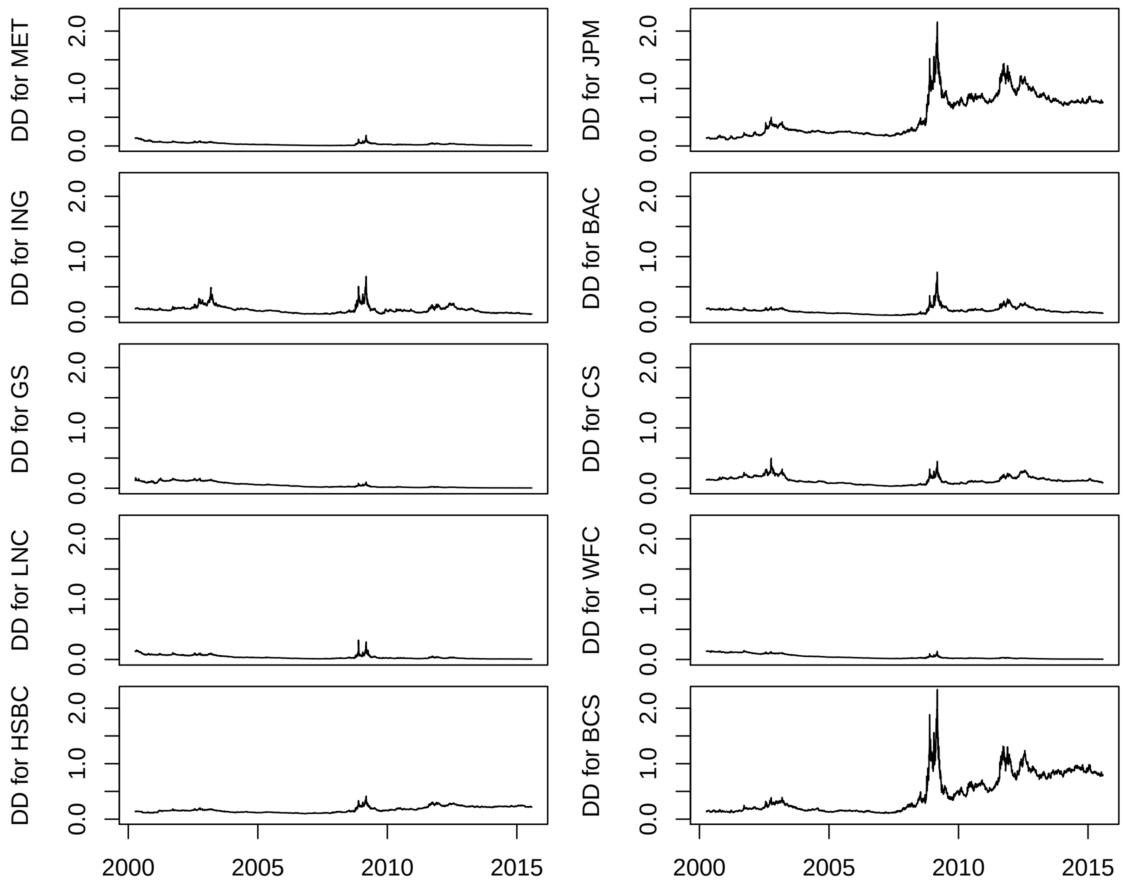

Figure 7: Risk Premia of the Insurance.

4.6 Scoring the Stability of Financial Institutions

The three measures above, that are EAL, DD and $(\Delta DD)^2$, can be used to rank the financial institutions. They define rankings with different interpretations: by the implied financial leverage for EAL, by the distance-to-default as evaluated by the market for DD, and as a measure of the magnitude of speculation for $(\Delta DD)^2$. It is important to get multiple rankings with different interpretations. For instance, this will allow to distinguish an institution with small excess asset/liability ratio

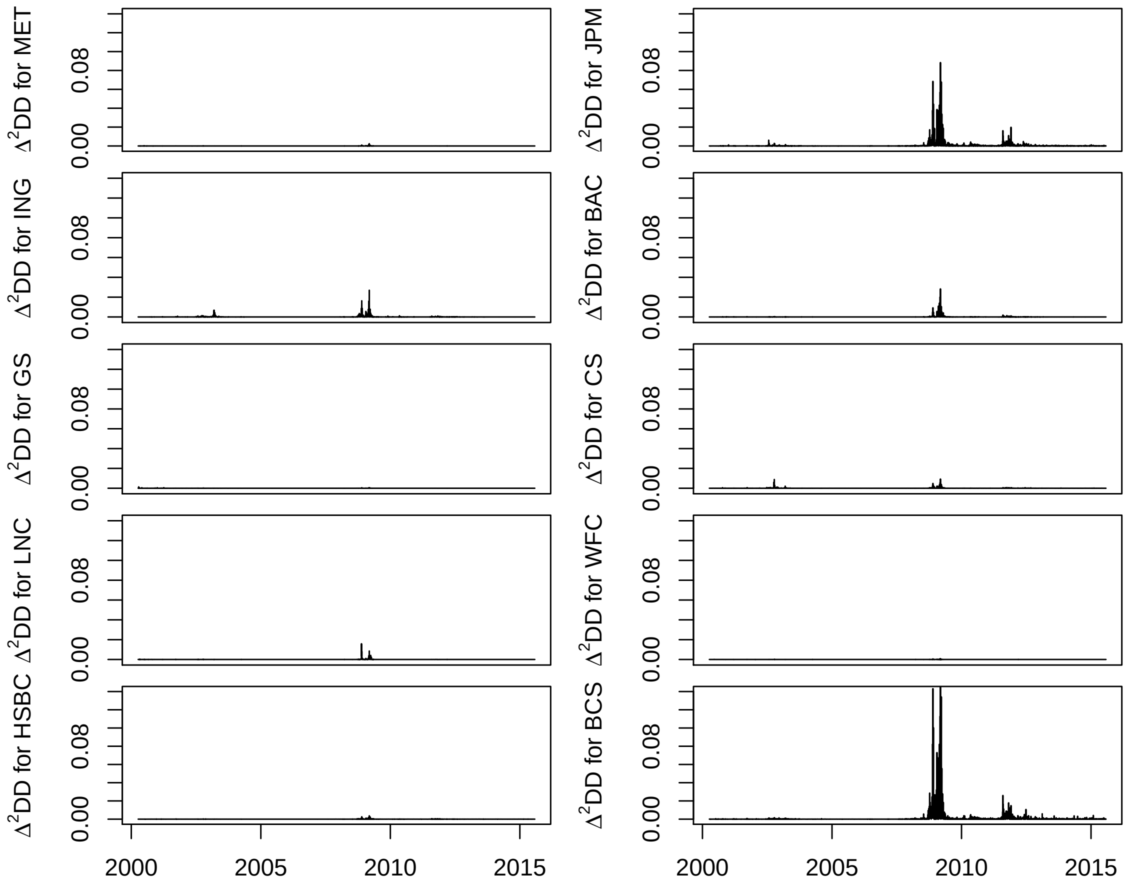

Figure 8: Squared Change in Default Risk Premia.

and small insurance premium, which is not necessarily risky for default, from an institution where both are large and which can be riskier. These rankings are a solvency ranking, a ranking for the default insurance, and a ranking for speculative asset, respectively.

The comparison of the financial institutions can be cross-sectional at a given date, or be performed with respect to both institution and time. We successively consider the two approaches.

Table 5 reports the rankings associated with the three risk measures EAL (first column), DD (second column) and $(\Delta DD)^2$ (third column), respectively, before the crisis (August 2006 - July 2007), during the crisis (August 2008 - July 2009) and after the crisis (August 2010 - July 2011). The grey numbers in parentheses correspond to the average risk measure over the given period. The numbers in black correspond to the ranks based on the numbers in parentheses. The solvency ranks are given by increasing order of EAL measures, the cost of default insurance by decreasing order of DD measures and the speculative assets ranks by decreasing order of $(\Delta DD)^2$. The higher the rank, the more secure and less speculative is the financial institution. As expected, the insurance companies MET and LNC introduced in our set of institutions are uniformly in the top, even if we observe a downgrade of two levels before and after the crisis for MET. Barclays and JP Morgan are the least secure and also the most speculative. Whereas improving its solvency and cost of default insurance ranks, ING is still speculative. These results show that the EAL and DD measures provide almost identical rankings while $(\Delta DD)^2$ provides slightly different rankings.[16]

Let us now provide ratings of these ten financial institutions that will allow for a comparison with respect to both institution and time. Let us collect all the values $\{EAL_{i,t}\}_{t=1,\dots,T}^{i=1,\dots,n}$, $\{DD_{i,t}\}_{t=1,\dots,T}^{i=1,\dots,n}$ and $\{\Delta DD^2{}_{i,t}\}_{t=1,\dots,T}^{i=1,\dots,n}$, and calculate selected quantiles of the associated sample distribution. These quantiles can be used to define a risk segmentation, with different ratings associated with the segments. The rating categories are defined in Table 6 with 10 different ratings. We have noted them as usual by aaa, aa, ..., but their interpretations differ from those by S&P and Moody's for instance, since they are based on different measures. In particular, we denote c$^-$

[16]The correlation between EAL and DD rankings is 0.996, while the correlation between $(\Delta DD)^2$ rankings and the rankings produced by either EAL and DD is 0.90.

