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Welfare Reversals in a Monetary Union*

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Abstract

We show that the welfare costs of business cycles in a monetary union can be higher under incomplete financial markets than under complete markets. A monetary union with home bias, sticky prices and country-specific shocks is a second-best environment in which the structure of financial markets shapes the extent of risk-sharing but also the welfare costs of nominal rigidities. If the Marshall-Lerner condition is met, complete financial markets increase the volatility of terms of trade, and that of inflation rates. The corresponding increase in welfare losses from nominal rigidities can overturn the welfare gains from a better sharing of risks.

Keywords: Monetary Union, Financial Markets Incompleteness, Sticky Prices, Fiscal and Monetary Policy.

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1 Introduction

In this paper, we quantify the welfare costs of business cycles in a monetary union with country-specific shocks, sticky prices, and home bias, under alternative financial market structures. What we have in mind is the achievement of monetary unification in Europe, often regarded as part of a process of integrating goods and financial markets. In particular, the integration of financial markets has long been considered as a crucial condition to foster the adoption of a common currency. The idea dates back to the contributions of Ingram (1969) and Mundell (1973), and places financial markets and their ability to provide insurance mechanisms against country-specific shocks at the heart of the definition of optimum currency areas. Our main result is that, in a second-best environment such as a monetary union with sticky prices, perfect risk-sharing through complete financial markets does not necessarily produce welfare gains. The result holds true for a wide range of empirically plausible parameter values and is robust to various policy configurations. It therefore makes an important warning against the simplistic and potentially misleading logic according to which structural policies aimed at deepening the integration of the financial markets should be the only priority when working toward monetary unification.

Formally, the paper builds on a two-country model of a currency area with sticky prices in the spirit of Benigno (2004). It is broader however, as it allows for home bias in private consumption, incompleteness or completeness of international financial markets, and embeds an explicit fiscal policy set-up, including distortionary labor income and sales taxes, public debt and public spending in the utility function. Sales taxes are aimed at closing efficiency gaps implied by the assumption of monopolistic competition. The labor income tax rate is set to keep public debt at a certain level in the long run, and public expenditure is the potentially active policy instrument. Our model is pretty close to the model considered by Pappa and Vassilatos (2007). However, Pappa and Vassilatos (2007) restrict their attention to specifications and parameter values under which the structure of financial markets is irrelevant in determining the equilibrium. In their model, as in Galí and Monacelli (2005), trade is balanced at each period. We proceed in a more general framework where trade imbalances arise endogenously and where the structure of international asset markets matters for the way these imbalances are financed, and thus for the determination of the equilibrium and the corresponding welfare losses from fluctuations.[1]

Completeness of international financial markets usually yields welfare gains by allowing agents to share risks perfectly across agents, countries and states of nature. As a counterpart, in multi-country models with representative agents, international relative prices are the favored

[1] Our model also shares some of the features of models analyzing the joint design of fiscal and monetary policy in a monetary union, such as Beetsma and Jensen (2005), Ferrero (2009), and Galí and Monacelli (2008).

external adjustment channel. When the Marshall-Lerner condition is satisfied, real exchange rates are more volatile under complete than under incomplete financial markets (see de Paoli (2009)). In particular, let us consider a second-best environment, characterized by country-specific shocks, price stickiness and fewer policy instruments than targets. In such an environment, as in monetary unions for instance, this excess of real exchange rate volatility can be costly for the following reason.

In new Keynesian models, Calvo contracts introduce a wedge between efficient (flexible prices) and sticky price fluctuations in country-level labor efforts. Fluctuations in producer prices' inflation rates lead this wedge to fluctuate, deteriorating households' welfare. In a closed economy, there is no trade-off between stabilizing the inflation rate and the output gap when the economy faces productivity shocks, and the first-best can be achieved. In a monetary union with sticky prices, the dynamics of national producer prices inflation rates not only depend on the output gap and expected inflation, but also on a terms-of-trade gap. The terms-of-trade gap depends on relative inflation rates and on structural shocks in a particular way. If inflation rates are fully stabilized, the gap is necessarily different from zero, while if the terms-of-trade gap is closed, inflation rates cannot be fully stabilized. This specificity of new Keynesian Phillips curves in monetary unions makes this environment a second-best world where national inflation rates and the terms-of-trade gap cannot be stabilized at the same time (see Benigno (2004)). Because of this policy trade-off, all equilibria result in positive volatility of national inflation rates (unless prices are flexible), even when monetary policy is designed optimally. Therefore, factors affecting the volatility of terms of trade, such as the structure of financial markets, also impact the volatility of inflation rates in equilibrium, whatever the monetary policy followed by the central bank.

