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The Productive Government Spending Multiplier, In and Out of the Zero Lower Bound

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The Productive Government Spending Multiplier, In and Out of the Zero Lower Bound

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Abstract

Recently, a series of papers have argued that output multipliers of government spending can be potentially large during times when the Zero Lower Bound on nominal interest rates is binding (Christiano et al. (2011)). This literature generally considers "excess-savings" liquidity traps and identifies the reaction of real interest rates —that follows the effect of government purchases on marginal cost and, hence, inflation —as the main channel of propagation. Here, I show that taking explicitly into account the fact that government spending is productive can mitigate this result. The higher the share of productive government spending in total stimulus spending, the lower the gap between the government spending multipliers in and out of the Zero Lower Bound. Furthermore, a sufficient share of productive government spending in total stimulus spending will imply a higher multiplier when the Zero Lower Bound is not binding. It follows that the government spending multiplier need not be unusually large when the economy is in an "excess-savings" liquidity trap. In a "expectations-driven" liquidity trap (Mertens & Ravn (2010)) however, the government spending multiplier will be larger than in normal times for a sufficient share of productive government spending. But for this to happen, a rise in inflation is still needed. While the predictions of the model with an "expectations-driven" liquidity trap are difficult to compare with the data, I show that the model with an "excess-savings" liquidity trap is at odds with recent empirical evidence on the behavior of key macroeconomic variables in a recession. In contrast, the simple New-keynesian model augmented with a sufficient share of productive government spending is qualitatively consistent with aforementioned evidence.

"So I would suggest to you that in the fact that interest rates are down not up in Britain, down not up in the United States you see that it is economic weakness that is the greatest threat to the long-run fiscal health of both of our nations and that is why addressing economy weakness, in part through necessary public investment starting from cancelling imprudent and excessive slashing of public budgets has to be the first and major priority." Larry Summers, 2012

"In a depressed economy, with the government able to borrow at very low interest rates, we should be increasing public investment." Paul Krugman, 2012

"We can create some room to invest in things that make America stronger, like rebuilding America's infrastructure." Timothy F. Geithner, 2012

1 Introduction

The topic of government spending multipliers has received considerable attention since the last financial crisis compelled governments to engage in stimulative fiscal policy. A lot of work has been done on the empirical as well as on the theoretical side. On the empirical side, studies using Vector Auto Regressions (VARs) on post-World War II samples seem to converge towards a government spending multiplier on output in the neighborhood of [0.8;1.7]. But most of these studies implicitly assume that there is one unique, time-invariant government spending multiplier. Some exceptions can nevertheless be found, such as Perotti (2005) and Bilbiie et al. (2008). Those papers look at the difference between multipliers before and during the Volcker-Greenspan period (1983-2008). They find that fiscal policy was less effective in the second part of the sample. But recent evidence has been put forward by Auerbach & Gorodnichenko (2012) and Bachmann & Sims (2012) in favor of a multiplier that is higher in recession times, while close to zero in normal times[1]. Those papers use non-linear VAR techniques that allow the multiplier to be dependant on some definition of the output gap.

On the theoretical side, researchers have been struggling to reconcile those results with existing models. In fact, considering the basic RBC model, government consumption crowds out private consumption du to the wealth

[1] On the other hand, considering a narrative approach, Owyang et al. (2013) find no or little evidence of larger government spending multipliers in periods of economic slack.

effect implied by future taxes and the output multiplier is strictly less than one. This appears to be in contrast with, for instance, Blanchard & Perotti (2002) who find a positive effect of government spending on consumption and thus an output multiplier larger than 1. Therefore, there has been several attempts to devise models yielding a positive effect of government spending on consumption. One first solution is to add some non-Ricardian features to the baseline New-Keynesian model. In particular, one can add "Rule of thumb" consumers (Galí et al. (2007)), who —in contrast to Ricardian consumers who choose their consumption path with respect to their permanent income—consume their real wage each period. In this setting, an increase in government spending will induce higher real wages which, in turn, will induce constrained individuals to consume more. A straightforward feature of this model is that the impact of government spending on consumption will be an increasing function of the share of constrained individuals. Nevertheless, the fact reamins that the government must finance its spending. If the government must run a balanced budget, it has to raise taxes at the same time, which will drive down the consumption of ricardian households. Therefore, one feature that is also needed to have a positive government consumption multiplier is that the expenditure is more or less debt financed. In this framework, several "special" features are then needed to yield the expected result. This is perhaps why another approach has been given far more attention. If we stay in the familiar, one representative agent environment, then —unless you assume non-separable preferences over consumption and leisure in a model with sticky prices as in Bilbiie (2011)—you end up with a negative multiplier on consumption. The reason is that an increase in government spending will imply higher labor demand, yielding higher equilibrium hours and real wages. Higher wages will lead to higher expected marginal costs, which, in turn, will lead to higher inflation. If we suppose that the Central Bank follows a Taylor rule such as to ensure determinacy of a unique rational expectations equilibrium, then it has to set its interest rate so that it will react more than one for one with inflation. Therefore, the increase in inflation generated by government consumption will lead to a higher nominal interest rate, which will reduce consumption through the Euler Equation. But this mechanism is muted when the interest rate is stuck at the Zero Lower Bound. In this case, higher inflation leads to a lower (more negative) real interest rate, which stimulates consumption today, again through the Euler Equation. Woodford (2011) provides a comprehensive survey of this literature.

However, those studies usually assume wasteful (or, perhaps, utility enhancing) government spending, with some notable exceptions (Baxter & King (1993), Leeper et al. (2010)). But usually, when engaging into stimulative fiscal policy in times of economic stress, the government is not merely hiring people to dig holes then fill it. As Bachmann & Sims (2012) have recently shown, the mix of government consumption / government investment usually shifts during recessions. In fact, during such times the government tend to spend more in investment than in consumption/public goods[2]. In addition, some economists participating in the lively economic debate on the solutions to the Great Recession, among which Paul Krugman and Larry Summers (see the quotations in the preamble), have advocated increases in public investment. The argument is that you can kill two birds with one stone : in the short term you increase aggregate demand and in the medium/long term you enhance growth prospects through better infrastructure. In contrary to utility enhancing government consumption, government investment usually enter the aggregate production function through the stock of government capital. One might then conjecture that, for a given number of hours worked, marginal cost will decrease for the representative firm because of the induced increase in marginal productivity of labor. Indeed, Bachmann & Sims (2012) show that labor productivity rises after a government spending shock in a typical recession. Therefore, if marginal cost is expected to decrease due to the increase in public capital, this effect might counteract the rise in marginal cost due to higher real wages and lower marginal productivity due to decreasing returns with respect to labor in the production function.

Following Eggertsson & Woodford (2003), it is also common in this literature to consider "excess-savings" liquidity traps. The shock that drives the economy into a liquidity trap is generally assumed to affect the discount rate of the representative agent and typically has a simple markov structure. Agents want to save more, and since aggregate savings are zero, a fall in output and inflation are necessary to bring back savings to zero. Another route has been taken by Mertens & Ravn (2010) who, building on Schmitt-Grohe et al. (2001), consider an expectation driven liquidity trap.

[2]This has been exemplified by the recent case of the united states. See for example Leeper et al. (2010). One can also invoke the French "Grand Emprunt" of 2010 which is aimed at investment in various areas of public infrastructure (academic research, transport services, energy and numeric technology for the main ones). More recently, Chinese authorities have approved a Rmb 1 trillion plan of infrastructure investment after concerns over a "hard-landing" of the Chinese economy became increasingly put forward by analysts. We can also cite the recent example of Sweden, which included public infrastructure in its new stimulus policy.

In this setting, the results are essentially the reverse to the ones obtained under an excess-savings liquidity trap. Since, as Mertens & Ravn (2010) have argued, the effects of government spending in a liquidity trap depend crucially on the nature of the underlying shock, I will also study the effects of productive government spending in an expectation-driven liquidity trap. In fact, expectation-driven liquidity traps occur under high probability of persistence for the underlying shock, which, as I will show later, can be considered as more in adequation with the ongoing episode.

The paper will be structured as follows. In section 2, I present some recent empirical evidence on the effects of government spending in a recession. In section 3, I develop a DSGE model with productive government spending. In section 4, I investigate the effects of government spending in the model with flexible prices. In section 5, I do the same thing in a New-Keynesian model with sticky prices. In section 6, I investigate these effects at the Zero Lower Bound, distinguishing between an excess-savings and an expectations-driven liquidity trap. In section 7, I study the American Recovery and Reinvestment Act of 2009 to get a rough estimate of the share of productive government spending and the likely magnitude of the implied output multiplier. Finally, in section 8 I study the optimal government spending policy at the Zero Lower Bound.

2 Some Empirical Evidence

There is by now some form of consensus on the idea that the effects of government spending are larger when the economy is depressed. This is all the more true when this depression push the Central Bank to lower its main interest rate up to the effective Zero Lower Bound. Empirically, it is hard to gauge the magnitude of the multiplier when nominal rates have only been pinned at 0 three times in recent history (Most of developped countries during the Great Depression, United States in the Great Recession, Japan during the "Lost Decade(s)"). One way to circumvent this lack of data is to look at cross-country evidence, as has been done by Almunia et al. (2009). They find an output multiplier as large as 2.5. The other approach is to conjecture that, while the Zero Lower Bound may not be a binding constraint, the effects of government spending might be different in a depressed economy. This is what Bachmann & Sims (2012) and Auerbach & Gorodnichenko (2012) do. Using a non-linear SVAR, they calculate the response of main macroeconomic variables to a government spending shock during a typical recession. I will focus here on the response of the variables that are central to the mechanisms at work in the class of models most commonly used for macroeconomic policy discussion : New-Keynesian models. I will therefore focus on the responses of prices, private productivity and consumption to a government spending shock in a typical recession. Before that, I want to display an interesting result of Bachmann & Sims (2012). In Figure 8 (in the Appendix), we can see that when government spending increases in the recession, the share of government investment usually increases also.[3] This feature is not taken into account in papers looking at the effects of government spending at the Zero Lower Bound.

