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We analyze the effects of large war episodes (world wars) on the macroeconomic dynamics of four advanced countries (France, Germany, the UK and the U.S.) using a structural small open economy model estimated with Bayesian techniques. Our dataset is taken from Piketty and Zucman (2014) and goes back to the XIX-th century. A typical war episode is well captured by the following combination of exogenous shocks: productivity and investment efficiency shrink, public spending rises massively, trade collapses, the discount factor goes up first and then falls significantly, and capital depreciation increases sharply. The rise in government spending is partly financed by monetization and public debt rises. After a few years, labor income taxation increases and capital income taxation falls. Our estimations also produce results about the size of government spending and tax multipliers, and the size of the welfare losses from fluctuations.

Keywords: Fluctuations, War, Trade, Taxes, Public Debt, Bayesian estimations, Multipliers, Welfare.

1 Introduction

In this paper, we investigate the systematic effects of large war episodes on the dynamics of key macroeconomic variables, with a special focus on external trade and the monetary and fiscal strategies adopted during major war episodes.

In the literature, the macroeconomics of war mostly focuses on the effects of large increases in military and defense spending along with the consequences of the corresponding financing schemes. On the theoretical side, the seminal contribution of Braun and McGrattan (1993), as well as McGrattan and Ohanian (2010) show that the standard neo-classical business cycle model fed with exogenous shocks on government spending, investment and taxes can properly account for the effects of WWII in the U.S. or in the UK. Martin (2012) models large war episodes as large government spending shocks and shows that the financing schemes observed during WWI and WWII, mostly based on public debt and inflation in the short-run, is consistent with the optimal distortion-smoothing policy with limited commitment. The paper echoes the contribution of Ohanian (1997), who shows the great importance of financing schemes in the effects of war episodes on output and welfare. On the empirical side, the focus is also very much on government spending, as the “narrative approach” highlights the role of war/peace episodes in the identification of unexpected fiscal shocks (see Ramey and Shapiro (1997), Ramey (2011b), Ramey (2011a) and more recently Ben Zeev and Pappa (2014)). However, if the major focus on public spending seems justified for the U.S. especially during WWII where output grew significantly, additional elements need to be considered to account for the dynamics of macroeconomic aggregates, in particular in countries where output shrank during war episodes.

Among these additional factors, productivity shocks, capital depreciation shocks or the open economy dimension can be considered. In particular, Francis and Ramey (2006) disentangle the extent to which macroeconomic fluctuations were due to technological vs. non-technological shocks in the U.S. history, and highlight the importance of technological shocks, especially before WWII. Besides, Auray, Eyquem and Jouneau-Sion (2014) show that capital depreciation shocks à la Ambler and Paquet (1994) crucially contribute to the macroeconomic dynamics induced by major war episodes.\(^1\) Finally, some literature highlights the importance of external trade and finance to account for the dynamics of

\(^1\)See also Furlanetto and Seneca (2014) regarding the importance of capital depreciation shocks over the business cycle in normal times.
economies during wars. Devereux and Smith (2007) analyze the Franco-Prussian 1871 war indemnity through the lens of the transfer problem, invoking the 1929 Keynes-Ohlin controversy. The possibility to account for such large transfers must be considered when analyzing periods involving large war episodes or immediate post-war periods. More generally, war episodes most often induce large disruptions in trade, that contribute to the dynamics of key domestic macroeconomic variables (see Anderton and Carter (2001) or more recently Glick and Taylor (2010)).

Our approach of the macroeconomics of large war episodes consists in estimating a medium-scale small open economy model on a set of nine macroeconomic time series for France, Germany, the UK and the U.S. The model is pretty standard and assumes trade in some components of demand (consumption and investment), sticky prices, sticky wages, and incomplete international financial markets. Other assumptions are taken from the Smets and Wouters (2005) model, except for the introduction of money in the utility function, the absence of a systematic Taylor-type monetary policy rule, and the introduction of distorsionary taxes.

The data is almost entirely taken from Piketty and Zucman (2014) and ranges from the late XIX-th century to the mid-2000’s. It provides a large quantity of useful information concerning our four countries (France, Germany, the UK and the U.S.), including national income, the share of (public plus private) consumption in national income, net exports to national income, debt to national income and the GDP deflator inflation rate. Depending on countries, our dataset also includes the labor income tax rate, the capital income tax rate, monetary aggregates, the profit rate, private investment, government investment, the pre-tax return on capital, employment, government debt nominal yields or the real wage inflation rate.

The model considers nine potential driving forces for the business cycle, among which standard shocks (productivity, public spending, investment efficiency, discount factor) and shocks that are designed to capture important features of the economy during war episodes. In particular, we consider foreign output shocks to account for the dynamics of external trade, capital depreciation shocks to capture the potential destruction of (public and private) physical capital, and fiscal and money growth rate shocks, that capture the
behavior of governments. Precisely, we assume that those shocks are purely exogenous components of the model because we want to remain agnostic about the way countries of our sample finance war-related public spending. In other words, we abstract from any kind of systematic fiscal or monetary policy rules, and let the data speak.\(^4\) The model is estimated using Bayesian techniques and we obtain point estimates for key structural parameters like those that relate to price and wage stickiness, investment adjustment cost or the trade elasticity, as well as estimates about the dynamics of driving processes.

Our main results derive from the exploitation of our Bayesian estimations, especially from the smoothed dynamics of exogenous variables.

First, we offer some narratives about the macroeconomic effects of large war episodes like WWI and WWII. Beyond country-specific features, a common pattern emerges from our graphed exogenous variables. Productivity and investment efficiency shrink, public spending rises significantly in the four countries of our sample, trade collapses, the discount factor goes up first and then falls significantly, and capital depreciation increases sharply. Those narratives are consistent with most contributions of the literature. The main specificities uncovered by our results concern the monetary and fiscal strategies adopted in the different countries. France and the UK use monetization more systematically than Germany or the U.S., and tax rates were adjusted according to various patterns.

Second, based on this first-pass analysis, we proceed more systematically and quantify the dynamic interactions between our exogenous variables using a Panel VAR in which a dummy variable capturing war episodes is included. Our Panel VAR estimates at hand, we identify the effects of war shocks as shocks to the dummy variable using a Cholesky decomposition of the variance-covariance matrix, and report the Impulse Response Functions (IRFs). This exercise confirms our narrative results and uncovers a robust financing scheme for the observed rise in government spending and investment. On average, the countries of our sample let public debt rise initially, monetize some of the current public deficits, and eventually raise the labor income tax and lower the capital income tax, although with a 4 to 6 years lag. Those results are quite in accordance with those of Ohanian (1997) and Martin (2012), except they apply more generally to both WWI and WWII, and to countries other than the U.S.

Third, we make use of our model to compute the estimated value of government spending and tax multipliers. Our estimated model produces public spending output multiplier

\(^4\)The only fiscal rule imposed is that lump-sum taxes react very mildly to deviations of debt to GDP from its steady-state value, to insure the stationarity of this variable in the long run.
that lie in the range of values found in the empirical literature, between 0.85 for France and 1.08 for the UK (see Ramey (2011a) and references therein). The U.S. are found to have a multiplier of 1.06, slightly above one, and Germany is at 0.9. The estimated degrees of price and wage stickiness, the responsiveness in labor supply and the steady-state shares of public consumption in GDP account for the cross-country differences. We also find tax output multipliers that lie in the middle of the interval suggested by empirical studies, between 0.5 and 3, depending on countries and horizons considered. The average 4 years tax multiplier is around 1.05, in line with the estimates of Barro and Redlick (2011). For the U.S. however, and to a lesser extent for the UK, the tax multiplier is found to be substantially lower. Higher public spending multipliers and somehow smaller tax multipliers compared to the literature lead to less pessimistic conclusions about the efficiency of tax-financed increases, especially in the UK and the U.S.

Fourth, using our smoothed variables, we proceed to an analysis of the welfare costs of fluctuations over different historical periods. Our main finding is that, compared to post-war welfare losses, the welfare losses from fluctuations are 2.5 to 4 times larger over the whole sample, and 3.5 to 6 times larger before 1945. Working on post-war data thus significantly underestimates the welfare losses from fluctuations. We also provide quantitative evidence that the large swings in macroeconomic time series induced by war episodes produce massive instantaneous welfare losses, as large as 15% to 17% of current consumption, and that the pre-1945 period featured a lot more volatility than the post-war period.

The paper is organized as follows. Section 2 describes our small open economy model. Section 3 presents our dataset, the estimation strategy, and reports our estimates. Section 4 offers some narratives about the dynamics of exogenous variables during war episodes, and characterizes the systematic effects of these events on macroeconomic variables. Section 5 is devoted to the analysis of public spending and tax multipliers. Section 6 presents an analysis of the welfare losses from business cycles, as well as a quantification of the instantaneous welfare losses produced by large war episodes. Section 7 concludes.