Table 6: **Rating Categories**

	EAL	DD	ΔDD^2
aaa	$\cdot \geq q_{99\%}^{EAL}$	$\cdot \leq q_{1\%}^{DD}$	$\cdot \leq q_{1\%}^{\Delta DD^2}$
aa	$q_{99\%}^{EAL} > \cdot \geq q_{97\%}^{EAL}$	$q_{3\%}^{DD} > \cdot \geq q_{1\%}^{DD}$	$q_{3\%}^{\Delta DD^2} > \cdot \geq q_{1\%}^{\Delta DD^2}$
a	$q_{97\%}^{EAL} > \cdot \geq q_{95\%}^{EAL}$	$q_{5\%}^{DD} > \cdot \geq q_{3\%}^{DD}$	$q_{5\%}^{\Delta DD^2} > \cdot \geq q_{3\%}^{\Delta DD^2}$
bbb	$q_{95\%}^{EAL} > \cdot \geq q_{90\%}^{EAL}$	$q_{10\%}^{DD} > \cdot \geq q_{5\%}^{DD}$	$q_{10\%}^{\Delta DD^2} > \cdot \geq q_{5\%}^{\Delta DD^2}$
bb	$q_{90\%}^{EAL} > \cdot \geq q_{80\%}^{EAL}$	$q_{20\%}^{DD} > \cdot \geq q_{10\%}^{DD}$	$q_{20\%}^{\Delta DD^2} > \cdot \geq q_{10\%}^{\Delta DD^2}$
b	$q_{80\%}^{EAL} > \cdot \geq q_{70\%}^{EAL}$	$q_{30\%}^{DD} > \cdot \geq q_{20\%}^{DD}$	$q_{30\%}^{\Delta DD^2} > \cdot \geq q_{20\%}^{\Delta DD^2}$
ccc	$q_{70\%}^{EAL} > \cdot \geq q_{60\%}^{EAL}$	$q_{40\%}^{DD} > \cdot \geq q_{30\%}^{DD}$	$q_{40\%}^{\Delta DD^2} > \cdot \geq q_{30\%}^{\Delta DD^2}$
cc	$q_{60\%}^{EAL} > \cdot \geq q_{50\%}^{EAL}$	$q_{50\%}^{DD} > \cdot \geq q_{40\%}^{DD}$	$q_{50\%}^{\Delta DD^2} > \cdot \geq q_{40\%}^{\Delta DD^2}$
c	$q_{50\%}^{EAL} > \cdot \geq q_{40\%}^{EAL}$	$q_{60\%}^{DD} > \cdot \geq q_{50\%}^{DD}$	$q_{60\%}^{\Delta DD^2} > \cdot \geq q_{50\%}^{\Delta DD^2}$
c^-	$\cdot \leq q_{40\%}^{EAL}$	$q_{60\%}^{DD} \leq \cdot$	$q_{60\%}^{\Delta DD^2} \leq \cdot$

the last rating to avoid the notation d meaning default. This is compatible with the recent regulation on financial stability, the segment c^- corresponding to *watching segment* (or *resolution segment*).

The ratings are derived by using the conversion Table 6. We report the appropriate ratings in Table 7. When we compare to the whole period of observations, we see that even in the period August 2006-August 2007, that is, between the drop in the price of the US real estate and the announcement of frozen funds by Bnp Paribas and AXA, only two institutions were bbb, and the speculative ratings systematically smaller than the two other ratings, which might be an advanced indication of a future bubble crash. The bad ratings during the crisis period are not surprising. Finally, Table 7 shows the slow recovery after the crisis, with still very high risks on HSBC, JPM and BCS.

Table 7: **Ratings**

	Solvency	Cost of Default Ins.	Speculative Assets
	A. Before the crisis (Aug 2006-July 2007)		
MET	bbb	bbb	b
ING	ccc	ccc	c
GS	bb	bb	cc
LNC	bbb	bbb	ccc
HSBC	c	c	c
JPM	c^-	c^-	c^-
BAC	b	b	cc
CS	b	b	cc
WFC	bb	bb	b
BCS	c	c	c^-
	B. During the crisis (Aug 2008-July 2009)		
MET	b	ccc	c^-
ING	c^-	c^-	c^-
GS	b	b	c^-
LNC	ccc	ccc	c^-
HSBC	c^-	c^-	c^-
JPM	c^-	c^-	c^-
BAC	c^-	c^-	c^-
CS	c	c^-	c^-
WFC	b	b	c^-
BCS	c^-	c^-	c^-
	C. After the crisis (Aug 2010-July 2011)		
MET	bb	bb	cc
ING	cc	cc	c^-
GS	bbb	bbb	ccc
LNC	bb	bb	cc
HSBC	c^-	c^-	c^-
JPM	c^-	c^-	c^-
BAC	c	c	c^-
CS	c	c	c^-
WFC	bb	bb	cc
BCS	c^-	c^-	c^-

4.7 Systemic Risk Stress Tests and Capital Requirements

The level of required capitals are usually based on measures of risk such as Value-at-Risk (VaR), or expected shortfalls. In the recent regulation, it is necessary to distinguish the components due to idiosyncratic and systemic risks. They are usually computed following stresses on latent factors (see European Banking Authority (2016) for the governance of a stress test exercise). Let us focus on VaR computation for expository purpose. The VaR is conditional, depends on the conditioning information set, and possibly on the stresses introduced on some of the conditioning variables. The computation of the VaR in the SDFM is made easy since the function h is strictly increasing.

Let us consider the expression of the return on short horizons. We have:

$$Y_{i,t}^s(F_t^s, \varepsilon_{i,t}^s) = h(eal_{i,t-1} + \hat{\alpha}_i + \hat{\beta}_i F_t^s + \hat{\sigma}_{i,t}\varepsilon_{i,t}^s; \hat{\omega}) - h(eal_{i,t-1}; \hat{\omega})\,,$$

where the superscript s indicates stressed variables. These stresses concern only the future values, the parameters and previous state variables being fixed at their median filtered values. We will apply the following basic stress corresponding to different VaR interpretations.

The *standard supervisory global VaR* at 5% is:

$$\mathrm{VaR}_{i,t}^g = h(eal_{i,t-1} + \hat{\alpha}_i + \hat{\beta}_i\hat{\gamma}\hat{F}_{t-1} - 2\sqrt{\hat{\beta}_i^2\hat{\eta}_t^2 + \hat{\sigma}_{i,t}^2}; \hat{\omega}) - h(eal_{i,t-1}; \hat{\omega})\,.$$

In the regulation for Financial Stability, the computation of VaRs is done with adverse stresses. We define below the stresses on F_t and $\varepsilon_{i,t}$ by means of appropriate quantiles deduced from estimated SDFM. Let us focus on a computation with a short view of F_t value (Point In Time, PIT). In this view, we can compute the stressed effect of idiosyncratic risk as:

$$\mathrm{VaR}^{id}_{i,t} = h(eal_{i,t-1} + \hat{\alpha}_i + \hat{\beta}_i \hat{\gamma} \hat{F}_{t-1} - 2\hat{\sigma}_{i,t}; \hat{\omega}) - h(eal_{i,t-1}; \hat{\omega}) \,,$$

and the specific effect of the systemic factor by stressing additionally F_t. This second stressed VaR is:

$$\mathrm{VaR}^{syst}_{i,t} = h(eal_{i,t-1}+\hat{\alpha}_i+\hat{\beta}_i(\hat{\gamma}\hat{F}_{t-1}-2\hat{\eta}_t)-2\hat{\sigma}_{i,t}; \hat{\omega})-h(eal_{i,t-1}+\hat{\alpha}_i+\hat{\beta}_i\hat{\gamma}\hat{F}_{t-1}-2\hat{\sigma}_{i,t}; \hat{\omega}) \,,$$

Intuitively, $\mathrm{VaR}^g_{i,t}$ is with respect to the information set:

$$(F_{t-1}, \eta_t, \sigma_{i,t}, Y_{i,t-1}, a_{i,t-1}) \,,$$

whereas the other stressed VaR measures include also in the information set the values F_t and $\epsilon_{i,t}$ which are possibly stressed at the 5% quantile. Note that the coefficients $\hat{\beta}_i$ are usually positive, therefore:

$$\sqrt{\hat{\beta}_i^2 \hat{\eta}_t^2 + \hat{\sigma}^2_{i,t}} < \hat{\beta}_i \hat{\eta}_t + \hat{\sigma}_{i,t} \,,$$

and $\mathrm{VaR}^g_{i,t} > \mathrm{VaR}^{id}_{i,t} + \mathrm{VaR}^{syst}_{i,t}$. Thus the associated aggregate required capital functions of the opposite of the VaR is strictly higher with the stressed approach than with the standard one. Following this practice we can compute for every institution and time, the values $\mathrm{VaR}^{id}_{i,t}$ and $\mathrm{VaR}^{sys}_{i,t}$.