The effect on marginal cost
In a standard New Keynesian model without productive government spending marginal cost rises unambiguously after a government spending shock. Since marginal cost is just a markup over the inverse of the marginal productivity of labor, with decreasing returns to scale increased utility-enhancing government spending implies a higher labor demand and thus lower marginal productivity of labor, which translates into higher marginal cost. Higher labor demand also induces higher real wages, which further increases marginal cost. In contrast, Bachmann & Sims (2012) find a positive effect of government spending on private productivity in a recession (see Figure 9 in the Appendix).

The effect on prices
If we feed the effect on marginal cost that has been found empirically in a New Keynesian model (through the New Keynesian Phillips Curve that relates inflation to marginal cost), we would find a deflation due to lower marginal costs. Since in the baseline New Keynesian model government spending drives up marginal cost, it generates inflation. In fact, Auerbach & Gorodnichenko (2012) find a negative effect of government spending on prices in a recession (see Figure 10 in the Appendix).

The effect on consumption
Suppose now we feed the decrease of prices in a New Keynesian model with a binding constraint on nominal rates. This would yield a negative multiplier on private consumption through higher real interest rates (since nominal

[3] What we generally call "government spending" is termed Government Consumption Expenditures and Gross Investment (GCEGI) in NIPA tables. Using the acronym, Figure 8 shows that the ratio GI/GCEGI increases in a recession.

rates are pinned at zero, deflation means more negative inflation and thus higher real rates). However, Auerbach & Gorodnichenko (2012) show that private consumption is usually crowded in and not out in a recession, and thus *a fortriori* when the Zero Lower Bound is binding (see Figure 11).

These empirical findings put into question the mechanisms through which large government spending multipliers are usually obtained. The necessary rise in marginal cost and prices in particular is not supported by the data. Since productive government spending seems to represent a higher fraction of government spending in a recession, we might want to ask the following question : to what extent can the introduction of productive government spending bring the model closer (at least qualitatively) to the data?

3 A Model with Productive Government Spending

The model I will study will be based almost entirely on Christiano et al. (2011) except for a few details. In particular, I will introduce productive government spending that enters contemporaneously in the private production function as in Barro (1990) or, more recently, Linnemann & Schabert (2006). In order to keep constant returns to scale, I assume decreasing returns with respect to the labor factor.

3.1 Households

There is a large number of identical households, who solve the following maximization problem:

$$\max_{C_t, N_t, B_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{[C_t^{\gamma} L_t^{1-\gamma}]^{1-\sigma} - 1}{1-\sigma} + v(G_t^U) \right\}, \tag{1}$$

where C_t denotes a Dixit-Stiglitz aggregator of differenced private goods with elasticity of substitution equal to ν. L_t denotes hours of leisure and the household is subject to a time endowment constraint:

$$N_t + L_t \leq 1,$$

where N_t is aggregate hours worked. Finally, G_t^U is utility-enhancing government spending. The form taken by the function $v(.)$ will be discussed in section 8. I postpone the discussion since the inclusion of this feature will have no effect on the aggregate dynamics of this economy. Because the utility the consumer gets from consumption is additively separable and since the aggregator is homothetic, the maximization problem can be broken down in two sub-problems. The first one is the static problem of consumption allocation through varieties given a level of total expenditures. This yields the following demand function for each good:

$$C_t(\omega) = C_t \left(\frac{P_t(\omega)}{P_t} \right)^{-\nu}$$

For a given level of total consumption, the demand of variety ω is a decreasing function of this variety's relative price, with ν being the elasticity of substitution between varieties. The second one is the dynamic problem of total consumption and labor supply subject to a budget constraint, which writes:

$$(1 + I_t)^{-1} B_{t+1} + P_t C_t = B_t + W_t N_t - T_t + \mathcal{P}_t,$$

where B_t denote one period riskless bonds, yielding an interest rate I_t. P_t,C_t,N_t and W_t denote, respectively, the price level, consumption, hours worked and the nominal wage. Finally, $\mathcal{P}_t$ denotes profits accruing from firms. The household uses his net income (labor income plus profits minus lump-sum taxes) and the bonds carried over from period $t-1$ to pay for current consumption and bonds giving one unit of the numéraire in period $t+1$. Besides giving more simple analytic expressions for the multiplier(s), it has also being argued by economists working on public finance that introducing distortionary taxation in a model where it is not desired can lead to erroneous results. Therefore, lump-sum taxation may be considered as the least harmful specification for tax policy in a DSGE model.[4] The first order conditions for the problem yield the following equations:

$$U_C(C_t, L_t) = \beta \mathbb{E}_t \left\{ (1 + I_t) \frac{P_t}{P_{t+1}} U_C(C_{t+1}, L_{t+1}) \right\}, \tag{2}$$

$$U_L(C_t, L_t) = U_C(C_t, L_t) \frac{W_t}{P_t}. \tag{3}$$

The first one is the Euler Equation governing the intertemporal allocation of consumption as a function of interest rates, while the second one is the intratemporal condition linking consumption and hours worked. We now turn to the description of the representative firm.

[4]See Werning (2007) on this subject. In particular, he argues that distortionary taxation is only relevant in a model of heterogeneous agents. Since I develop a representative agent model, lump-sum taxation is the most desired form of taxation.

3.2 Firms

There is a mass one of firms (indexed by ω), each producing a differentiated good with the same technology:

$$Y_t(\omega) = (G_t^P)^\zeta (N_t(\omega))^\eta,$$

with the condition that $\eta + \zeta = 1$. For simplicity, I abstract with technological shocks. The fact that G_t^P enters the private production function is meant to capture the productive government spending part of total government spending, which also includes utility-enhancing government spending. Modeling government spending as appearing in the production function has been quite standard since Barro (1990). While in this paper government spending appears contemporaneously in the production function, in most papers (including Baxter & King (1993) and Leeper et al. (2010)), productive government spending appears as an incremental investment in a stock of public capital. However, the dynamics of the model I am studying—along with the process of the government spending shock—can be seen as a good approximation to the ones that would be obtained in a model with a stock of public capital.[5] Further, for a Cobb-Douglas production function, modeling productive government spending as an additional input is observationaly equivalent to, say, government spending appearing in the total factor productivity term or as a transportation cost which would be decreasing in government spending on infrastructure.

The elasticity of output to productive government spending

Economists have tried to evaluate empirically the relevance of this modeling assumption by running regressions on log-linear production functions including government spending. The earliest attempt at this exercise is Aschauer (1989), who finds a high elasticity of output with respect to government capital. Following this paper, many others attempts at estimating this parameter have been conducted. On this issue, Romp & de Haan (2007) provide a comprehensive survey. In Table 1, I summarize the different reported values for the parameter ζ, along with the method employed by the authors. The

[5]Specifically, consider a model with $Y_t = (K_t^G)^\zeta (N_t)^\eta$, where K_t^G is the stock of public capital, which evolves according to $K_{t+1}^G = (1-\delta^G)K_t^G + PI_t^G$, where δ^G is the depreciation rate and PI_t^G public investment. In log-linearized terms this gives $k_{t+1}^G = (1-\delta^G)k_t^G + \delta^G pi_t^G$. For a shock of Δpi in period $t = T$, we have $k_{T+n}^G = (1-\delta^G)^n k + \delta^G (1-\delta^G)^{n-1}\Delta pi$. Now, for government spending process of $g_{t+1}^P = \rho_g g_t^P + \varepsilon_t$, a shock of $\Delta\varepsilon$ at date $t = T$ gives $g_{T+n}^P = \rho_g^n g^P + \rho_g^{n-1}\Delta\varepsilon$. Therefore, with $\rho_g = 1 - \delta^G$ and $\Delta\varepsilon = \delta^G \Delta pi$, we get the same dynamics for the two variables following an exogenous shock.

Authors	**Method**	**elasticity**
Aschauer (1989)	Production function	0.39
Bonaglia et al. (2000)	Production function	0.05 (insignificant)
Boscá et al. (2002)	Cost/profit function	0.08
Cadot et al. (2006)	Production function	0.08
Serven & Calderon (2004)	Production function	0.16
Charlot & Schmitt (1999)	Production function	0.3 to 0.4
Bougheas et al. (2000)	Quadratic Cost function	0.36 to 2.06
Everaert & Heylen (2001)	VECM	0.14
Everaert & Heylen (2004)	Production function	0.31
Kamps (2005)	Production function	0.22
Stephan (2001)	Production function	0.11
Stephan (2003)	Production function	0.38 to 0.65

Table 1: Summary table : Empirical studies for the elasticity of output to government capital. Based on Romp & de Haan (2007)

most used method is the one where public capital appears as an input into the production function. But this method relies on implicit assumptions that are questionable. An alternative way is to assume that public capital enters the cost function, the latter being decreasing with respect to public capital. Finally, some authors used Vector Autoregression (VAR) on time series data, which has the advantage of not imposing causal links among the variables under study. It also deals with the main critic adressed to the first approach: public capital can be driven by output growth as well as the reverse. Taking this simultaneity into account reveals that, in fact, some of the effect picked up by production function approaches goes from output to public capital. It is not a panacea though, since restrictions on the specification to be estimated are necessary (like the ordering of variables to get the response of a structural shock). As can be seen from Figure 1, the three methods yield consistent results, with $\zeta \in [0, 0.4]$.

