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Abstract

When an interested party controls both the acquisition and the transmission of evidence and some of her information is unverifiable, a tension between selective revelation and benefits of transparency arises. The interested party might hence find it optimal to reveal unfavorable evidence even though disclosure is discretionary. While this revelation policy yields to more accurate decisions, informativeness is non monotone in the extent of voluntary disclosure. The model provides a rationale for adverse announcements such as product recalls and earning warnings and implications for mandatory disclosure laws.

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1 Introduction

In many social and economic interactions, an individual seeks to influence the opinion of others. For instance, an employee aims at convincing superiors of her virtues, a seller at assuring buyers of the quality of her products, and a CEO at persuading investors of the financial soundness of her firm. While the interested party will generally have a better idea than the public on the merits of her position, she may or may not be in possession of verifiable information on the matter. When this is the case, she should reveal favorable pieces of evidence and conceal unfavorable ones. This selective communication policy has indeed been documented in a variety of settings[1] and constitutes the building block of most theoretical works on discretionary disclosure.[2] Nevertheless, interested parties also release unfavorable information, admit mistakes or offer counterproductive arguments.

Our daily life offers anecdotal evidence.[3] Moreover, well-documented examples involving large economic actors exist. Firms voluntarily release adverse announcements on their profitability such as earning warnings (Skinner, 1994) and sellers disclose defective attributes of their goods that can be so severe to warrant a recall (Jarrell and Peltzman, 1985).[4] A multitude of factors influence these decisions,[5] which might partially account for the mixed evidence on the extent of these voluntary disclosures and the consequent allegedly negative reaction.[6] At the same

[1]See Miller (2002) and Kothari, Shu and Wysocki (2009) for corporate disclosures, Dedman et al. (2008) for drug approval notifications, and Mathios (2000) and Jin and Leslie (2003) for information on product quality.

[2]See for instance Jovanovic (1982); Verrecchia (1983); Dye (1985); Jung and Kwon (1988); Shin (1994); Shavell (1994); Shin (2003); Acharya, DeMarzo and Kremer (2011).

[3]Students point out faults in their own assignments, political parties reveal scandals involving their members, job applicants mention previous entrepreneurial failures...

[4]In these settings, the cost of false reporting is usually very high due to anti-fraud laws. It is hence common in the literature to consider firms' statements as hard evidence. While the regulatory framework makes the provision of some information compulsory, firms' discretion concerns information that falls outside its scope or the timing of revelation: i.e. a manager who discovers an earnings surprise can choose whether to release the news or to wait until the quarterly report. Note also that in some technology-based industries, non-financial information is the most important source of price discovery (Amir and Lev, 1996).

[5]See Skinner (1997) and Rupp and Taylor (2002) on legal liability, Aboody and Kasznik (2000) on CEOs' compensation schemes and Tucker (2010) on future coverage by market experts.

[6]Skinner (1994) and Kasznik and Lev (1995) document that the returns of warning firms are lower than those of non-warning ones. Tucker (2007) and Pukthuanthong (2010) find that this short-run difference fades or even reverts over a longer time frame. Jarrell and Peltzman (1985) document a persistent negative effect of automobile recalls on stock prices while Hoffer, Pruitt and Reilly (1988) find no such evidence using the same data. Salin and Hooker (2001) document mixed reactions to food recalls. Dedman et al. (2008) document the tendency of pharmaceutical companies to over-report successes relative to failures in the drug approval process but also a positive reaction to the few announcements whose content is plainly negative. A major challenge in all these studies consists in separating evidence that is unfavorable in absolute terms and relative to market expectations.

time, empirical evidence suggests that the market rewards transparency to some extent. Several surveys identify building a reputation for transparent reporting as the main rationale for earning warnings (Tucker, 2010) and warning firms may surpass silent ones with comparable current performances (Tucker, 2007; Pukthuanthong, 2010). Similarly, the negative stock price reaction to product recalls is weaker when they are voluntary than when they are initiated by regulatory bodies (Hingorani, Shastri and Shastri, 1994; Chen and Nguyen, 2013). When explaining possible reasons for the absence of a negative reaction to a food recall by IBP Inc, Salin and Hooker (2001) postulate that "the microbiological contamination in the ground beef was identified before reaching consumers, a fact that could have bolstered investors' confidence in IBP's management of foods safety risks." Consistent with the idea that transparency may act as a positive signal, high-quality brands tend to preempt consumers and authorities by speeding up a recall while low-quality brands tend to delay it (Eilert and Jayachandran, 2014). Interestingly, Kasznik and Lev (1995) suggest that firms would sometimes be willing to disclose adverse news if the market were to update appropriately but refrain from doing so due to the fear of an excessively negative reaction.

This paper builds on the observation that when interested parties control both the process of information acquisition and its transmission, a tension between strategic information retention and benefits of transparency arises. Thus, once we take into account that information is endogenous, these somehow counter-intuitive communication practices might actually be rational. Indeed, the better the news an interested party expects, the stronger are her incentives to invest in information. By disclosing, she can prove her prior confidence in obtaining favorable evidence, regardless of the news content, and reveling negative information might be better than being perceived as uninformed. I develop a simple model that formalizes this argument and aims at accounting for why the extent of voluntary disclosure varies in content and quantity. It also aims at assessing the informativeness of the decisions the public ultimately takes and the impact of some possible regulatory interventions.

I consider a game of persuasion in which the state consists of Bernoulli trials whose likelihood of success is privately know by the sender and unverifiable (the sender's type). At a cost, the sender can gather verifiable information on whether some trial succeeds (good news) or fails (bad news). She can then freely and credibly disclose the news she gathered to the receiver. In the context of the examples above, we might interpret soft information as a firm's propensity to deliver good products or a manager's ability to carry out successful projects. Extensive product testing or rigorous internal auditing practices generate hard evidence, while residual uncertainty remains on future outcomes.

In equilibrium, only types with a high enough probability of obtaining good news find it worthwhile to acquire information. Thus, the presence of uninformed types prevents a worst-case inference by the receiver in the event of nondisclosure. At the same time, by disclosing, the sender can certify that her type is above the information acquisition cutoff. When the signaling value of unfavorable evidence offsets its negative content, there exists an equilibrium in which informed senders disclose both good and bad news. This equilibrium coexists with the more conventional one in which informed senders disclose good news but conceal bad news. The game may also have an equilibrium in which no type acquires information and an equilibrium in which all types acquire information and disclose unselectively. The receiver's reaction to good, bad and no news represents an important source of multiplicity and stigma against unfavorable evidence might cause its withholding.

Overall, equilibria in which the sender discloses bad news exist when information is not too expensive and the prior over types is not too favorable. Indeed, if nondisclosure is met with high skepticism, the benefit of claiming ignorance cannot be too large and the reputational value of bad news dominates. Skepticism in the absence of news is high when: the cost of information is small, so that only very low types elect to remain uninformed; or when the prior is sufficiently unfavorable, so that, even if the cost is high and hence also relatively high types do not acquire information, the average quality of uninformed types remains low. Thus, in accordance with the predictions of other works,[7] adverse market conditions as captured in the model by a low prior can encourage the release of bad news. Equilibria in which the sender discloses bad news also exhibit a higher extent of overall voluntary disclosure. Indeed, when both favorable and unfavorable pieces of evidence are valuable, not only the sender reveals more states but also a larger fraction of types elect to acquire information.

Restricting myself to the uniform case, I then assess the amount of information that the receiver obtains in equilibrium. Equilibria with concealment and revelation of bad news always

[7] See for instance Jung and Kwon (1988) and Acharya, DeMarzo and Kremer (2011).

coexist and, consistent with intuition, the receiver forms more accurate opinions in the latter than in the former. Thus, laws that force interested parties to *reveal* any acquired evidence are beneficial to the public in that they facilitate coordination on more informative equilibria. At the same time, due to the same signaling considerations that induce the sender to release bad news, informativeness is non-monotone in the overall extent of voluntary disclosure. The receiver's opinions are the most accurate when an intermediate cost level dissuades low enough types from acquiring information. Laws that force interested parties to *acquire and disclose* information can therefore decrease the amount of information that ultimately reaches the public. Conversely, if the cost level is set by a profit-maximizing certifier, information acquisition is too low from the public's point of view, due to rent extraction considerations.

The main insights of the model are robust to several alternative assumptions. In particular, equilibria in which the sender reveals unfavorable evidence also exist if in the absence of disclosure evidence becomes public with some delay,[8] if the sender can eliminate residual uncertainty by acquiring additional information and if information acquisition can be inconclusive.[9]

Related literature The unraveling argument that yields to full separation (Grossman, 1981; Milgrom, 1981; Matthews and Postlewaite, 1985) fails in the model because information is costly[10] and partly unverifiable. Retention of verifiable evidence occurs whenever uninformed senders, who cannot differentiate themselves from silent informed ones, would benefit from separation.[11] Similar to Matthews and Postlewaite (1985) and Shavell (1994) and in contrast to most of the literature on discretionary disclosure, the partition between informed and uninformed senders arises endogenously. In Matthews and Postlewaite (1985), there is no ex-ante heterogeneity, so that either all types or none elect to acquire information. In Shavell (1994), senders differ in their cost of acquiring information. That source of heterogeneity becomes irrelevant at the disclosure stage but in a repeated setting it would induce the sender to withhold more information to develop a reputation for ignorance, as in Grubb (2011). In this paper, instead,

[8] For instance, in the context of consumption goods, consumers will eventually discover the quality of products even in the absence of a seller's disclosure by means of experience or of a compulsory recall, if warranted.

