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The Effects of Management and Provision Accounts on Hedge Fund Returns –

Part II : The Loss Carry Forward Scheme

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Abstract

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In addition to active portfolio management, hedge funds are characterized by the allocation of portfolio performance between the external investors and the management firm accounts. This allocation can take different forms, such as the Loss Carry Forward scheme, and some of them can be coupled with performance smoothing techniques. This paper shows that this additional smoothing component might explain some empirical facts observed on the distribution and the dynamics of hedge fund returns.

Keywords: Hedge Fund, Sharpe Performance, Manager Incentive, Loss Carry Forward, Performance Smoothing.

1 Introduction

In addition to an active portfolio management[1], hedge funds (HF) are characterized by the allocation of portfolio performance between the external investors and the management firm accounts. There exist almost as many account allocation schemes as hedge funds shares. This explains why any precautionary investor, regulator, or researcher should study in details the prospectus of the funds, and in particular the fee structure. This paper completes the discussion of the effect of the High Water Mark (HWM) allocation scheme in Darolles, Gourieroux (2013). The HWM scheme basically describes the allocation between the account invested by external clients, called class A units, and the account invested by the management firm, called class B units. The Loss Carry Forward Scheme introduced in this paper can in addition include a provision account used to smooth the performance of the class A account.

We describe the Loss Carry Forward (LCF) allocation scheme in Section 2 and the dynamics of the allocation between the A, B accounts and the reserve account C. We also characterizes the returns of the different accounts for a given trajectory of the total portfolio return. This additional smoothing component might increase the impact of the fee structure on the hedge fund return characteristics. Section 3 compares the portfolio and fund returns for the LCF allocation schemes, when the portfolio returns are independent and identically Gaussian distributed. The i.i.d. Gaussian assumption on portfolio returns corresponds to a rather exogenous portfolio management. This assumption allows us to focus on the way the hedge fund manager will account for the existence of multiple accounts in his/her management strategy. We emphasize the special role of the provision account in this scheme. Section 4 contains conclusions. Proofs are gathered in Appendices.

2 The Loss Carry Forward Scheme

We first introduced the basic LCF scheme without provision account. We then consider a more complex scheme including a provision account.

[1] The active management includes the possibility for the hedge fund manager to invest in illiquid assets, in derivatives, in junk assets, and last but not least to borrow in such assets to increase their leverage.

2.1 The basic scheme

The basic Loss Carry Forward (LCF) allocation scheme is parametrized by a performance fee rate α, a hurdle rate $y_{h,t}$, and a reset time T. The difference with the HWM scheme described in Darolles, Gourieroux (2013) is the definition of the predeterminated path dependent scheme.

(a) Allocation between A and B accounts

Let us first consider two accounts, with respective values A_t, B_t at date t, $t = 0, ..., T$. The A account is invested by external clients while the B account is invested by the management firm. The contractual hurdle rate is denoted by $y_{h,t}, y_{h,t} \geq 0$, and is assumed to be predetermined and observable at date t [see Darolles, Gourieroux (2013)]. The global portfolio value $A_t + B_t$ is invested and provides at the end of the period a return y_{t+1}. The change in portfolio value $(A_t + B_t)y_{t+1}$ can be positive, or negative. The possibility of negative return has to be considered seriously for HF, especially when they use a high leverage ratio, i.e. borrow a lot on financial markets. This change in total portfolio value has to be allocated between the two accounts. As in the HWM framework [see Darolles, Gourieroux (2013)], the performance fee is not charged if the fund is globally in a deficit of performance, called loss carry forward[2] (LCF). This measure of deficit is recursively defined by $LCF_0 = 0$ and:

$$\begin{aligned} LCF_t &= -\left[LCF_{t-1} + A_{t-1}(y_t - y_{h,t-1})\right]^{-}, & (2.1) \\ &= -A_{t-1}\left[y_t - (A_{t-1}y_{h,t-1} - LCF_{t-1})/A_{t-1}\right]^{-}, & (2.2) \end{aligned}$$

where $X^- = \max(-X, 0)$. The LCF is always nonpositive and corresponds to the cumulated negative performance. The hurdle rate $y_{h,t-1}$ is fixing an objective for the portfolio return. If this objective is not reached, that is if $y_t < y_{h,t-1}$, this is considered as a loss and the measure of deficit increases. The LCF_t becomes negative if y_t is not large enough to cover (potential) previous losses.

Then, the allocation depends on LCF and is driven by the following updating equations:

$$\left\{ \begin{array}{l} A_{t+1} = A_t(1 + y_{t+1}) - \alpha A_t\left[y_{t+1} - (A_t y_{h,t} - LCF_t)/A_t\right]^{+}, \\ B_{t+1} = B_t(1 + y_{t+1}) + \alpha A_t\left[y_{t+1} - (A_t y_{h,t} - LCF_t)/A_t\right]^{+}, \end{array} \right. \qquad (2.3)$$

[2]The loss carry forward is an accounting technique that applies the current year's losses to future years gains in order to reduce tax liability.

where α, $\alpha > 0$, is the performance rate. Thus the management firm (B account) receives a bonus, if the portfolio return is sufficiently large, i.e. if $y_{t+1} > A_t y_{h,t} - LCF_t$, receive nothing otherwise.

