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The Effects of Management and Provision Accounts on Hedge Fund Returns – Part I : The High Water Mark Scheme

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Abstract

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A characteristic of hedge funds is not only an active portfolio management, but also the allocation of portfolio performance between different accounts, which are the accounts for the external investors and an account for the management firm, respectively. Despite a lack of transparency in hedge fund market, the strategy of performance allocation is publicly available. This paper shows that, for the High Water Mark Scheme, these complex performance allocation strategies might explain empirical facts observed in hedge fund returns, such as return persistence, skewed return distribution, bias ratio, or implied increasing risk appetite.

Keywords: Hedge Fund, Sharpe Performance, Manager Incentive, Risk Appetite, High Water Mark.

1 Introduction

The applied literature has shown that the return dynamics of individual hedge funds[1] (HF) are very different from the return dynamics of more standard assets such as stocks, currencies, or mutual funds. The HF return dynamics can depend on the management style, but generally, feature persistence, especially at short term and in extreme returns [Agarwal, Naik (2000), Koh, Koh, Teo (2003), Getmanski, Lo, Makarov (2004)], local asymmetries around zero, called bias ratio in the literature [Abdulali (2006), Bollen, Pool (2009), Darolles, Gourieroux, Jasiak (2009)], very heavy tails, for instance for Convertible Arbitrage or Fixed Income Arbitrage funds; moreover, some HF returns are weakly correlated with major asset market returns [Fung, Hsieh(1999)]. These empirical facts reflect an underlying nonlinear dynamic of HF return, which can be explained by:
i) The frequent path dependent updating of the portfolio associated with the fund [see e.g. Lo (2008)];
ii) The procedure used to allocate the performance between different accounts, that are the investor's account and the account of the management firm.
Since the sequence of portfolio updatings and allocations are not observable by the econometrician and the standard investor[2], the management style and its effect on returns are difficult to analyse. On the other hand, the procedures used to allocate the total performance between the different accounts are precisely described in the prospectus written at the creation of the fund and validated by the appropriate authorities. The aim of this paper and of its companion paper [Darolles, Gourieroux (2013)] is to discuss the possible effects of these rather complex procedures and to see if they can partly explain empirical facts observed on individual HF returns[3].
In Section 2, we provide an example of allocations between accounts used in practice. We

[1]Note that we are interested in the return dynamics of individual hedge funds, not in hedge funds indices.

[2]They are known by the fund manager and partly known by large investors, who profit of due diligence, or investors in US funds reporting their holdings on Form 13F with the Security Exchange Commission (SEC). This creates asymmetric information on HF markets.

[3]Performance based fees (also called incentive fees) are characteristics of hedge funds; they are much less frequent for mutual funds. For instance, in 1999 only 108 out of a total 6.716 bond and stock mutual funds used incentive fees [Elton, Gruber, Blake(2001)]. Moreover by law the mutual funds must use a special form of incentive fees known as fulcrum fee (see the 1970 amendment to the Investment Company Act of 1940). Typically, the fulcrum fees are centered around an index[4] and have upper and lower limits in size. Such constraints do not exist for HF.

consider the rather standard high-water mark (HWM) scheme. The presence of several accounts can imply significant differences between the return of the managed porfolio and the published HF return. We describe in detail the nonlinear filter to pass from the portfolio return to the published HF return.

Section 3 compares the portfolio and fund returns when the portfolio returns are independent and identically Gaussian distributed. The i.i.d. Gaussian assumption on portfolio returns corresponds to a rather exogenous portfolio management, whereas the hedge fund manager will account for the existence of multiple accounts in his/her management strategy.

In Section 4, we discuss the mean-variance efficient portfolio management according to the account of interest. If the fund performance has to be maximized, the management differs from the standard mean-variance management of the global portfolio. More precisely, the allocation scheme between accounts has a significant impact on the optimal portfolio management. There exists a theoretical literature in the introduction of multiple accounts as an incentive for the hedge fund manager[5]. However, this question is often considered under rather irrealistic assumptions such as continuous time incentives, whereas the barrier effects apply monthly [see e.g. Goetzmann, Ingersoll, Ross (2003), Kouwenberg, Ziemba (2007)], competitive hedge fund market, whereas each hedge fund has a specific design and its secondary market is not very active [see e.g. Christoffersen, Musto, Yilmaz (2013)], two periods instead of multiperiod optimization [see e.g. Christoffersen, Musto, Yilmaz (2013)], risk-neutral manager [Paganeas, Westerfield (2009)], binary returns [Christoffersen, Musto, Yilmaz (2013)], or rather ad-hoc account description, which does not correspond to the account allocations proposed in the hedge fund industry [Kazemi, Li (2009), Aragon, Nanda (2012)]. We try in this section to stay as close as possible to the actual hedge fund designs and to focus on their dynamic features. Section 5 contains conclusions. Proofs are gathered in Appendices.

[5] There exists also a more empirical literature studying the links between the risk taken by the hedge fund manager, often summarized by means of the HF return volatility, and some characteristics of the HF, such as proxies for the optional feature of the compensation scheme [see e.g. Kazemi, Li (2009)]. These analysis are often based on the rather simple static linear regression techniques and thus neglect the complexity of the compensation scheme, especially its dynamics and nonlinear features.

2 High-Water Mark allocation scheme

There exist almost as many account allocation schemes as hedge funds shares, which explains why any precautionary investor, regulator, or researcher[6] should study in details the prospectus of the funds. We describe below a standard scheme used to allocate the performance between the account invested by external clients, called class A units in the following, and the account invested by the management firm, called class B units[7].

This allocation scheme is parametrized by an allocation rate, called performance fee rate, a return benchmark, called hurdle rate, and a validity period corresponding to the duration between consecutive resets of class B account. These parameters differ according to the fund share.

2.1 Allocation between A and B accounts

Let us first consider two accounts, with respective values A_t, B_t at month t, $t = 0, ..., T$. The contractual hurdle rate is denoted by $y_{h,t}, y_{h,t} \geq 0$, and is assumed to be predetermined and observable at date t. The contractual hurdle rate is a benchmark introduced to define the performance allocations. This hurdle can be set to zero [see e.g. Panageas, Westerfield (2009)], or to a cash return like the 1-month London Interbank Offered Rate (LIBOR)[8]. The maximal value reached on the past by account A is discounted at rate $y_{h,t}$ and called the high-water mark (HWM). This HWM is first computed at date t by:

$$HWM_t = \max_{0 \leq \tau \leq t} \left[A_\tau \prod_{\tau^*=\tau}^{t} (1 + y_{h,\tau^*}) \right], \; t = 0, ..., T-1. \tag{2.1}$$

and then compare to A_{t+1} at date $t+1$. The fee schedule is endogenous[9] as a function of past successes, but is entirely defined at date t, due to the choice of the predetermined hurdle rate. We deduce that:

$$HWM_t = \max \left[HWM_{t-1}, A_t \right] (1 + y_{h,t}), \; t = 1, ..., T-1, \tag{2.2}$$

[6]Typically, it is misleading to consider as an homogenous class the set of funds reporting a high-water mark benchmark in the standard Lipper/TASS database [see e.g. Aragon, Nanda (2012)].

[7]To simplify, we assume that there is neither redemption, nor subscription after the inception date and no misreporting of the data. The changes observed in the values of the different accounts come from the evolution of the portfolio return only.

[8]The hurdle rate has to be defined in the same currency as the fund reference currency, e.g. US Dollar, Euro, Yen, ...

[9]Exogenous HWM of the type $HWM_t = HWM_0 \prod_{\tau=1}^{t} (1 + y_{h,\tau})$ are often assumed in the HF literature [see e.g. Hodder, Jackwerth (2007)]. Such HWM schemes correspond to the fulcrum scheme for mutual funds, but are very different from the actual HWM for hedge funds.

with initial condition $HWM_0 = A_0(1 + y_{h,0})$.