In this environment, we show that alternative financial market structures result in two key effects. First, complete financial markets provide a better sharing of risks among members of the monetary union, and neutralize income effects. This results in welfare gains for the households. Second, as long as the Marshall-Lerner condition is met, complete financial markets result in more volatile relative prices (terms of trade) and more volatile national producer prices inflation rates, magnifying the welfare costs from nominal rigidities, and resulting in welfare losses. We show that the welfare costs from more volatile inflation rates can overturn the welfare gains from a better sharing of risks, reversing the traditional result according to which complete financial markets generate net welfare gains. This welfare reversal arises for a wide range of plausible parameterizations, and under various policy configurations. From a methodological point of view, the result is immune to the critique raised by Kim and Kim (2003), since a second-order approximation of equilibrium conditions is used to solve all models. All solutions are computed around the same steady-state and welfare comparisons are made using the conditional expectation of households aggregate welfare.[2]

According to our sensitivity analysis, the occurrence of welfare reversals crucially depends on the trade elasticity and the degree of price stickiness. First, the reversal arises when complete financial markets lead to more volatile terms of trade, which is the case only when depreciations (increases in competitiveness) generate trade surpluses, i.e. when the Marshall-Lerner condition is met. We show analytically that the latter depends on the trade elasticity. A sufficient condition is that the trade elasticity is larger than one. As long as this condition is satisfied, complete financial markets induce larger movements in terms of trade, which, given our environment, results in more volatile national inflation rates. However, this condition is not sufficient to obtain welfare reversals. For the welfare costs induced by complete financial markets and more volatile national inflation rates to overturn the welfare gains from a better sharing of risks, prices have to be sticky enough. According to our computations and for the chosen calibration of the model, welfare reversals arise when the Calvo parameter is greater than 0.33, i.e. when the frequency of price changes is larger than 1.5 quarters. Even though both parameters are subject to empirical controversies, we argue that the conditions for welfare reversals to arise are loose enough to be empirically plausible. Importantly, the fact that complete financial markets produce lower welfare losses from fluctuations when prices are flexible is in clear accordance with previous quantifications of the welfare gains of risk-sharing (see for instance Benigno (2009) and Van Wincoop (1999)). With flexible prices, the trade-off originating from the monetary union configuration vanishes and only the welfare gains from a better sharing of risks remain.

Lastly, we conduct an extensive robustness exercise and quantify the welfare losses from fluctuations from business cycles under complete and incomplete financial markets for various combinations of monetary and fiscal policies. None of these policy arrangements is able to reverse the ranking of welfare losses that arises when public spending policies are passive, at least for the parameterizations considered. In addition, our result is robust to alternative financing schemes for the public expenditure, and also arises when taxes are lump-sum.

The paper is structured as follows. Section 2 describes the various assumptions of the model, the policy set-up, the solution technique and the welfare computation as well as the baseline calibration. Section 3 analyzes the effects of productivity shocks when public-spending policies are passive. It also reports the quantitative properties of the model, including the welfare losses from fluctuations under alternative financial market structures. Additionally, Section 3 discusses the conditions under which welfare reversals arise. Section 4 proceeds to an ex-

[2] Because a second-order approximation of models is used, the conditional expectation of welfare is different from its steady-state value. We precisely make use of this difference to quantify the welfare losses from fluctuations.

tensive sensitivity analysis with respect to parameter values. Section 5 tests the robustness of the result to various monetary and fiscal policy arrangements. Section 6 concludes.

