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Nonlinear Pricing and Exclusion
II. Must-Stock Products

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Abstract

We adapt the exclusion model of Choné and Linnemer (2014) to reflect the notion that dominant firms are unavoidable trading partners. In particular, we introduce the share of the buyer's demand that can be addressed by the rival as a new dimension of uncertainty. Nonlinear price-quantity schedules allow the dominant firm to adjust the competitive pressure placed on the rival to the size of the contestable demand, and to distort the rival supply at both the extensive and intensive margins. When disposal costs are sufficiently large, this adjustment may yield highly nonlinear and locally decreasing schedules, such as "retroactive rebates".

JEL codes: L12, L42, D82, D86

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1 Introduction

The notions of market power and dominance in competition law are multi-faceted. They are often modeled by assuming incumbency and commitment power. Following this approach, we have explored the exclusionary properties of nonlinear pricing in a companion paper, Choné and Linnemer (2014). Building on the methodology developed there, we now introduce another common feature of dominant firms, namely the fact that those firms often are unavoidable trading partners –at least to a certain extent and over a certain period of time. For instance, the European Commission has observed that:

> "Competitors may not be able to compete for an individual customer's entire demand because the dominant undertaking is an unavoidable trading partner at least for part of the demand on the market, for instance because its brand is a "must stock item" preferred by many final consumers or because the capacity constraints on the other suppliers are such that a part of demand can only be provided for by the dominant supplier."[1]

Adapting the model of our companion paper, we assume here that the rival can address only a fraction of the buyer's demand. The contestable share of the buyer's demand depends on the characteristics of the rival good, which are not known when the buyer and the dominant firm agree on a price-quantity schedule. We therefore treat this parameter as a new dimension of uncertainty, on top of the rival's cost and product quality. We are thus able to relate the shape of the optimal price-quantity schedule to the joint distribution of rival characteristics, and to describe exclusionary effects in a way that is both transparent and consistent with the recent practice of competition agencies.

By lowering the price of contestable units, the dominant firm forces the rival to match lower prices, which, depending on its cost efficiency, he may not be able to do profitably. The average price of contestable units, also known as the "effective price", is therefore negatively related to the competitive pressure placed on the rival. We introduce the notion of elasticity of entry to measure the rival's sensitivity to competitive pressure, i.e., by how much (in percentage terms) the probability that the rival can profitably serve the contestable demand decreases as

[1] Communication on abusive exclusionary conduct by dominant undertakings (2009/C 45/02). This line of reasoning appears in a number of landmark European cases, such as *Van den Bergh Foods Limited* (distribution of ice-cream products in Ireland), Court of First Instance (T-65/98) 23 October 2003; *British Airways* (air tickets) Court of First Instance (Third Chamber, Case C-95/04 P), 15 March 2007; *Michelin* (new and retreaded tyres for trucks and buses) Judgment of the Court of First Instance (Third Chamber, T-203/01), 30 September 2003. For an economic perspective on the case see Motta (2009); *Tomra* (reverse vending machines for containers) C-549/10 P Judgment of the Court of First Instance (Third Chamber), 19 April 2012; *Intel* (Central Processing Units, CPU).Commission Decision COMP/C-3 /37.990 of 13 May 2009. Jing and Winter (2014) discuss a Canadian case involving the scanner-based information company Nielsen.

the effective price set by the dominant firm falls. For any given size of the contestable demand, the elasticity reflects the extent to which more pressure placed on the rival (i.e., lower effective prices) translates into more exclusion.

Next, we explain how using a nonlinear schedule helps the dominant firm to adjust the competitive pressure placed on the rival to the size of the contestable demand. In particular, the optimal schedule is linear only when the elasticity is constant, which occurs if and only if the size of the contestable demand and the rival's efficiency index are independent. When the elasticity of entry increases with the contestable market share, the dominant firm wants to place less pressure on larger competitors, and hence the effective price increases with the number of units. This tends to make the optimal schedule concave. Concavity, in the present context, is associated with complete exclusion of efficient rival types. In contrast, in the one-dimensional model of Choné and Linnemer (2014), globally concave schedules are associated with distortions of the rival supply at the intensive margin only.

The new dimension of uncertainty allows for much richer patterns. When the elasticity is non-monotonic in the contestable share, optimal schedules may exhibit highly nonlinear shapes and even admit decreasing parts. The distortion of the rival supply, here, can be at both the extensive and the intensive margins. We find so-called "retroactive rebates" when the dominant firm places more pressure on rival types with intermediate size than with small or large size. Such rebates, also called "all-units discounts", are granted for all purchased units once a quantity threshold is reached. They induce downward discontinuities in price-quantity schedules –a pattern that has received much attention from antitrust enforcers.[2]

The analysis must be modified when the buyer is allowed to dispose of or to resell unconsumed units because strong quantity rebates, and a fortiori decreasing parts in a price-quantity schedule, might induce her to purchase unneeded units of incumbent good with the sole purpose of pocketing the rebates. We know from our companion paper that this extreme form of buyer opportunism is never seen in equilibrium. Yet the mere possibility of such a behavior may constrain the shape of optimal schedules when disposal costs are weak. In fact, in the present context, the constraint is active only in the presence of a *super-efficient* rival, i.e., a rival who is so efficient that the buyer, had she purchased enough units of incumbent good to meet her entire demand, would nevertheless want to supply her contestable demand from the rival and hence would prefer throwing part of the purchased units away rather than consuming them. Driving a super-efficient rival out of the market is not possible under a non-conditional schedule. To do so, the dominant firm needs to let the price of the incumbent good depend on the quantity supplied from the rival.

The introduction of a new dimension of uncertainty in Choné and Linnemer (2014) is made possible by assuming that the buyer's demand is inelastic and by abstracting away from

[2] See, among others, Waelbroeck (2005) and Faella (2008).

horizontal differentiation. Recall that under a non-conditional schedule (whereby the price of the incumbent good depends only on the quantity of that good), the buyer purchases the efficient quantity of rival good given that of incumbent good. The quantity sold by the rival is thus indirectly controlled by the price-quantity schedule. Granting rebates for the incumbent good allows the dominant firm to lower the buyer's incentives to supply from the rival, but this comes at a cost: the buyer purchases inefficiently many units of incumbent good given the rival supply. Hence the "buyer opportunism" phenomenon at the heart of our companion paper. The demand specification adopted here dramatically simplifies the analysis as in equilibrium the quantities of the two goods sum up to the total demand and each quantity is efficient given the other (no buyer opportunism). Technically, the dominant firm has a single instrument to screen out multidimensional types, a situation we handle with a method analogous to the profile demand technique of Wilson (1993).

A number of papers have adopted a demand specification close to the one used here, but all of them assume perfect information. Colon and Mortimer (2013) study the relationships between the chocolate candy manufacturer Mars Inc. and a retail vending operator in downtown Chicago. They provide empirical evidence that all-units discounts foreclose competition: "Specifically, the retailer can increase profits by substituting a Hershey product for a Mars product, but the threat of losing the rebate discourages him from doing so." DeGraba (2013) models the competitor as being capacity constrained as we do. His approach is in the tradition of the "naked exclusion" literature (Rasmusen, Ramseyer, and Wiley (1991) and Segal and Whinston (2000)). Also in a complete information framework (with a timing similar to ours), Feess and Wohlschlegel (2010) study all-units rebates and Chao and Tan (2013) compare all-units discounts, quantity forcing, and three-part tariff. Finally, Figueroa, Ide, and Montero (2014) analyze a model close to ours under one-dimensional uncertainty, the rival's production capacity being known. They show that restricting the buyer-incumbent coalition's ability to share rents through transfers limits the scope for inefficient exclusion.

The paper is organized as follows. Section 2 presents the model. Section 3 proceeds to the pointwise maximization of the virtual surplus and shows how to obtain two-part tariffs and concave schedules. Section 4 characterizes the implementable second-best allocations and presents a general method to construct optimal schedules. Section 5 examines a couple of typical schedules, such as retroactive rebates. Section 6 introduces conditional schedules and explains the role of disposal costs. Section 7 concludes.

2 Model

As in Choné and Linnemer (2014), we consider a dominant firm and a competitor who successively interact with a large buyer. The main modeling difference is that the buyer's utility function now involves a second dimension of uncertainty, namely the share of the buyer's demand that can be addressed by the rival.

Buyer's demand The buyer cannot *consume* more that s_E units of the rival good and one unit of both goods altogether. When she consumes $x_E \le s_E$ units of the rival good and x_I units of the incumbent good, she earns a gross profit of $v_E x_E + v_I x_I$ as long as $x_E + x_I \le 1$. This specification corresponds to the special case of Choné and Linnemer (2014) where the convex function $h(x_E, x_I)$ is zero when $x_E \le s_E$ and $x_E + x_I \le 1$, and $+\infty$ otherwise. The two goods are vertically differentiated when v_E and v_I differ. The specification, however, does not capture horizontal differentiation; in this respect, the reader will notice a number of formal analogies with the limit case of the quadratic example in Choné and Linnemer (2014) where the imperfect substitution parameter, σ, equals one.

As in our companion paper, the buyer may *purchase* more units than she needs, and dispose of the unconsumed units at some cost or possibly resell them on a secondary market. To account for the latter possibility, we allow the per unit disposal cost γ_k to be negative, but assume that reselling entails a productive inefficiency, i.e., the total costs $\gamma_I + c_I$ and $\gamma_E + c_E$ are always nonnegative. Finally, the utility derived by the buyer from purchasing quantities q_E and q_I of rival and incumbent good is given by

$$V(q_E, q_I; s_E, v_E) = \max_{(x_E, x_I) \in X(q_E, q_I)} v_E x_E + v_I x_I - \gamma_E(q_E - x_E) - \gamma_I(q_I - x_I) \tag{1}$$

where $X(q_E, q_I)$ reflects the constraints $x_E \le q_E$, $x_I \le q_I$, $x_E + x_I \le 1$, and $x_E \le s_E$: the buyer cannot consume more of each good than the purchased quantity, more of both goods together than her total requirement, and more of good E than the contestable demand.

When $q_E \le s_E$ and $q_E + q_I \le 1$, the buyer consumes all the purchased units. The "no-disposal region" is located below the dashed line on Figures 1a to 1c. In this region, the buyer utility is simply $v_E q_E + v_I q_I$.

Efficiency The total surplus $W(q_E, q_I) = V(q_E, q_I; s_E, v_E) - c_E q_E - c_I q_I$ is maximal in the no-disposal region because producing units to throw them away is obviously inefficient. In this region, the total surplus equals $\omega_E q_E + \omega_I q_I$, where $\omega_E = v_E - c_E$ and $\omega_I = v_I - c_I$ denote the unit surplus of each good. The efficient quantities, therefore, depend on ω_E and s_E, but not on the magnitude of the disposal costs:

$$(q_E^{**}(\omega_E, s_E), q_I^{**}(\omega_E, s_E)) = \begin{cases} (s_E, 1 - s_E) & \text{if } \omega_E > \omega_I \\ (0, 1) & \text{if } \omega_E < \omega_I. \end{cases} \tag{2}$$

The welfare isolines in the no-disposal region are straight lines of slope $-\omega_E/\omega_I$. When $\omega_I > \omega_E$, it is efficient that the incumbent firm serves all of the buyer's demand, and the maximum of W is achieved at point A', see Figure 1a. When $\omega_E > \omega_I$, it is efficient that the rival serves the contestable demand, and the maximum of W is at point A, see Figures 1b and 1c.

