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Episodes of War and Peace in an Estimated Open Economy Model

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Abstract

We analyze the effects of large war episodes (world wars) on the macroeconomic dynamics of four advanced countries (France, Germany, the UK and the U.S.) by means of an estimated open-economy model. The model allows wars to produce specific effects on the economy through capital depreciation, sovereign default and a military draft. These effects, together with large surges in public spending and debt, and significant drops in labor taxes, account for the bulk of fluctuations during wars. We also use our estimations to discuss the size and state-dependence of output multipliers, and the size of welfare losses from fluctuations.

Keywords: Fluctuations, War, Trade, Taxes, Public Debt, Bayesian estimations, Multipliers, Welfare.

JEL Classification: E31, E32, E62, F41, H56, H63.

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1 Introduction

In this paper, we propose a novel approach to investigate the systematic effects of large war episodes on the dynamics of key macroeconomic variables, that consists in estimating a medium-scale model on a set of macroeconomic time series for France, Germany, the UK and the U.S. The model is inspired from the canonical model of Smets and Wouters (2005), but is extended along many dimensions. First, we allow for trade in some components of demand (private consumption and investment). Second, we consider incompleteness in international financial markets to account for potential wealth transfers. Third, we include a full set of policy instruments: the money growth rate, public spending, public debt and distorsionary taxes on labor and capital. Fourth, the specific effects of large war episodes are captured by a dummy variable that opens the possibility for a larger depreciation of capital, a military draft, a default on sovereign debt and a shift in households preferences toward private/public consumption expenditure. Obviously, this model can be criticized as it does not take into account explicitly all the dimensions along which war episodes affect the economy: goods rationing, financial markets disruptions, changes in national borders, among other aspects. However, it makes a significant step forward in this task, and features enough flexibility to capture business cycle dynamics during normal times as well as during war episodes. In addition, our approach is very much in line with the wedge approach proposed by Chari, Kehoe and McGrattan (2007), according to which "shocks" may not necessarily be structural shocks but may capture features of the economy that are not modeled explicitly.

We have four different datasets for France (1898-2006), Germany (1880-2008), UK (1870-2005) and the U.S. (1871-2010). For each country, a first time series is the dummy for war episodes. Each dataset then mixes data from Barro and Ursùa (2011) and data from Piketty and Zucman (2014) and is made of eight time series: GDP, consumption, investment, total consumption expenditure (public and private) to national income, net exports to national income, debt to national income, total tax receipts to national income and the inflation rate.[1] As in Leeper, Plante and Traum (2010), our annual datasets are HP-filtered with a rather large value of $\lambda = 400$. This approach has the advantage to preserve large swings in macroeconomic variables while avoiding the issue of fiscal and macroeconomic variables having their own trends. The model is estimated using Bayesian techniques and considering nine potential driving forces: war shocks, productivity and investment efficiency shocks, foreign demand (trade) shocks, money growth rate and public

[1]See Data Section for a full description.

spending shocks, labor and capital income tax rate shocks, and a measurement error shock on GDP. We assume that those shocks are purely exogenous components of the model especially because we want to remain agnostic about the way countries of our sample finance war-related increases in public spending. In other words, we abstract from any kind of systematic fiscal or monetary policy rules, and let the data speak. For each of the four countries considered, we obtain point estimates for key structural parameters like those that relate to price and wage stickiness, investment adjustment cost, the inverse of the Frisch elasticity, the trade elasticity or the Edgeworth complementarity/substitutability parameter, as well as estimates about the dynamics of driving processes.

Our main results derive from the exploitation of our Bayesian estimations, especially from the smoothed dynamics of exogenous variables. First, we quantify the dynamic interactions between our exogenous variables using a SVAR analysis. We identify the effects of a war shock (a shock on the depreciation of capital, affected by our war dummy) on other exogenous shocks (or variables directly affected by other shocks) using a Cholesky decomposition of the variance-covariance matrix with the war shock ordered first. This exercise reveals that the effects implied by our war shock (capital depreciation, draft, default and switch in preferences) do not account for all the effects of wars. War shocks are associated with an contemporaneous increase in public spending, public debt and a fall in labor income taxes in all countries. Other effects are country-specific. Productivity, investment efficiency and foreign demand rise in the U.S. case, capturing an improvement in GDP and net exports, while an opposite pattern is observed for other countries. Our SVAR analysis also uncovers a variety of adjustment patterns for money growth and the capital income tax rate.

Second, since an increase in public spending is a robust feature of war episodes across countries, we make use of our estimates to compute the estimated value of government spending output multipliers, with three results. First, the value of estimated multipliers is in the range of what is found in the empirical literature (see Ramey (2011a) and references therein). Second, we find that multipliers during war times are significantly different from multipliers during normal times. This is especially true for France and the UK. Third, wartime multipliers are *lower* than multipliers in normal times in most countries (France, Germany and the UK) but *larger* in the U.S. These results seem to back the results of Auerbach and Gorodnichenko (2012) more than those of Owyang, Ramey and Zubairy (2013), even though our focus remains strictly on large war periods. Indeed, the question of state-dependent multipliers has been addressed recently with two different sets of conclusions. Auerbach and Gorodnichenko (2012) show that multipliers are larger during slack periods while Owyang et al. (2013) find no particular difference for the U.S. but a significant one for Canada. Our results contribute to the debate by showing that wartime multipliers are more different than similar and larger in the U.S. during war episodes.

Finally, we use our smoothed variables to proceed to an analysis of the welfare costs of fluctuations over different historical periods and with or without war episodes. As usual in the macroeconomic literature, welfare losses are computed against the steady state. We find that, compared to post-1950 welfare losses, the welfare losses from fluctuations are 2.5 to 16 times larger over the whole sample. In addition, compared to losses over the whole sample, welfare losses are around 20% lower when ignoring the effects of war shocks, and 40% when removing war years from the sample. We also provide quantitative evidence that the large swings in macroeconomic time series induced by war episodes produce massive instantaneous welfare losses, as large as 28% to 32% of current consumption for WWII in France and Germany respectively (2% and around 10% respectively for the UK and the U.S.).

Our paper relates to many contributions on the macroeconomic dynamics induced by war episodes. On the theoretical side, many papers model wars are as large public finance shocks. The seminal contribution of Braun and McGrattan (1993), as well as McGrattan and Ohanian (2010) show that the standard neoclassical business cycle model fed with exogenous shocks on government spending, investment and taxes can properly account for the effects of WWII in the U.S. or in the UK. Martin (2012) models large war episodes as large government spending shocks and shows that the financing schemes observed during WWI and WWII, mostly based on public debt and inflation in the short-run, are consistent with the optimal distortion-smoothing policy with limited commitment. This last result echoes the contribution of Ohanian (1997), who shows the great importance of financing schemes in the effects of war episodes on output and welfare. On the empirical side, the focus is also very much on government spending, as the "narrative approach" highlights the role of war/peace episodes in the identification of unexpected fiscal shocks (see Ramey and Shapiro (1997), Ramey (2011b), Ramey (2011a) and more recently Ben Zeev and Pappa (2015)). However, if the major focus on public spending seems justified for the U.S. especially during WWII where output grew significantly, additional elements need to be considered to account for the dynamics of macroeconomic aggregates, in particular in countries where output shrank during war episodes like France or Germany.

Among these additional factors, productivity shocks, capital depreciation shocks or the open economy dimension must be considered. In particular, Francis and Ramey (2006) disentangle the extent to which macroeconomic fluctuations were due to technological *vs.* non-technological shocks in the U.S. history, and highlight the importance of technological shocks, especially before WWII. Besides, Auray, Eyquem and Jouneau-Sion (2014) show that capital depreciation shocks *à la* Ambler and Paquet (1994) crucially contribute to the macroeconomic dynamics induced by major war episodes.[2] Finally, some literature highlight the importance of external trade and finance to account for the dynamics of economies during wars. Devereux and Smith (2007) analyze the Franco-Prussian 1871 war indemnity through the lens of the transfer problem, invoking the 1929 Keynes-Ohlin controversy. The possibility to account for such large transfers must be considered when analyzing periods involving large war episodes or immediate post-war periods, as it is the case in our model with incomplete international asset markets. More generally, war episodes most often induce large disruptions in trade, that contribute to the dynamics of key domestic macroeconomic variables (see Anderton and Carter (2001) or more recently Glick and Taylor (2010)), a channel that our model is fully able to take into account.[3]

The paper is organized as follows. Section 2 describes our open economy model. Section 3 presents our dataset, the estimation strategy, and reports our estimates. Section 4 characterizes the systematic effects of war episodes on other exogenous variables. Section 5 is devoted to the analysis of public spending output multipliers during war times and during normal times. Section 6 presents an analysis of the welfare losses from business cycles, as well as a quantification of the instantaneous welfare losses produced by large war episodes. Section 7 concludes.

2 The model

We consider a small open economy model with a government in charge of fiscal and monetary policy. The model builds on Smets and Wouters (2005) but is extended along many dimensions. First, it considers an open economy with trade in consumption and investment in goods, home bias, and an incomplete international financial market. Second, we

[2]See also Furlanetto and Seneca (2014) regarding the importance of capital depreciation shocks over the business cycle in normal times.

[3]Focusing on a different issue, Martin, Mayer and Thoenig (2008) identify the impact of international trade on the occurrence of conflicts. To sum up, data show that the sign and the magnitude of the relationship between trade openness and armed conflicts depend on the specific characteristics of trade flows and agreements but that major conflicts reduce international trade flows.

introduce money in the utility function and monetary policy is not conducted through a Taylor-type rule, but is affected by exogenous money growth rate shocks. In addition, seignorage revenues are transferred to the government. Both assumptions are consistent with a non-trivial role for monetary policy and for a potential financing of public spending through seignorage revenues. Third, the government levies distorsionary taxes on labor and capital income. Both tax rates are assumed to follow exogenous processes and are not calibrated to correct steady-state monopolistic distortions. The government spends on domestic consumption goods, invests in domestic public capital goods, issues government bonds, levies taxes and has access to seignorage revenues when issuing money. Finally, we introduce a dummy for war episodes that gives rise to a larger depreciation rate of capital, and opens the possibility of a military draft, a default on sovereign debt, and a shift in households preferences regarding public spending.

2.1 Equilibrium conditions

2.1.1 Households

We consider a unit continuum of households indexed in j that maximize their lifetime welfare

$$\mathcal{W}_t = E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \left(\frac{\left(c_s\left(j\right) + \nu g_s\right)^{1-\sigma_c}}{1-\sigma_c} + \chi_m \frac{\left(m_s\left(j\right)/p_s\right)^{1-\sigma_m}}{1-\sigma_m} - \chi_n \frac{\left(n_s\left(j\right)\right)^{1+\psi}}{1+\psi} \right) \right\} \tag{1}$$

In the welfare index, total consumption is a bundle of individual consumption of the private good $c_t\left(j\right)$ and the total consumption of public good g_t. When $\nu > 0$, private and public goods are substitutes while $\nu < 0$ implies complementarity.[4] We introduce the possibility that this Edgeworth complementarity parameter takes different values whether countries are engaged in wars or not, that is

$$\nu = \left(1 - \Delta\right) \nu_{norm} + \Delta \nu_{war} \tag{2}$$

where Δ is a dummy variable that equals one during war times and zero in normal times. Regarding other variables of the welfare index, $m_t\left(j\right)/p_t$ is the level of real money balances, and $n_t\left(j\right)$ is the total amount of labor supply, expressed in hours. Parameters σ_c and σ_m are the degrees of risk-aversion with respect to total consumption and real money balances, ψ is the inverse of the Frisch elasticity of labor supply.

[4]See Bouakez and Rebei (2007) and references therein for a discussion of the empirical relevance of this assumption.

The budget constraint of agent j is

$$\begin{aligned} e_t f_t(j) + b_t(j) + m_t(j) + p_t\left(c_t(j) + i_t(j) + ac_t^f(j)\right) &= e_t r^* f_{t-1}(j) + r_{t-1} b_{t-1}(j) \\ &+ m_{t-1}(j) + \left(\left(1-\tau_t^k\right) r_t^k z_t(j) - p_t ac_t^z(j) + p_t \tau_t^k \delta_t\right) k_{t-1}(j) \\ &+ \left(1-\tau_t^n\right) w_t(j) n_t(j) + \varphi_t(j) + tax_t(j) \end{aligned} \tag{3}$$

where e_t is the nominal exchange rate, $f_t(j)$ denotes the nominal value of foreign bonds returning a (constant) risk-less rate r^* between period t and $t+1$ and $b_t(j)$ denotes the nominal value of government bonds returning a risk-less rate r_t between period t and $t+1$. Further, p_t is the consumption price level, $i_t(j)$ is the investment in physical capital, τ_t^n is the labor income tax rate, $w_t(j)$ is the nominal wage rate paid to type-j labor, τ_t^k the tax rate on capital income, that comes with a tax deduction on depreciated capital, r_t^k is the gross return on the capital stock, $z_t(j)$ is the utilization rate of private capital, δ_t the time-varying depreciation rate of capital. Finally, $ac_t^f(j) = (\phi_f/2)\left(e_t f_t(j)/p_t - ef(j)/p\right)^2$ is an adjustment cost on real net foreign assets, $ac_t^z(j) = (\phi_z/2)\left(z_t(j)-1\right)^2$ is a utilization rate adjustment cost and $\varphi_t(j)$ denotes the profits from monopolistic firms paid to household j. An additional constraint to this optimization problem is the law of capital accumulation

$$k_t(j) = (1-\delta_t) k_{t-1}(j) + \zeta_t \left(1 - ac_t^i(j)\right) i_t(j) \tag{4}$$

where

$$ac_t^i(j) = \frac{\phi_i}{2}\left(\frac{i_t(j)}{i_{t-1}(j)} - 1\right)^2 \tag{5}$$

is an investment adjustment cost. In the dynamics of capital accumulation, ζ_t is an investment shock and δ_t the depreciation rate of capital. The investment shock evolves according to an autoregressive process

$$\zeta_t = (1-\rho_\zeta) + \rho_\zeta \zeta_{t-1} + \varepsilon_t^\zeta \tag{6}$$

while depreciation is:

$$\delta_t = \delta\left(1 + \Delta p^\delta\right) \tag{7}$$

As in Auray et al. (2014), the idea is to capture war capital destruction through higher capital depreciation, where p^δ captures the size of war-related capital destruction.

