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Why Do Homogeneous Firms Export Differently? A Density Externality Approach of Trade.[☆]

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Abstract

While the reason why the average exporting firm has a higher productivity than a purely domestic firm is now well understood, the theoretical literature has remained silent on why firms do not enter foreign markets according to an exact hierarchy, as predicted by models à la Méltz. To this aim, this paper proposes a new model of export choice in which the interactions between firms are characterized by density externalities. This type of interaction is closely related to a Mean Field Game. After showing that such an interaction is included in standard monopolistic competition, in the short run, we show how homogeneous firms can export differently. Moreover in this model, it remains true that more productive firms export *on average* to less attractive countries. Thence, our model displays a non-exact hierarchy of trade, as the findings of [Eaton et al. \(2011\)](#) suggest.

Keywords: Export Choice, Dispersion in Strategies, Density Externalities, Trade Externalities, Sequential Exporting.

JEL classification: F1.

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1. Introduction

Over the last decade, the [Melitz \(2003\)](#) model has become the standard framework in international trade. Including heterogeneity in marginal costs in a monopolistic competition model *à la* [Krugman \(1979, 1980\)](#) with as usual a fixed production cost and a fixed export cost, it explains self-selection. The popularity of the model is explained by its ability to reproduce a well-established empirical fact, namely the self-selection of firms into export markets. In the data, the average productivity of firms is shown to be increasing in the cost to enter the destination market. In the [Melitz \(2003\)](#) model, this tendency is explained by the presence of increasing costs that prevent firms with insufficient productivity from entering difficult markets. Namely, the model provides a rationale for the strong empirical evidence that exporters tend to be more productive than firms that sell exclusively domestically. In this model, with foreign countries ranked by their attractiveness, [Melitz \(2003\)](#) predicts an exact hierarchy of trade, meaning that if a firm exports to the k th most attractive market, then it should export to the $k - 1$ th most attractive country as well. This prediction is consistent with the data, *on average* ([Bernard and Jensen, 1999](#); [Bernard et al., 2007](#); [Mayer and Ottaviano, 2008](#); [Berthou and Fontagne, 2008](#); [Manova and Zhang, 2009](#)). A corollary of this prediction is that two firms that have the same productivity should export to the same set of markets (since we admit homogeneity in terms of demand and fixed costs otherwise). However, recent empirical work ([Eaton et al., 2011](#)) shows that this model is not fully consistent with the data. While it is true that the average productivity is decreasing in market access, firms do not enter markets according to an exact hierarchy. For instance, two firms which are identical in terms of their productivity can export to different markets. In other words, conditional on firms' productivity, firm-level data display a large amount of heterogeneity in terms of export destinations.³

³For instance, [Eaton et al. \(2011\)](#) report that only 17,699 out of 34,035 French exporters export to Belgium. As Belgium is the most attractive country from a French firm's point of view, then, according to Melitz, 100% of French exporters should export to Belgium.

Our main goal is to provide a model in which homogeneous firms can export differently. To this end, we propose a model of export choice with firms which are homogeneous in terms of their marginal cost. Our main theoretical point is to demonstrate that homogeneous firms export differently in the presence of density externalities. This kind of interaction between firms simply means that firms' profits are negatively correlated to how tough the competition is. We first show that such interactions are present in a simple monopolistic competition model *in the short run*.

In the first part of the paper, we present the simplest approach of our model with a static version. In this model, firms play a two-stage game. They first choose a unique destination to export to and once the destination is set, they compete monopolistically with other exporters to this destination. The outcome of the model is mixed strategies within a class of perfectly homogeneous firms. Moreover, in order to study the role of productivity, we use comparative statics. By varying the marginal cost, we show that the more productive firms are, the further they export on average.⁴ Combined with the fact that within a given type of productivity, there is a dispersion in destinations, the outcome of our model is close to what [Eaton et al. \(2011\)](#) call a "non-exact hierarchy".

In the recent literature, two models ([Eaton et al., 2011](#); [Chaney, 2011](#)) deal with the issue that we address. In both papers, firms with identical marginal costs can serve different sets of foreign markets. The common feature of these two papers is that they introduce a second dimension of heterogeneity between firms that helps explain heterogeneous choices in the markets served, conditional on their productivity. In the [Eaton et al. \(2011\)](#) model, the second dimension of heterogeneity that is added to the standard Melitz framework is market- and firm-specific heterogeneity in entry costs and demand. It is done by incorporating [Arkolakis \(2010\)](#) formulation of market access. Namely, [Eaton et al. \(2011\)](#) introduce firm- and country-specific entry costs and idiosyncratic demand shocks. In this model, even if two firms are homogeneous in terms of their marginal

⁴Here, distance stands for an inverse measure of attractiveness.

costs, they differ in the demand that they face and in the fixed cost that they incur to export to a given location. This extra dimension of heterogeneity explains why two identically productive firms make different choices regarding the decision to enter a given market. Only the one with the highest market-specific component of demand and/or lowest market-specific component of fixed cost enters.

[Chaney \(2011\)](#) also adds a second dimension of heterogeneity between firms. He argues that firms can meet trading partners in two ways. First, by direct search which is modelled as a geographically biased random search in his model. Second, once a firm has acquired trading contacts in foreign locations, it can develop a new network from these locations. So, in [Chaney \(2011\)](#)'s model, firms differ in their ability to develop a network of consumers in a given market. This dimension of heterogeneity explains the heterogeneity of export choices across firms belonging to the same type of productivity. Firms export to those markets where they are able to develop their network. Once again, as in [Eaton et al. \(2011\)](#), heterogeneous strategies within a same type of productivity are driven by heterogeneity in characteristics among firms, even if marginal costs are the same.

In both of these papers, mixed strategies are explained by an additional dimension of heterogeneity. Even if two firms are identical in terms of their marginal costs, they do not share the same market-specific demand and fixed cost ([Eaton et al., 2011](#)), or the same ability to develop their network of trading partners ([Chaney, 2011](#)).