It should be noted, however, that except for one study, all the reported estimates are statistically significantly different from 0. But maybe a paper that exhibits no effect of public capital on output would not be published at all: it is highly probable that there exists a "publication bias". Bom & Ligthart (2008) explicitly takes this bias into account when they conduct their meta-analysis of empirical studies on productivity of public capital. Their dataset consists of 76 papers, 51 of which have been published. Estimates of ζ vary from -0.175 to 0.917, with a mean of 0.193. Using fixed effects to correct for publication bias, their method yields a corrected estimate of 0.08 for ζ. In the remainder of the paper, this will be my preferred value.

Now moving to the behavior of the representative firm, profit maximization by each producer, taking into account the demand function for its product yields the following pricing condition:

$$P_t(\omega) = \mathcal{M}\frac{W_t}{\eta N_t(\omega)^{\eta-1}(G_t^P)^{\zeta}}, \tag{4}$$

where $\mathcal{M}$ is the desired steady state markup over marginal cost. This shows the channel through which government spending is going to influence the marginal cost of the representative firm. It will have an indirect effect coming from W_t and N_t as well as a direct effect coming from G_t^P. While a rising G_t^P after a government spending shock will tend to decrease marginal cost, an increase in W_t or N_t will raise it. Each additional unit of labor is less productive (because of diminishing returns) and is paid more in real terms.

3.3 Public Sector

The public sector is composed of two entities : the government, that collect lump-sum taxes which it spends in the same period; and the central bank, which sets the nominal interest rate. The government is imposed to have a balanced budget every period, so that

$$T_t = G_t^U + G_t^P$$

every period, where G_t^U is utility-enhancing government spending and G_t^P is productive government spending. The central bank is assumed to follow a simple Taylor rule of the following form:

$$I_t = \psi(\Pi_t)$$

which we will specify later in log-linearized terms.

3.4 Equilibrium

Since all firms are identical, they will all choose the same amount of labor, therefore, we can omit the ω index in N_t in the remainder. But the equilibrium will not be perfectly symmetric as in the flexible price case, because there will be relative price dispersion arising from different timing of price changes among firms. However, up to a first order approximation, the aggregate production function will be the same as in the flexible price case.

Finally, everything that is produced in each period is either consumed by the representative household or by the government. Therefore, we must have

$$Y_t = C_t + G_t^U + G_t^P.$$

Utility-enhancing and productive government spending have weights with respect to output of, respectively, α_W and α_P. It follows that the share of consumption is equal to $1 - \alpha_U - \alpha_P$. Consistent with post-WW II US data, I will set $\alpha_c = 0.8$, $\alpha_P = 0.03$ and $\alpha_U = 0.17$.

3.5 The Structure of the Shock

We have two variables for government spending. Let $\hat{G}_t$ denote deviation of total government spending from Steady State, then $\hat{G}_t = g_t^P + g_t^U$.

Assumption 1. *Let $\lambda \in$ (0,1) be the share of government investment in total government spending. Then we have: $g_t^P = \lambda \hat{G}_t$ and $g_t^U = (1 - \lambda)\hat{G}_t$*

I assume further that $\hat{G}_t \sim AR(1)$. I make this assumption to make the interpretation of my model comparable to the results of Bachmann & Sims (2012). Basically, their result for an increased share of government investment in a recession will translate in a higher value of λ in my model. What will be instructive will be to look at the multiplier for different values for λ, keeping in mind that λ may be higher than usual in a recession.

4 Log-Linearized Dynamics with Flexible Prices

In most of the theoretical literature on the macroeconomic effects of government spending, the latter is assumed to be wasteful. Therefore, the main effect goes through increased labor demand since not all firms can adjust prices to meet the higher demand. This goes on top of the increase in labor supply (due to the negative wealth effect on consumption) to produce more output. Then, since marginal cost and thus inflation increase, the central bank increase nominal rates, which further reduces private consumption. Since government spending can be productive in my model (provided that $\lambda > 0$), other neoclassical and new Keynesien effects will emerge. To first single out the one specific to the neoclassical paradigm, I first study the model with flexible prices.

As is standard in the DSGE literature, I study a log-linearization of the model around its deterministic steady state. The latter is pretty standard, so I do not reproduce it here. One feature of it although will be important for the remainder. As in Christiano et al. (2011), the steady state value of N will determine the aggregate supply response. Using the values in Christiano et al. (2011) for the preference and technology parameters, I find $N \simeq 0.4$. In what follows, for each variable X_t, let x_t denote $\frac{X_t - X}{X}$, where X is the steady state value of the variable X. A special case is nominal interest rates, with $i_t = \frac{1+I_t-(1+I)}{1+I}$. The resource constraint now reads

$$\begin{aligned} y_t = \zeta g_t^P + \eta n_t =& \alpha_c c_t + \alpha_U g_t + \alpha_P g_t^P \\ =& \alpha_c c_t + \overline{g}\hat{G}_t \end{aligned} \tag{5}$$

where I have defined $\overline{g} = \alpha_U(1-\lambda) + \alpha_P \lambda$. From this equation it is clear that the output multiplier will be given by:

$$\frac{1}{g}\frac{\partial y_t}{\partial \hat{G}_t} = \frac{\overline{g}}{g} + \frac{\alpha_c}{g}\frac{\partial c_t}{\partial \hat{G}_t}.$$

Therefore, whether the output multiplier is large or not will depend exclusively on the impact of government spending on consumption. For this reason, I will focus on the effects of government spending on consumption in the next two sections. Log-linearization of equations (2) and (3) gives

$$c_t = \mathbb{E}_t c_{t+1} - (i_t - \mathbb{E}_t \pi_{t+1}) + \mu(1-\rho_G)\hat{G}_t \tag{6}$$

$$w_t = c_t + \varphi n_t, \tag{7}$$

where

$$\begin{aligned} \mu &= -\frac{U_{CL}N}{U_C}\frac{\overline{g} - \zeta\lambda}{\eta} > 0 \\ \overline{g} &= \alpha_U(1-\lambda) + \alpha_P\lambda \\ \varphi &= \frac{N}{1-N}, \end{aligned}$$

and leisure and hours are linked through $l_t = -\frac{N}{1-N}n_t$. Moving finally to the supply side of this economy, log-linearization of equation (4) gives :

$$mc_t = w_t - \zeta g_t^P + (1-\eta)n_t \tag{8}$$

Using now equations (7), (5) and (8) to substitute for w_t and n_t, we can express marginal cost as a function of private consumption and government spending :

$$\kappa \cdot mc_t = \Theta_1 c_t + \Theta_2 \hat{G}_t \tag{9}$$

where :

$$\kappa = \frac{(1-\phi)(1-\beta\phi)}{\phi}\frac{\eta}{\eta+\nu(1-\eta)}$$
$$\Theta_1 = \kappa\left\{1+\frac{\varphi+1-\eta}{\eta}\alpha_c\right\}$$
$$\Theta_2 = \kappa\left\{\frac{\varphi+1-\eta}{\eta}\alpha_U(1-\lambda)+\left[\frac{\varphi+1-\eta}{\eta}(\alpha_P-\zeta)-\zeta\right]\lambda\right\}$$

For Θ_2, the first term in the bracket is the effect of utility-enhancing government spending on marginal cost. Since utility-enhancing marginal cost only results in higher labor demand, it unambiguously raises marginal cost. The right term captures the effect of productive government spending on marginal cost. Given that $\zeta = 0.08$, we have $\zeta > \alpha_P$, the aggregate demand effect of higher productive government spending is more than compensated by the aggregate supply effect : all else equal, productive government spending has a negative effect on marginal cost. We can now derive the private consumption government spending multiplier. Noting that, under flexible prices, marginal cost will not deviate from its steady state value ($i.e\ mc_t = 0$) we have:

$$c_t = -\frac{\Theta_2}{\Theta_1}\hat{G}_t \tag{10}$$

Therefore, since $\Theta_1 > 0$, the private consumption multiplier will be positive iff $\Theta_2 < 0$. This is true if government spending has a negative impact on marginal cost for a given consumption. Looking at the expression for Θ_2, it is clear that it is verified for a sufficiently high value of λ, the share of productive government spending in total stimulus spending. In particular, for the polar case of only productive government spending, $\lambda = 1$, the multiplier is given by the following expression:

$$\frac{\partial c_t}{\partial \hat{G}_t} = \frac{\zeta+\frac{\varphi+1-\eta}{\eta}(\zeta-\alpha_P)}{1+\frac{\varphi+1-\eta}{\eta}\alpha_c} > 0 \tag{11}$$

The mechanism is the following : since G_t^P enters the production function, a rise in government spending generates a rightward shift in labor demand (see equation (8)). Therefore, for a given level of hours, firms pay a higher wage because workers are now more productive. On the consumer side, there is a negative wealth effect on consumption because of the higher taxes needed to pay for higher government spending. But this effect is more than compensated by the higher real wage : the net effect is a leftward shift

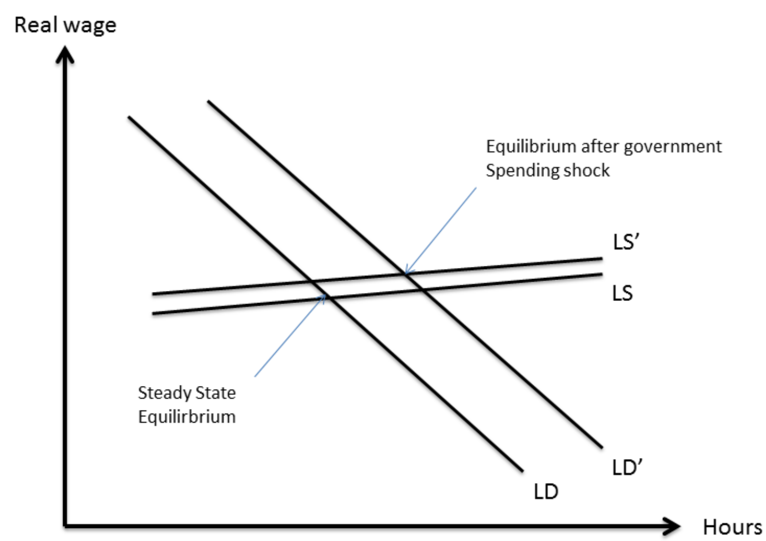


Figure 1: Labor Market after a Government Spending Shock. Flexible Prices and Government Investment Only

of the labor demand locus (see equation (7)) and thus a rise in private consumption. The mechanisms are summarized by Figure 1.