Households maximize the welfare function subject to the budget constraints with respect to consumption, government and foreign bonds, as well as the quantity of money.

First-order conditions imply

$$\beta E_t\left(\frac{u_{c,t+1}(j)}{u_{c,t}(j)}\frac{r_t}{\pi_{t+1}}\right) = 1 \tag{8}$$

$$\beta E_t\left(\frac{u_{c,t+1}(j)}{u_{c,t}(j)}\frac{e_{t+1}r_t^*}{e_{t+1}\pi_{t+1}\left(1+\phi_f\left(f_t^r(j)-f^r\right)\right)}\right) = 1 \tag{9}$$

$$u_{m,t}(j) - u_{c,t}(j)\left(\frac{r_t-1}{r_t}\right) = 0 \tag{10}$$

where $\pi_t = p_t/p_{t-1}$ is the CPI inflation rate, $u_{c,t}(j)$ is the marginal utility of consumption, $u_{m,t}(j)$ the marginal utility of real money balances, and $f_t^r(j) = e_t f_t(j)/p_t$ the real value of net foreign assets. The different types of labor offered by households are imperfectly substitutable making them monopolistic wage-setters. They take firms' labor demands

$$n_t(j,\omega) = \left(\frac{w_t(j)}{w_t}\right)^{-\theta^w} n_t(\omega) \tag{11}$$

into account along with their budget and capital accumulation constraints when optimizing. Further, we assume sticky wages whereby wage-setters face an individual probability $1-\eta^w$ to be allowed to re-optimize and a probability η^w to keep their previous-period nominal wage unchanged. The corresponding optimal nominal wage $\mathsf{w}_t(j)$ is thus

$$\sum_{i=0}^{\infty}(\beta\eta^w)^i E_t\left(n_{t+i}(j)\left(\frac{\theta^w}{\theta^w-1}\frac{u_{n,t+i}(j)}{u_{c,t+i}(j)}+(1-\tau^n)\frac{\mathsf{w}_t(j)}{p_{t+i}}\right)\right) = 0 \tag{12}$$

and the dynamics of nominal wages are

$$w_t^{1-\theta^w} = \eta^w(w_{t-1})^{1-\theta^w} + (1-\eta^w)\,\mathsf{w}_t(j)^{1-\theta^w} \tag{13}$$

From now on we assume perfect risk-sharing among households of the domestic economy. Households j are thus identical and we drop the j indices. Defining $q_t p_t \lambda_t$ as the Lagrange multiplier associated with the capital accumulation constraint, first-order conditions with respect to the capital stock, and investment are respectively

$$\beta E_t\left(\frac{u_{c,t+1}}{u_{c,t}}\left(q_{t+1}(1-\delta_{t+1}) + \left(1-\tau_{t+1}^k\right)\frac{r_{t+1}^k}{p_{t+1}} + \tau_{t+1}^k\delta_{t+1}\right)\right) = q_t \tag{14}$$

$$q_t\zeta_t\left(1-\frac{\phi_i}{2}\gamma_t^2 - \phi_i\gamma_t(1+\gamma_t)\right) + \beta E_t\left(\frac{u_{c,t+1}}{u_{c,t}}q_{t+1}\zeta_{t+1}\phi_i\gamma_{t+1}(1+\gamma_{t+1})^2\right) = 1 \tag{15}$$

$$\left(1-\tau_t^k\right)r_t^k - \phi_z(z_t-1) = 0 \tag{16}$$

where $\gamma_t = (i_t/i_{t-1}) - 1$ is the growth rate of private investment. Consumption and capital goods are made of domestic (d) and foreign (f) goods. Adjustment costs are paid in units of that composite good as well. The latter is defined as

$$x_t = \left((1-\alpha)^{\frac{1}{\mu}} x_{d,t}^{\frac{\mu-1}{\mu}} + \alpha^{\frac{1}{\mu}} x_{f,t}^{\frac{\mu-1}{\mu}}\right)^{\frac{\mu}{\mu-1}},\ x = \left\{c, i, ac^f, ac^z, ac^i\right\} \tag{17}$$

where $1-\alpha$ is the degree of home bias and μ the elasticity of substitution between domestic and foreign goods. The corresponding price index is

$$p_t = \left((1-\alpha) p_{d,t}^{1-\mu} + \alpha \left(e_t p_{f,t}\right)^{1-\mu}\right)^{\frac{1}{1-\mu}} \tag{18}$$

where $p_{d,t}$ and $p_{f,t}$ respectively denote the prices of domestic and foreign goods, expressed in units of local currency, and where e_t is the nominal exchange rate. Optimal expenditure on each good implies

$$x_{d,t} = (1-\alpha)\left(\frac{p_{d,t}}{p_t}\right)^{-\mu} x_t = (1-\alpha)\left(1-\alpha+\alpha s_t^{1-\mu}\right)^{\frac{\mu}{1-\mu}} x_t \tag{19}$$

$$x_{f,t} = \alpha\left(\frac{e_t p_{f,t}}{p_t}\right)^{-\mu} x_t = \alpha\left((1-\alpha) s_t^{\mu-1}+\alpha\right)^{\frac{\mu}{1-\mu}} x_t \tag{20}$$

for $x = \left\{c, i, ac^f, ac^z, ac^i\right\}$ where $s_t = e_t p_{f,t}/p_{d,t}$ stands for terms of trade.

2.1.2 Firms

There is a continuum of final good producers indexed in ω, with the following production function

$$y_t(\omega) = a_t k_t^s(\omega)^{\iota}\left(\left(1-\Delta p^{\ell}\right)\ell_t(\omega)\right)^{1-\iota} \tag{21}$$

where k_t^s is a measure of capital services used in production and $\left(1-\Delta p^{\ell}\right)\ell_t$ is the amount of labor that firms use to produce. Total labor is affected by a draft that lowers the amount of labor used in private firms during war times. Finally, a_t is a productivity measure following an autoregressive process

$$a_t = (1-\rho_a) + \rho_a a_{t-1} + \varepsilon_t^a \tag{22}$$

Cost minimization implies

$$k_t^s(\omega) = \frac{\iota}{1-\iota}\frac{w_t}{r_t^k}\ell_t(\omega) \tag{23}$$

which can be used to derive an expression of the nominal marginal cost

$$mc_t(\omega) = mc_t = \frac{\left(r_t^k\right)^{\iota}(w_t)^{1-\iota}\left((1-\iota)\left(1-\Delta p^{\ell}\right)+\iota\right)}{a_t\iota^{\iota}(1-\iota)^{(1-\iota)}\left(1-\Delta p^{\ell}\right)^{1-\iota}} \tag{24}$$

The adjustment of production prices is also subject to Calvo contracts. Re-setters face the following problem

$$\max_{\mathsf{p}_{d,t}(\omega)} \sum_{i=0}^{\infty} (\beta\eta^p)^i E_t\left(\frac{u_{c,t+i}}{u_{c,t}}\frac{p_t}{p_{t+i}}\left(\mathsf{p}_{d,t}(\omega)\, y_{t+i}(\omega) - mc_{t+i}y_{t+i}(\omega)\right)\right) \tag{25}$$

taking into account the demand addressed to firm ω

$$y_{d,t}(\omega) = \left(\frac{p_{d,t}(\omega)}{p_{d,t}}\right)^{-\theta^p} y_{d,t} \tag{26}$$

The optimal pricing condition is thus

$$\sum_{i=0}^{\infty} (\beta\eta^p)^i E_t\left(\frac{u_{c,t+i}}{p_{t+i}}y_{t+i}(\omega)\left(\mathsf{p}_{d,t}(\omega) - \frac{\theta^p}{\theta^p-1}mc_{t+i}\right)\right) = 0 \tag{27}$$

while the dynamics of prices are given by

$$p_{d,t}^{1-\theta^p} = \eta^p\left(p_{d,t-1}\right)^{1-\theta^p} + (1-\eta^p)\,\mathsf{p}_{d,t}(\omega)^{1-\theta^p} \tag{28}$$

2.2 Monetary policy, government and aggregation

The government is in charge of both monetary, public spending and fiscal policy and its budget constraint is

$$\begin{aligned} b_t + m_t + \tau_t^n w_t n_t - \Delta p^{\ell} w_t \ell_t + \tau_t^k\left(r_t^k z_t - p_t\delta_t\right)k_{t-1} + tax_t \\ = r_{t-1}\left(1-\Delta p^b\right)b_{t-1} + m_{t-1} + p_{d,t}\left(g_t + i_t^g\right) \end{aligned} \tag{29}$$

The total revenues from labor income taxation are lowered during war times as governments have to pay drafted people. Notice also that the possibility of defaulting on public debt during war times is taken into account, p^b being the size of default. We choose to be agnostic about how governments implement fiscal and monetary policies, and assume exogenous processes for public spending, tax rates and the money growth rate, therefore letting the data speak:

$$
\begin{aligned}
g_t &= (1-\rho_g)\, g + \rho_g g_{t-1} + \varepsilon_t^g && (30)\\
\tau_t^n &= (1-\rho_{\tau^n})\, \tau^n + \rho_{\tau^n} \tau_{t-1}^n + \varepsilon_t^{\tau^n} && (31)\\
\tau_t^k &= (1-\rho_{\tau^k})\, \tau^k + \rho_{\tau^k} \tau_{t-1}^k + \varepsilon_t^{\tau^k} && (32)\\
\mu_t^m &= (1-\rho_m) + \rho_m \mu_{t-1}^m + \varepsilon_t^m && (33)
\end{aligned}
$$

where $\mu_t^m = m_t/m_{t-1}$. Further, as in Auray et al. (2014), the government adjusts the amount of public investment to secure a given constant level of public unproductive capital k^g

$$
i_t^g = \delta_t k^g \tag{34}
$$

The stability of public debt to GDP in the long run is insured by having the lump-sum tax tax_t adjust slowly

$$
tax_t = \phi_b \left(b_t^r/y_t - b^r/y\right) \tag{35}
$$

where the value of ϕ_b is set to the minimum value that ensures stationarity.

We define an equilibrium as a path of endogenous variables that, conditional on the dynamics of exogenous variables, satisfies households and firms first-order conditions, and that ensures market clearing.

Equilibrium in the labor markets implies

$$
\ell_t = \int_0^1 \ell_t(\omega)\, d\omega = \int_0^1 \int_0^1 n_t(j,\omega)\, dj d\omega = \Upsilon_t^w n_t \tag{36}
$$

where $\Upsilon_t^w = \int_0^1 \left(w_t(j)/w_t\right)^{-\theta^w} dj$ denotes the dispersion of wages. Equilibrium in the physical capital market gives

$$
k_t^s = \int_0^1 k_t^s(\omega)\, d\omega = z_t k_{t-1} \tag{37}
$$

and equilibrium in goods markets implies

$$
y_t = (1-\alpha)\left(\frac{p_{d,t}}{p_t}\right)^{-\mu} y_t^d + \alpha\left(\frac{p_{d,t}}{p_t^*}\right)^{-\mu} y_t^* + g_t + i_t + i_t^g \tag{38}
$$

where $y_t^d = c_t + i_t + \phi_f \left(f_t^r - f^r\right)^2/2 + \left(\phi_z (z_t-1)^2/2\right) k_{t-1}$ and y_t^* is the foreign counterpart of y_t^d, subject to autoregressive shocks

$$y_t^* = (1 - \rho_{y^*})\, y^* + \rho_{y^*} y_t^* + \varepsilon_t^{y^*} \tag{39}$$

Finally, the aggregate production function is

$$\Upsilon_t^p y_t = a_t \left(z_t k_{t-1}\right)^{\iota} \left(\Upsilon_t^w \left(1 - \Delta p^{\ell}\right) n_t\right)^{1-\iota} \tag{40}$$

where $\Upsilon_t^p = \int_0^1 \left(p_{d,t}\left(\omega\right)/p_{d,t}\right)^{-\theta p} d\omega$ denotes the dispersion of producer prices. This small open economy model is closed by the net foreign asset equation, obtained by consolidating households, firms and government budget constraints

$$f_t^r = \left(\frac{e_t}{e_{t-1}}\right) \frac{r_t^* f_{t-1}^r}{\pi_t} + nx_t \tag{41}$$

where nx_t represents net exports, defined as

$$nx_t = \frac{p_{d,t}}{p_t}\left(y_t - g_t - i_t^g\right) - c_t - i_t - \frac{\phi_f}{2}\left(f_t^r - f^r\right)^2 - \frac{\phi_z}{2}\left(z_t - 1\right)^2 k_{t-1} \tag{42}$$

For future use, let us finally define national income as the sum of GDP plus net foreign income less total (public and private) capital depreciation

$$y_t^n = y_t + r_t^* f_t^r - \delta_t \left(k_{t-1} + k^g\right) \tag{43}$$

2.3 Discussion

We build the above model to capture the dynamics of developed economies over a very long course of history. As such, it is necessarily incomplete along some dimensions. Of course, the dynamics during war episodes is characterized by centralization and state-regulated production of goods, goods rationing, financial markets breakdown, wealth transfers among belligerents, changes in national borders, etc... No macroeconomic model can account for all the specificities of the four countries under investigation. We thus see our model as a way to discipline the data rather than an actual attempt to model all the aspects of wars, even though the introduction of state-dependent capital depreciation and households preferences and the inclusion of war-related military draft and sovereign defaults are steps into the right direction. Similarly, none of the considered countries is a small open economy. Here again, capturing accurately trade relations during war times would require country-specific. Above all, the effort of modeling war-specific dynamics would fail to capture regular business cycle moments from the data. Our approach here can therefore be brought in relation with the proposal of Chari et al. (2007), who argue that models can be used to uncover the dynamics of wedges: exogenous shocks capture all the dynamics from the data that are not modeled explicitly. Subject to the above caveats, we argue that the above model is flexible enough to capture relevant macroeconomic dynamics both during war times and during normal times.