A similar exercise can be performed by considering long term values of the function instead of the spot values. This exercise, often called Through The Cycle (TTC) is generally applied with the unconditional mean of F_t (i.e. zero), instead of $\hat{\gamma}\hat{F}_{t-1}$ and the unconditional variance of $\hat{F}_t$ (i.e. one), instead of $\hat{\eta}_t$. We provide in Figure 9

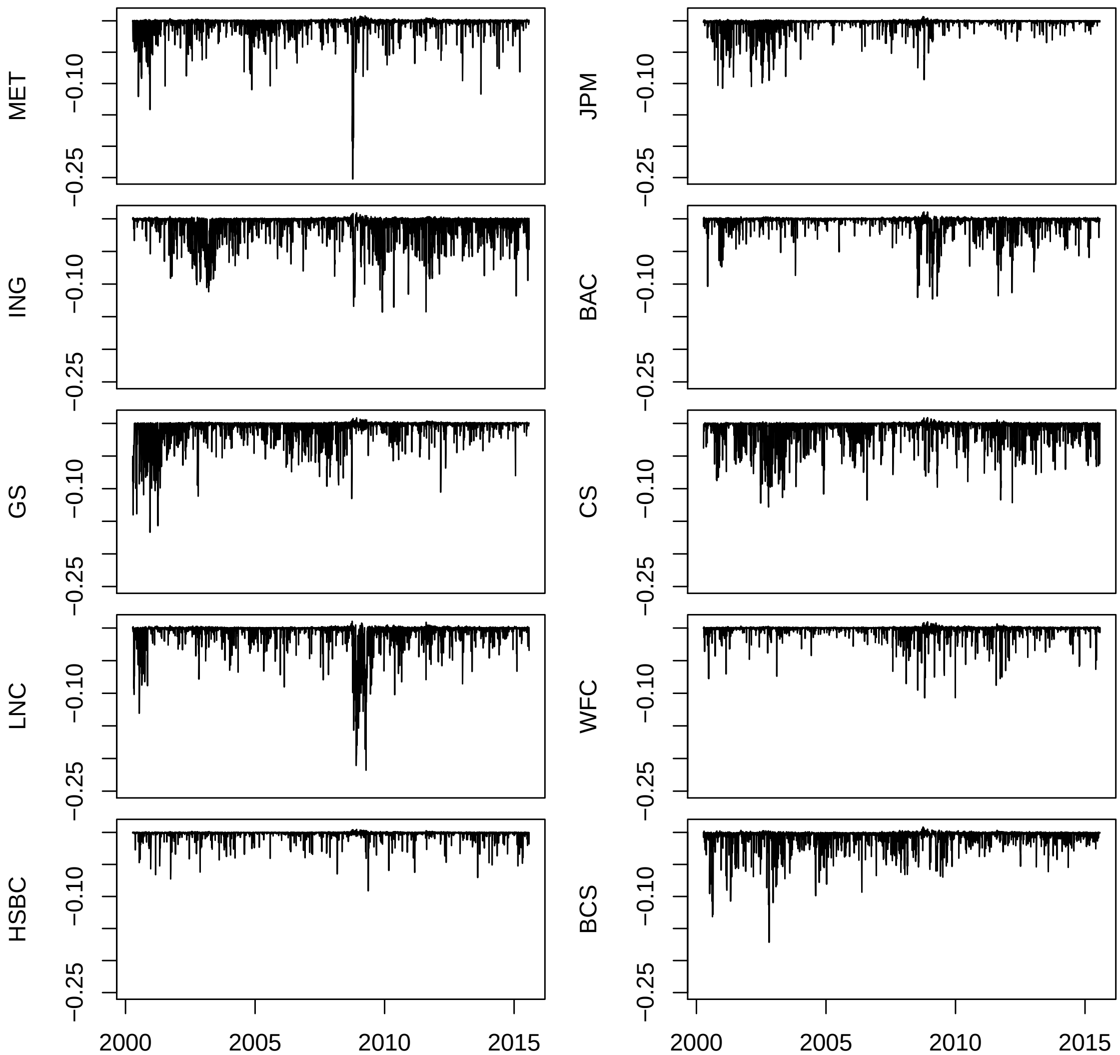

Figure 9: Idiosyncratic 5% VaR.

idiosyncratic and in Figure 10 the systemic 5% PIT VaRs.

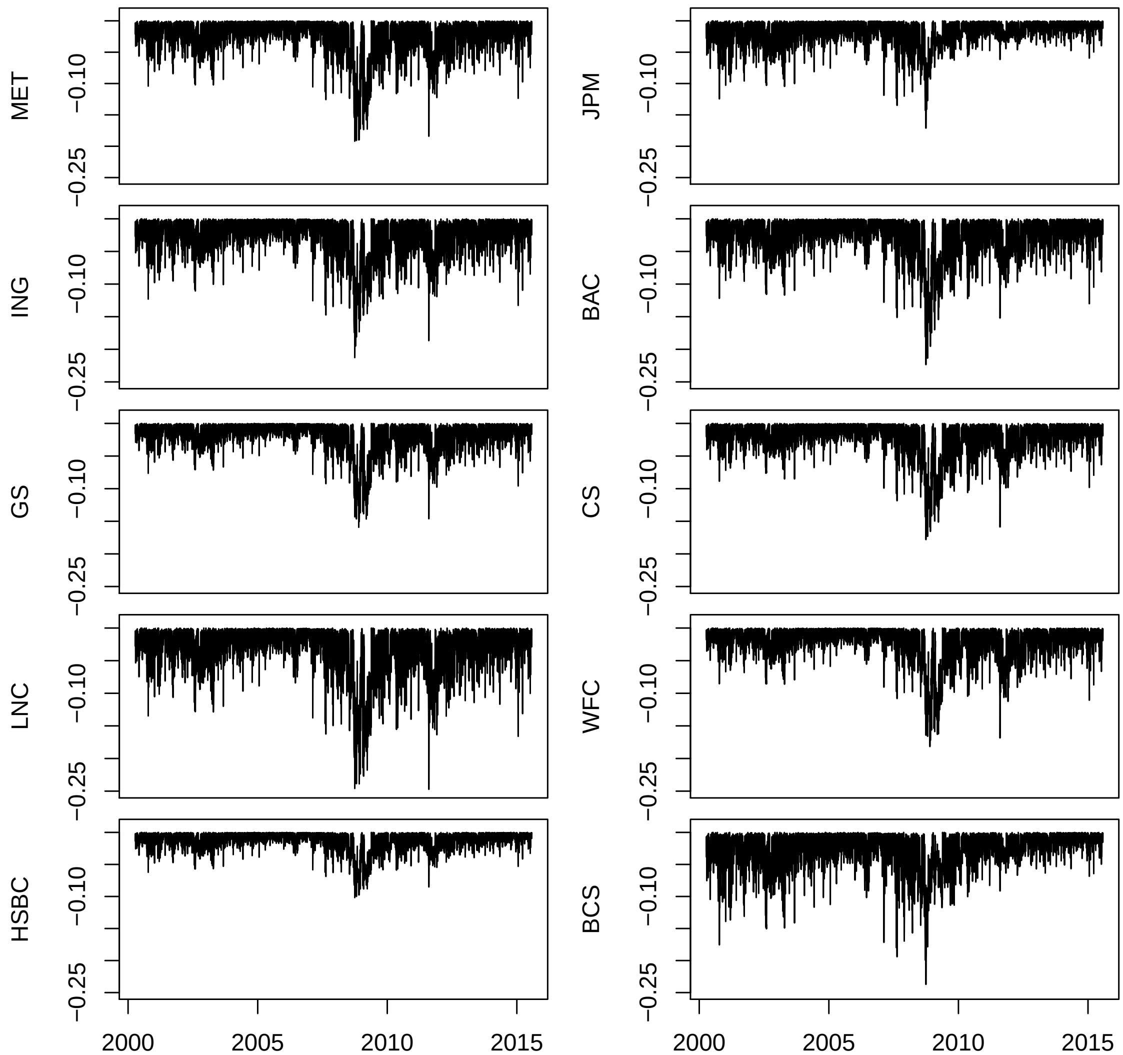

Figure 10: Systemic 5% VaR.