The fee rate α, $\alpha = 20\%$, say, is often presented at a first place when promoting a fund, whereas the complicated formulas (2.2), (2.3) can only be revealed by the careful reading of the prospectus. Therefore a naive investor may have the impression that the management firm receives at date $t+1$ the quantity $(A_t + B_t)y_{t+1}(1+\alpha)$. This is clearly not the case. The payment to the management firm includes some incentives to get extreme positive performance in order to increase the bonus and to optimize the reduction of tax liability. As important as the fee rate is of course the choice of the hurdle rate and its dynamics.

At short term horizon equal to 1, the future account values involve the payoff of a European call written on y_{t+1}, with predetermined path dependent strike equal to $y_{0,t} = (A_t y_{h,t} - LCF_t)/A_t$.

The recursive equations (2.3) are valid on period $\{0, T-1\}$. At reset time T, the management account is reset to the contractual initial value B_0 and the LCF reset to zero.

If the reset time is $T = 1$, the LCF is always set to zero, $y_{0,t} = y_{h,t}$, and the recursive equation (2.3) can be simplified and becomes:

$$\left\{ \begin{array}{l} A_{t+1} = A_t(1+y_{t+1}) - \alpha A_t \left[y_{t+1} - y_{h,t}\right]^+, \\ B_{t+1} = B_t(1+y_{t+1}) + \alpha A_t \left[y_{t+1} - y_{h,t}\right]^+. \end{array} \right. \tag{2.4}$$

that corresponds to the HWM scheme [see Darolles, Gourieroux (2013)]. Therefore, the HWM and LCF schemes are equivalent for a unitary reset time. The dependence of the change of account value $\Delta A_{t+1} = A_{t+1} - A_t$ (resp. $\Delta B_{t+1} = B_{t+1} - B_t$) with respect to net portfolio return y_{t+1} is described in Figure 1 (resp. Figure 2).

[Insert Figure 1: ΔA_{t+1} as a function of y_{t+1} (unitary reset time)]

[Insert Figure 2: ΔB_{t+1} as a function of y_{t+1} (unitary reset time)]

When $T = 1$, the value of the class A unit is a continuous increasing function of the net portfolio return with a change of slope at threshold $y_{0,t}$. The payoff on B account is a convex function of the return. This convexity property shows the incentive mechanism.

(b) The case of a zero hurdle rate

Larger the hurdle rate, greater is the incentive for the fund manager to take risk and to increase the leverage in order to get a high bonus. In the HF industry the hurdle rate is generally positive and indexed on some basic rate such as the LIBOR. However, a significant number of HF set the hurdle rate to zero, that is do not adjust for a riskfree rate. We consider this special LCF scheme in this section to better highlight the link with the HWM framework.

Proposition 1: The HWM and LCF schemes are identical for zero hurdle rate, with $LCF_t = A_t - HWM_t$.

Proof: see Appendix 1.

For a nonzero hurdle rate, the HWM and LCF approaches differ by their discounting scheme and the dependence of ΔA_{t+1} (resp. ΔB_{t+1}) with respect to net portfolio return y_{t+1} is more complex.

Proposition 2: For zero hurdle rate, there exists a one-to-one relationship between the trajectories of the portfolio return y_t and the return of the investors' account $y_{A,t}$. More precisely, we can deduce the underlying portfolio return as a deterministic function:

$$y_t = g(y_{A,t}, y_{A,t-1}, ..., y_{A,1}, A_0), \; say.$$

Proof: By the transformation in Figure 1, we have:

$$A_t \;\; = \;\; A_{t-1} + b(y_t, y_{0,t-1}),$$

and by recursive substitution:

$$A_t \;\; = \;\; b^*(y_t, A_{t-1}, A_{t-2}, ..., A_0), \; say,$$

where b^* is one-to-one in the first argument y_t. Thus, by introducing return $y_{A,t}$, we deduce the formula of Proposition 2.

QED

The return $y_{A,t}$ on the investors' account is regularly reported by the HF manager and use to promote the fund. They do not report the underlying portfolio return y_t in order not to reveal clearly their portfolio management, but also the actual level of fees. As a consequence, the academic literature is often using the return $y_{A,t}$ as a proxy of y_t, that is neglects the effect of the management fee. Proposition 2 shows that we are able to derive the underlying portfolio return from the return of account A by simply inverting the filter, which defines the accounts allocation. Even if the data on portfolio return are not made directly observable by the fund manager, we can recursively reconstruct them. Of course the relation between y_t and $y_{A,t}$ is not static, and no deterministic link of the type $y_t = g^*(y_{A,t})$, say, will be detected by a joint plot of $(y_t, y_{A,t})$. When the hurdle rate is nonzero, we still have a one-to-one relationship conditional on the knowledge of the hurdle rate history, that is:

$$y_t = g(y_{A,t}, y_{A,t-1}, ..., y_{A,1}, y_{h,t-1}, y_{h,t-2}, ..., A_0),\ say.$$

2.2 An allocation scheme with provision account

More sophisticated allocation scheme can include a third account, called provision account[3]. This scheme involves additional allocation parameters characterizing the allocation between the external investors' A account and the provision C account.

(a) Allocation between A, B and C accounts

Let us now consider three accounts, with respective values A_t, B_t and C_t at date t, $t = 0, ..., T$. The global portfolio value $A_t + B_t + C_t$ is invested and provides at the end of the period a return denoted by y_{t+1}. Then, the change in total portfolio value is $(A_t + B_t + C_t)y_{t+1}$. As in Section 2.1., we first assume that the return on B account is always allocated to the corresponding class. We only consider how $(A_t + C_t)y_{t+1}$ has to be allocated between the three accounts depending on some predetermined regimes.

