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Why is the Government Spending Multiplier Larger at the Zero Lower Bound ? Not (Only) Because of the Zero Lower Bound

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Abstract

I develop a New Keynesian model with search and matching frictions, in which the government buys goods produced by the firms. I solve the non-linear model globally and examine the magnitude of government spending multipliers in and out of the ZLB. I distinguish the cases of Nash-bargained and rigid real wages. The model with Nash-bargained wages gives results that do not not differ qualitatively from the existing literature. It generates a high multiplier at the ZLB due to a higher elasticity of real marginal cost to aggregate demand in those conditions—which is at odds with empirical evidence. The model with rigid real wages exhibits *job rationing* (Michaillat (2012a)) and also delivers a higher multiplier at ZLB than in normal times. In this setup however, the multiplier is not larger because real marginal cost is more responsive to aggregate demand, but just the opposite. With a slack labor market, it is easy to recruit and real marginal cost is *less* responsive to aggregate demand. This is in line with available empirical evidence.

1 Introduction

Four years after the American Recovery and Reinvestment Act has been passed, the debate about government spending multipliers is still lively among academics. The main question that is being asked is the following : can temporarily higher government spending boost the economy in a downturn? The standard way to measure the effectiveness of such a policy is to see if it is successful at raising output more than proportionally; in other words : to see if the government spending multiplier is large. On this subject, there is mounting empirical evidence pointing towards bigger government spending multipliers in periods of recession. Using non-linear Vector Auto-Regression methods, Bachmann & Sims (2012) and Auerbach & Gorodnichenko (2012) show that the government spending multiplier is higher when some measure of the output gap is higher than usual.[1] On the theoretical side, attempts to explain this are still going on. Canzoneri et al. (2012), building on a model à la Curdia & Woodford (2010), show that countercyclical financial frictions can make government spending quite effective during recessions, all the more so when it is financed by debt. Focusing on the labor market, Michaillat (2012b) shows that increasing public employment has a larger effect on total private employment in a recession than in an expansion. The reason is that since there is job rationing in a recession and the labor market tightness is low, public employment has a low crowding out effect on private employment in a recession. Aside from those two papers (to the best of my knowledge), most of the papers have been focused on episodes where the Zero Lower Bound (ZLB henceforth) is a binding constraint (see Eggertsson & Woodford (2003), Woodford (2011) or Christiano et al. (2011)). This encompasses very few of the episodes covered by the sample of Auerbach & Gorodnichenko (2012). The mechanism that is typically put forward in those papers is the following : by increasing inflation through higher government spending, the government can reduce real interest rates since the nominal rate is pinned at zero. This will induce people to consume more today, generating more inflation and thus more consumption. At the end of this virtuous cycle stands an output multiplier roughly three times as large as in normal times (Christiano et al. (2011)). All of these papers use a New-Keynesian model in which prices are set as a markup over current and future expected marginal costs[2].

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[1]Owyang et al. (2013), however, using US and Canadian data along with a narrative approach find little evidence for state-dependant multipliers of government spending.

[2]One notable exception is Rendahl (2012), who uses a neoclassical model with a frictional labor market. By lowering unemployment today and tomorrow, government spending increases current output further. This generates a virtuous cycle, which yields a multiplier of about 2.

Now while the ZLB does not always bind in a recession, the ZLB itself is always the *consequence* of a recession. In fact, it has been a binding constraint only three times in recent history : in most of developed countries during the Great Depression, in the United States and EuroZone in the Great Recession and in Japan during the "Lost Decade(s)". Three periods which are associated with severe recessions. I do not need to provide a figure showing that unemployment usually rises in a recession. Moreover, Michaillat (2012a) shows that job rationing, *i.e* unemployment that is not due to search and matching frictions, is more prevalent in times of recession. Surprisingly, there is no reference to the dismal state of the labor market in the mainstream literature about the impact of government spending at the ZLB. One might then wonder : is it really this important? Both Albertini & Poirier (2013) and Rendahl (2012) show that it matters a lot. In this paper, I also argue that the answer is yes, depending on how wages are set. With Nash-bargained wages, the model exhibits the same mechanisms as the baseline New-Keynesian one, so that search and matching frictions add little to the story. Furthermore, I show that the implied behavior of key macroeconomic variables is at odds with empirical evidence, as is the case with the baseline New-Keynesian model. In fact, empirical evidence tends to show that labor market adjustment occurs largely through the extensive margin in a recession (see van Rens (2012)). Since hiring is essentially costless, one might then conjecture that the elasticity of marginal cost with respect to government spending will be low in a recession, and *a fortriori* when the ZLB binds. Since there is a high degree of slack in the labor market, a recession might precisely be a period in which the cost of putting additional resources to use is lower. I show that the model with rigid real wages is consistent with this intuition. It exhibits *job rationing* (Michaillat (2012a)) and a lower elasticity of real marginal cost to aggregate demand in bad times. The higher than normal multiplier does not make extensive use the usual virtuous cycle of consumption and real interest rates, but relies mostly on the fact that it is easier to put resources to use in a recession so severe that the nominal rate is pinned at zero.

I first describe in section 2 the usual mechanism through which one obtains a higher than normal multiplier at ZLB in the context of a simple New Keynesian model. I then show that it implies a behavior of main macroeconomic variables at odds with empirical evidence. In section 3, I develop the New Keynesian model with search and matching frictions. I also present the calibration and solution algorithm. In sections 4 and 5 I show the results for Nash-bargained and rigid real wages. I conclude in section 6.

2 High multipliers in the standard New Keynesian model and empirical evidence

Most of the New Keynesian models that have been used to show the possibility of high government spending multipliers can be summarized by the following system of three equations :

$$c_t = E_t c_{t+1} - \Phi_r \left(i_t - E_t \pi_{t+1} \right), \tag{1}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \left(\Theta_c c_t + \Theta_g g_t \right), \tag{2}$$

$$i_t = \max \left(0; \phi_\pi \pi_t \right). \tag{3}$$

where Φ_r, Θ_c and Θ_c are positive functions of underlying parameters. The first equation is the Euler equation of the representative consumer. It states that, given expected consumption tomorrow, consumption today is a decreasing function of the expected real interest rate. The second equation is the New Keynesian Phillips curve, and states that for given expected inflation, inflation today is an increasing function of real marginal cost. Here, I have replaced the real marginal cost by its expression as a function of private consumption and government spending.

The common practice is to model a shock on the discount factor so that the representative agent wants to save more today. With everyone being identical in this economy, net aggregate savings are zero so a fall in output is needed to counteract the need to save. If the shock is big enough so that interest rates have to go all the way down to zero, the economy finds itself in a liquidity trap, with output and inflation being lower than their steady state value.

Since this system has no endogenous state variable, all endogenous control variables jump to their new steady state on impact. Then, they revert back to their steady state value as soon as the shock expires. Let us assume that the shock as a simple markov structure and stays at its liquidity trap level with probability p. During the period when the economy is in the liquidity trap, the dynamics can be summarized by the following system of two equations :

$$(1-p)c_t = p\Phi_r \pi_t \tag{4}$$

$$(1-\beta p)\pi_t = \kappa \left(\Theta_c c_t + \Theta_g g_t \right) \tag{5}$$

where I have used the fact that $\mathbb{E}_t c_{t+1} = pc_t$, $\mathbb{E}_t \pi_{t+1} = p\pi_t$ and $i_t = 0$. One can see from equation (4) that private consumption is an increasing function of inflation. Since the nominal interest rate is zero, higher inflation reduces the real interest rate and induces the representative household to consume more today. Then, equation (5) indicates that higher consumption

generates higher inflation which, in turn, will generate higher consumption. This virtuous cycle goes on until the level of consumption is consistent with the Euler equation.

But how do we get inflation to increase in the first place? One way to do this is to increase government spending.[3] In this framework, government spending acts as a trigger for this virtuous cycle.