At period t, the global portfolio value $A_t + B_t$ is invested and provides at the end of the period a return net of base management fees[10] denoted by y_{t+1}. Then, the change in total portfolio value $(A_t + B_t)y_{t+1}$ is allocated between the two accounts. The performance fee is not charged if the fund is globally in a deficit of performance with respect to the high-water mark. Thus, this allocation depends on the location of:

$$A_t(1 + y_{t+1}), \tag{2.3}$$

with respect to the predetermined HWM_t as follows:

1. if $HWM_t \geq A_t(1 + y_{t+1})$,

$$\begin{cases} A_{t+1} = A_t(1 + y_{t+1}), \\ B_{t+1} = B_t(1 + y_{t+1}). \end{cases} \tag{2.4}$$

2. If $HWM_t < A_t(1 + y_{t+1})$,

$$\begin{cases} A_{t+1} = A_t(1 + y_{t+1}) - \alpha[A_t(1 + y_{t+1}) - HWM_t], \\ B_{t+1} = B_t(1 + y_{t+1}) + \alpha[A_t(1 + y_{t+1}) - HWM_t], \end{cases} \tag{2.5}$$

   where $0 < \alpha < 1$ is the (high-water mark) performance fee rate. This performance fee rate varies from 15% to 50%, with an increase in recent years [see e.g. Fung, Hsieh(1999), Table 2, Zuckerman (2004)]. It is equal to 20% for the Quantum Fund reported in Goetzmannn, Ingersoll, Ross (2003).

The updating equations (2.4)-(2.5) can also be written as:

$$\begin{cases} A_{t+1} = A_t(1 + y_{t+1}) - \alpha A_t \left[y_{t+1} - (HWM_t - A_t)/A_t\right]^+, \\ B_{t+1} = B_t(1 + y_{t+1}) + \alpha A_t \left[y_{t+1} - (HWM_t - A_t)/A_t\right]^+, \end{cases} \tag{2.6}$$

where $X^+ = \max(X, 0)$, to highlight the presence of an option component. When the fund gains enough value, the manager is paid and the strike price increases, but when the fund loses money, the strike price remains unchanged and the manager retains his/her option at the old strike price.

At short term horizon equal to 1, the future account values involve the payoff of a European call option[11] written on y_{t+1}, with predetermined path dependent strike equal to

[10] The base management fee is generally proportional to the asset value managed by the fund. Without loss of generality, we take them into account by considering portfolio return net of base management fee.

[11] Or of a European put option if we note that coefficient $-\alpha$ is negative and account for the put-call parity relationship.

$y_{0,t} = (HWM_t - A_t)/A_t$. At larger horizon, we get a sequence of European calls with changing strike prices. Both rolling effect and path dependent strike show that the option interpretation of the account allocation is significantly different from the simplified European call interpretations introduced for instance in Kouwenberg and Ziemba (2007) or Diez de los Rios, Garcia (2008), eq. (2.5), which neglects path dependence.

For a zero hurdle rate, the recursive equation for account A can also be written as:

$$A_{t+1} = A_t(1 + y_{t+1}) - \alpha(HWM_{t+1} - HWM_t)^+, \tag{2.7}$$

which shows that the fund manager receives a fraction of the increase in HWM as compensation.

In practice, the management firm is periodically paid by means of the management account, generally at the end of the year. The recursive equations are valid on a period $\{0, T-1\}$ of a given length T corresponding to the duration between consecutive resets, i.e. 0 and T. At time T, the management account is reset to the initial fixed[12] contractual value B_0 and the HWM reset[13] to $A_T(1 + y_{h,T})$. Since the allocation scheme may create nonstationary features, this practice breaks down possible explosive behavior.

If the reset time is $T = 1$, the HWM is equal to $A_t(1 + y_{t+1})$ and regimes (2.4) and (2.5) are active depending if the portfolio management out- or underperforms the hurdle. We get $y_{0,t} = y_{h,t}$ and the HWM disappears in equation (2.6) that becomes:

$$\begin{cases} A_{t+1} = A_t(1 + y_{t+1}) - \alpha A_t \left[y_{t+1} - y_{h,t}\right]^+, \\ B_{t+1} = B_t(1 + y_{t+1}) + \alpha A_t \left[y_{t+1} - y_{h,t}\right]^+, \end{cases} \tag{2.8}$$

and is reset at each date. In this setup a fixed proportion α of the return above the hurdle is allocated to class B at each period, which corresponds to a standard fulcrum scheme.

To summarize, the evolutions of account values depend on the portfolio management, that is, the sequence of portfolio returns (y_t), and on the allocation design characterized by hurdle rate $(y_{h,t})$, performance fee rate α, and reset time[14] T. The dynamic system is

[12] That is, this contractual value is not discounted.

[13] There exist funds with different reset times for the HWM and the B account.

[14] It is important to distinguish the reset time and the termination date of an hedge fund. Whereas most hedge fund management contracts do not have a pre-specified termination date, a reset time is often indicated. The presence of a reset time has significant implications on fund management and returns, and has to be taken into account. By implicitely assuming an infinite reset time, a part of the literature considered rather unrealistic models [see e.g. Panageas, Westerfield (2009)]. Typically, in a continuous time framework, the reset time will imply jumps of an endogenous size at predetermined dates.

recursive, since value (A_t) has an autonomous dynamic, and value (B_t) is fixed later. Let us finally remark that the value of account A can decrease and even become smaller than the initial value A_0, or negative. Therefore, the HF can fail[15] before the contractual reset time T. We will consider in the theoretical analysis that the fund fails if A_t becomes negative before reset time. From equation (2.6), we see that the portfolio return is necessarily larger than -1 before the potential failure time, and account B is positive. From this theoretical point of view, fund failure arises as the consequence of an abnormal negative return. In practice, it is also possible that the fund manager decides to liquidate the fund if the losses on account A are too large, even if A_t is still positive, or if his/her fees B_t are too small.

2.2 Discussion of the High-Water Mark scheme

Let us now discuss scheme (2.1) – (2.5). Since $HWM_t \geq A_t$, regime (2.4) applies if the spread between the net portfolio return and the hurdle $y_{t+1} - y_{h,t}$ is negative. If the spread is negative, the total loss is allocated proportionally to each account. If the spread is positive and small, regime (2.4) still applies and the same return is applied to accounts A and B. If the spread is positive and large enough to hit the HWM, the allocation rule is no longer proportional. The gain is shared between accounts A and B, with an allocation more favorable for the managing firm [see (2.5)]. The values of accounts A and B can increase or decrease, but the effect of net portfolio return y_{t+1} is no longer symmetric.
If the reset time is $T = 1$, the dependence of $\Delta A_{t+1} = A_{t+1} - A_t$ (resp. $\Delta B_{t+1} = B_{t+1} - B_t$) with respect to net porfolio return y_{t+1} is described in Figure 1 (resp. Figure 2).