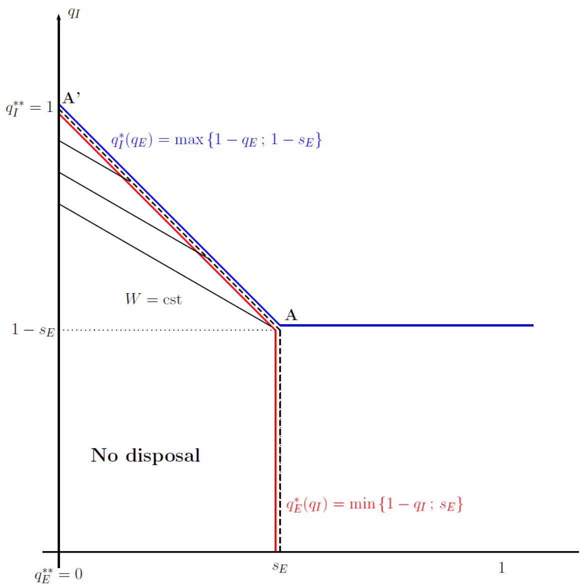

Figure 1a: Conditionally efficient quantities when $\omega_E < \omega_I < v_E + \gamma_E$. Welfare maximum at A'

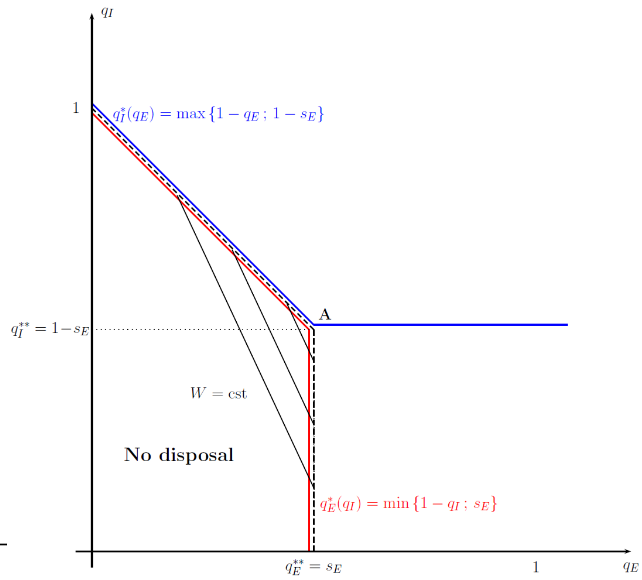

Figure 1b: Conditionally efficient quantities when $\omega_I < \omega_E < v_I + \gamma_I$. Welfare maximum at A

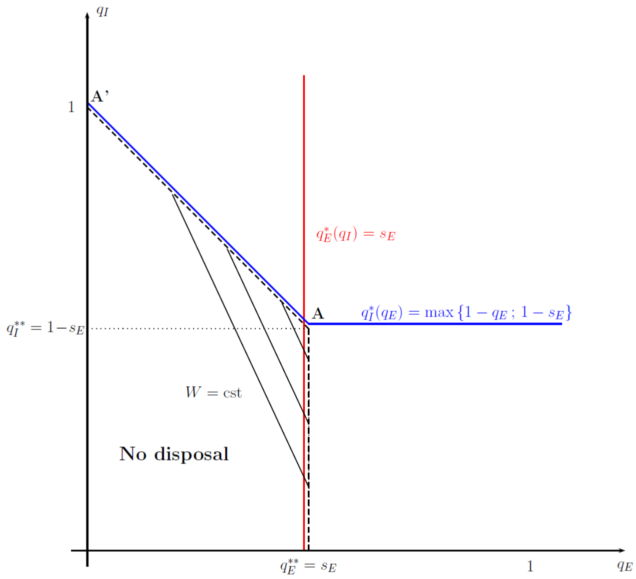

Figure 1c: Super-efficient rival: $\omega_E > v_I + \gamma_I$. Welfare maximum at A.

The conditionally efficient quantity of rival good, i.e., the quantity q_E that maximizes the social welfare W given q_I, is

$$q_E^*(q_I; \omega_E, s_E) = \begin{cases} \min\{1 - q_I, s_E\} & \text{if } \omega_E < v_I + \gamma_I \\ s_E & \text{otherwise.} \end{cases}$$

Consider first the situation where the rival is inefficient or moderately efficient, specifically $\omega_E < v_I + \gamma_I$. If the buyer has purchased more than $1 - s_E$ units of incumbent good, it is efficient that she consumes those units and, if necessary, complete her supply with units of rival good to meet her demand. It follows that when q_I is larger than one, the conditionally efficient quantity of rival good is zero, see Figures 1a and 1b; in the terminology of our companion paper, the rival is not super-efficient.

In contrast, when the surplus created by the rival good is very high, $\omega_E > v_I + \gamma_I$, efficiency requires that the buyer disposes of the purchased units of incumbent good in excess of $1 - s_E$ and supplies all of the contestable part of her demand from the rival. In this case, the conditionally efficient quantity of rival good, $q_E^*(q_I)$, is s_E however large q_I becomes, see Figure 1c; the rival firm, in other words, is super-efficient. When γ_I tends to $-c_I$, super-efficiency becomes equivalent to standard efficiency. When disposal costs are infinite, there are no super-efficient rival types.

It is easy to check that the conditionally efficient quantity of incumbent good, $q_I^*(q_E)$, is given by

$$q_I^*(q_E) = \begin{cases} \max\{1 - q_E, 1 - s_E\} & \text{if } \omega_I < v_E + \gamma_E \\ 1 & \text{otherwise.} \end{cases}$$

For further reference, it is worthwhile noticing along the upper part of the boundary of the no-disposal region, i.e., along the segment (AA') on Figures 1a to 1b, each quantity is efficient given the other. This property is due in large part to the absence of horizontal differentiation. In the presence of such differentiation, for instance in the quadratic example of our companion paper, the curves $q_E = q_E^*(q_I)$ and $q_I = q_I^*(q_E)$ cross at a single point (the efficient allocation); in contrast, here, they coincide along a whole segment. This property has strong implications for the comparison the allocation under conditional and non-conditional schedules, see Section 6).

Timing and information The timing of the game and the informational setup are very similar to those employed in our companion paper with the major difference that a new dimension of uncertainty is introduced.

At the first stage of the game, the buyer B and the dominant firm I design a price-quantity schedule to maximize (and split) their joint expected surplus, knowing the characteristics of the incumbent good, i.e., the constant marginal cost c_I and willingness to pay v_I. At this stage, however, they do not know the characteristics of the rival good, which now include the size of the contestable demand, s_E, on top of the constant marginal cost c_E and the willingness to pay v_E. In most of the paper, we concentrate on *non-conditional* schedules $T(q_I)$, for which the price of the incumbent good only depends on the quantity of that good.[3]

At the second stage of the game, the buyer and the rival discover c_E, s_E and v_E, the terms

[3]Conditional schedules, whereby the price $T(q_I, q_E)$ also depends on the purchased quantity of rival good, are studied in Section 6.

of the agreement between B and I being common knowledge. B and E jointly decide on a transfer p_E and quantities q_E and q_I. As in our companion paper, the negotiation takes place under complete information and is modeled as Nash bargaining where β denotes the rival's bargaining power.

Purchase decisions At the last stage of the game, the buyer and the rival choose the quantities to maximize their joint surplus

$$S_{BE} = \max_{q_E, q_I} V(q_E, q_I) - T(q_I) - c_E q_E, \tag{3}$$

with no consideration for the incumbent's cost or profit. The latter equation shows that under a non-conditional schedule $T(q_I)$, the quantity of rival good is efficient given that of the incumbent good, formally $q_E = q_E^*(q_I)$; in particular, no unit of the rival good is produced and disposed of. As a result, to save on disposal and production costs, the buyer and the rival, who negotiate under perfect information, never trade more than s_E units. It follows that the inequality $q_E \leq s_E$ holds for all values of (c_E, s_E, v_E); hence an alternative interpretation of the model where the non-contestable share of the market reflects a rival's capacity constraint rather than a demand characteristics. Under both interpretations, the rival firm can address the fraction s_E of the buyer's demand.

Without loss of generality, the competitor's outside option is normalized to zero. As to the buyer, she may source exclusively from the incumbent, so her outside option is

$$V_B^0 = \max_{q_I \geq 0} V(0, q_I) - T(q_I). \tag{4}$$

The reservation utility V_B^0, which depends on the price schedule by (4), is endogenous but independent from ω_E. The surplus created by the the buyer and the rival firm can thus be written as

$$\Delta S_{BE} = S_{BE} - V_B^0. \tag{5}$$

Denoting by $\beta \in (0,1)$ the competitor's bargaining power vis-à-vis the buyer, we derive the competitor's and buyer's profits:

$$\begin{array}{rccccc} \Pi_E = & \beta & \Delta S_{BE} & & \\ \Pi_B = & (1-\beta) & \Delta S_{BE} & + & V_B^0. \end{array} \tag{6}$$

If $\beta = 0$, the competitor has no bargaining power and may be seen as a competitive fringe from which the buyer can purchase any quantity at price c_E. On the contrary, the case $\beta = 1$ happens when the competitor has all the bargaining power vis-à-vis the buyer.

Virtual surplus Ex ante, the buyer and the incumbent design the price schedule to maximize their expected joint surplus, equal to the total surplus minus the profit left to the competitor:

$$\mathbb{E}_{c_E,s_E,v_E}\Pi_{BI} = \mathbb{E}_{c_E,s_E,v_E}\left\{W(q_E, q_I; c_E, v_E) - \Pi_E\right\}, \tag{7}$$

where q_E and q_I are solution to (3) and Π_E is given by (6). The expectation is to be taken against the distribution of the rival good's characteristics (c_E, s_E, v_E). As all the purchased units of rival good are consumed, the surplus depends on the uncertain cost and preference parameters c_E and v_E through the difference $\omega_E = v_E - c_E$. It follows that only the joint distribution of s_E and ω_E matters, and we adopt the following set of assumptions on that distribution.

Assumption 1. *The cumulative distribution function of s_E, denoted by G, admits a positive and continuous density function g on $[\underline{s}_E, \bar{s}_E]$.*

The conditional distribution of ω_E given s_E has a positive density $f(\omega_E|s_E)$ on its support $[\underline{\omega}_E, \overline{\omega}_E]$, with $\underline{\omega}_E < \omega_I < \overline{\omega}_E$. The hazard rate $f/(1-F)$ is nondecreasing in ω_E.

Following Choné and Linnemer (2014), we observe that the rival's rent is convex in ω_E, with its derivative being βq_E. We then integrate the rent by parts with respect to ω_E, for each size of the contestable demand s_E, and rewrite the buyer-incumbent objective as a function of the virtual surplus

$$\mathbb{E}_{s_E,\omega_E}\Pi_{BI} = \mathbb{E}_{s_E,\omega_E}S^{\mathrm{v}}(q_E, q_I; \omega_E, s_E) - \Pi_E(\underline{\omega}_E, s_E),$$

where

$$S^{\mathrm{v}}(q_E, q_I; \omega_E) = W(q_E, q_I; \omega_E) - \beta q_E \frac{1 - F(\omega_E)}{f(\omega_E)}.$$

Under a non-conditional schedule, the buyer purchases the efficient quantity of rival good given that of incumbent good, $q_E = q_E^*(q_I)$. Accordingly, we start by maximizing the virtual surplus for each value of ω_E and s_E subject to the constraint that $q_E = q_E^*(q_I)$, which we call the "relaxed problem". The solution of the relaxed problem is the second-best allocation provided that it is implementable with a non-conditional schedule.

3 Elasticity of entry

To concentrate on the role of the heterogeneity about contestable demand, we assume in this and the next two sections that disposal costs are very large, $v_I + \gamma_I > \overline{\omega}_E$, so the rival is never super-efficient. Under this circumstance, the conditionally efficient quantity of rival good is given by $q_E^*(q_I) = \min(1 - q_I, s_E)$, and the constraint $q_E = q_E^*(q_I)$ is the equation of the boundary of the no-disposal region, as shown on Figures 1a and 1b.

The relaxed problem For each s_E and ω_E, we maximize the virtual surplus subject to the constraint that $q_E = \min(1 - q_I, s_E)$, i.e., along the boundary of the no-disposal region. Since the virtual surplus increases with q_I, the maximum is achieved on the upper part of that boundary, namely on the segment (AA'). Along this segment, we have $q_E + q_I = 1$ and, given the expression of the virtual surplus, it is convenient to parameterize the problem with q_E rather with q_I. We thus rewrite the virtual surplus on (AA') as

$$S^{\mathrm{v}}(q_E, q_I; \omega_E) = \omega_E q_E + \omega_I q_I - \beta q_E \frac{1 - F(\omega_E)}{f(\omega_E)} = \omega_I + s^{\mathrm{v}}(s_E, \omega_E) q_E,$$

where we define the virtual surplus per unit of rival good as

$$s^{\mathrm{v}}(s_E, \omega_E) = \omega_E - \beta \frac{1 - F(\omega_E)}{f(\omega_E)} - \omega_I.$$

The unit virtual surplus can in turn be rewritten as

$$s^{\mathrm{v}}(s_E, \omega_E) = \omega_E[1 - \beta/\varepsilon(\omega_E|s_E)] - \omega_I, \tag{8}$$

where we define the elasticity of entry $\varepsilon(\omega_E|s_E)$ as the percentage decrease in the probability that the rival efficiency index is above some threshold when that threshold rises by 1%:

$$\varepsilon(\omega_E|s_E) = \frac{\omega_E f(\omega_E|s_E)}{1 - F(\omega_E|s_E)} = -\frac{\partial \ln [1 - F(\omega_E|s_E)]}{\partial \ln \omega_E}. \tag{9}$$

Lemma 1. *Assume that the rival is never super-efficient, $v_I + \gamma_I > \overline{\omega}_E$. Then the virtual surplus achieves its maximum subject to $q_E = q_E^*(q_I)$ at the point $(q_E, q_I = 1 - q_E)$ given by*

$$q_E(s_E, \omega_E) = \begin{cases} 0 & \text{if } \omega_E \leq \hat{\omega}_E(s_E) \\ s_E & \text{otherwise,} \end{cases}$$

where $\hat{\omega}_E(s_E) \in (\omega_I, \overline{\omega}_E)$ is the unique solution to

$$\frac{\hat{\omega}_E(s_E) - \omega_I}{\hat{\omega}_E(s_E)} = \frac{\beta}{\varepsilon(\hat{\omega}_E(s_E)|s_E)}. \tag{10}$$

The fraction of efficient types that are inactive increases with the rival's bargaining power vis-à-vis the buyer and decreases with the elasticity of entry.