Unlike these recent contributions, our model provides an endogenous explanation, in the short run, of this feature of the data. We argue that, in the presence of density externalities (see explanations in the online appendix), homogeneous firms may differ in export choices. The presence of density externalities allows us to apply a methodology which is closely related to Mean Field Games (MFG hereafter). So, we first show that in our static two-step model, the outcome of the monopolistic competition stage displays sufficient features to apply our methodology. Namely, we identify the presence of density

externalities in the profit function, which simply means that profits are decreasing in the density of competitors. At the same time, as it seems natural, profits are decreasing in the distance of the destination. Thus, firms face a trade-off. They first have an incentive to export to close economies in order to avoid paying high transport costs, but they anticipate that these destinations are precisely those to which a high number of competitors will export. Then, they also have incentives to export to remote countries in order to escape competition. This trade-off is the key feature of our model. An equilibrium is a distribution of firms with respect to their destination choice (we assume a continuum of possible destinations), such that both effects of the trade-off compensate for each other. In other words, an equilibrium is characterized by a situation in which all firms with the same productivity obtain the same profit. Another important result is that our model displays trade externalities. With such interactions between firms, changing the trade cost in a given region impacts other regions. Namely, trade liberalization in a given region prompts firms to export to this region and to desert others. Thus, a trade liberalization leads to a reallocation of firms with respect to their destination. This leads to non-obvious implications of trade liberalization.

The results of our model owe their properties to two key elements. First, we assume that each firm exports to a unique destination and that the number of firms is fixed. This allows having positive profits at equilibrium. This assumption is key while being empirically motivated as well notably by [Heid et al. \(2010\)](#) and [Albornoz et al. \(2012\)](#) who pay attention to the dynamics of exports, or more generally to expansion at the firm level. Using Chinese and Argentinean data respectively, they show that the export choices of firms feature a sequential entry into foreign markets. Thus, contrary to what is implied in long-run models with free entry, firms do not choose all their destinations simultaneously, but rather one at a time. The second key feature is our methodology. Density externalities are local interactions. Hence, there are interactions between firms, but at the same time, there is a continuum of firms. In other words, interactions between firms are of mean-field type. An interaction of the mean-field type is defined as

a situation in which agents make their decision based upon a given statistic (here the distribution of exporters according to their export choice), but each agent has a marginal impact on this statistic and thus cannot influence the strategy of other players by herself. As a consequence, each agent is atomized in the continuum and chooses a strategy that depends on the distribution of players' characteristics. MFG is a simple way of introducing interactions between agents.⁵ Instead of taking the whole set of possible interactions between agents into account (as is done in oligopoly models where interactions are strategic), players consider the influence of others as summed up by their distribution.

In the second part of the paper, we add periods to our baseline model. In this dynamic framework, each firm chooses a destination per period (we still assume that the number of firms is fixed). With no fixed cost of exporting, we show that our model tends to yield the same predictions as [Melitz \(2003\)](#). Namely, when time tends to infinity, firms with the same type of productivity tend to export to the same basket of destinations. Moreover, we show by comparative statics that more productive firms still export further on average. Therefore, our argument is that what we observe in the data is a snapshot of the convergence towards the [Melitz \(2003\)](#) equilibrium. A first corollary is that as time runs, firms with the same productivity tend to have more and more similar strategies in terms of destinations. A second corollary is that the older the firm is as an exporter, the more its basket of destinations sticks to the [Melitz \(2003\)](#) prediction. Adding a fixed cost of exporting to this dynamic framework has several implications. First, the model never converges to an exact hierarchy in trade. There exists a unique period when the game stops. Namely even with profits driven to zero, there are mixed strategies at equilibrium. Second, the period when the game stops is increasing with productivity. As we assume that firms choose a unique destination per period, we find that more productive firms export to more countries. Generally, our theoretical argument is that models with free entry cannot explain mixed strategies without adding an extra dimension of heterogene-

⁵For more details about MFG, see the online appendix.

ity.

The remainder of the paper is organized as follows: Section 2 presents the body of the model, Section 3 presents some results of this baseline model. Section 4 discusses some implications of symmetric and asymmetric trade liberalization, especially on welfare. Section 5 presents the dynamic framework.

2. The Baseline Model

Let us consider a world composed of a Home country and a continuum of foreign countries distributed on a Hotelling line $[0; X]$. Foreign countries are ranked with respect to their distance x to the Home country. As we assume that countries are identical in terms of size and demand, x stands for an inverse measure of market potential from the viewpoint of a domestic firm.⁶

We assume two goods in the economy. One homogeneous produced under perfect competition in all countries and one horizontally differentiated produced only by domestic firms. This means that we assume no local firms in foreign countries.⁷

2.1. Supply Side

In the Home country, there is a unit mass of *ex ante* homogeneous and risk-neutral firms. As our primary goal is to explain why homogeneous firms export differently, we assume that they all produce a variety of the horizontally differentiated good at the same marginal cost c . These firms have the possibility of exporting to a single foreign country. To export to x , they have to incur a transport cost $\tau x > 0$ for each unit of exported good knowing that, once the destination is selected, they will be engaged in a monopolistic competition only with other domestic firms that export to the same destination.

⁶More exactly, x captures the relative attractiveness of countries from the viewpoint of a domestic firm. Thus, x captures more precisely the notion of multilateral trade resistance.

⁷See appendices for a proof that adding local firms does not change the nature of our results.

The linearity of the transportation cost with respect to distance can be relaxed without any difficulty. See Subsection 3.3 for a discussion about the tractability of our model.

Thus, within a simple framework, domestic firms play the following two-step game:

1. They choose a unique country x to export to
2. They compete (monopolistically) in quantity in country x with all firms that export to this location

Therefore, the present model is a short-run model with no free entry. As a consequence, there is no general equilibrium effect and our focus is only on identifying a theoretical reason that explains mixed strategies among homogeneous firms.

The fact that firms choose a unique destination is key in our model as it forbids free entry and then allows profits to be positive at equilibrium. If we allowed firms to simultaneously choose all destinations where they earn positive profits, then our model would become standard again and we would not find mixed strategies at equilibrium. So, this assumption is key for our results and is empirically supported. Among others, [Albornoz et al. \(2012\)](#) show that most firms start by exporting to a single destination and then, for those which survive as exporters, export to another destination. This dynamics of trade is described as "sequential exporting". Notice that the model can be derived by assuming that each firm chooses a number $n > 1$ of destinations. Instead of choosing a scalar, firms would choose a vector of size n destinations. The nature of the results would be the same unless the model obtained added intractability.

Once firms have selected their destination x , we denote respectively $q_i(x)$ and $Q(x)$ the quantities sold in country x for variety i and for all varieties available in country x , namely the total quantity exported. Hence, we have:

$$Q(x) = \int_{I_x} q_i(x) di$$

with I_x the set of available varieties in x . As I_x also represents the density of firms

exporting to x , it will be endogenously determined by the first stage of the game.