What is key then is the elasticity of real marginal cost with respect to government spending. This latter is determined in two steps. The first step is the partial equilibrium effect of government spending on real marginal cost. Using the notation above, this effect is of magnitude Θ_g. Observe that this partial equilibrium effect is the same in normal times and when the economy is in a liquidity trap. Implicit here is the fact that the labor market, the workings of which determine the magnitude of this effect, functions in the same way whether the economy is at the ZLB or not. Since prices are set as a markup over the marginal cost, whatever happens to real marginal costs translates into inflation. The second step is when the Euler equation kicks in and it is when the mechanisms that operate at the ZLB start diverging from the ones in normal times.

In normal times, higher inflation will trigger a more than proportional response of the nominal interest rate (since $\Phi_\pi > 1$). The real interest rate will rise and thus private consumption will decline, putting downward pressure on inflation. The general equilibrium effect of government spending on inflation will then be lower than Θ_g. At the ZLB however, the initially higher inflation will not trigger a rise but a decline in the real interest rate, generating more consumption and the virtuous cycle begins. The general equilibrium effect of government spending on inflation will then be higher than Θ_g. One sees that the ZLB is just an amplification mechanism of the initial effect on the real marginal cost. Were this effect to be negative, the multiplier at the ZLB would be lower than in normal times.[4]

The bottom line is that for the mechanism put forward in the literature to be relevant empirically, the labor market structure should be such that the partial equilibrium effect of higher government spending on real marginal

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[3]One could think of many other policies that would generate inflation. For example, expectations of more accomodative monetary policy can do the job (Eggertsson (2008)). It has also been argued that New Deal policies during the Great Depression have been efficient due to this virtuous circle Eggertsson (2012)

[4]Suppose for example that part of government spending enters the production function as an exogenous input. One can think of this as public capital that depreciates over one period. In this framework, higher government spending can trigger a negative multiplier at the ZLB if the share of public investment over total government spending is high enough. See Bouakez et al. (2014)

cost is positive. A binding ZLB is a rare event, that occurs *exclusively* when the economy is in a recession. Therefore, to get an estimate of the partial equilibrium effect of government spending on real marginal cost, we can look at the impact of government spending in a typical recession. Naturally, the Impulse Response one gets from the data gives us information about the general equilibrium response of prices and wages. But on the maintained hypothesis that the "'true"' model is a New Keynesian one, both partial and general equilibrium reactions of prices and wages will have the same sign. The only difference between the two will be the magnitude of the response. For example, a government spending policy that has a partial equilibrium effect of zero on consumption will have a general equilibrium effect of zero, since aggregate demand does not react. A policy that has a positive partial equilibrium effect on inflation (say, higher wasteful government spending) will have the effect to increase the response of monetary policy through a higher nominal and real rate. With lower aggregate demand, the effect of government spending on prices will still be positive, but dampened.

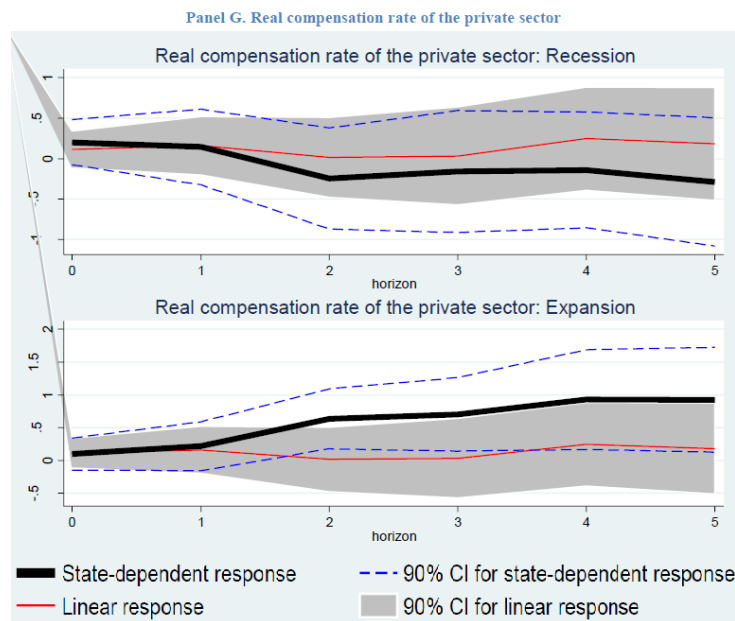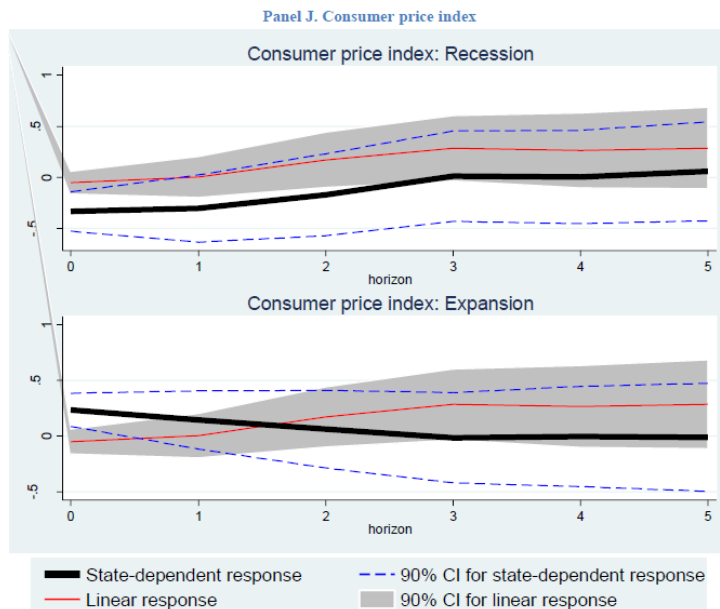Figure 1: Effects of a government spending shock on real wages. Source : Auerbach & Gorodnichenko (2012).

Figure 2: Effects of a government spending shock on private consumption. Source : Auerbach & Gorodnichenko (2012).

One can see from Figure 1 that the prediction of the baseline New Keynesian model that higher government spending has a positive effect both in normal and recession times seems to be at odds with empirical evidence. Indeed, real wages seem to react less to a government spending shock when the economy is in a recession : the error bands of the state-dependent response always contain the x-axis. Therefore, the response of real wages is not statistically different from zero. If we feed this partial equilibrium result into the baseline New Keynesian model with a ZLB, the latter would amplify nothing : the output multiplier would be equal to one. Moreover, one can see from Figure 2 that prices seem to respond less to a government spending shock in a recession than in an expansion. Again, this contradicts the usual mechanism at the ZLB, which builds on an initial inflationary effect. This initial effect does not seem to be operative.

At this point, it is clear that a successful theory for explaining the fact that government spending has higher multiplier effects at the ZLB needs at least two ingredients. First, the multiplier on output should be higher when the ZLB is binding. Second, the impact effect of government spending on both prices and real wages should be lower in a recession than in an expansion. The question that inevitably comes to mind is : what is the baseline New Keynesian model missing? The crux of the issue is likely to lie in the setup of the labor market. With unemployment being usually higher than normal

in a recession, the assumption that labor markets function in the same way is highly questionable. With this in mind, I study next a New Keynesian model with a labor market that is subject to search and matching frictions.

3   A New Keynesian model with search and matching frictions

In this section, I augment the baseline New Keynesian model with a frictional labor market along the lines of Mortensen & Pissarides (1994).