2 A two-country currency area

We model a two-country cashless currency area with a common central bank. Each country is populated by a unit mass of households, a unit mass of firms that are specialized in the production of differentiated varieties of goods and a government. Households have access to national government bonds and a single international financial asset – a one-period bond – so that the international financial market is incomplete. Access to the international financial market is also costly for households. Trading international bonds implies paying portfolio-management costs. Households' preferences are biased towards domestic goods. Production prices follow a Poisson arrival process, as in Calvo (1983). Governments raise taxes on sales and labor income. Public spending is financed by taxes or by issuing public one-period bonds. In this section, we describe and discuss the most important assumptions of the model.

2.1 Households

The monetary union is composed of two areas of identical size, namely the home country (h) and the foreign country (f). The representative household of country $i \in \{h, f\}$ maximizes a welfare index

$$W_0^i = E_0 \left[\sum_{t=0}^{\infty} \beta^t u \left(c_t^i, g_t^i, n_t^i \right) \right], \ 0 < \beta < 1, \tag{1}$$

with first-order derivatives u_c, $u_g \geqslant 0$, $u_n \leqslant 0$ and with second-order derivatives u_{cc}, u_{cg}, u_{gg}, $u_{nn} \leqslant 0$, u_{cn}, $u_{gn} = 0$, subject to the following budget constraint

$$b_t^i + b_{g,t}^i + p_t^i \left(c_t^i + ac_t^i \right) = r_{t-1} b_{t-1}^i + r_{g,t-1}^i b_{g,t-1}^i + \left(1 - \zeta_t^i \right) w_t^i n_t^i + \varphi_t^i, \tag{2}$$

and a standard transversality condition on wealth. In Equation (1), the parameter β is the subjective discount factor, c_t^i is the level of consumption, g_t^i is the level of public spending in country i and n_t^i is the level of hours worked. In Equation (2), b_t^i is the holding of one-period international nominal bonds at the end of period $t-1$ that pays a gross nominal rate of interest r_t between periods $t-1$ and t and $b_{g,t}^i$ is the holding of one-period local government nominal bonds at the end of period $t-1$ that pays a gross nominal rate of interest $r_{g,t}^i$ between periods $t-1$ and t. The consumer price index (CPI) in country i is denoted p_t^i, the nominal wage is w_t^i, the tax rate on labor income is ζ_t^i, and $\varphi_t^i = \int_0^1 \varphi_t^i(k)dk$ is the profit paid by monopolistic firms to national households.

In the optimization problem described above, households have access to a single international asset. Because both countries are affected by country-specific (idiosyncratic) shocks, the possible future states of the economy are numerous. The international financial market is thus incomplete and risk-sharing is imperfect. As underlined by Schmitt-Grohé and Uribe (2003), this situation leads to non-stationary dynamics originating in the dynamics of net foreign assets. This happens because any temporary idiosyncratic shock leads to permanent wealth transfers, affecting consumption, labor supply, inflation rates, or terms of trade. Therefore, we introduce an additional assumption to ensure stationary dynamics when the international financial market is incomplete. We assume that ac_t^i is a quadratic adjustment cost that households have to pay to change their net foreign asset position. The latter is expressed in units of the consumption good and defined as

$$ac_t^i = \frac{\chi}{2}\left(\frac{b_t^i}{p_t^i} - \frac{b^i}{p^i}\right)^2, \; \chi > 0, \tag{3}$$

where b^i/p^i is the real steady-state level of net foreign assets. First-order conditions combine to yield

$$\beta E_t\left[\frac{r_t}{1+\chi(b_t^i/p_t^i - b^i/p^i)}\frac{u_{c^i,t+1}}{\pi_{t+1}^i u_{c^i,t}}\right] = 1, \tag{4}$$

$$\beta E_t\left[r_{g,t}^i\frac{u_{c^i,t+1}}{\pi_{t+1}^i u_{c^i,t}}\right] = 1, \tag{5}$$

$$-\frac{u_{n^i,t}}{u_{c^i,t}} - \left(1-\zeta_t^i\right)\omega_t^i\frac{p_{i,t}}{p_t^i} = 0, \tag{6}$$

where $\pi_t^i = p_t^i/p_{t-1}^i$ is the CPI inflation rate and $\omega_t^i = w_t^i/p_{i,t}$ is the producer price index (PPI) based real wage. According to Equation (4), buying (respectively selling) bonds affects negatively (resp. positively) the individualized interest rate on which households base their consumption smoothing decision, so that *(i)* households belonging to a creditor country face lower nominal interest rates than households in the debtor country and *(ii)* households return to their initial position in the long run. Equation (5) summarizes the arbitrage between national (government) bonds and the international bond. Equation (6) is a standard open-economy labor supply relation.