5 Conclusion

We have introduced a structural dynamic factor model (SDFM), which accounts for linear common factor, stochastic volatilities, and also to their nonlinear impacts on equity returns, when the distance-to-default is small. This leads to a state space model, in which the state dynamics is stationary and the measurement equation possibly creates nonstationarity. Due to the structural Value-of-the-Firm model, defining the measurement equation, some latent variables such as the asset/liability ratio have an economic interpretation in terms of balance sheets. The SDFM allows to deduce from the observation of equity returns only approximations of these financial ratios, the so-called implied asset/liability ratios. The model also allows for defining different risk measures and associated rankings: a solvency ranking, a cost of default insurance ranking and a speculative assets ranking. It also provides a useful tool to disentangle the stressed required capitals for systemic and idiosyncratic risks by means of the associated stressed VaR.

Clearly, such an approach is only based on market data. It would be nice to use jointly the information on returns with the information on the balance sheets. As mentioned in the main text, this is difficult challenge. First, the observation frequencies are not the same; second, it is difficult to deduce from the balance sheet the financial ratio involved in Merton's model, that are the part of debt to be recovered immediately and the part of asset sufficiently liquid. Finally, several lines of the balance sheet of a bank contain financial assets, such as stock, bonds, or derivatives, which are often marked-to-market values. Thus, they are also partly market data.

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A ABC Filtering and Smoothing

Since neither the observation density, nor the transition kernel are available in closed form in the SDFM, we use ABC filtering and smoothing to impute the hidden states in a SDFM.

In this appendix we describe the principle of ABC filtering (Calvet and Czellar, 2011, 2015a; Jasra et al., 2012)[17] and we propose a new ABC smoothing method, using an ABC filtering in a first step.

A.1 ABC Filtering

The idea of ABC filtering is to replace in a particle filtering algorithm the unavailable conditional distribution $f(Y_t|x_t)$ by the distance of simulated observations $\{\tilde{Y}_t^{(i)}\}_{i=1}^N$ to the empirical observation Y_t. The distance is measured by a strictly positive kernel $K : \mathbb{R}^{\dim y} \to \mathbb{R}$ integrating to unity, where $\dim y$ denotes the dimension of y. The algorithm is described below.

The empirical distribution of Step 2 particles $\{\tilde{x}_t^{(i)}\}_{i=1}^N$ finitely estimates the distribution $f(x_t|Y_{1:t-1})$ and the empirical distribution of $\{\tilde{Y}_t^{(i)}\}_{i=1}^N$ estimates the distribution $f(Y_t|Y_{1:t-1})$, which can be used for forecasting purposes. In Step 3, we use the quasi-Cauchy kernel K and plug-in bandwidth h_t (Calvet and Czellar, 2015a):

$$K(u) = (1 + c\, u^\top u)^{-(\dim u+3)/2} \text{ and } h_t^*(N) = \left[\frac{c^2 B(K) \dim u}{N\, \tilde{P}_t}\right]^{1/(\dim u+4)}, \qquad \text{(A.1)}$$

where $\dim u$ is the dimension of u, $c = \pi^{1+1/\dim u}/\{2\Gamma[(\dim u+3)/2]\}^{2/\dim u}$, $B(K) =$

[17] Calvet and Czellar (2011) called the ABC filtering State-Observation Sampling (SOS).

$2\,\Gamma(\dim u/2+3)\,\Gamma[(\dim u+3)/2]/[\sqrt{\pi}\Gamma(\dim u+3)]$, $\Gamma(\cdot)$ is the gamma function, and

$$\tilde{P}_t = \frac{2\,\mathrm{tr}(\tilde{\Sigma}_t^{-2}) + [\mathrm{tr}(\tilde{\Sigma}_t^{-1})]^2}{2^{\dim u+2}\,\pi^{\dim u/2}\,[\det(\tilde{\Sigma}_t)]^{1/2}}\,, \tag{A.2}$$

where $\tilde{\Sigma}_t$ is the sample covariance matrix calculated with the simulated $\{\tilde{Y}_t^{(i)}\}_{i=1,\dots,N}$ in Step 2 of the ABC filter.

ABC Filtering Algorithm

Step 1 (Initialization): At date $t=0$, simulate $x_0^{(i)}$, $i=1,\dots,N$, from the initial density $f(x_0)$.

For $t=1,\dots,T$, iterate Steps 2-4.

Step 2 (Sampling): Simulate a **state-observation pair** $(\tilde{x}_t^{(i)}, \tilde{Y}_t^{(i)})$ from $f(x_t, Y_t|x_{t-1}^{(i)}, Y_{1:t-1})$, $i=1,\dots,N$.

Step 3 (Importance weights): Observe Y_t and compute the weights

$$\omega_t^{(i)} = \frac{1}{h_t^{\dim y}} K\left(\frac{\tilde{Y}_t^{(i)} - Y_t}{h_t}\right),\ i=1,\dots,N.$$

Step 4 (Multinomial resampling): Draw $x_t^{(1)},\dots,x_t^{(N)}$ from $\tilde{x}_t^{(1)},\dots,\tilde{x}_t^{(N)}$ with probabilities $\frac{\omega_t^{(1)}}{\sum_i \omega_t^{(i)}},\dots,\frac{\omega_t^{(N)}}{\sum_i \omega_t^{(i)}}$.

Moreover, the empirical distribution of Step 4 particles $\{x_t^{(i)}\}_{i=1}^N$ finitely estimates the distribution $f(x_t|Y_{1:t})$.

To impute the distribution of $x_{1:T}$ using observations $Y_{1:T}$, we use an ABC particle smoothing method, which we now explain.

A.2 ABC Smoothing

If the transition density $f(x_t|x_{t-1})$ were available in closed form, we could use a standard particle smoothing algorithm (Godsill et al., 2004) as given below.

Godsill et al. (2004)'s algorithm is based on Bayes' rule and on the assumption that $f(x_{t+1}|x_t)$ is available in closed-form:

$$f(x_t|x_{t+1}, Y_{1:T}) = \frac{f(x_t|Y_{1:T})f(x_{t+1}|x_t)}{f(x_{t+1}|Y_{1:t})}\,. \tag{A.3}$$

Godsill et al. (2004)'s Smoothing Algorithm

Step 1 (Particle filtering): Use a particle filter to obtain an approximate particle representation of $f(x_t|Y_{1:t})$ at each date $t = 1, \ldots, T$. Denote these particles by $\{x_t^{(i)}\}_{t=1,\ldots,T}^{i=1,\ldots,N}$.

For $m = 1, \ldots, M$, replicate Steps 2-4.

Step 2 (Positioning of the backward simulation): Choose $\tilde{x}_T^{(m)} = x_T^{(i)}$ with probability $1/N$.

Step 3 (Backward simulation): For $t = T-1, \ldots, 1$ and each $i = 1, \ldots, N$

(i) compute the importance weights

$$\omega_{t|t+1}^{(i,m)} = f(\tilde{x}_{t+1}^{(m)}|x_t^{(i)}), \quad i = 1, \ldots, N\,;$$

(ii) choose $\tilde{x}_t^{(m)} = x_t^{(i)}$ with probability $\omega_{t|t+1}^{(i,m)}$.

Step 4 (Path drawing): $\tilde{x}_{1:T}^{(m)} = (\tilde{x}_1^{(m)}, \ldots, \tilde{x}_T^{(m)})$ is an approximate realization from $f(x_{1:T}|Y_{1:T})$.

Formula (A.3) suggests that at each date t, we need a filter $\{x_t^{(i)}\}_{i=1,\ldots,N}$ estimating $f(x_t|Y_{1:T})$. This is provided in Step 1 of the smoothing algorithm. The numerator in formula (A.3) then suggests that, given a date $t+1$ state x_{t+1} and observations $Y_{1:T}$, we can impute the hidden states x_t in a backward manner by reweighting the particles by $f(x_{t+1}|x_t^{(i)})$. This backward reweighting procedure is summarized in Steps 2-4 in Godsill's algorithm.

However, in our model, $f(x_{t+1}|x_t)$ is not available in closed form, and a standard smoothing method is inapplicable. We therefore construct a variant of Godsill et al. (2004)'s smoothing algorithm which is applicable in models in which the transition kernel is intractable, but can be easily simulated from.

We consider the joint distribution of $(\hat{x}_{t+1}, x_t)$ where $\hat{x}_{t+1}$ is a pseudo-particle generated from $f(x_{t+1}|x_t)$:

$$f(\hat{x}_{t+1}, x_t|x_{t+1}, Y_{1:t}) = \frac{\delta(\hat{x}_{t+1} - x_{t+1}) f(\hat{x}_{t+1}, x_t|Y_{1:t})}{f(x_{t+1}|Y_{1:t})}, \tag{A.4}$$

where δ denotes de Dirac distribution on $\mathbb{R}^{\dim x}$.