We consider below an allocation process based on a modified LCF measure of performance deficit. In this case, the LCF can be interpreted as the negative part of a virtual provision

[3] In HF literature, this account is called reserve account. It seems preferable to avoid this terminology, which will become misleading if some Basel type of regulation is applied to HF in a near future.

account (whereas the value of the actual provision account has to be always positive). Hence, at any date t, the sum $LCF_t + C_t$ is only impacted by one of its two components, the other one being zero. The LCF starts to be negative when the provision account is empty and C_t starts to be positive when the LCF is null . For expository purpose, the allocation scheme is described below in two steps to highlight the smoothing technique.

i) Three accounts - no smoothing

A proportion β of the change in the A+C accounts value up to the hurdle rate, that is $(A_t + C_t)(y_{t+1} - y_{h,t})$, is allocated to the provision account, under the positivity constraint on this account. The loss carry forward is defined by:

$$LCF_{t+1} = -\left[LCF_t + C_t + \beta(A_t + C_t)(y_{t+1} - y_{h,t})\right]^{-}, \tag{2.5}$$

and the corresponding provision account value is:

$$C_{t+1} = \left[LCF_t + C_t + \beta(A_t + C_t)(y_{t+1} - y_{h,t})\right]^{+}, \tag{2.6}$$

with initial conditions $LCF_0 = C_0 = 0$. Then, the values of accounts A and B are deduced from the dynamics of the provision account by the following equations:

$$\begin{cases} A_{t+1} = (A_t + C_t)(1 + y_{t+1}) - C_{t+1}, \\ B_{t+1} = B_t(1 + y_{t+1}). \end{cases} \tag{2.7}$$

By construction, the provision account value (resp. the LCF) is always nonnegative (resp. nonpositive). Moreover, only one of the LCF and C value can be different from zero at any given date.

When $C_t = 0$, equation (2.5) reduces to the standard LCF recursive equation (2.1). When $C_t > 0$ (and $LCF_t = 0$), a capital appreciation $(A_t + C_t)(y_{t+1} - y_{h,t}) > 0$ will increase the value of the provision account, whereas the LCF will stay equal to zero. Finally, if $C_t > 0$ and there is a large capital depreciation up to the hurdle rate, the provision account is set to zero and the complete return allocated to the A account.

ii) Three accounts with smoothing

We now add to the previous allocation scheme the smoothing component. This effect is obtained through a change in the recursive equation (2.6) giving the C account dynamics. We assume that a proportion of the provision account is allocated to the external investors' and management firm accounts in case of bad portfolio performance. The recursive system becomes:

$$LCF_{t+1} = -\left[LCF_t + C_t + \beta(A_t + C_t)(y_{t+1} - y_{h,t})\right]^{-}, \tag{2.8}$$

for the LCF,

$$C_{t+1} = [1 - \varphi_A(y_{t+1}) - \varphi_B(y_{t+1})] \left[LCF_t + C_t + \beta(A_t + C_t)(y_{t+1} - y_{h,t})\right]^+ , \tag{2.9}$$

for the provision account, and:

$$\begin{cases} A_{t+1} = (A_t + C_t)(1 + y_{t+1}) + [\varphi_A(y_{t+1}) - 1] \left[LCF_t + C_t + \beta(A_t + C_t)(y_{t+1} - y_{h,t})\right]^+ , \\ B_{t+1} = B_t(1 + y_{t+1}) + \varphi_B(y_{t+1}) \left[LCF_t + C_t + \beta(A_t + C_t)(y_{t+1} - y_{h,t})\right]^+ , \end{cases} \tag{2.10}$$

for A and B accounts, where the smoothing functions φ_A, φ_B are positive and such that $\varphi_A + \varphi_B \leq 1$.

A simple scheme assumes constant smoothing functions $\varphi_A(y) = \varphi_A$, $\varphi_B(y) = \varphi_B$, say. For instance, if φ_A and φ_B are such that $\varphi_A + \varphi_B = 1$, and if moreover $\beta = 1$, the provision account is always empty, and the scheme reduces to the standard LCF scheme with two accounts described in Section 2.2.

However, more sophisticated smoothing functions are introduced in the hedge fund industry. For instance, we can fix a predetermined level[4] $y_{0,t} < 0$, different from the hurdle rate, and define the smoothing functions as:

$$\varphi_A(y_{t+1}) = \varphi_B(y_{t+1}) = \frac{1}{2} \min \left[1, \left(\frac{y_{t+1}}{y_{0,t}} \right)^+ \right]. \tag{2.11}$$

Thus, if $y_{t+1} < y_{0,t}$, we get $\varphi_A(y_{t+1}) = \varphi_B(y_{t+1}) = \frac{1}{2}$, and a full use of the provision account to smooth A (and B) return. If $y_{0,t} < y_{t+1} < 0$, we have a partial smoothing. Finally, if $y_{t+1} > 0$, we get $\varphi_A(y_{t+1}) = \varphi B(y_{t+1}) = 0$ and the previous account is feeded to insure the fund against future potential losses.