More generally, for a sufficiently high value of the share of productive government spending in total stimulus spending, the negative wealth effect will be more than compensated by the higher real wage. Formally, for $\lambda \geq \lambda^*$, the private consumption multiplier will be positive. Using the parameters values of Christiano et al. (2011), I find $\lambda^* \simeq 0.65$.

The Euler equation (6) defines the natural rate of interest of this economy. It is given by the following expression:

$$r_t^* = \left[\mu + \frac{\Theta_2}{\Theta_1}\right](1 - \rho_G)\hat{G}_t \tag{12}$$

where $\rho_G < 1$ is the autoregressive coefficient on the government spending shock process. Both μ and Θ_2 are decreasing functions of λ, so the natural interest rate will be a decreasing function of λ. To see why, assume first that $\sigma = 1$, so that $U_{CL} = U_{LC} = 0$ and there is no complementarity in labor and consumption. This implies that $\mu = 0$. I have shown that the private consumption multiplier is positive for $\Theta_2 < 0$, and since $\rho_G < 1$, the government spending shock fades away each period so that consumption growth will be negative. To be consistent with this path for consumption, the real interest rate should be negative. With general preferences however, μ will be different from zero. For λ such that the aggregate demand effect of government spending dominates, which implies $\mu > 0$, higher government spending today means lower expected government spending next period, and thus lower expected inflation, which finally translates into higher real rates. If λ is sufficiently high and $\mu < 0$ instead, higher government spending today implies a further negative real interest rate.

I have therefore highlighted a mechanism through which there can be a positive response of private consumption under flexible prices. A former attempt has been done by Linneman, using only the complementarity between leisure and consumption. But it has been shown that to obtain a positive multiplier under such a setup you must violate the underlying concavity conditions and thus assume that consumption is an inferior good (see Bilbiie), which sounds highly improbable.

In most of the recent literature, it has been emphasized that to evaluate properly the magnitude of government spending multipliers you cannot do away with the response of monetary policy. Moreover, my main interest in this paper is to study the sign and magnitude of the government spending multiplier at the Zero Lower Bound. This is why I turn now to a model with sticky prices.

5 Log-Linearized dynamics with sticky prices

I now assume Calvo pricing for the reprensentative firm. A portion $1 - \phi$ change their price at a given period, while the remaining ϕ keep them fixed. The optimal price that firms will charge, which I will denote P_t^o, is then given implicitly by the following equation:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \phi^s Q_{t,t+s} Y_{t,t+s} [P_t^o - (1-\tau^L)\mathcal{M}.MC_{t,t+s}^o] = 0 \tag{13}$$

where $Q_{t,t+s} = \beta^s \frac{U_C(C_{t+s},L_{t+s})}{U_C(C_t,L_t)}$ is the stochastic discount factor and $Y_{t,t+s}$ if the amount produced at period $t+s$ if the price set at period t is still in place (which happens with probability ϕ^s). Similarly, $MC_{t,t+s}^o$ is the marginal cost of producing one more unit of output at period $t+s$ if the price set at period t is still in place. Finally, τ^L is the optimal subsidy that verifies $(1-\tau^L)\mathcal{M} = 1$. As is well known, equation (13) imply dynamics for inflation that are given by the following equation:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \cdot mc_t \tag{14}$$

where κ is the elasticity of inflation to marginal cost for given expected future inflation. It can be seen as a measure of the degree of price stickiness. The higher ϕ, the more firms have to keep their price fixed. A higher ϕ imply in turn a lower κ : variations of marginal cost are less reflected through prices and thus prices are more sticky. Equation (14) is usually known as the New Keynesian Phillips Curve.

Substituing for marginal cost using equation (8), we get inflation as a function of one period ahead expected inflation, private and government consumption :

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \Theta_1 c_t + \Theta_2 \hat{G}_t \tag{15}$$

Together with the Euler equation (2) and the log-linearized version of the Taylor rule ($i_t = \phi_\pi \pi_t$), this equation completly determines the dynamics of the economy. We can use the flexible price equilibrium to simplify the system and express it with the consumption gap $x_t \equiv c_t - c_t^*$ (c_t^* being the flexible price deviation of consumption from steady state) and inflation. I end up with the following system

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \Theta_1 x_t \tag{16}$$

$$x_t = \mathbb{E}_t x_{t+1} - (i_t - \mathbb{E}_t \pi_{t+1} - r_t^*) \tag{17}$$

We can see that government spending only affects this system through the natural rate of interest. We can express this system in the form $\mathbb{E}_t z_{t+1} = \mathbf{\Gamma} z_t + \mathbf{\Psi} \hat{G}_t$, with $z_t = (x_t \ \pi_t)'$. Using the method of undetermined coefficients and conjecturing $z_t = \Omega \cdot \hat{G}_t$ I get :

$$\begin{bmatrix} x_t \\ \pi_t \end{bmatrix} = \frac{1}{det} \begin{bmatrix} \left(\frac{1}{\beta} - \rho_G\right)\left(\mu + \frac{\Theta_2}{\Theta_1}\right)(1-\rho_G) \\ \left(\mu + \frac{\Theta_2}{\Theta_1}\right)(1-\rho_G)\frac{\Theta_1}{\beta} \end{bmatrix} \hat{G}_t \tag{18}$$

where $det = det(\rho_G \cdot I - \Gamma) > 0$. We can see that the positive effect of government spending on the "‘consumption gap"’ is conditional on the sign of $\mu + \Theta_2/\Theta_1$. Assume first that $\lambda = 0$ and thus $\Theta_2 < 0$. In this case, $\partial x_t / \partial \hat{G}_t < 0$ and since

$$\frac{\partial c_t}{\partial \hat{G}_t} = \frac{\partial x_t}{\partial \hat{G}_t} + \frac{\partial c_t^*}{\partial \hat{G}_t},$$

the multiplier is lower with sticky prices than with flexible prices. To get the intuition for this result, consider the equation for labor demand : $w_t =$

$mc_t + \zeta\lambda\hat{G}_t - (1-\eta)n_t$. In the flexible prices case, $mc_t = 0$ and government spending has a higher effect on wages than in the sticky prices case, where movements in mc_t counteract this effect. Higher wages translate into higher private consumption through the labor supply equation : $w_t = c_t + \varphi n_t$. As a consequence, wages react more than hours in the flexible prices case. The fact that $\Theta_2 < 0$ also triggers a deflation which calls for lower nominal rates and higher consumption, but this effect is too small to be larger than the one at play under flexible prices. This appears clearly in the expression derived for the government consumption multiplier under sticky prices:

$$\frac{\partial c_t}{\partial \hat{G}_t} = \frac{(1-\beta p)(1-p)\mu - (\phi_\pi - p)\Theta_2}{(1-\beta p)(1-p) + (\phi_\pi - p)\Theta_1}$$

When $\Theta_2 < 0$, the elasticity of the numerator being higher than the one for the denominator, a higher ϕ_π enhances this effects and delivers a higher multiplier. It is this effect which will play in the opposite direction when the economy is at the zero lower bound. Consider now the case with complementarity and λ is such that the representative agent works more after increased government spending, meaning that $\mu > 0$. Since hours react more in the sticky prices environment and hours are complementary with consumption, the multiplier will be higher when prices are sticky. In contrast with the zero lower bound case, μ will play a larger role with respect to Θ_2 in this framework. In fact, for $p = 0.8$ as in Christiano et al. (2011), Θ_2 is negative enough to prevent the multiplier to be negative but not to be decreasing with respect to λ.

Another feature of the model that should be noted is that, for a sufficiently high value of λ, both μ and Θ_2 will be negative. In this case, government spending has an unambiguous negative effect on inflation, as can be seen from equation (18). Specifically, it can be shown that for

$$\hat{G}_t \geq -\beta \frac{1-\beta}{\phi_\pi(\mu\Theta_1 + \Theta_2)(1-\rho_G)} > 0$$

the economy hits the zero lower bound. Since I am only interested in the impact response of output and consumption to a government spending shock, I will not have to calibrate the magnitude of the shock and accordingly this caveat will not be important.

As a conclusion, while government investment tends to generates an output multiplier higher than 1, it also pushes the economy towards the Zero Lower Bound. As far as the New Keynesian model is concerned—and contrary to the citations that I have highlighted in the preamble, government investment as a means of stimulating the economy should be used with caution. This warning is all the more true in the context of an economy that is already stuck at the Zero Lower Bound, as we will now see.