3.1   The Labor Market

Workers and firms matches are given by the following matching function:

$$m_t = m \cdot s_t^\eta v_t^{1-\eta},$$

where s_t is the pool of job seekers and v_t is the number of vacancies posted. Let $\theta_t \equiv \frac{v_t}{s_t}$ denote the labor market tightness. Firm-worker matches are destroyed at an exogenously given rate s, therefore the pool of job seekers at t is given by $s_t = 1 - (1-s)N_{t-1}$ (the size of the labor force is normalized to 1). They find work with probability $f(\theta_t) \equiv \frac{m_t}{s_t} = m\theta_t^{1-\eta}$ and firms fill a vacancy at a rate $q(\theta_t) \equiv \frac{m_t}{v_t}$. The number of unemployed people is then given by $u_t = 1 - n_t$. To recruit, the firm pays a cost of $r \cdot A_t$. Therefore, the recruiting expenses are given by:

$$\frac{rA_t}{q(\theta_t)}[N_t(i) - (1-s)N_{t-1}(i)].$$

The household's employment rate is given by the following law of motion:

$$N_t = (1-s)N_{t-1} + \Big[1 - (1-s)N_{t-1}\Big]f(\theta_t). \tag{6}$$

3.2   The Representative Household

The household is assumed to be large and solve the following maximization program:

$$\max_{C_t, B_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\prod_{j=0}^{t} \xi_j \right) \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{N_t^{1+\varphi}}{1+\varphi} \right\},$$

where ξ_t is a preference shock and φ is the inverse of the Frisch labor supply elasticity. As in Merz (1995), workers pool their income before choosing consumption and so the budget constraint reads:

$$P_t C_t + B_t = P_t N_t W_t + R_{t-1} B_{t-1} + \mathcal{P}_t,$$

where P_t is the price level, C_t is real consumption, B_t are nominal one-period riskless bonds, R_t is the gross nominal interest rate, W_t is the real wage and $\mathcal{P}$ are nominal profits distributed by firms. The Lagrangian then writes:

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\prod_{j=0}^{t} \xi_j \right) \left\{ \frac{\left[W_t N_t + \frac{R_{t-1}B_{t-1}+\mathcal{P}_t-B_t}{P_t} \right]^{1-\sigma}}{1-\sigma} - \chi \frac{N_t^{1+\varphi}}{1+\varphi} \right.$$
$$\left. - V_{N,t} \left[N_t - (1-s)N_{t-1} + \left[1 - (1-s)N_{t-1} \right] f(\theta_t) \right] \right\}.$$

The first order conditions with respect to C_t yields:

$$\lambda_t = \beta R_t \mathbb{E}_t \left\{ \xi_{t+1} \frac{\lambda_{t+1}}{\Pi_{t+1}} \right\} \tag{7}$$

where $\Pi_t = \frac{P_t}{P_{t-1}} - 1$ and $\lambda_t = C_t^{-\sigma}$

3.3 The Representative Firm

As in Michaillat (2012b), there is no entry nor exit into production of consumption goods. The monopolistically competitive firm—indexed by i—posts vacancies to recruit workers, who, once employed produce according to the following production function:

$$Y_t(i) = A_t N_t(i)^\zeta, \tag{8}$$

The firm is assumed to face costs when changing its price as in Rotemberg (1982) and knows the demand facing its product, with elasticity ϵ. It posts vacancies v_t to recruit workers. The Lagrangian of the firm then writes:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \left(\prod_{j=0}^{t} \xi_j \right) \beta^t \lambda_t \left\{ \left(\frac{Pt(i)}{P_t} \right)^{1-\epsilon} Y_t - W_t L_t(i) - \frac{\phi}{2} \left(\frac{Pt(i)}{P_t} - 1 \right)^2 - r A_t v_t \right.$$
$$\left. - V_{J,t}(i)[N_t(i) - (1-s)N_{t-1}(i) - q(\theta_t)v_t] + mc_t(i) \left[N_t(i)^\zeta - \left(\frac{Pt(i)}{P_t} \right)^{-\epsilon} Y_t \right] \right\}$$

where $mc_t(i)$ is the Lagrange multiplier on the firm's production function —which will be equal to real marginal cost—and $V_{J,t}$ is the Lagrange multiplier on the low of motion of employment. Since every firm is identical, the equilibrium will be symmetric as far as firms are concerned and therefore I can drop the index i. The first order condition with respect to P_t then gives the standard New Keynesian Phillips Curve:

$$\epsilon mc_t = \epsilon - 1 + \phi \frac{\Pi_t}{\Pi} \left(\frac{\Pi_t}{\Pi} - 1 \right) - \beta \phi \mathbb{E}_t \left\{ \xi_{t+1} \frac{\lambda_{t+1}}{\lambda_t} \frac{Y_{t+1}}{Y_t} \frac{\Pi_{t+1}}{\Pi} \left(\frac{\Pi_{t+1}}{\Pi} - 1 \right) \right\}, \tag{9}$$

where Π is the steady state inflation rate. Likewise, combining the first order condition with respect to N_t and v_t yields:

$$\zeta A_t mc_t N_t^{\zeta-1} = W_t + \frac{rA_t}{q(\theta_t)} - \beta(1-s)\mathbb{E}_t \left\{ \xi_{t+1} \frac{rA_{t+1}}{q(\theta_{t+1})} \frac{\lambda_{t+1}}{\lambda_t} \right\}. \qquad (10)$$

At equilibrium, what the marginal employee produces (the lhs of equation (10)) must be equal to what it costs for the firm : the current real wage, recruiting expenses and saved expenses from having to hire an additional worker tomorrow.

3.4 Fiscal and Monetary Policy

The government finances an exogenous stream of expenses G_t by levying non-distortionary lump-sum taxes. In contrast to Michaillat (2012b), government spending does not take the form of public employees. While public employees do represent a large share of government spending in the data, I am interested here—as is most of the literature on the effects of government spending —in the effects on aggregate output of the purchase of goods by the government. In fact, public employment did not represent a large share of the American Recovery and Reinvestment Act of 2009, if anything at all.[5] The budget constraint of the government then reads:

$$T_t + \frac{B_t}{P_t} = G_t + \frac{R_t-1}{P_t} B_{t-1}$$

The Monetary Authority sets the gross nominal interest rate according to:

$$R_t = \max\left\{ 1, \frac{\Pi}{\beta} \left(\frac{\Pi_t}{\Pi} \right)^{\phi_\pi} \right\} \qquad (11)$$

3.5 Equilibrium

Substituting the definition of real profits in the household's budget constraint and combining the result with the government budget constraint, one gets the resource constraint of this economy:

$$Y_t \left[1 - \frac{\phi}{2} \left(\frac{\Pi_t}{\Pi} - 1 \right)^2 \right] = C_t + G_t + \frac{rA_t}{q(\theta_t)}[N_t - (1-s)N_{t-1}]. \qquad (12)$$

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[5]With spending reversals on the state level, one can even argue that the net effect of ARRA on public jobs might be negative.

3.6 Wage Setting

Let Z_t and S_t denote, respectively, the vector of all control and state variables. All the parameters of the model are represented by the vector Θ. I assume the following general form for the real wage:

$$\frac{W_t}{P_t} = \mathcal{W}(Z_{t+1}, Z_t, S_{t+1}, S_t; \Theta) \tag{13}$$

In what follows, I will distinguish between flexible/Nash-bargained and rigid real wages.

3.7 Model Solution and Calibration

The model requires a large shock on the discount factor to drive the economy at the ZLB. Therefore, I do not rely on log-linear approximations around a deterministic steady state, as is usually done. Being based on a Taylor expansion of the first order conditions, these approximations are only valid in a small neighborhood of the steady state. In fact, it has been shown that the usual discount factor shock takes the economy too far from the steady state for those approximations to remain valid (see Braun et al. (2012)). Christiano & Fisher (2000) argue that a special case of projection methods, the Parameterized Expectations Algorithm (PEA) is the most efficient one to approximate models with occasionally binding constraints. Accordingly, I solve the model globally using this algorithm, as in Albertini et al. (2014).