Proof. By linearity of the virtual surplus, the maximization problem generically yields a corner solution, either $q_E = 0$ or $q_E = s_E$. In the former case, the maximum is at point A, see Figure 2a; in the latter, it is at A', see Figure 2b. On the figures, the dashed lines represent the isolines of the virtual surplus. At the maximum, we have $q_E = s_E$ (respectively $q_E = 0$) when $s^{\mathrm{v}} > 0$ (resp. $s^{\mathrm{v}} < 0$). The unit virtual surplus s^{v} is positive if and only if

$$\frac{\omega_E - \omega_I}{\omega_E} > \frac{\beta}{\varepsilon(\omega_E|s_E)}.$$

The left-hand side increases in ω_E, and the right-hand side is non-increasing in ω_E by Assumption 1, which yields the uniqueness of a solution (10). Moreover, the virtual surplus per unit is negative for $\omega_E = \omega_I$ and positive for $\omega_E = \overline{\omega}_E$. Hence the existence of a solution to equation (10) lying between ω_I and $\overline{\omega}_E$. Straightforward comparative statics shows that $\hat{\omega}_E$ increases with β and decreases with ε. $\square$

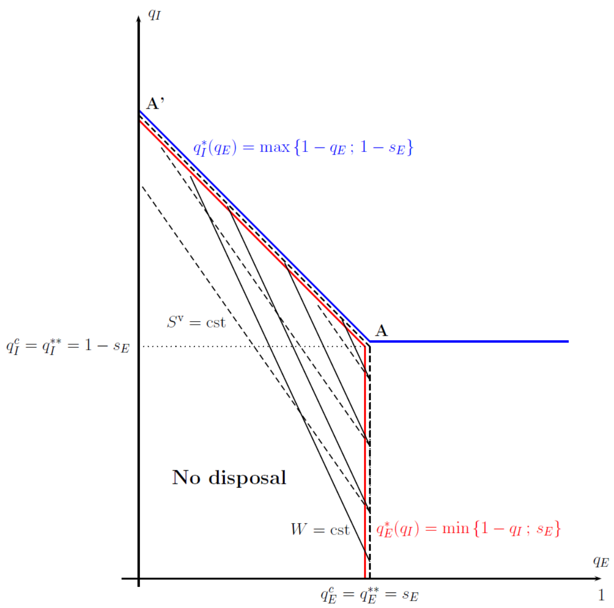

Figure 2a: Entry: $\omega_E > \hat{\omega}_E(s_E) > \omega_I$

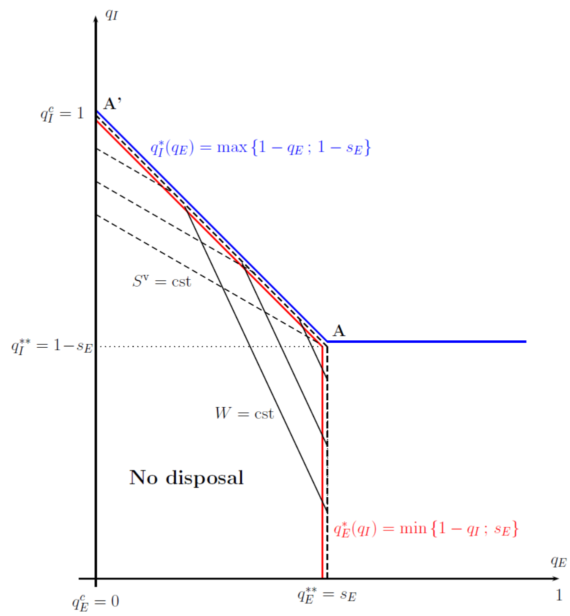

Figure 2b: Inefficient exclusion: $\omega_I < \omega_E < \hat{\omega}_E(s_E)$

We interpret the threshold $\hat{\omega}_E(s_E)$ as the height of the entry barrier that the buyer and the incumbent would want to erect if the size of the contestable demand s_E were known. When the elasticity of entry increases (respectively decreases) with s_E, the barrier is lower (resp. higher) as the size of the contestable demand rises. The next lemma relates the variations of $\varepsilon(\omega_E|s_E)$ with s_E to the primitives of the model.

Lemma 2. *The random variables s_E and ω_E are independent if and only if the elasticity of entry, $\varepsilon(\omega_E|s_E)$, does not depend on s_E. If the elasticity of entry increases (decreases) with s_E, then ω_E first-order stochastically decreases (increases) with s_E.*

Proof. The elasticity of entry varies with s_E in the same way as the hazard rate h given by

$$h(\omega_E|s_E) = \frac{f(\omega_E|s_E)}{1 - F(\omega_E|s_E)}.$$

We have

$$\int_{\underline{\omega}_E}^{\omega_E} h(x|s_E)\,\mathrm{d}x = -\ln[1 - F(\omega_E|s_E)].$$

If the elasticity of entry does not depend on (increases with, decreases with) s_E, the same is true for the hazard rate, and hence also for the cdf $F(\omega_E|s_E)$, which yields the results.[4] $\square$

Two-part tariffs When s_E and ω_E are independent, the threshold $\hat{\omega}_E(s_E)$ is flat, as represented on Figure 3a: the rival serves all of the contestable demand when ω_E is higher than the constant level of $\hat{\omega}_E$, and is inactive otherwise. We now show that this allocation is implementable by a two-part tariff with slope $(v_I - \hat{\omega}_E)$.

Proposition 1. *When s_E and ω_E are independent, the buyer and the incumbent sell contestable units of incumbent good at price $(v_I - \hat{\omega}_E)$.*

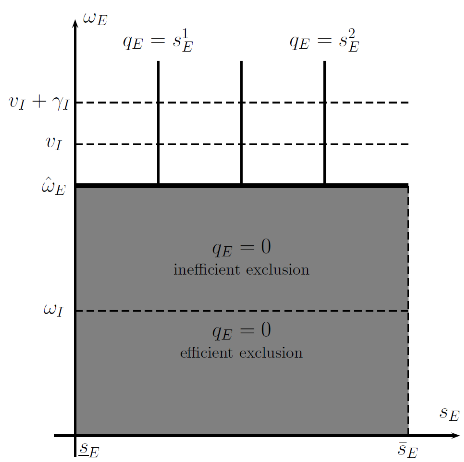

Figure 3a: Quantity purchased from the rival

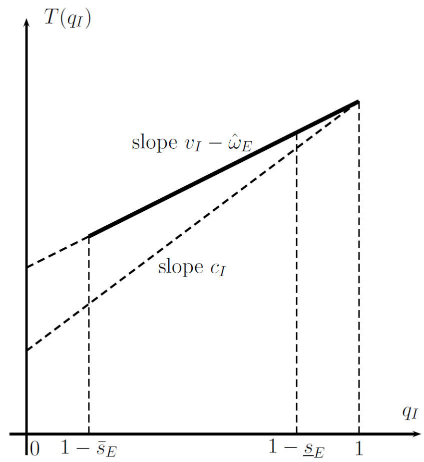

Figure 3b: Optimal price schedule

Proof. We compute the surpluses S_{BE}, V_B^0, and ΔS_{BE} given by (3), (4) and (5) for a two-part tariff with slope $(v_I - \hat{\omega}_E)$, i.e., for $T(q_I) = T(1) + (v_I - \hat{\omega}_E)(q_I - 1)$. We first recall that under such a non-conditional schedule q_E is conditionally efficient and no unit of rival good is left unconsumed, hence $q_E \leq s_E$. Next, we observe that the marginal price of the incumbent good, $v_I - \hat{\omega}_E$, is above $-\gamma_I$ by assumption, and hence the buyer consumes all the purchased units of incumbent good, too. Finally, since the price $v_I - \hat{\omega}_E$ is lower than the willingness to pay v_I, the buyer purchases as much units of incumbent good as necessary to meet its total demand, hence $q_E + q_I = 1$. The surplus created with the rival can thus be rewritten as

$$S_{BE} = v_E q_E + v_I q_I - c_E q_E - T(1) - (v_I - \hat{\omega}_E)(q_I - 1) = v_I - T(1) + (\omega_E - \hat{\omega}_E)q_E. \quad (11)$$

For the same reasons, we get $V_0^B = v_I - T(1)$, and hence $\Delta S_{BE} = (\omega_E - \hat{\omega}_E)q_E$, showing that the buyer supplies the contestable part of her demand from the rival if and only if $\omega_E > \hat{\omega}_E$. It

[4]The variable ω_E first-order stochastically decreases (increases) with s_E if and only if $F(\omega_E|s_E)$ increases (decreases) with s_E.

follows that the proposed tariff allows the buyer and the incumbent to solve the rent-efficient tradeoff when the elasticity of entry $\varepsilon(\omega_E|s_E)$ and the corresponding entry threshold $\hat{\omega}_E(s_E)$ do not depend on the size of the contestable demand s_E. $\square$

The contestable units are sold below cost as $v_I - \hat{\omega}_E < v_I - \omega_I = c_I$. Such a tariff is represented on Figure 3b assuming that $\hat{\omega}_E < v_I$. It may well be the case, however, that $\hat{\omega}_E$ is larger than v_I while being lower than $v_I + \gamma_I$ to respect the assumption made in this section; in this case, contestable units of incumbent units would be sold at a negative price.

Effective price We define the "effective price" of the incumbent good as the average incremental price of the last units :

$$p^{\mathrm{e}}(x) = \frac{T(1) - T(1-x)}{x}. \tag{12}$$

This is the price the rival must match to supply the contestable demand when the buyer has the same willingness to pay for the two goods, $v_E = v_I$. The effective price, therefore, is negatively related to the competitive pressure placed on the rival: the lower the effective price, the higher the entry barrier and the more pressure placed on the rival.

For the two-part tariff studied above, the effective price is constant and equal to $p^{\mathrm{e}} = v_I - \hat{\omega}_E$. Decreasing p^{e} is equivalent to increasing the entry threshold $\hat{\omega}_E$, and thus the probability that the rival is driven out of the market. The elasticity of entry measures the sensitivity of the rival to competitive pressure.

Nondecreasing effective price From now on, we consider cases where the elasticity of entry varies with s_E and hence two-part tariffs are no longer optimal: the optimal tariff must exhibit some curvature. We start with the case where the elasticity increases with s_E: larger competitors, i.e., competitors with a larger contestable demand, are more sensitive to competitive pressure. Under this circumstance, the efficiency-rent tradeoff leads the buyer and the incumbent to place *less* competitive pressure on larger competitors.

Proposition 2. *When the elasticity of entry $\varepsilon(\omega_E|s_E)$ increases with s_E, the buyer and the incumbent set the effective price $p^{\mathrm{e}}(s_E)$ at $v_I - \hat{\omega}_E(s_E)$. The price schedule is concave in the neighborhood of $q_I = 1$. It is globally concave if $\hat{\omega}_E$ is concave or moderately convex in s_E. The equilibrium features inefficient exclusion; partial foreclosure is not present.*

Proof. When $\varepsilon(\omega_E|s_E)$ increases with s_E, the threshold $\hat{\omega}_E$ given by (10) decreases with s_E, see Figure 4a. Suppose the buyer and the incumbent set the effective price $p^{\mathrm{e}}(s_E)$ at $v_I - \hat{\omega}_E(s_E)$, which increases in s_E. From the definition of the effective price, (12), we recover the price schedule as

$$T(q_I) = T(1) + (v_I - \hat{\omega}_E)(q_I - 1),$$

where $\hat{\omega}_E$ is evaluated at $s_E = 1 - q_I$. The same observations as in the proof of Proposition 1 yield the expression (11) for the surplus S_{BE}. Since $\hat{\omega}_E$ decreases in q_E, the surplus is maximum either at $q_E = 0$ or at $q_E = s_E$. The rival makes no sales if $\omega_E < \hat{\omega}_E(s_E)$ and serves all the contestable demand if $\omega_E > \hat{\omega}_E(s_E)$.