[Insert Figure 1: ΔA_{t+1} as a function of y_{t+1} (unitary reset time)]

[Insert Figure 2: ΔB_{t+1} as a function of y_{t+1} (unitary reset time)]

The value of the class A unit is a continuous increasing function of the net portfolio return with a change of slope at threshold $y_{0,t}$. The payoff on B account is a convex function of the return, which might be an incentive for the fund manager to take risk, i.e. to produce large positive returns at some date to feed account B. These extreme positive returns might

[15] A HF fails when the fund manager decides to liquidate the fund and gives back the remaining asset under management to investors. The decision for liquidation is not contractual, but is at the discretion of the fund manager.

have to increase in time due the increasingness of the high-water mark as function of past successes. This misleading intuition has been challenged by Carpenter(2000), Ross(2004), Panageas, Westerfield(2009) [see also the discussion in Section 4].
Let us also discuss this scheme if the fund manager invests only in a riskfree asset, $y_{t+1} = y_{f,t}$, with a riskfree return larger than the hurdle, $y_{f,t} \geq y_{h,t}$, say[16]. Since $A_t(1 + y_{f,t}) \geq A_t(1 + y_{h,t}) = HWM_t$, the fund manager would profit systematically of such a static riskfree investment. Surprisingly, this account allocation scheme is often used in the HF industry with a zero hurdle rate $y_{h,t} = 0$.

2.3 The returns and effective performance fees

A major point in the discussion of fund returns is the definition of returns in case of several accounts. Indeed, the following returns can be introduced:
i) the total portfolio net return y_{t+1},
ii) the return on B account[17]: $y_{B,t+1} = (B_{t+1} - B_t)/B_t$,
iii) the return on A account: $y_{A,t+1} = (A_{t+1} - A_t)/A_t$.
The fund returns available in the standard Hedge Funds Research (HFR) or Lipper-Tass databases are returns $(y_{A,t})$ corresponding to class A units. They can feature dynamics very different from the dynamics of (y_t) and $(y_{B,t})$. For instance, return $y_{A,t}$ is always smaller or equal to the total net porfolio return y_t. It coincides with it at some endogeneous periods, and is strictly below, otherwise. It can be important in the analysis to distinguish the reported HF return $y_{A,t}$ and the underlying total portfolio return y_t. As an illustration, the methodology proposed in Henriksson, Merton (1981) [see also Glosten, Jagannathan (1994), Agarwal, Naik (2004), Diez de los Rios, Garcia (2008)] to detect the market timing ability of a portfolio manager consists in running a regression of the HF return on a market return and on a put option payoff written on this market return, say, and to test if the optional component is significant[18]. Applied to reported HF return y_t, this optional effect will likely appear as a consequence of the HWM scheme, even if this effect is not present in the total portfolio return, that is, if the portfolio manager shows no market timing ability.

[16] Under no arbitrage opportunity, this means that the contractual riskfree rate has been fixed at a level strictly smaller than the market riskfree rate.
[17] We have to choose a contractual positive B_0 initial value to give a meaning to this return.
[18] This methodology has to be applied on individual hedge funds, not on HF indices, to get this interpretation.

This might explain why "this option like payoff (effect) is not restricted only to trend followers and risk arbitrageurs, but is a feature on a wide range of hedge funds strategies" [Agarwal, Naik (2004), p. 66]. Anyway, the first equation in (2.6) shows that the observed return $y_{A,t}$ is a complicated nonlinear function of $y_t, y_{t-1}, ..., y_{h,t}, y_{h,t-1}, ...$, function which is known from the prospectus[19].

The ex-post performance allocation rate, i.e.:

$$\alpha_t = \frac{B_{t+1} - B_t}{A_{t+1} + B_{t+1} - (A_t + B_t)} = \frac{B_t y_{B,t}}{A_t y_{A,t} + B_t y_{B,t}}, \tag{2.9}$$

is not constant in time, can be erratic and rather different from the announced rate α. An effective performance allocation rate can be computed on a larger period to smooth the $\alpha_t' s$, for instance on the period $[0, T]$ corresponding to the time between resets. This effective performance allocation is:

$$\hat{\alpha}_T = \frac{B_T - B_0}{A_T + B_T - (A_0 + B_0)}, \tag{2.10}$$

and can also be different from α even for large T. Rate $\hat{\alpha}_T$ is likely strictly larger than α, since the total loss is assigned to account A, when the portfolio underperforms. It can even be larger due to the nonlinear allocation filtering which can create a convexity effect (see Appendix 1).

3 The effects of the scheme on i.i.d. Gaussian portfolio returns

In this section, we assume a zero riskfree rate, a zero hurdle rate $y_{h,t} = 0$, and *i.i.d.* Gaussian net portfolio returns $y_t \sim N(m, \sigma^2)$, where m (resp. σ^2) is the path-independent expected return (resp. volatility). Thus, we assume a constant hedge fund leverage ratio [see Getmanski, Lo, Makarov (2004), eq. 10] and do not consider the additional uncertainty associated with the hurdle. Except in the special case of unitary reset time in the standard allocation scheme (see Appendix 1), a theoretical analysis of the dynamics of bank accounts is difficult due to the nonlinear path dependent allocation schemes. The dynamic properties are discussed below by means of simulation studies.

[19] In Getmanski, Lo, Makarov (2004), the observed return $y_{A,t}$ is written as a Moving Average $MA(2)$ process of the underlying porfolio return. This moving average representation is a linar stochastic approximation of the actual known nonlinear deterministic relation existing between the returns. Its interpretation, which neglects nonlinearity, can be misleading.

In the standard High-Water Mark allocation scheme with zero hurdle, the joint dynamics of Class A value and high-water mark is characterized by the bivariate recursive system:

$$\begin{cases} A_{t+1} = A_t(1+y_{t+1}) - \alpha[A_t(1+y_{t+1}) - HWM_t]^+, \\ HWM_{t+1} = \max\left[HWM_t, A_t(1+y_{t+1}) - \alpha(A_t(1+y_{t+1}) - HWM_t)^+\right]. \end{cases} \tag{3.1}$$

with given initial condition (A_0, HWM_0). The bivariate process (A_t, HWM_t) is a Markov process. The joint transition distribution of (A_t, HWM_t) involves two partly degenerate distributions. Therefore, the joint bivariate transition is given by[20]:

$$\begin{aligned} & f_t(a_{t+1}, HWM_{t+1}) \\ & = \Big\{ I_{a_{t+1} > HWM_t} \times \tfrac{1}{(1-\alpha)A_t\sigma\sqrt{2\pi}} \varphi\left[\tfrac{a_{t+1} - A_t(1+m) + \alpha[A_t(1+m) - HWM_t]}{(1-\alpha)A_t\sigma}\right] \\ & + I_{a_{t+1} < HWM_t} \times \tfrac{1}{A_t\sigma\sqrt{2\pi}} \varphi\left[\tfrac{a_{t+1} - A_t(1+m)}{A_t\sigma}\right] \Big\} \otimes \epsilon_{(HWM_{t+1} = Max(HWM_t, a_{t+1}))}, \end{aligned} \tag{3.2}$$

where $\epsilon_{(.)}$ denotes a point mass, φ the probability density function (pdf) of the standard normal distribution and $\otimes$ the tensor product.
To illustrate the consequences of the allocation scheme on accounts returns, let us consider risky returns following a Gaussian distribution with mean $m = 1\%$, and volatility $\sigma = 3.46\%$. We set the performance fee rate at $\alpha = 20\%$. The initial values of the accounts are $A_0 = 100$, $B_0 = 10$ and the reset time is set to $T = 72$ months= 6 years.

[Insert Figure 3: Return Dynamics]

The return dynamics for y_t, $y_{A,t}$, $y_{B,t}$ are given in Figure 3. The return on management account is much more volatile than the underlying portfolio return and we observe the clustering for positive returns corresponding to the threshold effect of the HWM. The trajectories of y_t and $y_{A,t}$ are quite close[21]: the HWM effect is seen by the smoothing of peaks of y_t trajectories for the account A. These evolutions can be summarized in different ways. First, we compare the historical distribution of returns y_t and $y_{A,t}$. Second, we consider the associated autocorrelogram.

[20] Note that $y_{t+1} > y_{0,t}$, iff $A_{t+1} > HWM_t$.