(*b*) Returns and Asset Values

By analogy with the standard scheme, we can consider different returns. The most important ones are:

i) The total portfolio return: y_{t+1};

ii) The return for class A account: $y_{A,t+1} = (A_{t+1} - A_t)/A_t$;

iii) The return associated with both A and C accounts: $y_{A,C,t+1} = (A_{t+1} + C_{t+1} - (A_t + C_t))/(A_t + C_t)$.

[4] The level $y_{0,t}$ can be constant and set for example to -1% to smooth small negative returns.

Indeed, it is important to distinguish the net asset value (NAV) for class A, i.e. A_t, and the value including also the provision account, i.e. $A_t + C_t$. The net asset value A_t is provided for at least two purposes. This is the accounting value which has to be introduced by the investors in their balance sheet. This is also the benchmark for the selling price proposed by the fund management to an investor who wants to redeem its investment. This NAV A_t is a kind of bid price (i.e. selling price), which is smaller or equal to the "fair value" of the fund equal to $A_t + C_t$.
Clearly, the provision account creates a "conditional return smoothing" when passing from y_t to y_t^A, to follow the terminology of Bollen, Pool (2008). However, this (known) smoothing is much more complicated than usually described in the academic literature [see e.g. Bollen, Pool (2008), eq 7].

3 The effects of the scheme on i.i.d. Gaussian portfolio returns

In this section, we assume a zero riskfree rate, a zero hurdle rate $y_{h,t} = 0$, and *i.i.d.* Gaussian net portfolio returns $y_t \sim N(m, \sigma^2)$, where m (resp. σ^2) is the path-independent expected return (resp. volatility). Thus, we assume a constant hedge fund leverage ratio [see Getmanski, Lo, Makarov (2004), eq. 10] and do not consider the additional uncertainty associated with the hurdle. Except in the special case of unitary reset time in the standard allocation scheme for which the LCF and HWM coincide [see Darolles, Gourieroux (2013)], a theoretical analysis of the dynamics of bank accounts is difficult due to the nonlinear path dependent allocation schemes. The dynamic properties are discussed below by means of simulation studies.

3.1 The Loss Carry Forward allocation scheme (without provision account)

From Proposition 1, we know that the LCF scheme is identical to the HWM scheme for a zero hurdle rate. The associated $LCF^* = LCF$ trajectory is given in the fourth panel of Figure 3.

We display in Figure 3 the trajectories of the two account values, the HWM, the implied $LCF_t^* = A_t - HWM_t$ (see Proposition 1) and the relative weights of both accounts, i.e. the ratio $w_t = \frac{A_t}{B_t}\frac{B_0}{A_0}$.

[Insert Figure 3: Trajectories of Account Values, HWM and LCF^* (without provision account)]

Due to the selected performance fee rate of the portfolio management, the two account values are increasing, but this increase is larger for the management account than for the investor's account. We also observe that the ratio w_t is decreasing in time and clearly different from the announced $1 - \alpha = 80\%$.

3.2 The allocation scheme with provision account

We display in Figure 4 the trajectories of the three accounts A, B, C, and the LCF. We consider independent risky returns following a Gaussian distribution with mean $m = 1\%$, and variance $\sigma^2 = 1\%$, set the provision rate at $\beta = 25\%$, and use the smoothing functions (2.21) with level $y_0 = -1\%$.

[Insert Figure 4: Trajectories of Account Value and LCF (with provision account)]

The return dynamics for y_t, $y_{A,t}$, $y_{B,t}$ are provided in Figure 5. We observe that the presence of a provision account smooths the investor's account return. This makes more marketable the published HF returns $y_{A,t}$ by reducing the value of the usual fund risk indicators such as the return volatility.

[Insert Figure 5: Return Dynamics (with provision account)]

i) Historical distribution of returns

As in the HWM allocation scheme [see Darolles, Gourieroux (2013)], the return dynamics can be summarized in different ways. First, we compare the historical distributions of returns y_t and $y_{A,t}$ in Figure 6. In presence of a provision account, the two sides of the distribution are modified. The left side (corresponding to negative return) is moved to the right, that is, we get less negative returns, especially around zero. Moreover, the right part is also impacted, due to the smoothing rule used in this simulation. The high positive returns are less frequent, but the probability to observe small positive returns increases. Thus, the provision account implies right skewness and discontinuity on the return distribution, which is clearly seen on the histogram of $y_{A,t}$ provided in the second panel of Figure 6. The discontinuity is less pronounced with return computed on two consecutive periods (3^d panel of Figure 6), which is compatible with the observation by Bollen, Pool (2009) that the discontinuity can disappear when the horizon increases. These empirical facts have already been documented in the literature. However, they have been explained by either fraud [Abdulali (2006)], misreporting of returns, if the manager fully report gains, but delays reporting losses [see e.g. Bollen, Pool (2009)], survivorship bias [Brown, Goetzmann, Ibbotson (1999)], or backfilling bias, when both superior and inferior performers stop reporting [Ackermann, McEnally, Ravenscraft (1999)]. In fact, the bias ratio is likely a consequence of the (transparent) design of the allocation scheme between the three accounts.