[9] When there is always a possibility that interested parties might be uninformed, forcing disclosure by means of laws or contracts becomes harder. Thus, strategic information revelation considerations occupy a central role even in settings in which these instruments might be available to the public or the regulator.

[10] As in a canonical signaling model (Spence, 1973), the cost acts as separating device. However, the decision to acquire information is equally costly for all types and unobservable.

[11] See the general results of Mathis (2008).

heterogeneity in quality induces senders to disclose more information to develop a reputation for competence.[12] Moreover, the presence of payoff-relevant soft information generates opposite policy implications.[13] The model of Teoh and Hwang (1991) also features heterogeneity in quality but the arrival of news is exogenous, the time-line more articulated and the payoff structure more flexible.[14] By withholding initial favorable information and disclosing unfavorable one, a sender might signal her confidence in future good news and hence her quality. The two models generate opposite predictions on the quality of firms that disclose good news.[15] When senders differ in their bias (Dziuda, 2011), instead, they will disclose favorable arguments bundled with some unfavorable ones. Finally, because the model this paper considers is static, reputational arguments behind transparency that are typical of repeated games[16] are absent.

2 Model

2.1 Players, information structure and timing of the game

The game has two players: a sender (S) and a receiver (R). S aims at maximizing the action R takes, while R wants her action to match the true state. The true state is $\omega = \omega_1 + \omega_2$, where $\omega_1 \in \Omega_1 = \{0, 1\}$ and $\omega_2 \in \Omega_2 = \{0, 1\}$ are the returns of two independent random variables $\tilde{\omega}_1$ and $\tilde{\omega}_2$, which can either succeed (yield 1) or fail (yield 0).[17]

The probability of success of each random variable is $q \in Q = [0, 1]$. Nature draws q from a common knowledge distribution with support Q and finite mean (μ) and variance (σ^2). I denote

[12] In contrast to the literature on delegated expertise (Bourjade and Jullien, 2011), competence has no intrinsic value. That is, the sender does not derive utility from proving her ability in receiving information.

[13] Matthews and Postlewaite (1985) identify a countervailing effect of laws that make the disclosure of acquired information compulsory. This effect does not occur in this paper both because information acquisition is costly, so that even in the absence of these laws the sender might conceal evidence, and because the receiver can punish information retention by means of an adverse inference on the sender's soft information, so that the sender finds it less attractive to prove her ignorance. In Shavell (1994), the main source of inefficiency lies in the acquisition of wasteful information and mandatory disclosure rules eliminate it. In this paper, the policy discussion centers on the accuracy of the receiver's opinions but mandatory disclosure may rather encourage information acquisition.

[14] In particular, their main result requires a complementarity between news and quality, i.e. that good news is worth more when the sender's type is high.

[15] The issue has received less empirical attention than the question of whether it is high or low quality firms that disclose bad news, on which two models agree. This paper predicts that good(bad) news should have high(low) persistence.

[16] See for instance Welling (1991) on voluntary product recalls.

[17] The binary nature of evidence makes the model tractable and renders unambiguous what bad news is. When the state is continuous, bad news is any realization below some endogenously determined cutoff (Jung and Kwon, 1988).

by $f(q)$ its pdf, which is continuous, differentiable, and strictly positive in the interior of Q. At the initial stage S privately observes q, which I will refer to as S's type. An additional source of asymmetric information lies in the possibility for S to incur a cost $k \in \left(0, \bar{k}\right)$ and learn ω_1, where $\bar{k} \equiv 2(1-\mu)$.[18] I denote by $i \in I = \{0, 1\}$ S's choice on whether to acquire information ($i = 1$) or not ($i = 0$).

q and i are unobservable to R and unverifiable. Instead, S can credibly reveal ω_1 before R chooses her action a. Let $\emptyset$ represent the event of no communication. Also, let $M = \{0, 1, \emptyset\}$ represent the message space[19] and $M^i_{\omega_1}$ represent the set of messages available to S when her information choice is i and the outcome of $\tilde{\omega}_1$ is ω_1. Thus, $M^0_{\omega_1} = \{\emptyset\}$ and $M^1_{\omega_1} = \{\omega_1, \emptyset\}$.[20]

S's payoff is $U^S = a - ik$ and R's payoff is $U^R = -(\omega - a)^2$. Because U^R depends on ω_2, the higher R's assessment of q, the higher the action she is willing to choose.[21] Thus, reputational concerns arise endogenously. The timing of the game is the following:

0. nature draws q, ω_1 and ω_2.
1. S observes q and chooses whether to spend k to learn ω_1;
2. S chooses a message m; R observes m and chooses an action a;
3. ω_1 and ω_2 become public and payoffs realize.[22]

2.2 Strategies and equilibrium concept

A pure strategy of S specifies an information acquisition decision $\boldsymbol{i(q)} : Q \to I$ and a communication policy to adopt when informed $\boldsymbol{m(q, \omega_1)} : Q \times \Omega_1 \to M^1_{\Omega_1}$. I will denote by $\boldsymbol{m_r}$ the policy that prescribes revealing both good news ($\omega_1 = 1$) and bad news ($\omega_1 = 0$) and by

[18]This upper bound for k guarantees the existence of an equilibrium in which some types indeed acquire information.

[19]Because the conflict of interests between S and R is maximal, including soft information in the message space would not affect the information that S reveals in equilibrium.

[20]$M^i_{\omega_1}$ does not depend on q for ease of notation, albeit M^i_1 and M^i_0 are empty sets when $q = 0$ and $q = 1$, respectively. We can solve this small inconsistency by stipulating that $M^i_1 \equiv \{\emptyset\}$ when $q = 0$ and $M^i_0 \equiv \{\emptyset\}$ when $q = 1$.

[21]While R takes a single action, the game is isomorphic to one in which she takes actions a_1 and a_2 and payoffs are $U^S = a_1 + a_2 - k$ and $U^R = -(\omega_1 - a_1)^2 - (\omega_2 - a_2)^2$. As long as no additional information arrives in between, a_1 and a_2 need not to occur simultaneously. The two specifications are no longer equivalent if R observes ω_1 before choosing a_2 no matter the disclosure decision of S, an extension I consider in section 5.1.

[22]One may add a last stage in which R can revise her action, which would then be equal to the true state.

$\boldsymbol{m_c}$ the policy that prescribes revealing good news but concealing bad news. A pure strategy of R is a mapping $\boldsymbol{a}(\boldsymbol{m}) : M \to \mathbb{R}$. I will denote by $f_R(\cdot|m)$ R's posterior over Q that message m induces. The relevant solution concept is perfect Bayesian equilibrium in pure strategies,[23] which requires that:

- $\forall\ q \in Q$ and $\omega_1 \in \Omega_1$, $i^*(q)$ maximizes $\mathbb{E}\left[a^*(m)|q, i\left(q\right), \boldsymbol{m}^*\left(q, \boldsymbol{\omega_1}\right), \boldsymbol{a}^*(\boldsymbol{m})\right] - i(q)k$ and $m^*(q,\omega_1)$ maximizes $a^*(m)$ over $M^1_{\omega_1}$;

- $\forall\ m \in M$, $a^*(m)$ maximizes $\mathbb{E}_R\left[U^R|m\right]$,[24] where $\mathbb{E}_R\left[\cdot|\cdot\right]$ represents the conditional expectation operator induced by R's belief system;

- R's belief system derives from Bayes' rule after any m that has positive probability according to $\boldsymbol{i}^*(\boldsymbol{q})$ and $\boldsymbol{a}^*(\boldsymbol{m})$; besides, after any $m \in \Omega_1$ that has zero probability, it assigns:[25]
  - probability one to the events that $i = 1$ and $\omega_1 = m$;
  - probability zero to the event that $q = 0$ if $m = 1$ and probability zero to the event that $q = 1$ if $m = 0$.

I also adopt the convention that S elects to acquire information and to remain silent whenever she is indifferent.

3 Equilibrium

3.1 Preliminary observations

Lemma 1. *In any equilibrium, S's disclosure policy does not depend on her type.*

Proof. Because R cannot observe S's type, $\boldsymbol{a}\left(\boldsymbol{m}\right)$ is independent of q. Hence, the result follows directly from the definition of equilibrium. $\square$

[23] Focusing on pure strategies is essentially without loss of generality. There is neither an equilibrium in which R randomizes nor one in which a positive measure of types randomize at the information acquisition stage. Moreover, an equilibrium that prescribes randomization at the disclosure stage exists only for non-generic parameters.