[21] It could be rather misleading to analyse the correlation between both returns in this dynamic framework. For instance, for a unitary reset time, we would have $y_{A,t} = y_t - \alpha y_t^+$. We see immediately that the conditional correlation between $y_{A,t}$ and y_t for "small" return $y_{A,t} < 0$ (resp. "large" return $y_{A,t} > 0$) is equal to 1 [resp. 1], whereas the unconditional correlation between the returns is positive, but significantly smaller than 1, with a value function of α.

[Insert Figure 4: Historical Distributions of Returns]

The smoothed historical distributions of y_t and $y_{A,t}$ are given in the first panel of Figure 4 and the histogram of $y_{A,t}$ in the second panel. The presence of the management account explains the negative drift observed when passing from a positive portfolio return y_t to account A return. Indeed, the left part of the distribution is not impacted by the allocation scheme, whereas the right part is. The probability to observe high return is lower; the return distribution becomes more concentrated and skewed.
The nonlinear autoregressive effect due to the HWM barrier is difficult to detect from a standard linear analysis of serial dependence, but also from an analysis of the linear dependence between squared returns (see Figure 6). We observe a cycle effect in both autocorrelograms[22], which is just significant.

[Insert Figure 5: ACF and Squared ACF on Returns]

Let us now compare the characteristics of HF returns $y_{A,t+1}$, for different values of the performance fee rate α, $\alpha = 0\%, 10\%, 20\%, 50\%$, the limiting case $\alpha = 0\%$ corresponding to $y_{A,t+1} = y_{t+1}$. We fix the initial values to $A_0 = 100$, $B_0 = 10$. Finally, we set m to 1%, consider different underlying annualized Sharpe performance ratio[23] for the portfolio return $P = \sqrt{12} \times m/\sigma = 0.5, 1, 1.5$ and different reset times for the fund, i.e. $T = 24$ (2 years), 48(4 years), 72(6 years).

[Insert Table 1: Statistics on $y_A(T)$]

Table 1 provides the mean, variance, annualized Sharpe performance, skewness, excess kurtosis and $5\% - 95\%$ quantiles of the average class A return on period $(0, T)$, that is $y_A(T) = (A_T - A_0)/(TA_0)$. These summary statistics are obtained with $S = 10000$ replications for each Monte-Carlo design.
For a zero performance fee, the return of class A unit is equal to the return of the underlying portfolio, i.e. $y_A(T) = \frac{1}{T}\left[\prod_{t=1}^{T}(1 + y_t) - 1\right]$. For horizon $T \neq 1$, this return is no longer Gaussian and a convexity effect appears in the computation of the mean and the variance. For instance, we get:

$$E\left[y_A(T)\right] = \frac{1}{T}\left\{(1+m)^T - 1\right\} \simeq 1 + m + \frac{T-1}{2}m^2, \tag{3.3}$$

[22] This is a consequence of the threshold autoregressive effects in the HWM dynamics [see Tong (1983)].
[23] The Sharpe performance ratio measures the annualized excess return per unit of annualized risk.

for small mean m, and:

$$\begin{aligned}
V\left[y_A(T)\right] &= \frac{1}{T^2} V\left[\prod_{t=1}^{T}(1+y_t)\right] \\
&= \frac{1}{T^2}\left\{E\left[\prod_{t=1}^{T}(1+y_t)^2\right] - \left(E\left[\prod_{t=1}^{T}(1+y_t)\right]\right)^2\right\} \\
&= \frac{1}{T^2}\left\{\left[\sigma^2+(1+m)^2\right]^T - (1+m)^{2T}\right\} \\
&\simeq \frac{1}{T^2}\left[T(m^2+\sigma^2+2m) + \frac{T(T-1)}{2}(m^2+\sigma^2+2m)^2 - T(m^2+2m)\right. \\
&- \left.\frac{T(T-1)}{2}(m^2+2m)^2\right] \\
&\simeq \frac{\sigma^2}{2} + (T-1)2m\sigma^2,
\end{aligned}$$

for small m, σ of a same magnitude. The convexity effects on these moments and the associated Sharpe ratio can be checked on all rows of Table 3 corresponding to $\alpha = 0$. As expected from the design of management fees, the return distribution is shifted to the left. Thus, the mean, median and quantiles diminish when α increases. There is also a diminution of risk, since this distribution becomes more concentrated as observed on the values of the standard deviation and kurtosis. Finally, the distribution is right skewed for $\alpha = 0$, due to the convexity effect describe above, but the skewness diminishes when α increases due to the option interpretation of the HWM.

4 Endogeneous portfolio management

By considering i.i.d. Gaussian portfolio return in Section 3, we have implicitely assumed that the portfolio manager was investing in a kind of market portfolio, and in particular that his/her management strategy does not account for the existence of multiple accounts. The aim of this section is to discuss how the dynamics of account returns is modified with an endogenous investment strategy. In practice, the fund manager will account for an incentive mix such as reporting of good investor's performance, benefiting from the HWM on the management account, and controlling the risk of fund closure. In this section, we focus on mean-variance myopic strategies without taking into account the risk of fund closure. The strategies differ by the account value which is chosen as the main target. We consider the case of unitary reset times, where explicit strategies can be derived and analysed.

For illustration, let us assume that the fund manager invests only in a riskfree asset with zero riskfree rate and in a risky asset with i.i.d. Gaussian returns[24], denoted by y_t^*. With unitary reset time and the hurdle rate equal to the riskfree rate $y_{h,t} = y_{f,t} = 0$, the allocation between A and B accounts is given by (2.8):

$$\left\{ \begin{array}{l} A_{t+1} = A_t(1 + y_{t+1}) - \alpha A_t(y_{t+1})^+, \\ B_{t+1} = B_t(1 + y_{t+1}) + \alpha A_t(y_{t+1})^+. \end{array} \right. \tag{4.1}$$

where y_{t+1} is the portfolio return. Let us now consider the portfolio allocation at date t. The total budget is allocated between the two assets: $W_t = A_t + B_t = a_{0,t} + a_t$, where $a_{0,t}$ (resp. a_t) is the value invested in the riskfree asset (resp. risky asset). At date $t+1$, the portfolio value becomes:

$$W_{t+1} = a_{0,t} + a_t(1 + y_{t+1}^*) = W_t + a_t y_{t+1}^*.$$

We deduce the portfolio return as:

$$y_{t+1} = \frac{W_{t+1} - W_t}{W_t} = \delta_t y_{t+1}^*, \tag{4.2}$$

where $\delta_t = a_t/(A_t + B_t)$ denotes the fraction invested in risky asset. By substitution in (4.1), we get:

$$\left\{ \begin{array}{l} A_{t+1} = A_t + \delta_t \left[A_t y_{t+1}^* - \alpha A_t (y_{t+1}^*)^+\right], \\ B_{t+1} = B_t + \delta_t \left[B_t y_{t+1}^* + \alpha A_t (y_{t+1}^*)^+\right], \end{array} \right. \tag{4.3}$$

and

$$A_{t+1} + B_{t+1} = (A_t + B_t)(1 + \delta_t y_{t+1}^*). \tag{4.4}$$

Let us now consider a myopic mean-variance investor[25], with absolute risk aversion[26] η. The optimal allocation depends on the account he/she is interested in.

i) If the account of interest is the total account $A + B$, the optimal allocation is the standard mean-variance efficient allocation [Markovitz (1952)] given by:

$$\delta_t^* = \frac{1}{A_t + B_t} \frac{1}{\eta} \frac{E(y_{t+1}^*)}{V(y_{t+1}^*)}. \tag{4.5}$$

[24]This assumption is compatible with the standard Black-Scholes model.