This algorithm consists in approximating the expectations functions of the model by a simple polynomial function of the state variables. Beginning with a first guess of the coefficients relating the expectations functions to the polynomials, I can compute the policy rules relating the endogenous variables to the state variables. Using these along with the transition equations of the state variables, I can compute the expectations using a Gauss-Hermite quadrature. I then regress those expectations on the state variables to update the value of the coefficients in front of the polynomials. I iterate on these coefficients until the difference at successive iterations is small enough. I explain the algorithm in greater detail in the Appendix B.

I now move to the calibration of the model. The model is calibrated at quarterly frequency. As in Michaillat (2012b), I assume decreasing returns with respect to labor and set $\zeta = 0.66$. The elasticity of substitution across goods is equal to $\epsilon = 11$, which yields a markup of 10 %. I set $\beta = 0.994$ and $\sigma = 1$. I set $\eta = 0.5$ for the elasticity of the matching function with respect to unemployment. The exogenous separation rate is set to $s = 0.11$, in between the values considered by Blanchard & Gali (2010) and Ravenna &

Walsh (2010). The Frisch elasticity of labor supply is set to $\varphi = 1$. Following Michaillat (2012b), steady state unemployment is set to 6.4%, which yields an employment level of $N = 0.936$. The matching efficiency parameter is set so that, at steady state, $\theta = 0.43$. This yields $m = 0.9715$. As in Silva & Toledo (2009), I impose that the recruiting cost amounts to 4.3% of the real wage of a newly recruited worker, which yields $r = 0.0263$. The share of government spending with respect to output is set to the conventional value of 20%. The calibrated parameters are summarized in Table 1.

Table 1: **Parameters**

Variables	Symbol	Value	Source
Discount factor	β	0.994	Standard
Risk aversion coefficient	σ	1	Standard
Elast. of subst. between goods	ϵ	11	Standard
Frisch elasticity	φ	1	Standard
Annual steady state inflation	π	0.02	Standard
Production function elasticity	ζ	0.66	Michaillat (2012b)
Matches/seekers elasticity	η	0.5	standard
Separation rate	s	0.11	Blanchard & Gali (2010)
Steady state unemployment	u	0.064	Michaillat (2012b)
Steady state tightness	θ	0.43	Michaillat (2012b)
Price adjustment	ψ	61	Michaillat (2012b)
Matching efficiency	m	0.9715	Target $\theta = 0.43$
Recruiting cost	r	0.0263	Silva & Toledo (2009)
Response to inflation	ϕ_π	1.5	Standard
Government spending share	g	0.2	Convention

4 Labor Market Dynamics at ZLB with Flexible Wages

In this section, I assume that real wages are set by a Nash bargaining process. Let $V_{N,t}$ and $V_{J,t}$ denote, respectively, the value of employment for the household and the value of a job for a firm. The first one is equal to the Lagrangian multiplier in front of the employment transition equation in the consumer program. Likewise, $V_{J,t}$ is equal to the Lagrangian multiplier in front of the employment transition equation in the firm program. The real wage is the one that maximizes the joint surplus of the representative firm

and household, *i.e*

$$\mathcal{W}(Z_{t+1}, Z_t, S_{t+1}, S_t; \Theta) = \arg\max_{\mathcal{W}} V_{N,t}^{\mu} V_{J,t}^{1-\mu},$$

where μ is the bargaining power of the household. As is standard, the outcome of the bargaining process is a real wage that can be expressed as an average of the household and firm outside option. Formally, the real wage is given by:

$$\mathcal{W}(Z_{t+1}, Z_t, S_{t+1}, S_t; \Theta) = \mu\left(b + \chi\frac{N_t^{\varphi}}{\lambda_t}\right) + (1-\mu)\left(mc_t\zeta A_t N_t^{\zeta-1} + \beta(1-s)\mathbb{E}_t\xi_{t+1}\frac{\lambda_{t+1}}{\lambda_t}r\theta_{t+1}\right),$$

where b is the replacement rate of the unemployment benefits.[6] I report in Figure 3 the estimated policy rules as a function of the preference shock. To do this, I keep all the other state variables—the technology and government spending shock and last period employment—at their steady state value. I then plot the main endogenous variables as a function of the preference shock.

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[6]I set a standard value of $b = 0.4$ for the replacement rate. The bargaining power of the household is set to $\mu = 0.5$ so that the Hosios condition holds. Finally, $\chi = 0.2788$ is set so as to balance the steady state wage equation.

Figure 3: Policy rules for the model with flexible wages.
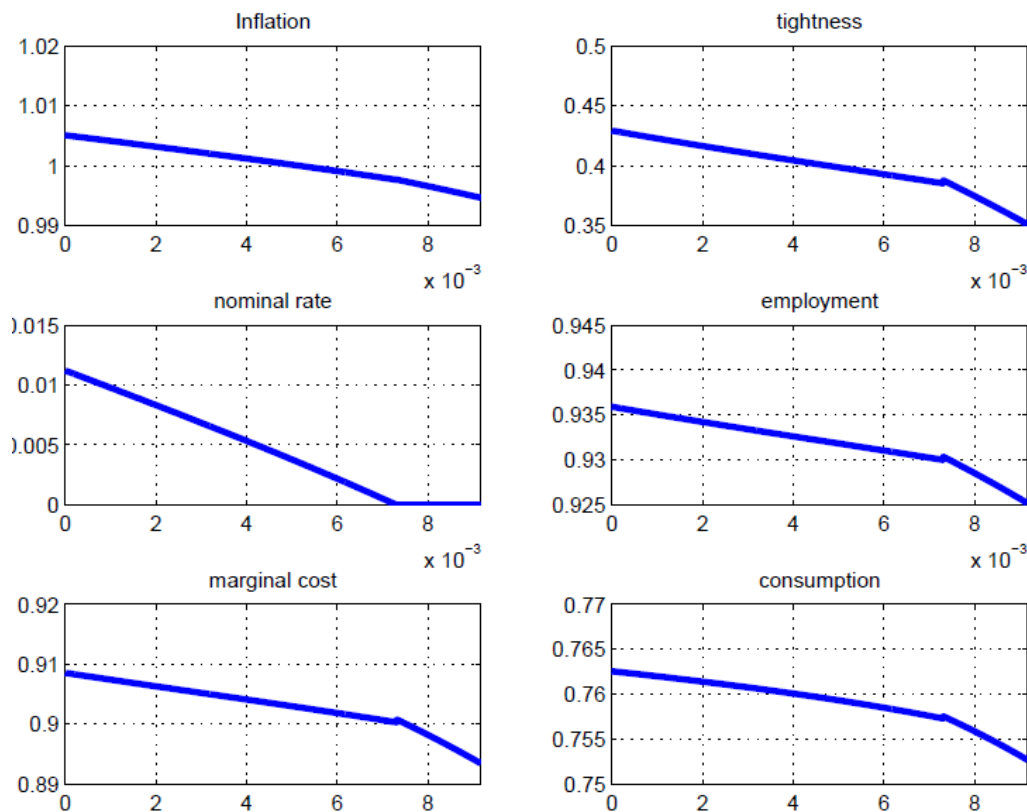
One can see that for a sufficiently large preference shock, the economy undergoes deflation and the nominal interest rate goes all the way down to the Zero Lower Bound. Except maybe for inflation, all the other endogenous variables exhibit a sharp kink when the nominal interest rate is pinned at zero. This suggests that, as has been extensively pointed out in the literature, things are very different at the ZLB. If we look at the labor market, whenever the shock is sufficiently large to send the economy at the ZLB, it generates a larger fall in employment or labor market tightness. In line with the discussion of the baseline New Keynesian model, the real marginal cost behaves very differently in and out of the Zero Lower Bound. The slope of the policy rule is much steeper when the nominal rate is pinned at zero. Take two levels of the preference shock, one being large enough to send the economy in a liquidity trap and the other not sufficiently large. When one augments both shocks by, say, 10%, the first shocks generates a larger fall in real marginal cost. In other words, real marginal cost is more elastic to the preference shock, conditional on the fact that this latter is sufficient to

generate a liquidity trap. With this in mind, I will now compute the government spending multiplier inside and outside a liquidity trap[7]. The results are reported in Figure 4. I focus on the spending multiplier on output, which is defined as GDP (private and public consumption) plus the recruitment and price adjustment cost. I also report the results for GDP, which are comparable to the ones usually reported in the literature. Since the latter generally uses log-linear approximations of the New Keynesian model, the real resources used when the price deviates from its steady state value are equal to zero up to a first order approximation. In the end, in these models, GDP equals output.