The entire program of a firm is the following:

$$\max_{x, q_i(x)} \pi_i(x, q_i(x)) = \max_{x, q_i(x)} \{(p_i(x) - c - \tau x)q_i(x)\}$$

with π_i the profit of the firm i and $p_i(x)$ the inverse demand it faces in country x .

The first stage of the game consists in choosing x , while the second stage determines $q_i(x)$. In the present paper, we solve this model with a quadratic utility function, but it can be derived with other functional forms (see Subsection 3.3).

2.2. Demand

In each foreign country, the population size is normalized to one and the demand side is summarized by a representative consumer with homogeneous preferences. The upper-tier utility function of this representative consumer is assumed to be quasi-linear such that:

$$U(z, \mathbf{q}(x)) = z + \mathcal{U}(\mathbf{q}(x))$$

with z the consumption of the Hicksian composite good produced under perfect competition and used as numeraire. We define $\mathbf{q}(x)$ as the vector of consumption of varieties of the horizontally differentiated good: $\mathbf{q}(x) = (q_s(x))_{s=0}^{I_x}$.

The lower-tier utility of consuming the differentiated good is given by a continuum quadratic utility function, similarly to Melitz and Ottaviano (2008):

$$\mathcal{U}(\mathbf{q}(x)) = \int_{I_x} \left(a q_i(x) ds - \frac{b}{2} q_i(x)^2 \right) di - \frac{\gamma}{2} \left(\int_{I_x} q_i(x) di \right)^2$$

As the parameter a represents the consumer's intrinsic evaluation of the differentiated good, it can stand for a measure of the quality of the good (Di Comite et al., 2012). We assume it to be the same for all varieties, which implies that firms sell a good that is only horizontally differentiated. Parameter b reflects substitutability between varieties. It is

assumed to be a constant and equal for each variety, meaning that no variety is more or less substitutable to another. At the extreme, when $b = 0$, consumers only care about the level of total consumption $\int_{I_x} q_i(x) di$. Finally, γ captures the demand linkage between varieties. A higher γ means that varieties are less differentiated and the marginal utility of consuming a unit of variety i decreases more rapidly with the consumption of any variety $j \neq i$. We assume a, b, γ to be the same for all varieties and in all countries. As a consequence, the demand function addressed to each single firm will be strictly the same in each country.

With Y the exogenous income of consumers, the budget constraint is the following:

$$z + \int_{I_x} p_i(x) q_i(x) di \leq Y$$

Given this environment, the marginal utility of income can be normalized to one (see [Spence \(1976\)](#) and [Neary \(2009\)](#) for further explanations), and each firm faces the same downward sloping demand function in each x :

$$p_i[q_i(x), Q(x)] = a - bq_i(x) - \gamma Q(x)$$

Last, we assume that firms take $Q(x)$ as given, which translates the fact that they do not anticipate their impact on aggregates. In this sense, there is no strategic interaction, and firms compete monopolistically with other firms exporting to the same destination.

3. Results

The model is solved by backward induction. Stage 2 determines an equilibrium quantity sold by firm i in location x . This immediately gives firm i 's profits as a function of the density of firms exporting to x . Thus, we identify the presence of density externalities. Then, stage 1 determines an equilibrium distribution of firms in destination x (or, equivalently, an equilibrium number of varieties exported to foreign country x) denoted

$\mu(x)$.⁸

3.1. Stage 2: Monopolistic Competition

The second stage of the model is standard. We assume monopolistic competition between firms in each country. The density $\mu(x)$ is taken as given. The equilibrium in stage 2 consists in finding optimal quantities in x for each firm that exports to this country. Then, given that a firm exports to x , stage two solves the following program:

$$\max_{q_i(x)} \pi_i(x) = (p_i[q_i(x), Q(x)] - c - \tau x)q_i(x)$$

Each firm takes the aggregate $Q(x)$ as given. It yields the following best response function:

$$q_i(x) = \frac{a - c - \tau x - \gamma Q(x)}{2b}$$

As all firms are homogeneous in terms of their marginal cost, added to the fact that best response functions are linear in $Q(x)$, all firms that export to the same destination choose the same quantity. Let $q(x)$ be this quantity. Hence, the total quantity sold in each country is simply $Q(x) = \mu(x)q(x)$. The quantity produced by a firm exporting in x is:

$$q(x) = \frac{a - c - \tau x}{2b + \gamma\mu(x)} \quad (1)$$

Not surprisingly, quantities in equilibrium are decreasing in both x , namely the distance to the destination, and $\mu(x)$ the density of competitors that a firm faces.

With this quantity, the equilibrium profit made by a firm that exports to x is simply:

$$\pi(x) = bq(x)^2 \quad (2)$$

As the equilibrium profit is strictly increasing in quantities, thus the equilibrium profit made by a firm exporting to x is itself a decreasing function of both distance (at a given

⁸Thus $\mu(x) = I_x$.

transport cost level) and the density of firms exporting to x .

3.2. Stage 1: MFG

From the reduced form of profits given by the resolution of stage 2 (see equation 2), it appears that there is a trade-off from the point of view of the firm. This trade-off is captured by the decreasing relationships between profits and both x , the distance, and $\mu(x)$. This trade-off translates respectively the will to export to close country and the will to escape competition. As transport costs are lower for close countries, firms anticipate that more firms will export to these countries. Thus, to escape competition, firms have an incentive to export to more remote countries.

For a given equilibrium in stage 2, firms choose a destination such that:

$$\max_x b \left(\frac{a - c - \tau x}{2b + \gamma \mu(x)} \right)^2$$

This program shows that firms select a destination for their exports in accordance with their preferences and the strategies of others, summarized by the density of competitors. Therefore, the suited spatial equilibrium in stage 1 is a Nash equilibrium. However, with an infinite number of agents and interactions summarized by a probability density function, a Nash equilibrium consists in finding a density of firms $\mu^*(x)$ such that:⁹

$$\begin{cases} \pi(x) = \bar{\pi} \\ \int_0^{\bar{x}} \mu^*(x) dx = 1 \\ \mu^*(x) \geq 0 \end{cases} \quad (3)$$

Where $\bar{\pi} \in \mathbb{R}^+$.