3 Estimation

We estimate our model using Bayesian methods, adopting the standard approach of An and Schorfheide (2007). This implies obtaining the posterior distribution of our estimated parameters based on the linear approximation of the model's solution around the steady state using the Kalman filter. A major advantage of the approach is that it allows for the extraction of the dynamics of shocks, as well as the historical paths of endogenous variables. We will therefore have a complete quantitative evaluation of the model with respect to the data.

Steady state. We analyze the linearized dynamics of the model around a symmetric steady-state without inflation, implying zero net foreign assets.[5] Our goal is to identify the sources of macroeconomic fluctuations with a specific interest in war periods, focusing on France, Germany, the UK and the U.S. The model is flexible enough to allow for cross-country differences and remains agnostic on the behavior of the government, considering variations of policy instruments as purely exogenous.

Data. Before presenting our estimation results, we describe our dataset. In the model section, we have introduced a dummy variable for war episodes. This variable is treated as an exogenous shock, and the dummy variable is included as an observed variable. Further, given the considerable likelihood of measurement error in the data, we add a measurement error shock on GDP. These two shocks added to our structural shocks: productivity, investment, foreign demand, public spending, labor income tax rate and capital income tax rate provide a total number of 8 shocks. Exact identification thus requires 8 time series. We have four different datasets for France (1898-2006), Germany (1880-2008), UK (1870-2005) and the U.S. (1871-2010). For each country, a first time series is the dummy for war episodes. Our focus is on world wars so the variable is $\Delta = 1$ during WWI and WWII observations (years) and $\Delta = 0$ otherwise. Notice that this time series will

[5]Details about the steady state are given in Appendix A.

be approximated by an AR1 process in the estimation, even though the observable is a discrete variable. Each dataset then includes GDP and consumption per capita in 2006$, investment as a percentage of national income (i_t/y_t^n), total consumption expenditure (public and private) as a percentage of national income ($\left(c_t+g_t\right)/y_t^n$), net exports as a percentage of national income (nx_t/y_t^n), debt to national income (b_t^r/y_t^n), total tax receipts as a percentage of national income and the inflation rate (π_t). GDP and consumption per capita are taken from the dataset of Barro and Ursùa (2011). Other variables are taken mostly from Piketty and Zucman (2014). For the U.S., total tax receipts are taken from Mitchell before 1947 and FRED after 1947. For Germany, total tax receipts are taken from Flandreau and Zumer (2004) before 1913 and from Piketty and Zucman (2014) from 1950 onwards. The inflation rate is based on the CPI for France and Germany, and on the GDP deflator for the UK and the U.S., a distinction that our open economy model handles easily. As in Correia, Neves and Rebelo (1992), data in levels (GDP and consumption per capita) and inflation (*i.e.* 1+inflation/100) are taken in logs and ratios are taken in levels before HP-filtering the time series using $\lambda = 400$.[6] Our dataset for Germany has missing points that are handled as follows.[7] Series are first interpolated with a spline before being filtered. Then the interpolated missing observations are removed, to be properly handled as missing observations by the estimation algorithm. We use the Dynare built-in estimation routine, that copes with missing observations treating them as unobserved states and using the Kalman filter to infer their values during estimation.

Calibrated parameters. Before estimating the model, we choose to fix the value of a variety of parameters, either to preserve the model's steady state or because of weak identification from the data. The time unit is the year. The steady-state discount factor is $\beta = 0.96$, producing an average 4.17% real interest rate. The risk-aversion parameters on (total) consumption and real money balances are respectively $\sigma_c = 1.5$ and $\sigma_m = 1.5$. The steady-state capital depreciation rate is $\delta = 10\%$, the adjustment cost on utilization

[6]As explained in Leeper et al. (2010), Bayesian estimation including fiscal variables is problematic as fiscal variables may have their own trend over long periods of economic history. HP-filtering resolves this tricky issue and avoids the need to build balanced-growth models that are obviously inconsistent with fiscal and more generally macroeconomic variables featuring their own trends (due to structural change for instance). However, the question of the value of λ matters because a too low value of λ may lead to underestimate quite dramatically the size of economic downturns during war episodes while too large values might result in near-unit root processes. The datasets resulting from the use of $\lambda = 400$ are reported in Appendix B and show that none of these caveats apply when using $\lambda = 400$. Series remain stationary and large swings in macroeconomic time series are preserved.

[7]Investment and consumption expenditure to national income have missing points from 1914 to 1924, and from 1939 to 1949. Net exports to national income have missing points from 1919 to 1924 and from 1939 to 1949. Inflation data in 1922 and 1923 are considered missing. Total tax receipts are missing from 1914 to 1949.

is $\phi_z = 0.15$ and the capital share is $\iota = 0.36$. The import share is $\alpha = 0.15$ and the adjustment cost on foreign assets is $\phi_f = 0.025$, as in Ghironi and Melitz (2005). The value of the labor disutility parameter χ_n is adjusted to normalize hours of work to one, which makes our choice for the calibrated value of the wage mark-up ($1/(\theta^w - 1) = 20\%$) almost inessential (see Appendix A for details). Finally, the real balances utility parameter is set to $\chi_m = 0.05$, to produce steady-state real balances to GDP between 0.2 and 0.25, in accordance with (unreported) data. Those parameters remain fixed across countries and their values are summarized in Table 1 below.

Table 1: Calibrated common parameter values

Discount factor	$\beta = 0.96$
Risk-aversion (consumption)	$\sigma_c = 1.5$
Risk-aversion (real money balances)	$\sigma_m = 1.5$
Capital depreciation	$\delta = 0.10$
Utilization adjustment cost	$\phi_z = 0.15$
Capital share	$\iota = 0.36$
Import share	$\alpha = 0.15$
Adjustment cost on foreign assets	$\phi_f = 0.0025$
Real money utility parameter	$\chi_m = 0.05$
Wage mark-up	$1/(\theta^w - 1) = 0.2$
Labor disutility parameter	χ_n adjusted to get $n = 1$

Country-specific calibrated parameters. Remaining calibrated parameters are country specific, to account for differences in the tax systems, the levels of public consumption to GDP (κ) or public debt to GDP. Our calibrated values are based on data taken from the Piketty and Zucman dataset that are not necessarily used to estimate the model. Using average values of tax rates in France, we set the steady state tax rates to $\tau^n = 0.3$ and $\tau^k = 0.22$. The average government investment rate, taken from our French data is $\kappa^i = 0.0253$. We adjust public consumption to GDP $\kappa = 0.1346$ to match exactly the average level of debt to GDP (75%) over our sample. Last, we set $\theta^p = 6$ to match average net profits (20%). In the absence of empirical evidence for profits in Germany, we set $\theta^p = 6$. German data suggest $\kappa^i = 0.0164$ and $\kappa = 0.1490$. The average level of capital income taxation is $\tau^k = 0.2$. We adjust $\tau^n = 0.2926$ to match the average level of debt to GDP (45%) over the sample. Concerning the UK, our time series average indicate a profit rate of 25%, implying $\theta^p = 5$. Average government investment to GDP over our sample is $\kappa^i = 0.0263$. We assume $\tau^k = 0.2$ and $\tau^n = 0.3$, and adjust $\kappa = 0.1138$ to hit the 97% average debt-to-GDP ratio of our data sample. Finally, for the U.S., based on sample averages, we set $\kappa^i = 0.0190$, impose $\kappa = 0.10$ with $\tau^k = 0.35$ and adjust $\tau^n = 0.1678$

to hit the average debt-to-GDP ratio of our sample (55%). Those parameter values are summarized in Table 2 below.

Table 2: Calibrated country-specific parameter values

	France	Germany	UK	U.S.
Share of public investment in GDP (κ^i)	0.0253	0.0164	0.0263	0.0190
Share of public consumption in GDP (κ)	0.1346	0.1490	0.1138	0.1000
Labor income tax rate (τ^n)	0.3000	0.2926	0.3000	0.1678
Capital income tax rate (τ^k)	0.2200	0.2000	0.2000	0.3500
Steady-state price mark-up ($1/(\theta^p - 1)$)	0.2000	0.2000	0.2500	0.2000

Estimated parameters. Remaining parameters of the model are estimated. Our priors are as follows. The inverse of the Frisch elasticity ψ is a Normal with mean 2 and standard deviation 0.25 for all countries. The Calvo parameters on prices η^p and on wages η^w are Betas, with prior means 0.25 and standard deviations 0.1.[8] The Edgeworth complementarity parameters (ν_{norm}, ν_{war}) are estimated separately but with the same priors. They are Normals with prior means 0 and standard deviations 0.25. The percentage of public debt defaulted during war episodes p^b, the proportion of the workforce that is drafted p^ℓ and the percentage increase in the depreciation rate of capital induced by war episodes p^δ are assumed to be Betas with prior means 0.15 and standard deviations 0.1. Following Smets and Wouters (2003), the investment adjustment cost parameter ϕ_i is a Normal with prior mean 5 and standard deviation 1.5. As in the standard international RBC literature (see Backus, Kehoe and Kydland (1993) and the abundant subsequent literature), the trade elasticity is a Normal with prior mean 1.5 and a 0.25 standard deviation. The persistence of forcing processes (including the measurement error shock on GDP) are Betas with prior means 0.7 and standard deviations 0.2 and the standard deviations of innovations are Inverse Gamma distributions, with prior means 0.1 and infinite standard deviations.

Estimation results. Table 3 and Table 4 report the estimated parameters with 95% confidence intervals for each of the four countries, based on 500 000 replications of the MH algorithm where the first 100 000 were discarded and the scale parameter adjusted to get a 25% acceptance rate.

Prior and posterior distributions are reported in Appendix C and show that the estimation procedure provides enough information to significantly update prior distributions.

[8]We did try to estimate different values of the Calvo parameters during war times and during normal times but Calvo parameters during war times were weakly identified.

Table 3: Country-specific estimated parameters (France and Germany)

	Priors			Post. (France)			Post. (Germany)		
	Dist.	Mean	Sd.	Mean	Inf	Sup	Mean	Inf	Sup
Structural parameters									
Inv. Fisch elasticity (ψ)	N	2.00	0.25	1.11	1.00	1.23	1.80	1.46	2.11
Calvo prices (η^p)	B	0.25	0.10	0.04	0.02	0.07	0.43	0.36	0.50
Calvo wages (η^w)	B	0.25	0.10	0.03	0.01	0.06	0.33	0.19	0.46
Edgeworth comp. (ν_{norm})	N	0.00	0.25	-0.10	-0.14	-0.05	-0.10	-0.19	-0.02
Edgeworth comp. (ν_{war})	N	0.00	0.25	0.50	0.30	0.71	0.17	-0.01	0.35
War draft (p^ℓ)	B	0.15	0.10	0.05	0.00	0.09	0.06	0.00	0.11
War default (p^b)	B	0.15	0.10	0.09	0.04	0.14	0.17	0.03	0.29
War capital depreciation (p^δ)	B	0.15	0.10	0.06	0.01	0.10	0.13	0.01	0.26
Investment adj. cost (ϕ_i)	N	5.00	1.50	1.77	1.09	2.44	1.91	0.90	2.91
Trade elasticity (μ)	N	1.50	0.25	2.30	2.06	2.53	1.16	1.02	1.30
Persistence									
Productivity (ρ_a)	B	0.70	0.20	1.00	1.00	1.00	0.80	0.64	1.00
Public spending (ρ_g)	B	0.70	0.20	0.31	0.18	0.45	0.73	0.58	0.87
Investment (ρ_ζ)	B	0.70	0.20	0.15	0.04	0.25	0.50	0.35	0.64
Foreign demand (ρ_{y^*})	B	0.70	0.20	0.87	0.84	0.91	0.66	0.51	0.80
Meas. error (ρ^y_{err})	B	0.70	0.20	0.53	0.40	0.66	0.78	0.69	0.88
Labor tax (ρ_{τ^n})	B	0.70	0.20	1.00	1.00	1.00	0.41	0.26	0.55
Capital tax (ρ_{τ^k})	B	0.70	0.20	0.58	0.47	0.69	0.44	0.29	0.59
Money growth (ρ_m)	B	0.70	0.20	0.66	0.56	0.77	0.82	0.76	0.88
War (ρ_{war})	B	0.70	0.20	0.85	0.76	0.94	0.83	0.75	0.91
Standard deviations of shocks									
Productivity	IG	0.10	Inf	0.08	0.07	0.08	0.05	0.03	0.06
Public spending	IG	0.10	Inf	0.16	0.14	0.18	0.10	0.08	0.12
Investment	IG	0.10	Inf	0.38	0.26	0.51	0.23	0.13	0.34
Foreign demand	IG	0.10	Inf	0.72	0.60	0.85	0.15	0.13	0.18
Meas. error	IG	0.10	Inf	0.08	0.07	0.09	0.07	0.06	0.08
Labor tax	IG	0.10	Inf	0.03	0.03	0.03	0.20	0.17	0.23
Capital tax	IG	0.10	Inf	0.10	0.08	0.11	0.25	0.21	0.28
Money growth	IG	0.10	Inf	0.04	0.03	0.05	0.03	0.02	0.03
War	IG	0.10	Inf	0.18	0.16	0.20	0.17	0.15	0.19
Marginal data density									
					1358.55			1588.52	

Notes: Results based on 500 000 replications of the MH algorithm. Standard deviations are expressed in percents. N, B and IG respectively denote Normal, Beta and Inverse Gamma distributions. Marginal data density is the harmonic mean.