To prove concavity in the neighborhood of $q_I = 1$, we differentiate the above expression twice with respect to q_I, which yields $T''(q_I) = 2\hat{\omega}'_E + (1 - q_I)\hat{\omega}''_E$. The term $\hat{\omega}'_E$, which is negative for any q_I, tends to make the tariff concave. Assuming that $\hat{\omega}''_E(0)$ is not infinite, we get $T''(1) = 2\hat{\omega}'_E(0) < 0$, hence the concavity at the top. □

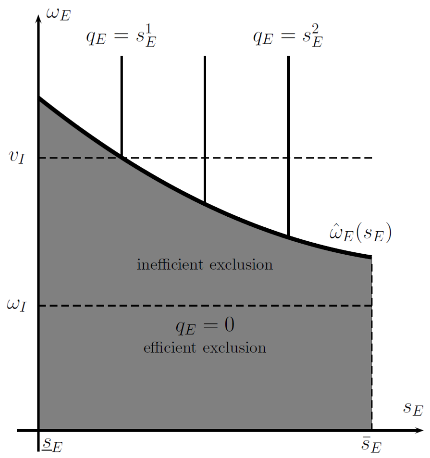

Figure 4a: Second best with $\varepsilon(\omega_E|s_E)$ increasing in s_E

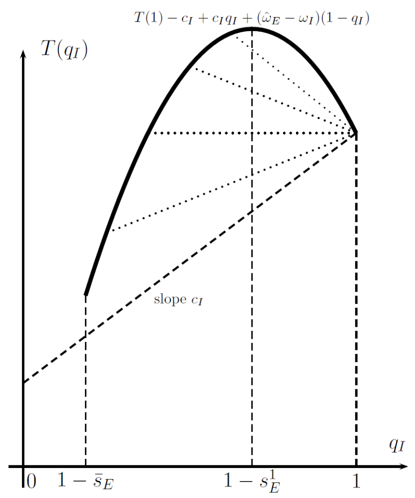

Figure 4b: Optimal price schedule with $\underline{s}_E = 0$ and $v_I < \hat{\omega}_E(0)$

Proposition 2 assumes that the elasticity of entry is nondecreasing in the size of the contestable demand. According to Lemma 2, this assumption implies that rival types with larger s_E tend to generate a lower surplus ω_E and hence are more sensitive to competitive pressure. The buyer and the dominant firm therefore exert less pressure on larger rival types, and the optimal effective price $p^{\text{e}}(q_E) = [T(1) - T(1 - q_E)]/q_E$ increases with q_E. Geometrically, the effective price is the slope of a chord drawn from the point $(1, T(1))$. The chords, represented by the dotted lines on Figure 4b, are indeed steeper as the number of concerned units rises: they are upwards-slopping for large values of q_E, approximately flat for intermediate values, and decreasing for low values. The latter property happens here the figure assumes $\hat{\omega}_E(0) > v_I$, implying that the effective price $p^{\text{e}}(q_E)$ is negative for low values of q_E, which gives the buyer strong incentives to supply exclusively from the dominant firm when the contestable market is small. For any concave schedule, the effective price (i.e., the slope of the chord) is monotonic, but the reverse is not true in general: the monotonicity of the effective price is a weaker property than concavity.

4 Derivation of the optimal allocation

When the elasticity of entry is not constant or increasing in the size of the contestable demand, solving the problem for each s_E separately does not yield an incentive compatible allocation. To illustrate, suppose that the entry barrier $\hat{\omega}_E$ depends on s_E as shown on Figure 5. In such a case, the solution to the relaxed problem, which is zero below the dotted line and s_E above, is not incentive compatible. Indeed, the rival of type $B = (\omega_E^B, s_E^B)$ would be inactive and hence would earn zero profit, while the rival $A = (\omega_E^A, s_E^A)$, with $s_E^A < s_E^B$ and $\omega_E^A = \omega_E^B$, would serve all of the contestable demand. It follows that type B would have an incentive to mimic type A and to sell s'_E rather than s_E.

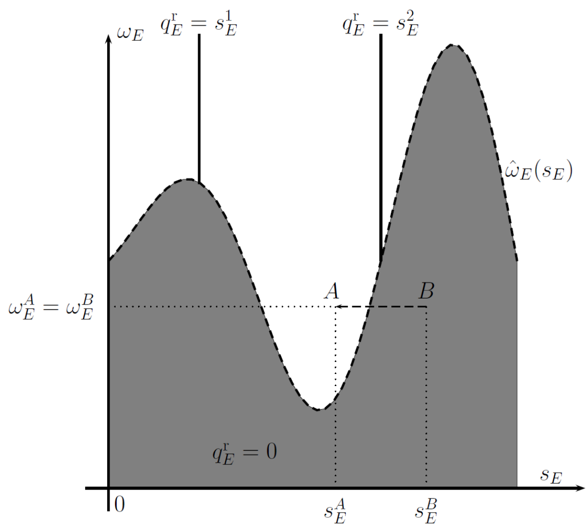

Figure 5: The relaxed solution is not implementable

To solve the problem, we therefore need a proper characterization of implementable quantity allocations. After providing such a characterization, we explain how to construct the optimal allocation. The main idea is that configurations like the one represented on Figure 5 give rise to quantity distortions at the intensive margin (partial foreclosure), for which an appropriate first-order condition must be derived.

Implementable allocations The buyer and the competitor maximize their joint surplus under a given price-quantity schedule $T(q_I)$. We know that the quantity of rival good is conditionally efficient, hence $q_E \leq s_E$. As disposal costs are infinite by assumption in this section, all units are consumed, $q_E + q_I \leq 1$. Finally, if the effective price of incumbent units is below her willingness to pay, i.e., if $p^{\mathrm{e}}(q) \leq v_I$ for all q,[5] the buyer finds it optimal to supplement her supply from the rival with enough incumbent units to meet her total purchase

[5]This inequality will be checked below.

requirements. Under this circumstance, we can replace q_I with $1 - q_E$ and rewrite the buyer-rival problem (3) as

$$S_{BE} = \max_{q_E \leq s_E} \omega_E q_E + v_I(1 - q_E) - T(1 - q_E). \tag{13}$$

A quantity function $q_E(s_E, \omega_E)$ is implementable with a non-conditional price schedule if and only if there exists a function $T(q_I)$ such that $q_E(s_E, \omega_E)$ is solution to (13) for all (s_E, ω_E).

The buyer and the rival, when solving (13), hit the constraint $q_E \leq s_E$ when the rival efficiency index ω_E is large. This is because q_E is nondecreasing in ω_E (recall that βq_E is the derivative of the rival's rent which is convex in ω_E). Formally, for any $s_E > 0$,there exists a threshold $\Psi(s_E)$ such that the buyer supplies all the contestable units from the competitor, $q_E(s_E, \omega_E) = s_E$, if and only if $\omega_E \geq \Psi(s_E)$. We define the *boundary line* $\omega_E = \Psi(s_E)$ associated to the quantity function $q_E(s_E, \omega_E)$ by

$$\Psi(s_E) = \inf\{x \in [\underline{\omega}_E, \overline{\omega}_E] \mid q_E(x, s_E) = s_E\},$$

with the convention $\Psi(s_E) = \overline{\omega}_E$ when the above set is empty. Above the boundary line, $q_E(s_E, \omega_E)$ equals s_E; below that line, $q_E(s_E, \omega_E)$ is independent from s_E.

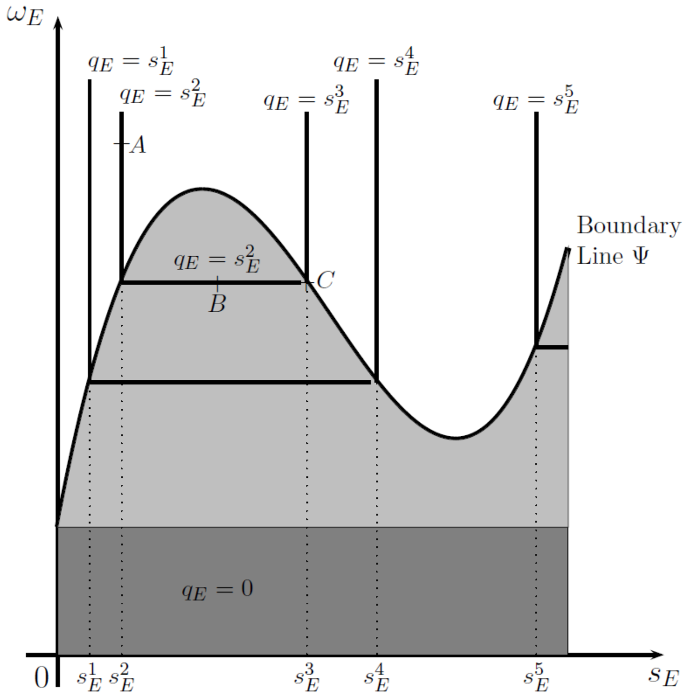

Figure 6: Implementable quantity function (isolines)

As shown on Figure 6, an implementable quantity function is entirely described by the associated boundary line. The bunching sets, i.e., the sets on which the quantity $q_E(s_E, \omega_E)$ is constant, are determined by the boundary line. They can be of three types: (i) vertical segments above decreasing portions of the boundary line (e.g. $q_E = s_E^3$ and $q_E = s_E^4$ on the

Figure); (ii) two-dimensional areas whose left border is vertical, being included either in the ω_E-axis (then $q_E = 0$, see the shaded area on Figure 6) or in a vertical part of the boundary line (see the light shaded area on Figure 13b); (iii) "L"-shaped unions of a vertical segment and a horizontal segment intersecting on an increasing portion of the boundary line (e.g. $q_E = s_E^1$, $q_E = s_E^2$ and $q_E = s_E^5$). For instance, the types represented by points A and B on Figure 6 belong to such a L-shaped bunch: both types have $q_E = s_E^2$, with A (respectively B) corresponding to a corner solution (resp. an interior solution) of (13). Note that type C does not belong to the same bunch, see below.

Increasing parts of the boundary function thus translate into horizontal bunching segments or into two-dimensional bunching areas, and hence into partial foreclosure: $0 < q_E(s_E, \omega_E) < s_E$ for some types located below the boundary. (To illustrate, type B on Figure 6 sells $q_E = s_E^2$, which is lower than the size of its contestable market.) In such regions, the constraint $q_E \leq s_E$ is slack: increasing s_E does not allow the competitor to enter at a larger scale and q_E does not depend on s_E.

Lemma 3. *A quantity function $q_E(.,.)$ is implementable if and only if there exists a boundary function $\Psi(.)$ defined on $[0,1]$ such that*

$$q_E(s_E, \omega_E) = \begin{cases} \min\{\ x \leq s_E \mid \Psi(y) \geq \omega_E \text{ for all } y \in [x, s_E]\} & \text{if } \Psi(s_E) > \omega_E, \\ s_E & \text{if } \Psi(s_E) \leq \omega_E. \end{cases} \tag{14}$$

When (13) has multiple solutions, the above definition selects the highest. For instance, type C on Figure 6 is indifferent between s_E^2 and s_E^3 and, by convention, is assumed to choose s_E^3. Notice that the quantity function jumps from s_E^2 to s_E^3 at C; more generally, implementable quantity functions are discontinuous along decreasing parts of their boundary line.

Proof. Starting from a boundary line Ψ, we derive the associated quantity function $q_E(s_E, \omega_E)$ with the bunching pattern described above. We then check in Appendix A that replacing q with $q_E(s_E, \omega_E)$ in the following equation

$$T(1) - T(1-q) = (v_I - \omega_E)q + \Delta S_{BE}(s_E, \omega_E), \tag{15}$$

where

$$\Delta S_{BE}(s_E, \omega_E) = \int_{\underline{\omega}_E}^{\omega_E} q_E(s_E, x)\, \mathrm{d}x, \tag{16}$$

unambiguously defines a non-conditional price-quantity schedule, $T(q)$. We also check that the buyer and the rival facing that schedule indeed agree on the considered quantity function $q_E(s_E, \omega_E)$.