The first equation of the system indicates that an equilibrium in such a game is a situation in which each firm receives the same total profit wherever it exports to because the two

⁹See, for instance, [Lions \(2009\)](#) and [Cardaliaguet et al. \(2012\)](#) for a proof.

parts of the trade-off identified above compensate each other. The second equation of the system ensures that all firms choose where to export from an endogenous set of destinations. It is natural since profits are positive, no firm has incentives to stay out of the game. The third simply imposes that the distribution be positive everywhere.

$\bar{x}$ is the threshold country beyond which no firm of productivity c exports. This threshold immediately gives the support of the distribution $[0; \bar{x}]$.

Therefore an equilibrium for this model is given by a distribution $\mu(x)$, the size of the support for this distribution $[0; \bar{x}]$, and the equilibrium profit $\bar{\pi}$.

Proposition 1. *There exists a unique firm distribution with respect to their destination choice such that $\forall x \in [0; \bar{x}]$*

$$\mu^*(x) = \frac{a - c - \tau x}{\gamma \sqrt{\frac{\pi}{b}}} - \frac{2b}{\gamma} \quad (4)$$

with $\bar{x}$ the threshold country beyond which no firm exports:

$$\bar{x} = \frac{a - c - 2b \sqrt{\frac{\pi}{b}}}{\tau} \quad (5)$$

and with $\bar{\pi}$ the equilibrium profit such that:

$$\bar{\pi} = b \left(\frac{\gamma \tau + 2b(a - c) - \sqrt{\gamma^2 \tau^2 + 4b\gamma\tau(a - c)}}{4b^2} \right)^2 \quad (6)$$

The first appealing result sketched in Proposition 1 is that the distribution of firms with respect to their destination choice is never degenerated and is far from a Dirac mass. In other words, it means that homogeneous firms in terms of productivity always export differently in the short run. This feature is explained by an escape competition effect captured by the presence of density externalities stemming from the monopolistic competition equilibrium profits in stage 2. Of course, firms want to export to the most attractive countries in order to avoid paying high transport costs, but at the same time, because they have the same profit function (*i.e.* they are homogeneous), they anticipate that these markets will be coveted by their competitors, and thus that competition will be tougher. Thus, to escape competition, firms have an incentive to play differently by exporting to more remote countries. Interestingly, in the presence of congestion effects,

the dispersion at equilibrium is explained by the fact that agents are identical and not by heterogeneity.

Now, let us analyze the outcome more precisely. In this baseline model, we decided to present the simplest model with homogeneous firms in order to focus on the mechanism explaining why, even in this case, there is dispersion at equilibrium. Nevertheless, one can give a first intuition of the influence of firm productivity on the destination choice. Notice that the slope of $\mu^*(x)$ with respect to distance is given by:

$$\frac{\partial \mu^*(x)}{\partial x} = -\frac{\tau}{\gamma \sqrt{\frac{\pi}{b}}} \quad (7)$$

First, notice that, at a given transport cost level, the slope is flatter for a higher $\bar{\pi}$. If we refer to equation 6, it appears that the equilibrium profit is a decreasing function of c . It means that for a higher productivity (lower c), the equilibrium profit is higher, which in turns implies a lower influence of distance on trade.

Moreover, plugging equation 6 into equation 5 gives:

$$\bar{x} = \min\left\{\frac{\sqrt{\gamma^2 \tau^2 + 4b\gamma\tau(a-c)} - \gamma\tau}{2b\tau}; X\right\}$$

This threshold country beyond which no firm exports is itself a decreasing function of c . Thus, for a lower c , the distribution is not only flatter, but the support for the size distribution is longer. These two features mean that, *on average*, more productive firms export further, which is consistent with the data. Combined with the fact that the distribution is not a Dirac mass, this leads to an outcome which is close to a non-exact hierarchy in trade: within a given type of productivity, firms export differently and comparative statics show that lowering the marginal production cost increases $\bar{x}$ and makes the distribution flatter. In figure 1, we show two distributions with two different

marginal costs.¹⁰ The solid line depicts a distribution with a higher marginal cost than the dotted line.

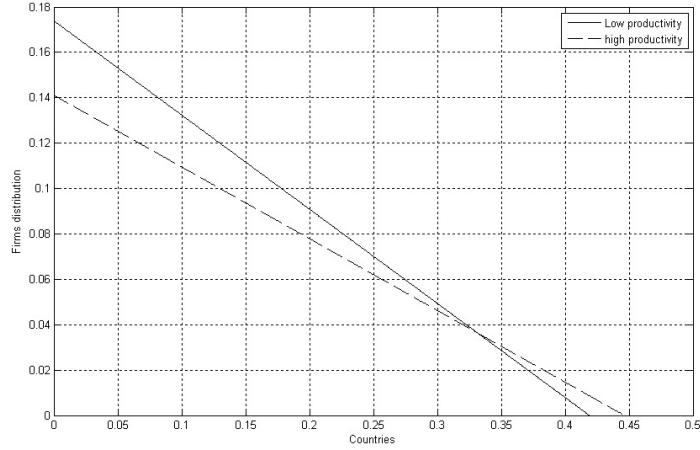


Figure 1: Distributions of firms with different productivities

Using equation 2 and 6, it comes that the quantity sold by firms does not depend on distance. As profits have to be the same wherever firms export to, they have to sell the same quantity whatever their destination. Let $\bar{q}$ be this quantity:

$$\bar{q} = \frac{\gamma\tau + 2b(a - c) - \sqrt{\gamma^2\tau^2 + 4b\gamma\tau(a - c)}}{4b^2} \quad (8)$$

Thus, the intensive margin is not a function of distance which reflects, once again, the fact that at equilibrium the distance effect and the competition toughness compensate for each other. Notice also that the price faced by firm i in each country x can be expressed directly as a function of $\mu^*(x)$:

$$p_i(\mu^*(x)) = a - (b + \gamma\mu^*(x))\bar{q}$$

¹⁰The value of the parameters is set randomly. We show graphs in order to illustrate the analytical results. We just give values that ensure positive profits at equilibrium.