[24] Given the form of U^R and the definition of conditional expectation, this condition implies $a^*(m) = \mathbb{E}_R\left[\omega|m\right]$.

[25] These off-the-equilibrium-path restrictions are not part of the usual definition of perfect Bayesian equilibrium. They reflect the assumptions that S can disclose only when informed and only truthful information. The solution concept is de facto equivalent to that of sequential equilibrium (Kreps and Wilson, 1982), whose formal definition however applies only to finite games.

Thus, we can safely ignore the dependence of $\boldsymbol{m}(q, \boldsymbol{\omega_1})$ from q. Given a disclosure policy $\boldsymbol{m}(\boldsymbol{\omega_1})$ and an action policy $\boldsymbol{a}(\boldsymbol{m})$, the expected value of information for type q is

$$v(q) \equiv \mathbb{E}\big[\boldsymbol{a}(\boldsymbol{m})\big|\, q, i(q) = 1, \boldsymbol{m}(\boldsymbol{\omega_1})\big] - a(\emptyset).$$

In equilibrium, type q acquires information if and only if $v(q) \geq k$.

Lemma 2. *There exists no equilibrium in which some types acquire information and S conceals both good and bad news.*

Proof. If $m^*(\omega_1) = \emptyset \;\forall\; \omega_1 \in \Omega_1$, $v(q) = 0 < k$ and the result follows. □

Lemma 3. *There exists no equilibrium in which some types acquire information and S conceals good news but discloses bad news.*

Proof. See section A.1 in the appendix. □

If S were to disclose only bad news, low types would have stronger incentives to acquire information than high types. After an adverse disclosure R would revise the expectation of both $\tilde{\omega_1}$ and $\tilde{\omega_2}$ downward, so that S would prefer to remain silent.

Lemma 4 (Optimality of good news)**.** *In any equilibrium in which some types acquire information S discloses good news.*

Proof. This is a direct corollary of lemma 2 and 3. □

Lemma 5 (Single-crossing)**.** *If in equilibrium a given type $\breve{q}$ acquires information, all types higher than $\breve{q}$ do so.*

Proof. See section A.2 in the appendix. □

Because good news is worth more than bad news, information is more valuable to high types.

3.2 Cutoff equilibria

By lemma 5, in any equilibrium in which only a subset of types acquire information, $\boldsymbol{i}^*(\boldsymbol{q})$ takes a cutoff form. Accordingly, types below some threshold q^* remain uninformed.

Proposition 1 (Concealing equilibrium)**.** *There exists an equilibrium in which only types $q \geq q_c^*(k) \in (0,1)$ acquire information and S discloses good news but conceals bad news.*

Proof. If such an equilibrium exists, it must be that $a^*(1) = 1 + \mathbb{E}\big[q|q \geq q^*, \omega_1 = 1\big]$ and $a_c^*(\emptyset) = \mathbb{E}\big[\omega|m = \emptyset, \boldsymbol{m_c}\big]$.[26] The value of information for type q^* is then

$$v_c(q^*) = q^*\Big(1 + \mathbb{E}\big[q|q \geq q^*, \omega_1 = 1\big] - \mathbb{E}\big[\omega|m = \emptyset, \boldsymbol{m_c}\big]\Big), \tag{VIc}$$

which is positive and continuous in q^*. As $v_c(0) = 0 < k$ and $v_c(1) = 2(1-\mu) \equiv \bar{k} > k$, by the intermediate value theorem there exists a $q_c^* \in (0,1)$ such that $v_c(q_c^*) = k$. By lemma 5, no type wants to deviate at the information acquisition stage. As for the disclosure stage, $a^*(1) > a_c^*(\emptyset)$ and hence $m^*(1) = 1$ is optimal. To ensure that $m^*(0) = \emptyset$ is optimal, take $f_R^*(q|0)$ to be a degenerate distribution at 0, so that $a_c^*(\emptyset) > a^*(0) = 0$. □

In the concealing equilibrium unraveling fails because the presence of uninformed types prevents R from maximal skepticism after nondisclosure. Regardless of the shape of $f(q)$, there always exist R's beliefs that ensure S prefers to pool with uninformed types when she receives bad news. Yet, S might be willing to disclose provided that R updates appropriately.

Lemma 6 (Optimality of bad news)**.** *Fix $q^* \in (0,1)$ as exogenous. This reduced game has an equilibrium in which S discloses both good and bad news if and only if*

$$2\mathbb{E}\big[q|q < q^*\big] < \mathbb{E}\big[q|q \geq q^*\big] - \frac{\mathbb{V}\big[q|q \geq q^*\big]}{1 - \mathbb{E}\big[q|q \geq q^*\big]}. \tag{OBN}$$

Proof. Suppose that such an equilibrium exists. Then, it must be that $a_r^*(\emptyset) = \mathbb{E}\big[\omega|m = \emptyset, \boldsymbol{m_r}\big] = 2\mathbb{E}\big[q|q < q^*\big]$, $a^*(0) = \mathbb{E}\big[q|q \geq q^*, \omega_1 = 0\big]$ and $a^*(1) = 1 + \mathbb{E}\big[q|q \geq q^*, \omega_1 = 1\big]$. As $a^*(1) > a^*(0)$, the disclosure policy is sequentially rational if and only if $a_r^*(\emptyset) < a^*(0)$, which

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[26]

$$\begin{aligned}\mathbb{E}\big[\omega|m = \emptyset, \boldsymbol{m_c}\big] &\equiv \mathbb{P}\big(q < q^*|m = \emptyset, \boldsymbol{m_c}\big)2\mathbb{E}\big[q|q < q^*\big] + \mathbb{P}\big(q \geq q^*|m = \emptyset, \boldsymbol{m_c}\big)\mathbb{E}\big[q|q \geq q^*, \omega_1 = 0\big]\\ &= \frac{2\int_0^{q^*} qf(q)\,\mathrm{d}q + \int_{q^*}^1 (1-q)qf(q)\,\mathrm{d}q}{1 - \int_{q^*}^1 qf(q)\mathrm{d}q}.\end{aligned} \tag{1}$$

corresponds to condition (OBN).[27] $\square$

The informational content of an adverse disclosure is threefold. First of all, it proves that $\omega_1 = 0$. In addition, it certifies that $q \geq q^*$. Finally, it indicates that, conditional on $q \geq q^*$, S's type is more likely to be low than high. Thus, S elects to disclose bad news when twice the right truncated mean of q at q^* is less than the left truncated mean adjusted by the downward updating that bad news entails.

The size of this adjustment depends in an intuitive way on the shape of $f(q)$. For instance, consider the limit case $q^* = 0$ and rewrite the rhs of (OBN) as

$$\mathbb{E}\left[q|\omega_1 = 0\right] = \mu - \underbrace{\frac{\sigma^2}{\mu}}_{\text{dispersion index}} \times \underbrace{\frac{\mu}{1-\mu}}_{\text{likelihood of good news}} \equiv \underaccent{\tilde}{k}. \tag{2}$$

The greater the dispersion over types and the higher the likelihood of a success relative to a failure, the stronger the downward updating is after bad news. Similarly, the upward updating after good news is

$$\mathbb{E}\left[q|\omega_1 = 1\right] = \mu + \frac{\sigma^2}{\mu}.\text{[28]} \tag{3}$$

When $q^* > 0$, the same interpretation applies, but the relevant moments are the mean and variance of the left-truncated distribution. Inequality (OBN) is a joint condition on $f(q)$ and on q^*, which depends on k in equilibrium.

Proposition 2 (Revealing equilibrium)**.** *For k in some interval $(\underaccent{\tilde}{k}, \hat{k})$ there exists an equilibrium in which only types $q \geq q_r^*(k) \in (0,1)$ acquire information and S discloses both good and bad news.*

Proof. Taking the limit for $q^* \to 0^+$ of the lhs and rhs of condition (OBN) yields $0 < \underaccent{\tilde}{k}$, where

[27] For a derivation of the rhs, use the identity $\mathbb{E}\left[x^2\right] = \left(\mathbb{E}\left[x\right]\right)^2 + \mathbb{V}\left[x\right]$ in

$$\mathbb{E}\left[q|q \geq q^*, \omega_1 = 0\right] = \frac{\int_{q^*}^1 q\,(1-q)\,f\,(q)\,\mathrm{d}q}{\int_{q^*}^1 (1-q)\,f\,(q)\,\mathrm{d}q}.$$

[28] For a derivation of this expression, take $q^* = 0$ in:

$$\mathbb{E}\left[q|q \geq q^*, \omega_1 = 1\right] = \frac{\int_{q^*}^1 q^2 f\,(q)\,\mathrm{d}q}{\int_{q^*}^1 q f\,(q)\,\mathrm{d}q} = \mathbb{E}\left[q|q \geq q^*\right] + \frac{\mathbb{V}\left[q|q \geq q^*\right]}{\mathbb{E}\left[q|q \geq q^*\right]}.$$