2 The model

We consider a small open economy model with a government in charge of fiscal and monetary policy. The model builds on Smets and Wouters (2005) but is extended along some dimensions. First, it considers an open economy with trade in consumption and invest-
ment in goods, home bias, and an incomplete international financial market. Second, we introduce money in the utility function and monetary policy is not conducted through a Taylor-type rule, but is affected by exogenous money growth rate shocks. In addition, seignorage revenues are transferred to the government. Both assumptions are consistent with a non-trivial role for monetary policy and for a potential financing of public spending through seignorage revenues. Third difference with Smets and Wouters (2005), the government levies distorsionary taxes on labor and capital income. Both tax rates are assumed to follow exogenous processes and that are not calibrated to balance steady-state monopolistic distorsions. The government spends on domestic consumption goods, invests in domestic public capital goods, issues government bonds, levies taxes and has access to seignorage revenues when issuing money.

2.1 Equilibrium conditions

2.1.1 Households

We consider a unit continuum of households indexed in \( j \) that maximize their lifetime welfare

\[
W_t = E_t \left\{ \sum_{s=t}^{\infty} \left( (c_s(j) - h_s(j)) \frac{1}{1 - \sigma_c} \right) + \chi_m \left( \frac{m_s(j)}{p_s} \right) \frac{1}{1 - \sigma_m} - \chi_n \left( \frac{n_s(j)}{1 + \psi} \right) \right\},
\]

(1)

where \( c_t(j) \) denotes consumption, \( h_t(j) = h_c c_{t-1}(j) \) a stock of external consumption habits, \( m_t(j) / p_t \) real money balances, \( n_t(j) \) the hours of work supplied and \( \beta_t \) the time-varying discount factor, affected by autoregressive shocks

\[
\beta_t = (1 - \rho_\beta) + \rho_\beta \beta_{t-1} + \epsilon_t^\beta
\]

(2)

The budget constraint of agent \( j \) is

\[
e_t f_t(j) + b_t(j) + m_t(j) + \pi_t \left( c_t(j) + i_t(j) + ac_t^f(j) \right) = e_t r^* f_{t-1}(j) + r_{t-1} b_{t-1}(j) + m_{t-1}(j) + \left( (1 - \tau^k_t) r^k_t z_t(j) - p_t ac_t^z(j) + p_t \tau^k_t \delta_t \right) k_{t-1}(j) + (1 - \tau^n_t) w_t(j) n_t(j) + \varphi_t(j) + tax_t(j)
\]

(3)

where \( e_t \) is the nominal exchange rate, \( f_t(j) \) denotes the nominal value of foreign bonds returning a (constant) risk-less rate \( r^* \) between period \( t \) and \( t + 1 \) and \( b_t(j) \) denotes the nominal value of government bonds returning a risk-less rate \( r_t \) between period \( t \) and \( t + 1 \).
Further, $p_t$ is the consumption price level, $i_t(j)$ is the investment in physical capital, $\tau_t^n$ is the labor income tax rate, $w_t(j)$ is the nominal wage rate paid to type-$j$ labor, $\tau_t^k$ the tax rate on capital income, that comes with a tax deduction on depreciated capital, $r_t^k$ is the gross return on the capital stock, $z_t(j)$ is the utilization rate of private capital, $\delta_t$ the time-varying depreciation rate of capital. Finally, $ac_t^i(j) = (\phi_f/2) (e_t f_t(j)/p_t - e_t f_t(j)/p)^2$ is an adjustment cost on real net foreign assets, $ac_t^z(j) = (\phi_z/2) (z_t(j) - 1)^2$ is a utilization rate adjustment cost and $\varphi_t(j)$ denotes the profits from monopolistic firms paid to household $j$. An additional constraint to this optimization problem is the law of capital accumulation

$$k_t(j) = (1 - \delta_t) k_{t-1}(j) + \zeta_t (1 - ac_t^i(j)) i_t(j)$$  \(4\)

where

$$ac_t^i(j) = \frac{\phi_i}{2} \left( \frac{i_t(j)}{i_{t-1}(j)} - 1 \right)^2$$  \(5\)

is an investment adjustment cost. In the dynamics of capital accumulation, $\zeta_t$ is an investment shock and $\delta_t$ the depreciation rate of capital. Both evolve according to autoregressive processes

$$\zeta_t = (1 - \rho_{\zeta}) + \rho_{\zeta} \zeta_{t-1} + \epsilon_t^\zeta$$  \(6\)

$$\delta_t = (1 - \rho_{\delta}) + \rho_{\delta} \delta_{t-1} + \epsilon_t^\delta$$  \(7\)

Households maximize the welfare function subject to the budget constraints with respect to consumption, government and foreign bonds, as well as the quantity of money. First-order conditions imply

$$E_t \left( \beta_{t+1} \frac{u_{c,t+1}(j)}{u_{c,t}(j)} \frac{r_t}{\pi_{t+1}} \right) = 1$$  \(8\)

$$E_t \left( \beta_{t+1} \frac{u_{c,t+1}(j)}{u_{c,t}(j)} \frac{e_{t+1} r_t^*}{e_{t+1} \pi_{t+1} (1 + \phi_f (f_t^r (j) - f_r))} \right) = 1$$  \(9\)

$$u_{m,t}(j) - u_{c,t}(j) \left( \frac{r_t - 1}{r_t} \right) = 0$$  \(10\)

where $\pi_t = p_t/p_{t-1}$, $u_{c,t}(j)$ is the marginal utility of consumption, $u_{m,t}(j)$ the marginal utility of real money balances, and $f_t^r(j) = e_t f_t(j)/p_t$ the real amount of net foreign assets. The different types of labor offered by households are imperfectly substitutable
making them monopolistic wage-setters. They take firms’ labor demands

\[ n_t(j, \omega) = \left( \frac{w_t(j)}{w_t} \right)^{-\theta^w} n_t(\omega) \]  

into account along with their budget and capital accumulation constraints when solving their optimization program. Further, we assume sticky wages whereby wage-setters face an individual probability \(1 - \eta^w\) to be allowed to re-optimize and a probability \(\eta^w\) to keep their previous-period nominal wage unchanged. The corresponding optimal nominal wage \(w_t(j)\) is thus

\[ \sum_{i=0}^{\infty} (\beta_{t+i} \eta^w)^i E_t \left( n_{t+i}(j) \frac{\theta^w}{\theta^w - 1} \frac{u_{t+i}(j)}{u_{c,t+i}(j)} + (1 - \tau^w) \frac{w_t(j)}{p_{t+i}} \right) = 0 \]  

and the dynamics of nominal wages is

\[ w_{t+1}^{1-\theta^w} = \eta^w (w_{t-1})^{1-\theta^w} + (1 - \eta^w) w_t(j)^{1-\theta^w} \] 

From now on we assume perfect risk-sharing among households of the domestic economy. Households \(j\) are thus identical and we drop the \(j\) indices. Defining \(q_t \rho_t \lambda_t\) as the Lagrange multiplier associated with the capital accumulation constraint, first-order conditions with respect to the capital stock, and investment are respectively

\[ E_t \left( \beta_{t+1} \frac{u_{c,t+1}}{u_{c,t}} \left( q_{t+1} (1 - \delta_{t+1}) + (1 - \tau^k_{t+1}) \frac{r^k_{t+1}}{p_{t+1}} + \tau^k_{t+1} \delta_{t+1} \right) \right) = q_t \]  

\[ q_t \zeta_t \left( 1 - \frac{\phi_i}{2} \gamma_t^2 - \phi_i \gamma_t (1 + \gamma_t) \right) + E_t \left( \beta_{t+1} \frac{u_{c,t+1}}{u_{c,t}} q_{t+1} \zeta_{t+1} \phi_i \gamma_{t+1} (1 + \gamma_{t+1})^2 \right) = 1 \]  

where \(\gamma_t = (i_t/i_{t-1}) - 1\) is the growth rate of private investment. Consumption and capital goods are made of domestic \((d)\) and foreign \((f)\) goods. Adjustment costs are paid in units of that composite good as well. The latter is defined as

\[ x_t = \left( (1 - \alpha) \frac{1}{\mu} x_{d,t}^{\mu-1} + \alpha \frac{1}{\mu} x_{f,t}^{\mu-1} \right)^{\frac{\mu}{\mu-1}}, \quad x = \{ c, i, ac^f, ac^z, ac^i \} \]  

where \(1 - \alpha\) is the degree of home bias and \(\mu\) the elasticity of substitution between domestic
and foreign goods. The corresponding price index is

\[ p_t = \left( (1 - \alpha) p_{d,t}^{1-\mu} + \alpha (e_t p_{f,t})^{1-\mu} \right)^{\frac{1}{1-\mu}} \] (18)

Optimal expenditure on each good implies

\[ x_{d,t} = (1 - \alpha) \left( \frac{p_{d,t}}{p_t} \right)^{-\mu} x_t = (1 - \alpha) \left( 1 + \alpha \frac{1}{s_t^{1-\mu}} \right)^{\frac{\mu}{1-\mu}} x_t \] (19)

\[ x_{f,t} = \alpha \left( \frac{e_t p_{f,t}}{p_t} \right)^{-\mu} x_t = \alpha \left( 1 + \alpha s_t^{\mu-1} + \alpha \right)^{\frac{\mu}{1-\mu}} x_t \] (20)

for \( x = \{ c, i, ac^f, ac^z, ac^i \} \) where \( s_t = e_t p_{f,t}/p_{d,t} \) stands for terms of trade.