Table 4: Country-specific estimated parameters (UK and U.S.)

	Priors			Post. (UK)			Post. (U.S.)		
	Dist.	Mean	Sd.	Mean	Inf	Sup	Mean	Inf	Sup
Structural parameters									
Inv. Fisch elasticity (ψ)	N	2.00	0.25	1.10	1.01	1.20	0.76	0.73	0.79
Calvo prices (η^p)	B	0.25	0.10	0.05	0.02	0.08	0.13	0.10	0.17
Calvo wages (η^w)	B	0.25	0.10	0.04	0.01	0.06	0.07	0.04	0.11
Edgeworth comp. (ν_{norm})	N	0.00	0.25	-0.04	-0.11	0.02	0.13	0.08	0.18
Edgeworth comp. (ν_{war})	N	0.00	0.25	0.22	0.11	0.34	0.04	-0.13	0.19
War draft (p^ℓ)	B	0.15	0.10	0.06	0.01	0.11	0.13	0.05	0.21
War default (p^b)	B	0.15	0.10	0.05	0.02	0.09	0.18	0.10	0.27
War capital depreciation (p^δ)	B	0.15	0.10	0.10	0.06	0.14	0.02	0.00	0.04
Investment adj. cost (ϕ_i)	N	5.00	1.50	0.50	0.26	0.73	0.35	0.17	0.52
Trade elasticity (μ)	N	1.50	0.25	2.24	1.92	2.56	2.28	2.07	2.50
Persistence									
Productivity (ρ_a)	B	0.70	0.20	0.99	0.99	1.00	1.00	1.00	1.00
Public spending (ρ_g)	B	0.70	0.20	0.67	0.58	0.76	0.47	0.36	0.58
Investment (ρ_ζ)	B	0.70	0.20	0.19	0.08	0.31	0.30	0.17	0.44
Foreign demand (ρ_{y^*})	B	0.70	0.20	0.85	0.79	0.92	0.98	0.97	0.98
Meas. error (ρ^y_{err})	B	0.70	0.20	0.53	0.42	0.65	0.42	0.29	0.56
Labor tax (ρ_{τ^n})	B	0.70	0.20	1.00	1.00	1.00	1.00	1.00	1.00
Capital tax (ρ_{τ^k})	B	0.70	0.20	0.59	0.50	0.69	0.43	0.32	0.53
Money growth (ρ_m)	B	0.70	0.20	0.63	0.55	0.71	0.68	0.61	0.76
War (ρ_{war})	B	0.70	0.20	0.82	0.75	0.90	0.82	0.75	0.90
Standard deviations of shocks									
Productivity	IG	0.10	Inf	0.03	0.03	0.03	0.04	0.03	0.04
Public spending	IG	0.10	Inf	0.08	0.07	0.09	0.09	0.08	0.10
Investment	IG	0.10	Inf	0.08	0.05	0.11	0.06	0.03	0.08
Foreign demand	IG	0.10	Inf	0.27	0.21	0.33	0.27	0.22	0.31
Meas. error	IG	0.10	Inf	0.05	0.04	0.05	0.05	0.04	0.05
Labor tax	IG	0.10	Inf	0.02	0.02	0.02	0.03	0.02	0.03
Capital tax	IG	0.10	Inf	0.07	0.06	0.08	0.07	0.06	0.08
Money growth	IG	0.10	Inf	0.02	0.01	0.02	0.02	0.02	0.02
War	IG	0.10	Inf	0.16	0.15	0.18	0.16	0.15	0.18
Marginal data density									
					2451.52			2522.60	

Notes: Results based on 500 000 replications of the MH algorithm. Standard deviations are expressed in percents. N, B and IG respectively denote Normal, Beta and Inverse Gamma distributions. Marginal data density is the harmonic mean.

Inverse of Frisch elasticities of labor supply are all below the prior mean, ranging between 0.76 for the U.S. (the most responsive labor market according to our estimates) and 1.80 for Germany. Calvo price and wage parameters are also quite low, around 0.05-0.1 except for Germany, where they reach 0.43 for prices and 0.33 for wages. In any case, estimated parameters are consistent with prices and wages adjusting over less than 7 quarters on average (and even less than 5 quarters in most cases), numbers that are consistent with estimates based on quarterly datasets and models.

The estimates of Edgeworth complementarity parameters deserve some extended comments. Recall that a negative (positive) number indicates complementarity (substitutability) and results in altered dynamics of private consumption and investment dynamics with respect to models that do not consider public spending in the utility function. This parameter is particularly important in the transmission of public spending shocks, as will become clear later on. We do not claim that the assumption of Edgeworth complementarity/substitutability captures deep microfounded features of households preferences but we do think that it captures quite accurately the transmission mechanism of public spending shocks to private consumption (see Bouakez and Rebei (2007) for a discussion). The latter may in fact go through financial constraints, subsistence points, goods rationing or precautionary motives, but including each of these effects would require substantial additional modeling efforts and result in even larger models to estimate. Interestingly, two opposite patterns are uncovered by our estimations. For France, Germany and the UK, ν is negative and rather low in absolute terms during normal times, indicating moderate complementarity, and ν is positive and rather large during war times, pointing to substitutability. For the U.S., ν is positive both during normal and war times but larger during normal times. This different pattern for the U.S. will result in different predictions regarding the size and state-dependence of fiscal multipliers. Finally, if ν_{norm} is and statistically significant for all countries and ν_{war} significant for France and the UK (in the Bayesian sense that the data is informative about this parameter), while ν_{war} is less significant for Germany and the U.S., as shown by larger confidence intervals.

War-specific parameters pertaining to the size of the labor draft point to moderate values for France, Germany and the UK (5 to 6%) and a much larger value for the U.S. (13%). War sovereign default parameters lie between 5% (UK) and 18% (U.S.). War-related capital destruction parameters are estimated to be small in the U.S. (2% of the pre-war stock of public and private capital), larger for France and the UK (10%) and much larger for Germany (13%). Theses estimates are consistent with what we know about war destruction and highlight the difference between wars fought on the national soil (that destroy capital) and wars fought on foreign soil (that destroy less capital but require large military drafts).

Estimated investment adjustment costs ϕ_i are rather low for UK and the U.S. (0.5 and 0.35 respectively) and a bit larger for France and Germany (1.77 and 1.91). The last parameter that does not relate to exogenous processes is the trade elasticity. This parameter is usually a source of empirical controversies in the literature. Micro-econometric evidence suggests large values (between 6 and 10), and macro-econometric results suggest much lower values, typically around unity. Unsurprisingly, our results comfort macro-econometric studies, as the trade elasticity is 1.16 for Germany and around 2.5 for other countries.

We do not comment extensively the characteristics of estimated parameters governing the dynamics of exogenous shocks as most persistence estimates are in line with common values found in the empirical literature. Two notable exceptions are productivity and labor tax shocks, that both exhibit close-to-unity values. While this may reflect the large importance of these shocks (or wedges in the terminology of Chari et al. (2007)), this may also be the result of using a rather large value of the HP-filter. Given the scope of the paper and its focus on war periods, however, an approach that would smooth macroeconomic fluctuations too much and result in less persistent dynamics would have the highly undesirable side-effect to lessen the importance and size of macroeconomic fluctuations during war episodes.

4 Exogenous and policy dynamics after a war shock

Is a war shock (*i.e.* a shock on Δ) in the estimated model enough to account for the macroeconomic effects of a war? There are good reasons to think that this is not the case. First, war episodes are associated with large resource shifts from the private to the military sector, which can be reflected in the dynamics of productivity and investment efficiency. Second, as stated explicitly in the model exposition, our estimations remain agnostic about how other exogenous variables, and more particularly policy variables react to a war episode.

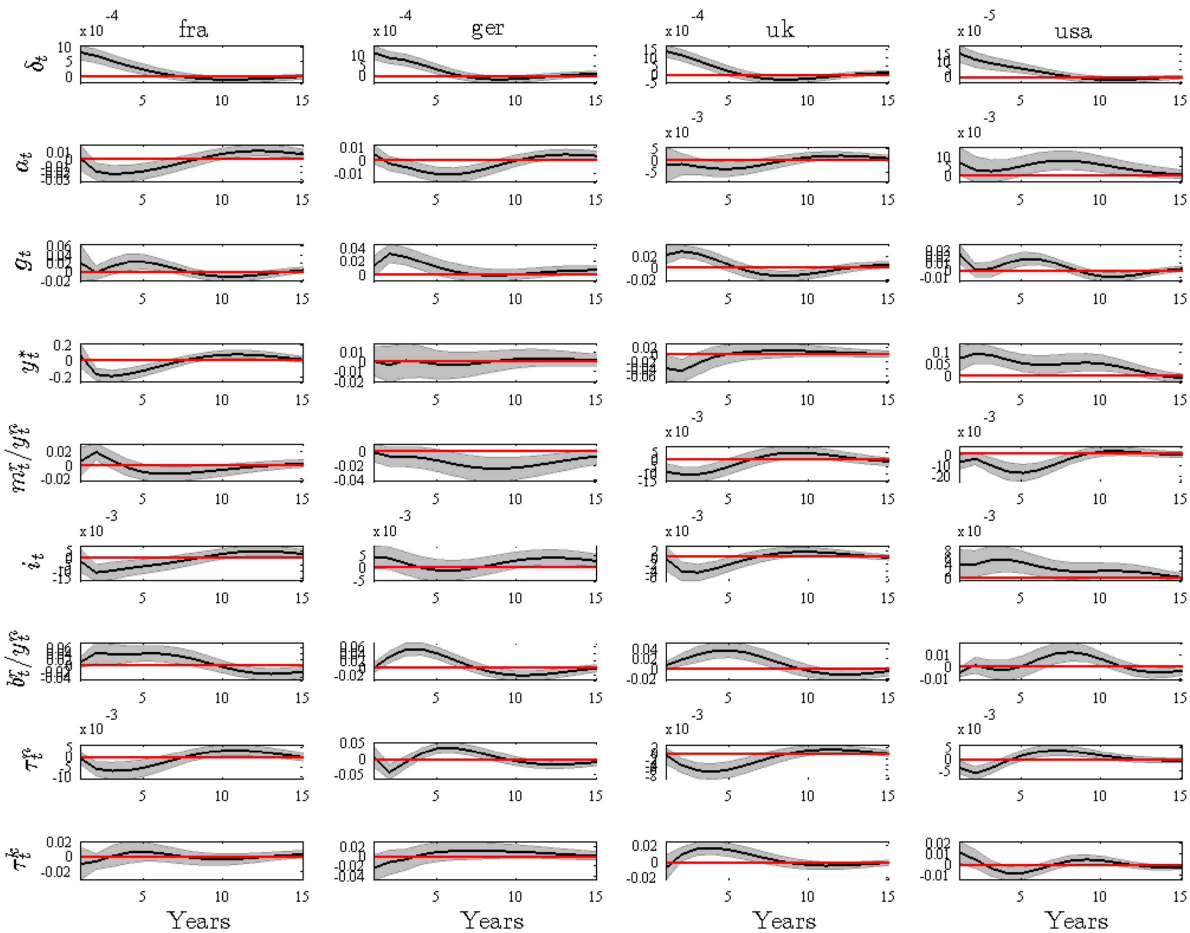
We thus proceed to a systematic analysis of the effects of a war shock on other exogenous variables. We consider eight key smoothed variables taken from the estimations together with the depreciation rate, that shifts only during war war periods, and estimate a SVAR. Five of them are exogenous variables (productivity, public spending, foreign output, the labor income tax rate, and the capital income tax rate), and three of them result directly from exogenous shocks (real money holdings to national income, private investment and public debt to national income). Lag selection points to a one-lag specification. We then identify a war shock as a shock to the depreciation rate δ_t using a Cholesky decomposition of the variance-covariance matrix. The depreciation rate δ_t is ordered first which means that it affects all variables within the current period while it remains unaffected by other variables within the current period. The set of variables included and the ordering are thus:

$$Y_t = \left\{\delta_t, a_t, g_t, y_t^*, m_t^r/y_t^n, i_t, b_t^r/y_t^n, \tau_t^n, \tau_t^k\right\}$$

Figure 1 below reports the IRFs produced by our SVAR estimations after a one standard deviation shock on the depreciation rate of capital identified using a Cholesky decomposition of the variance-covariance matrix, as well as one standard deviation confidence intervals, *i.e.* 84% confidence intervals, based on 2000 bootstrap replications.

What are the effects of a world war episode on macroeconomic variables? The answer depends on variables. Some react similarly across countries and some react differently. In all countries, a war episode raises public spending and public debt to GDP, and leads governments to lower the labor income tax rate. This last effect may effectively result from government decision, but in a wedge perspective *à la* Chari et al. (2007), it can also capture the larger labor force participation that is usually observed during war episodes (an effect that is not modeled explicitly in the our framework). The responses of other variables differ across countries. In the U.S. a typical war raises output, which is captured by an increase in productivity and investment efficiency. In other countries, opposite effects on output are observed and captured by a persistent fall in productivity and investment efficiency. In the U.S., net exports improve during war periods while a less clear-cut pattern is observed for other countries. Consequently, our SVAR captures this increase by a positive reaction of foreign demand from a U.S. perspective after a war shock. For France and the UK, the response of foreign demand is statistically significant and negative: war episodes depress trade and the trade balance. For Germany, the response of foreign demand is muted and statistically not significant. Finally, the responses of real money balances and capital income taxes are not suggestive of a systematic pattern across countries. We find that France experiences an increase in real money balances (evidence of monetization) while real money balances fall in other countries. The capital income tax rate exhibits a muted and not significant response in France and Germany while it increases in the UK and the

Figure 1: Impulse Response Functions to a one standard deviation shock on δ_t



Shaded areas correspond to one standard deviation confidence intervals (16^{th} and 84^{th} percentiles) based on 2000 bootstrap replications.

U.S. after a war shock.