[25]This corresponds to the two periods behavior analyzed in Christoffersen, Musto, Yilmaz (2013).

[26]We assume that the risk aversion is constant. Thus, the fund manager does not change his/her risk aversion as function of the size of the managed portfolio, or his/her past successes.

Under the i.i.d. Gaussian assumption, the value invested in the risky asset is time dependent. As usual, the portfolio manager is proportionally investing less in risky asset, when $A_t + B_t$ increases. This total change in portfolio value, $(A_t + B_t)\delta_t^* y_{t+1}^* = \frac{1}{\eta}\frac{E(y_{t+1}^*)}{V(y_{t+1}^*)} y_{t+1}^*$, is i.i.d. Gaussian, whenever y_{t+1}^* is i.i.d. Gaussian.

ii) If the account of interest is account B, the efficient allocation becomes:

$$B_t \delta_{B,t}^* = \frac{1}{\eta} \frac{E[y_{t+1}^* + \alpha\gamma_t (y_{t+1}^*)^+]}{V[y_{t+1}^* + \alpha\gamma_t (y_{t+1}^*)^+]}, \tag{4.6}$$

where $\gamma_t = A_t/B_t$. As expected, the allocation is different from the standard allocation δ_t^*. It changes in time due to the evolution of both accounts (A_t, B_t). Moreover, the ratio between this allocation and the standard one shows a double effect: the effect of portfolio size, which diminishes from $A_t + B_t$ to B_t and implies an increase of the quantity invested in the risky asset; the effect of the optional component depends on time and tail distribution of the underlying return. The global effects is unclear.

For instance, if γ_t is large, the investment in risky asset will become very small. Contrary to a usual belief, it is not guaranteed that giving an option to the fund manager makes him/her willing to take risk, even if he/she focus on the management account. This is compatible with the recent literature on incentives, in which several authors arrive to similar conclusions for instance by changing the utility function [Ross(2004)], introducing an infinite horizon [Panageas, Westerfield (2009)], or considering an option on the portfolio itself, not on the HWM [Carpenter(2000)]. As noted in this literature, if the value of account B becomes large, that is "if the HF manager has a substantial personal investment in the fund, this will inhibit excessive risk taking" [Fung, Hsieh (1999)]". This can lead to surprising consequences: for instance, at initial date 0, a small value of B_0 can be an incentive to take risk at the beginning; equivalently, introducing more frequent reset times with rather small B_0 can be an incentive to take risk regularly (ceteris paribus, i.e. for fixed gamma).

In addition to this size effect, there is the optional feature since account B is a portfolio in the underlying asset and a call written on this asset. As noted in Hodder, Jackwerth (2007), this "generates risk-taking below the HWM, when the manager tries to assure that his/her incentive option will finish in the money". But "at performance levels modestly above the HWM, he/she reverses that strategy and opts for very low risk positions to lock in the option payoff".

iii) If the account of interest is account A, the efficient allocation is:

$$A_t \delta^*_{A,t} = \frac{1}{\eta} \frac{E[y^*_{t+1} - \alpha(y^*_{t+1})^+]}{V[y^*_{t+1} - \alpha(y^*_{t+1})^+]}. \tag{4.7}$$

This allocation depends on the evolution of account A only. The change in account value is:

$$\begin{aligned} A_{t+1} - A_t &= A_t \delta^*_{A,t}[y^*_{t+1} - \alpha(y^*_{t+1})^+] \\ &= cst\ [y^*_{t+1} - \alpha(y^*_{t+1})^+]. \end{aligned}$$

If the risky return is i.i.d. Gaussian, this change in value is still i.i.d., but no longer Gaussian.

iv) Finally, the fund manager can also own at date t a fraction ν_t of the fund, i.e. of account A [see e.g. the discussion in Fung, Hsieh (1999), or Kouwenberg, Ziemba (2007)]. Then his/her account of interest is $\nu_t A_{t+1} + B_{t+1}$, which leads to a mix of cases *ii)* and *iii)* above, if ν_t is taken exogenous.

In practice, it is difficult to know what is really the criterion selected by the fund manager. This is likely a mix, which takes into account his/her individual wealth, that is account B, and probably a fraction of account A. But he/she has also to account for the rankings of fund managers, which are regularly published in the press, and are a strong incentive for considering the preferences of fund investors[27]. To illustrate the consequences of these portfolio managements on accounts returns, we consider risky returns following a Gaussian distribution with mean $m = 1\%$, and volatility $\sigma = 3.46\%$. We set the performance fee rate at $\alpha = 25\%$, with unitary reset time and the absolute risk aversion at $\eta = 0.08$. The initial values of the accounts are $A_0 = 100$, $B_0 = 10$. The length of the simulation period is $T = 72$. The explicit expressions of the mean and variance-covariance matrix of $[y^*_{t+1}, (y^*_{t+1})^+]$ are derived in Appendix 2. They are used to compute the optimal allocations.

Figure 6 displays the dynamics of efficient allocation in risky asset for the three strategies, that are δ^*_t, $\delta^*_{A,t}$, $\delta^*_{B,t}$.

[Insert Figure 6: Efficient Allocation in Risky Asset]

[27]see Chevalier, Ellison (1997) for a deeper discussion of the agency conflict between fund investors and fund companies.

The size effect is dominant in the three situations, where the allocation in risky asset diminishes in time. This shows the main role of the reset frequencies. If this frequency is the year, this might explain the empirical fact around Christmas discussed in Agarwal, Daniel, Naik (2011). We provide in Figure 7 the historical distributions of account A return when the managed portfolio is the market itself [$\delta_t = 1$], and for endogenous portfolio management with objectives $A+B$ and A, respectively. An endogenous portfolio management has clearly two effects: an increase of the discontinuity at zero and a more concentrated distribution.

[Insert Figure 7: Historical Distribution of Returns]

However, the myopic mean-variance behaviour is not sufficient to create highly significant short term correlation on returns as shown on Figure 8. The serial correlation, which can be observed on real data, are more likely due to either the nonlinear dynamics of the basic assets introduced in the portfolio, or a non myopic, intertemporal portfolio management [see Darolles, Gourieroux (2013)]. In this respect, it could be interesting to reproduce the same simulation exercice with a market return conditionally Gaussian, but including an ARCH effect. Indeed, this volatility effect could create linear serial correlation after passing by the nonlinear filter of HWM and provision account.

[Insert Figure 8: ACF on Return]

5 Conclusion

The selected HWM scheme for allocating gains and profits between the investor's account and management account has a significant impact on the performance of the investor's account. This effect is twofold. There is a direct effect on account A return due to the nonlinear scheme, especially the barrier effect included in the HWM. There is an additional indirect effect, when the fund manager ajusts his/her portfolio management to this scheme. These effects explain a part of the empirical facts observed on hedge fund returns, such as the skewness of the return distribution, its discontinuity at zero, or some cyclical serial correlation. The special type of nonlinearity involved in this scheme can also lead to misleading interpretations for the analysis using thresholds effect, such as the study of market timing ability, or the comparison of unconditional correlations with correlations restricted to period of poor (or large) performances.

The hedge fund industry is known for its lack of transparency. Surprisingly, a lot of information is available in the prospectus of a fund, especially the scheme of allocation between the different accounts. A wise investor should analyse the consequences of these schemes on the performance of his own account before any investment in hedge funds. Similarly, it is important to take into account these schemes in the academic study of HF returns and of the behaviour of HF portfolio managers. In other terms, we have to correct the results for the management account bias and the provision account bias, and these corrections will differ due to the variability of schemes followed by individual hedge funds.