$\underline{k}$ is defined in (2). As both sides are continuous in q^*, either the inequality is satisfied for all q^*s or there exists at least a q^* for which the lhs and rhs are equal. In the former case, let $\tilde{q}^*$ equal 1. In the latter, let $\tilde{q}^*$ denote the lowest of such q^*s. In a candidate equilibrium of the type described in the proposition, the value of information for type q^* is

$$v_r(q^*) = q^*\big(1 + \mathbb{E}\big[q|q \geq q^*, \omega_1 = 1\big]\big) + (1 - q^*)\big(\mathbb{E}\big[q|q \geq q^*, \omega_1 = 0\big]\big) \\ - 2\mathbb{E}\big[q|q < q^*\big], \tag{VId}$$

which is positive and continuous in q^*. Besides, $v_r(0) = \underline{k}$. Let us define $\tilde{k} \equiv v_r(\tilde{q}^*)$. As k varies from $\underline{k}$ to $\tilde{k}$, by the intermediate value theorem there exists a $q_r^* \in (0, \tilde{q}^*)$ such that $v_r(q_r^*) = k$. When $q^* = q_r^*$, no type wants to deviate at the information acquisition stage and the disclosure policy is sequentially rational by lemma 6. Take $\underset{\sim}{k} = \underline{k}$ and $\hat{k} = min\{\tilde{k}, \bar{k}\}$ whenever $\tilde{k} > \underline{k}$; otherwise, take $\underset{\sim}{k} = \tilde{k}$ and $\hat{k} = \underline{k}$.[29] $\square$

As the benefit of concealing bad news vanishes when all types are informed, condition (OBN) holds at least for q^* low enough. A moderate k ensures that q^* is sufficiently low and, as a consequence, that a revealing equilibrium exists. At the same time, k must be non-negligible to discourage low enough types from acquiring information.[30] If condition (OBN) is always satisfied, the revealing equilibrium exists for any $k > \underline{k}$, where $\underline{k}$ is defined in equation (2). This is the case when $f(q)$ is uniform, because the rhs of (OBN) is bounded below by q^* and the lhs is exactly q^*, and, more generally, for any pdf which is nonincreasing. This is also the case whenever $\mu < 1/2$ and $f(q)$ satisfies the following regularity condition, essentially stating that incentives to conceal information do not decrease with q^*:[31]

Assumption 1 (Monotonicity)**.** If condition (OBN) is violated for some $\breve{q}^*$, it is also violated for any $q^* > \breve{q}^*$.

[29] For all distributions I considered, $\bar{k} > \tilde{k} > \underline{k}$. Should $\tilde{k} = \underline{k}$, note the interval of existence of the revealing equilibrium would still be non-degenerate, as there is no $f(q)$ for which $v_r(q^*)$ is constant.

[30] As proposition 5 will show, in the parameter region $k \leq \underset{\sim}{k}$ there exists an equilibrium in which all types acquire and disclose information. It can be thought of as a revealing equilibrium in which $q_r^* = 0$.

[31] The property trivially holds when $f(q)$ is non-increasing. When $f(q)$ is increasing, $\mathbb{E}\big[q|q \geq q^*\big] - \mathbb{E}\big[q|q < q^*\big]$ is decreasing in q^* (Harbaugh and Rasmusen, 2013), which suggests that the property again holds, at least provided the second term of the rhs of condition (OBN) is small or does not decrease too fast. In the class of unimodal distributions, few counterexamples obtain by considering distributions with large kurtosis and right skewness.

Corollary 1. *When $f(q)$ is nonincreasing, or when $\mu < 1/2$ and assumption 1 holds, the revealing equilibrium exists for any $k > \underline{k}$.*

Proof. See section A.3 in the appendix. □

Because R's belief about q^* affects S's incentives to acquire information, the information acquisition cutoff of the concealing and the revealing equilibrium might not be uniquely defined.[32] In order to deal with this potential multiplicity, I will make use of the following assumptions:[33]

Assumption 2 (Uniqueness of cutoffs)**.**

$q_c^*(k)$ is unique for any $k < \bar{k}$. (2a)

$q_r^*(k)$ is unique for any $k > \underline{k}$. (2b)

Assumption 2b allows establishing a sort of inverse of corollary 1 for increasing distributions, and more generally, for distributions whose mean exceeds 1/2.

Corollary 2. *Under assumption 2b, when $\mu > \frac{1}{2}$ the revealing equilibrium does not exist for k sufficiently large.*

Proof. See section A.4 in the appendix. □

In the revealing equilibrium, S discloses more states than in the concealing equilibrium. For the extent of voluntary disclosure to be unambiguously higher, also a larger fraction of types must elect to acquire information. The following proposition establishes that this is the case, because incentives to acquire information are stronger when bad news is valuable (lemma 7 in the proof). Indeed, if informed types find it optimal to disclose unfavorable evidence, it must be that types below the cutoff benefit from pooling with those above. Then, the revealing policy decreases the reservation value from remaining uninformed.

Proposition 3 (Cutoffs comparison)**.** *An information acquisition cutoff of the revealing equilibrium obtains as cutoff of the concealing equilibrium for a lower value of k. Provided the latter is unique (assumption 2a), $q_r^*(k) < q_c^*(k)$.*

Proof. See section A.5 in the appendix. □

[32] That is, $v_c(q^*)$ and $v_r(q^*)$, which are defined in equation (VIc) and (VId), might be non-monotone in q^*.

[33] For all distribution I considered, $v_c(q^*)$ is strictly quasi-concave and $v_r(q^*)$ is strictly quasi-convex. Assumptions 2a and 2b follow directly from these two properties. Naturally, when $v_c(q^*)$ and $v_r(q^*)$ are strictly monotone, as it is the case when $f(\cdot)$ is uniform, $q_c^*(k)$ and $q_r^*(k)$ are always unique.

3.3 Pooling equilibria

Proposition 4 (Silent equilibrium)**.** *There exists an equilibrium in which no type acquires information if and only if* $k > 1 - 2\mu$.

Proof. To prove the *if* part, suppose that such an equilibrium exists. Take $f_R^*(q|1)$ and $f_R^*(q|0)$ to be degenerate distributions at $\epsilon > 0$, where ϵ is arbitrarily close to zero. Then, it must be that $a^*(\emptyset) = 2\mu$, $a^*(1) = 1 + \epsilon$ and $a^*(0) = \epsilon$. Because $a^*(0) < a^*(\emptyset)$, $m^*(0) = \emptyset$ necessarily. When $\mu \geq 1/2$, $a^*(1) \leq a^*(\emptyset)$. Therefore, $m^*(1) = \emptyset$ and $v(q) = 0 \; \forall \; q$. When $\mu < 1/2$, instead, $a^*(1) > a^*(\emptyset)$. Therefore, $m^*(1) = 1$ and $v(q) = q(1 - 2\mu)$. Thus, whenever $k > 1 - 2\mu$, type 1 does not want to deviate from $i^*(1) = 0$ (and hence, no type). As disclosure is a zero probability event, we indeed identified an equilibrium. To prove the *only if* part, note that we chose R's beliefs that make information acquisition the least attractive and, nonetheless, high enough types prefer to acquire information whenever $k \leq 1 - 2\mu$. □

The silent equilibrium exists when the prior is favorable enough relative to the cost of information. When $\mu \geq \frac{1}{2}$, it exists even under free information. Unraveling fails because of R's adverse inference on ω_2 after the disclosure of ω_1.

Proposition 5 (Skeptical equilibrium)**.** *There exists an equilibrium in which all types acquire information if and only if* $k \leq \underline{k}$. *In this equilibrium* S *discloses both good and bad news.*

Proof. To prove the *if* part, suppose that the equilibrium described in the proposition exists. Take $f_R^*(q|\emptyset)$ to be a degenerate distribution at 0, so that $a^*(\emptyset) = \mathbb{E}_R\big[\omega|q = 0\big] = 0$. As $a^*(1) \geq 1$ and $a^*(0) = \underline{k} > 0$, S does not want to deviate at the disclosure stage. Besides, $v(0) = a^*(0)$. Thus, when $k \leq \underline{k}$ type 0 does not want to deviate from $i^*(0) = 1$ (and hence, no type). As nondisclosure is a zero probability event, we indeed identified an equilibrium. To prove the *only if* part, note that we chose R's beliefs that make information acquisition the most attractive and, nonetheless, low enough types prefer to remain uninformed whenever $k > \underline{k}$. Also, no other disclosure policy can be part of an equilibrium in which all types acquire information.[34] □

The skeptical equilibrium is the reverse of the silent equilibrium in that it hinges on R's beliefs being the most pessimistic after nondisclosure instead of after disclosure.

[34] If $m^*(0) = \emptyset$, $v(0) = 0$ and if $m^*(1) = \emptyset$, $v(1) = 0$, which contradicts $v(q) \geq k \; \forall \; q$.