I assume that the shock is big enough for the economy to stay one period at the Zero Lower Bound. I also assume that the government spending shock is small enough so that it does not influence the length of the ZLB spell. In fact, by generating inflation, a large government spending shock will reduce the length of the ZLB spell. Several reasons can be put forward to justify a small government spending shock. First, it simplifies the interpretation of the mechanisms, which is my focus here. Second, it facilitates the comparison with the large literature on the effects of government spending shock, which usually relies on shocks that are infinitely small[8]. Finally, fiscal packages usually represent a minor share of total GDP. As C.Brown puts it when talking about the Great Depression : "*Fiscal policy, then, seems to have been an unsuccessful recovery device in the 'thirties—not because it did not work, but because it was not tried*".

I do not consider larger preference shocks —which would make the economy stay at the Zero Lower Bound for a larger number of periods—because then the magnitude of the shock would exceed the bounds of the grid on which the policy rules are interpolated. Considering larger shocks will then generate increasing approximation errors. With this in mind, for the results to be comparable with the model featuring rigid real wages, I choose the magnitude of the shock in the following manner. There is an interval of values for the preference shock that make the economy stay just one period at the Zero Lower Bound. I take the magnitude of the shock to be the average

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[7]Suppose the shock(s) occur at date $t = 0$. Let $z_t \in Z_t$ be any endogenous variable. I let $z_{t,\xi}^g$ denote the path of this variable conditional on the preference and government spending shock. I also let z_t^g denote the path of this variable conditional on the preference shock only. Then, the multiplier is given by $\frac{z_{t,\xi}^g - z_t^g}{g_1 - g}$, where g is government spending at steady state and g_1 is government spending one period after the shock. It follows that $g_1 - g$ is the size of the government spending shock

[8]By definition, $\frac{\partial y_t}{\partial g_t}$, which is the focus of this literature, is the reaction of output to an infinitely small deviation of government spending with respect to its steady state value.

14

Figure 4: Government spending multiplier for the model with flexible wages, in and out of ZLB
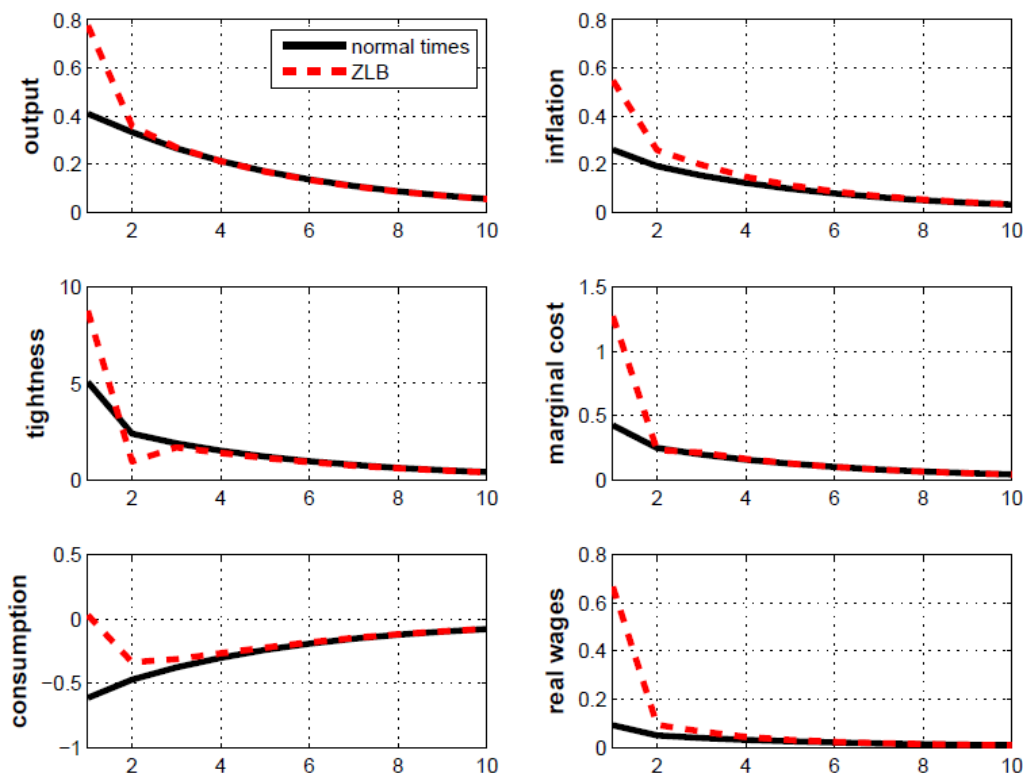
of the values in this interval.[9]

One can see from Figure 4 that the government spending multiplier at ZLB is indeed higher than in normal times. Since labor market tightness and the real wage are more elastic with respect to government spending at ZLB, real marginal cost reacts more than in normal times to a rise in government expenses. What follows is that inflation rises more than it does in normal times, generating a (small) crowding in effect on private consumption through a lower real interest rate. Unlike in the baseline New Keynesian model, producing additional output requires more employees, which is costly. Therefore, the mapping between the effect on real marginal cost, inflation and consumption is not as straightforward in this setup. Eventually, the additional output demanded will not be produced in equilibrium since it is too costly to do so. This is why the response of consumption is so small,

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[9]Denoting by ϵ^ξ the exogenous shock on the AR(1) process for ξ_t, this yields a value of $\epsilon^\xi \simeq 0.0085$.

despite higher than normal inflation and a zero nominal rate.

Furthermore, note that the multiplier on output is less than one despite the small crowding in of consumption.[10] Here, the price adjustment cost plays an important role. Recall that the multiplier is computed as the path of each variable with the preference and government spending shock, minus the one with only a preference shock. The one with only a preference shock exhibits deflation. Therefore, by dampening deflation, the government spending shock actually has the effect to reduce the amount paid to change prices, which reduces total output through the resource constraint. Since private consumption is barely crowded in, the response of GDP is slightly higher than one[11].

The bottom line is that, like the baseline New Keynesian model, the model with Nash-bargained wages is at odds with what can be seen in the data. In the data, inflation and real wages seem to react less to a government spending shock in a recession. In the appendix, I show what happens when the preference shock is not large enough for the ZLB to become binding (see Figure 7). In this setup, inflation, real wages and real marginal cost react more to the government spending shock in the recession than in normal times, with the difference being of small magnitude. As in the baseline New Keynesian model then, the labor market structure is responsible for the sign of the effect on key macroeconomic variables and the binding ZLB plays an amplifying role on aggregate demand effects.[12] In this section, I have focused on the mechanisms through which government spending can have stimulative effect in a New-Keynesian model with a frictional labor market and Nash Bargained wages. For a related study that also considers the case of extended unemployment benefits, see Albertini & Poirier (2013).

5   Labor Market Dynamics at ZLB with Rigid Wages

I have shown in the previous section that a New Keynesian model augmented with search and matching frictions exhibits roughly the same mechanisms as

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[10]This comes from the fact that the shock sends the economy at the ZLB for just one period. With one more period at ZLB, the effect on output is close to one and the crowding in effect on consumption is stronger, but with higher approximation errors.

[11]By construction, the GDP multiplier is just one plus the multiplier on private consumption.