In this model, $\mu^*(x)$ stands for the extensive margin which is, as mentioned above, decreasing in distance which is consistent with the findings of Bernard and al. (2007). Thus, even if each firm sells the same quantity wherever it exports to, the markup over the marginal cost is increasing with distance.¹¹ This is a direct consequence of the decreasing shape of $\mu^*(x)$ translating a lower competition toughness in remote countries. Another interesting result lies in parameter a . First, notice that an increase in a has exactly the same impact as a decrease in c . What matters is the level of $(a - c)$. Namely, a higher a leads to a larger distribution support and a flatter distribution. As a is the intrinsic valuation of the good by consumers, a can be interpreted as a measure of quality (Di Comite et al., 2012). In other words, our model predicts that trade in high quality goods generates more profits, integrates further markets, and is less sensitive to distance than trade in low quality goods. The latter theoretical result is consistent with recent empirical findings highlighted by Martin and Mayneris (2013) and Fontagné and Hatte (2013). The interpretation of parameter γ is simple. A higher γ makes competition tougher. This prompts firms to escape competition by exporting to more difficult countries. As a consequence, the threshold country increases even if profits decrease. This comes from the argument in our model according to which trade, in the short run, is a way to escape competition.¹²

Last, but not least, the dependence of the outcome on transport costs is not straightforward. As we can expect, a lower τ implies higher profits, a larger size of the distribution support and a flatter distribution. Figure 2 displays two distributions with two different transport costs which allows us to use comparative statics to analyze the effect of trade liberalization on destination choice.

As can be seen, $\mu^*(x)$ is flatter with low trade costs. This has a twofold consequence.

¹¹we define this markup as: $\frac{p_i(\mu^*(x)) - c}{c}$

¹²See the graphs in the appendices.

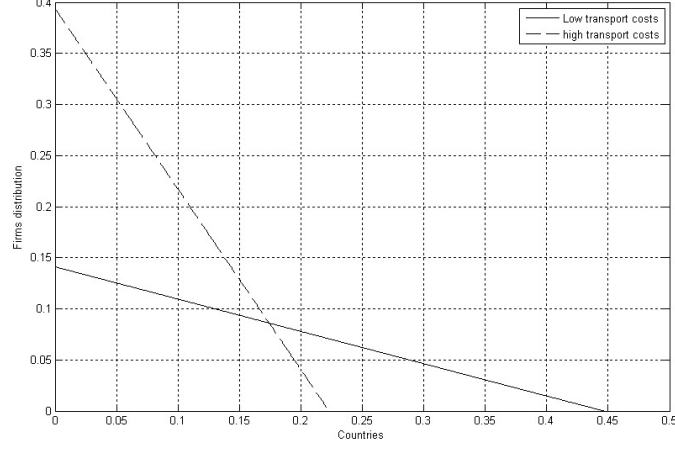


Figure 2: Distributions of firms with different transport costs

Remote countries host more exporters but at the same time, close countries have a lower density of exporters. In other words, trade liberalization leads to firms reallocating their destinations, with implications for welfare which are not straightforward.

3.3. Comments

To be solvable, the model needs only two characteristics: the second stage of the game must provide a reduced form of profits that is i) a negative function of distance and ii) a negative function of the density of players that choose the same strategy. Namely, transport costs must be positive and the demand function must be downward sloping. These two characteristics lead to the trade-off that firms face in the model above: firms like to export close in order to avoid paying transport costs and escape competition.

To sum up, the only required conditions are:

$$\pi(x) = \pi(x, \mu(x))$$

$$\text{with } \frac{\partial \pi}{\partial x} < 0 \text{ and } \frac{\partial \pi}{\partial \mu(x)} < 0$$

Thus, our methodology is highly tractable and supports any specifications of transport costs and demand that respect these two conditions which seem natural. These conditions

do not only allow the model to be solvable, but keep the nature of the results unchanged. If the pay-off is a decreasing function of the distribution of firms that choose the same strategy, then mixed strategies between identical players is the outcome. Finally, if all other things being equal, the pay-off is decreasing with distance, then the distribution found at the first stage of the game is decreasing in distance as well. Both assumptions together in the model lead to a non-exact hierarchy in trade.

4. Trade Liberalization and Welfare Analysis

4.1. Symmetric Trade Liberalization

The consequence of a trade liberalization can be analyzed in terms of welfare. In this subsection, we consider the case where τ falls, meaning that the attractiveness of all countries increases proportionally to their distance. In every x , welfare is defined as:¹³

$$W(x) = \frac{b}{2}\mu(x)q(x)^2 + \frac{\gamma}{2}(\mu(x)q(x))^2$$

First of all, notice that the less attractive the country, the lower the consumer surplus. Of course, this result is entirely driven by the fact that we assume no local firms in all countries, but we can point out the fact that even with a uniform distribution of local firms, the surplus would be lower in less attractive markets because less firms would export to these countries and each exporting firm would sell lower quantities.

Second, a trade liberalization does not have the same impact on each country. Welfare is increasing in both $\mu(x)$, taken as the number of available varieties in country x , and $q(x)$. For remote countries, trade liberalization leads unambiguously to higher welfare: it increases both $\mu(x)$ and $q(x)$. For close economies, things are more ambiguous. On the one hand, it tends to raise $q(x)$ which enhances welfare, but on the other hand, it tends to decrease $\mu(x)$. The intuition is the following: a fall in trade costs causes a shift in the distribution of firms according to their export destination. The outcome

¹³With no local firms, welfare is the consumer surplus.

becomes flatter, meaning that some firms desert close countries for remote countries. At the extreme, when $\tau = 0$, the distribution is uniform on the entire plausible support $[0; X]$, the quantity sold by each firm is the same and then the consumer surplus is strictly the same in all countries. In this model, a trade liberalization tends to equalize consumer surplus across countries, but does not have the same impact on each country. It is harmful for the most attractive countries, but beneficial for the least attractive ones. To see that the impact of trade liberalization is different between countries, let us notice two things. First, $q(x) = \bar{q}, \forall x$. Thus the intensive margin is the same in each country and a lower τ unambiguously raises it. Second, the first derivative of $\mu(x)$ displays the following feature:

$$\frac{\partial \mu^*(x)}{\partial \tau} \leq 0 \Leftrightarrow x \geq -\frac{\bar{q}'(a-c)}{y - \tau \bar{q}'}$$

with $\bar{q}' = \frac{\partial \bar{q}}{\partial \tau} < 0$. Thus the consequences of trade liberalization on welfare depend on x . This non-monotonicity of the impact of trade liberalization is based on the fact that, in our model, trade liberalization leads to a reallocation in firms' export destinations. Firms tend to go further on average which leads to a flatter distribution and in turn to a more equal welfare across countries. Finally, notice that the total welfare (the sum of all countries' welfare) is decreasing in τ .¹⁴