4 Selection and welfare

Results on the revealing and the skeptical equilibrium jointly imply that equilibria in which S discloses bad news exist when the cost of information is sufficiently low and the prior is sufficiently unfavorable. Under these circumstances, R meets no disclosure with high skepticism, so that S's benefit from claiming ignorance is small and the reputational value of information dominates.

Proposition 6 (Regions with revelation of bad news)**.** *Under assumption 1 and 2, when $\mu \leq 1/2$ an equilibrium in which S discloses bad news always exists, while when $\mu > 1/2$ it exists if and only if k is sufficiently small.*

Proof. See section A.6 in the appendix. □

As equilibria in which S conceals bad news also exist and this multiplicity is driven by an indeterminacy in R's off-the-equilibrium-path beliefs,[35] a natural question arises on whether R indeed benefits from S's unselective disclosure policy. In what follows, I will restrict my attention to the uniform case and show that this intuitive prediction is correct. In particular, for any cost level, the unique most informative equilibrium prescribes that S discloses bad news.

Proposition 7 (Most informative equilibrium)**.** *When $f(\cdot)$ is uniform, the skeptical equilibrium exists if and only if $k \leq \underline{k}$, the revealing equilibrium exists if and only if $k > \underline{k}$ and each of the two equilibria is most informative in its existence region.*

Proof. See section A.7 in the appendix. □

R favors equilibria in which S discloses bad news not only because S reveals both states but also because S acquires information more often. Thus, disclosure rules that make the revelation of acquired evidence compulsory are beneficial to R or innocuous.[36] At the same time, because incentives to acquire information differ across types and only a subset of the payoff-relevant information is certifiable, the information status of S itself works as a separating device and R is

[35]Because at the disclosure stage S's net payoff from any message does not depend on q, all equilibria satisfy most common refinements for signaling games.

[36]As noted by Matthews and Postlewaite (1985), mandatory disclosure makes the sender's ignorance credible. But in the revealing equilibrium, ignorance is already credible, while in the skeptical equilibrium, no sender aims at being perceived as ignorant. Instead, the concealing equilibrium no longer exists. The sole instance in which this intervention would actually decrease informativeness is if, rather arbitrarily, it caused the silent equilibrium to prevail.

not necessarily better-off when a higher fraction of types acquire information.[37] And indeed, R's maximal payoff obtains in the revealing equilibrium, namely, when a fraction of types remain uninformed.

Corollary 3 (Most informative cost)**.** *R's maximal payoff obtains in the revealing equilibrium for some $\dot{k} > \underline{k}$.*

Proof. See section A.8 in the appendix. □

Thus, laws that make acquisition and disclosure of information compulsory can reduce informativeness.[38]

As for S, due to R's rational expectations, in any equilibrium she obtains the prior mean of ω net of the expected cost of information. Thus, ex-ante, she favors the silent equilibrium, even though such an equilibrium looks rather fragile ex-post.[39] If one interprets the cost of information as a pure waste, the socially optimal k balances S's preferences for cost minimization and R's preferences for informativeness.[40] Alternatively, if k represents a transfer S offers to a third party in exchange for evidence, $\dot{k}$ maximizes not only R's utility but also total welfare. This latter interpretation allows studying the profit-maximizing choice of the information intermediary. If the intermediary can only offer a unique contract consisting of a fixed fee and a disclosure policy,[41] her profits coincide with the expected cost of information of the correspondent equilibrium. She would then choose to enforce the revealing equilibrium.[42] Moreover, due to rent extraction considerations, she would exclude a large fraction of types from certification and hence charge a too high fee from R's point of view.

[37] This is also the reason why for an arbitrary distribution R may in principle prefer the concealing equilibrium to the revealing equilibrium even though $q_r^*(k) < q_c^*(k)$.

[38] In Teoh and Hwang (1991), in which information is exogenous, it is mandatory disclosure that may decrease informativeness, because it destroys equilibria in which different types adopt different disclosure policies.

[39] Were R to update appropriately, high enough types would be willing to acquire information and disclose. Moreover, as R would also benefit, it is unclear why she would hold the very peculiar beliefs that sustain the silent equilibrium in the first place.

[40] Results are then sensitive to the weight the social planner attaches to U^R relative to U^S or, equivalently, to the exact magnitude of R's loss function as measured by $\lambda > 0$ in $U^R = -\lambda(\omega - a)^2$. In the uniform case, one can show that the social optimum obtains in the skeptical equilibrium (with $k^O = 0$) when $\lambda = 1$ and then shifts to the revealing equilibrium with $k^O \in \left(\underline{k}, \dot{k}\right)$ as λ increases.

[41] I am hence precluding the intermediary from offering menus to screen types based on their willingness to pay for certification and their preferences for the disclosure or concealment of bad news.

[42] Given the intermediary's choice of k, the skeptical equilibrium does not exist, while the concealing equilibrium is ruled out by her disclosure policy. Arguably, the silent equilibrium still exists, but if it prevails, the intermediary's choices are simply irrelevant. Multiplicity issues apart, note that identical results obtain if the certifier simply sells evidence to S without fixing the disclosure policy.

Proposition 8 (Presence of an information intermediary). *A profit-maximizing certifier charges $k^* > \dot{k}$ and adopts the revealing policy.*

Proof. See section A.9 in the appendix. □

Even if the certification market is stylized, these results demonstrate that in settings where not all of the payoff-relevant information is certifiable, in contrast to Lizzeri (1999) and Farhi, Lerner and Tirole (2013), the profit-maximizing policy may not be opaque.

5 Extensions

In this section, I demonstrate that the rationale behind adverse voluntary disclosures this paper identifies is robust to some alternative modeling specifications and I discuss some further applications.

5.1 Sequential actions

I have assumed that R takes a single action which reflects her simultaneous beliefs on $\tilde{\omega}_1$ and $\tilde{\omega}_2$. I now consider the alternative game in which R takes two actions a_1 and a_2 sequentially and, regardless of S's disclosure decision, the outcome of $\tilde{\omega}_1$ becomes public before R chooses a_2. The objectives of S and R become $U^S = a_1 + a_2 - k$ and $U^R = -(\omega_1 - a_1)^2 - (\omega_2 - a_2)^2$, respectively, and the timing is the following:

0. nature draws q, ω_1 and ω_2.
1. S observes q and chooses whether to spend k to learn ω_1;
2. S chooses a message m_1; R observes m_1 and chooses an action a_1;
3. ω_1 becomes public and R chooses an action a_2.

As even in the absence of disclosure ω_1 works as a signal of q, the value of information is *weaker* than in the baseline model. Thus, a more stringent condition than $k < \bar{k}$ might be required to ensure the existence of equilibria in which some types acquire information.[43] At the

[43]In a cutoff equilibrium, the value of information for type $q = 1$ as $q^* \to 1$ is now $\bar{k} - \frac{\sigma^2}{\mu}$.

same time, because S cannot permanently conceal evidence, incentives to disclose bad news are *stronger*. Indeed, condition (OBN) becomes

$$\mathbb{E}\big[q|q<q^*\big]+\mathbb{E}\big[q|q<q^*,\omega_1=0\big]<\mathbb{E}\big[q|q\geq q^*,\omega_1=0\big], \tag{4}$$

which is satisfied whenever (OBN) is so, because the rhs is the same and the lhs is lower. Finally, differently from the baseline model, there might exist equilibria in which information acquisition does not take the cutoff form, because S's payoff from remaining uninformed depends on q.[44]

The following proposition mirrors proposition 2 and corollary 1 and shows that a revealing equilibrium still exists. In the uniform case, it implies that the revealing equilibrium exists for any $k\in(\frac{1}{3},\frac{5}{6})$ and, due to a reduction in the value of information, the information acquisition cutoff is higher than in the baseline model.[45]

Proposition 9 (Sequential actions)**.** *For k in some interval, the game with sequential actions admits a revealing equilibrium. If condition (4) is always satisfied for any q^*, which is always the case when $f(\cdot)$ is nonincreasing, the revealing equilibrium exists for any* $k\in\Big(min\big\{\underline{k},\bar{k}-\frac{\sigma^2}{\mu}\big\},max\big\{\underline{k},\bar{k}-\frac{\sigma^2}{\mu}\big\}\Big)$.

Proof. See section A.10 in the appendix. □

This specification allows for more flexible dynamics and, in particular, it can predict a temporary negative reaction to voluntary adverse disclosures followed by an even stronger negative reaction against involuntary ones. These findings, as well as the fact that it is high-quality firms that tend to voluntarily release negative information, are consistent with empirical works on product recalls.[46] To derive even richer predictions, one might take the model a step forward by assuming that $U^S=a_1+\gamma a_2-k$, where $\gamma>0$ captures in reduced form the relative importance

[44] A type q who remains uninformed obtains $a_1(\emptyset)+qa_2(\emptyset,\omega_1=1)+(1-q)a_2(\emptyset,\omega_1=0)$. Thus, lemma 5 might no longer hold. In particular, there might exist equilibria in which the receiver punishes undisclosed bad news and high types dare to remain uninformed, in the same spirit of Teoh and Hwang (1991). However, differently from their framework, the disclosure policy of informed senders remains type independent, i.e. lemma 1 still holds.