This suggests a variant of Godsill et al. (2004)'s algorithm in which the weights $\omega_{t|t+1}^{(i)}$ should be replaced by $\delta(\hat{x}_{t+1}^{(i)} - x_{t+1})$ and pseudo-particles $\hat{x}_{t+1}^{(i)}$ are generated from $f(x_{t+1}|x_t^{(i)})$. However, the Dirac function gives degenerate weights. Therefore, we replace it by the positive kernel defined in (A.1)[18]. The new smoothing algorithm, called Approximate Bayesian Computation (ABC) smoothing, is as follows.

[18]The quasi-Cauchy kernel with plug-in bandwidth satisfies an optimality property given in Calvet and Czellar (2015a). The optimality property is likely not to be satisfied in the smoothing context.

ABC Smoothing Algorithm

Step 1 (Particle filtering): Use a particle filter to obtain an approximate particle representation of $f(x_t|Y_{1:t})$ at each date $t = 1, \ldots, T$. Denote these particles by $\{x_t^{(i)}\}_{t=1,\ldots,T}^{i=1,\ldots,N}$.

For $m = 1, \ldots, M$, replicate Steps 2-4.

Step 2 (Positioning of the backward simulation): Choose $\tilde{x}_T^{(m)} = x_T^{(i)}$ with probability $1/N$.

Step 3 (Backward simulation): For $t = T-1, \ldots, 1$ and each $i = 1, \ldots, N$

(i) generate a pseudo-particle $\hat{x}_{t+1}^{(i,m)}$ from $f(x_{t+1}|x_t^{(i)})$;

(ii) compute the importance weights:

$$\omega_{t|t+1}^{(i,m)} = \frac{K_{h_t}\big(\tilde{x}_{t+1}^{(m)} - \hat{x}_{t+1}^{(i,m)}\big)}{\sum_{i'=1}^{N} K_{h_t}\big(\tilde{x}_{t+1}^{(m)} - \hat{x}_{t+1}^{(i',m)}\big)}, \quad i \in 1, \ldots, N\,;$$

(iii) choose $\tilde{x}_t^{(m)} = x_t^{(i)}$ with probability $\omega_{t|t+1}^{(i,m)}$.

Step 4 (Path drawing): $\tilde{x}_{1:T}^{(m)} = (\tilde{x}_1^{(m)}, \ldots, \tilde{x}_T^{(m)})$ is an approximate realization from $f(x_{1:T}|Y_{1:T})$.

In Step 1 of the ABC smoothing algorithm, we apply an ABC filter to generate particles $\{x_t^{(i)}\}_{t=1,\ldots,T}^{i=1,\ldots,N}$. In Step 3, we use the kernel and bandwidth defined in (A.1), where we naturally replace $\tilde{\Sigma}_t$ by the sample covariance matrix calculated with the simulated $\{\hat{x}_t^{(i)}\}_{i=1,\ldots,N}$ in Step 3 of the ABC smoothing algorithm.

In all our applications, we use $N = 10,000$ particles in both ABC filtering and smoothing algorithms and generate $M = 100$ paths $(\tilde{F}_t^{(m)}, \{\tilde{\sigma}_{i,t}\}_{i=1}^{n}, \tilde{\eta}_t, t = 1, \ldots, T)$, $m = 1, \ldots, M$ in the ABC smoothing algorithm.

B Smoothed Hidden States for the SDFM

We provide in this section the smoothed hidden states for the SDFM.

In Figures 11-14, we report the smoothed 90% confidence bands for F_t, whose bounds are the 5th and 95th percentiles of the sample distribution of the smoothed $\{\tilde{F}_t^{(m)}\}_{m=1}^{M}$ using the SDFM model. Within each confidence band, we plot with a continuous line the median smoothed F_t. We plot in Figure 12 the smoothed median F_t obtained via SDFM against the smoothed median F_t obtained via DFM.

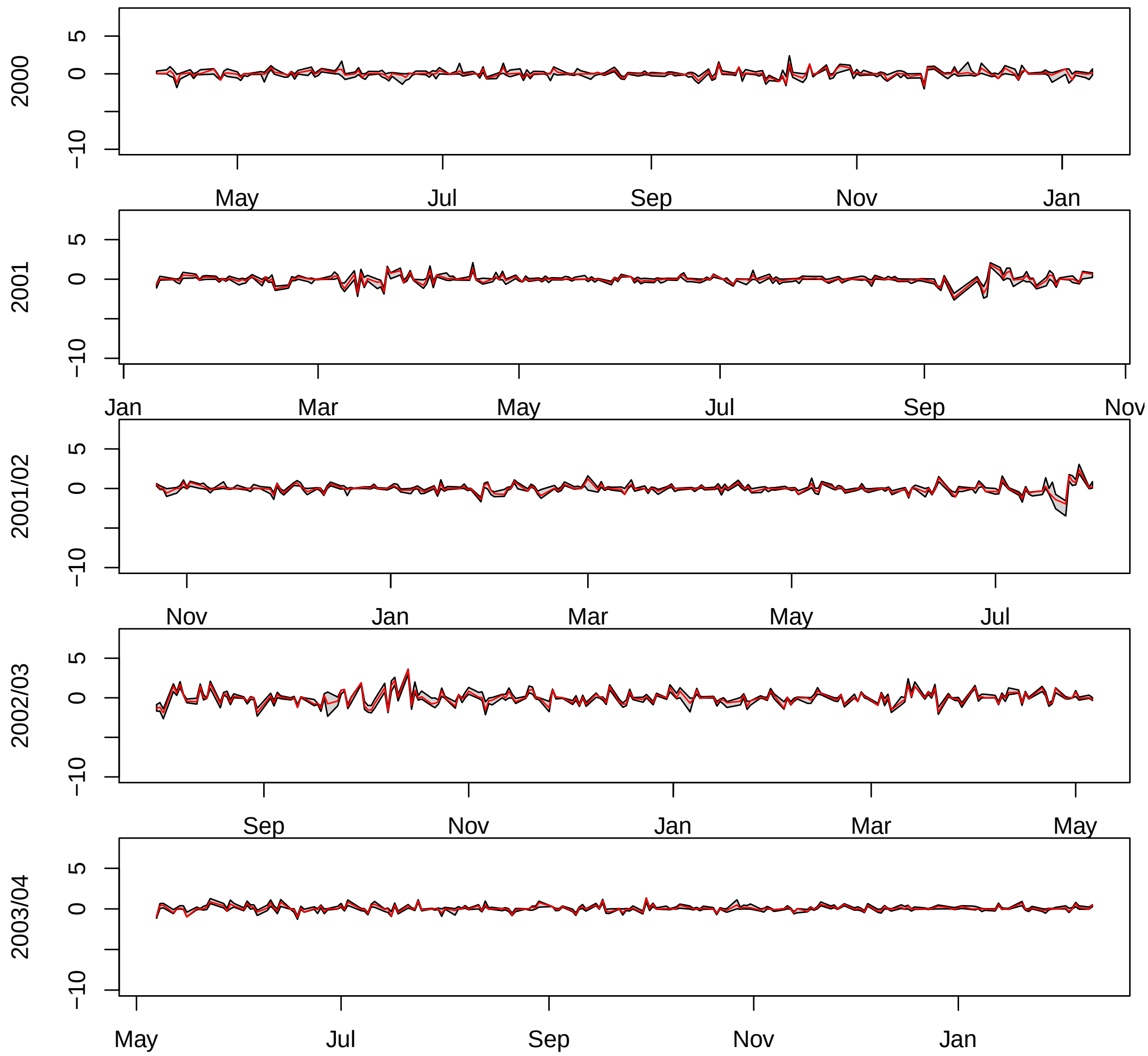

Figure 11: Smoothed F_t for the Period Between April 6, 2000 and February 10, 2004. The 90% confidence bands for F_t are delimited by the 5th and 95th percentiles of the sample distribution of the smoothed $\{\tilde{F}_t^{(m)}\}_{m=1}^{M}$ using the SDFM model. Within each confidence band we plot with a continuous line the median smoothed F_t.