[Insert Figure 6: Historical Distributions of Returns (with provision account)]

ii) Return dynamics

The nonlinear autoregressive effect due to the provision account is still difficult to detect from a simple linear analysis of serial dependence (see Figure 7), even if the cycle effect due to the threshold autoregressive dynamics (2.10) [see Tong (1983)] becomes more significant. This cycle effect implies in particular negative autocorrelations at periodic lags. This dependence created by the account allocation scheme is not able to explain the positive short term persistence emphasized in the HF literature [see e.g. Agarwal, Naik (2000), Getmanski, Lo, Makarov (2004)], but is compatible with the negative autocorrelation detected in Bollen, Pool (2009), when lagged returns are just above zero[5].

[Insert Figure 7: ACF and Squared ACF on Returns (with provision account)]

iii) Summary statistics on return

Let us now compare the characteristic of HF returns $y_{A,t+1}$, for different values of the provision rate β assigned to account C, $\beta = 0\%$, 5%, 10%, 20%; the limiting case $\beta = 0\%$ corresponds to $y_{A,t+1} = y_{t+1}$. All other parameters are set to the values used to compute Table 1.

[5]A linear analysis of serial correlation can also be rather misleading. Indeed conditional serial correlations can be very different. For instance, it is equal to zero when $y_{A,t}$ is sufficiently large, since $y_{A,t} = y_t$, but will become significant when $y_{A,t}$ is small, due to the effect of the optional component which depends on the past. These different levels of conditional serial correlations are just consequences of the HWM schemes. We cannot necessarily conclude that a "manager smooths more likely losses than gains" [Bollen, Pool (2008), (2009)].

[Insert Table 1: Statistics on $y_A(T)$ (with provision account)]

We observe that the distribution is shifted to the left when the β parameter increases, but this shift is less pronounced than in the scheme without provision account. Moreover, the risk parameters also diminish when the β parameter increases. In consequence, the Sharpe ratio is stable, and then is less sensitive to the management fee politics. The skewness and kurtosis parameters also decrease with β.

4 Conclusion

The LCF scheme used for allocating gains and profits between the investor's account, management account and provision account has a significant impact on the performance of the investors' account. The first effect is related to the nonlinearity of the scheme, especially the barrier effects, An additional effect is introduced by the smoothing component associated with the provision account. These two effects explain a part of the empirical facts observed on hedge fund returns, such as the skewness of the return distribution, it discontinuity at zero, or some cyclical serial correlation.

We see that the complexity of the formulas defining the allocation schemes and also the diversity of these schemes, which depend on the choice of the free rate, sequence of hurdle rate, the rate of the capital appreciation/depreciation and the smoothing functions. This diversity makes difficult the comparison of what is proposed by different funds. From a regulatory point of view, there is a need for a standardization of these allocation schemes, that is of the way the "bonuses" of the HF management firms are computed.

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Appendix 1
Proof of Proposition 1

i) Let us first consider the HWM scheme and denote by $LCF_t^* = A_t - HWM_t$ the implied LCF associated with this scheme. The recursion for the HWM scheme is:

$$\begin{cases} A_{t+1} = A_t(1+y_{t+1}) - \alpha A_t \left[y_{t+1} - (HWM_t - A_t)/A_t\right]^+, \\ HWM_{t+1} = \max\left(HWM_t, A_{t+1}\right), \end{cases}$$

or equivalently,

$$\begin{cases} A_{t+1} = A_t(1+y_{t+1}) - \alpha A_t \left(y_{t+1} + \frac{LCF_t^*}{A_t}\right)^+, \\ LCF_{t+1}^* = -\left(LCF_t^* + A_{t+1} - A_t\right)^-. \end{cases}$$

We get the two following regimes:

- Regime 1: $LCF_t^* + A_t y_{t+1} > 0$,

  with:

$$A_{t+1} = A_t(1+y_{t+1}) - \alpha A_t \left(y_{t+1} + \frac{LCF_t^*}{A_t}\right). \tag{1.1}$$

  Then:

$$\begin{aligned} LCF_t^* + A_{t+1} - A_t &= LCF_t^* + A_{t+1} - \alpha A_t \left(y_{t+1} + \frac{LCF_t^*}{A_t}\right) \\ &= (1-\alpha)(LCF_t^* + A_t y_{t+1}) > 0. \end{aligned}$$

  We deduce that:

$$LCF_{t+1}^* = 0. \tag{1.2}$$

- Regime 2: $LCF_t^* + A_1 y_{t+1} < 0$.

  We get:

$$A_{t+1} = A_t(1+y_{t+1}). \tag{1.3}$$

  Thus, $LCF_t^* + A_{t+1} - A_t = LCF_t^* + A_t y_{t+1} < 0$, and we deduce that:

$$LCF_{t+1}^* = LCF_t^* + A_t y_{t+1}. \tag{1.4}$$

ii) Let us now consider the recursion for the LCF scheme:

$$\begin{cases} A_{t+1} = A_t(1+y_{t+1}) - \alpha A_t \left[y_{t+1} + LCF_t/A_t\right]^+ \\ LCF_{t+1} = -\left(LCF_t + A_t y_{t+1}\right)^-. \end{cases}$$

We get the two following regimes:

- Regime 1: $LCF_t + A_t y_{t+1} > 0$,

$$\begin{cases} A_{t+1} = A_t(1 + y_{t+1}) - \alpha A_t \left[y_{t+1} + LCF_t / A_t \right], \\ LCF_{t+1} = 0. \end{cases} \tag{1.5}$$

- Regime 2: $LCF_t + A_t y_{t+1} < 0$,

$$\begin{cases} A_{t+1} = A_t(1 + y_{t+1}), \\ LCF_{t+1} = LCF_t + A_t y_{t+1}. \end{cases} \tag{1.6}$$

The recursive equations $(1.1)-(1.4)$ are identical to the equations $(1.5)-(1.6)$. Proposition 1 follows by noting that the initial values of the LCF and implied LCF are the same: $LCF_0^* = A_0 - HWM_0 = 0$, $LCF_0 = 0$.