4.2. Asymmetric Trade Liberalization and Trade Externalities

In the previous subsection, we have demonstrated that a fall in trade costs has a non-monotonous impact on welfare due to destination reallocation in the short run. This reallocation mechanism is also interesting if the trade liberalization is asymmetric, in the sense that it concerns only a single region. In order to do so, let us marginally modify our setting by assuming that firms can now export either to $x \geq 0$ or to $x < 0$ such that $x \in [-X; X]$. Let us also assume that exporters have to incur a transport cost τ if they

¹⁴The total welfare can be written as follows:

$$\bar{W} = \int_0^{\bar{x}} W(x) dx$$

export to $x > 0$ and a transport cost ϕ if they export to $x < 0$. Thus, they pay τx on each unit sold when they export to any $x > 0$ and $\phi|x|$ when they export to any $x < 0$. Let us define $\bar{x}^+$ and $\bar{x}^-$ threshold countries beyond which no firm exports for respectively positive values of x and negative values of x . The equilibrium conditions become:

$$\begin{cases} \pi(x) = \bar{\pi} \\ \int_{\bar{x}^-}^{\bar{x}^+} \mu^*(x) dx = 1 \\ \mu^*(x) \geq 0 \end{cases} \quad (9)$$

With the same methodology as above, we find the following distribution:

$$\mu^*(x) = \begin{cases} \frac{a-c+\phi x}{\gamma\sqrt{\frac{\pi}{b}}} - \frac{2b}{\gamma}, & \text{if } x < 0 \\ \frac{a-c}{\gamma\sqrt{\frac{\pi}{b}}} - \frac{2b}{\gamma}, & \text{if } x = 0 \\ \frac{a-c-\tau x}{\gamma\sqrt{\frac{\pi}{b}}} - \frac{2b}{\gamma}, & \text{if } x > 0 \end{cases}$$

Threshold countries are given by:

$$\bar{x}^- = \max\left\{-X; \frac{a-c-2b\sqrt{\frac{\pi}{b}}}{-\phi}\right\} \quad (10)$$

$$\bar{x}^+ = \min\left\{X; \frac{a-c-2b\sqrt{\frac{\pi}{b}}}{\tau}\right\} \quad (11)$$

Notice that, as profits are the same for all destinations, we reasonably have $\bar{x}^+ > |\bar{x}^-|, \forall \phi > \tau$. Moreover, if $\phi = \tau$ then the distribution is symmetric with respect to 0. We obtain in particular $\bar{x}^+ > |\bar{x}^-|$ and $\mu^*(x) = \mu^*(x'), \forall x = |x'|$.

Let us define $\Phi = \frac{\phi\tau}{\phi+\tau}$. As previously, there exists a unique solution of profits which is positive and reasonably decreasing in transport costs:

$$\bar{\pi} = b \left(\frac{\gamma\Phi + 2b(a-c) - \sqrt{\gamma^2\Phi^2 + 4b\gamma\Phi(a-c)}}{4b^2} \right)^2 \quad (12)$$

As $\bar{\pi}$ is the equilibrium profit of all firms, notice that the profit of a firm exporting to a positive x (respectively negative x) depends of course on τ (respectively ϕ), but also on ϕ (respectively τ). This feature has direct implications for the concern of asymmetric trade liberalization.

Let us consider that only τ falls. As in the previous section, it increases $\bar{x}^+$ and flattens $\mu^*(x)$ for positive values of x . The most interesting thing is that, even if ϕ remains the same, both $\bar{x}^-$ and $\mu^*(x), \forall x < 0$ are affected. It is clear from equation 12 that a fall in τ increases equilibrium profits. It directly comes from equation 10 that the absolute value of $\bar{x}^-$ decreases. Thus, some countries are deserted by firms in the negative region. Let us go further by analyzing welfare. Consequently to what has just been shown, asymmetric trade liberalization concerning only τ has two impacts on welfare. The positive part of the distribution support (concerned by trade liberalization) undergoes the same welfare evolution as in section 4.1, while for the negative part it unambiguously decreases welfare. Let us define $W^-(x)$ the welfare in x given that x is negative.

$$W^-(x) = \frac{b}{2} \left(\frac{a-c-\phi x}{\gamma} \bar{q} - \bar{q}^2 \frac{2b}{\gamma} \right) + \frac{\gamma}{2} \left(\frac{a-c-\phi x}{\gamma} - \bar{q} \frac{2b}{\gamma} \right)^2$$

with $\bar{q} = \sqrt{\frac{\bar{\pi}}{b}}$. $W^-(x)$ is linked to τ through $\bar{q}$. We get:

$$\frac{\partial W^-(x)}{\partial \bar{q}} = -\frac{3}{2}b \frac{a-c-\phi x}{\gamma} - 4yb^2 \left(\frac{1-\gamma}{\gamma} \right)$$

which is negative under the assumption $\gamma \leq 1$.

As $W^-(x)$ is negatively correlated to $\bar{q}$, which itself is a negative function of τ , then $W^-(x)$ is a positive function of τ . It simply means that asymmetric trade liberalization (here, a lower τ) unambiguously leads to a decrease in welfare in the other region. In figure 3, we show distributions of firms with different τ . The black line stands for symmetric transport costs ($\phi = \tau$), while the two blue lines represent a decrease in τ , with the solid blue line depicting a lower τ than the dotted blue line. For the three distributions, ϕ remains the same.