Let us now investigate the accuracy of the forecasted VaR for F_t using DFM on the in-sample period April 6, 2000 and July 31, 2015. Using an ABC particle filter with

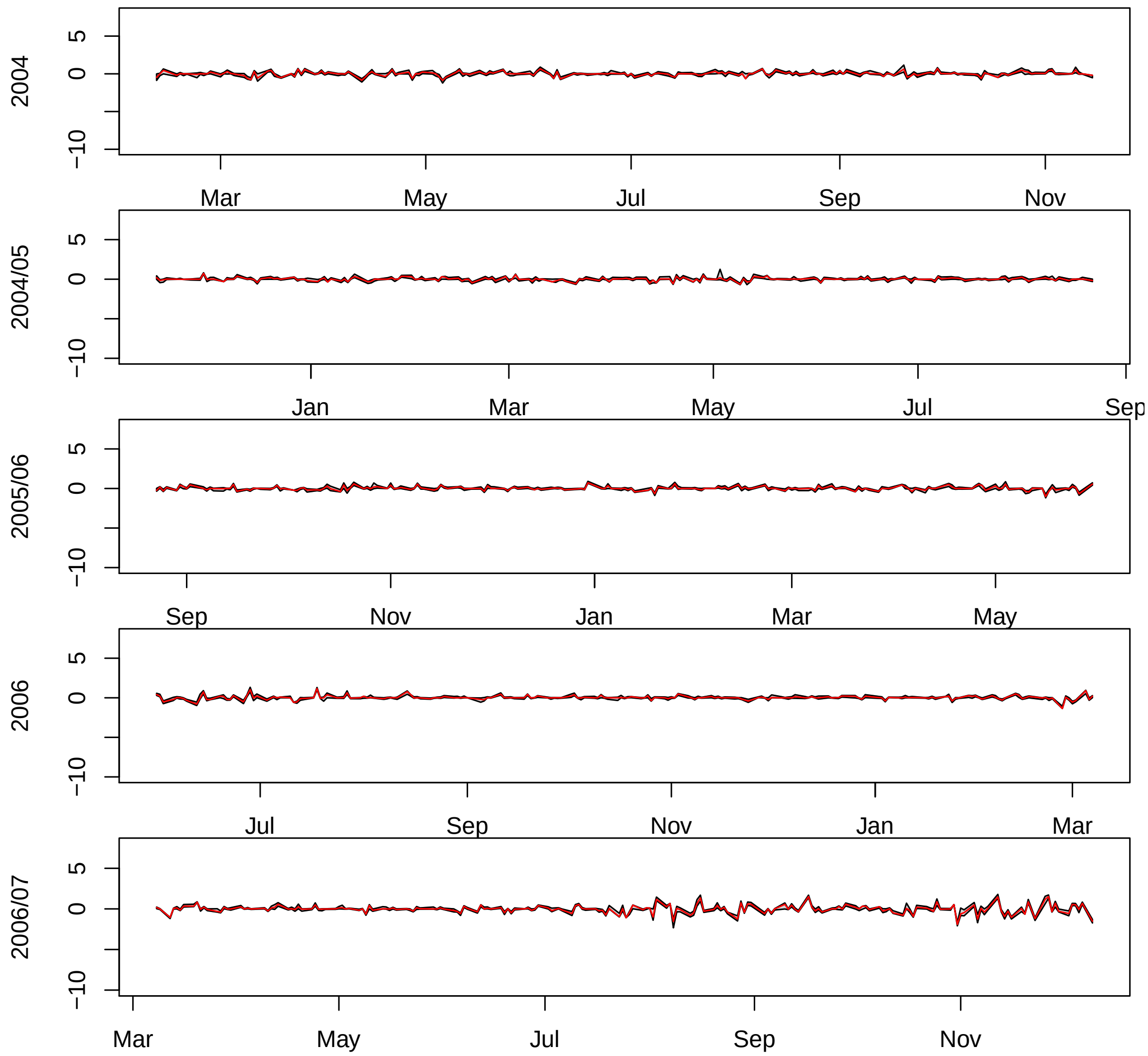

Figure 12: Smoothed F_t for the Period Between February 11, 2004 and December 10, 2007. The 90% confidence bands for F_t are delimited by the 5th and 95th percentiles of the sample distribution of the smoothed $\{\tilde{F}_t^{(m)}\}_{m=1}^{M}$ using the SDFM model. Within each confidence band we plot with a continuous line the median smoothed F_t.

$N = 10,000$ we first forecast the 0.05th and 0.95th quantiles of $\sum_{h=1}^{d} F_{t+h}|Y_{1:t}$ for $d = 1, 5, 10, 20, 40, 60$ (1-day, 5-day, 10-day, 20-day, 40-day and 60-day horizons)

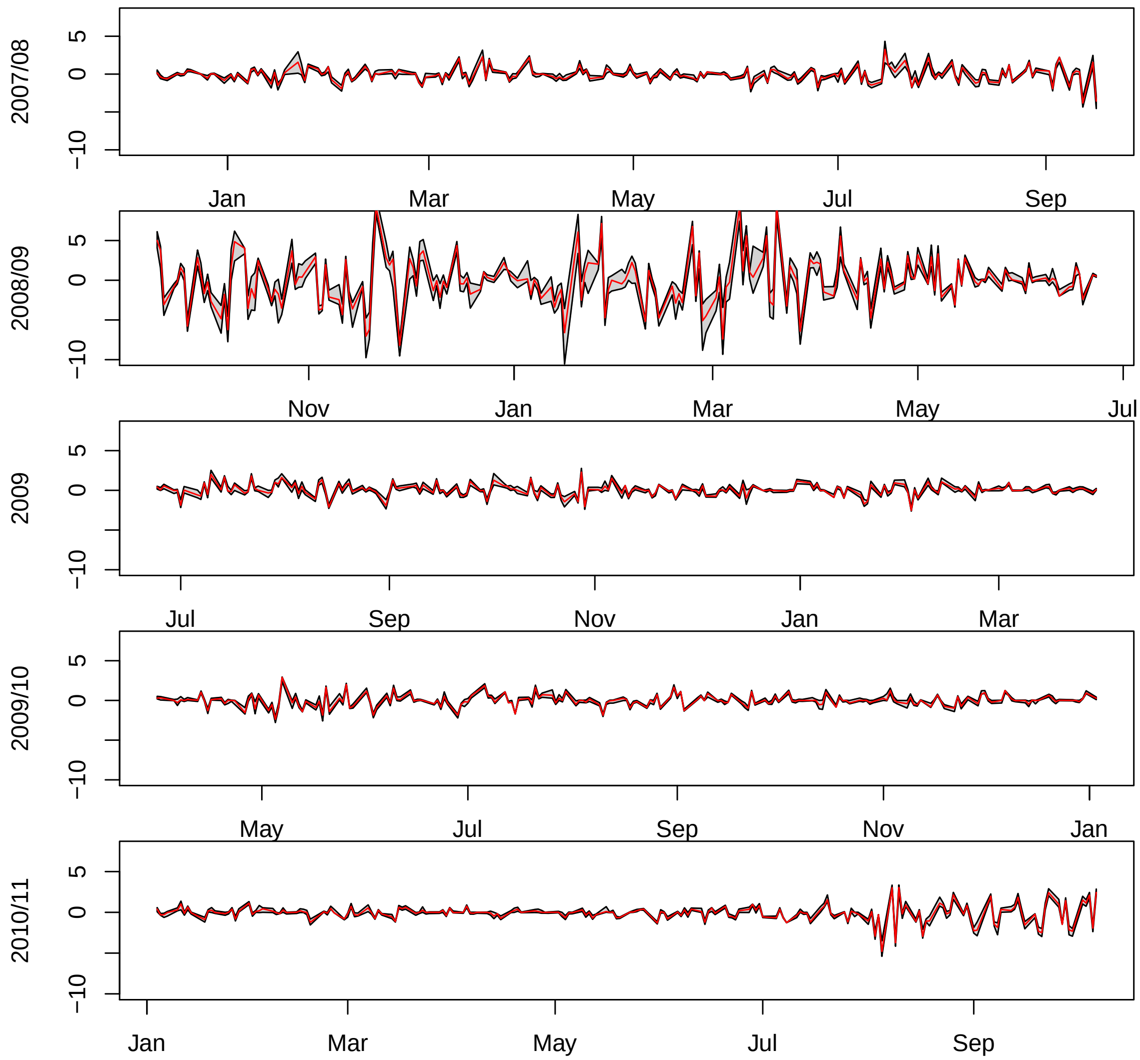

Figure 13: Smoothed F_t for the Period Between December 11, 2007 and October 7, 2011. The 90% confidence bands for F_t are delimited by the 5th and 95th percentiles of the sample distribution of the smoothed $\{\tilde{F}_t^{(m)}\}_{m=1}^{M}$ using the SDFM model. Within each confidence band we plot with a continuous line the median smoothed F_t.

that are the VaRs for the systematic risk component. Figure 16 reports the VaR forecasts of cumulative F_t along with the pseudo-true F_t values corresponding to the