Finally applying (15) for $q = s_E$ and $\omega_E = \Psi(s_E)$, we find $T(1) - T(1-q) = [v_I - \Psi(s_E)]\, q$, implying that the effective price $p^{\mathrm{e}}(q)$ is below v_I. This condition ensures that the buyer purchases enough units to meet her demand, $q_E + q_I = 1$, see footnote 5. □

Construction of the optimal allocation We now explain intuitively how to correct the height of the entry barrier $\hat{\omega}_E(s_E)$ when s_E is unknown. A formal construction of the optimal boundary line $\omega_E = \Psi(s_E)$ is presented in Appendix B.[6]

Consider a type (s_E, ω_E) with $\omega_E > \hat{\omega}_E(s_E)$. If the virtual surplus is always positive at the right of this point, there is no objection to setting $q_E = s_E$. In contrast, if the virtual surplus is negative at the right of this point, setting $q_E = s_E$ implies that q_E will have to be positive in an area where the virtual surplus is negative. We show in appendix that the expected virtual surplus on horizontal bunching segments is zero, as under the standard ironing procedure. Denoting by (AB) such a segment (see Figure 7b), we get

$$\mathbb{E}(\ s^{\mathrm{v}} \mid [AB]\ ) = 0, \tag{17}$$

with the boundary conditions that the virtual surplus is positive at A and zero at B. This leads to the following construction of the optimal boundary line $\omega_E = \Psi(s_E)$. We first draw the line $\omega_E = \hat{\omega}_E(s_E)$. For $s_E = \bar{s}_E$, we set $\Psi(\bar{s}_E) = \hat{\omega}_E(\bar{s}_E)$. Then we consider lower values of s_E. If $\hat{\omega}_E$ decreases at $\bar{s}_E$, we stick to the original entry barrier $\hat{\omega}_E$, as long as it remains decreasing. When $\hat{\omega}_E$ starts increasing (possibly at $\bar{s}_E$), we know that there is horizontal bunching. Equation (17) provides a unique value for $\Psi(s_E)$. If the candidate boundary hits the line $\omega_E = \hat{\omega}_E(s_E)$ at some lower value of s_E, it must be on a decreasing part of that line and, from that value on, the optimal boundary again coincides with $\hat{\omega}_E$ (as long as $\hat{\omega}_E$ remains decreasing). Proposition 3, proved in Appendix B, presents sufficient conditions for the above construction to yield the optimal allocation.

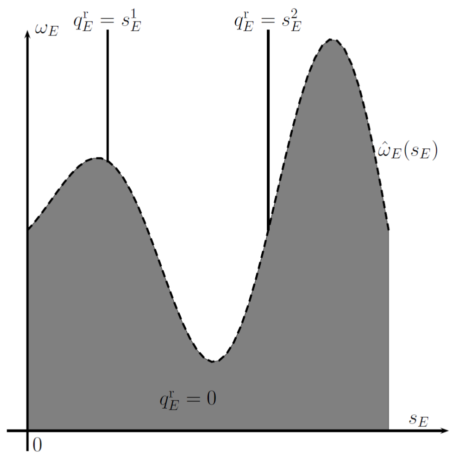

Figure 7a: The relaxed solution locally decreases with s_E.

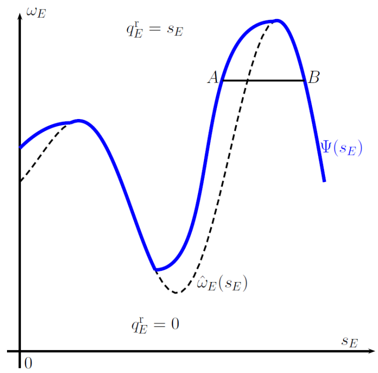

Figure 7b: Entry barrier $\hat{\omega}_E$ for known s_E (dashed). Optimal boundary Ψ (solid)

[6] Deneckere and Severinov (2009) propose a similar method for solving a general class of screening problems, which relies on a characterization of “isoquants”.

Proposition 3. *Assume that one of the following conditions holds: (i) The conditional density $f(\omega_E|s_E)$ is nondecreasing in ω_E; or (ii) the elasticity of entry is bounded from below and from above by $\underline{\varepsilon}$ and $\bar{\varepsilon}$ satisfying*

$$\beta(\bar{\varepsilon} - \underline{\varepsilon})^2 \leq 4\underline{\varepsilon}\bar{\varepsilon}. \tag{18}$$

Then the optimal boundary line Ψ can be constructed from the following properties:

1. $\Psi(\bar{s}_E) = \hat{\omega}_E(\bar{s}_E)$;
2. $\Psi = \hat{\omega}_E$ *where* Ψ *is non-increasing;*
3. Ψ *is given by (17) where it is increasing.*

The sufficient conditions of Proposition 3 are fairly mild. Under Assumption 1, the construction yields the optimal allocation provided that condition (18) holds, i.e., the range of the entry elasticity is not too wide. For instance, if the rival has all the bargaining power vis-à-vis the buyer ($\beta = 1$), the elasticity of entry $\varepsilon(\omega_E|s_E)$ may vary freely between 0.6 and 3 in the set of possible rival types.

When none of the two sufficient conditions holds, the above construction may not yield a function of s_E, as is the case in the example shown on Figure 8a.[7] Under this circumstance, the optimal allocation features two-dimensional bunching, see the shaded region D pictured on Figure 8b. The constant value of the rival quantity on the bunching region, denoted by $\hat{s}$ on the picture, is determined by the first-order condition that the expected unit virtual surplus is zero on that region, $\mathbb{E}(s^{\mathrm{v}}|D) = 0$.

5 The shape of optimal price schedules

After mentioning a set of general properties we exhibit circumstances under which optimal price-quantity schedules have convex parts and feature retroactive rebates.

General properties In Appendix C, we prove a number of properties that link the shape of the price schedule to that of the boundary line. In particular, we check that flat parts of the boundary line correspond to linear parts of the schedule (see Figure 3a and 3b) and increasing parts of the boundary line correspond to convex parts of the schedule (see Figures 9a and 9b as well as the portion A_1A_3 on Figures 10a and 10b). An upward discontinuity in the boundary line, that induces two-dimensional bunching, corresponds to a convex kink in the schedule, see Figures 8a and 8b, as well as Figures 13a and 13b in the appendix.

[7] We have constructed this example as follows: the elasticity of entry is $\underline{\varepsilon} = 1.75$ for $s_E > \tilde{s}$ and $\bar{\varepsilon} = 9.5$ for $s_E < \tilde{s}$, with $\tilde{s} = .7$; the size of the contestable demand belongs to $(0, \tilde{s})$ with probability .95 and to $(\tilde{s}; 1)$ with probability .05, and is uniformly distributed on each of the two intervals. Finally $\omega_I = \underline{\omega}_E = 1$.

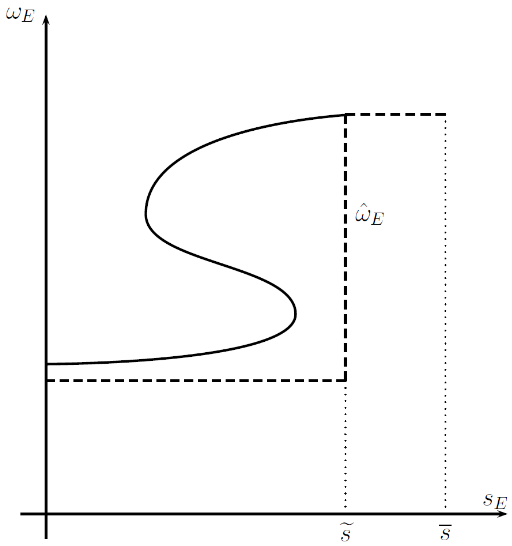

Figure 8a: Entry barrier for known s_E (dashed). Left extremity of bunching interval from (17) (solid line)

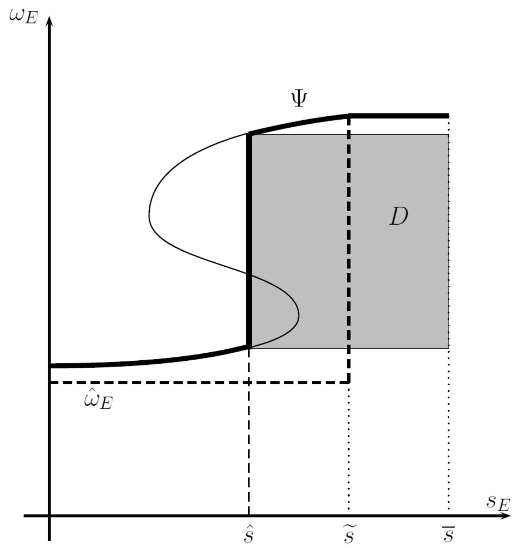

Figure 8b: Two-dimensional bunching area: $q_E = \hat{s}$ in D. Boundary line Ψ in bold.

In contrast, the curvature of the schedule may change along decreasing parts of the boundary: the schedule is concave near local maxima of the boundary line and convex near local minima. Local maxima of the boundary line thus correspond to inflection points of the tariff. An example is the point $A3$ on Figures 10a and 10b.

Convex price schedules We now turn to the case where the elasticity of entry $\varepsilon(\omega_E|s_E)$ decreases with s_E. Under this circumstance, the efficiency-rent tradeoff leads the buyer and the incumbent to place *more* competitive pressure on larger competitors: the threshold $\hat{\omega}_E(s_E)$ is monotonically increasing in s_E.

If q_E were equal to s_E above this threshold and zero below, the quantity purchased from the rival would locally decrease with s_E, which is impossible. Hence the presence of bunching along the s_E-dimension. Following the constructive method presented of Section 4, we find bunching intervals such as the horizontal interval (AC) represented on Figure 9a. A rival whose type belongs to (AC) sells s_E^1 units, where s_E^1 denotes the left extremity of the bunching interval. The unit virtual surplus $s^{\mathrm{v}}(s_E, \omega_E)$ is positive on (AB) and negative on (BC), as ω_E is above (resp. below) $\hat{\omega}_E(s_E)$ in these respective regions. The buyer and the incumbent must leave a positive rent to rival types in (BC) because those types can serve less than their contestable demand (i.e., "mimic" types with lower s_E). Under the sufficient conditions of Proposition 3, the virtual surplus is zero in expectation on bunching segments

$$\int_{s_E}^{\bar{s}_E} s^{\mathrm{v}}(s, \omega_E) f(\omega_E|s) g(s)\,\mathrm{d}s = 0,$$

and the above equation defines an increasing relationship between ω_E and s_E, denoted by

$\omega_E = \Psi(s_E)$ on Figure 9a.

Proposition 4. *Assume that $\varepsilon(\omega_E|s_E)$ decreases with s_E. Then the optimal tariff is convex. The equilibrium outcome exhibits inefficient exclusion, in the form of both full and partial foreclosure.*

Proof. For all $s_E \in [\underline{s}_E, \bar{s}_E]$, consider a rival type (s'_E, ω_E) with $\omega_E = \Psi(s_E)$ and $s'_E > s_E$. For such a type, the solution of the buyer-competitor problem (13) is interior. Assuming no two-dimensional bunching, the solution is given by the first-order condition $T'(1-s_E) = v_I - \Psi(s_E)$ or $T'(q_I) = v_I - \Psi(1-q_I)$, which increases with q_I because Ψ is an increasing function. We conclude that the price-quantity schedule T is convex. The analysis holds in the presence of two-dimensional bunching as well, with the minor difference the price schedule is locally non-differentiable (it admits a convex kink). $\square$

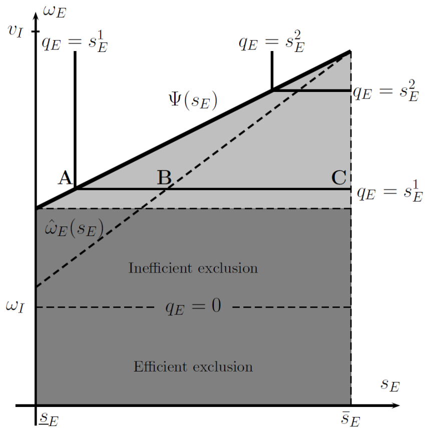

Figure 9a: Entry barrier for known s_E (dashed), second-best threshold $\Psi(s_E)$ (bold) with decreasing elasticity

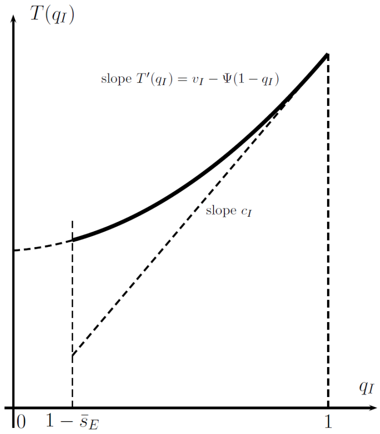

Figure 9b: Optimal price schedule with $\underline{s}_E = 0$ and $v_I > \hat{\omega}_E(1)$.