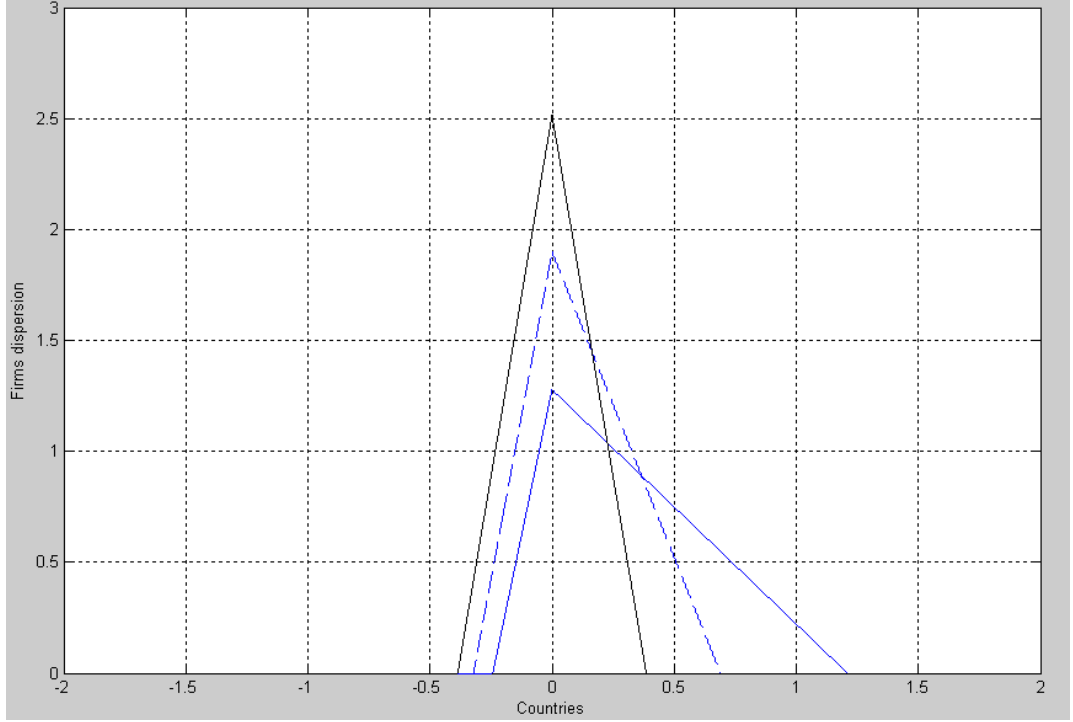


Figure 3: Trade externalities

So, in this model, due to the reallocation of destinations by firms, trade liberalization in any given region does not matter only for this region. It leads firms to export to this given region and to desert others. Therefore, this model displays trade externalities. Based on this feature, we can plead for an international coordination of trade policy.

5. Adding Periods

The model presented above is static. With no free entry, profits are not driven to zero. This allows us to have positive profits at equilibrium and to apply our methodology in the first stage. According to the theoretical argument explaining why standard monopolistic

competition models *à la* Melitz (2003) cannot account for a non-exact hierarchy of trade without adding an extra dimension of heterogeneity, a congestion effect seen through the presence of density externalities must be taken into account. By choosing a destination at a time, homogeneous firms are forced to have different strategies in order to escape competition. This is what we have emphasized in the previous section by presenting the model in a single period version.

This section aims to present the same model with several periods in order to emphasize the link between our model and the literature. The number of firms is assumed to be fixed, but we allow each firm to choose a supplementary destination at each period. The results of this section suggest that as time runs, homogeneous firms tend to have more and more similar baskets of destinations. Thus, we argue that what we observe in the data is a snapshot of the convergence towards the equilibrium of the Melitz (2003) model. Our model, when time tends to infinity, tends to have the same conclusions as Melitz (2003).

Suppose that we add periods to our model. At each period t , firms choose a unique destination and carry on to export to the destinations chosen in $t - n, n = [1, \dots, t - 1]$. Since profits remain positive, firms do not have incentives to quit markets to which they already export. Moreover, we assume that firms live infinitely so that there is neither an exogenous rate of entry nor exit.¹⁵ Thus, we assume that the number of firms is fixed. At each period t , firm i faces the following downward sloping demand function in each x :

$$p_{i,t}(x) = a - bq_{i,t}(x) - \gamma Q_t(x)$$

$$\text{with } Q_t(x) = \left(\sum_{n=1}^{t-1} \mu_{t-n}(x) + \mu_t(x) \right) q_t(x)$$

¹⁵See in online appendix the same framework with an exogenous entry/exit rate. We show that the nature of results is the same.

The definition of $Q_t(x)$ is the number of firms that export to x , namely those which exported before ($\sum_{n=1}^{t-1} \mu_{t-n}(x)$) and those which make the decision of exporting to x at t ($\mu_t(x)$), times quantity sold at equilibrium. As in the single period version of the model, the quantity sold in each x by each firm at t is the same. Let $q_t(x)$ be this quantity.

$$q_t(x) = \frac{a - c - \tau x}{2b + \gamma \left(\sum_{n=1}^{t-1} \mu_{t-n}(x) + \mu_t(x) \right)}$$

Stage 1 at each period t consists in finding $\mu_t(x)$, taking $\sum_{n=1}^{t-1} \mu_{t-n}(x)$ as given. At each period, the game is solved by respecting the analogical conditions of system 3. The profits of each firm at each period must be the same. The total profits are different *between* periods, but are the same for each firm at each period. Namely, at each period, we must respect $\pi_t(x) = \bar{\pi}_t$, with $\pi_t(x) = b(q_t(x))^2$. This yields the following distribution of firms at t :

$$\pi_t(x) = \bar{\pi}_t \Leftrightarrow \mu_t(x) = \frac{a - c - \tau x}{\gamma \sqrt{\frac{\bar{\pi}_t}{b}}} - \frac{2b}{\gamma} - \sum_{n=1}^{t-1} \mu_{t-n}(x)$$

A simple computation yields the following distributions:

$$\begin{aligned} \mu_1(x) &= \frac{a - c - \tau x}{\gamma \sqrt{\frac{\bar{\pi}_1}{b}}} - \frac{2b}{\gamma} \\ \mu_t(x) &= (a - c - \tau x) \left(\frac{\sqrt{\frac{\bar{\pi}_{t-1}}{b}} - \sqrt{\frac{\bar{\pi}_t}{b}}}{\gamma \sqrt{\frac{\bar{\pi}_{t-1}}{b}} \sqrt{\frac{\bar{\pi}_t}{b}}} \right), \forall t > 1 \end{aligned}$$

At each time t , there exists a threshold country $\bar{x}_t$ beyond which no firm exports. As previously showed, this threshold is found by solving $\mu_t(\bar{x}_t) = 0$. It comes that:

$$\begin{aligned} \bar{x}_1 &= \frac{a - c - 2b \sqrt{\frac{\bar{\pi}_1}{b}}}{\tau} \\ \bar{x}_t &= \frac{a - c}{\tau}, \forall t > 1 \end{aligned}$$

Notice that $\bar{x}_t > \bar{x}_1, \forall t > 1$. Thus, this threshold country $\bar{x}_t$ is increasing between the

first and the second period. This accounts for a higher incentive to export far when competition becomes tougher. From the second period, the threshold country stays the same and is simply decreasing in both c and τ . All implications are discussed in the single period version of the model.