QED

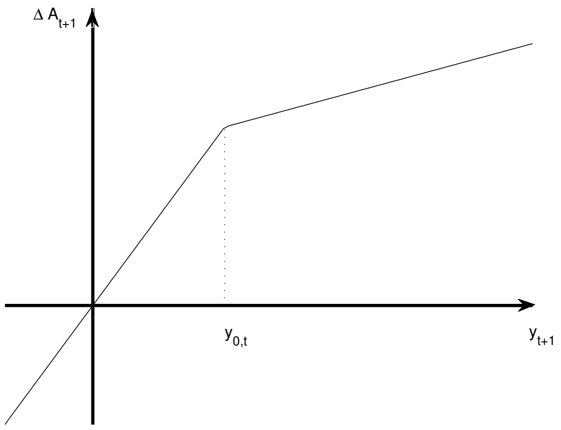

Figure 1: ΔA_{t+1} as a function of y_{t+1} (unitary reset time)

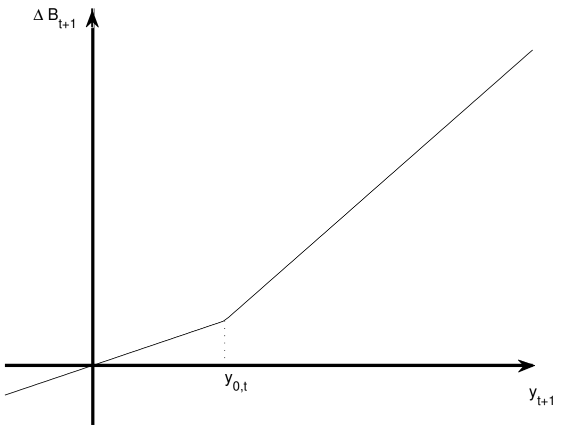

Figure 2: ΔB_{t+1} as a function of y_{t+1} (unitary reset time)

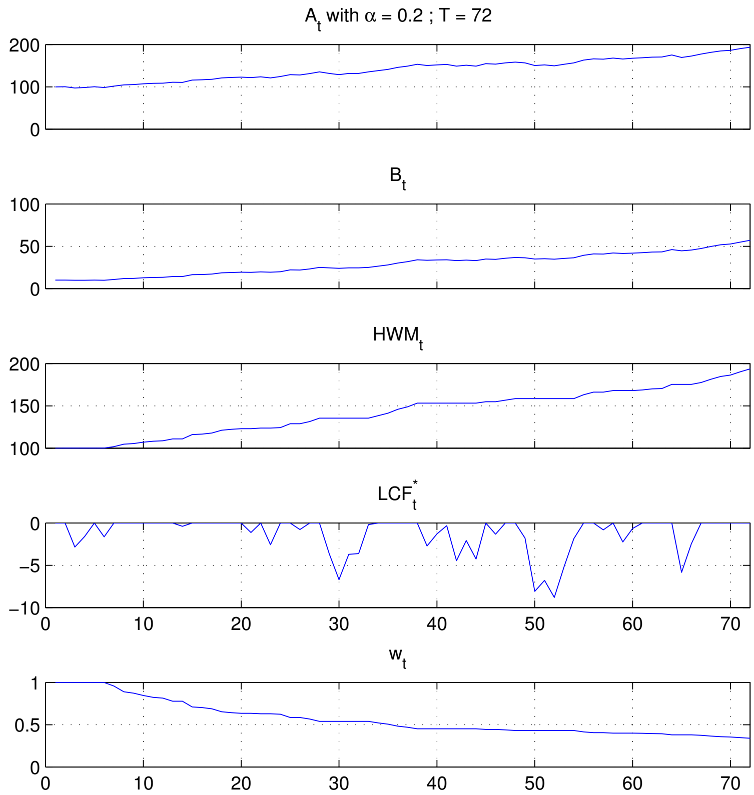

Figure 3: Trajectories of Account Values, HWM, LCF^* (no provision account)

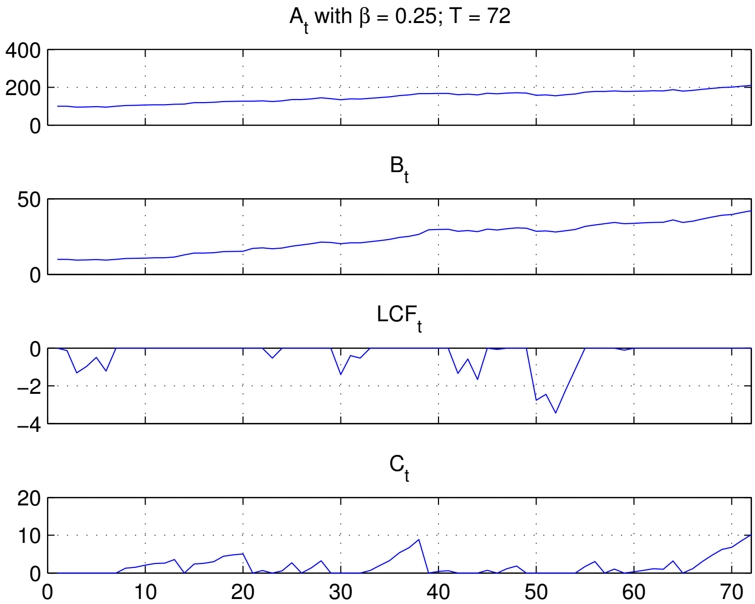

Figure 4: Trajectories of Account Values and LCF (with provision account)