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Appendix 1
Long term analysis of HWM allocation scheme

In HWM scheme (2.8), the dynamics of A account does not depend on the periodic reset of B account. The Net Asset Value (NAV) dynamics can be written as:

$$A_{t+1} = \left[1 + y_{t+1} - \alpha(y_{t+1} - y_{h,t})^+\right] A_t, \tag{1.1}$$

and (A_t) is an autoregressive process with stochastic autoregressive coefficient. Let us assume $y_{h,t} = 0$, and i.i.d. portfolio returns, with $y_t > -1/(1-\alpha)$. We can write:

$$A_{t+1} = \exp\left[\log(1 + y_{t+1} - \alpha y_{t+1}^+)\right] A_t, \tag{1.2}$$

and by recursive substitution:

$$A_t = A_0 \exp\left[\sum_{\tau=1}^{t} \log(1 + y_\tau - \alpha y_\tau^+)\right]. \tag{1.3}$$

Following the approach used in Nelson (1990), Bougerol, Picard (1992), we can determine the Lyapunov exponent of process (A_t) as follows. We have:

$$A_t = A_0 \exp\left[t \frac{1}{t} \sum_{\tau=1}^{t} \log(1 + y_\tau - \alpha y_\tau^+)\right] \tag{1.4}$$

$$\simeq A_0 \exp\left[tE\log(1 + y_t - \alpha y_t^+)\right], \tag{1.5}$$

for large t, by the Law of Large Number. Thus, the long term return on class A account is:

$$r_{\infty,A} = \lim_{t\to\infty} \frac{1}{t} \log(A_t/A_0) = E\log(1 + y_t - \alpha y_t^+). \tag{1.6}$$

Since $log(1+x) \leq x$, we note that:

$$r_{\infty,A} \leq E(y_t - \alpha y_t^+) = Ey_t - \alpha E y_t^+ \leq (1-\alpha)Ey_t. \tag{1.7}$$

As expected, this rate is strictly smaller than the long term rate on the portfolio crudely adjusted for performance rate α, i.e. $(1-\alpha)Ey_t$. It can also be significantly smaller than $Ey_t - \alpha E(y_t^+)$, with a difference which increases with the variability on (y_t).

Appendix 2

First- and Second-Order Moments of the Truncated Normal

Let us consider a Gaussian variable with mean m and unitary variance 1. The variable can be written as: $Y = m + U$, $U \sim N(0,1)$.

i) First-Order Moments

We have:

$$\begin{aligned} E[Y^+] &= E[(m+U)^+] \\ &= \int_{-m}^{\infty} (m+u)\varphi(u)du \\ &= m\int_{-m}^{\infty} \varphi(u)du + \int_{-m}^{\infty} u\varphi(u)du \\ &= m[1-\Phi(-m)] - \int_{-m}^{\infty} \frac{d\varphi(u)}{du}du \\ &= m\Phi(m) + \varphi(m), \end{aligned}$$

where φ [resp. Φ] is the pdf [resp. cdf] of the standard normal, by using the symmetry of the standard normal. Therefore: $[EY, EY^+] = [m, m\Phi(m) + \varphi(m)]$.

ii) Second-Order Moments

Let us consider the expected squared variables, that are: $E[Y^2]$, $E[YY^+]$, $E[(Y^+)^2]$, and introduce $Y^- = Max(-Y, 0)$. We have: $Y = Y^+ - Y^-$ and $E[Y^-Y^+] = 0$. Thus:

$$\begin{aligned} E[Y^2] &= 1 + m^2 \\ E[YY^+] &= E[(Y^+)^2]. \end{aligned}$$

Therefore, all second-order moments are directly deduced from the quantity $E[(Y^+)^2]$. We get:

$$\begin{aligned} E[(Y^+)^2] &= E[((m+U)^+)^2] \\ &= \int_{-m}^{\infty} (m+u)^2\varphi(u)du \\ &= m^2\int_{-m}^{\infty} \varphi(u)du + 2m\int_{-m}^{\infty} u\varphi(u)du + \int_{-m}^{\infty} u^2\varphi(u)du \\ &= m^2\Phi(m) + 2m\varphi(m) - \int_{-m}^{\infty} ud\varphi(u) \\ &= m^2\Phi(m) + 2m\varphi(m) - u\varphi(u)]_{-m}^{\infty} + \int_{-m}^{\infty} \varphi(u)du, \\ &= m^2\Phi(m) + m\varphi(m) + \Phi(m). \end{aligned}$$

We deduce:

$$
\begin{aligned}
V\begin{bmatrix} Y \\ Y^+ \end{bmatrix} &= E\left[\begin{pmatrix} Y \\ Y^+ \end{pmatrix}\begin{pmatrix} Y, & Y^+ \end{pmatrix}\right] - E\begin{pmatrix} Y \\ Y^+ \end{pmatrix} E\begin{pmatrix} Y, & Y^+ \end{pmatrix} \\
&= \begin{pmatrix} 1 & \Phi(m) \\ \Phi(m) & m^2\Phi(m) + m\varphi(m) + \Phi(m) - [m\Phi(m) + \varphi(m)]^2 \end{pmatrix}.
\end{aligned}
$$

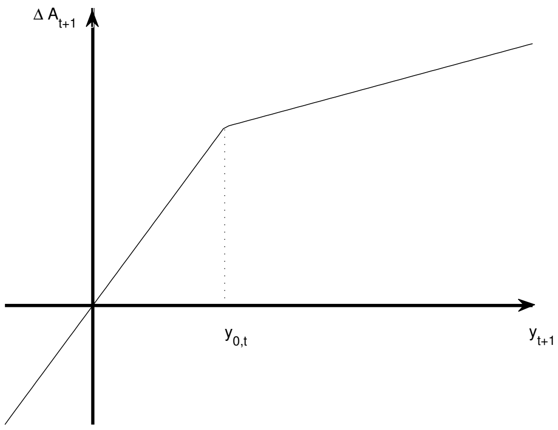

Figure 1: ΔA_{t+1} as a function of y_{t+1} (unitary reset time)

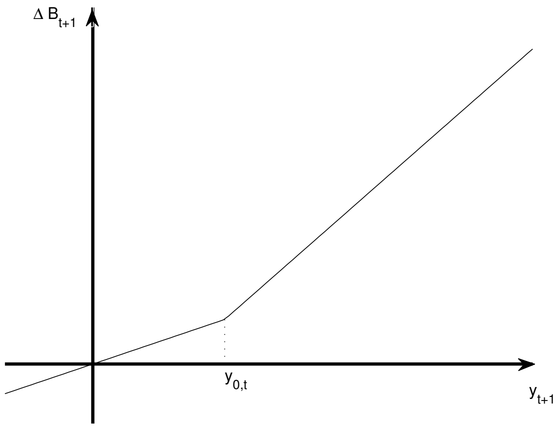

Figure 2: ΔB_{t+1} as a function of y_{t+1} (unitary reset time)