The price schedule plays the role of a barrier to expansion. When $T(q_I)$ is convex in q_I, the objective of the buyer-rival pair, $(\omega_E - v_I)q_E - T(1-q_E)$, is concave in q_E. The buyer and the rival compare the surplus created by an extra unit of rival good, ω_E, with the surplus foregone by consuming one unit less of incumbent good, $v_I - T'(1-q_E)$. The light-shaded area on Figure 9a represents the set of types for which the solution is interior, $0 < q_E(s_E, \omega_E) < s_E$, and hence the rival is partially foreclosed from the market, i.e., the quantity distortion is at the intensive margin.

"Retroactive" price schedules We now show that the dominant firm uses "retroactive rebates", also known as "all-units discounts" if a simple condition on the elasticity of entry is satisfied. Rebates are said to be retroactive when (i) they are granted provided that the buyer reaches a certain quantity threshold; and (ii) they apply to *all* the purchased units, not only to the units above the threshold. Such rebates induce downwards discontinuities in price-quantity schedules. Figure 11a shows the most simple retroactive price schedule. The slopes of the two segments correspond to the unit prices that are applied to all units depending on whether or not the quantity threshold $\bar{q}_I$ is attained.

We assume here that the elasticity of entry is first decreasing then increasing as the size of the contestable demand rises: competitors with intermediate size are less sensitive to competitive pressure than competitors with small or large size. Under this circumstance, the efficiency-rent tradeoff leads the buyer and the incumbent to place strong competitive pressure on competitors with intermediate size and less pressure on small or large competitors: the entry barrier $\hat{\omega}_E(s_E)$ is hump-shaped as shown on Figure 10a.

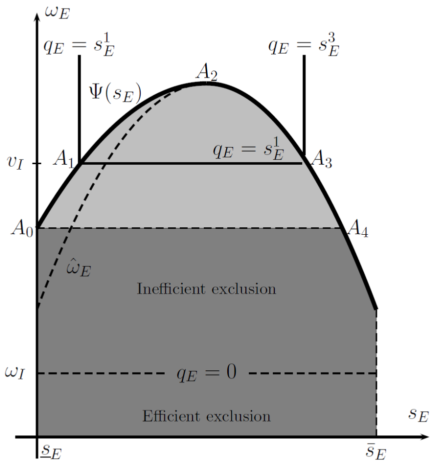

Figure 10a: Entry barrier for known s_E (dashed), second-best threshold (solid) with U-shaped elasticity

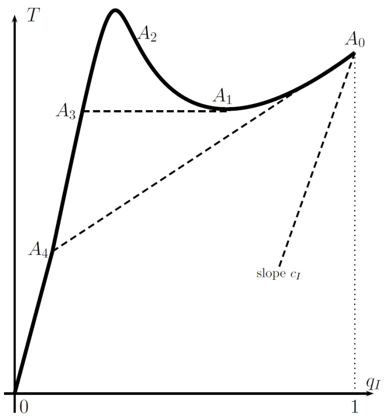

Figure 10b: Optimal price schedule with $\underline{s}_E = 0$ and $\bar{s}_E = 1$

We rely on Figures 10a and 10b to explain the shape of the optimal price schedule in this instance. Above the solid curve on Figure 10a, the competitor serves all of the contestable demand. In the light-shaded area below the solid curve, the quantity purchased from the competitor does not depend on the size of the contestable market. For instance, a rival whose type lies on the horizontal segment (A_1A_3) sells s_E^1 units. On such an interval, the unit virtual surplus is negative at the right of the dashed line $\omega_E = \hat{\omega}_E(s_E)$ and positive at its left.

When the size of the contestable demand is small, specifically between A_0 and A_2 , the equation of the light-shaded area's upper boundary, $\omega_E = \Psi(s_E)$, follows from the condition that the expected virtual surplus is zero on the bunching intervals such as the interval (A_1A_3). As already noticed, the quantity chosen by the buyer and the competitor is an interior solution of their surplus maximization and is therefore given by the first-order condition: $T'(1-s_E) = v_I - \Psi(s_E)$; the price-quantity schedule T is convex in this region, see Figure 10b.

Between A_2 and A_4, we recover the tariff by expressing that the quantity purchased from the rival is constant on the bunching segments. For example, if the rival is at A_3, the buyer-rival pair is indifferent between buying s_E^1 or s_E^3: $(\omega_E - v_I)s_E^1 - T(1-s_E^1) = (\omega_E - v_I)s_E^3 - T(1-s_E^3)$. As $T(1-s_E^1)$ is known, one can infer $T(1-s_E^3)$. At points A_1 and A_3, we have $\omega_E = v_I$, and hence $T(1-s_E^1) = T(1-s_E^3)$. It is readily confirmed that $T'' = 0$ at A_2, i.e., T has an inflection point, see point 4 of Lemma C.1.

Thus, a U-shaped elasticity of entry yields an optimal price-quantity schedule that is neither globally concave nor globally convex. The decreasing part of the price schedule gives the buyer a strong incentive to supply from the incumbent beyond the point A_1.

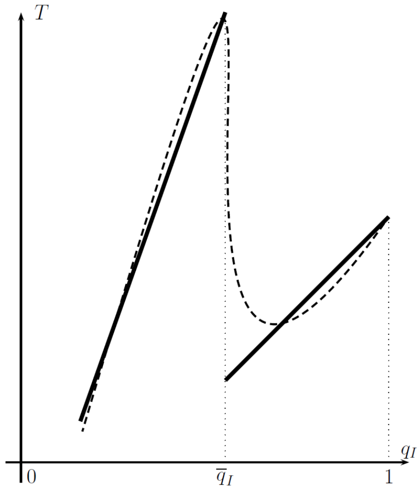

Figure 11a: Retroactive rebate approximating an optimal price schedule

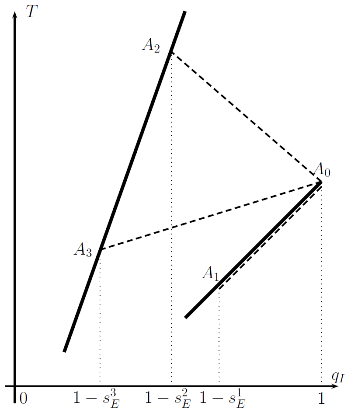

Figure 11b: Retroactive rebate when the contestable demand takes three values

When the distribution of types is continuous, the optimal price schedule is continuous. If instead the size of the contestable demand takes a finite number of values, a price schedule with a retroactive rebate, such as the one superimposed on Figure 11a, is optimal. Specifically, suppose that the support of s_E consists of three points $s_E^1 < s_E^2 < s_E^3$ and that the distribution of ω_E given s_E is such that $\hat{\omega}_E(s_E^2) > \max(\hat{\omega}_E(s_E^1), \hat{\omega}_E(s_E^3))$. Then the rent-efficiency tradeoff leads the buyer and the incumbent to place more (less) competitive pressure on the rival types with contestable demand s_E^2 (s_E^1 and s_E^3). This can be done with the schedule shown on Figure 11b. Critical on this figure are the slopes of the three dashed lines, which reflect the pressure put at each level.[8] Rival types with contestable market share s_E^2 or s_E^3 serve all of the contestable demand when they generate a high surplus ω_E, sell s_E^1 units when they generate a moderate surplus (partial foreclosure) or are inactive when they generate a low surplus.

The analysis so far has assumed that disposal costs are very large. To illustrate, suppose

[8] The quantity threshold (the discontinuity point in the schedule) can take any value between $1 - s_E^2$ and $1 - s_E^1$.

that the magnitude of disposal costs, γ_I, is smaller than the absolute value of the slope between A_1 and A_2 on Figure 11b. (This is the case for instance under free disposal, i.e., if $\gamma_I = 0$.) Then, to pocket the rebate, the buyer who has purchased s_E^2 units from the rival finds it ex post optimal to purchase $q_I = 1 - s_E^1$ from the dominant firm and to dispose of $s_E^2 - s_E^1$ units of incumbent good. This opportunistic behavior generates inefficient production and disposal costs and violates the equality $q_E + q_I = 1$ which has been taken for granted since the beginning of Section 4.

6 Disposal costs and conditional schedules

We now investigate how the magnitude of disposal costs affects the equilibrium outcome under a conditional and a non-conditional schedule.

Starting with a *conditional* schedule $T(q_E, q_I)$, we rewrite the buyer-rival problem (3) as

$$S_{BE} = \max_{q_E \le s_E} \omega_E q_E - \tau(q_E), \tag{19}$$

with $\tau(q_E) = -\max_{q_I} v_I q_I - T(q_I, q_E)$. By considering schedules of the form $T(q_E, q_I) = c_I q_I + P(q_E)$ and letting the penalty function P vary, we generate any function τ of q_E, and hence the same set of implementable quantity allocations as by letting the non-conditional schedule $T(q_I)$ vary in (13). At the same time, we are sure that the buyer purchases the conditionally efficient quantity of incumbent good, $q_I = q_I^*(q_E)$, irrespective of the magnitude of disposal costs. The construction presented in Sections 4 to 5, therefore, yields the optimal allocation under a conditional schedule for all $\gamma_I \ge -c_I$. Many schedules implement that allocation as c_I can be replaced with any price between $-\gamma_I$ and c_I while maintaining $q_I = q_I^*$.

Turning back to non-conditional schedules, we recall from Choné and Linnemer (2014) that the buyer's ability to dispose of unconsumed units at cost γ_I constrains the marginal price to be above $-\gamma_I$, formally $T' \ge -\gamma_I$. For instance, under free disposal ($\gamma_I = 0$), the portion of the schedule between A_1 and A_3 on Figure 10b is irrelevant, and can be replaced with a flat part between these two points. Indeed the point A_1 dominates any point in $(A_1 A_3)$ from the buyer's ex post perspective. As a second example, suppose that the slope of the tax schedule is $-\gamma_I$ at point B_1 on Figure 12b. Then the nonlinear portion of the schedule tariff above the segment $(B_1 B_2)$ is immaterial because the buyer prefers B_1 to any point between B_1 and B_2. More generally, changing T into $\hat{T}$

$$\hat{T}(q_I) = \inf_{q \ge q_I} T(q) + \gamma_I(q - q_I). \tag{20}$$

trims portions of the schedule that decrease more rapidly that γ_I. Intuitively, the modified schedule $\hat{T}$ offers the buyer the possibility of purchasing less units and to pay the corresponding

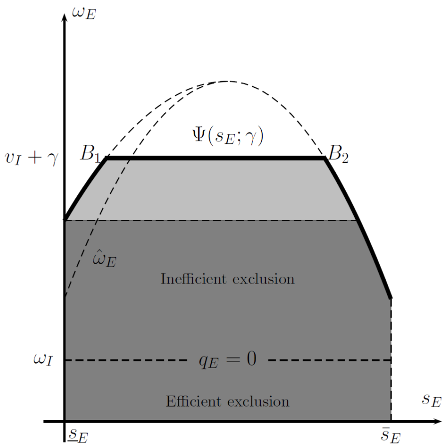

Figure 12a: Truncated second-best threshold (solid) with U-shaped elasticity

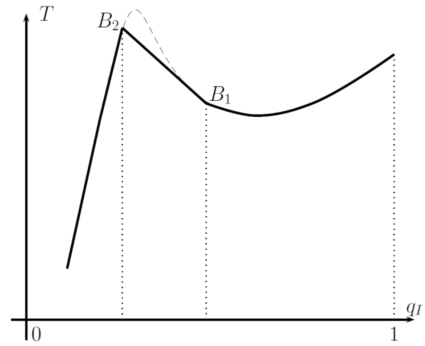

Figure 12b: Optimal price schedule under $\gamma_I = \infty$ (dashed); trimmed schedule (solid)

disposal costs directly to the dominant firm. It is immediate to check that the slope of $\hat{T}$ is never lower than $-\gamma_I$.