### 2.1.2 Firms

There is a continuum of final good producers indexed in \( \omega \), with the following production function

\[ y_t(\omega) = a_t k_t^s(\omega)^\ell_t(\omega)^{1-\ell} \] (21)

where \( k_t^s \) is a measure of capital services used in production and \( a_t \) is a productivity measure following an autoregressive process

\[ a_t = (1 - \rho_a) + \rho_a a_{t-1} + \varepsilon_t^a \] (22)

Cost minimization implies

\[ k_t^s(\omega) = \frac{\ell_t}{1 - \ell_t} \frac{w_t}{r_t} \ell_t(\omega) \] (23)

which can be used to derive an expression of the nominal marginal cost

\[ mc_t(\omega) = mc_t = (a_t)^{-1} \left( r_t^k \right)^\ell (w_t)^{1-\ell} (1 - \ell)^{-1} (1 - \ell)^{(1-\ell)} \] (24)

The adjustment of production prices is also subject to Calvo contracts. Re-setters face the following problem

\[ \max_{p_{d,t}(\omega)} \sum_{i=0}^{\infty} (\beta_{i+1} \eta^p)^i E_t \left( \frac{u_{c,t+i}}{p_t} \frac{p_t}{p_t+i} (p_{d,t}(\omega) y_{t+i}(\omega) - mc_t+i y_{t+i}(\omega)) \right) \] (25)
taking into account the demand addressed to firm $\omega$

$$y_{d,t}(\omega) = \left(\frac{p_{d,t}(\omega)}{p_{d,t}}\right)^{-\theta^p}y_{d,t} \quad (26)$$

The optimal pricing condition is thus

$$\sum_{i=0}^{\infty} (\beta_{t+i} \eta^p)^{i} E_t \left( \frac{u_{c,t+i}}{p_{t+i}} y_{t+i}(\omega) \left( p_{d,t}(\omega) - \frac{\theta^p}{\theta^p - 1} m_{t+i} \right) \right) = 0 \quad (27)$$

while the dynamics of prices is given by

$$p_{d,t}^{1-\theta^p} = \eta^p (p_{d,t-1})^{1-\theta^p} + (1 - \eta^p) p_{d,t}(\omega)^{1-\theta^p} \quad (28)$$

### 2.2 Monetary policy, government and aggregation

The government is in charge of both monetary, public spending and fiscal policy and its budget constraint is

$$b_t + m_t + \tau^n w_t n_t + \tau^k (r^k - p_t \delta_t) k_{t-1} + t a x_t = r_{t-1} b_{t-1} + m_{t-1} + p_{d,t} (g_t + i^g_t) \quad (29)$$

The amount spent by the government on domestic consumption goods $g_t$ is exogenous and subject to autoregressive shocks

$$g_t = (1 - \rho_g) + \rho_g g_{t-1} + \varepsilon^g_t \quad (30)$$

Further, as in Auray et al. (2014), the government adjusts the amount of public investment to secure a certain constant level of public unproductive capital $k^g$

$$i_t^g \zeta^g_t = \delta k^g \quad (31)$$

The government finances its expenditure by levying tax, issuing public debt or by increasing the amount of circulating money, thereby raising seignorage revenues. Monetary policy is assumed to be conducted through a simple exogenous money growth rule

$$\mu^m_t = (1 - \rho_m) + \rho_m \mu^m_{t-1} + \varepsilon^m_t \quad (32)$$
where $\mu_t^m = m_t/m_{t-1}$. We also consider exogenous tax rate dynamics

$$
\tau_t^n = (1 - \rho_{\tau^n}) \tau^n + \rho_{\tau^n} \tau_{t-1}^n + \varepsilon_t^n \tag{33}
$$

$$
\tau_t^k = (1 - \rho_{\tau^k}) \tau^k + \rho_{\tau^k} \tau_{t-1}^k + \varepsilon_t^k \tag{34}
$$

The stability of public debt to GDP in the long run is insured by having the lump-sum tax $\text{tax}_t$ to adjust very slowly over time

$$
\text{tax}_t = \phi_b \left( \frac{b_t^*}{y_t} - b^*/y \right) \tag{35}
$$

where the value of $\phi_b$ is set to the minimum value that ensures stationarity.

We define an equilibrium as a path of endogenous variables that, conditional on the dynamics of exogenous variables, satisfies households and firms first-order conditions, and that ensures market clearing.

Equilibrium in the labor markets implies

$$
\ell_t = \int_0^1 \ell_t(\omega) \, d\omega = \int_0^1 \int_0^1 n_t(j, \omega) \, dj \, d\omega = \Upsilon_t w_t n_t \tag{36}
$$

where $\Upsilon_t w_t = \int_0^1 (w_t(j)/w_t)^{-\theta} \, dj$ denotes the dispersion of wages. Equilibrium in the physical capital market gives

$$
k_t^s = \int_0^1 k_t^s(\omega) \, d\omega = z_t k_{t-1} \tag{37}
$$

and equilibrium in goods markets implies

$$
y_t = (1 - \alpha) \left( \frac{p_d t}{p_t} \right)^{-\mu} y_t^d + \alpha \left( \frac{p_d t}{p_t} \right)^{-\mu} y_t^* + g_t + i_t^g \tag{38}
$$

where $y_t^d = c_t + i_t + \phi f (f_t^r - f^r)^2 / 2 + (\phi_z (z_t - 1)^2 / 2) k_{t-1}$ and $y_t^*$ is the foreign counterpart of $y_t^d$, subject to autoregressive shocks

$$
y_t^* = (1 - \rho y^*) y^* + \rho y^* y_t^* + \varepsilon_t^y \tag{39}
$$

Finally, using other aggregate conditions, the aggregate production function is

$$
\Upsilon_t y_t = a_t (z_t k_{t-1})^\gamma (\Upsilon_t w_t n_t)^{1-\gamma} \tag{40}
$$
where $\Upsilon_p^t = \int_0^1 (p_{d,t}(\omega)/p_{d,t})^{-\theta_p} d\omega$ denotes the dispersion of producer prices. This small open economy model is closed by the net foreign asset equation, obtained by consolidating households, firms and government budget constraints

$$f_t^r = \left(\frac{e_t}{e_{t-1}}\right) \frac{r_t^s f_{t-1}^r}{\pi_t} + nx_t$$

(41)

where $nx_t$ represents net exports, defined as

$$nx_t = \frac{p_{d,t}}{p_t} (y_t - g_t - i_t^p) - c_t - i_t - \frac{\phi_f}{2} (f_t^r - f^r)^2 - \frac{\phi_z}{2} (z_t - 1)^2 k_{t-1}$$

(42)

3 Estimation

We estimate our model using Bayesian methods, adopting the standard approach of An and Schorfheide (2007). This implies obtaining the posterior distribution of our estimated parameters based on the linear approximation of the model’s solution around the steady state using the Kalman filter. A major advantage of the approach is that it allows for the extraction of the dynamics of shocks, as well as the historical paths of endogenous variables. We will therefore have a complete quantitative evaluation of the model with respect to the data.

Steady state. We analyze the linearized dynamics of the model around a symmetric steady-state without inflation, implying zero net foreign assets.\(^5\) Our goal is to identify the sources of macroeconomic fluctuations with a specific interest in war periods, focusing on France, Germany, the UK and the U.S. The model is flexible enough to allow for cross-country differences and remains agnostic on the behavior of the government, considering variations of policy instruments as purely exogenous.

Calibrated parameters. Before estimating the model, we choose to fix the value of a variety of parameters, either to preserve the model’s steady state or because of weak identification from the data. The model is yearly. The steady-state discount factor is $\beta = 0.96$, producing an average 4.17% real interest rate. The risk-aversion parameters on consumption and real money balances are respectively $\sigma_c = 1.5$ and $\sigma_m = 1.5$, and the external habit parameter is $h_c = 0.7$. The steady-state capital depreciation rate is $\delta = 10\%$, the adjustment cost on utilization is $\phi_z = 0.15$ and the capital share is $\iota = 0.36$. The import share is $\alpha = 0.15$ and the adjustment cost on foreign assets is $\phi_f = 0.025$, as

\(^5\)Details about the steady state are given in Appendix A.
in Ghironi and Melitz (2005). Those values correspond to the consensual values reached by the DSGE literature. The value of the labor disutility parameter $\chi_n$ is adjusted to normalize hours of work to one, which makes our choice for the calibrated value of the wage mark-up ($1/ (\theta_w - 1) = 20\%$) basically inessential (see Appendix A for details). Finally, the real balances utility parameter is set to $\chi_m = 0.05$, to produce steady-state real balances to GDP between 0.2 and 0.25, in accordance with the data (considering deposits plus coins and bills over national income). Those parameters remain fixed across countries and their values are summarized in Table 1 below.