6 The Government Spending Multiplier at the Zero Bound

6.1 Government Spending in an Excess Savings Liquidity Trap

I will now study the impact of government spending when the economy is stuck at the zero lower bound. for the formalization of this situation, I follow Eggertsson (2011). Specifically, let us assume that the discount factor is now time-varying and given by $\beta_t = \frac{1}{1+\delta_t}$. The reason for including time variation in the discount rate is to introduce a shock (a desire to save more) that will make the zero lower bound a binding constraint. Under those assumptions, the Euler Equation becomes:

$$c_t = \mathbb{E}_t c_{t+1} - [I_t - \mathbb{E}_t \pi_{t+1} - \delta_t] + \mu(1-p)\hat{G}_t. \tag{19}$$

I assume further, as has become standard, that δ_t follows a Markov structure. If there is a shock to the discount rate ($\delta^L < 0$), where L stands for "Low") the probability that it will persist next period is p, and with probability $1-p$, it reverts to its steady state value. Furthermore, once the discount rate returns to its steady state, it stays there afterwards. To assure the comparability of the results established in the previous section, I will assume $p = \rho_G$. I also modify the Taylor rule in the following way:

$$I_t = \delta_t + \phi_\pi \pi_t \tag{20}$$

I also assume that government spending reacts positively to the discount rate shock, *i.e* $\hat{G}^L \geq 0$. Under these assumptions, the IS and New Keynesian Phillips curves now read:

$$c^L = pc^L + [p\pi^L + \delta^L] + \mu(1-p)\hat{G}^L \tag{21}$$

$$\pi^L = \beta p \pi^L + \Theta_1 c^L + \Theta_2 \hat{G}^L \tag{22}$$

One can view equation (21) as an Aggregate Demand (AD henceforth) relationship, while (22) stands for an Aggregate Supply (AS henceforth) relationship. Rewriting these equations with π^L as a function of c^L, it is clear that both AS and AD are increasing functions of c^L. Now taking $c^L = 0$, we see that the intercept of AD is positive while the one for AS is negative. Therefore, for an equilibrium to exist the two lines should cross, which requires that the slope of AD is steeper than for AS. Formally, this condition translates into:

$$(1-p)(1-\beta p) - p\Theta_1 > 0$$

In the remainder, I will refer to the left hand side of this equation as z. Looking at the equilibrium without automatic government spending stimulus ($i.e\ \hat{G}^L = 0$), this is the condition that guarantees that there is deflation and a fall in consumption when there is a discount rate shock that takes the economy to the zero lower bound. In fact, under $\hat{G}^L = 0$, c^L and π^L are given by:

$$c^L = \frac{1-\beta p}{z}\delta^L \tag{23}$$

$$\pi^L = \frac{\Theta_1}{z}\delta^L \tag{24}$$

Since individuals want to save more, real rates should be increasing to be consistent with these expectations : there is deflation. But since there is no capital in this economy, net savings are zero in equilibrium, so a decrease in income is needed to pull savings down. This is why the AS and AD curves will be situated in the southwest part of the (c^L, π^L) space. These are depicted in Figure 2.

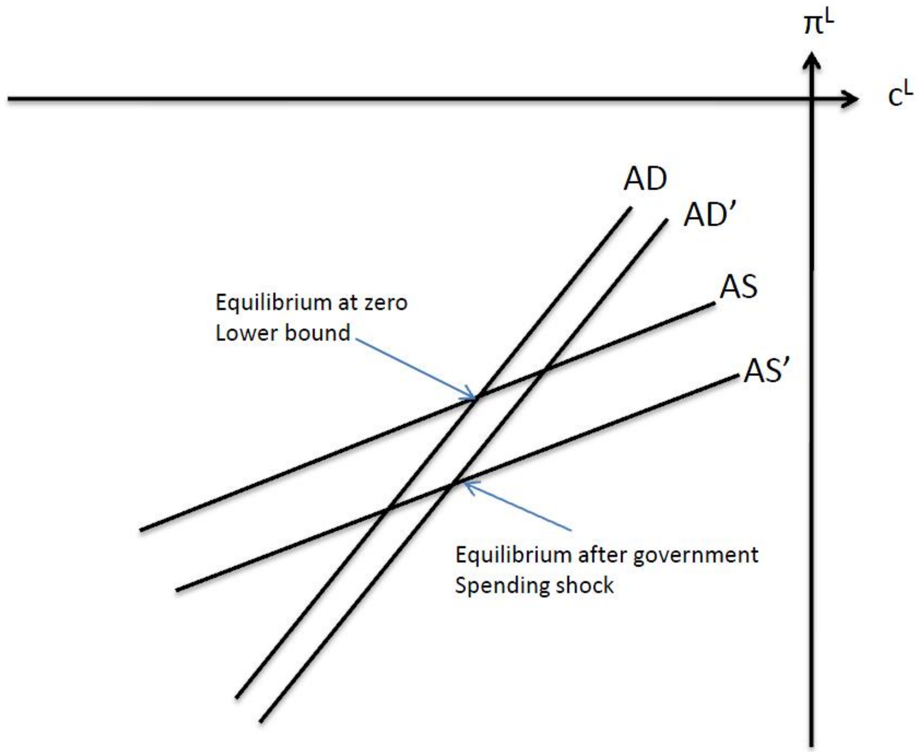


Figure 2: Effects of a government spending shock in an excess-savings liquidity trap.

In drawing the AS' curve I have assumed that λ is sufficiently high for Θ_2 to be negative, so that the AS curve shifts right after a government spending shock. As can be seen from the diagram, for private consumption to be crowded out after a government spending shock, the rightward shift in AS should be relatively larger than the one in AD. To see the conditions needed for this to hold, I reproduce here the elasticities of inflation with respect to government spending in both AD and AS equations, taking consumption as given.

$$\left(\frac{\partial \pi^L}{\partial \hat{G}^L}\right)^{AD} = -\frac{1-p}{p}\mu \qquad \left(\frac{\partial \pi^L}{\partial \hat{G}^L}\right)^{AS} = \frac{\Theta_2}{1-\beta p}$$

By definition, μ is a decreasing function of λ. It is positive for $\lambda = 0$ and negative or $\lambda \geq \lambda^{**}$. Under my preferred specification, I get $\lambda^{**} \simeq 0.78$. The lines in Figure 2 are drawn for $\lambda < \lambda^{**}$. As μ increases, the shift in AD diminishes in magnitude. Eventually, for $\lambda > \lambda^{**}$, AD will shift left and consumption will be even more crowded out after a government spending shock. Conversely, as λ increases, Θ_2 becomes more negative and so the rightward shift in AS is amplified.

One can wonder the magnitude of the value of λ needed for government spending to become contractionary at the zero lower bound. First, combining equations (21) and (22), I obtain an analytical expression for the private consumption multiplier of government spending:

$$\frac{\partial c^L}{\partial \hat{G}^L} = \frac{1}{z}\left[(1-p)(1-\beta p)\mu + p\Theta_2\right] \tag{25}$$

By assumption, the parameters are such that $z > 0$. For $\lambda = 0$, both terms inside the brackets are positive and the private consumption government spending is positive. Conversely, for $\lambda = 1$ both terms are negative, so the private consumption government spending multiplier is negative. Given that both terms inside the brackets are decreasing functions of λ, so for a sufficiently high value of λ, the term in brackets will become negative. Taking $p = 0.8$ as in Christiano et al. (2011), I get that for $\lambda \geq 0.64$, private consumption is crowded out after a government spending shock. What is the main driver of the opposite sign of consumption's response with respect to what can be found in the literature? In Eggertsson (2011) for example, there is also a rightward shift in AD but AS shifts left (in this case, $\Theta_2 > 0$ and government spending generates inflation). The rightward shift in AD is due to the same reasons here. Therefore, in Eggertsson (2011), aggregate demand effects more than compensate aggregate supply ones. For a sufficiently high value of λ, the opposite holds in my framework.

Even if in reality λ is not enough to overturn the effects of government spending at the zero lower bound, what the introduction of government spending shows is that the gap between the multiplier in normal and bad times is shrinking in λ. In fact, the multiplier in normal times is decreasing in λ because of the negative effect on μ that is not compensated by a more negative Θ_2, but it stays positive. In contrast, the multiplier in bad times decreases much more rapidly with respect to λ and eventually becomes negative.

I want now to see how the introduction of productive government spending affects the magnitude of the large output multiplier obtained by Christiano et al. (2011). Using the resource constraint, I can get an expression for the output multiplier of government spending that fully nests the one obtained by Christiano et al. (2011):

$$\frac{\partial y^L}{\partial \hat{G}^L} = \frac{\overline{g}(1-p)(1-\beta p)(1+\sigma(\gamma-1)) - (1-g)\Delta_1}{(1-p)(1-\beta p) - (1-g)\Delta_1} \tag{26}$$
$$\Delta_1 = \kappa\left(\frac{1}{1-g} + \frac{\varphi}{\eta}\right) \qquad \Delta_2 = \kappa\left(\frac{\overline{g}}{1-g} + \frac{\varphi\zeta\lambda}{\eta}\right),$$

where I have defined $g = \alpha_U + \alpha_P$. For $\alpha_P = \lambda = \zeta = 0$, $\alpha_U = 1 - \alpha_c$, $\eta = 1$ and all government spending is utility-enhancing. With these restrictions, I get an output multiplier of 3.1 as in Christiano et al. (2011). First of all, introducing productive government spending as appearing in the production function is equivalent to assuming decreasing returns to labor to keep constant returns to scale. Keeping $\lambda = 0$ for the moment (stimulus spending in only composed of utility-enhancing spending) but imposing $\alpha_P = 0.03$, $\zeta = 0.08$ and $\eta = 1 - \zeta$ has a non negligible effect on the parameter κ. From roughly 0.028 in Christiano et al. (2011), it equals 0.018 here. This parameter governs the elasticity of inflation with respect to marginal cost and is given by the following expression, which I reproduce here:

$$\kappa = \frac{(1-\phi)(1-\beta\phi)}{\phi} \cdot \frac{\eta}{\eta + \nu(1-\eta)}.$$

The second term of this expression enters only with decreasing returns to labor. In fact, with $\eta = 1$, the expression is the same as in Christiano et al. (2011). It can also be shown that the elasticity of the numerator of equation (26) with respect to κ is greater than the one of the denominator : a lower κ will imply a lower multiplier, whether there is productive government spending in the stimulus or not. Once again, this shows the reliance of large output multipliers on the marginal cost channel. Quantitatively, introducing productive government spending but keeping it out of the stimulus yields an output multiplier of 1.55, half of the one derived in Christiano et al. (2011). With a fiscal stimulus package composed equally of productive and utility-enhancing government spending, such that $\lambda = 1/2$, I get an output multiplier of 0.55. In this case, stimulus spending delivers a lower multiplier than in the normal, out of the zero lower bound case which is equal to 0.57. With a fiscal stimulus package composed equally of productive and utility-enhancing government spending, the multipliers in good and bad times are very close to each other, the latter being slightly higher.