[45] The value of information (equation (10) in the proof) is now $v_r(q^*)=\frac{1}{6}\left(8-\frac{4}{2-q^*}+3q^*-\frac{4}{1+q^*}\right)$, which is increasing in q and lower than $v_r(q^*)$ as defined in equation (6).

[46] See Hingorani, Shastri and Shastri (1994); Salin and Hooker (2001); Chen and Nguyen (2013); Eilert and Jayachandran (2014).

of reputation.[47]

5.2 Exhaustive information acquisition

I have imposed that S can acquire verifiable information on $\tilde{\omega}_1$ but not on $\tilde{\omega}_2$. This assumption is natural if the second outcome realizes after the communication stage or represents an attribute outside S's reach.[48] In some settings, S might be able to certify both, for instance when outcomes represent testable characteristics of a good on sale. When after having acquired information on the first attribute S can do the same for the second one, always at a cost k, the revealing equilibrium carries through provided information is sufficiently expensive.

Proposition 10 (Sequential information acquisition)**.** *In the uniform case, when $k > \frac{2}{9}\left(5-\sqrt{7}\right) \cong 0.523$ the revealing equilibrium remains one of the alternative game in which S can acquire information sequentially on both projects.*

Proof. See section A.11 in the appendix. □

The intuition behind this result is that the second piece of evidence contains no signaling value beyond its content and is hence less valuable to S. As long as R is able to distinguish the two attributes when S discloses, the result does not depend on S being forced to acquire evidence in an established order, i.e. first on $\tilde{\omega}_1$ and then on $\tilde{\omega}_2$.[49] Moreover, the region of existence of the revealing equilibrium would simply enlarge if information acquisition was not sequential, i.e. if S had to choose whether to learn ω_2 without first observing ω_1.[50]

5.3 Probabilistic information acquisition

I have maintained that S always obtains evidence whenever she invests. I now consider the possibility that information acquisition is unsuccessful. More precisely, when S pays k, she

[47] For instance, Eilert and Jayachandran (2014) also find that firms use voluntary recalls more often for brands that are important to their revenues than for brands that are more diversified.

[48] For instance, $\tilde{\omega}_2$ may represent a taste parameter of consumers that becomes known only with experience.

[49] Naturally, when S is free to choose, the game might have some new equilibria and surely symmetric ones in which S acquires evidence only on $\tilde{\omega}_2$. See Fishman and Hagerty (1990) for a setting in which attributes are instead indistinguishable to R.

[50] As the proof shows, the new relevant incentive constraint is that of types who initially obtained bad news. If ω_1 was unknown, a smaller k would suffice to deter further information acquisition.

discovers the outcome of $\tilde{\omega}_1$ only with probability $p \in (0,1)$.[51] For simplicity, I focus on a binary type space and assume that S is of type $h = q_h \in (0,1)$ with probability $\alpha \in (0,1)$ and of type $l = q_l < q_h$ with complementary probability. When the value of reputation is sufficiently high ($q_h > 2q_l$) and information acquisition is sufficiently effective ($p > \hat{p}$), the revealing equilibrium exists for an intermediate cost range, as before. S finds it optimal to disclose unfavorable evidence when the likelihood of obtaining information is high and adverse selection is severe ($\hat{p}$ is increasing in α) because R's skepticism in the absence of news is then strong.

Proposition 11 (Probabilistic information acquisition)**.** *When $q_h > 2q_l$, $p > \hat{p}$ and $k \in (k_1, k_2]$, there exists an equilibrium in which only the high-quality type invests in information and S discloses both good and bad news. $\hat{p}$ is increasing in α.*

Proof. See section A.12 in the appendix. □

6 Conclusion

In the model, a sender who elects to disclose unselectively de facto relinquishes control over the flow of information that becomes public. Thus, its insights may extend to other information revelation practices, such as those of brands that let consumers post reviews on their online store without any censoring.[52] Even when the cost of setting up such a review system is negligible, signaling might work through other channels. For example, suppose that the presence of naive consumers prevents maximal skepticism in the absence of information. Then, setting up the review system is more costly for low-quality brands, because poor reviews dissuade naive consumers from buying.

Besides, strategic considerations will evolve over the course of repeated interactions. For instance, think of the sender as a seller who in each period brings a new experience good to the market. A seller with a higher propensity to deliver good products might no longer have stronger incentives to test once she has proven that she is trustworthy. Moreover, disclosure entails an additional strategic dimension due to its impact on future costs of information acquisition.

[51] An older version of the paper covers the case in which the probability of obtaining evidence increases with S's investment.

[52] See for instance www.philips-shop.fr/store.

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A Appendix

A.1 Proof of lemma 3

Suppose that such an equilibrium exists and let $\breve{Q} \subseteq Q$ be the set of types for which $i^*(q) = 1$. Then, it must be that $a^*(1) \leq a^*(\emptyset) < a^*(0)$.[53] Therefore

$$v(q) = (1 - q)\left(a^*(0) - a^*(\emptyset)\right).$$

As $v(q)$ is continuous and strictly decreasing in q, with $v(1) = 0$ and $v(q) \geq k \; \forall \; q$ in $\breve{Q}$, it must be that $\breve{Q} = [0, \breve{q}]$, where $\breve{q} < 1$ solves $v(q) = k$. Then, $a^*(\emptyset)$ is a convex combination of $2\mathbb{E}\left[q|q > \breve{q}\right]$ and $(1 + \mathbb{E}\left[q|q \leq \breve{q}, \omega_1 = 1\right])$. As $a^*(0) = \mathbb{E}\left[q|q \leq \breve{q}, \omega_1 = 0)\right] \leq \mathbb{E}\big[q|q \leq \breve{q}\big]$, $a^*(\emptyset) > a^*(0)$, yielding a contradiction.

[53] If the second inequality were not strict, there would be no type for which $i^*(q) = 1$ is optimal.

A.2 Proof of lemma 5

Consider a candidate equilibrium in which type $\breve{q}$ acquires information and let $\breve{Q} \subseteq Q$ be the set of types for which $i^*(q) = 1$. Lemma 4 implies that $m^*(1) = 1$, which in turn implies that $a^*(1) > a^*(\emptyset)$. Suppose first that $m^*(0) = \emptyset$. The value of information for type q is then

$$v(q) = q\big(a^*(1) - a^*(\emptyset)\big),$$

which is increasing in q. Suppose that $m^*(0) = 0$, instead. The value of information for type q is then

$$v(q) = qa^*(1) + (1-q)a^*(0) - a^*(\emptyset),$$

where $a^*(1) = 1 + \mathbb{E}\big[q|q \in \breve{Q}, \omega_1 = 1\big]$ and $a^*(0) = \mathbb{E}\big[q|q \in \breve{Q}, \omega_1 = 0\big]$. Because $\mathbb{E}\big[q|q \in \breve{Q}, \omega_1 = 0\big] \leq 1$, $a^*(1) > a^*(0)$. Thus, $v(q)$ is again increasing. As $v(\breve{q}) \geq k$ by assumption, $v(q) > k$ $\forall$ $q > \breve{q}$.

A.3 Proof of corollary 1

This corresponds to case $\tilde{q}^* = 1$ in the proof of proposition 2, after noting that:

- if $f'(q) \leq 0$, $\mathbb{E}\big[q|q < q^*\big] \leq q^*/2$, so that condition (OBN) is satisfied $\forall\, q^*$;
- as $q^* \to 1^-$, the lhs and rhs of condition (OBN) converge to 2μ and 1, respectively. Therefore, when $\mu < 1/2$ assumption 1 implies that condition (OBN) is satisfied $\forall\, q^*$.

When $\tilde{q}^* = 1$, $\tilde{k} = v_r(1) = \bar{k}$ and the revealing equilibrium exists for all $k \in (\underline{k}, \bar{k})$.

A.4 Proof of corollary 2

The lhs and rhs of condition (OBN) are continuous in q^*. Taking the limit for $q^* \to 0^+$ and $q^* \to 1^-$ on both sides yields $0 < \underline{k}$ and $2\mu > 1$, respectively. Thus, there exists a $\hat{q}^* \in (0,1)$ such that condition (OBN) is not satisfied for $q^* \geq \hat{q}^*$. As $v_r(q^*)$ is continuous with $v_r(0) = \underline{k}$ and $v_r(1) = \bar{k}$, assumption 2b implies that $v_r(\hat{q}^*) < \bar{k}$. Besides, it implies that for any $k \in \big(max\,\{\underline{k}, v_r(\hat{q}^*)\}\,, \bar{k}\big)$, the (unique) solution to $v_r(q^*) = k$ obtains for $q^* > \hat{q}^*$. But as condition (OBN) is violated, a revealing equilibrium does not exist in this interval.

A.5 Proof of proposition 3

To prove the result, I will make use of the following lemma.