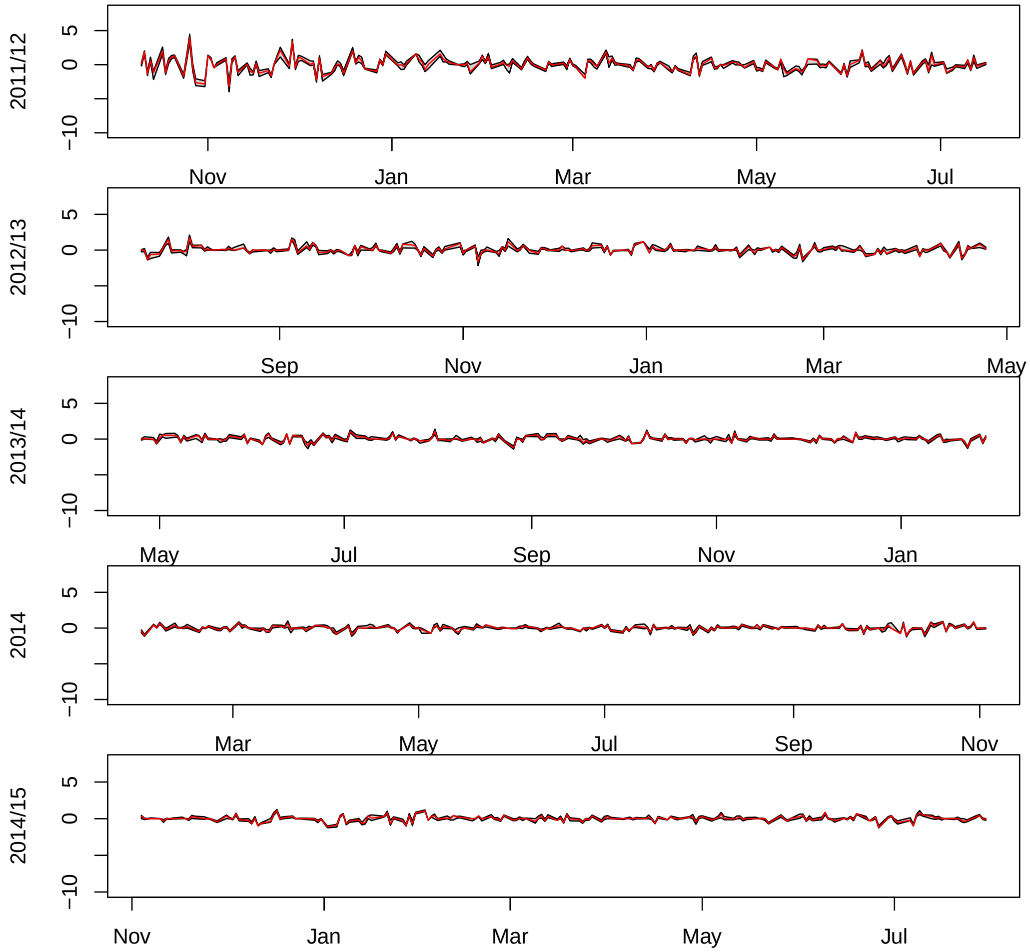

Figure 14: Smoothed F_t for the Period Between October 10, 2011 and July 30, 2015. The 90% confidence bands for F_t are delimited by the 5th and 95th percentiles of the sample distribution of the smoothed $\{\tilde{F}_t^{(m)}\}_{m=1}^{M}$ using the SDFM model. Within each confidence band we plot with a continuous line the median smoothed F_t.

median smoothed F_t obtained via the SDFM.

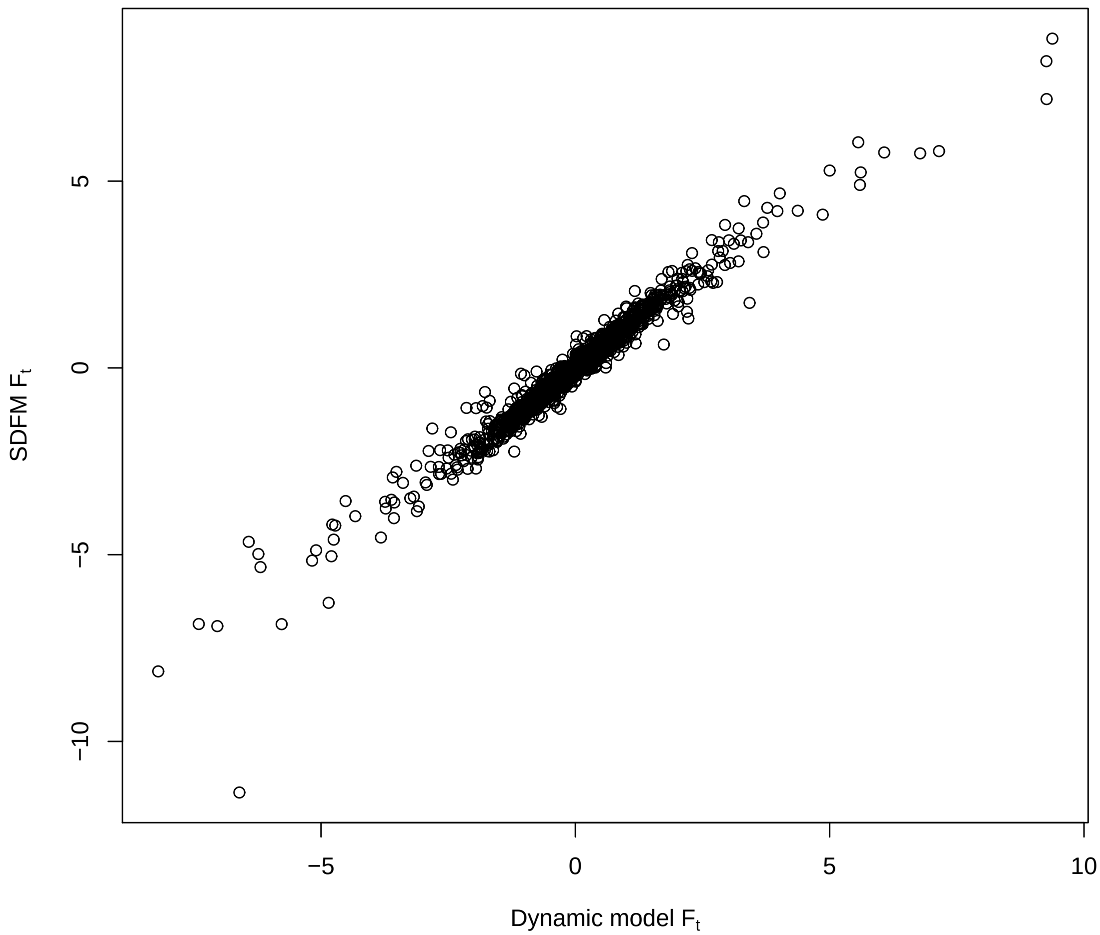

Figure 15: F_t of the SDFM Against F_t of the DFM. This figure plots the smoothed median of F_t obtained via the SDFM against the smoothed median of F_t obtained via the DFM.