Our SVAR exercise shows that our war shock does not replicate all the dynamic effects of a typical war episode. In all countries of our sample, war episodes also increase public spending and debt and are accompanied with a fall in the labor income tax rate. The effect on output is positive in the U.S., negative in other countries. The responses of productivity and investment measures in each country capture with these effects on output. The effects on trade are contrasted: positive in the U.S., negative in France and in the UK, muted in Germany. Finally, we do not uncover common financing schemes across countries: the response of real money balances and capital income tax rates are heterogeneous across countries of our sample.

5 Public spending output multipliers

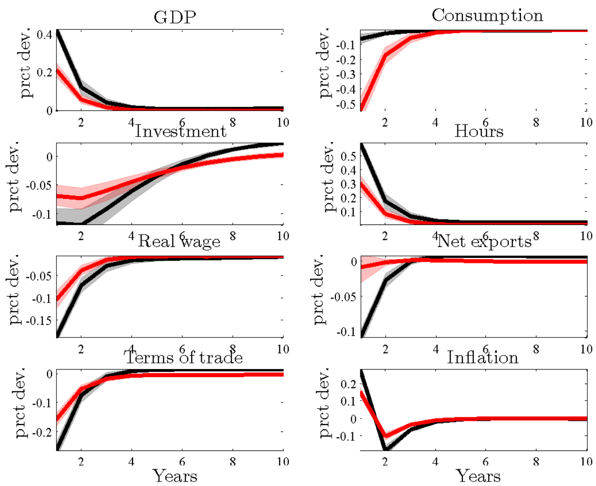
Our previous analysis makes clear that the rise in government spending during war episodes is a robust and commonly observed feature of war periods. In this section, we thus make use of our model and Bayesian estimations to compute Impulse Response Functions (IRFs hereafter) to a public spending shock and retrieve the values of public spending output multipliers. We are interested in the predicted absolute values of multipliers and in determining whether they are significantly different during normal times and during war episodes.

Computations are conducted based on non-linear simulations of the model. In all cases, the economy is initially in the steady state. In the case of normal times, we just run a 1% public spending shock and compute IRFs and multipliers. In the case of war episodes, we set the war dummy to one for 6 years, the average duration of a war episode, and run a 1% public spending shock in period 1. The IRFs and multipliers are computed using the difference between the resulting dynamics and those obtained from a 6 years war dummy shock alone. Figure 2 below reports the IRFs of to a 1% public spending shock during normal times and during war times.

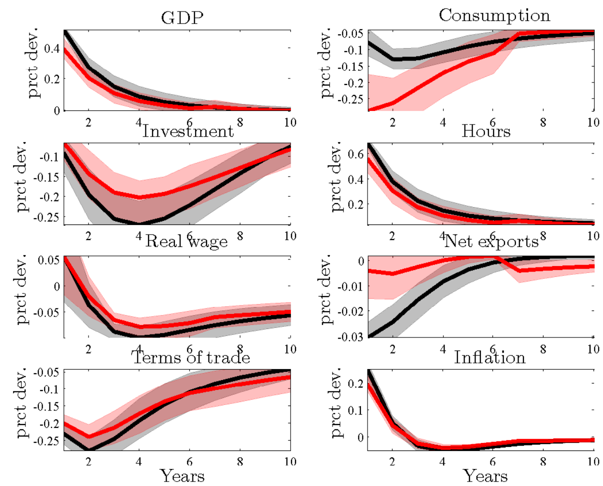
In all countries and independently of the state of the economy, a public spending shock raises GDP through higher demand for goods, pushing producer prices inflation up. The rise in public spending generates a fall in the permanent income of households. They react by increasing labor supply, which contributes to depreciate the real wage. They also cut consumption and investment. After the shock, the rise in domestic demand and prices leads the terms of trade to appreciate, which partly diverts private expenditure towards

Figure 2: Impulse Response Functions to a 1% public spending shock

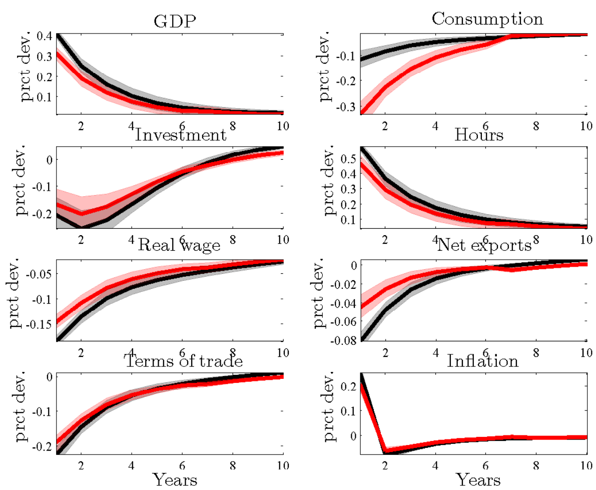
(a) France



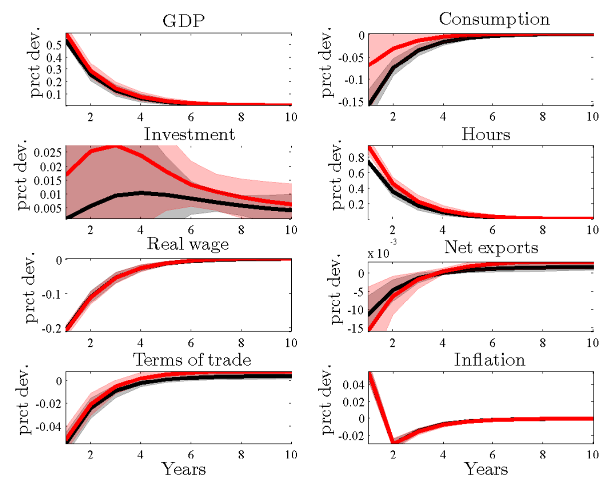
(b) Germany



(c) UK



(d) U.S.



Black: normal times. Red: War episodes. Shaded areas correspond to one standard deviation confidence intervals (16^{th} and 84^{th} percentiles) based on 120 replications of our non-linear simulations. Each replication draws a vector of the estimated parameters from the posterior distributions reported in Appendix C and computes IRFs.

foreign goods and contributes to depress the trade balance, along with the fall in the total level of private spending (consumption and investment). The strength and persistence of these effects differ substantially across countries and states of the economy. Compared to other countries, public spending shocks are less persistent in France (Panel 2a) so their effects are more short-lived. Germany (Panel 2b) is more strongly affected by nominal rigidities so IRFs exhibit a hump-shaped pattern (investment, real wages and the terms of trade in particular). The U.S. (Panel 2d) have a lower steady-state share of public spending in GDP and a much larger estimated value of the elasticity of labor supply so the effects of the shock on output are magnified and those on inflation attenuated.

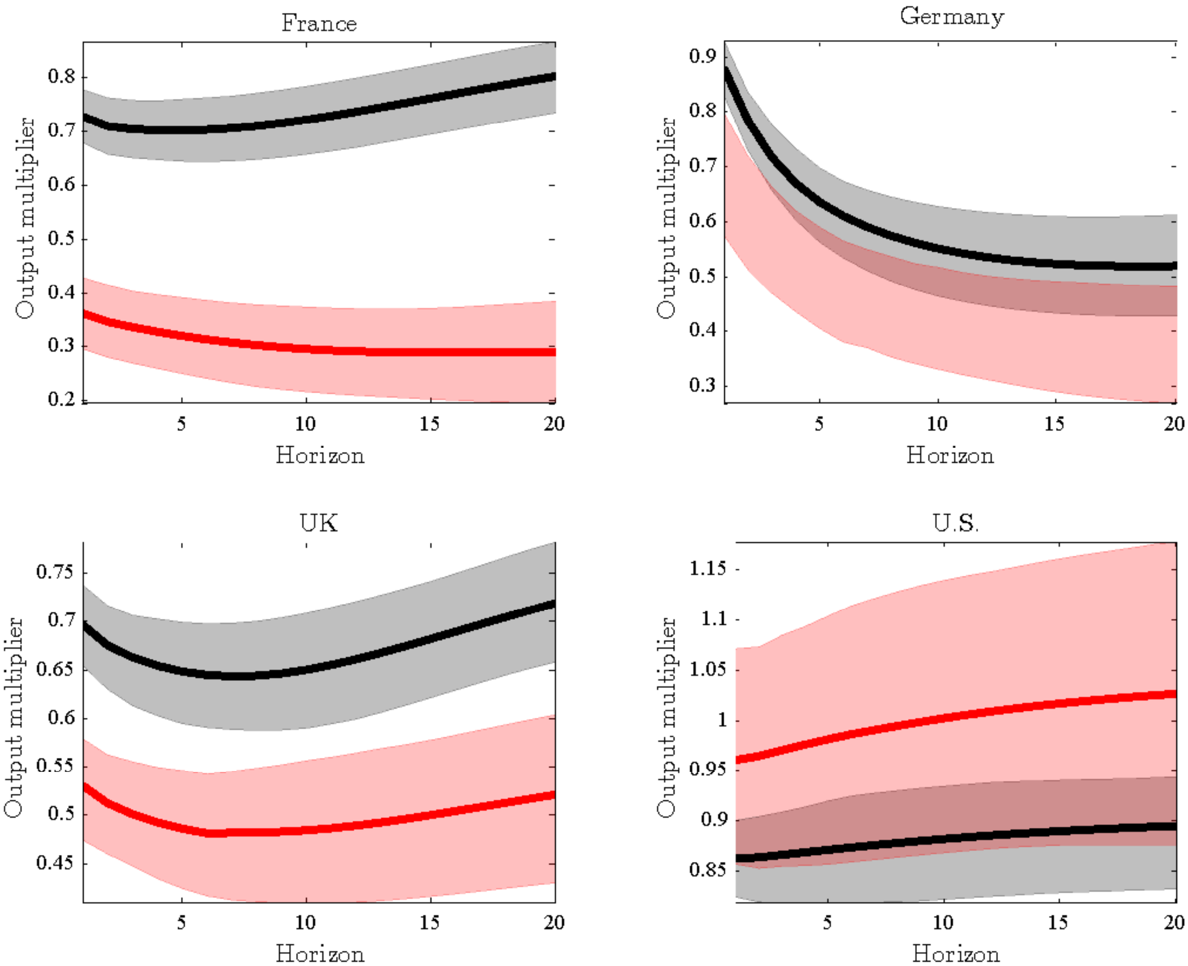
IRFs are also qualitatively different because of different estimated values of the Edgeworth parameter ν. When ν is negative, private and public spending are complementary so an increase in public spending also lowers the marginal utility of private consumption, pushing households to increase consumption. This effect partially or totally (if ν is negative and sufficiently large) offsets the negative wealth effect and consumption is crowded *in* instead of being crowded *out*. An opposite effect is at work when ν is positive and deepens the negative wealth and crowding-out effects. Figure 2 shows this quite clearly as countries with larger estimated values of the Edgeworth parameter (substitutability) experience larger contractions of private consumption after the shock and consequently smaller contractions in private investment. The response of private consumption however bears the largest weight in aggregate demand, which tends to alleviate the pressure on output and reduces the positive response of output and inflation. Real wages fall less, inducing hours worked to increase less, consistently with the response of output. Finally, the responses of external variables (terms of trade and net exports) are also dampened when ν is larger.

Overall, when ν is large, public spending output multipliers are smaller while lower values of ν are associated with larger values of the multiplier. France, Germany and the UK have larger estimated values of ν_{war} and lower values of ν_{norm}. As a consequence public spending output multipliers are lower during war times and larger during normal times. As opposed to those countries, the U.S. have lower estimated values of ν_{war} and larger values of ν_{norm}, implying larger multipliers during war episodes. The results are more directly reported in Figure 3, that shows the estimated present value (public spending output) multipliers (PVM) at the horizon h:

$$\nabla^{y}_{t+h} = \frac{\sum_{i=0}^{h} \beta^i \Delta y_{t+i}}{\sum_{i=0}^{h} \beta^i \Delta g_{t+i}} \tag{44}$$

for $h = 1$ to $h = 20$.

Figure 3: Public spending output present-value multipliers



Black: normal times. Red: War episodes. Shaded areas correspond to one standard deviation confidence intervals (16^{th} and 84^{th} percentiles) based on 120 replications of our non-linear simulations. Each replication draws a vector of the estimated parameters from the posterior distributions reported in Appendix C and computes multipliers.

Estimated multipliers during normal times lie between 0.7 and 0.85 in all countries, values that are consistent with those found in the literature. Indeed, Ramey (2011a) recalls that most empirical studies find public spending output multipliers typically between 0.6 and 1.2. Based on defense spending evidence and military conflict episodes, Barro and Redlick (2011) find that output multipliers are well below unity for the U.S, around 0.6. Over a larger panel of countries, Ilzetzki, Mendoza and Végh (2013) find that multipliers in high-income countries are 0.39 on impact and 0.66 in the long run. Our point estimates and confidence intervals are consistent with those numbers.

Recently, the question of state-dependent multipliers has been addressed with two different set of conclusions. Auerbach and Gorodnichenko (2012) find that multipliers are larger during slack periods while Owyang et al. (2013) find no particular difference for the U.S. but a significant one for Canada. In our model, multipliers differ during war episodes because when the war-dummy is one, Edgeworth parameters differ from their value in normal times, capturing different transmission mechanisms for shocks on public consumption. We exploit this non-linearity in our simulations and, as reported in Figure 3, find that wartime multipliers are more different than similar. Indeed, confidence intervals do not overlap for France and the UK, and overlap only marginally for Germany and the U.S. Further, for France, Germany and the UK, we find that multipliers are *lower* during war times than during normal times. For the U.S., the outlier of our sample, the wartime multiplier is *larger*. Our results thus contribute to the debate on state-dependent multipliers, even though our focus is on war periods rather than on slack/boom periods, which makes our results difficult to confront directly with existing evidence.