[12]In this respect, consumption is a special case. Since it depends exclusively on the path of real interest rates, consumption is typically crowded out by government spending in normal times and crowded in when the ZLB binds.

the baseline New Keynesian model in explaining why government spending is more efficient at stimulating output at the Zero Lower Bound. To sum it up, the real wage reacts too much to a government spending shock. This is in contrast to what can be observed in the data. We can draw a parallel here with the literature initiated by Shimer (2005) and Hall (2005). In fact, it has been known for a long time that Nash Bargaining generates real wages that are too reactive with respect to overall economic conditions. This entails that the conventional search and matching model is not able to replicate the observed volatility of unemployment. A corollary of this is that the search and matching model with Nash Bargained wages does not do a good job at explaining high unemployment in recessions. To explain the occurrence of high peaks of unemployment (which can amount to 25% of the labor force as in the Great Depression) one needs to assume some form of real wage rigidity as in Michaillat (2012a) or Petrosky-Nadeau & Zhang (2013).

Accordingly, I now assume that real wages do not react perfectly to economic conditions. Specifically, I follow Blanchard & Gali (2010) and assume that

$$\mathcal{W}(Z_{t+1}, Z_t, S_{t+1}, S_t; \Theta) = \omega A_t^{\gamma},$$

where γ indexes the extent of real wage rigidity. The lower gamma, the less the real wage reacts to variation in technology[13]. By construction, such a form for real wage rigidities implies that the effects of government spending on real wages will be nil. It is however consistent with the findings of Auerbach & Gorodnichenko (2012). I report below in Figure 5 the estimated policy rules of the model with real wage rigidity.

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[13]Following Michaillat (2012b) I set $\gamma = 0.5$ and $\omega = 0.6116$ is set so as to balance the steady state job creation equation.

Figure 5: Policy rules for the model with rigid wages.
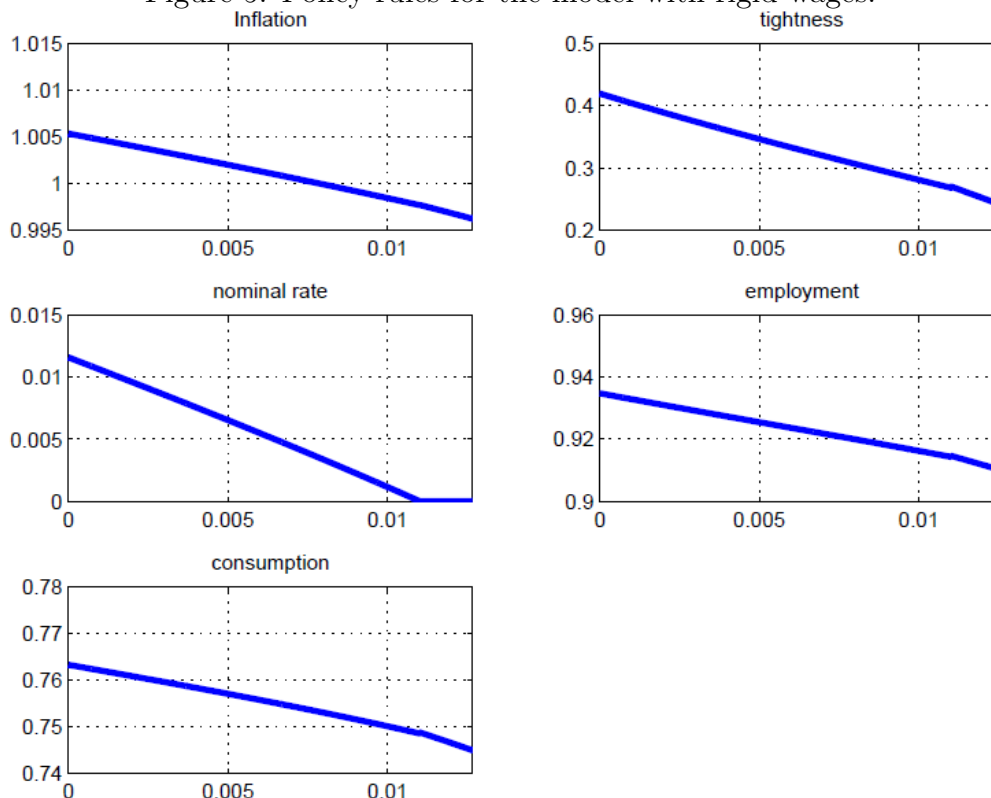
One can see that, aside from the nominal interest rate, the kink is less pronounced than in the model with Nash Bargained wages. This suggests that the dynamics at the ZLB might not be as different as in the model with flexible wages. In particular, real marginal costs do not seem to be more elastic when the preference shock is large enough to send the economy at the ZLB. To gauge the dynamic effects of government spending in this setup, I report below the impulse response functions in and out of a liquidity trap. To highlight the role played by job rationing, I also assume that there is a potentially large technology shock. In fact, the occurrence of job rationing in the model is due to the fact that the real wage is higher than the marginal productivity of would-be employees. With a negative technology shock then, the real wage will fall slowly while the marginal product will react in full proportion. By a 'large' productivity shock I mean a shock that has the effect to reduce the length of the ZLB spell from two to one period[14].

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[14]In the simulation, I set $\epsilon^a = -0.005$. Given this value for ϵ^a, I look for the mean value

Figure 6: Government spending multiplier for the model with rigid wages, in and out of ZLB.
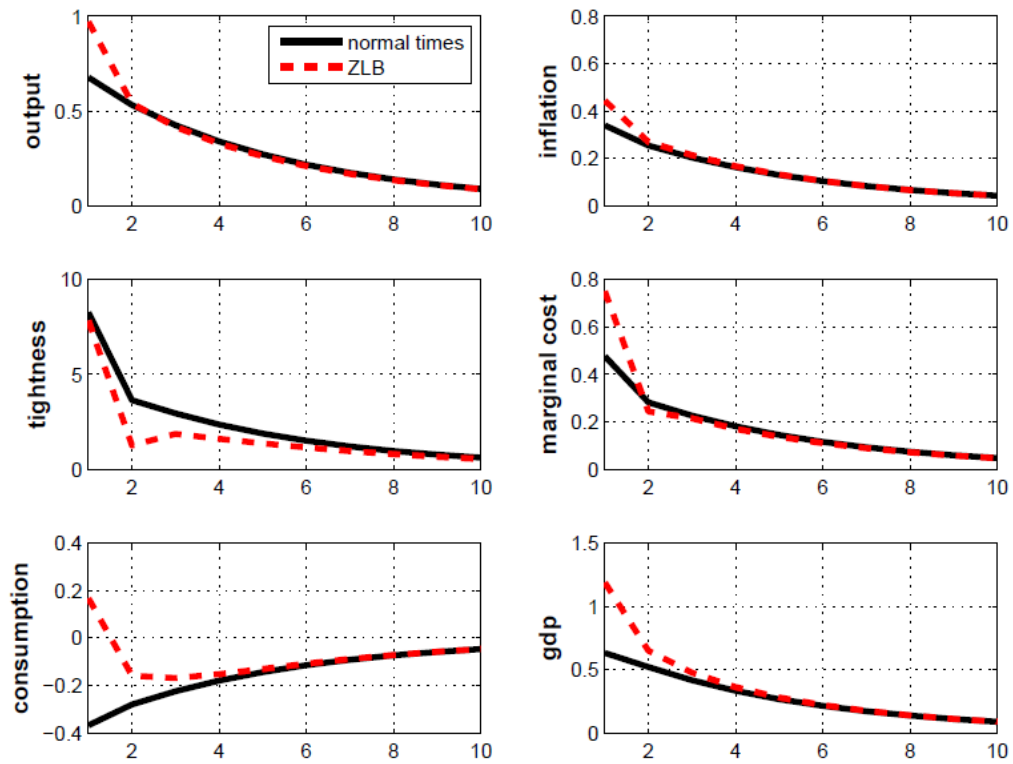
What stands out of Figure 6 is the fact that the government spending multiplier on output is still higher than the one in normal times. One can also see that the crowding in effect on private consumption is higher than with Nash Bargained wages. Indeed, since the effect of government spending on labor market tightness —and, thus, on recruitment costs—is smaller than in normal times, firms respond by employing more people to meet the additional demand. Because it is cheaper to put additional resources to use, firms can meet the increased demand coming from the representative consumer who reacts to a lower real rate. They can do so without incurring disproportionately larger than normal real marginal costs, which translates into a slightly larger than normal response of inflation. Here again, the deadweight cost associated to price changes plays a role. Because higher government spending mitigates the deflation coming from the preference shock, the response of the price adjustment cost is negative after a government spending shock, so that the response of output is lower than the one of gdp.[15] At the end of the day, what really drives the results is the fact that it is easier to recruit in a deep recession. As a consequence, firms are more willing to employ additional people to meet the increased demand.