<table>
<thead>
<tr>
<th>Table 1: Calibrated common parameter values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor</td>
</tr>
<tr>
<td>Risk-aversion (consumption)</td>
</tr>
<tr>
<td>Risk-aversion (real money balances)</td>
</tr>
<tr>
<td>External habit parameter</td>
</tr>
<tr>
<td>Capital depreciation</td>
</tr>
<tr>
<td>Utilization adjustment cost</td>
</tr>
<tr>
<td>Capital share</td>
</tr>
<tr>
<td>Import share</td>
</tr>
<tr>
<td>Adjustment cost on foreign assets</td>
</tr>
<tr>
<td>Real money utility parameter</td>
</tr>
<tr>
<td>Wage mark-up</td>
</tr>
<tr>
<td>Labor disutility parameter</td>
</tr>
</tbody>
</table>

*Country-specific calibrated parameters.* Remaining calibrated parameters are country specific, mostly to account for differences in the tax systems, the levels of public consumption to GDP ($\kappa$) or public debt to GDP. Using average values of tax rates in France, we set the steady state tax rates to $\tau^n = 0.3$ and $\tau^k = 0.22$. The average government investment rate, taken from our French data is $\kappa^i = 0.0253$. We adjust public consumption to GDP $\kappa = 0.1346$ (against an average 0.1538 in the data) to match exactly the average level of debt to GDP (75%) over our sample. Last, we set $\theta^p = 6$ to match average net profits (20% in our sample). In the absence of empirical evidence for profits in Germany, we set $\theta^p = 6$. German data suggest $\kappa^i = 0.0164$ and $\kappa = 0.1490$. The average level of capital income taxation in our data implies $\tau^k = 0.2$. We adjust $\tau^n = 0.2926$ to match the average level of debt to GDP (45%) in our sample. Concerning the UK, our time series average indicate a profit rate of 25%, implying $\theta^p = 5$. Average government investment to GDP over our sample is $\kappa^i = 0.0263$. We assume $\tau^k = 0.2$ and $\tau^n = 0.3$, and adjust $\kappa = 0.1138$ to hit the 97% average debt-to-GDP ratio of our data sample. Finally, for the U.S., based on sample averages, we set $\kappa^i = 0.0190$, impose $\kappa = 0.10$ with $\tau^k = 0.35$ and adjust $\tau^n = 0.1678$ to
hit the average debt-to-GDP ratio of our sample (55%). In our estimations, we also fix the value of the inverse of the Frisch elasticity.\textsuperscript{6} In accordance with macro estimates, we set $\psi = 2$ for France and Germany and assume more responsiveness in hours worked for the UK and the U.S., setting $\psi = 1$. Those parameter values are summarized in Table 2 below.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>France</th>
<th>Germany</th>
<th>UK</th>
<th>U.S.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share of public investment in GDP ($\kappa^i$, in %)</td>
<td>2.53</td>
<td>1.64</td>
<td>2.63</td>
<td>1.90</td>
</tr>
<tr>
<td>Share of public consumption in GDP ($\kappa$, in %)</td>
<td>13.46</td>
<td>14.90</td>
<td>11.38</td>
<td>10.00</td>
</tr>
<tr>
<td>Labor income tax rate ($\tau^n$, in %)</td>
<td>30.00</td>
<td>29.26</td>
<td>30.00</td>
<td>16.78</td>
</tr>
<tr>
<td>Capital income tax rate ($\tau^k$, in %)</td>
<td>22.00</td>
<td>20.00</td>
<td>20.00</td>
<td>35.00</td>
</tr>
<tr>
<td>Steady-state price mark-up ($1/(\theta^p - 1)$, in %)</td>
<td>20.00</td>
<td>20.00</td>
<td>25.00</td>
<td>20.00</td>
</tr>
<tr>
<td>Inverse of the Frisch elasticity ($\psi$)</td>
<td>2.00</td>
<td>2.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

**Estimated parameters.** Remaining parameters of the model are estimated. Our choices for priors about estimated parameters are the following. Calvo parameters $\eta^p$ and $\eta^w$ are Beta distributions with prior mean 0.5 and variance 0.1. The trade elasticity $\mu$ is a Normal with prior mean 1.5, as in the standard international RBC literature (see Backus, Kehoe and Kydland (1993) and the abundant subsequent literature), and the variance is 0.25. For the investment adjustment cost parameter, we follow Smets and Wouters (2003) and assume that $\phi_i$ is a Normal with prior mean 5 and variance 1.5. Finally, the persistence of forcing processes are Beta distributions with prior means 0.7 and variances 0.2 and the standard deviations of innovations are Inverse Gamma distributions, with prior means 0.1 and infinite variances.

**Data.** Before presenting our estimation results, we describe our dataset. With nine shocks perfect identification requires nine time series. Unless stated otherwise, our data come from Piketty and Zucman (2014)’s effort to unify existing data and build national accounts on a very long period of time. For France (1898-2006), Germany (1871-2008), UK (1870-2005) and the US (1871-2010), we have time series for national income in real terms, expressed in billions of 2008 national currency ($y_t$), consumption expenditure (public and private) as a percentage of national income (($c_t + g_t)/y_t$), net exports as a percentage of national income ($nx_t/y_t$), debt to national income ($b^r_t/y_t$ – completed with Carmen Reinhart’s data) and the inflation rate ($\pi_t$). Remaining time series are country-specific.

\textsuperscript{6}We did try to estimate this parameter but identification tests suggested that estimates suffered from weak identification, a result that is already well documented in the literature (see the discussion in Justiniano and Preston (2010) for example).
For France we have net profits to national income ($\phi t / yt$), the labor and capital income tax rates in percents ($\tau^a_t$ and $\tau^k_t$) and the sum of deposits plus coins and bills, as a percentage of national income ($mt / yt$). For Germany, we have the pre-tax return on capital in percents ($r^k_t$), the tax rate on capital income in percents ($\tau^k_t$), government investment ($i^g_t / yt$) and domestic private investment ($i^d_t / yt$), both as a percentage of national income. For the UK, the five common time series are completed with net profits to national income ($\phi t / yt$), total tax receipts to national income, deposits plus coins and bills as a percentage of national income ($mt / yt$) and the number of persons employed ($nt$). For the US, our five reference time series are augmented with the annual yield on government debt in percents ($rt$), total government receipts as a percentage of national income (data from Mitchell before 1947 and FRED after 1947), the savings rate and the real wage inflation rate in percents ($w^r_t / w^r_{t-1}$ – data from MeasuringWorth, computed using nominal hourly production compensation in the manufacturing sector and the GDP deflator, see Officer and Williamson (2014)). As in Correia, Neves and Rebelo (1992), data in levels (national income, employment) and rates (inflation, wage inflation, interest rates) are taken in logs and ratios are taken in level before HP-filtering the time series using $\lambda = 400$. Missing data points are handled as follows. Series with missing data are first interpolated with a spline before HP-filtering. Then the interpolated missing observations are removed, to be properly handled as missing observations by the estimation algorithm. We use the Dynare built-in estimation routine, that copes with missing observations treating them as unobserved states and using the Kalman filter to infer their values.

**Estimates.** Table 3 and Table 4 report the estimated parameters with 95% confidence intervals for each of the four countries, based on 500,000 replications of the MH algorithm where the first 100,000 were discarded and the scale parameter adjusted to hit a 25% acceptance rate target.

Prior and posterior distributions are reported in Appendix B and show that the estimation procedure provides enough information to significantly update prior distributions. Price Calvo parameters governing the extent of price stickiness lie between 0.26 (for France) and 0.82 (for the U.S.), with tight confidence intervals, implying that firms reset prices

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7For net profits and capital income tax rates, clear outliers in 1944 and 1945 are considered as missing points. Deposits and coins and bills data are missing in 1941 and from 1974 to 1980. See Piketty and Zucman (2014) and references therein for further details.

8For Germany, consumption expenditure to national income, return on capital, the capital income tax rate, government investment and domestic private investment have missing points from 1914 to 1924, and from 1939 to 1949. Net export to national income have missing points from 1919 to 1924 and from 1939 to 1949. Finally, inflation data in 1922 and 1923 are considered as missing.
### Table 3: Country-specific estimated parameters (France and Germany)