Further, as has been shown in Christiano et al. (2011), the multiplier is quite large in economies where the cost of being in the zero lower bound state is high. In particular, the cost of being in the zero lower bound state is high if the discount rate shock is expected to last for long, that is if p is high. In fact, the cost of being in the zero lower bound is summarized by $\frac{1}{z}$, and it is clear that z is a decreasing function of p. As z is the denominator of the consumption multiplier at the zero lower bound, a lower z implies a higher multiplier. This is how Christiano et al. (2011) obtain a large multiplier. But the amplification effect can work in the opposite sense. If λ is high enough for government spending to crowd out private consumption, the result will be a large but negative multiplier on output, as can be seen in Figure 3 for example.

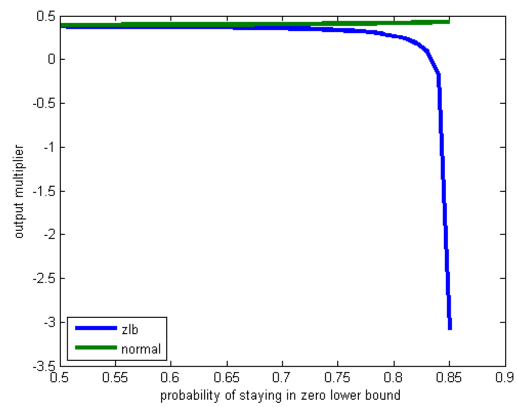


Figure 3: Output multipliers of government spending. $\lambda = 0.7$

Another feature that makes the multiplier large in Christiano et al. (2011) is the degree of price flexibility. A higher degree of price flexibility imply that after a government spending shock inflation will rise more and real rates will decrease by more. In my framework, for a sufficiently high λ, a higher degree of price flexibility will imply a larger positive effect on real rates, and thus a larger negative effect on private consumption.

As has been extensively stressed in the literature on fiscal policy at the zero lower bound, the main problem when the nominal rate is pinned at zero is a lack of demand. Increasing demand through government spending is thus a straightforward solution. In my framework, as λ increases, fiscal stimulus becomes more tilted towards a supply side policy rather than an aggregate demand management policy. This is consistent with the results of Bachmann & Sims (2012). It is then no surprise that government spending with a sufficient share of productive government spending is not so much efficient at the zero lower bound. As has been shown by Eggertsson (2011), supply side policies such as a cut in labor taxes are contractionary at the zero lower bound through the same effect I have highlighted : an increase in real rates through lower prices that crowds out inflation. But there is another reason for why policies that look promising at first glance (such as a cut in labor taxes) are in fact contractionary at the zero lower bound. Building on Schmitt-Grohe et al. (2001), Mertens & Ravn (2010) shows that the nature of the shock that brought the economy to the zero lower bound matters also. In particular, they study the effects of fiscal policy in an expectation driven liquidity trap.

6.2 Government Spending in an Expectation Driven Liquidity Trap

Suppose now that δ_t remains the same and so is still strictly positive. I describe the mechanisms without government spending first. What drives the economy to the zero lower bound is a sudden change in beliefs, which are not related to fundamentals. Let us say that agents expect a temporary but persistent drop in income. They will want to consume less as a consequence. For the aggregate resource constraint to be satisfied, both p_t and y_t decrease. The decrease in prices generates higher real interest rates, prompting the representative household to increase its savings. But since savings are zero in this economy, an further decrease in prices is needed to reduce desired savings. As a consequence, output and prices decrease further, but the decrease in price raises the incentives to save, so a further drop in output is needed. This deflationary spiral ends when output has decreased enough for real interest rates to equate consumption to output.

The existence of sunspot equilibria because of the zero lower bound is linked to the global indeterminacy inherent in the new keynesian model (see Schmitt-Grohe et al. (2001)). This is why Mertens & Ravn (2010) study the effects of government spending with a non-linear new keynesian model. However, they show that qualitative results carry over with the log-linear approximation of the model. In the context of the log-linear approximation, the algebra is the same as in the previous subsection and the opposite results depend on different assumptions for the underlying parameters. Since δ^L is now positive, for there to be deflation and a fall in consumption in the liquidity trap scenario, those parameters must imply $z < 0$. By the definition of z, it is clear that higher values of p push z into negative territory. In my framework, z becomes negative for $p \geq 0.85$. Does a value of p of this magnitude makes sense? The expected duration of the zero lower bound episode is given by $\frac{1}{1-p}$. If one period is considered as a quarter, then the expected duration of the liquidity trap scenario for $p = 0.85$ is a little more than one year and a half. With hindsight we now that the Fed Funds rate has been pinned at zero for nearly 4 years, and from recent declarations of Fed chairman Ben Bernanke, it is expected to remain so for at least a couple of years. A liquidity trap duration of 4 years imply $p \simeq 0.94$. Therefore, $p \to 1$ does not seem irrelevant, if anything it is more relevant than the values considered by Christiano et al. (2011).

What are then the effects of government spending in a sunspot driven liquidity trap? As before, it depends whether government spending is more tilted towards aggregate demand management than aggregate supply. In the former case, which is the one studied by Mertens & Ravn (2010), higher utility-enhancing government spending generates higher taxes and thus a decrease in income. Households want to save less as a result, so more deflation is needed to bring back savings to zero. This effect more than compensate the inflationary effect highlighted by Christiano et al. (2011). With a sufficient share of productive government spending however, the opposite holds. Higher productive government spending generates higher incomes, so households want to save more. To pull savings down, a rise in inflation is needed, which prompts households to consume more today. As in the last subsection, we can draw an AS-AD diagram. With $z < 0$, the AS locus will now be steeper than the AD one. As before, for $\Theta_2 < 0$ both AS and AD will shift right after a government spending shock. Provided the intercept of AS is higher than the one for AD[6], we have the following diagram:

[6]It can be shown that it implies the following equation : $\frac{\Theta_2}{1-\beta p} > -\frac{\delta^L + \mu(1-p)\hat{G}^l}{p}$.

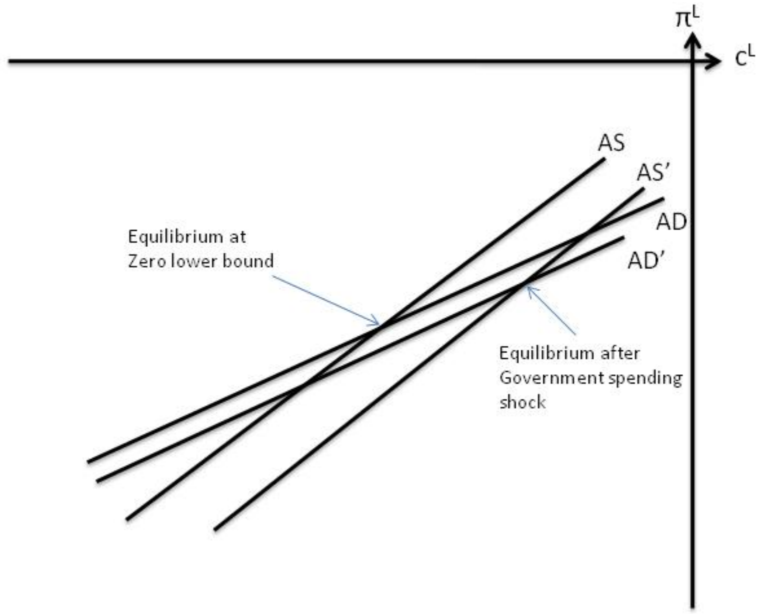


Figure 4: Effects of a government spending shock in an expectation-driven liquidity trap.