According to recent research (see van Rens (2012)), this is exactly what firms tend to do in a recession. In his discussion of Ohanian & Raffo (2012), he shows that in a recession, labor market adjustment occurs almost exclusively through the extensive margin, *i.e* through employment rather than hours worked. Furthermore, the model with rigid real wages is more in line with the estimated effects of government spending in a recession (see Figure 1) than the model with flexible wages. By construction, real wages do not react to a government spending shock in a recession. I report the reactions of the other variables to a preference shock that is not large enough to push the economy at the ZLB in the appendix (see Figure 8). This situation can be interpreted as a "normal" recession.

5.1 Discussion

Nevertheless, the model with rigid real wages has one important flaw : real wages are rigid whatever the state of the economy. In particular, the model implies that a government spending shock in normal times will have no effect on the real wage. This is clearly at odds with what we see in the data .

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of ϵ^ξ for which the economy stays at the Zero Lower Bound for one period. This gives $\epsilon^\xi = 0.015$.

[15]The response of recruitment cost plays a small quantitative role in the aggregate resource constraint.

Indeed, one can see from Figure 1 that real wages tend to rise after a government spending shock in normal times, with the response being significantly different from zero (statistically). The outcome of this is that the multiplier in normal times in the model with rigid real wages is biased upwards. With the real wage not reacting to higher government spending, real marginal cost rises less than it should, thereby dampening the inflationary pressure. This latter calls for a reaction from the Central Bank (since we are in normal times, the Taylor rule is still active) which generates a rise in the real rate that depresses consumption. Rigid real wages have then the effect to underestimate the crowding out effect of government spending on consumption in normal times. This leads the model to underestimate the difference in magnitude of the spending multiplier in good and bad times. Were real wages able to rise in normal times, the difference between the multipliers in and out of the ZLB would be higher. At the end of the day, the model with rigid real wages is then better suited to capture the dynamics of an economy in a recession with a possibly binding ZLB.

One possible remedy to this shortcoming would be to assume downward rigid nominal wages. Intuitively, since there is deflation when the economy reaches the ZLB, with downward rigid nominal wages the real wage would be too high : this would generate job rationing. With the real wage already higher than it should be, the upward pressure following a government spending shock should be mild. In addition, the real wage would be free to rise after a government spending shock in normal times. I am currently working on the introduction of a downwardly rigid nominal wage.

6 Conclusion

In this paper I have focused on how the setup of the labor market plays a role in the transmission mechanisms of government spending in a liquidity trap. I have shown that when one takes into account the fact that a liquidity trap is always associated with an unemployment crisis, higher government spending can be efficient at stimulating output. This comes not from the fact that government spending is inflationary, but from the fact that putting resources to use is essentially costless in a severe recession.

My goal in this paper has been to devise a model consistent with available empirical evidence on the behavior of key macroeconomic aggregates in a typical recession . In this regard, the model fails to replicate the fact that private consumption seems to be crowded in a recession (and not only in a liquidity trap, as in the model I have developed, see Auerbach & Gorodnichenko (2012)). To get an idea of how the model could be amended to cope with this

shortcoming, one can argue that aside from higher unemployment, a liquidity trap is also usually associated with disrupted credit/financial markets. Using a model in which higher government spending has a negative effect on credit spreads, Canzoneri et al. (2012) show that higher government spending can increase private consumption in a downturn. By reducing the credit spread, credit constrained agents increase their consumption more than Ricardian agents reduce theirs. In a paper closely related to mine, Wieland (2013) studies the effect of technology shock in a liquidity trap. He shows that taking financial frictions into account is essential for the dynamics of the New Keynesian model at the Zero Lower Bound to be consistent with the data. I leave this for future research.

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A Figures

Figure 7: Government spending multiplier for the model with flexible wages, in and out of recession
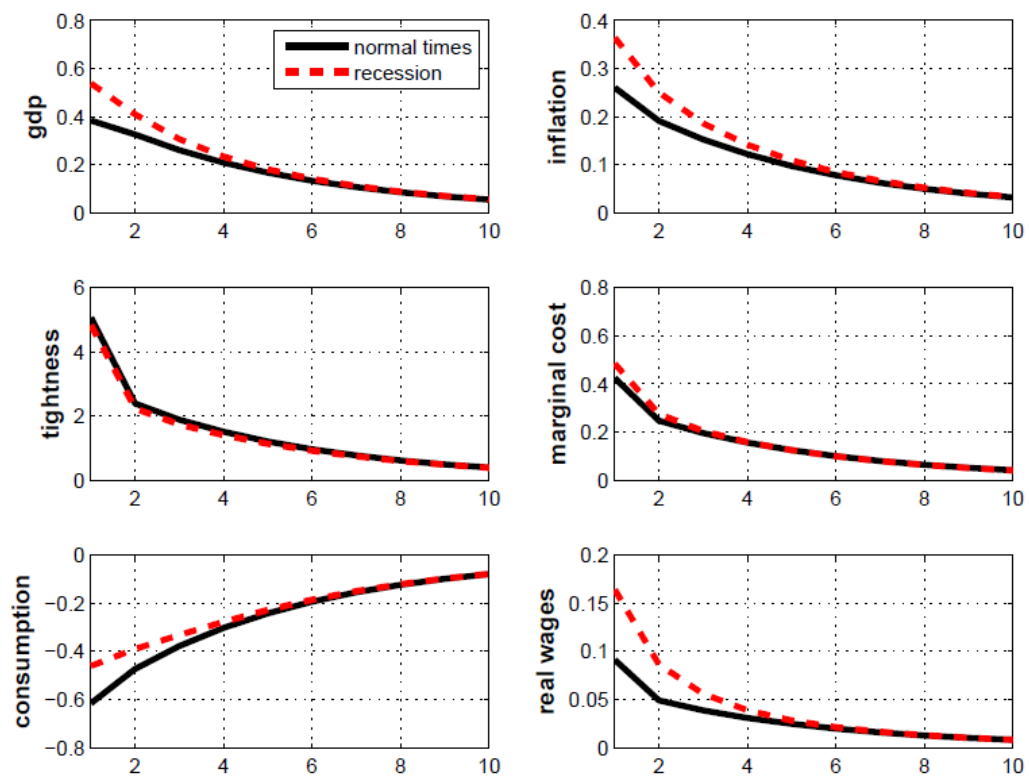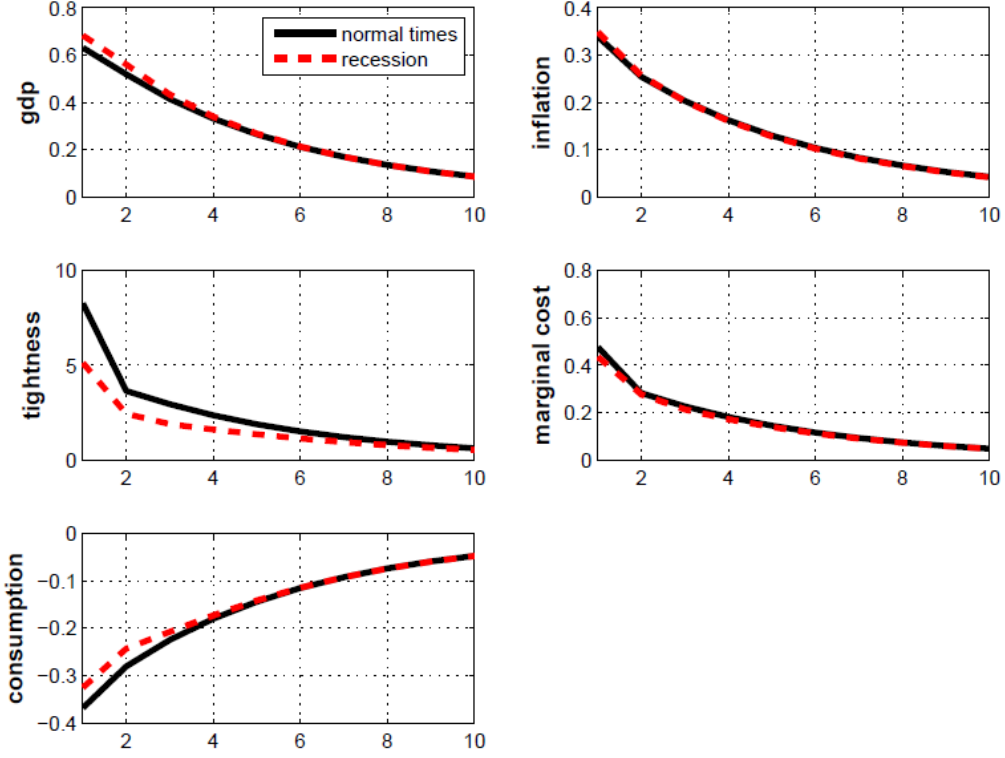
Figure 8: Government spending multiplier for the model with rigid wages, in and out of recession