<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>Priors</th>
<th>Post. (France)</th>
<th>Post. (Germany)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dist.</td>
<td>Mean</td>
<td>Var.</td>
</tr>
<tr>
<td>Calvo parameter – prices ($\eta^w$)</td>
<td>$B$</td>
<td>0.50</td>
<td>0.10</td>
</tr>
<tr>
<td>Calvo parameter – wages ($\eta^m$)</td>
<td>$B$</td>
<td>0.50</td>
<td>0.10</td>
</tr>
<tr>
<td>Investment adj. cost ($\phi_m$)</td>
<td>$N$</td>
<td>5.00</td>
<td>1.50</td>
</tr>
<tr>
<td>Trade elasticity ($\mu$)</td>
<td></td>
<td>1.50</td>
<td>0.25</td>
</tr>
<tr>
<td>Pers. – productivity ($\rho_a$)</td>
<td>$B$</td>
<td>0.70</td>
<td>0.20</td>
</tr>
<tr>
<td>Pers. – public spending ($\rho_g$)</td>
<td>$B$</td>
<td>0.70</td>
<td>0.20</td>
</tr>
<tr>
<td>Pers. – investment ($\rho_i$)</td>
<td>$B$</td>
<td>0.70</td>
<td>0.20</td>
</tr>
<tr>
<td>Pers. – foreign demand ($\rho_{y^f}$)</td>
<td>$B$</td>
<td>0.70</td>
<td>0.20</td>
</tr>
<tr>
<td>Pers. – discount factor ($\rho_d$)</td>
<td>$B$</td>
<td>0.70</td>
<td>0.20</td>
</tr>
<tr>
<td>Pers. – capital depreciation ($\rho_d$)</td>
<td>$B$</td>
<td>0.70</td>
<td>0.20</td>
</tr>
<tr>
<td>Pers. – labor income tax ($\rho_{r^w}$)</td>
<td>$B$</td>
<td>0.70</td>
<td>0.20</td>
</tr>
<tr>
<td>Pers. – capital income tax ($\rho_{r^m}$)</td>
<td>$B$</td>
<td>0.70</td>
<td>0.20</td>
</tr>
<tr>
<td>Pers. – money growth ($\rho_m$)</td>
<td>$B$</td>
<td>0.70</td>
<td>0.20</td>
</tr>
<tr>
<td>Std – productivity shocks</td>
<td>$IG$</td>
<td>0.10</td>
<td>$Inf$</td>
</tr>
<tr>
<td>Std – public spending shocks</td>
<td>$IG$</td>
<td>0.10</td>
<td>$Inf$</td>
</tr>
<tr>
<td>Std – investment shocks</td>
<td>$IG$</td>
<td>0.10</td>
<td>$Inf$</td>
</tr>
<tr>
<td>Std – foreign demand shocks</td>
<td>$IG$</td>
<td>0.10</td>
<td>$Inf$</td>
</tr>
<tr>
<td>Std – discount factor shocks</td>
<td>$IG$</td>
<td>0.10</td>
<td>$Inf$</td>
</tr>
<tr>
<td>Std – capital depreciation shocks</td>
<td>$IG$</td>
<td>0.10</td>
<td>$Inf$</td>
</tr>
<tr>
<td>Std – labor income tax shocks</td>
<td>$IG$</td>
<td>0.10</td>
<td>$Inf$</td>
</tr>
<tr>
<td>Std – capital income tax shocks</td>
<td>$IG$</td>
<td>0.10</td>
<td>$Inf$</td>
</tr>
<tr>
<td>Std – money growth shocks</td>
<td>$IG$</td>
<td>0.10</td>
<td>$Inf$</td>
</tr>
</tbody>
</table>

Notes: Results based on 500,000 replications of the MH algorithm. Standard deviations are expressed in percents. N, B and IG respectively denote Normal, Beta and Inverse Gamma distributions. Marginal data density is the harmonic mean. Prior and posterior distributions are available in Appendix B.
Prior and posterior distributions are available in Appendix B. Marginal data density is the harmonic mean. Prior and posterior distributions are available in Appendix B.

<table>
<thead>
<tr>
<th>Table 4: Country-specific estimated parameters (UK and U.S.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Priors</td>
</tr>
<tr>
<td>Dist.</td>
</tr>
<tr>
<td>Calvo parameter – prices ($\eta^w$)</td>
</tr>
<tr>
<td>Calvo parameter – wages ($\eta^w$)</td>
</tr>
<tr>
<td>Investment adj. cost ($\phi_i$)</td>
</tr>
<tr>
<td>Trade elasticity ($\mu$)</td>
</tr>
<tr>
<td>Pers. – productivity ($\rho_a$)</td>
</tr>
<tr>
<td>Pers. – public spending ($\rho_g$)</td>
</tr>
<tr>
<td>Pers. – investment ($\rho_\zeta$)</td>
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<tr>
<td>Pers. – foreign demand ($\rho_y$)</td>
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<tr>
<td>Pers. – discount factor ($\rho_\beta$)</td>
</tr>
<tr>
<td>Pers. – capital depreciation ($\rho_b$)</td>
</tr>
<tr>
<td>Pers. – labor income tax ($\rho_{\tau^l}$)</td>
</tr>
<tr>
<td>Pers. – capital income tax ($\rho_{\tau^c}$)</td>
</tr>
<tr>
<td>Pers. – money growth ($\rho_m$)</td>
</tr>
<tr>
<td>Std – productivity shocks</td>
</tr>
<tr>
<td>Std – public spending shocks</td>
</tr>
<tr>
<td>Std – investment shocks</td>
</tr>
<tr>
<td>Std – foreign demand shocks</td>
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<tr>
<td>Std – discount factor shocks</td>
</tr>
<tr>
<td>Std – capital depreciation shocks</td>
</tr>
<tr>
<td>Std – labor income tax shocks</td>
</tr>
<tr>
<td>Std – capital income tax shocks</td>
</tr>
<tr>
<td>Std – money growth shocks</td>
</tr>
</tbody>
</table>

Marginal data density: 2069, 2019

Notes: Results based on 500,000 replications of the MH algorithm. Standard deviations are expressed in percents. N, B and IG respectively denote Normal, Beta and Inverse Gamma distributions. Marginal data density is the harmonic mean. Prior and posterior distributions are available in Appendix B.
on average every 1.3 to 6 years. While a bit large for the U.S., estimated values for this parameter are still reasonable with respect to the literature, although the latter more often relies on quarterly models and data to uncover estimates. Wage Calvo parameters lie between 0.35 (for Germany) and 0.75 (for the U.S.).

Estimated investment adjustment costs $\phi_i$ are well above the prior mean value in the UK and the U.S. (13.25 and 10.44 respectively) but well below for France and Germany (0.87 and 0.46 respectively). This result is most certainly due to the different calibrations of the labor supply elasticity and the estimated degrees of price and wage stickiness. The last parameter that does not relate to exogenous processes is the trade elasticity. This parameter is usually the source of empirical controversies in the literature. Micro-econometric evidence suggests large values (between 6 and 10), and macro-econometric results suggest much lower values, typically around unity. Our results clearly comfort macro-econometric studies, as the trade elasticity is very close to unity for all countries, slightly but significantly above one for Germany (1.16) and the UK (1.09) and slightly but significantly below one for France (0.85) and the U.S. (0.93).

We do not comment extensively the characteristics of estimated parameters governing the dynamics of exogenous shocks but most persistence estimates are in line with common values found in the empirical literature. One notable exception is the persistence of money growth rate shocks for France, the UK and the U.S. (below 0.15 and as low as 0.01 for the U.S.) while estimates are usually larger in the literature, more in line with the persistence estimated for Germany (0.73).

4 The macroeconomic effects of a war

4.1 The dynamics of exogenous variables

Before we further analyze the effects of war episodes on key macroeconomic variables, we examine in Figure 1 to 4 the dynamics of exogenous variables and provide some preliminary narratives about these effects.

Figure 1 shows that during both WWI and WWII, France experienced a slowdown in productivity, a rise in public spending and a sharp fall in foreign output. Real money balances increased significantly during both world wars but most impressively during WWII, suggesting that France inflated its public debt away at this time. Investment efficiency rose before world wars and fell during war episodes, slowing down private investment. In
Figure 1: Estimated dynamics for exogenous variables (deviations) – France
Figure 2: Estimated dynamics for exogenous variables (deviations) – Germany
Figure 3: Estimated dynamics for exogenous variables (deviations) – UK
Figure 4: Estimated dynamics for exogenous variables (deviations) – U.S.
addition, war periods implied large upward shifts in the discount factor, and major spikes in the capital depreciation rate. The monetary and fiscal strategy of France during world wars was complemented by an important fall in the labor income tax rate. The capital income tax rate fell during WWI and rose massively during WWII.

As shown by Figure 2, the dynamics of the German economy during war episodes shares some similarities with the French one, but is also specific along some dimensions. Productivity fell during war episodes. The interwar period is also marked by a very large fall in productivity in 1929. Public spending rose massively during both world wars, but the increase starts slightly before the war in the case of WWII. The dynamics of foreign output shocks suggests that war episodes depressed trade, although Germany experienced a late fall during WWII, preceded by a large positive “shock” in the immediate pre-war period, that corresponds to the massive transfers from invaded territories just before and at the beginning of the war. Noticeably, the size of negative foreign output shocks experienced during war episodes is much smaller for Germany than it is in the French case. Investment efficiency was affected by both world war episodes but a little bit like productivity, not as much as during the 1929 Great Depression. Germany did also experience significant shifts in the discount factor and, like France, a rise of the capital depreciation rate during war episodes. However, the 1929 depression had larger effects on the capital depreciation rate. Fiscal and monetary dynamics during world wars show that Germany adopted alternative strategies, raising first and then lowering the labor income tax rate during WWI (the opposite during WWII) and lowering the capital income tax rate during WWI. The monetization of deficits during world wars does not appear clearly from the dynamics of real money balances, and does not seem to be part of the German fiscal strategy. Interestingly, our model accurately captures the 1923 hyperinflation episode during which inflation was higher than the money growth rate, inducing real money balances to collapse dramatically this particular year.

Now turning to the UK, Figure 3 shows that productivity followed a different pattern compared to France and Germany, as it fell during WWI and rose during WWII. As in France and Germany however, public spending went up significantly and foreign output fell, suggesting a worldwide economic slump, producing a fall in external trade that affected the three European countries of our sample. Further, as in France and Germany, WWII produced large pre-war upward spikes in investment efficiency followed by significant dips. As for France, the discount factor shows large upward shifts, and just like France and Germany, the capital depreciation rate shows large spikes. The fiscal
and monetary strategy consisted of a monetization of deficits, illustrated by the rise of real money balances. The labor income tax rate fell during war episodes and rose in the immediate post-war period, the capital income tax rate shown opposite movements, going up during war times and down after war periods. Overall this strategy has a lot in common with the French strategy.