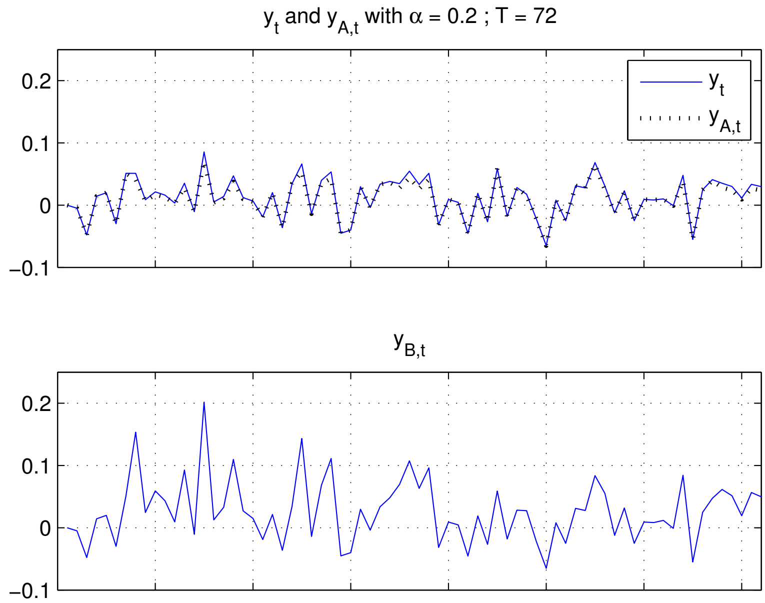

Figure 3: Return Dynamics

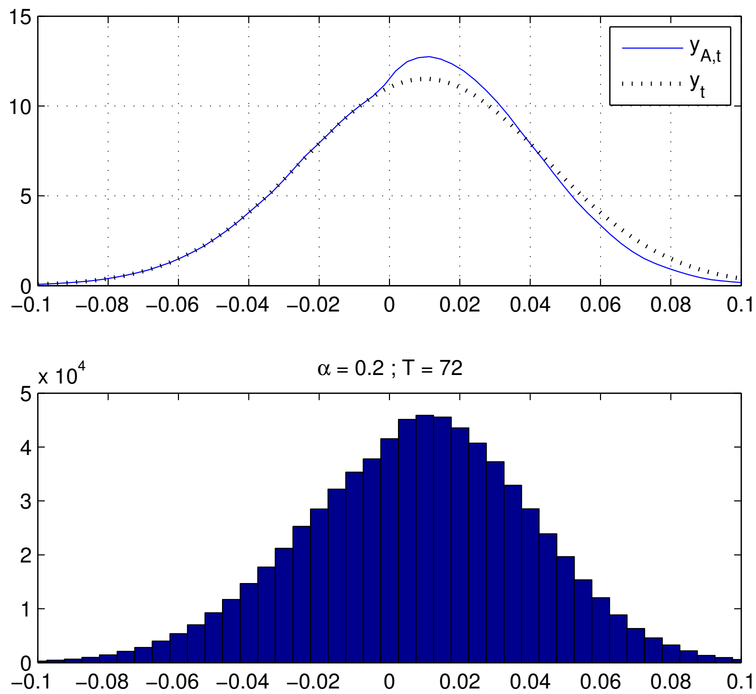

Figure 4: Historical Distributions of Returns

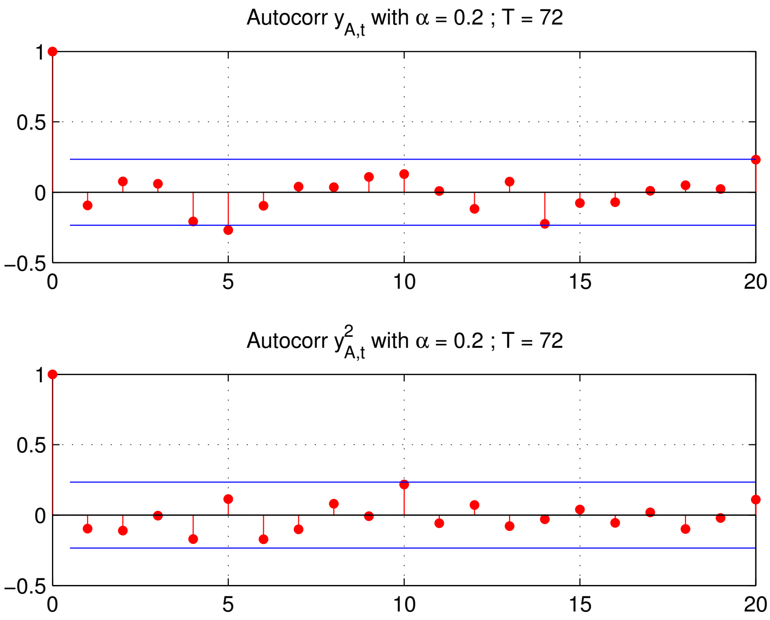

Figure 5: ACF on Return and Squared Return

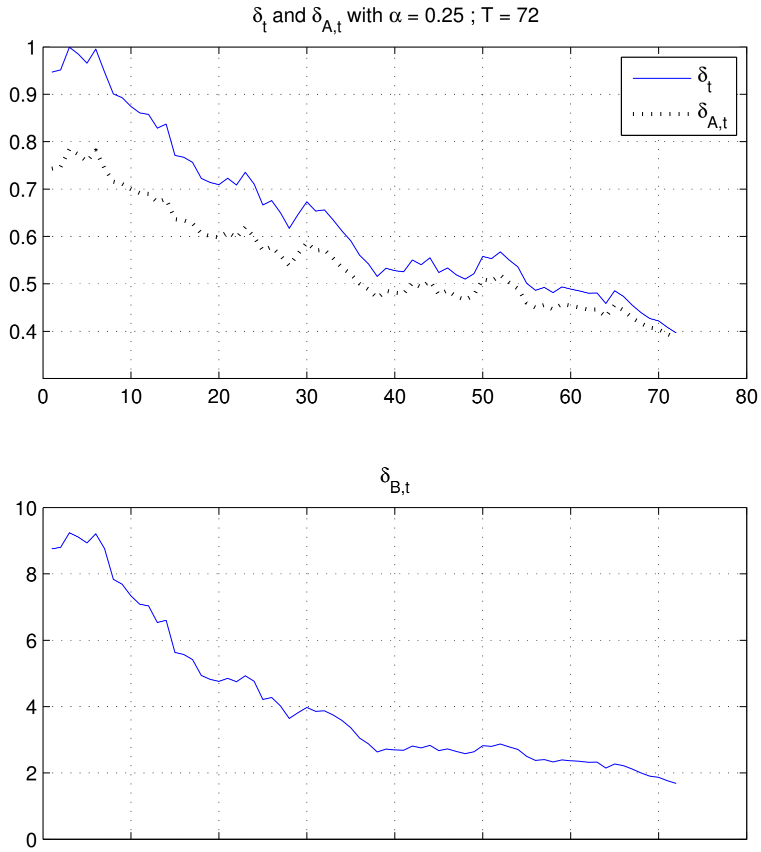

Figure 6: Efficient Allocation in Risky Asset

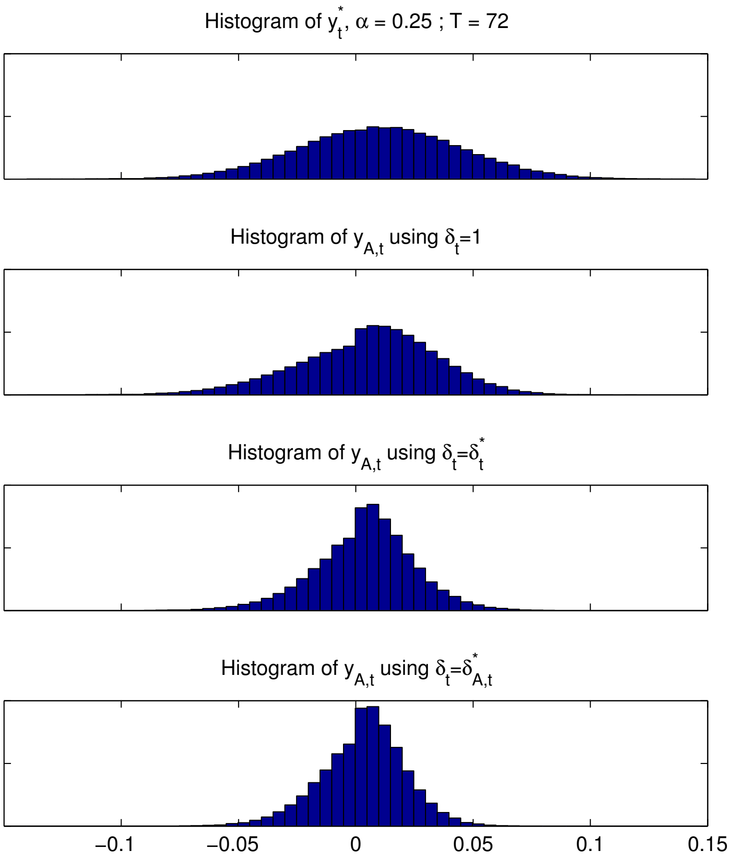

Figure 7: Historical Distribution of Returns

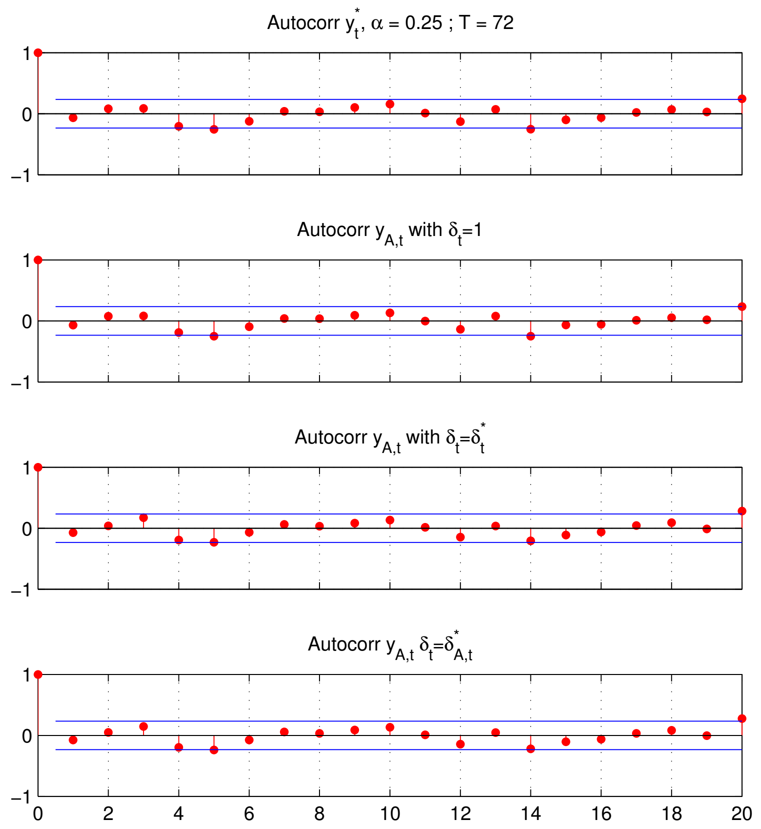

Figure 8: ACF on Return

Panel A: $T = 24$ **(2 years)**

Incentive fee α level	Mean	SD	Sharpe	Median	Skew	Exc. Kurt.	5%-Quant.	95%-Quant.
Sharpe ratio = 0.5								
0%	0.0116	0.0187	0.4375	0.0082	1.0815	2.0117	-0.0130	0.0472
10%	0.0095	0.0166	0.4028	0.0069	0.9282	1.4905	-0.0132	0.0406
20%	0.0074	0.0147	0.3585	0.0056	0.7649	1.0259	-0.0134	0.0346
50%	0.0020	0.0100	0.1408	0.0018	0.1794	0.0138	-0.0142	0.0188
Sharpe ratio = 1								
0%	0.0114	0.0091	0.8867	0.0104	0.5458	0.4780	-0.0021	0.0279
10%	0.0099	0.0081	0.8644	0.0092	0.4594	0.3658	-0.0023	0.0244
20%	0.0084	0.0072	0.8337	0.0079	0.3605	0.2722	-0.0026	0.0211
50%	0.0044	0.0047	0.6541	0.0044	-0.0748	0.2169	-0.0036	0.0121
Sharpe ratio = 1.5								
0%	0.0113	0.0060	1.3325	0.0109	0.3794	0.2175	0.0021	0.0220
10%	0.0100	0.0053	1.3234	0.0096	0.3242	0.1782	0.0017	0.0194
20%	0.0087	0.0047	1.3077	0.0084	0.2604	0.1514	0.0013	0.0168
50%	0.0049	0.0030	1.1832	0.0049	-0.0416	0.2505	0.0001	0.0098

Panel B: $T = 48$ **(4 years)**

Incentive fee α level	Mean	SD	Sharpe	Median	Skew	Exc. Kurt.	5%-Quant.	95%-Quant.
Sharpe ratio = 0.5								
0%	0.0129	0.0170	0.3809	0.0094	1.4832	3.5525	-0.0073	0.0462