As opposed to the situation of incomplete financial market, we also consider a situation with complete international financial markets. In such an economy, households have access to Arrow-Debreu securities that are traded before policy choices are made, and risk-sharing is perfect (see Senay and Sutherland (2007, 2011)). First-order conditions with respect to private consumption and international financial assets thus imply

$$\beta E_t\left[r_t\frac{u_{c^h,t+1}}{\pi_{t+1}^h u_{c^h,t}}\right] = 1, \text{ and } \frac{u_{c^f,t}}{u_{c^h,t}} = \Gamma_0 q_t, \tag{7}$$

where Γ_0 is a constant reflecting initial relative wealths and $q_t = p_t^f/p_t^h$ is the real exchange rate, while Equations (5) and (6) remain unchanged. Households equalize their wealth and share risk across time and across states of the economy. Hence, temporary shocks do not imply wealth transfers. Net foreign assets move only to the extent that relative prices move but the quantity of assets held by each household remains constant in equilibrium. Relative marginal utilities of consumption are thus determined by relative CPIs only.

After optimizing for aggregate consumption, households optimize the composition of the consumption bundle (see Galí and Monacelli (2005) or Pappa and Vassilatos (2007)). Households consume both domestic and foreign goods. Both goods are imperfectly substitutable with elasticity of substitution μ. In addition, households' preferences are biased towards local goods. We denote $1-\alpha^i$ as the share of goods produced in country h in the aggregate consumption of the household living in country i. In equilibrium, $\alpha_h < 1/2$ and $1-\alpha_f < 1/2$ also happen to be the shares of imported goods in GDP and are natural measures of trade openness. The aggregate consumption bundle of the household living in country i is thus

$$c_t^i = \left((1-\alpha_i)^{\frac{1}{\mu}} \left(c_{h,t}^i\right)^{\frac{\mu-1}{\mu}} + \alpha_i^{\frac{1}{\mu}} \left(c_{f,t}^i\right)^{\frac{\mu-1}{\mu}} \right)^{\frac{\mu}{\mu-1}}, \mu > 0, \tag{8}$$

and the companion consumption price index is

$$p_t^i = \left((1-\alpha_i) \left(p_{h,t}^i\right)^{1-\mu} + \alpha_i \left(p_{f,t}^i\right)^{1-\mu} \right)^{\frac{1}{1-\mu}}. \tag{9}$$

In each country, firms produce differentiated varieties k. Goods are bundles of varieties with elasticity of substitution θ. The corresponding consumption bundles are

$$c_{h,t}^i = \left(\int_0^1 c_{h,t}^i(k)^{\frac{\theta-1}{\theta}} dk \right)^{\frac{\theta}{\theta-1}} \text{ and } c_{f,t}^i = \left(\int_0^1 c_{f,t}^i(k)^{\frac{\theta-1}{\theta}} dk \right)^{\frac{\theta}{\theta-1}}, \theta > 1, \tag{10}$$

where $c_{h,t}^i(k)$ (resp. $c_{f,t}^i(k)$) is the consumption of a typical good k of country h (resp. f) by the household of country i. Producers do not price-discriminate markets, so that

$$p_{h,t}^i = p_{h,t} = \left(\int_0^1 p_{h,t}(k)^{1-\theta} dk \right)^{\frac{1}{1-\theta}} \text{ and } p_{f,t}^i = p_{f,t} = \left(\int_0^1 p_{f,t}(k)^{1-\theta} dk \right)^{\frac{1}{1-\theta}}. \tag{11}$$