Lemma 7 (Value of information and bad news)**.** *For any $q^* \in (0,1)$, $v_r(q^*) > v_c(q^*)$ if and only if condition (OBN) is satisfied.*

Proof. When $q^* \in (0,1)$, $a_c^*(\emptyset)$ is a linear combination with strictly positive coefficients of $a_r^*(\emptyset)$ and $a^*(0)$, which are the lhs and rhs of condition (OBN), respectively. It follows that $a_r^*(\emptyset) < a_c^*(\emptyset) < a^*(0)$ whenever $a_r^*(\emptyset) < a^*(0)$ and opposite inequalities hold whenever $a_r^*(\emptyset) \geq a^*(0)$. Subtracting equation (VIc) from (VId) and rearranging yields

$$v_r(q^*) - v_c(q^*) \equiv (1-q^*)\big(a^*(0) - a_c^*(\emptyset)\big) + \big(a_c^*(\emptyset) - a_r^*(\emptyset)\big),$$

and hence the result. □

By definition of equilibrium, $v_c(q_c^*) = v_r(q_r^*) = k$ and q_r^* satisfies condition (OBN). As $v_c(q^*)$ is continuous in q^* with $v_c(0) = 0$ and $v_c(1) = \bar{k}$, assumption 2a implies that $v_c(q^*) > k \;\forall\; q^* > q_c^*$. Moreover, by lemma 7, $v_r(q_r^*) > v_c(q_r^*)$. Hence, $q_r^* < q_c^*$, as $q_r^* \geq q_c^*$ would imply the contradiction $k = v_r(q_r^*) > v_c(q_r^*) > k$.

A.6 Proof of proposition 6

The statement for the $\mu \leq 1/2$ case simply combines corollary 1 and proposition 5. The statement for the $\mu > 1/2$ case follows from propositions 2 and 5 and corollary 2 after noting that:

- because either $\underline{k} = \underset{\sim}{k}$ or $\underline{k} = \hat{k}$, $(\underset{\sim}{k}, \hat{k}) \cup (0, \underline{k}] = (0, \tilde{k})$, where $\tilde{k} = max\left\{\underline{k}, \hat{k}\right\}$; by corollary 2, $\tilde{k} < \bar{k}$;
- assumptions 1 and 2b imply that the revealing equilibrium does not exist for $k \geq \tilde{k}$. Indeed, because in this range $q_r^*(k)$ is unique and increasing, $q_r^*(k) > q_r^*(\tilde{k})$ and, by assumption 1, condition (OBN) is violated.

A.7 Proof of proposition 7

When $f(\cdot)$ is uniform, $\mu = 1/2$, $\sigma^2 = 1/12$, $\underline{k} = 1/3$ and $\bar{k} = 1$. Besides, in a cutoff equilibrium, $a(1) = 1 + \frac{2\left(1+q^*+(q^*)^2\right)}{3(1+q^*)}$, $a(0) = \frac{1}{3}(1+2q^*)$, $a_c(\emptyset) = \frac{1+3(q^*)^2+2(q^*)^3}{3+3(q^*)^2}$ and $a_r(\emptyset) = q^*$. Thus, equations (VIc) and (VId) simplify to

$$v_c(q^*) = \frac{4q^*\left(1+q^*+(q^*)^2\right)}{3\left(1+q^*+(q^*)^2+(q^*)^3\right)}, \tag{5}$$

$$v_r(q^*) = \frac{1+4q^*+(q^*)^2}{3+3q^*}. \tag{6}$$

Because $v_c(q^*)$ and $v_r(q^*)$ are monotone,[54] the results of section 3.2 imply that $q_c^*(k)$ and $q_r^*(k)$ are unique, with $q_r^*(k) < q_c^*(k)$, and that the revealing equilibrium exists if and only if $k > \underline{k}$. Results of section 3.3 imply that the skeptical equilibrium exists if and only if $k \leq \underline{k}$ while the silent equilibrium always exists.

The silent equilibrium is clearly the least informative, with[55]

$$\mathbb{E}\left[U_{si}^R\right] = -\int_0^1 \left(q^2(2-1)^2 + (1-q)^2(0-1)^2\right)\, dq = -2/3.$$

In the skeptical equilibrium, instead,[56]

$$\mathbb{E}\left[U_{sk}^R\right] = -\int_0^1 \left(q\left(q(2-5/3)^2+(1-q)(1-5/3)^2\right)+(1-q)\left(q(1-1/3)^2+(1-q)(0-1/3)^2\right)\right) dq = -2/9.$$

In a concealing equilibrium with cutoff q^*

$$\begin{aligned}\mathbb{E}\left[U_c^R\right] = &-\int_0^{q^*} \left(q^2(2-a_c^*(\emptyset))^2 + 2q(1-q)(1-a_c^*(\emptyset))^2 + (1-q)^2(0-a_c^*(\emptyset))^2\right) dq \\ &-\int_{q_*}^1 \left(q^2(2-a^*(1))^2 + q(1-q)(1-a^*(1))^2 + (1-q)q(1-a_c^*(\emptyset))^2 + (1-q)^2(0-a_c^*(\emptyset))^2\right) dq \\ = &-\frac{2\left(1+q^*+(q^*)^2+5(q^*)^3+2(q^*)^4+2(q^*)^5\right)}{9(1+q^*)\left(1+(q^*)^2\right)}.\end{aligned} \tag{7}$$

$\mathbb{E}\left[U_c^R\right]$ is decreasing in q^*,[57] with $\mathbb{E}\left[U_c^R | q^*=0\right] = \mathbb{E}\left[U_{sk}^R\right]$ and $\mathbb{E}\left[U_c^R | q^*=1\right] = \mathbb{E}\left[U_{si}^R\right]$. As

[54] Indeed, $v_c'(q^*) = \frac{4\left(1+2q^*+3(q^*)^2\right)}{3(1+q^*+(q^*)^2+(q^*)^3)^2} > 0$ and $v_r'(q^*) = \frac{3+2q^*+(q^*)^2}{3(1+(q^*))^2} > 0$.

[55] As no updating occurs, $a^*(\emptyset) = 2\mu = 1$.

[56] Equation (2) and (3) yield $a^*(0) = \underline{k} = 1/3$ and $a^*(1) = 5/3$, respectively.

[57] $\frac{d\mathbb{E}\left[U_c^R\right]}{dq^*} = -\frac{8(q^*)^2\left(1+q^*+(q^*)^2\right)\left(3+q^*+(q^*)^2+(q^*)^3\right)}{9(1+q^*+(q^*)^2+(q^*)^3)^2} < 0$.

in the revealing equilibrium $q_r^*(k) < q_c^*(k)$ and informed types reveal both good and bad news, $\mathbb{E}\left[U_c^R\right] < \mathbb{E}\left[U_r^R\right]$ necessarily, where $\mathbb{E}\left[U_r^R\right]$ represents R's expected utility in the revealing equilibrium.[58] Because the skeptical and the revealing equilibrium do not exist simultaneously, it follows that R favors the skeptical equilibrium for $k \leq \underline{k}$ and the revealing equilibrium for $k > \underline{k}$.

A.8 Proof of corollary 3

In a revealing equilibrium with cutoff q^*

$$
\begin{aligned}
\mathbb{E}\left[U_r^R\right] = &- \int_0^{q^*} \left(q^2(2-a_r^*(\emptyset))^2 + 2q(1-q)(1-a_r^*(\emptyset))^2 + (1-q)^2(0-a_r^*(\emptyset))^2\right) dq \\
&- \int_{q_*}^{1} \left(q^2(2-a^*(1))^2 + q(1-q)(1-a^*(1))^2 + (1-q)q(1-a^*(0))^2 + (1-q)^2(0-a^*(0))^2\right) dq \\
= &- \frac{2+q^*+3(q^*)^2+7(q^*)^3-(q^*)^4}{9(1+q^*)}. \qquad (8)
\end{aligned}
$$

As $\mathbb{E}\left[U_r^R\right]$ is concave in q^*,[59] with $\mathbb{E}\left[U_r^R | q^*=0\right] = \mathbb{E}\left[U_{sk}^R\right]$, $\mathbb{E}\left[U_r^R | q^*=1\right] = \mathbb{E}\left[U_{si}^R\right]$ and $\frac{dU_r^R}{dq^*}\Big|_{q^*=0} = \frac{1}{9}$,[60] it has a unique interior maximizer, which I denote by $\dot{q}_r^*$. Because $q_r^*(k)$ is unique and increasing in k with $q_r^*(\underline{k}) = 0$ and $q_r^*(\bar{k}) = 1$, there is a unique $\dot{k} > \underline{k}$ such that $\dot{q}_r^*$ obtains as equilibrium cutoff. This concludes the proof. For further reference, a numerical solution to $\frac{dU_r^R}{dq^*} = 0$ yields $\dot{q}_r^* \cong 0.11$. Solving $v_r(q^*) = k$, where $v_r(q^*)$ is defined in equation (6), yields the following closed form solution for the cutoff of the revealing equilibrium

$$
q_r^*(k) = \frac{1}{2}\sqrt{3}\sqrt{4-4k+3k^2} - \frac{1}{2}(4-3k). \qquad (9)
$$

A numerical solution to $q_r^*(k) = \dot{q}_r^*$ yields $k(\dot{q}_r^*) \equiv \dot{k} \cong 0.438$.