6 Welfare losses from fluctuations and war episodes

In this Section, we proceed to a systematic welfare analysis. Using the series of smoothed shocks, *i.e.* the series of shocks that perfectly replicate our data (disregarding measurement error shocks though), we simulate the model for each country. We proceed to 4 different exercises. First, we feed the model with all smoothed shocks on the total sample. Second, we feed the model with all shocks except war shocks, again on the total sample. Third, we feed the model with all shocks but remove observations that correspond to world war years from the sample. Finally, we remove all years before 1950. Table 5 reports Hicksian consumption equivalents, *i.e.* the consumption compensation that makes households indifferent between living in a world with fluctuations and living in an economy that remains in the steady state always.[9]

The magnitude of post-war welfare losses from fluctuations is perfectly in line with the order reported by Lucas (1987), between 0.085% and 0.23% of consumption equivalent. Over the total sample on the other hand, the model predicts substantially larger welfare losses. According to our estimates, focusing on post-war instead of the full sample underestimates welfare losses from fluctuations by a factor 2.5 (for the UK) to more than 15

[9]We use a second-order approximation to the utility function, assume away the role of real money balances and, given the construction of our datasets – all series have theoretically a zero mean –, neglect the potential level effects from fluctuations and concentrate on the volatility effects. See Appendix D for details.

Table 5: Welfare losses from fluctuations, in percents

	France	Germany	UK	U.S.
Total sample all shocks	1.3212	1.5192	0.3036	0.5116
Total sample no war shocks	1.1385	1.5202	0.2943	0.3702
Removing war years	0.5547	1.2798	0.2276	0.2694
Post 1950	0.0844	0.2290	0.1143	0.1312

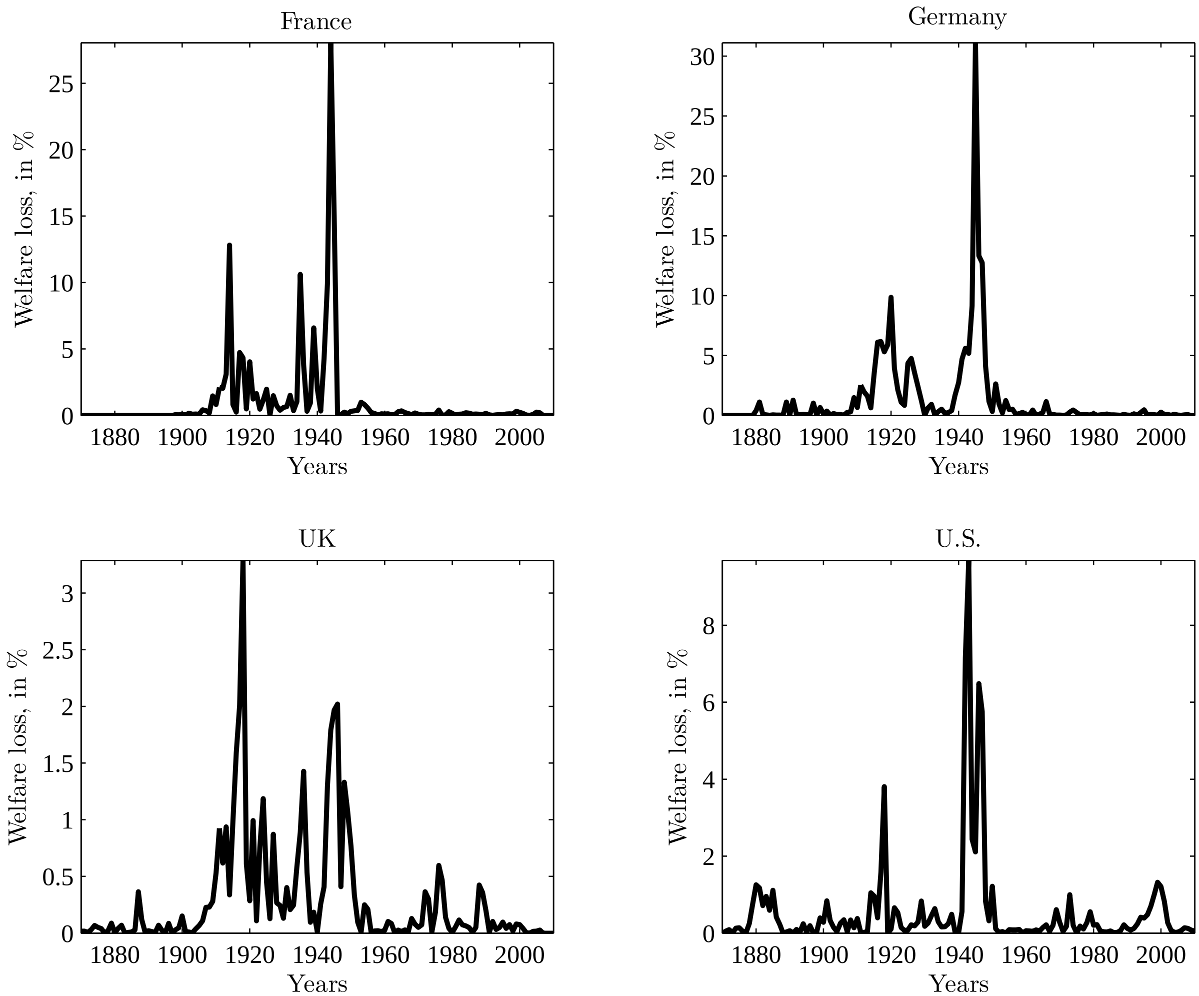
(for France). The reason behind this result is firstly the presence of war episodes. Welfare computations show that, compared to losses with all shocks on the whole sample, welfare losses are around 20% lower without war shocks and around 40% lower on average when removing war years from the sample. An exception is Germany, for which welfare losses do not fall so much when disregarding war shocks or war years, but the country experienced the 1923 hyperinflation episode and can consensually be considered engaged in war as early as 1936. However, the larger macroeconomic volatility during the pre-1945 period also explains a non-negligible fraction of the difference between losses over the whole sample and post-1950 welfare losses.

How large are the perceived welfare losses from war episodes in "real time"? To answer this question, we compute an instantaneous welfare measure and report the corresponding time series in Figure 4.[10] It shows that war episodes produce instantaneous welfare losses as large as 28% to 32% for WWII (numbers for France and Germany respectively). In other words, agents would have been willing to give up between a quarter and a third of their current consumption to avoid experiencing the effects of wars. In the UK and the U.S., the welfare impact of WWII is smaller, around 2% for the UK and a substantial 10% for the U.S. According to our calculations, the welfare losses from WWI range from less than 3-4% of current consumption for the UK and the U.S. to more than 10% for European countries.

Our model captures the effects on welfare through large shifts in hours worked and aggregate (private and public consumption). First, during war times, hours worked increase significantly, either because of the draft, that makes hours worked in the military sector rise, or because women enter the labor market, substantially raising the participation rate. These effects might be captured and reflected by the fall in labor income tax rates documented in Section 4. Second, private consumption falls substantially. Rationing and penury first come to mind, but the massive increase in public consumption

[10]Instead of taking the variances of variables as explained in the Appendix, we consider current squared deviations from the means and compute the welfare loss in the very same way.

Figure 4: Welfare losses from fluctuations, in percents



and public investment, producing large crowding out effects also contribute to lower private consumption. Last, public spending increases massively during war episodes, which also contributes to the welfare losses from fluctuations since public spending (and their deviations from the mean) enter negatively in the utility function of households.

7 Conclusion

War episodes induce large shifts in macroeconomic variables provoked by a series of potential causes: destruction, military draft, changes in national borders, large wealth transfers from a defeated country to a victorious one, interventions of governments that raise public spending, inflate the public and military sector, alter the structure of the labor market and impose financial and goods market restrictions. This paper proposed an estimated open economy model that would be suited to capture the dynamics of advanced countries like France, Germany, the UK and the U.S. both during normal times and during war episodes. Of course, all the aspects enumerated above could not be fully taken into account but some of them (capital destruction, draft, sovereign default) were, and were shown to be statistically significant. In addition, we shown that the macroeconomic effects of war episodes were not fully accounted for by these only elements, and needed to be completed with other shocks: an increase in public spending and public debt, a fall in the labor income tax rate, and, depending on countries, a fall/rise in productivity, investment efficiency and foreign demand (trade). Finally, our estimation results were used to derive results about the size and state-dependence of public spending output multipliers and about the welfare losses from fluctuations in a long-run perspective.

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A The steady state

We analyze the linearized dynamics of the model around the closed economy steady-state without inflation, where we also assume $s = 1$. The steady-state of the economy is one where the war dummy is zero, *i.e.* $\Delta = 0$. Further, by definition, $\gamma = 0$, and the dynamics of q gives $q = 1$. The condition on the utilization rate thus gives

$$r^k = \frac{\phi_z (z-1)}{1-\tau^k} \tag{45}$$

which plugged in the investment equation pins down z

$$z = \sqrt{1 + \frac{2\left(\beta^{-1} + \delta\left(1-\tau^k\right) - 1\right)}{\phi_z}} \tag{46}$$

as well as r^k. Next use the real marginal cost to get the real wage w

$$w = \left(\frac{(\theta-1)}{\theta} \iota^{\iota} (1-\iota)^{(1-\iota)} \left(r^k\right)^{-\iota}\right)^{\frac{1}{1-\iota}} \tag{47}$$

Further, we impose $a = n = 1$ and use

$$k = \frac{\iota}{1-\iota} \frac{w}{r^k z} \tag{48}$$

and the aggregate production function

$$y = (zk)^{\iota} \tag{49}$$

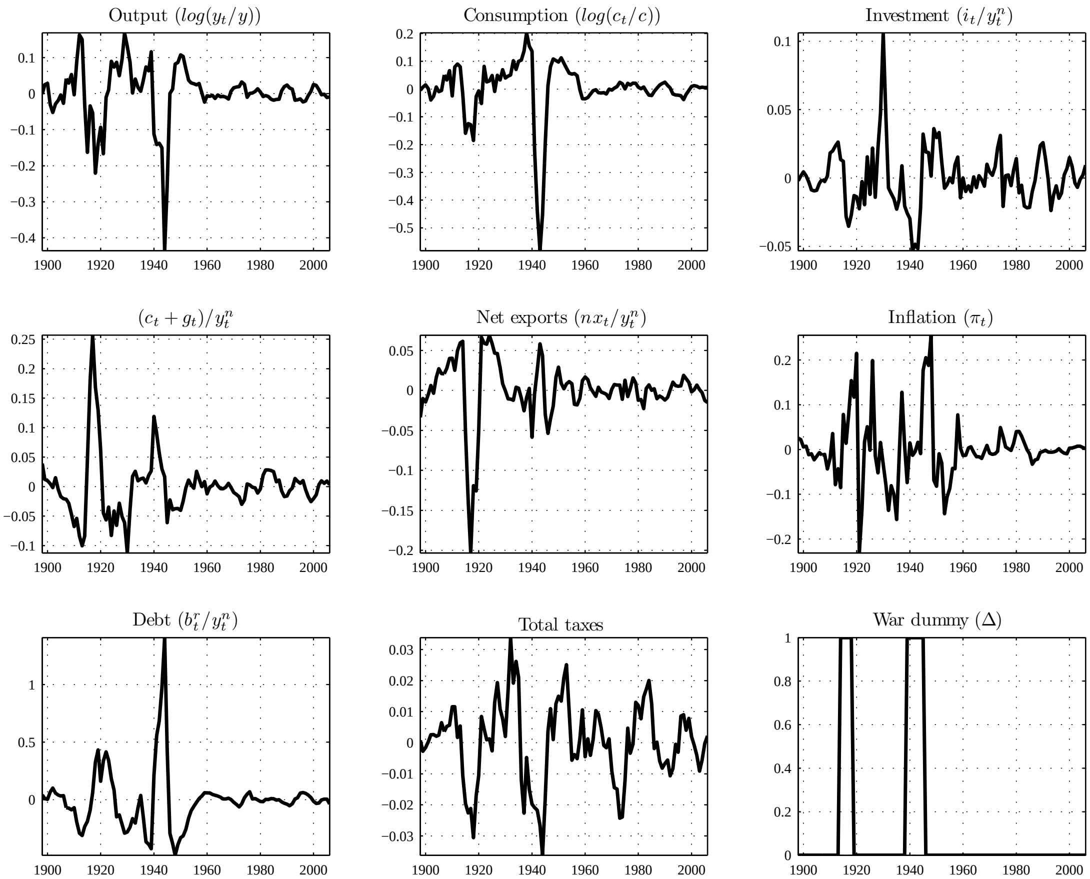
to get consumption

$$c = \left(1 - \kappa - \kappa_g^i\right) y - \delta k - \frac{\phi_z}{2} (z-1)^2 k \tag{50}$$