Accordingly, optimal variety demands depend on relative prices of goods, on relative prices of varieties and on the aggregate consumption level in each country

$$c_{h,t}^i(k) = (1-\alpha_i) \left(\frac{p_{h,t}}{p_t^i} \right)^{-\mu} \left(\frac{p_{h,t}(k)}{p_{h,t}} \right)^{-\theta} c_t^i \text{ and } c_{f,t}^i(k) = \alpha_i \left(\frac{p_{f,t}}{p_t^i} \right)^{-\mu} \left(\frac{p_{f,t}(k)}{p_{f,t}} \right)^{-\theta} c_t^i. \tag{12}$$

Lastly, the terms of trade are defined as the relative price of the foreign good in terms of the domestic good

$$s_t = \frac{p_{f,t}}{p_{h,t}}. \tag{13}$$

2.2 Firms

Each firm is specialized in the production of a single variety k of good according to a linear production function

$$y_t^i(k) = a_t^i \ell_t^i(k), \tag{14}$$

where productivity measures a_t^i evolve according to autoregressive processes with persistence ρ_a and are affected by iid innovations $\varepsilon_{a,t}^i$ with constant variance. The marginal cost of firm k in country i is

$$mc_t^i(k) = mc_t^i = w_t^i / a_t^i. \tag{15}$$

Production prices are governed by Calvo (1983) contracts. Let η^i denote the probability faced by producers of country i to be constrained to keep their prices unchanged. The corresponding optimal pricing scheme for producers allowed to reset is[3]

$$\overline{p}_{i,t}(k) = \Phi \frac{\sum_{v=0}^{\infty} \left(\eta^i \beta\right)^v E_t \left[y_{t+\nu}^i(k) u_{c^i,t+\nu} mc_{t+\nu}^i / p_{t+\nu}^i\right]}{\sum_{v=0}^{\infty} \left(\eta^i \beta\right)^v E_t \left[y_{t+\nu}^i(k) u_{c^i,t+\nu} / p_{t+\nu}^i\right]}, \; 0 < \eta^i < 1, \tag{16}$$

where $\Phi = \theta / \left((\theta - 1)(1 - \tau)\right) \geq 1$ and $y_t^i(k)$ is the aggregate demand faced by firm k. In Equation (16), τ is the (constant) tax rate on sales imposed by the government. The latter is intended to restore Pareto-optimal steady-state allocations (see next section for details). Aggregating among firms and assuming behavioral symmetry, the production price index in country i is

$$p_{i,t} = \left(\left(1 - \eta^i\right) \overline{p}_{i,t}(k)^{\;1-\theta} + \eta^i p_{i,t-1}^{1-\theta}\right)^{\frac{1}{1-\theta}}. \tag{17}$$

A simple recursive representation of the pricing conditions yields

$$\eta^i \pi_{i,t}^{\theta-1} + \left(1 - \eta^i\right) \left(\Phi x_{1,t}^i / x_{2,t}^i\right)^{\;1-\theta} = 1. \tag{18}$$

$$x_{1,t}^i - \eta^i \beta E_t \left[x_{1,t+1}^i \pi_{i,t+1}^{1+\theta} / \pi_{t+1}^i\right] - y_t^i u_{c^i,t} \left(w_t^i / p_{i,t}\right) / a_t^i = 0, \tag{19}$$

$$x_{2,t}^i - \eta^i \beta E_t \left[x_{2,t+1}^i \pi_{i,t+1}^{\theta} / \pi_{t+1}^i\right] - y_t^i u_{c^i,t} = 0, \tag{20}$$

where $\pi_{i,t} = p_{i,t} / p_{i,t-1}$ is the PPI inflation rate. Lastly, the dispersion of production prices, $\Upsilon_{i,t} = \int_0^1 \left(p_{i,t}(k) / p_{i,t}\right)^{-\theta} dk$, has the following dynamics

$$\Upsilon_{i,t} = \eta^i \Upsilon_{i,t-1} \pi_{i,t}^{\theta} + \left(1 - \eta^i\right) \left(\Phi x_{1,t}^i / x_{2,t}^i\right)^{-\theta}. \tag{21}$$

[3] Since Calvo (1983), this type of Poisson adjustment process is widely used in the macroeconomic literature to model staggered price setting behavior. As the assumption is by now very standard, we do not detail the calculations.