Finally the case of the U.S. is depicted in Figure 4. It also shares common features with European countries. War episodes triggered downward productivity breaks (especially WWII) when wars begin, followed by large rises in productivity at the end of wars. This pattern is consistent with the fact that GDP actually rose during war episodes in the U.S. As in other countries, the U.S. experienced large upward shifts in public spending. Foreign output went down for a few years but went up quite significantly in the immediate post-war periods, especially after WWI. Investment efficiency collapsed during war episodes, the discount factor rose significantly, especially during WWI, and the capital depreciation rate was affected. According to our estimations, the fiscal and monetary strategy was a bit different for WWI and for WWII. The first world war was characterized by a fall in real money balances, a rise in the labor income tax rate and a deep fall in the capital income tax rate. WWII is characterized by a moderate monetization of public debt, a small rise in the labor income tax rate, and a rise and fall of the capital income tax rate.

Beyond the specificities highlighted above, common features emerge about the macroeconomic dynamics induced by war episodes. Productivity and investment efficiency go down, public spending skyrockets, foreign output and trade are depressed, the discount factor goes up first and then falls significantly, and capital depreciation increases. The most striking differences between the four countries under consideration relate to the monetary and fiscal strategies: France and the UK used monetization more systematically than Germany or the U.S., and tax rates were adjusted according to various patterns. Although the immediate response of tax rates to war episodes does not show clear regularities, the most common lagged response seems to consist in raising the labor income tax rate and lowering the capital income tax rate. Nevertheless, the above analysis builds on visual inspection while a more systematic analysis can be conducted. Next section proposes a Structural VAR (SVAR) analysis of the dynamics of our four economies during war episodes.

4.2 SVAR analysis

With the above narrative in mind, we now proceed to a systematic analysis of the effects of a war. We consider nine key smoothed variables taken from the estimations together
with a dummy variable indicating war periods and estimate a VAR. We then identify a war shock as a shock to the dummy variable using a Cholesky decomposition of the variance-covariance matrix, and consider the IRFs. This can be done in the cross-section or in panel. We choose to report results for a panel VAR for two reasons. First, using the panel approach with pooled data is more powerful statistically. Second, it allows us to disentangle the macroeconomic effects that are robust among all countries from those that are less robust.

**Data and estimation.** Our dataset consists of smoothed variables produced by our Bayesian estimation. We consider six exogenous variables (productivity, public spending, foreign output, the labor income tax rate, the capital income tax rate and the capital depreciation rate), along with three endogenous variables that are affected by other exogenous shocks: real money holdings to GDP, private investment and public debt to GDP. By definition, all those variables are stationary – because computed from a stationary model – and lag selection indicates one lag for the VAR estimation, whether it is in cross-section or in panel with pooled data. Indeed, given that all variables – except our war dummy – are centered, we abstract from fixed or random effects, pool the data and estimate our panel VAR using OLS.

**Results.** Figure 5 below reports the IRFs produced by a panel VAR estimation after a one standard deviation shock on the war dummy identified using a Cholesky decomposition of the variance-covariance matrix, as well as one standard deviation confidence intervals, i.e. 84% confidence intervals, based on 2000 bootstrap replications.

By construction, the immediate impact of a war shock is muted. After the shock, Figure 5 shows that foreign output and investment are significantly and negatively affected respectively for 8 and 12 years. To a lesser extent, productivity is negatively affected but the impact is short-lived (3 to 4 years), and followed by a strong recovery effect that lasts between 4 and 6 years. Public spending increases significantly for 5 years, i.e. the average duration of a world war, feeding a long-lived rise in debt to GDP that lasts approximately 10 years. Public spending then self-correct significantly, as in Corsetti, Meier and Müller (2012). Consistently with the findings of Auray et al. (2014), capital depreciation rises significantly and persistently for at least 5 years, which once again corresponds to the average duration of a war episode in our sample. The initial 5 years rise is then followed by a 7 to 9 years correction, that contributes to the post-war recovery.

In terms of monetary and fiscal strategies, real money balances to GDP rise significantly for 5 to 6 years. In terms of tax rate adjustments, our SVAR approach does not uncover
Figure 5: IRFs and confidence intervals to a one standard deviation shock on the war dummy from a Panel SVAR (Cholesky)
a statistically significant impact response, reflecting the variety of strategies. However, a statistically significant lagged pattern emerges, according to which the labor income tax rate rises and the capital income tax rate falls 4 to 5 years after the war shock.

Overall, our analysis confirms the broad narratives developed in the previous paragraphs and robustly identifies the macroeconomic effects of war episodes: a sharp and persistent drop in investment efficiency and foreign output, i.e. trade, a short-lived productivity slowdown followed by a strong recovery, a significant increase in public spending that leads public debt to GDP to rise and an increase in the capital depreciation rate. The analysis also suggest that monetization was part of the “average” strategy to finance war related public deficits, and that the lagged response of tax rates consisted in raising the labor income tax rate and lowering the capital income tax rate. These findings are in accordance with those of Ohanian (1997) and Martin (2012), except they apply more generally to a typical war episode, and to countries other than the U.S.

5 Public spending and tax multipliers

In this section, we make use of our model and Bayesian estimations to compute the values of public spending and tax multipliers.

The Bayesian estimations of our model are performed on linearized variables around the steady-state. Using the estimated parameter values, we run model simulations and derive IRFs after unexpected shocks. Variables are then taken in levels, i.e. we take the exponential and multiply by the steady-state value, to compute cumulative multipliers. These are defined as the cumulative difference between the response in level of a variable – say output – and its steady-state value, scaled by the cumulative difference between the response in level of the instrument – say public spending – and its steady-state value. For instance, the cumulative output multiplier $h$ periods after a public spending shock is

$$\nabla^{y}_{g,t+h} = \sum_{i=0}^{h} \frac{\partial y_{t+i}/\partial \varepsilon_{g}^{t}}{\sum_{i=0}^{h} \partial g_{t+i}/\partial \varepsilon_{g}^{t}},$$

where $\varepsilon_{g}^{t}$ is the initial unexpected shock and where variables are expressed in levels. $\nabla^{y}_{g,t+h}$ represents the cumulated dollar gain in output for a cumulated dollar increase in public spending. We proceed similarly for tax rates. Multipliers report the cumulative difference in the target variable, output for example, normalized by the cumulative difference in government tax income after a shock on either the labor income tax rate or the capital
income tax rate. For instance, the labor income tax output multiplier is

\[
\nabla^y_{\tau^n, t+h} = \frac{\sum_{i=0}^{h} \partial y_{t+i}/\partial \tau^n_t}{\sum_{i=0}^{h} \partial r_{g_{t+i}}/\partial \tau^n_t};
\]

(44)

where \( r_g_t = \tau^n_t (w_t/p_t) n_t + \tau^k_t (r^k_t/p_t - \delta) k_{t-1} \) stands for government’s total tax receipts. \( \nabla^y_{\tau^n, t+h} \) is expected to be negative and represents the cumulated dollar output loss for a cumulated dollar increase in taxes produced by a rise in the labor income tax rate. Accordingly, we compute \( \nabla^y_{\tau^k, t+h} \), the capital income tax output multiplier. We also report debt-to-GDP and inflation multipliers, respectively \( \nabla^{b^r/y}_{g, t+h}, \nabla^{b^r/y}_{\tau^n, t+h}, \nabla^{b^r/y}_{\tau^k, t+h}, \nabla^{\pi_d}_{g, t+h}, \nabla^{\pi_d}_{\tau^n, t+h}, \nabla^{\pi_d}_{\tau^k, t+h} \), where \( b^r \) denotes the real amount of public debt. This definition of multipliers, especially for output multipliers, makes our results directly comparable with the empirical literature on fiscal multipliers.

Figure 6 reports the estimated public spending and fiscal multipliers over a 9 years horizon, and Table 5 gives their numerical values on impact, 4 years and 8 years after a shock.