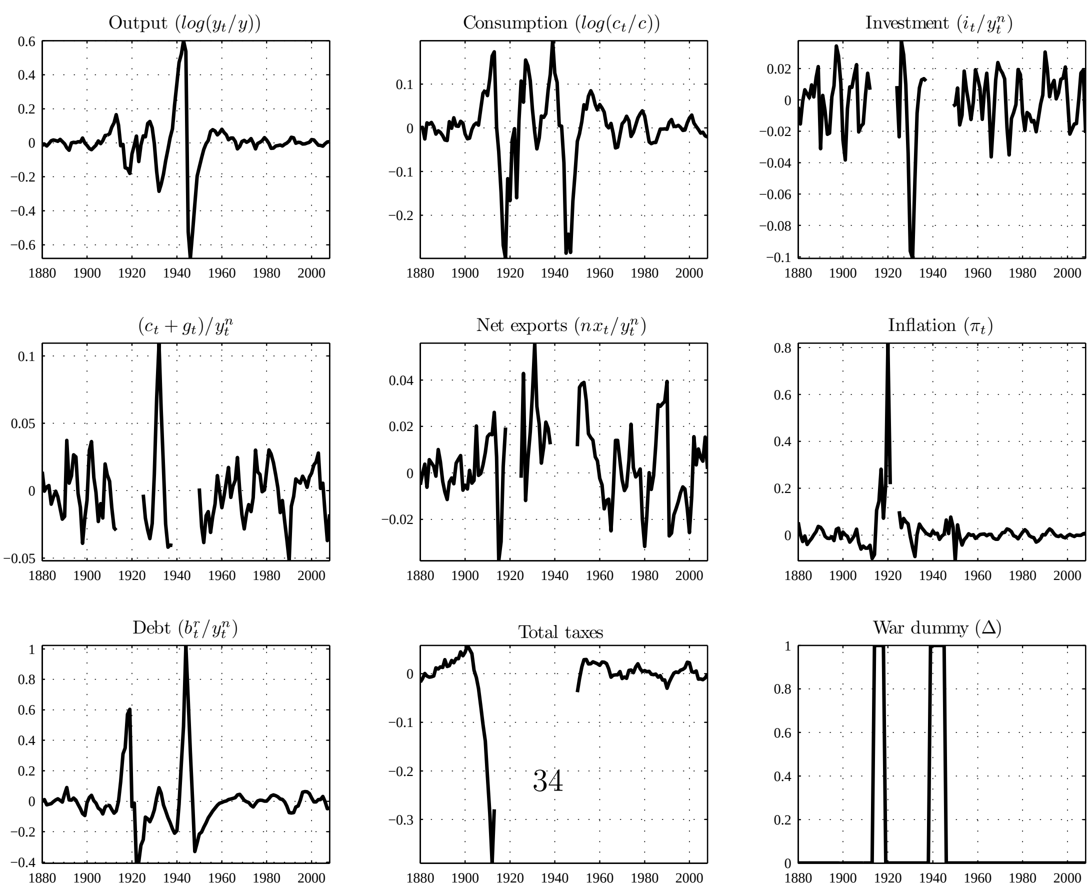
where $\kappa = g/y$ and $\kappa^i = i^g/y$ are the (imposed) steady-state share of public consumption and investment expenditure in GDP. Finally, the labor supply equation gives the value of χ_n that normalizes n to one

$$\chi_n = (c + \nu\kappa y)^{-\sigma_c} \left(\left(\theta^w - 1\right)\left(1 - \tau^n\right)/\theta^w\right) w \tag{51}$$

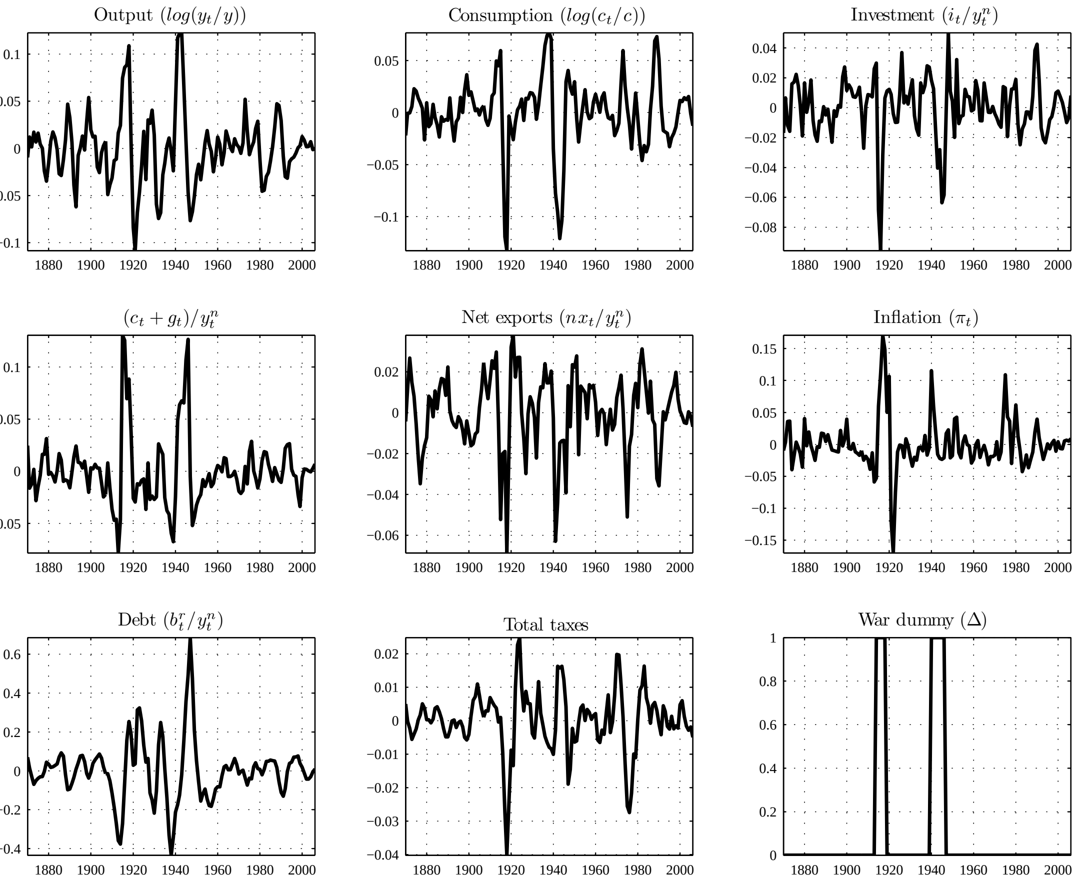
B Datasets



(a) France

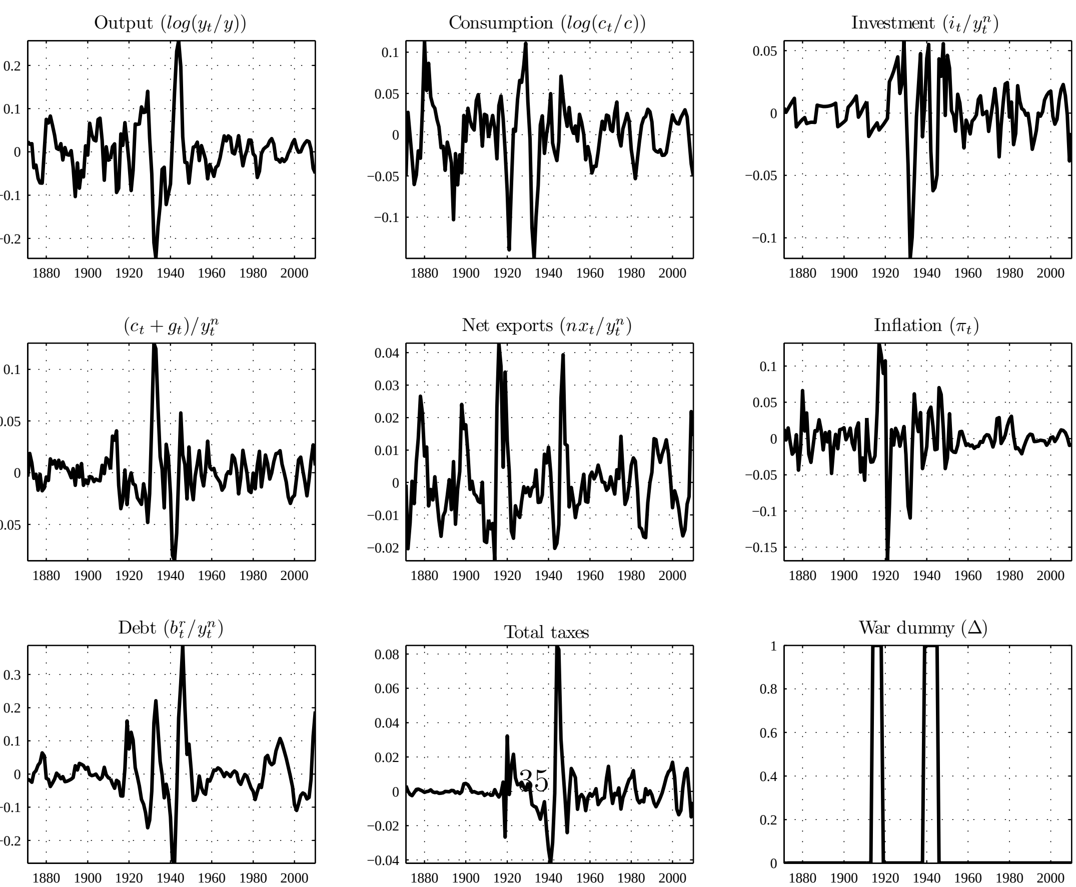


(b) Germany



Output ($log(y_t/y)$)
Consumption ($log(c_t/c)$)
Investment (i_t/y_t^n)
$(c_t+g_t)/y_t^n$
Net exports (nx_t/y_t^n)
Inflation (π_t)
Debt (b_t^r/y_t^n)
Total taxes
War dummy (Δ)
1880 1900 1920 1940 1960 1980 2000

(a) UK



Output ($log(y_t/y)$)
Consumption ($log(c_t/c)$)
Investment (i_t/y_t^n)
$(c_t+g_t)/y_t^n$
Net exports (nx_t/y_t^n)
Inflation (π_t)
Debt (b_t^r/y_t^n)
Total taxes
War dummy (Δ)
1880 1900 1920 1940 1960 1980 2000

(b) U.S.