2.3 Governments

In each country, a government finances real public spending g_t^i by levying taxes on national households labor income at the rate ζ_t^i, national firms sales at the rate τ or by issuing one-period government nominal bonds. These bonds can be subscribed by national households only. Public expenditure is fully home biased, and the corresponding demand of goods falls on national goods only. The budget constraint of government in country i in period t is

$$d_t^i - r_{g,t-1}^i d_{t-1}^i = p_{i,t} g_t^i - \tau \int_0^1 p_{i,t}(k)\, y_t^i(k)\, dk - \zeta_t^i w_t^i n_t^i. \tag{22}$$

2.4 Equilibrium

We assume symmetry in the home bias parameter and impose $\alpha_h + \alpha_f = 1$, so that $\alpha_h = 1 - \alpha_f = \alpha$. Defining aggregate output as $y_t^i = \left(\int_0^1 y_t^i(k)^{\frac{\theta-1}{\theta}} dk\right)^{\frac{\theta}{\theta-1}}$ for $i \in \{h, f\}$, an equilibrium is a sequence of quantities

$$\{\mathcal{Q}_t\}_{t=0}^{\infty} = \left\{y_t^h, y_t^f, c_t^h, c_t^f, n_t^h, n_t^f, \ell_t^h, \ell_t^f, b_t^h, b_t^f, b_{g,t}^h, b_{g,t}^f, d_t^h, d_t^f, ac_t^h, ac_t^f\right\}_{t=0}^{\infty}, \tag{23}$$

a sequence of prices

$$\{\mathcal{P}_t\}_{t=0}^{\infty} = \left\{p_t^h, p_t^f, w_t^h, w_t^f, mc_t^h, mc_t^f, p_{h,t}, p_{f,t}, x_{1,t}^h, x_{1,t}^f, x_{2,t}^h, x_{2,t}^f, \Upsilon_{h,t}, \Upsilon_{f,t}\right\}_{t=0}^{\infty}, \tag{24}$$

such that, conditionally on a sequence of productivity shocks $\{\xi_t\}_{t=0}^{\infty} = \left\{\xi_{a,t}^h, \xi_{a,t}^f\right\}_{t=0}^{\infty}$, of fiscal policies $\{\mathcal{G}_t\}_{t=0}^{\infty} = \left\{\zeta_t^h, \zeta_t^f, g_t^h, g_t^f\right\}_{t=0}^{\infty}$, and a monetary policy $\{\mathcal{R}_t\}_{t=0}^{\infty} = \{r_t\}_{t=0}^{\infty}$:

(i) For a given sequence of prices $\{\mathcal{P}_t\}_{t=0}^{\infty}$, $\{\mathcal{Q}_t\}_{t=0}^{\infty}$ satisfies households and firms optimality conditions and balances the governments budget constraints.

(ii) For a given sequence of quantities $\{\mathcal{Q}_t\}_{t=0}^{\infty}$, the sequence $\{\mathcal{P}_t\}_{t=0}^{\infty}$ clears final goods markets

$$y_t^h = (1-\alpha)\left(\frac{p_{h,t}}{p_t^h}\right)^{-\mu}\left(c_t^h + ac_t^h\right) + \alpha\left(\frac{p_{h,t}}{p_t^f}\right)^{-\mu}\left(c_t^f + ac_t^f\right) + g_t^h, \tag{25}$$

$$y_t^f = (1-\alpha)\left(\frac{p_{f,t}}{p_t^f}\right)^{-\mu}\left(c_t^f + ac_t^f\right) + \alpha\left(\frac{p_{f,t}}{p_t^h}\right)^{-\mu}\left(c_t^h + ac_t^h\right) + g_t^f, \tag{26}$$

labor markets,

$$n_t^i = \int_0^1 \ell_t^i(k) dk, \text{ for } i \in \{h, f\}, \tag{27}$$

the international financial market

$$b_t^h + b_t^f = 0, \tag{28}$$

and government bonds markets

$$d_t^i = b_{g,t}^i, \text{ for } i \in \{h, f\}. \tag{29}$$

Labor markets clearing conditions imply that the aggregate production function is