$\sqrt{\frac{\pi_1}{b}} = \bar{q}$ is still found in equation 8. The first period of this version is the same as the single period version. We now have to find a solution for each $\sqrt{\frac{\pi_t}{b}}, t > 1$. Following the same methodology as in the previous section:

$$\int_0^{\bar{x}_t} \mu_t(x) dx = 1 \Leftrightarrow \sqrt{\frac{\pi_t}{b}} = \frac{A \sqrt{\frac{\pi_t - 1}{b}}}{\gamma \sqrt{\frac{\pi_t - 1}{b}} - A}$$

with $A = \frac{(2-\tau^2)(a-c)^2}{2\tau}$ assumed to be positive (*i.e.* $\tau < \sqrt{2}$).

First, notice that $\sqrt{\frac{\pi_t}{b}} < \sqrt{\frac{\pi_t - 1}{b}}$. As adding periods here is strictly equivalent to allowing a supplementary mass one of firms to choose a unique destination, then competition is tougher as time runs. This implies that profits are decreasing over time. Second, $\sqrt{\frac{\pi_t}{b}}$ is positively correlated to A , which is itself positively linked to $(a - c)$ and negatively linked to τ . Thus, as in the single period version of the model, a higher $(a - c)$ and/or a lower τ imply a higher $\bar{x}_t$ and $\bar{\pi}_t$.

Of course, profits earned in x by any firm that exports to x are the same whenever it begun exporting to x . In other words, at any t a firm that has exported to x since t' earns the same profits in x as any other firm that has exported to x since t'' (for $t'' \neq t'$ and both t' and t'' lower than t). The profits of all firms at t are equal to $t \times \bar{\pi}_t$. Moreover, as $\bar{\pi}_t$ is decreasing over time, it comes that the distribution becomes less and less sensitive to distance.

5.1. Convergence Towards Melitz

A corollary to the Melitz (2003) model in such a framework would be that all firms belonging to a given type of productivity should export to all countries between 0 and

$\bar{x}$. This is the case here when time tends to infinity.

At any $t > 1$, the threshold country $\bar{x}_t$ is the same. Moreover, at each period, all firms choose a supplementary destination.¹⁶ Moreover, firms cannot choose the same destination twice, it would be irrational as we assume that they sell a single product. It comes that at time t , all firms export to t destinations between 0 and $\bar{x}$. Therefore, two firms observed at t are more likely to have similar baskets of destinations than if they were observed at $t' < t$. Hence, a prediction of our model is that as time runs, firms tend to have more similar strategies. Moreover, as mentioned above, $\bar{x}$ is increasing in productivity. The two statements together are equivalent to saying that trade follows an exact hierarchy when $t \rightarrow \infty$.

5.2. Fixed Costs

The predictions above remain the same when we add a fixed cost of exporting but in this latter case, there exists a period $\bar{t}$ at which the game stops.

To be clearer, assume that firms have to pay a given fixed cost f to export to any x (without loss of generality, we do not assume a fixed cost of producing). In this case, the total profits at time t (Π_t) become (with $\bar{\pi}_t$ the same as in the previous sections):

$$\Pi_t = t(\bar{\pi}_t - f)$$

As $\bar{\pi}_t$ is decreasing over time, then, there exists a unique period $\bar{t}$ which leads to zero profits. Thus, in the long run, firms export to $\bar{t}$ destinations.

As we know, $\bar{\pi}_t$ is increasing with the productivity of firms. Then, the threshold period $\bar{t}$ is itself increasing in firms' productivity. With a given fixed cost of exporting, the model then predicts that more productive firms export to more destinations in the long run.

Last, but not least, notice that as the game stops at $\bar{t}$, the model never converge towards Melitz's prediction. Of course, between $t = 0$ and $\bar{t}$ it is still true that firms tend to have

¹⁶As the total profits at t are $t \times \bar{\pi}_t$, all firms choose a destination. If a firm did not, its profits would be $(t - 1) \times \bar{\pi}_t$.

more and more similar baskets of destinations, but at $\bar{t}$ there still exists a dispersion.

To sum up this section, we found first that firms tend to have more and more similar strategies as time runs. Added to the fact that the size of the distribution support is increasing in productivity, the model, when time tends to infinity, is equivalent to an exact hierarchy of trade when there is no fixed cost of exporting. Moreover, with a fixed cost, the model predicts that the higher the productivity, the higher the number of destinations per firm in the long run.

6. Conclusion

With a short-run model of trade including mean field interactions in the choice of destinations, we explain why identical firms may serve different markets. This result is not driven by an additional dimension of heterogeneity between firms as in [Eaton et al. \(2011\)](#) and [Chaney \(2011\)](#). On average, less attractive markets remain served by more productive firms. A corollary to this is that trade does not follow an exact hierarchy, which is consistent with recent empirical findings.

In a two-stage model in which firms first choose a unique destination to serve and compete monopolistically in this destination, we show that, in the short run, monopolistic competition exhibits density externalities. This presence of density externalities allows us to solve the first stage following an MFG. This model is highly tractable, and to be solvable and keep the same nature of results, it requires only two characteristics: profits in equilibrium must be decreasing functions of both distance (which captures attractiveness here) and degree of competition. Namely, the more numerous the players that choose the same strategy, the lower the pay-off obtained by playing this strategy. The outcome of this model is mixed strategies among homogeneous players. By doing so, we provide a rationale for [Eaton et al. \(2011\)](#)'s findings. Here, homogeneous firms may export endogenously to different locations. Our main theoretical point is that with the free entry condition, a way to have mixed strategies among homogeneous players cannot

be found, without adding another dimension of heterogeneity. Mixed strategies can be found only in the short run, with positive profits. The analytical model presented here can be seen as an illustration. The key point of our proposition is contained in the methodology used to derive equilibrium in the first stage. We argue that the fact that we observe in the data that homogeneous firms serve different countries can be driven by an escape competition effect in the short run. Otherwise, we show that this methodology does not require strong restrictions to be applied. The only thing that we need is a profit function that is decreasing in both distance and competition toughness, conditions which are easily displayed by other models.

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