**Table 5: Numerical values of public spending and tax multipliers (impact, 4 years and 8 years after a shock)**

<table>
<thead>
<tr>
<th></th>
<th>France</th>
<th>Germany</th>
<th>UK</th>
<th>U.S.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Output</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \nabla^y_{\tau^n} )</td>
<td>0.85</td>
<td>0.59</td>
<td>0.38</td>
<td>0.90</td>
</tr>
<tr>
<td>( \nabla^y_{\tau^k} )</td>
<td>-0.24</td>
<td>-0.69</td>
<td>-1.27</td>
<td>-0.18</td>
</tr>
<tr>
<td>( \nabla^y_{\tau^n} )</td>
<td>-1.15</td>
<td>-2.23</td>
<td>-3.35</td>
<td>-0.94</td>
</tr>
<tr>
<td><strong>Debt-to-GDP</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \nabla^{b^r/y}_{g, t+h} )</td>
<td>-0.07</td>
<td>1.00</td>
<td>2.80</td>
<td>0.10</td>
</tr>
<tr>
<td>( \nabla^{b^r/y}_{\tau^n, t+h} )</td>
<td>-0.59</td>
<td>-1.63</td>
<td>-3.34</td>
<td>-0.56</td>
</tr>
<tr>
<td>( \nabla^{b^r/y}_{\tau^k, t+h} )</td>
<td>-0.40</td>
<td>-1.62</td>
<td>-3.84</td>
<td>-0.39</td>
</tr>
<tr>
<td><strong>Inflation</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \nabla^{\pi_d}_{g, t+h} )</td>
<td>0.27</td>
<td>0.08</td>
<td>0.04</td>
<td>0.25</td>
</tr>
<tr>
<td>( \nabla^{\pi_d}_{\tau^n, t+h} )</td>
<td>0.23</td>
<td>0.14</td>
<td>0.09</td>
<td>0.19</td>
</tr>
<tr>
<td>( \nabla^{\pi_d}_{\tau^k, t+h} )</td>
<td>0.56</td>
<td>0.09</td>
<td>0.09</td>
<td>0.23</td>
</tr>
</tbody>
</table>

We find that public spending output multipliers are below one on impact for France and Germany (0.85 and 0.90 respectively), and slightly above one for the UK and the U.S. (1.08 and 1.06 respectively). All multipliers decline smoothly over time, although more significantly for France and Germany. The data used in the Bayesian estimations are stationarized and filtered. This can explain why cumulative multipliers are not growing over time, as in some empirical papers of the literature working on time series in (log)levels or in papers taking time series in first difference. In spite of that, the reported values for
Figure 6: Public spending and tax multipliers

Solid: France, Dashed red: Germany, Dash-dotted: UK, Circles: U.S.
impact multipliers are clearly in line with the empirical literature. Ramey (2011a) recalls that most empirical studies find public spending output multipliers typically between 0.6 and 1.2 for the U.S.

In our set-up, price and wage stickiness act as amplifiers of the effects of government spending shocks. At the same time, our model assumes that part of the corresponding public deficit is financed through a lump-sum tax – actually just enough to make public debt to GDP a stationary variable (see Equation (35) in the presentation of the model). In our view, it thus makes sense that government spending multipliers lie above the lower bound of the literature – Barro and Redlick (2011) find that output multipliers as well below unity – but below the upper bound, given our methodology based on filtered data, and given that a small fraction of the increase in public spending is financed through lump-sum taxes.

The now discuss the factors that may account for the relative size of public spending output multipliers in the countries of our sample. First, our estimations assume that the UK and the U.S. have more elastic labor supplies, favoring a stronger positive response of hours worked after the crowding-out in private consumption triggered by the public spending shock. The real wage falls more and output increases more for a given size of the government spending shock. Second, the estimated degrees of price and wage stickiness are larger in the UK and in the U.S. than in France or Germany. It reduces the inflationary effects of government spending shocks in the UK or the U.S. and magnifies the short-run effects on output. Third, government size also contributes to scale the size of multipliers. For a given elasticity of output to public spending, a lower share of government spending in GDP, as assumed for the UK and the U.S., increases mechanically the value of the public spending output multipliers.

Some of those explanations are illustrated in Figure 7 below. Figure 7 reports the theoretical values of the impact output multipliers when varying one of the Calvo probability for prices, the Calvo probability for wages, the investment adjustment cost, the trade elasticity or the persistence of shocks ($\rho$). The default values for these parameters are $\eta^p = 0.5$, $\eta^w = 0.5$, $\phi_i = 5$, $\mu = 1$ and $\rho = 0.75$ to reflect the average country of our sample. Other parameters remain constant and are calibrated with the same objective.\footnote{The calibration of common parameters follows the values reported in Table 1. Public spending in GDP is $\kappa = 0.125$, public investment in GDP is $\kappa^i = 0.02$, mark-ups are $1/(\theta^p - 1) = 1/(\theta^w - 1) = 0.2$, and steady-state tax rates are $\tau^k = \tau^k = 0.25$.} This exercise is proposed for the two calibrated values of the inverse of the Frisch elasticity, $\psi = 1$ and $\psi = 2$.\footnote{The calibration of common parameters follows the values reported in Table 1. Public spending in GDP is $\kappa = 0.125$, public investment in GDP is $\kappa^i = 0.02$, mark-ups are $1/(\theta^p - 1) = 1/(\theta^w - 1) = 0.2$, and steady-state tax rates are $\tau^k = \tau^k = 0.25$.}
**Figure 7:** Theoretical values of output multipliers, as a function of key parameters

Solid red: $\psi = 2$, Dashed black: $\psi = 1$
Figure 7 first shows that labor supply elasticity produces larger spending multipliers on average. Multipliers are just below or above one when $\psi = 1$ while substantially lower when $\psi = 2$, when labor supply is less elastic. Second, spending output multipliers increase with the degree of nominal rigidity, except for wage stickiness when labor is more elastic ($\psi = 1$). In this case, the spending multiplier is basically insensitive to the degree of wage stickiness. Third, investment adjustment costs and the trade elasticity have only marginal quantitative effects on the size of spending output multipliers. Different labor supply elasticities and degrees of price and wage stickiness thus account for quantitative differences in the values of spending multipliers depicted in Figure 6. Finally, given that there is little cross-country heterogeneity in the estimated values of spending shocks persistence, this parameter does not explain cross-country differences in spending output multipliers.

Figure 6 also reports output multipliers for taxes. We find that tax output multipliers are between -0.13 and -0.24 for the labor income tax rate and are growing over the horizon considered, up to -1.27 for France. The cumulative labor income tax output multipliers over 8 years remains lower than one in absolute terms for the UK (-0.84) and for the U.S. (-0.57) while it is just above one in absolute terms for Germany (-1.04). Capital income tax output multipliers are much larger on impact, ranging from -0.4 for the U.S. to -1.17 for the UK, and reach much larger values over 8 years, especially for France (-3.35), Germany (-2.28) and the UK (-2.86). As for the labor income tax output multiplier, the capital income tax output multiplier over 8 years remains lower than one (-0.66) in the U.S.

One first observation concerning these values is that increasing the labor income tax has less contractionary effects than increasing the capital income tax. In the optimal taxation literature, the labor income tax base is less elastic to changes in tax rates than the capital income tax base. This argument is often used to prescribe low or even zero tax rates on capital income. It applies here and explains why capital income multipliers are larger than labor income tax multipliers. Further, in the empirical literature, tax output multipliers are found to range from -0.5 to -3, depending on experiments and identification methods (see Ramey (2011a) and references therein). Much of this range can actually be attributed to the implications of alternative identification schemes (see Favero and Giavazzi (2012)) and to small sample uncertainty (see Chahrour, Schmitt-Grohé and Uribe (2012)). Perotti (2012) shows that considering discretionary components of tax changes delivers tax output multiplier that are about halfway between the higher and lower bound, around 1.5. Barro and Redlick (2011) find similar magnitude for tax output multipliers, around 1.1. Our results comfort these halfway results. Capital income tax shocks produce a 4 years tax
output multiplier around 1.6 on average and labor income tax shocks an output multiplier round 0.5 on average. If tax shocks in the model result from shocks on each tax rate in equal proportions, the average 4 years tax output multiplier would be a little bit above one, around 1.05, very close to Barro and Redlick (2011).

According to Figure 7, smaller labor income tax output multipliers can be attributed to either more nominal rigidities, more investment adjustment costs, more substitutability between domestic and foreign goods or to less persistence in labor income tax shocks. Nominal rigidities favor a smaller response of the real wage and thus of hours worked. Larger investment adjustment costs delay the response of investment to any kind of shock. Both lower the short-term response of output. The persistence of shocks alters the magnitude of households and firms responses to shocks, and less persistence implies smaller tax multipliers. The way the parameters considered affect capital income tax output multipliers is essentially the same, except for wage stickiness, that induces larger output multipliers. Intuitively, nominal wage stickiness dampens the capital-labor substitution produced by capital income tax rate shocks, therefore magnifying the fall in investment and output.

An interesting outlier in our sample is the U.S., with significantly smaller estimated tax output multipliers. Comparing the U.S. point estimates reported in Table 4 with the estimates of other countries, the smaller tax multipliers can be attributed to larger nominal – especially price – rigidities, and larger investment adjustment costs. This is particularly true when comparing the U.S. to France and Germany. However, the difference between the U.S. and the UK, especially for the capital income tax multiplier, mostly comes from a lower estimated persistence of capital income tax shocks in the U.S. (0.04) compared to the UK (0.99). This is at least what Figure 7 shows about the effects of shocks persistence on the size of tax output multipliers. Given our methodology, it should thus not be concluded too strongly that the low value reported for the capital income tax output multiplier in the U.S. would necessarily prevail during a tax-financed increase in public spending. Indeed, the persistence of tax rate adjustments would be larger in this case, and would induce a larger capital income tax output multiplier in absolute terms.

Now turning to the values of debt-to-GDP multipliers, they make perfect sense when looking at the values of public spending and tax multipliers. For public spending shocks, they are positive on impact – except for France and the UK. They are positive and growing at longer horizons. As for tax shocks, debt to GDP multipliers are negative and growing over time in absolute terms. Finally, inflation multipliers are all positive on impact and feature relatively little persistence. The sign of inflation multipliers is consistent with the
effects of public spending and tax shocks. Public spending shocks increase demand, leading firms to increase their prices. In addition, real rental rates go up, which participates to increase the real marginal cost of firms and hence the inflation rate. Factor tax shocks reduce the supply of the factor that experiences the tax shock, making its equilibrium price (real wages or real rental rates) higher and contributes to increase real marginal costs and inflation. The size of government spending inflation multipliers is inversely related to the size of output multipliers. The larger the effects of spending shocks on output, the less inflationary those shocks. The size of tax inflation multipliers follows the size of output multipliers in absolute terms. The larger the impact of tax shocks on output, the larger the inflationary effects. This relation is also scaled by the degree of price and wage stickiness.