The following proposition shows that the constraint $T' \geq -\gamma_I$ is binding if and only if the efficiency-rent tradeoff leads to exclude some super-efficient rival types, i.e., $\hat{\omega}_E(s_E) \geq v_I + \gamma_I$ for some s_E. The intuition is that when the rival is not super-efficient, maximizing the virtual surplus and maximizing the *constrained* virtual surplus are the same thing, because the curves $q_E = q_E^*(q_I)$ and $q_I = q_I^*(q_E)$ coincide with the segment (AA') on Figures 1a and 1b.

Proposition 5. *The magnitude of the disposal costs, γ_I, affects the optimal allocation as follows:*

1. *The optimal conditional schedule $T(q_E, q_I)$ does not depend on γ_I;*

2. *When γ_I is larger than $\max \hat{\omega}_E - v_I$, the second-best allocation is the same whether the schedule is conditional or not;*

3. *When γ_I is smaller than $\max \hat{\omega}_E - v_I$, the optimal non-conditional schedule corresponding to $\gamma_I = \infty$ should be trimmed according to (20). The expected profit of the buyer-incumbent pair, $\mathbb{E}\Pi_{BI}$, is nondecreasing and total welfare is non-increasing in γ_I.*

Proof. The first point of the proposition has been demonstrated at the beginning of the section. To establish the second point, we derive the optimal boundary Ψ as explained in Section 4 and recover the schedule using (13)

$$T(1) - T(1 - s_E) = [v_I - \omega_E]s_E + \Delta S_{BE}(s_E, \omega_E), \tag{21}$$

with $\omega_E = \Psi(s_E)$. Differentiating with respect to s_E and observing that the terms coming from ω_E cancel out by the envelope theorem, we get

$$T'(1 - s_E) = v_I - \Psi(s_E) + \frac{\partial \Delta S_{BE}}{\partial s_E}. \qquad (22)$$

Since ΔS_{BE} is nondecreasing in s_E and that Ψ is below $\max \hat{\omega}_E$ (see Section 4), we get $T'(1 - s_E) \geq v_I - \max \hat{\omega}_E$. It follows that when γ_I is larger than $\max \hat{\omega}_E - v_I$, all units are sold at a price above $-\gamma_I$, and the buyer has no incentive to purchase unneeded units.

Suppose now that the rent-efficiency tradeoff would lead to the exclusion of super-efficient rival types. We know that this cannot happen under a non-conditional schedule, because the quantity purchased from the rival is conditionally efficient, hence $q_E = s_E$ for super-efficient rival types as shown on Figure 1c. This constraint is expressed in terms of boundary line by the inequality $\Psi \leq v_I + \gamma_I$. Accordingly, we replace $\Psi(s_E)$ with $\min(\Psi(s_E), v_I + \gamma_I)$. This truncation respects the bunching conditions on horizontal intervals and hence maximizes the expected virtual surplus on the set $\omega_E \leq v_I + \gamma_I$.[9] From (22) and the arguments below, we know that the constraint $T' \geq -\gamma_I$ is respected.

The truncated boundary line shifts upwards as γ_I rises, hence more inefficient exclusion and a lower welfare; at the same time, the profit of the buyer-incumbent pair rises as the constraint $T' \geq -\gamma_I$ is getting less stringent. □

When the buyer and the incumbent cannot condition prices on quantities purchased from the competitor, the buyer's ability to dispose of unneeded units of incumbent good limits the competitive pressure that can be placed on the rival, thus protecting super-efficient competitors from exclusion. Lower disposal costs, therefore, reduce the extent of inefficient market foreclosure. In the limit case where $\gamma_I = -c_I$, the constraint $T'(q_I) \geq -\gamma_I$ leaves no scope for anticompetitive exclusion.

7 Discussion

In this and our companion paper, we have examined nonlinear pricing through the lens of a static theory of harm. Our results shed light on the so-called "as-efficient competitor test" that checks whether a fictive rival firm, having the same production costs and selling a product of similar quality as the dominant firm, could profitably match the prices offered by that firm. The European Commission, describing its enforcement priorities, introduces the test as an instrument to screen out cases with few anticompetitive consequences.[10] The underlying logic has received a relative consensus on both sides of the Atlantic Ocean –see, e.g., the quote by

[9]Here we assume away the complications of two-dimensional bunching, which is possible in particular when the sufficient conditions of Proposition 3 are satisfied.

[10]European Commission (2009)

the Department Justice that opens our companion paper, as well as the support expressed by Shapiro and Hayes (2006).

In practice, the implementation of the test requires specifying the quantity range over which prices and costs should be estimated. It is natural to run the test over the contestable share of a buyer's demand because it is on that share that competition can possibly operate. In this paper, we have treated the number of contestable units as a random variable because "how much of the customer's purchase requirements can realistically be switched to a competitor" is a fundamentally uncertain figure.[11] Accordingly, the effective price is best represented as a function of the size of the contestable demand.

If larger rivals tend to be less efficient, the effective price governs the shape of the optimal price-quantity schedule and its exclusionary effects: either the rival can match the effective price and then serves the contestable demand, or it cannot and is driven out of the market. The logic prevailing in the one-dimensional model of Choné and Linnemer (2014) is different as the exclusionary effects are determined by the marginal price rather than by the effective price, and concavity is associated to partial, not complete, exclusion of rival types. In both models, however, nondecreasing and concave price schedules have the potential to generate anticompetitive market foreclosure. Competition agencies tend to see such schedules as relatively innocuous, often presuming they are justified by economies of scales. Our findings call for caution in this respect.

For other patterns of correlation between contestable demand and rival efficiency, the optimal effective price is non-monotonic, which generates partial exclusion of efficient rival types. Defendants in antitrust litigation commonly put forward that the alleged abuse did not prevent competitors from achieving a sizeable share of the market. Our analysis points out that antitrust enforcers are right to discard this line of defense as a positive market share is not incompatible with (partial) anticompetitive foreclosure.

Under non-conditional schedules, the buyer's ability to dispose of unneeded units limits the competitive pressure that can be placed on the rival. Higher disposal costs are associated to more exclusion and lower values of the expected total welfare. Antitrust authorities, therefore, should pay close attention to contracting provisions that help increase disposal costs. Conditional schedules, in effect, render disposal costs infinite and should therefore be subjected to close scrutiny.

[11] Assessed from an ex ante perspective, the distribution of uncertainty should reflect the players' beliefs at the time. For instance, in *Intel*, the Commission sought to determine "what volumes were actually *thought* to be at risk during the period examined" (emphasis added).

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Appendix

A Proof of Lemma 3

We first observe that $(v_I - \omega_E)q_E(s_E, \omega_E) + \Delta S_{BE}(s_E, \omega_E)$ is constant on q_E-isolines. Indeed, both $q_E(., \omega_E)$ and $\Delta S_{BE}(., \omega_E)$ are constant on horizontal isolines (located below the boundary line Ψ). On vertical isolines (above the boundary), $\Delta S_{BE}(s_E, .)$ is linear with slope s_E, guaranteing, again, that the above expression is constant. This shows that (15) unambiguously defines $T(1) - T(1-q)$ on the range of the quantity function $q_E(., .)$.[12]

We now prove that the buyer and the competitor, facing the above defined tariff T, agree on the quantity $q_E(s_E, \omega_E)$. We thus have to check that

$$\Delta S_{BE}(s_E, \omega_E) \geq (\omega_E - v_I)q' + T(1) - T(1-q') \tag{A.1}$$

for any $q' \leq s_E$. When q' is the range of the quantity function, we can write $q' = q_E(s'_E, \omega'_E)$ for some (s'_E, ω'_E), with $q' \leq s'_E$. Observing that $q' = q_E(q', \omega'_E)$ and using successively the monotonicity of ΔS_{BE} in s_E and its convexity in ω_E, we get:

$$\Delta S_{BE}(s_E, \omega_E) \geq \Delta S_{BE}(q', \omega_E) \geq \Delta S_{BE}(q', \omega'_E) + (\omega_E - \omega'_E)q',$$

which, after replacing $T(1) - T(1-q')$ with its value from (15), yields (A.1). To check (A.1) when q' is not in the range of the quantity function (q' belongs to a hole $(s^1_E s^2_E)$ as explained in Footnote 12), use (A.1) at s^1_E and the linearity of the tariff between s^1_E and q'.

[12] Notice that the range of q_E may be disconnected when Ψ is above $\overline{\omega}_E$ on some intervals. Specifically, if Ψ is above $\overline{\omega}_E$ on the interval $I = [s^1_E, s^2_E]$, then q_E does not take any value between s^1_E and s^2_E. In this case, we define T as being linear with slope $v_I - \overline{\omega}_E$ on the corresponding interval: $T(1-s^1_E) - T(1-q) = (v_I - \overline{\omega}_E)(q - s^1_E)$ for $q \in I$.

B Proof of Proposition 3

In Section B.1, we offer a convenient parametrization of horizontal bunching intervals. In Section B.2, we state and prove a one-dimensional optimization result, which serves to maximize the expected virtual surplus for a given level of ω_E. In Section B.3, we rewrite the complete problem as the maximization of the expected virtual surplus under monotonicity constraints. In Section B.4, we show that these constraints are not binding under fairly mild conditions.

B.1 Parameterizing horizontal bunching intervals

Consider an implementable quantity function q_E. For any ω_E, the function of one variable $q_E(.,\omega_E)$ is nondecreasing on $[0,1]$, being either constant or equal to the identity map: $q_E = s_E$. By convention, we call regions where it is constant "odd intervals", and regions where $q_E = s_E$ "even intervals".

We are thus led to consider any partition of the interval $[0,1]$ into "even intervals" $[s_{2i}, s_{2i+1})$ and "odd intervals" $[s_{2i+1}, s_{2i+2})$, where (s_i) is a finite, increasing sequence with first term zero and last term one.[13] We associate to any such partition the function of one variable that coincides with the identity map on even intervals, is constant on odd intervals, and is continuous at odd extremities. We denote by K the set of the functions thus obtained.

For any implementable quantity function q_E, the functions of one variable, $q_E(.,\omega_E)$, belong to K for all ω_E. Conversely, any quantity function such that $q_E(.,\omega_E)$ belong to K for all ω_E is implementable if and only if even (odd) extremities do not increase (decrease) as ω_E rises. Hereafter, we call the conditions on the extremities the "monotonicity constraints".

Even (odd) extremities constitute decreasing (increasing) parts of the boundary line. Odd intervals, $[s_{2i+1}, s_{2i+2})$, constitute horizontal bunching segments, or, more precisely, the horizontal portions of the L-shaped bunching regions.

B.2 A one-dimensional optimization result

In this section, we maximize a linear integral functional on the above-defined set K.

Lemma B.1. *Let $a(.)$ be a continuous function on $[0,1]$. Then the problem*

$$\max_{r\in K} \int_0^1 a(s)r(s)\,\mathrm{d}s$$

admits a unique solution r^* characterized as follows. For any interior even extremity s_E^{2i}, the function a equals zero at s_E^{2i} and is negative (positive) at the left (right) of s_E^{2i}. For any interior

[13] For notational consistency, we denote the first term of the sequence by $s_0 = 0$ if the first interval is even and by $s_1 = 0$ if the first interval is odd. Similarly, we denote the last term by $s_{2n} = 1$ if the last interval is odd and by $s_{2n+1} = 1$ if the last interval is even.

odd extremity s_E^{2i+1}, the function a is positive at s_E^{2i+1} and satisfies

$$\int_{s_E^{2i+1}}^{s_E^{2i+2}} a(s)\,\mathrm{d}s = 0. \tag{B.1}$$

If $a(1) > 0$, then $r^*(s) = s$ at the top of the interval $[0, 1]$. If $a(1) < 0$, then r^* is constant at the top of the interval.

Proof. Letting $I(r) = \int_0^1 a(x)r(x)\,\mathrm{d}x$, we have

$$I(r) = \sum_i \int_{x_{2i}}^{x_{2i+1}} xa(x)\,\mathrm{d}x + \sum_i x_{2i+1} \int_{x_{2i+1}}^{x_{2i+2}} a(x)\,\mathrm{d}x,$$

where the index i in the two sums goes from either $i = 0$ or $i = 1$ to either $i = n - 1$ or $i = n$, in accordance with the conventions exposed in Footnote 13. Differentiating with respect to an interior even extremity yields

$$\frac{\partial I}{\partial x_{2i}} = a(x_{2i}).[x_{2i-1} - x_{2i}].$$

The first-order condition therefore imposes $a(x_{2i}^*) = 0$. The second-order condition for a maximum shows that a must be negative (positive) at the left (right) of x_{2i}^*.