C Prior and posterior distributions

Figure 5: Priors and Posteriors - France

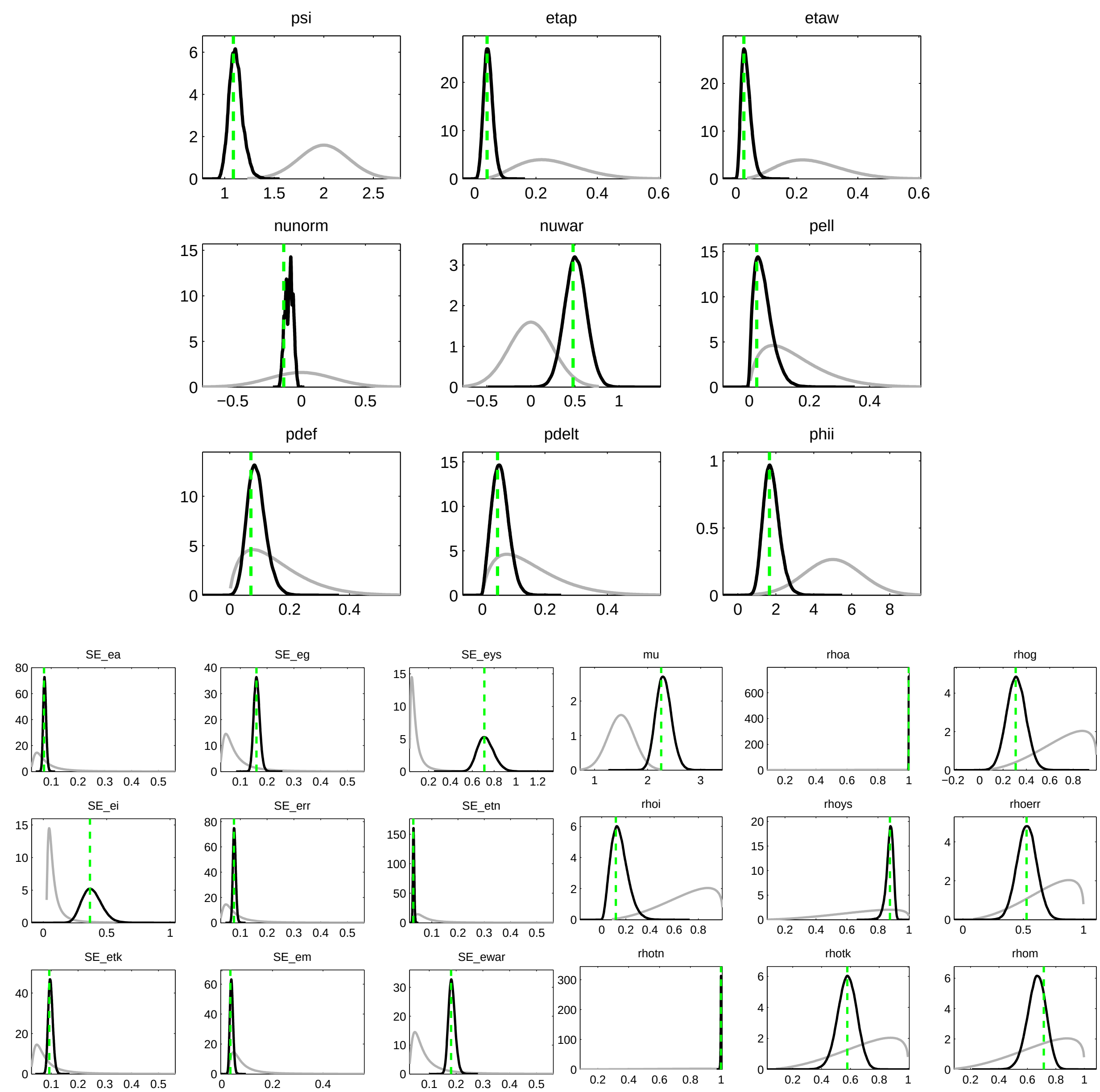


Figure 6: Priors and Posteriors - Germany

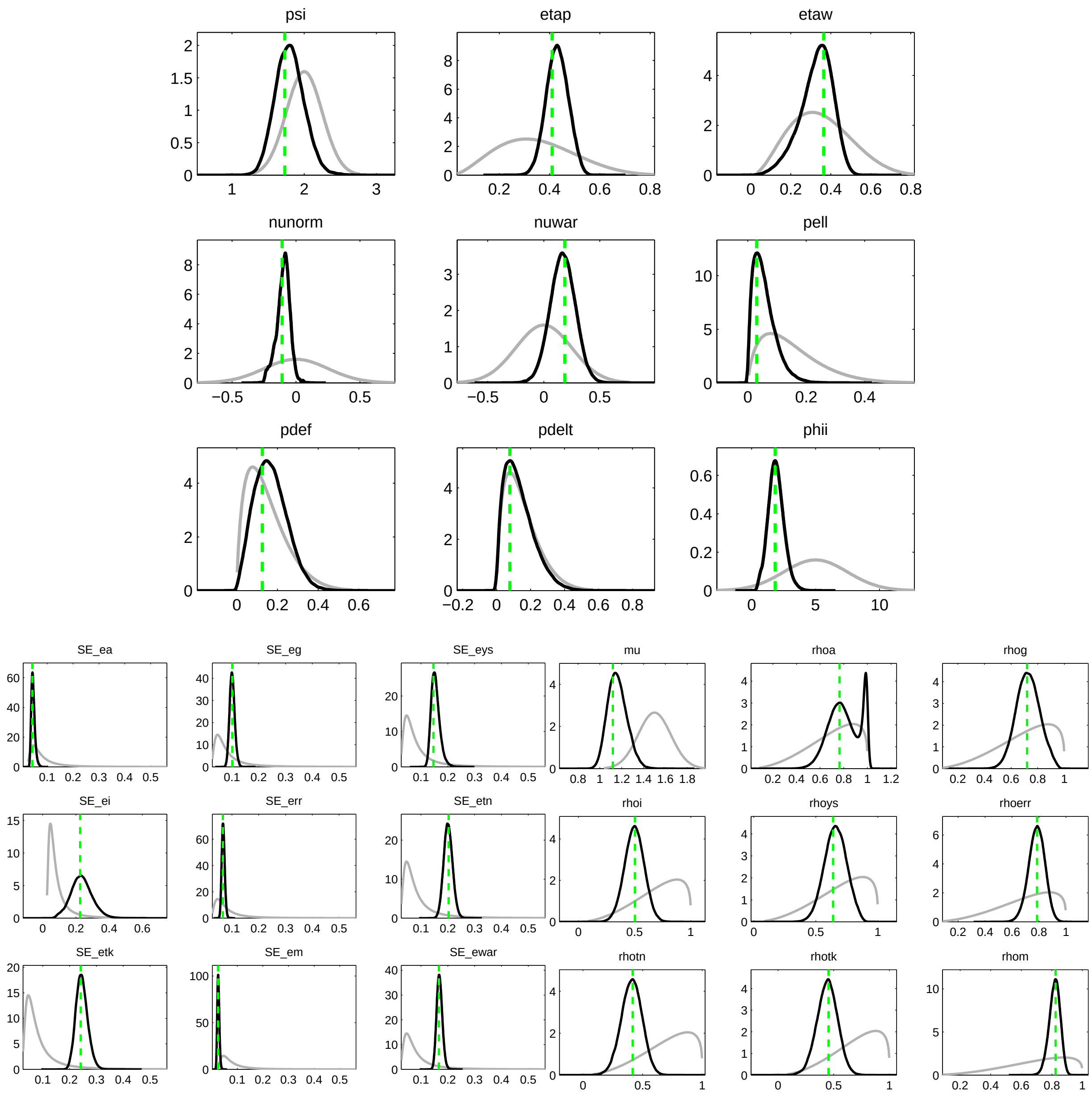


Figure 7: Priors and Posteriors - UK

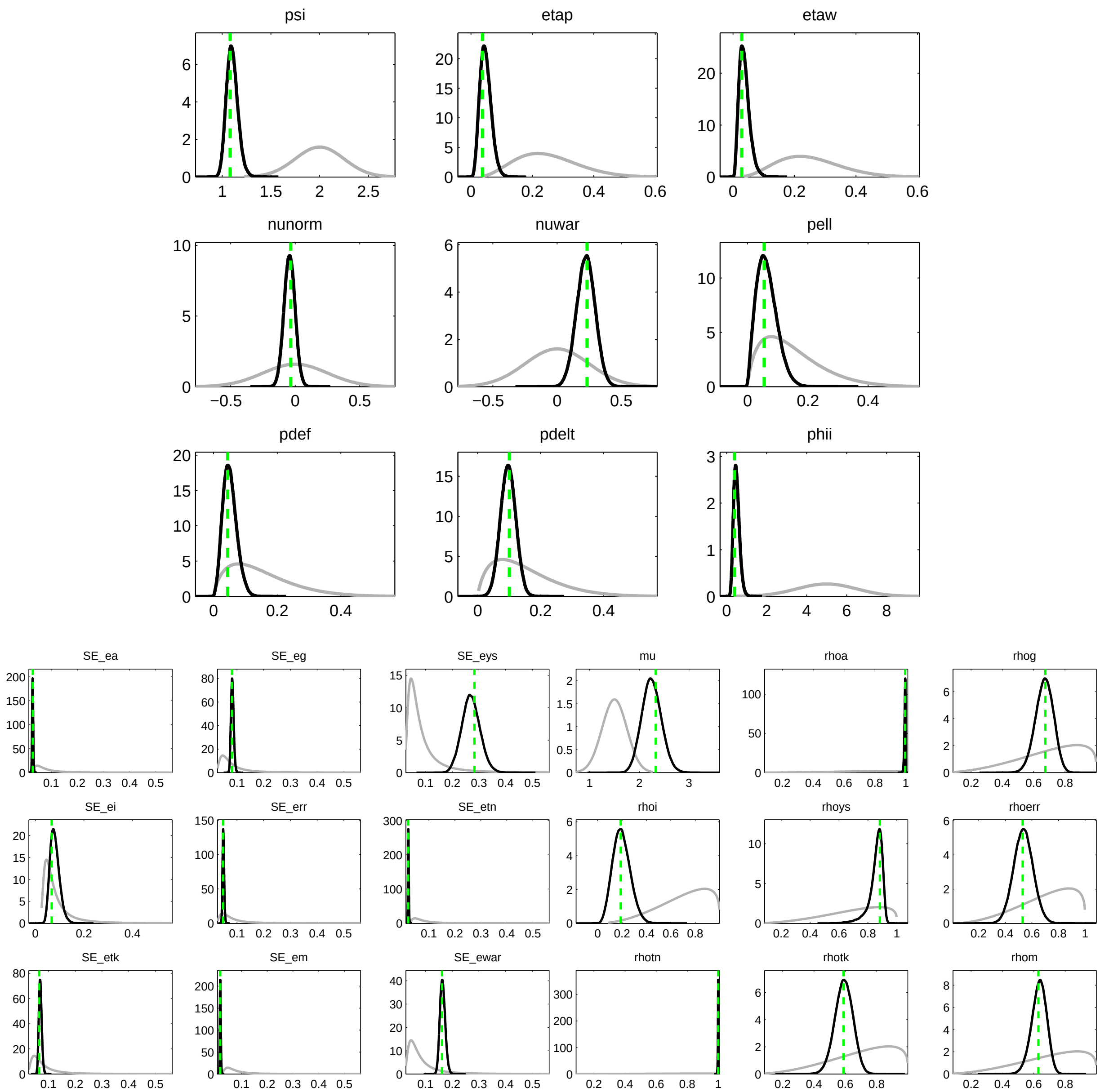
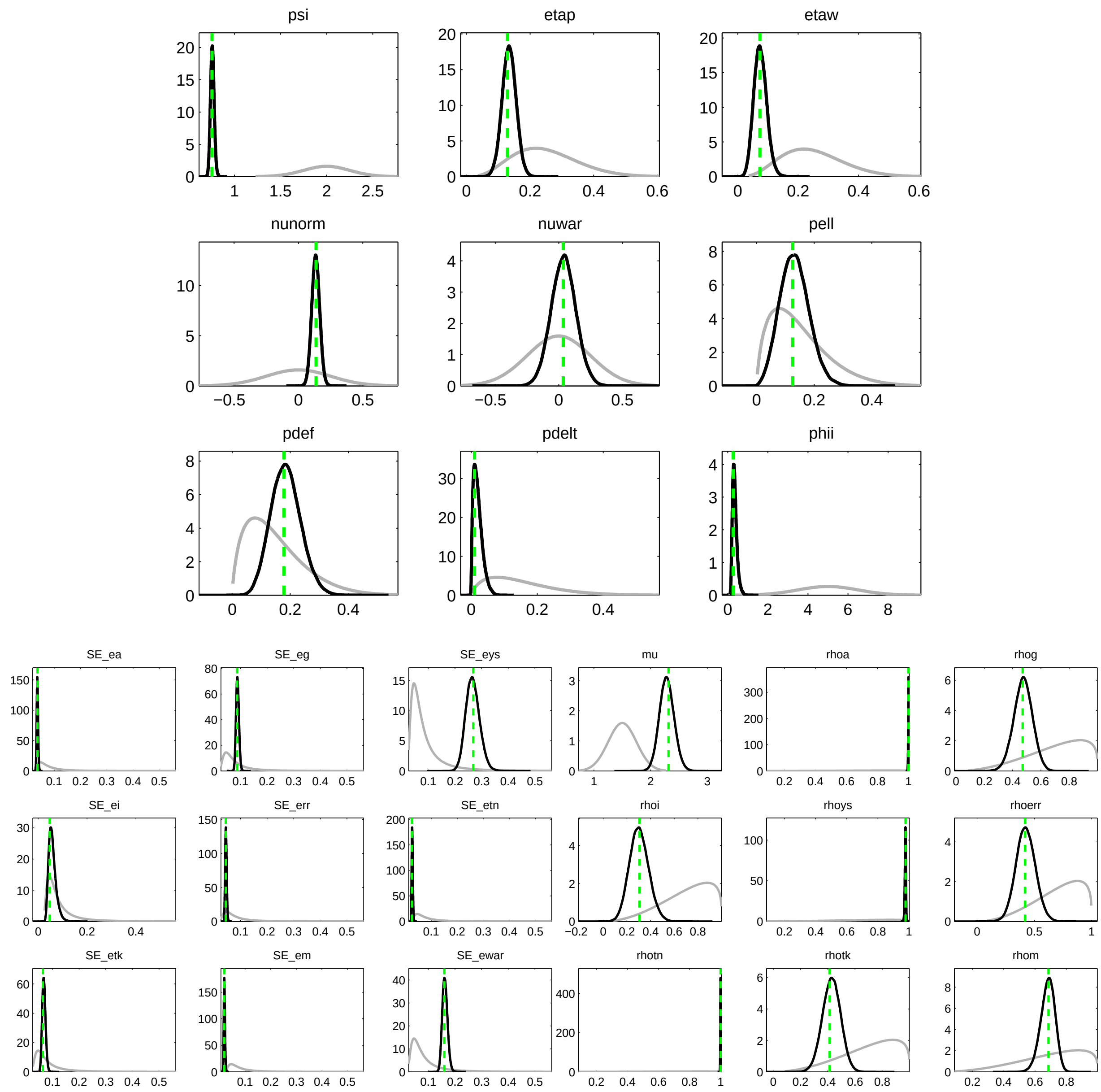


Figure 8: Priors and Posteriors - U.S.



D Welfare measure

The utility function is

$$\mathcal{U}_t = \frac{\widetilde{c}_t^{1-\sigma_c}}{1-\sigma_c} + \chi_m \frac{(m_t/p_t)^{1-\sigma_m}}{1-\sigma_m} - \chi_n \frac{n_t^{1+\psi}}{1+\psi} \tag{52}$$

where $\widetilde{c}_t = c_t + \nu_t g_t$. Neglecting the effect of real money balances

$$\mathcal{U}_t = \frac{\widetilde{c}_t^{1-\sigma_c}}{1-\sigma_c} - \chi_n \frac{n_t^{1+\psi}}{1+\psi} \tag{53}$$

Rewriting the utility function after assuming $\widehat{x}_t = \log(x_t/x)$ where x is the steady-state value of x:

$$\mathcal{U}_t \simeq \frac{\widetilde{c}^{(1-\sigma_c)} e^{(1-\sigma_c)\widehat{\widetilde{c}}_t}}{1-\sigma_c} - \chi_n \frac{n^{1+\psi} e^{(1+\psi)\widehat{n}_t}}{1+\psi} \tag{54}$$

Taking a second-order approximation gives

$$\mathcal{U}_t \simeq \mathcal{U} + \overline{c}^{(1-\sigma_c)} \left(\widehat{\widetilde{c}}_t + (1-\sigma_c)\widehat{\widetilde{c}}_t^2/2\right) - \chi_n n^{1+\psi}\left(\widehat{n}_t + (1+\psi)\widehat{n}_t^2/2\right) \tag{55}$$

Neglecting first-order (mean) effects:

$$\mathcal{U}_t \simeq \mathcal{U} + \overline{c}^{(1-\sigma_c)} (1-\sigma_c)\widehat{\widetilde{c}}_t^2/2 - \chi_n n^{1+\psi}(1+\psi)\widehat{n}_t^2/2 \tag{56}$$

Welfare thus writes

$$(1-\beta)\mathcal{W}_t \simeq (1-\beta)\mathcal{W} + \overline{c}^{(1-\sigma_c)}(1-\sigma_c)\, var\left(\widehat{\widetilde{c}}_t\right) - \chi_n n^{1+\psi}\frac{(1+\psi)}{2} var(\widehat{n}_t) \tag{57}$$

The consumption equivalent welfare losses from fluctuations Φ solve

$$\begin{aligned}(1-\beta)\left(\frac{(\widetilde{c}(1-\Phi/100))^{1-\sigma_c}}{1-\sigma_c} - \chi_n \frac{n^{1+\psi}}{1+\psi}\right) &= (1-\beta)\mathcal{W} + \overline{c}^{(1-\sigma_c)}(1-\sigma_c)\, var\left(\widehat{\widetilde{c}}_t\right) \\ &- \chi_n n^{1+\psi}\frac{(1+\psi)}{2} var(\widehat{n}_t) \end{aligned} \tag{58}$$