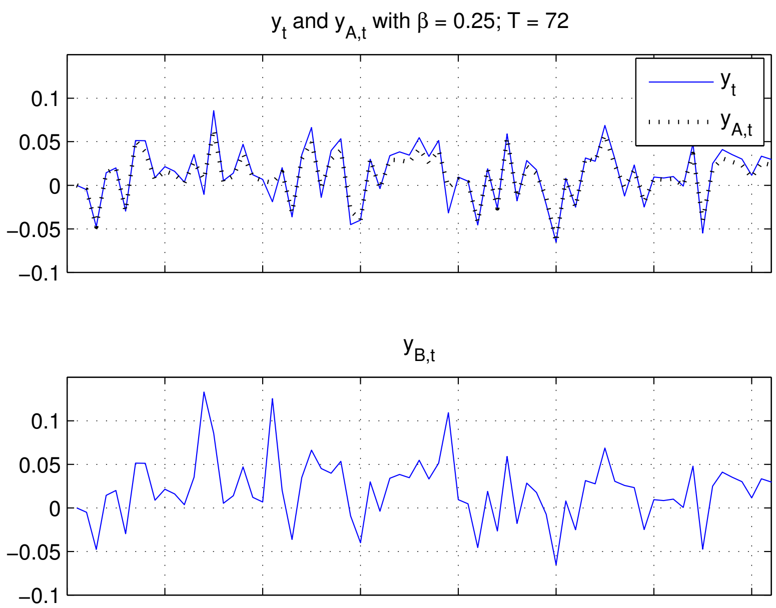

Figure 5: Return Dynamics (with provision account)

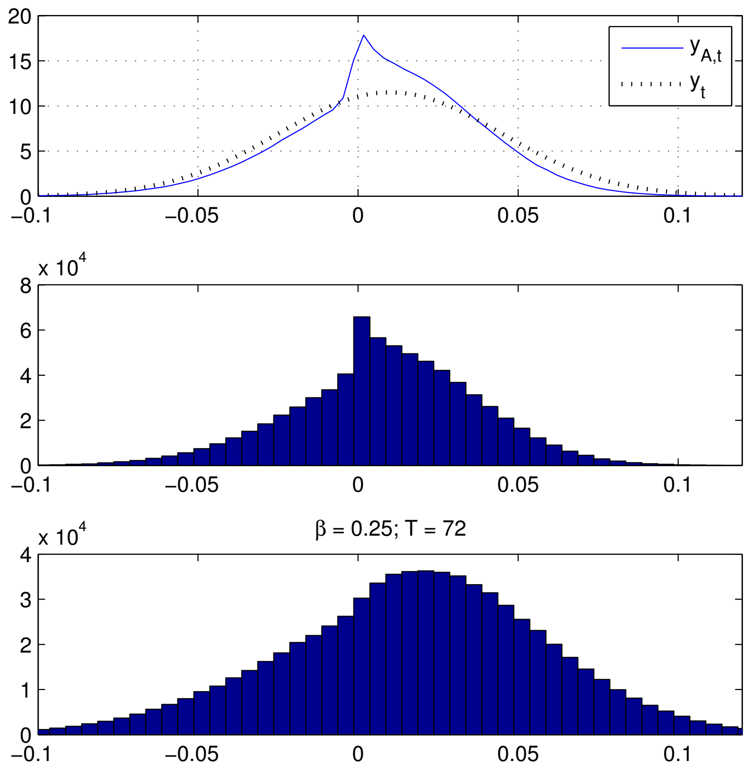

Figure 6: Historical Distributions of Returns (with provision account)

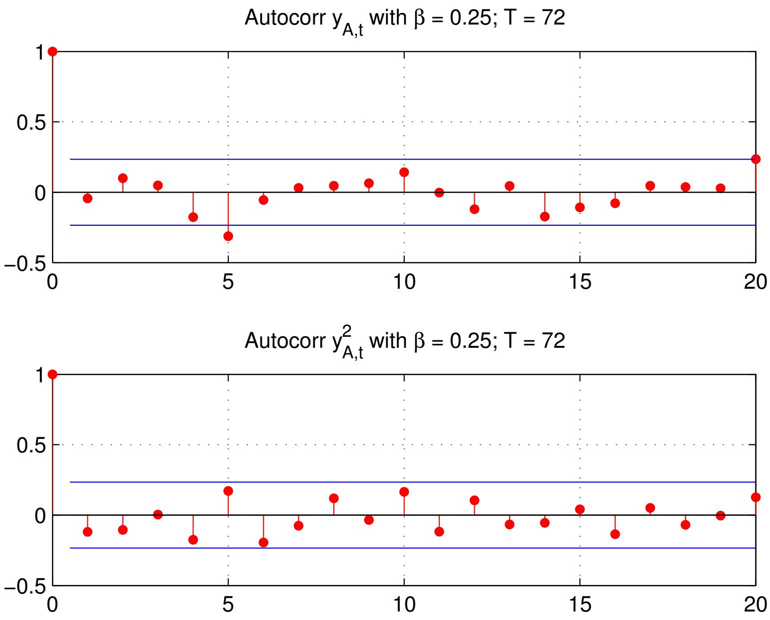

Figure 7: ACF on Return and Squared Return (with provision account)