Differentiating with respect to an interior odd extremity yields

$$\frac{\partial I}{\partial x_{2i+1}} = \int_{x_{2i+1}}^{x_{2i+2}} a(x)\,\mathrm{d}x.$$

The first-order condition therefore imposes $\int_{x_{2i+1}^*}^{x_{2i+2}^*} a(x)\,\mathrm{d}x$. The second-order condition for a maximum imposes that a is nonnegative at x_{2i+1}^*.

If $a(1) > 0$, then it is easy to check that $r^*(x) = x$ at the top, namely on the interval $[x_{2n}^*, x_{2n+1}^*]$ with x_{2n}^* being the highest zero of the function a and $x_{2n+1}^* = 1$. If the function a admits no zero, it is everywhere positive and hence $r^*(x) = x$ on the whole interval $[0, 1]$.

If $a(1) < 0$, then r^* is constant at the top, namely on the interval $[x_{2n-1}^*, x_{2n}^*]$, with $x_{2n}^* = 1$ and $\int_{x_{2n-1}^*}^1 a(x)\,\mathrm{d}x = 0$. If the integral $\int_y^1 a(x)\,\mathrm{d}x$ remains negative for all y, then r^* is constant and equal to zero on the whole interval $[0, 1]$. □

B.3 Solving the complete problem

The complete problem consists in maximizing the expected virtual surplus subject to the even (odd) extremities being nonincreasing (nondecreasing). The latter conditions are called hereafter the "monotonicity constraints".

Applying Lemma B.1 with $a(s_E) = s^{\mathrm{v}}(s_E, \omega_E)$ for any given ω_E, we find that the virtual surplus is zero at candidate even extremities: $s^{\mathrm{v}}(x_{2i}(\omega_E), \omega_E) = 0$ and is negative (positive) at the left (right) of these extremities. In other words, $\Psi = \hat{\omega}_E$ at candidate even extremities. Thus, as regards even extremities, the monotonicity constraints are never binding.

Lemma B.1 also implies that the virtual surplus is positive at odd extremities. At these extremities, we must therefore have $\Psi > \hat{\omega}_E$. By the first-order condition (B.1), the expected virtual surplus is zero on horizontal bunching intervals:

$$\mathbb{E}(s^{\mathrm{v}}|H) = 0, \tag{B.2}$$

where H is a horizontal bunching interval with extremities s_E^{2i+1} and s_E^{2i+2}. The virtual surplus on a bunching interval is first positive, then negative as s_E rises, and its mean on the interval is zero. The segment (AB) on Figure 7b is an example of horizontal bunching interval (in fact the horizontal part of an "L"-shaped bunching set). Unfortunately, the first-order condition (B.2) does not imply that candidate odd extremities $x_{2i+1}(\omega_E)$ are nondecreasing in ω_E: odd extremities might decrease with ω_E in some regions, generating two-dimensional bunching.

B.4 Sufficient conditions

We now check that each of the three conditions mentioned in Proposition 3 is sufficient for the odd extremities $s_E^{2i+1}(\omega_E)$ to be nondecreasing in ω_E. We can restrict attention to efficient rival types, $\omega_E \geq \omega_I$.[14] We rewrite equation (B.2) as $A(s_E^{2i+1}, \omega_E) = 0$ with

$$\begin{aligned} A(s_E^{2i+1}, \omega_E) &= \int_{s_E^{2i+1}}^{s_E^{2i+2}} s^{\mathrm{v}}(s, \omega_E) f(\omega_E|s) g(s)\,\mathrm{d}s \\ &= \int_{s_E^{2i+1}}^{s_E^{2i+2}} \left[(\omega_E - \omega_I) f(\omega_E|s) - \beta(1 - F(\omega_E|s))\right] g(s)\,\mathrm{d}s. \end{aligned}$$

The function A is nonincreasing in s_E^{2i+1}, as the virtual surplus is nonnegative at this point:

$$\frac{\partial A}{\partial s_E^{2i+1}}(s_E^{2i+1}, \omega_E) = -s^{\mathrm{v}}(s_E^{2i+1}, \omega_E) f(\omega_E|s_E^{2i+1}) g(s_E^{2i+1}) \leq 0.$$

Differentiating with respect to ω_E, we get

$$\frac{\partial A}{\partial \omega_E}(s_E^{2i+1}, \omega_E) = \int_{s_E^{2i+1}}^{s_E^{2i+2}} \left[(\omega_E - \omega_I) f'(\omega_E|s) + f(\omega_E|s) + \beta f(\omega_E|s)\right] g(s)\,\mathrm{d}s,$$

where we denote by f' the derivative of f in ω_E.

When f is nondecreasing in ω_E, or $f' \geq 0$, we have $\partial A/\partial \omega_E \geq 0$, and hence the odd extremities are nondecreasing in ω_E. We now examine successively the cases where the hazard

[14] For $\omega_E < \omega_I$, the virtual surplus is negative for all s_E and the solution is $q_E = 0$ for all s_E.

rate is nondecreasing in ω_E (a weaker condition than $f' \geq 0$) and the elasticity of entry is nondecreasing in ω_E (an even weaker condition).

We now assume that the hazard rate, $f/(1-F)$, is nondecreasing in ω_E, which can be expressed as $f' \geq -\varepsilon f/\omega_E$. Using $\omega_E \geq \omega_I$, we find that

$$\begin{aligned}
\frac{\partial A}{\partial \omega_E} &\geq \int_{s_E^{2i+1}}^{s_E^{2i+2}} \left[-(\omega_E - \omega_I)\frac{\varepsilon}{\omega_E} + 1 + \beta\right] f(\omega_E|s)g(s)\,\mathrm{d}s \\
&= \int_{s_E^{2i+1}}^{s_E^{2i+2}} \left\{\varepsilon \left[\frac{\omega_I}{\omega_E} - 1 + \frac{\beta}{\varepsilon}\right] + 1\right\} f(\omega_E|s)g(s)\,\mathrm{d}s.
\end{aligned}$$

On a horizontal interval H, the variable ω_E is constant, and only the elasticity ε may vary. Hence, the first order condition (B.2) yields: $\mathbb{E}(1-\beta/\varepsilon \,|\, H) = \omega_I/\omega_E$. The right-hand side of the above inequality is equal, up to a positive multiplicative constant, to $1 - \mathrm{cov}\,(\varepsilon, 1-\beta/\varepsilon|\, H)$. We now look for a sufficient condition for this expression to be nonnegative for any distribution of ε. Noting $m = \mathbb{E}(\varepsilon|H)$ the expectation of ε on H, the condition can be rewritten as

$$\mathbb{E}\left[(\varepsilon - m)\left(1 - \frac{\beta}{\varepsilon}\right)\bigg|\, H\right] \leq 1.$$

The function $(\varepsilon - m)(1-\beta/\varepsilon)$ is convex in ε. We denote by $[\underline{\varepsilon}, \bar{\varepsilon}]$ the support of the distribution of ε. For given values of $\underline{\varepsilon}$, $\bar{\varepsilon}$ and $m = \mathbb{E}(\varepsilon|H)$, the expectation of this convex function is maximal when the distribution of ε has two mass points at $\underline{\varepsilon}$ and $\bar{\varepsilon}$, associated with the respective weights $(\bar{\varepsilon} - m)/(\bar{\varepsilon} - \underline{\varepsilon})$ and $(m - \underline{\varepsilon})/(\bar{\varepsilon} - \underline{\varepsilon})$. We thus need to make sure that

$$(\bar{\varepsilon} - m)(\underline{\varepsilon} - m)\left(1 - \frac{\beta}{\underline{\varepsilon}}\right) + (m - \underline{\varepsilon})(\bar{\varepsilon} - m)\left(1 - \frac{\beta}{\bar{\varepsilon}}\right) \leq \bar{\varepsilon} - \underline{\varepsilon},$$

for any $m \in [\underline{\varepsilon}, \bar{\varepsilon}]$. The left-hand side of the above inequality is maximal for $m = (\underline{\varepsilon} + \bar{\varepsilon})/2$. It follows that the inequality holds for all $m \in [\underline{\varepsilon}, \bar{\varepsilon}]$ if and only if the condition (18) is satisfied.

C From the boundary function to the price schedule

Lemma C.1. *The shape of the boundary function Ψ and the curvature of the price schedule T are linked in the following way:*

1. *If Ψ is increasing (resp. constant) around s_E, then the tariff is strictly convex (resp. linear) around $1 - s_E$.*
2. *If Ψ decreases and is concave around s_E, then the tariff is concave around $1 - s_E$.*
3. *If Ψ decreases and is convex around s_E and s_E is close to a local minimum of Ψ, then the tariff is convex around $1 - s_E$.*

4. If Ψ has a local maximum at s_E, then the tariff has an inflection point at $1 - s_E$.

Proof. First, suppose that Ψ is nondecreasing on a neighborhood of s_E. Let s'_E slightly above s_E. Then $q_E = s_E$ is an interior solution of the buyer-rival pair's problem (13) for s'_E and $\omega_E = \Psi(s_E)$. It follows that the first order condition $\Psi(s_E) - v_I + T'(1 - s_E) = 0$ holds, implying property 1 of the lemma. The property holds when Ψ has an upward discontinuity at s_E, in which case the tariff has a convex kink at $1 - s_E$. To illustrate, Figures 13a and 13b consider the case where the boundary line is a nondecreasing step function with two pieces.

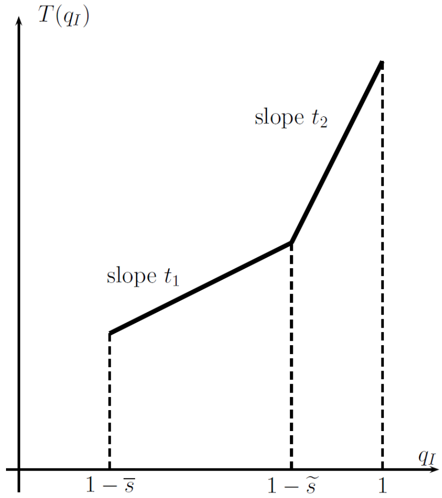

Figure 13a: Convex kink in the price schedule

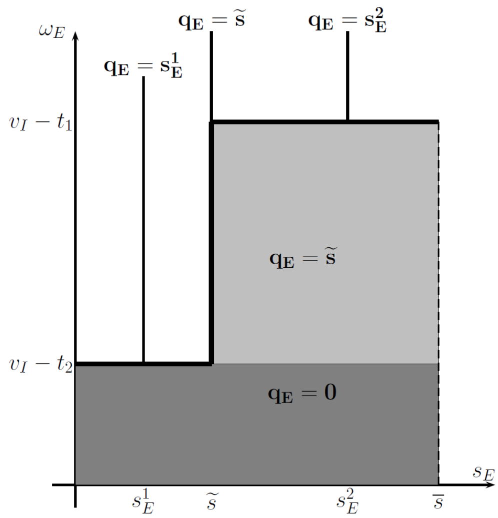

Figure 13b: Two-step increasing boundary line

Next, suppose that the boundary line decreases around s_E. Here we assume that Ψ is twice differentiable. We denote by $[\sigma(s_E), s_E]$ the set of value s'_E such that $q_E(s'_E, \omega_E) = \sigma(s_E)$, where $\omega_E = \Psi(s_E)$. The buyer-rival surplus $\Delta S_{BE}(s_E, \omega_E)$ is convex and hence continuous in ω_E. It can be computed slightly below or above $\Psi(s_E)$. At $(s_E, \Psi(s_E))$, the buyer and the rival are indifferent between quantities s_E and $\sigma(s_E)$:

$$\Delta S_{BE}(s_E, \Psi(s_E)) = [\Psi(s_E) - v_I]\sigma(s_E) - T(1 - \sigma(s_E)) = [\Psi(s_E) - v_I]s_E - T(1 - s_E).$$

Differentiating and using the first-order condition at $\sigma(s_E)$ yields

$$T'(1 - s_E) = -\Psi'(s_E)[s_E - \sigma(s_E)] - \Psi(s_E) + v_I.$$

Differentiating again yields

$$T''(1 - s_E) = \Psi''(s_E)[s_E - \sigma(s_E)] + \Psi'(s_E)[2 - \sigma'(s_E)]. \tag{C.1}$$

In the above equation, the two bracketed terms are nonnegative (use $\sigma' \leq 0$), and the slope Ψ' is negative by assumption, which yields item 2 of the lemma. Around a local minimum of Ψ, Ψ' is small, and the first term is positive, hence property 3. Property 4 follows from items 1 and 2. $\square$