As the probability of the shock tends towards 1 however, this effects is dampened because expected recovery get pushed more and more into the future. The effects of government spending on output for $p \geq 0.85$ is depicted in Figure 4

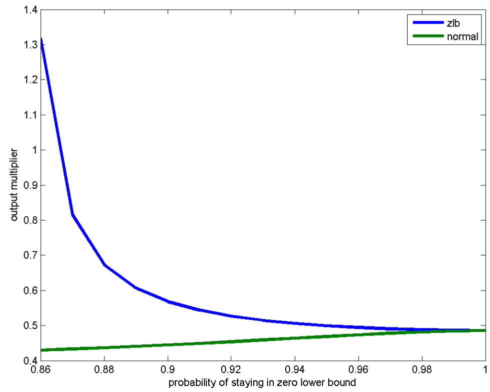


Figure 5: Output multipliers of government spending in a sunspot driven liquidity trap. $\lambda = 0.7$

To conclude, whether λ is indeed large enough for this opposite result to hold is ultimately an empirical question. Moreover, it depends on the fiscal package one want to study : some fiscal packages include a higher share of productive government spending than others. On this question, I will provide a simple empirical application based on the American Recovery and Reinvestment Act of 2009 in the next section. But what can be said for the moment is that *for government spending to yield the highest possible multiplier at the zero lower bound after excess savings, government spending should be just utility-enhancing. If the liquidity trap is due to pessimistic expectations, the more productive government spending the better.* This goes against the idea that if government spending is productive, then it will necessary be more efficient at the zero lower bound. As has been emphasised by Mertens & Ravn (2010), it depends on the shock that brought the economy to the zero lower bound. True, in the broadly discussed excess savings case, more income is generated in the future after investment in public infrastructure, but as in most mechanisms involved in the New Keynesian model, expectations play a fundamental role. Here, expected lower marginal costs in the future translate into lower prices in the short run, which dampens the inflationary effects of higher government spending.

6.3 Comparing the Models with Empirical Evidence

Let us begin with the excess-savings liquidity trap. The main message of this model is that when the nominal interest rate is pinned at zero, government spending can crowd in inflation and deliver a large output multiplier. Now, the zero lower bound on the nominal interest rate usually becomes a binding constraint in times of recession. The three occurrences of zero lower bound episodes (Great Recession and Depression, Japan during the Lost Decade) confirm this fact. In section 2, I have cited evidence that in recession times government spending has a negative effect on both marginal cost and inflation. In contrast, the underlying New-Keynesian model used by Christiano et al. (2011) delivers a positive effect on both inflation and marginal cost. If the excess-savings liquidity trap is instead appended to a model yielding a fall in marginal cost and inflation after a government spending shock, the presence of the liquidity trap delivers small output multipliers, which can be lower than outside of the liquidity trap. Furthermore, the value for the expected duration of the liquidity trap that Christiano et al. (2011) choose is hard to square with the observed duration of the ongoing zero lower bound episode.

I now turn to the expectation-driven liquidity trap. Considering a more relevant value for the expected duration of the liquidity trap, Mertens & Ravn (2010) reports exactly opposite results with respect to Christiano et al. (2011). It follows that, for a model with a sufficient share of productive government spending, the model with an expectation-driven liquidity trap delivers a positive private consumption multiplier. While the initial effect of a government spending shock is deflationary, there will be crowding in of private consumption only if there is inflation. The reason is that productive government spending raises actual and future incomes, inducing agents to want to save more. This, in turn, requires a rise in inflation to bring back savings to zero. The prediction that government spending has a positive effect on inflation due to the zero lower bound is hard to compare with existing empirical evidence. The papers investigating the effects of government spending at the zero lower bound, as Almunia et al. (2010) and IMF (2012) do not report the effect on prices. I do not know of any paper providing such results. It has also to be noted that the opposite results obtained by Mertens & Ravn (2010) with respect to Christiano et al. (2011) rely on the markov structure of the shock. If the shock is deterministic instead then, as Carlstrom et al. (2012) show, the multipliers obtained by Christiano et al. (2011) are smaller and do not flip signs.

In the last two paragraphs, I have argued that a model with a sufficient share of productive government spending as a share of total stimulus spending is consistent with existing empirical evidence of the effects of government spending in a typical postwar US recession. This is not conditional on any standard shock that would drive the economy into a recession (a negative technology shock for example). What are then the features of this model that are specific of an economy in recession. One is explicit and the other is hidden in the foundations. The first one has to do with the share of productive government spending in a recession. Both recent examples of fiscal stimulus packages cited the introduction and econometric evidence provided by Bachmann & Sims (2012) concur in yielding a higher share in recession times. The second one has to do with the labor market. One might expect that there a lot of unutilized resources during a recession. Therefore, with a pool of unemployed people, hiring new workers to produce the extra demand generated by the government is essentially costless (Michaillat (2012)). There is also empirical evidence showing that, indeed, labor market adjustment in a recession occurs largely on the extensive margin (see van Rens (2012)). In the model of Christiano et al. (2011), the elasticity of marginal cost to government spending depends negatively on the Frisch labor supply elasticity. In the model I have developed, this elasticity is equal to $1/\varphi$. Under the specification of Christiano et al. (2011), I get $\varphi = 0.47$ which yields an elasticity of approximately 2. The result is a quite low elasticity of marginal cost with respect to government spending. In fact, the higher φ, the steeper the labor supply curve and thus the higher the rise in wages for a given shift in labor demand. The empirical literature offers little support for such a high elasticity of labor supply, and quantitative papers usually assume $\varphi > 3$.

Both the presence of productive government spending and the high elasticity of labor supply act to reduce the elasticity of marginal cost to government spending. While the high elasticity of labor supply does most of the heaving lifting, productive government spending brings the "coup de grâce" to yield a negative elasticity of marginal cost to government spending. As I have shown, this negative elasticity is essential to get private consumption crowding out in the excess-savings liquidity trap. I therefore conjecture that a model *à la* Michaillat (2012) with productive government spending and an excess-savings liquidity trap will yield a lower output multiplier than the same model without a binding constraint on the nominal rate. I am currently working on this issue.

7 An Empirical Application : The American Recovery and Reinvestment Act of 2009

There has been much discussion about the American Recovery and Reinvestment Act of 2009. Some argued it would deepen the crisis, some where active proponents. My focus here is to get a rough estimate of λ, the share of productive government spending in total stimulus government spending. The composition of this fiscal stimulus package is summarized in Figure 6.

What is in the House Stimulus Plan?

Table 1: $825 Billion House Democratic Economic Stimulus Plan
Sources: BLS, BEA, Moody's Economy.com

	2009Q1	2009Q2	2009Q3	2009Q4	2010Q1	2010Q2	2010Q3	2010Q4	2009	2010	2009-10
Total Stimulus	0	89	118	135	142	132	116	94	341	484	825
Government Spending	0	39	58	79	96	105	98	76	175	375	550
Income Support	0	10	15	19	19	16	13	10	44	58	102
Unemployment Insurance Benefits	0	4	6	8	8	7	6	4	18	25	43
Food Stamps	0	2	3	4	4	3	2	2	9	11	20
COBRA Healthcare	0	4	6	7	7	6	5	4	17	22	39
Infrastructure Spending	0	1	6	16	29	41	41	27	22	136	159
Traditional Infrastructure	0	0	2	9	16	24	24	14	11	78	89
Non-Traditional Infrastructure	0	1	4	7	13	17	17	13	12	58	70
Aid to State Government	0	24	28	32	35	35	31	29	83	128	211
Medicaid Match	0	10	12	13	13	13	13	13	35	52	87
Fiscal Relief	0	10	10	12	14	14	10	9	32	47	79
Local School Districts	0	3	5	6	7	7	7	6	14	27	41
Law Enforcement	0	1	1	1	1	1	1	1	2	2	4
Healthcare/Education/Other	0	5	9	13	14	14	14	11	26	53	79
Tax Cuts	0	50	60	56	46	27	18	18	166	109	275
Business Tax Benefits	0	10	20	40	30	10	0	0	70	40	110
Inividual Tax Benefits	0	40	40	16	16	17	18	18	96	69	165

Figure 6: Decomposition of the American Recovery and Reinvestment Act of 2009. Source : Mark Zandi, Moody's Economy.com, 2009

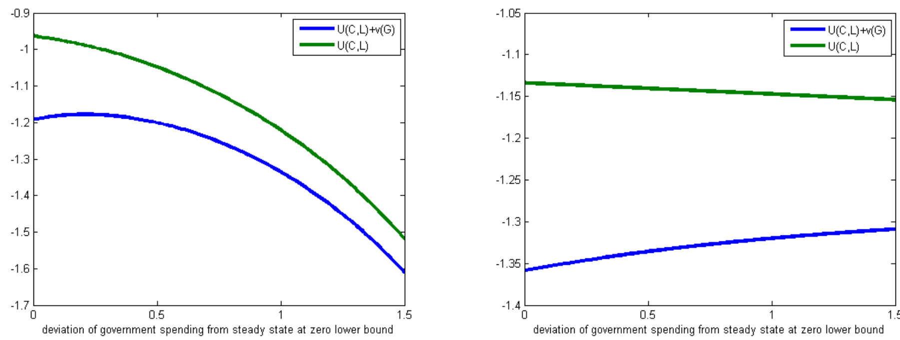
Considering total projected spending on 2009/2010, what can be considered as being government spending in the sense of the model I have developed? First of all, even though part of government spending is productive in my model, it can be shown that Ricardian equivalence still holds. Therefore, the income support part of the fiscal package can be ignored because it would be neutral in the model : income support now will be financed by higher future taxes and the representative household saves accordingly. This is a neutral transfer in this framework. What really matters in the model is real spending on goods. Similarly, I have assumed lump-sum taxation. Again, tax cuts can be considered as neutral, since lower lump-sum taxes today are expected to rise in the future so that permanent income is unchanged. This leaves only two categories : Infrastructure spending (ISp) and Aid to State Government (ASG). The former clearly fits my definition of productive government spending, while the second is ambiguous. In fact, the 211 billion $ that have been granted have not all been spent on goods. Part of it consists simply in transfers among different branches of the government (central and federal).