Overall, this section uncovers results that are in line with the literature, especially concerning the value of output multipliers. We find that public spending output multipliers are around 0.8-0.9 for France and Germany, and around 1.05-1.1 for the UK and the U.S. This is slightly larger than values reported by Barro and Redlick (2011), but remains well within the bounds of plausible empirical estimates. One can not exclude that these somehow larger spending multipliers are driven by the inclusion of world war episodes. Barro and Redlick (2011) themselves find that including WWII in their estimation sample increases spending multipliers by 0.1 in absolute terms. We also find tax multipliers that lie in the middle of the range of values found in the empirical literature, around 1.05-1.1. In addition, we uncover significant differences among countries, especially concerning the value of tax multipliers in the U.S., and to a lesser extent in the UK. Combining our results concerning spending and tax multipliers in the UK and in the U.S. leads to less pessimistic conclusions about the effects of tax-financed increases in public spending, especially if the labor income tax rate is increased. Pessimistic conclusions apply however for France and Germany.

6 Welfare analysis

In this last section, we proceed to a systematic welfare analysis. We rely on our welfare measure $W_t$ and on our path of smoothed variables to evaluate the welfare losses from business cycles over different periods, as well as the instantaneous welfare losses from particular historical episodes. We consider the full sample, the pre-1945 period and the post-1950 period. Table 6 reports the difference between the "observed" welfare and welfare in the steady state, expressed as a percentage of permanent steady-state consumption.
Table 6: Welfare losses from fluctuations, in % of permanent consumption

<table>
<thead>
<tr>
<th></th>
<th>France</th>
<th>Germany</th>
<th>UK</th>
<th>U.S.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total sample</td>
<td>0.4389</td>
<td>0.2931</td>
<td>0.3420</td>
<td>0.4541</td>
</tr>
<tr>
<td>Pre-1945</td>
<td>0.5156</td>
<td>0.4165</td>
<td>0.5137</td>
<td>0.7577</td>
</tr>
<tr>
<td>Post-1950</td>
<td>0.1090</td>
<td>0.1259</td>
<td>0.1293</td>
<td>0.1308</td>
</tr>
</tbody>
</table>

The magnitude of post-war welfare losses from fluctuations is perfectly in line with the order reported by Lucas (1987). Further, pre-war macroeconomic fluctuations were quite larger, producing substantially larger welfare losses. According to our estimates, focusing on post-war data underestimates welfare losses from fluctuations by a factor 2.5 to 4 compared to the welfare losses computed on the whole sample, and by a factor 3.5 to 6 compared to the pre-war period. The reason behind the results is the larger macroeconomic volatility during the pre-1945 period, and the fact that the period includes war and crisis episodes. To grasp both aspects, we compute the instantaneous welfare losses from fluctuations and express them in percents of steady-state current consumption. The results are graphed in Figure 8.

Figure 8 shows that war episodes produce instantaneous welfare losses as large as 15 to 17%. In other words, agents would have been willing to give up more than 15% of their current consumption to avoid experiencing the effects of wars on their welfare. According to our calculations, the welfare losses from WWI range from less than 3% of current consumption for the U.S. in 1919 to more than 8% for European countries. France stands at 4.4% in 1915 and experiences the largest loss during the immediate post-war, in 1921 with an 8% welfare loss. Germany exhibits a similar pattern as the largest loss during WWI reaches 4.2% in 1917, but a much larger loss of 10% in 1920. The losses from WWII are much larger for the U.S. as they reach 16.8% in 1943, but are followed by a 3.4% instantaneous welfare gain in the immediate post-war period, in 1947. The largest impact for France during WWII is an 8.8% loss in 1943, followed by a much larger 17% loss in 1948, before welfare gains from the recovery are witnessed, after 1950. Germany experiences the largest welfare loss in 1944, estimated at 12% of current steady-state consumption. The UK is also deeply affected by a 10% welfare loss in 1940, but the immediate post-war brings large welfare gains, peaking at 6.2% in 1946.

Our model captures the effects on welfare through large shifts in hours worked and consumption. First, during war times, hours worked increase significantly, either because of mobilization, that makes hours worked in the military sector rise, or because women enter the labor market, substantially raising the participation rate. Second, private con-
Figure 8: Instantaneous welfare losses from fluctuations, in % of current consumption – Historical decomposition
sumption falls substantially. Rationing and penury first come to mind, but the massive increase in public consumption and public investment, producing large crowding out effects also contribute to lower private consumption. Last, one could argue that the major increase in uncertainty magnifies precautionary savings, contributing to the fall in private consumption. Although our model is linearized, therefore missing this precautionary savings motive, the rise and fall of the discount factor identified in our estimations most likely captures these combined effects of war episodes on consumption, in addition to the standard effects of lower productivity and higher public spending on consumption and investment.

Finally, Figure 8 also shows some interesting numbers for other historical episodes, as the Great Depression. In the U.S., the second largest welfare loss after WWII is indeed the 1929 Great Depression, producing an instantaneous welfare loss estimated at 10.5% of current steady-state consumption. Germany is the most affected European country with a lagged 5% welfare loss in 1932. Another insight from Figure 8 is the massive drop in macroeconomic volatility after WWII, that rationalizes the substantial difference between pre-war and post-war welfare losses from fluctuations.

7 Conclusion

We offer a narrative and quantitative analysis of the effects of large war episodes (world wars) on four advanced countries: France, Germany, the UK and the U.S. Our analysis builds on a structural small open economy model estimated with Bayesian techniques on a dataset that goes back to the XIX-th century. During a war episode, trade collapses and public spending rises massively. This rise is partly financed by monetization and public debt rises. After a few years, labor income taxation increases and capital income taxation falls.

We also estimate the size of government spending and tax multipliers. While within the bounds of the empirical literature, our estimates of spending multipliers are large, and above one for the UK and the U.S. Average tax multipliers are slightly above one, and capital income tax multipliers are larger than labor income tax multipliers. In addition, tax multipliers are smaller in the UK and in the U.S. compared to France and Germany. These results lead to less pessimistic predictions about the effects of tax-financed increases in public spending, especially for the UK and the U.S.

Finally, we compute the welfare losses from fluctuations. We show that the large
swings in macroeconomic time series induced by war episodes produce massive instantaneous welfare losses, up to 15-17% of current consumption, and that the pre-1945 period featured a lot more volatility than the post-war period, producing larger welfare losses from fluctuations.

References


A  The steady state

We analyze the linearized dynamics of the model around the closed economy steady-state without inflation, where we also assume \( s = 1 \). By definition, \( d = 0 \), and the dynamics of \( q \) gives \( q = 1 \). The condition on the utilization rate thus gives

\[
 r^k = \frac{\phi_z (z - 1)}{1 - \tau^k} \tag{45}
\]

which plugged in the investment equation pins down \( z \)

\[
 z = \sqrt{1 + \frac{2 (\beta^{-1} + \delta (1 - \tau^k) - 1)}{\phi_z}} \tag{46}
\]

as well as \( r^k \). Next use the real marginal cost to get the real wage \( w \)

\[
 w = \left( \frac{(\theta - 1)}{\theta} \iota (1 - \iota)^{(1-\iota)} (r^k)^{-\iota} \right)^{\frac{1}{1-\iota}} \tag{47}
\]

Further, we impose \( a = n = 1 \) and use

\[
 k = \frac{\iota w}{1 - \iota r^k z} \tag{48}
\]

and the aggregate production function

\[
 y = (z k)^{\iota} \tag{49}
\]

to get consumption

\[
 c = (1 - \kappa - \kappa^i) y - \delta k - \frac{\phi_z}{2} (z - 1)^2 k \tag{50}
\]

where \( \kappa = g/y \) and \( \kappa^i = i^g/y \) are the (imposed) steady-state share of public consumption and investment expenditure in GDP. Finally, the labor supply equation gives the value of \( \chi_n \) that normalizes \( n \) to one

\[
 \chi_n = (c (1 - h_c))^{-\sigma_c} ((\theta^w - 1)(1 - \tau^w) / \theta^w) w \tag{51}
\]

B  Prior and posterior distributions

Figure 9: Priors and Posteriors for France - 1
**Figure 10:** Priors and Posteriors for France - 2

**Figure 11:** Priors and Posteriors for France - 3
Figure 12: Priors and Posteriors for Germany - 1

Figure 13: Priors and Posteriors for Germany - 2
Figure 14: Priors and Posteriors for Germany - 3

Figure 15: Priors and Posteriors for the UK - 1
Figure 16: Priors and Posteriors for the UK - 2

Figure 17: Priors and Posteriors for the UK - 3
Figure 18: Priors and Posteriors for the U.S. - 1

Figure 19: Priors and Posteriors for the U.S. - 2
Figure 20: Priors and Posteriors for the U.S. - 3