B   Computational Details

B.1   Summary of the model

The model can be summarized by the following set of equations:

$$N_t = (1-s)N_{t-1} + \left[1 - (1-s)N_{t-1}\right]f(\theta_t) \tag{14}$$

$$\lambda_t = C_t^{-\sigma} \tag{15}$$

$$C_t = G_t + [N_t - (1-s)N_{t-1}]\frac{rA_t}{q(\theta_t)} - A_t N_t^{\zeta}\left[1 - \frac{\phi}{2}(\Pi_t/\Pi)^2\right] \tag{16}$$

$$\lambda_t = \beta R_t \mathcal{E}_{t+1}^1 \tag{17}$$

$$\epsilon \cdot mc_t = \epsilon - 1 + \phi(\Pi_t/\Pi - 1)(\Pi_t/\Pi) - \beta\phi\mathcal{E}_{t+1}^2 \tag{18}$$

$$mc_t = \frac{1}{\zeta A_t N_t^{\zeta-1}}\left[\mathcal{W}(Z_{t+1}, Z_t, S_t; \Theta) + \frac{rA_t}{q(\theta_t)} - \beta(1-s)\mathcal{E}_{t+1}^3\right], \tag{19}$$

27

where the expectation functions are given by:

$$\mathcal{E}_{t+1}^1 = \mathbb{E}_t\left\{\xi_{t+1}\frac{\lambda_{t+1}}{\Pi_{t+1}}\right\} \tag{20}$$

$$\mathcal{E}_{t+1}^2 = \mathbb{E}_t\left\{\xi_{t+1}\frac{\lambda_{t+1}}{\lambda_t}\frac{Y_{t+1}}{Y_t}\frac{\Pi_{t+1}}{\Pi}\left(\frac{\Pi_{t+1}}{\Pi}-1\right)\right\} \tag{21}$$

$$\mathcal{E}_{t+1}^3 = \mathbb{E}_t\left\{\xi_{t+1}\lambda_{t+1}\frac{rA_{t+1}}{q(\theta_{t+1})}\right\} \tag{22}$$

The third expectation function's expression depends on how the real wage is set. If one assumes that real wages are rigid, then the real wage depends only on current technology, so that the expression does not change. When one assumes Nash bargained real wages however, the real wage depends on future state and control variables. Denoting $\mathcal{E}_{t+1}^{1,FW}$ the expectation function for flexible real wages, I have the following job creation equation

$$mc_t = \frac{1}{\zeta A_t N_t^{\zeta-1}}\left[b + \frac{\chi N_t^\varphi}{\lambda_t} + \frac{\frac{rA_t}{q(\theta_t)} - \beta(1-s)\frac{\mathcal{E}_{t+1}^{3,FW}}{\lambda_t}}{1-\mu}\right] \tag{23}$$

$$\mathcal{E}_{t+1}^{3,FW} = \mathbb{E}_t\left\{\xi_{t+1}\lambda_{t+1}\frac{rA_{t+1}}{q(\theta_{t+1})}(1-\mu f(\theta_{t+1}))\right\} \tag{24}$$

B.2   Solution Algorithm

The Parameterized Expectation Algorithm amounts to approximate the expectation functions by a simple polynomial function of the state variables. The polynomials I consider will be of the Chebychev type. Accordingly, for a state variable $s_t \in S_t$ let $C_i : s_t \longmapsto C_i(s_t)$ be the function that returns the Chebychev polynomial of order $i \in \mathbb{N}$ evaluated at the point s_t. I first build a linearly spaced grid of the state variables centered on the steady state value for each one. I then evaluate $C_i(.)$ at each point of the grid. For a given polynomial degree p (I choose $p = 3$ in the simulations) and for each grid point, I construct a modified grid which is composed of the products of $C_i(.)$ evaluated at different grid points, with the restriction that the product should be of degree less or equal than p. For example, I take the first grid point for each state variable, I evaluate each one with $C_1(.), \ldots C_p(.)$, keeping only the products of degree less or equal than p. This gives me the first line of the final grid. For the second line, I take the same points but the last one is the second grid point of the last state variable, and so and so on. I end up with a grid $\tilde{S} \in \mathbb{R}^{(p+1)^s}$, with s being the number of state variables.

The expectation functions are approximated by a simple function of the Chebychev polynomials of the state variables, namely:

$$\mathcal{E}_{t+1}^i = \tilde{S}_t \cdot \Xi^i, \quad i = \{1, 2, 3\},$$

where Ξ^i has no time subscript since it is a time-invariant "policy rule". Let Ξ denote $[\Xi^1, \Xi^2, \Xi^3]$. This is the object on which I will iterate until convergence. The endogenous variables will also be expressed as a function of the Chebychev polynomials of the states. It is sufficient to approximate the policy rules for two of the endogenous variables : the labor market tightness θ_t and the inflation rate Π_t. Given expectations and the states variables, all the other endogenous variables can be computed. Let Ω be the set of coefficients that relates θ_t and Π_t to the Chebychev polynomials of the state variables. The algorithm then works as follows:

1. Choose a value for the learning parameter $\zeta \in [0, 1]$ and the stopping criterion, ϵ.

2. Start with an initial guess for Ξ, say Ξ_0. As a first guess, I evaluate the expectations functions at steady state.

3. For each point of the grid on state values, compute the value of the expectations. Given a first guess for Ω_0, compute Ω using a Newton algorithm. [16]

4. Using Ω and the law of motion of the state variables, reevaluate the expectations functions using a Gauss-Hermite quadrature.

5. Regress these new expectations on the grid of state variables, which gives Ξ_1.

6. Compute $\hat{\Xi}_1 = \zeta \Xi_1 + (1 - \zeta)\Xi_0$

7. If $\left\| \frac{\hat{\Xi}_1 - \Xi_0}{\Xi_0} \right\| < \epsilon$ then stop. Else return to step 2, using $\hat{\Xi}_1$ and the last solution for Ω as guesses.

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[16]I actually compute two policy rules : one for when the economy is in normal times and one for when it is at the ZLB. This is done to precisely identify the kink in the policy rules.