In light of this, the most conservative value I can get for λ will be ISp/(ISp+ASG). It amounts to assuming that all the money allocated to Aid to State Government has been spent on goods. As a consequence, this ratio will give me a lower estimate for λ. Using the numbers in Figure 6, I get $\lambda = 0.43$. That is, at least 43% of government spending on goods has consisted in productive government spending. If we consider that half of Aid to State Government has been actually spent on goods, then I get $\lambda = 0.6$. Therefore, assuming this repartition of spending, the model says that for all values of $p \leq 0.85$, the effects of government spending is higher in the normal case, all the more so when p is close to 0.85. Were the share of productive government spending a little bit higher ($\lambda \simeq 0.64$), the effect would be a large negative multiplier for p close to 0.85. For $p \geq 0.86$, the multiplier is slightly higher in bad times (peaking at 0.6) and the two converge to the same value as p approaches 1.

8 Optimal Government Spending

In light of the effects of (productive) government spending on private consumption at the zero lower bound, one might wonder what the optimal level of government spending might be. Considering only utility-enhancing government spending, Christiano et al. (2011) report a value for optimal $\hat{G}^L$ of approximately 0.3. It is then optimal to increase government spending up to 30% in deviation from its steady state level. Following a government spending shock, both private consumption and hours worked increase. Then, considering only the effect of government spending on $U(C_t, L_t)$, it is negative. The reason is that, since the baseline specification assumes $\gamma = 0.29$ and $\sigma = 2$, consumption and leisure are complements. Then, since consumption rises and leisure falls after a government spending shock, utility decreases. But since government spending enters the utility function, it has a positive effect which declines as the deviation of government spending from steady state grows bigger. For small deviations, the direct utility effect of government spending dominates; then the opposite behavior of consumption and leisure trump this effect. This is why Christiano et al. (2011) get a hump-shaped pattern for utility as a function of deviations of government spending from steady state at the zero lower bound.

When productive government spending is added to the picture, things are very different. As I have shown, with a fraction of stimulus spending devoted to productive government spending, the effect on private consumption and on output is not that large. It can even turn negative. For λ roughly larger than 0.7, both private consumption and hours worked decrease. Since, for this share of productive government spending the shock looks more like a supply side policy, the representative agent is more productive and thus



(a) Optimal G, $\lambda = 0$ (b) Optimal G, $\lambda = 0.7$

Figure 7: Utility effect of government spending at the zero lower bound.

works less hours. This induces a fall in marginal cost, which itself imply a fall in prices and consumption. This effect further depress hours worked. But since the effects of government spending on consumption and hours worked are not that large, the point at which they overcome the positive effect on utility of utility-enhancing government spending is pushed farther away. It is then optimal to increase government spending up to 100% from its steady state value. As Bilbiie et al. (2012) have argued, the multiplier effects of government spending at the zero lower bound are a poor guide to judge whether this policy is optimal or not. In fact, while having a negative multiplier on consumption, increasing partly productive government spending is optimal here.

It is then clear that both results rely heavily on the presence of government spending in the utility function. Removing it by setting $\psi_g = 0$, the results with the presence of productive government spending are the same than without, but for different reasons. In the model without productive government spending, both consumption and hours worked increase. Since those two goods are substitutes, utility decreases. When there is a sufficient degree of productive government spending such that both consumption and hours worked decline, the same argument imply a declining utility. Therefore, with $\psi_g = 0$, the optimal government spending policy at the zero lower bound is to set $\hat{G}^L = 0$. The results are depicted in Figure (7).

9 Conclusion

Recent empirical evidence points towards higher multiplier of government spending in recession times. Since, in reality, the zero lower bound becomes a binding constraint mainly in recessions, the mutltiplier is likely to be high this time around. But aside from the constraint on the nominal rate giving an extra kick to an already high multiplier, in most of the literature it does all the work. In doing so, this literature runs into many problems. First of all, it cannot explain why mutlipliers are actually higher in recessions, since in most recessions the zero lower bound does not become a binding constraint. Secondly, the recent empirical evidence that I have mentionned shows the incapacity of government spending to generate inflation in times of recession. More generally, the rise in marginal cost that is needed to bring this up does not appear in empirical data on the effects of government spending in a recession. In this respect, the work of Rendahl (2012) is interesting as it unveils another mechanism through which government spending can get traction on the economy at the zero lower bound : since the zero lower bound is an environment of recession, there is likely to be pervasive unemployment. By making the future look brighter, spending increases today makes people want to consume more today which, in turn, raises future incomes *etc.* But this theory still runs into the fact that the zero lower bound is needed to generate a high multiplier.

In this paper, I have focused on a third issue concerning those kind of models. Since the share of productive government spending is typically higher in recession times, the effect of government spending on marginal cost should be lower. This conclusion is borne out by the data. A lower effect on marginal cost leads to a lower effect of government spending in an "‘excess-savings"’ liquidity trap. The government spending multiplier need not be unusually large at the zero lower bound, at least not for the reasons that have been put forward by most of the literature on the subject. I am not the first nor the only one to point to the difficulties of the new-keynesian model at the zero lower bound with empirical evidence. In a recent paper, Wieland (2012) shows that the emphasis on generating inflation at the zero lower bound yields to the conclusion that adverse supply shock are expansionary. By looking at oil shocks and the recent Japanese earthquake, he shows that adverse supply shocks are, indeed, contractionary and that adding financial frictions to the new-keynesian model helps reproduce the facts. In another line of research, Michaillat (2012) shows that, irrespective of the zero lower bound, fiscal policy multipliers on unemployment can be larger in recessions.

The bottom line is that, while empirical evidence seems to point to larger multiplier in recessions, a new-keynesian model with a zero lower bound does not seem to be the right way to rationalize it.

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A Figures

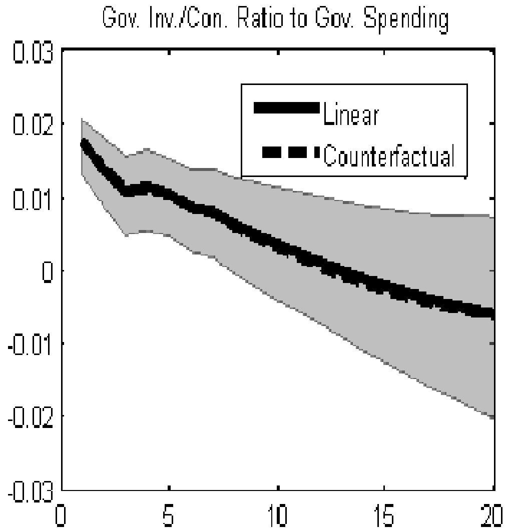

Figure 8: Response of Government investment/consumption ratio in a recession Source : Bachmann & Sims (2012).

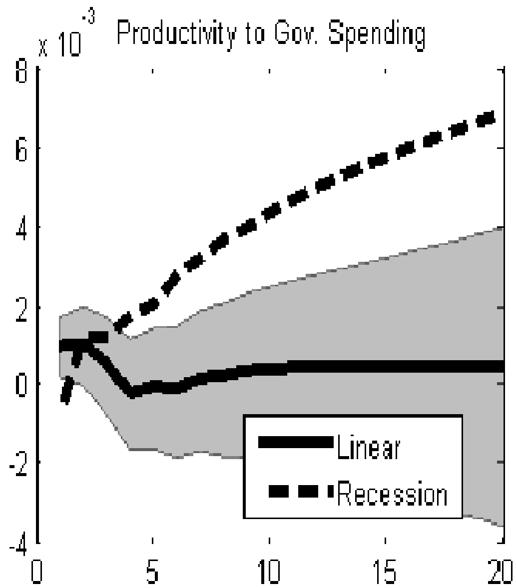

Figure 9: Response of Private Productivity in a recession Source : Bachmann & Sims (2012).

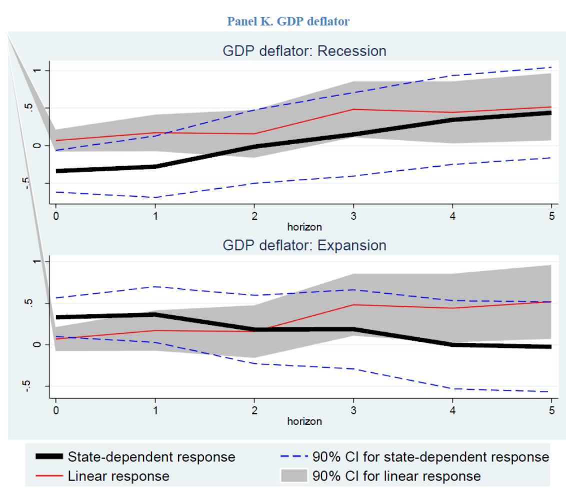

Figure 10: Response of Prices in a recession Source : Auerbach & Gorodnichenko (2012).

Panel B. Private consumption

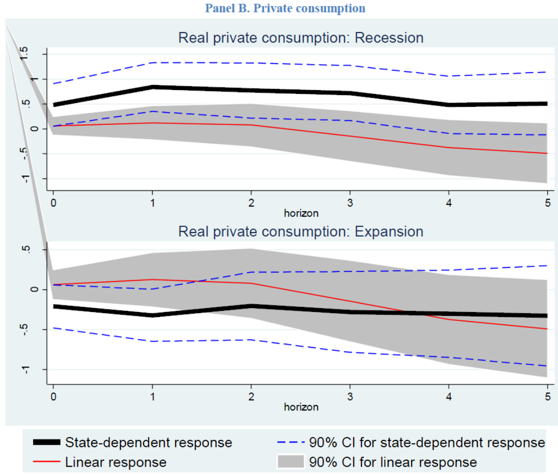

Figure 11: Response of Prices in a recession Source : Auerbach & Gorodnichenko (2012).