A.9 Proof of proposition 8

Denote by $\Pi(k)$ the expected cost of information in a given equilibrium. In the silent equilibrium $\Pi_{si}(k) = 0$, in the skeptical equilibrium $\Pi_{sk}(k) = k$, in the concealing equilib-

[58] Clearly, $\mathbb{E}\left[U_c^R\right] < \mathbb{E}\left[U_r^R\right]$ at q^* given. Because $q_r^*(k) < q_c^*(k)$ and $\mathbb{E}\left[U_c^R\right]$ is decreasing in q^*, the inequality is even more stringent at equilibrium cutoffs.

[59] $\frac{d^2U_r^R}{d^2q^*} = \frac{2}{9}\left(3q^* + \frac{4}{(1+q^*)^3} - 8\right) < 0$.

[60] $\frac{dU_r^R}{dq^*} = \frac{1}{9}\left(5 - \frac{4}{(1+q^*)^2} + q^*(3q^*-16)\right)$.

rium $\Pi_c(k) = (1 - q_c^*(k))k$ and in the revealing equilibrium $\Pi_r(k) = (1 - q_r^*(k))k$. Because $q_r^*(k) < q_c^*(k)$, $\Pi_c(k) \leq \Pi_r(k)$. Besides, as $\Pi_r(\underline{k}) = \underline{k}$[61] and the skeptical equilibrium does not exists for $k > \underline{k}$, $max\Pi_{sk}(k) \leq max\Pi_r(k)$. Maximizing $\Pi(k)$ hence boils down to maximize $\Pi_r(k)$. Replacing for $q_r^*(k)$ from equation (9), $\Pi_r(k)$ becomes

$$\Pi_r(k) = \left(1 - \left(\frac{1}{2}\sqrt{3}\sqrt{4 - 4k + 3k^2} - \frac{1}{2}(4 - 3k)\right)\right)k,$$

which is concave in k[62] with a stationary point in $k^* = \frac{1}{3}\left(2 + 2^{1/3} - 2^{2/3}\right) \cong 0.557$. Note that $k^* > \dot{k} \cong 0.438$.

A.10 Proof of proposition 9

In this candidate equilibrium with cutoff q^*, the value of information for type q is

$$\begin{aligned} v_r(q) = {} & q\big(1 + \mathbb{E}\big[q|q \geq q^*, \omega_1 = 1\big]\big) + (1-q)\big(\mathbb{E}\big[q|q \geq q^*, \omega_1 = 0\big]\big) \\ & - \mathbb{E}\big[q|q < q^*\big] - q\mathbb{E}\big[q|q < q^*, \omega_1 = 1\big] - (1-q)\mathbb{E}\big[q|q < q^*, \omega_1 = 0\big], \end{aligned} \tag{10}$$

which is increasing in q.[63] In particular, $v(q^*)$ is positive and continuous in q^*, with $v(0) = \underline{k}$ and $v(1) = \bar{k} - \frac{\sigma^2}{\mu}$. The proof then follows exactly the same steps as those of proposition 2 and corollary 1. The sole difference is that it is now a priori unclear whether $v(0) > v(1)$ or vice versa.

A.11 Proof of proposition 10

Assume that the revealing equilibrium carries through and denote by $a(m_1, m_2)$ R's action when S's messages about $\tilde{\omega}_1$ and $\tilde{\omega}_2$ are m_1 and m_2, respectively. Then, it must be that $a^*(m_1, \emptyset) = a^*(m_1)$, where $a^*(m_1)$ represents R's action policy in the revealing equilibrium of the baseline model. Besides, $a^*(\omega_1, \omega_2) = \omega_1 + \omega_2$. All one needs to check is that no informed type

[61] Indeed, $q_r^*(\underline{k}) = 0$.

[62] $\Pi_r^{'}(k) = \frac{1}{2}\left(6 - \frac{2\sqrt{3}\left(2+k\left(-3+3k+\sqrt{3}\sqrt{4+k(3k-4)}\right)\right)}{\sqrt{4+k(-4+3k)}}\right)$ and $\Pi_r^{''}(k) = -3 + \frac{\sqrt{3}(8-3k(8-3(2-k)k))}{(4-k(4-3k))^{3/2}} < 0$.

[63]

$$v_r'(q) = \big(1 + \mathbb{E}\big[q|q \geq q^*, \omega_1 = 1\big]\big) - \mathbb{E}\big[q|q \geq q^*, \omega_1 = 0\big] - \mathbb{E}\big[q|q < q^*, \omega_1 = 1\big] + \mathbb{E}\big[q|q < q^*, \omega_1 = 0\big] > 0.$$

profits from acquiring further evidence.[64] The best disclosure policy for a type who deviates is $m_2^*(1) = 1$ and $m_2^*(0) = \emptyset$,[65] while we can assume without loss of generality that $m_1^*(\omega_1) = \omega_1$.[66] A type q who obtains good news about $\tilde{\omega}_1$ does not wish to deviate if:

$$q(2 - (1 + \mathbb{E}\big[q|q \geq q_r^*, \omega_1 = 1\big])) < k, \tag{11}$$

while a type who obtains bad news does not wish to deviate if:

$$q(1 - \mathbb{E}\big[q|q \geq q_r^*, \omega_1 = 0\big]) < k. \tag{12}$$

As the lhs of (11) and (12) are increasing in q and $\mathbb{E}\big[q|q \geq q_r^*, \omega_1 = 0\big] < \mathbb{E}\big[q|q \geq q_r^*, \omega_1 = 1\big]$, no type elects to deviate when condition (12) holds for type 1. The condition simplifies to $k > \frac{2}{9}\left(5 - \sqrt{7}\right)$.[67]

A.12 Proof of proposition 11

Considerate a candidate equilibrium in which $i^*(h) = 1$, $i^*(l) = 0$, $m^*(1) = 1$ and $m^*(0) = 0$. Then, by Bayes' rule, it must be that $a^*(1) = 1 + q_h$, $a^*(0) = q_h$ and

$$a^*(\emptyset) = \frac{(1-p)\alpha 2q_h + (1-\alpha)2q_l}{1 - p\alpha}.$$

At the disclosure stage, $m^*(0) = 0$ is sequentially rational if and only if $a^*(0) > a^*(\emptyset)$, which simplifies to:

$$p > \hat{p} \equiv \frac{2(1-\alpha)q_l - (1-2\alpha)q_h}{\alpha q_h}.$$

[64] Even if S was free to begin her search on either attribute, in the equilibrium the proof constructs she would rather acquire information only on $\tilde{\omega}_1$ than only on $\tilde{\omega}_2$.

[65] Indeed, $a^*(1,1) > a^*(\omega_1, \emptyset) > a^*(\omega_1, 0)$.

[66] Indeed, due to an unraveling argument, $a^*(\emptyset, \omega_2) = a^*(0, \omega_2)$.

[67] Using $\mathbb{E}\big[q|q \geq q_r^*, \omega_1 = 0\big] = \frac{1}{3}(1 + 2q^*)$ and equation (9), condition (12) becomes:

$$2 - k - \sqrt{\frac{4}{3} - \frac{4k}{3} + k^2} < k.$$

Note that $\hat{p} < 1$ if and only if $q_h > 2q_l$. Besides, $\hat{p}$ is increasing in α.[68] As for the information acquisition stage, type h does not want to deviate if and only if

$$U^h_{i=1} \equiv p\left(q_h\, a^*(1) + (1-q_h)\, a^*(0)\right) + (1-p)\, a^*(\emptyset) - k \geq U^h_{i=0} = a^*(\emptyset),$$

which simplifies to

$$k \leq k_2 \equiv \frac{2p(1-\alpha)\,(q_h - q_l)}{1-p\alpha}.$$

Similarly, type l does not want to deviate if and only if

$$U^l_{i=0} = a^*(\emptyset) > U^l_{i=1} \equiv p\left(q_l\, a^*(1) + (1-q_l)\, a^*(0)\right) + (1-p)\, a^*(\emptyset) - k,$$

which simplifies to

$$k > k_1 \equiv \frac{p(1+(p-2)\alpha)\,(q_h - q_l)}{1-p\alpha}.$$

Finally, note that as $U^h_{i=1} > U^l_{i=1}$, $k_2 > k_1$, while $k_1 > 0$ whenever $a^*(0) > a^*(\emptyset)$.

68 $\frac{d\hat{p}}{d\alpha} = \frac{1-\frac{2q_l}{q_h}}{\alpha^2}$, which is positive whenever $q_h > 2q_l$. In particular, whenever $\alpha \leq \frac{q_h - 2q_l}{2q_h - 2q_l}$, $a^*(0) > a^*(\emptyset)$ is satisfied for any $p \in (0,1)$.