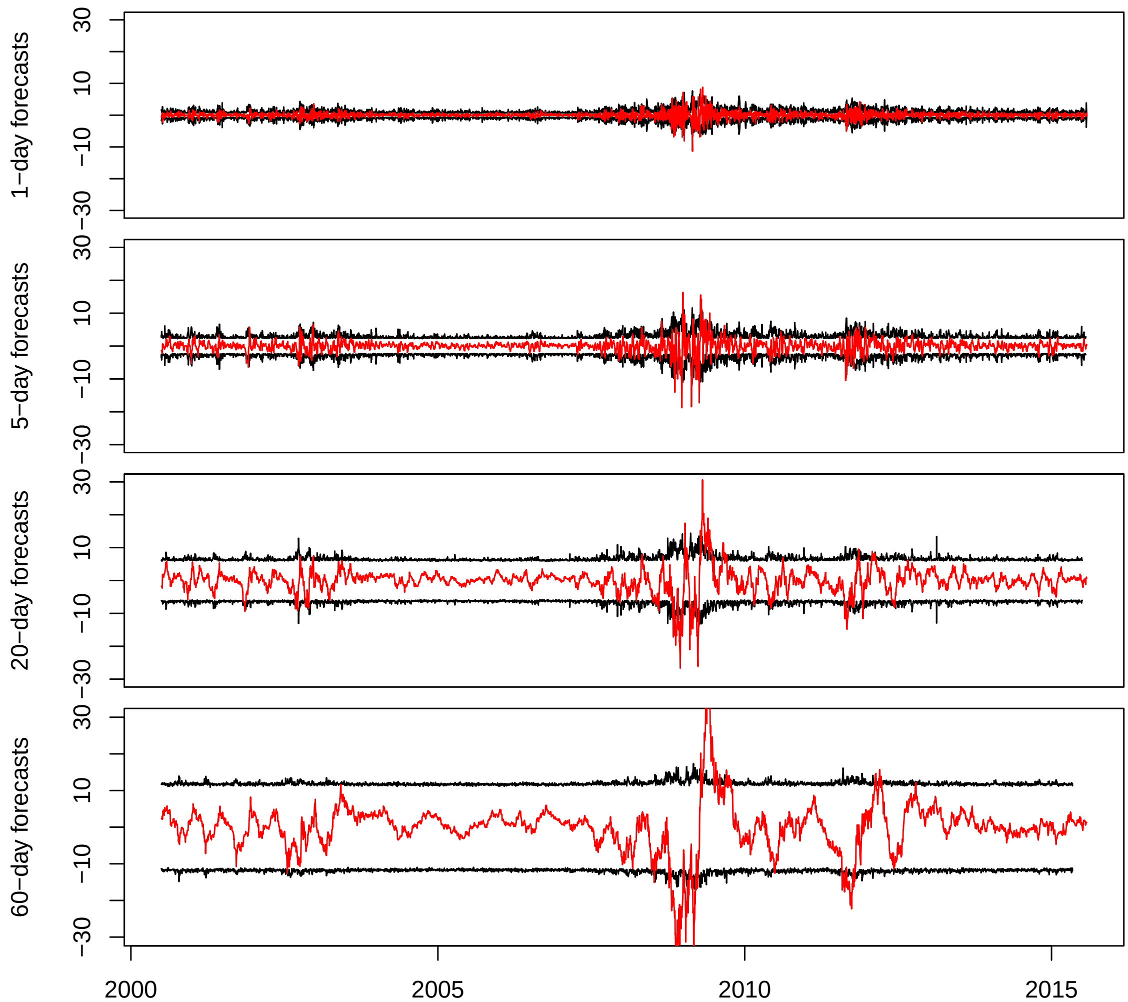

Figure 16: 1-day, 5-day, 20-day, and 60-day VaR Forecasts of Cumulative F_t. The filtered 5% and 95% quantiles of $\sum_{h=1}^{d} F_{t+h}|Y_{1:t}$ for $d = 1, 5, 10$ using the DFM model are reported in black and the pseudo-true F_t (median smoothed F_t obtained via the SDFM) are reported in red.

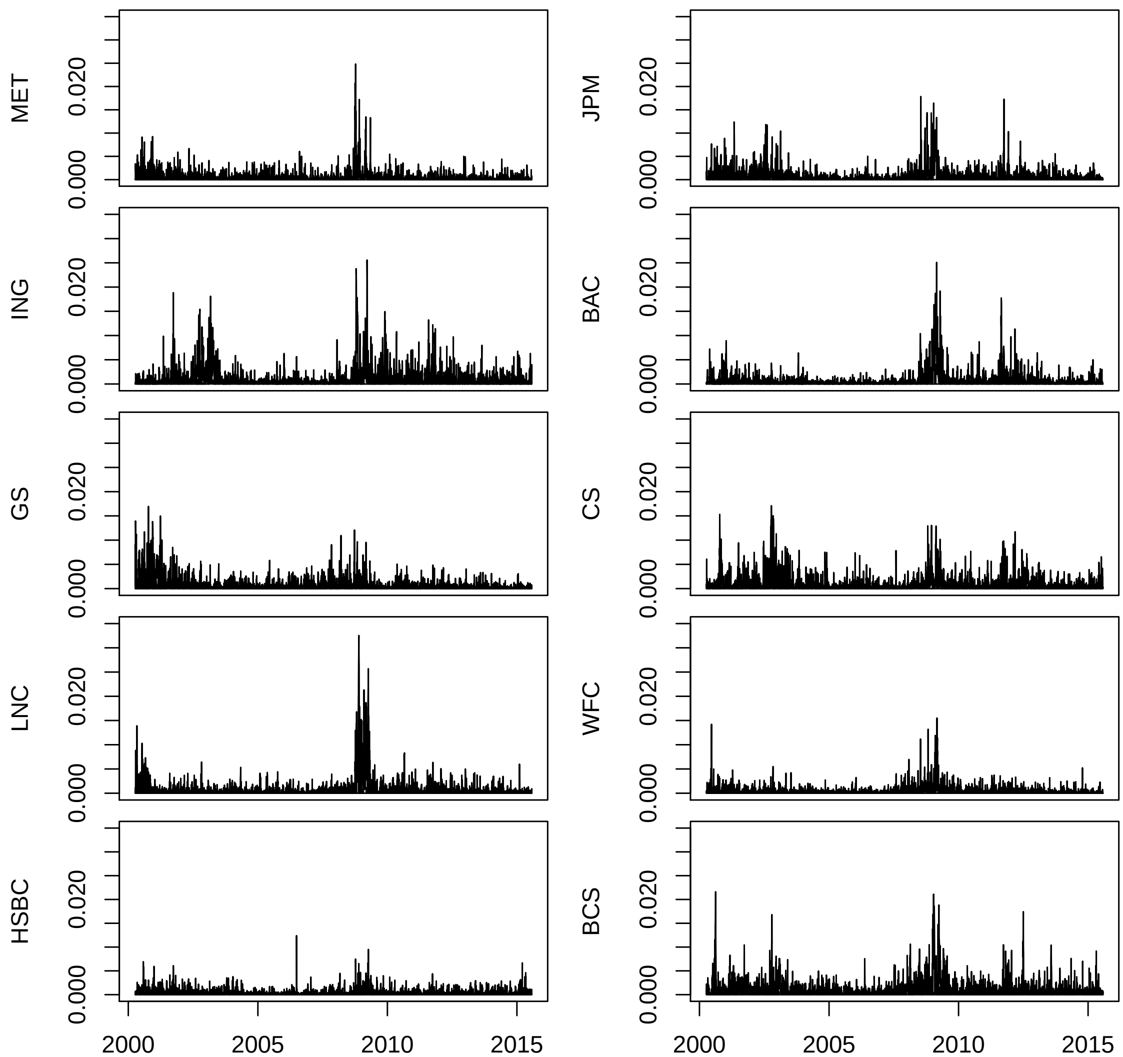

Figure 17: Smoothed $\sigma^2_{i,t}$ via SDFM. Smoothed 0.05th and 0.95th quantiles are reported with black lines and median is reported in red.

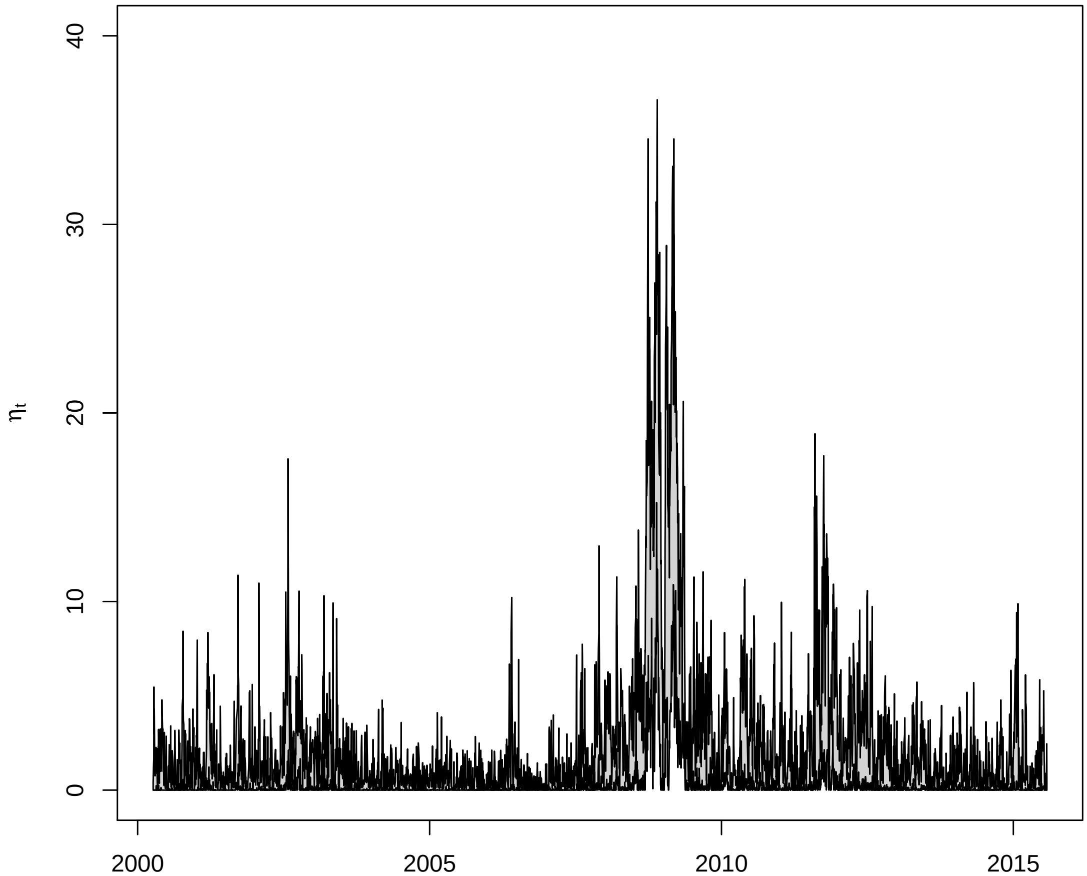

Figure 18: Smoothed η_t^2 via SDFM. Smoothed 0.05th and 0.95th quantiles are reported with black lines and median is reported in red.