Panel A: $T = 24$ **(2 years)**

Provision β level	Mean	SD	Sharpe	Median	Skew	Exc. Kurt.	5%-Quant.	95%-Quant.
Sharpe ratio = 0.5								
0%	0.0116	0.0187	0.4375	0.0082	1.0815	2.0117	-0.0130	0.0472
5%	0.0110	0.0181	0.4310	0.0079	1.0431	1.8856	-0.0130	0.0456
10%	0.0105	0.0175	0.4241	0.0076	1.0037	1.7617	-0.0130	0.0439
20%	0.0095	0.0165	0.4086	0.0070	0.9222	1.5221	-0.0131	0.0404
Sharpe ratio = 1								
0%	0.0114	0.0091	0.8867	0.0104	0.5458	0.4780	-0.0021	0.0279
5%	0.0110	0.0088	0.8840	0.0101	0.5216	0.4443	-0.0021	0.0269
10%	0.0106	0.0085	0.8810	0.0098	0.4967	0.4129	-0.0021	0.0259
20%	0.0098	0.0079	0.8736	0.0091	0.4449	0.3583	-0.0022	0.0240
Sharpe ratio = 1.5								
0%	0.0113	0.0060	1.3325	0.0109	0.3794	0.2175	0.0021	0.0220
5%	0.0110	0.0058	1.3344	0.0105	0.3615	0.1990	0.0020	0.0212
10%	0.0106	0.0056	1.3361	0.0102	0.3435	0.1827	0.0019	0.0204
20%	0.0098	0.0052	1.3385	0.0095	0.3073	0.1591	0.0018	0.0189

Panel B: $T = 48$ **(4 years)**

Provision β level	Mean	SD	Sharpe	Median	Skew	Exc. Kurt.	5%-Quant.	95%-Quant.
Sharpe ratio = 0.5								
0%	0.0129	0.0170	0.3809	0.0094	1.4832	3.5525	-0.0073	0.0462
5%	0.0124	0.0163	0.3787	0.0091	1.4406	3.3529	-0.0073	0.0444
10%	0.0118	0.0157	0.3763	0.0088	1.3976	3.1588	-0.0074	0.0425
20%	0.0108	0.0146	0.3703	0.0081	1.3101	2.7869	-0.0074	0.0389
Sharpe ratio = 1								
0%	0.0128	0.0081	0.7888	0.0120	0.6983	0.7160	0.0012	0.0280
5%	0.0124	0.0078	0.7901	0.0116	0.6794	0.6806	0.0011	0.0270
10%	0.0120	0.0076	0.7911	0.0112	0.6603	0.6468	0.0010	0.0259
20%	0.0111	0.0070	0.7925	0.0104	0.6213	0.5842	0.0009	0.0239
Sharpe ratio = 1.5								
0%	0.0128	0.0054	1.1905	0.0124	0.4623	0.2722	0.0047	0.0225
5%	0.0124	0.0052	1.1950	0.0120	0.4510	0.2590	0.0045	0.0217
10%	0.0120	0.0050	1.1993	0.0116	0.4398	0.2468	0.0044	0.0209
20%	0.0111	0.0046	1.2076	0.0108	0.4177	0.2260	0.0041	0.0193

Panel C: $T = 72$ **(6 years)**

Provision β level	Mean	SD	Sharpe	Median	Skew	Exc. Kurt.	5%-Quant.	95%-Quant.
Sharpe ratio = 0.5								
0%	0.0147	0.0181	0.3325	0.0104	2.1096	8.0992	-0.0046	0.0489
5%	0.0141	0.0173	0.3327	0.0100	2.0462	7.6148	-0.0047	0.0468
10%	0.0134	0.0165	0.3328	0.0096	1.9832	7.1516	-0.0047	0.0447
20%	0.0122	0.0150	0.3322	0.0088	1.8577	6.2843	-0.0047	0.0405
Sharpe ratio = 1								
0%	0.0147	0.0084	0.7090	0.0135	0.9212	1.5418	0.0031	0.0302
5%	0.0141	0.0081	0.7126	0.0130	0.8981	1.4693	0.0030	0.0289
10%	0.0136	0.0077	0.7162	0.0126	0.8749	1.3992	0.0029	0.0277
20%	0.0125	0.0071	0.7229	0.0117	0.8283	1.2663	0.0026	0.0254
Sharpe ratio = 1.5								
0%	0.0146	0.0056	1.0754	0.0141	0.6007	0.6736	0.0065	0.0245
5%	0.0141	0.0053	1.0822	0.0136	0.5871	0.6452	0.0063	0.0235
10%	0.0136	0.0051	1.0891	0.0131	0.5736	0.6180	0.0061	0.0226
20%	0.0126	0.0047	1.1026	0.0122	0.5469	0.5674	0.0056	0.0208

Table 1: Statistics on $y_A(T)$ (with provision account)