10%	0.0106	0.0145	0.3675	0.0080	1.2944	2.7091	-0.0075	0.0389
20%	0.0085	0.0123	0.3468	0.0066	1.0988	1.9638	-0.0077	0.0322
50%	0.0031	0.0074	0.2141	0.0027	0.4239	0.2953	-0.0082	0.0161
Sharpe ratio = 1								
0%	0.0128	0.0081	0.7888	0.0120	0.6983	0.7160	0.0012	0.0280
10%	0.0111	0.0070	0.7901	0.0104	0.6100	0.5503	0.0008	0.0240
20%	0.0094	0.0060	0.7864	0.0090	0.5141	0.4055	0.0005	0.0203
50%	0.0050	0.0035	0.7168	0.0049	0.1242	0.1760	-0.0006	0.0110
Sharpe ratio = 1.5								
0%	0.0128	0.0054	1.1905	0.0124	0.4623	0.2722	0.0047	0.0225
10%	0.0112	0.0046	1.2056	0.0109	0.4086	0.2061	0.0041	0.0195
20%	0.0096	0.0039	1.2178	0.0094	0.3512	0.1498	0.0035	0.0166
50%	0.0054	0.0022	1.2134	0.0054	0.1246	0.0832	0.0019	0.0092

Panel C: $T = 72$ **(6 years)**

Incentive fee α level	Mean	SD	Sharpe	Median	Skew	Exc. Kurt.	5%-Quant.	95%-Quant.
Sharpe ratio = 0.5								
0%	0.0147	0.0181	0.3325	0.0104	2.1096	8.0992	-0.0046	0.0489
10%	0.0120	0.0148	0.3312	0.0087	1.8247	6.0745	-0.0048	0.0401
20%	0.0096	0.0121	0.3242	0.0073	1.5464	4.3958	-0.0050	0.0326
50%	0.0038	0.0065	0.2424	0.0032	0.6854	0.9820	-0.0056	0.0154
Sharpe ratio = 1								
0%	0.0147	0.0084	0.7090	0.0135	0.9212	1.5418	0.0031	0.0302
10%	0.0125	0.0071	0.7227	0.0117	0.8129	1.2204	0.0026	0.0254
20%	0.0105	0.0059	0.7334	0.0099	0.7005	0.9372	0.0021	0.0211
50%	0.0055	0.0031	0.7238	0.0054	0.2917	0.3452	0.0007	0.0109
Sharpe ratio = 1.5								
0%	0.0146	0.0056	1.0754	0.0141	0.6007	0.6736	0.0065	0.0245
10%	0.0126	0.0047	1.1035	0.0122	0.5348	0.5452	0.0057	0.0208
20%	0.0107	0.0039	1.1304	0.0104	0.4668	0.4310	0.0049	0.0175
50%	0.0058	0.0020	1.1872	0.0057	0.2266	0.1934	0.0027	0.0092

Table 1: Statistics on $y_A(T)$