$$y_t^i \Upsilon_{i,t} = a_t^i n_t^i, \text{ for } i \in \{h, f\}. \tag{30}$$

Equation (30) illustrates that price stickiness implies some heterogeneity in individual production prices since $\Upsilon_{i,t} \geq 1$ and therefore a certain dispersion of labor demands, that is costly in terms of welfare. The dispersion of production prices implied by Calvo contracts induces inefficient fluctuations of hours worked in the equilibrium since a share $\Upsilon_{i,t} - 1 \geq 0$ of output is lost due to sticky prices, as compared to the situation of flexible prices where $\Upsilon_{i,t} = 1$ always.

Lastly, the aggregation of constraints yields the dynamics of net foreign assets

$$b_t^i - r_{t-1} b_{t-1}^i = tb_t^i. \tag{31}$$

where $tb_t^i = p_{i,t}\left(y_t^i - g_t^i\right) - p_t^i c_t^i$ is the trade balance of country i at time t.

2.5 Policy set-up

We describe governments and central bank behavior, as fiscal and monetary policies may altogether affect the economy.

2.5.1 Fiscal policy

In the model, monopolistic competition distorts the first-best steady-state allocation through mark-up pricing and results in lower steady-state output. As shown by Benigno and Woodford (2005), these first-order distortions are offset when $\Phi^* = \theta / \left(\left(\theta - 1\right)\left(1 - \tau^*\right)\right) = 1$, i.e. when $\tau^* = 1/\left(1 - \theta\right)$. Since $\theta > 1$, the optimal tax on sales τ^* is negative.

We assume that government spending is the main policy instrument. The assumptions regarding the way public spending policies are determined and their implications for fluctuations are discussed later in the paper. Following Galí, López-Salido and Vallés (2007), labor income taxes are adjusted by governments to meet a certain level of debt to GDP in the long run. We make this assumption to prevent the eventuality of unit roots on public debts under conditions discussed later.[4] Governments thus commit to the following labor income tax rule, which

$$\zeta_t^i = \zeta + \phi_{\zeta,d}\left(d_{y,t-1}^i - d_y\right), \tag{32}$$

where ζ is the steady-state level of taxes on labor income and $d_{y,t}^i = d_t^i / \left(p_{i,t} y_t^i\right)$ is the level of real public debt over GDP.[5] As in Galí et al. (2007), $\phi_{\zeta,y}$ controls the time horizon at

[4] These unit roots are very frequent when analyzing fiscal policy with public debt (see Ferrero (2009) among many others).

[5] We consider a symmetric set-up, implying that steady-state values as well as policy parameters are not country-specific.

which governments meet their commitment. Lastly the dynamics of real public debt over GDP writes:

$$d^i_{y,t} - r^i_{g,t-1}\left(y^i_{t-1}/y^i_t\right)\left(d^i_{y,t-1}/\pi_{i,t}\right) = g^i_t/y^i_t - \tau - \zeta^i_t w^i_t n^i_t/(p_{i,t}y^i_t). \tag{33}$$

2.5.2 Monetary policy

We consider two types of monetary policies: a simple monetary policy rule and a Ramsey optimal monetary policy. While the latter is analyzed as a robustness check of our results, the former is considered as the benchmark monetary policy. The most natural candidate is a Taylor-type rule. In this case, the central bank of the monetary union controls the nominal interest rate at time t. Changes in the nominal interest rate imply changes in country-specific real interest rates since prices are sticky and the transmission of monetary policy operates through an intertemporal substitution effect. As long as both countries face an identical degree of nominal rigidities, the aggregate block of the model behaves as a closed-economy version of the model. Because we consider productivity shocks, aggregate inflation and the aggregate output gap move in the very same direction. Therefore the inclusion of an output gap target in the rule would be redundant, and could only result in larger welfare losses (see Benigno (2004) and Ferrero (2009)). Consequently, we consider that the central bank only targets the aggregate CPI inflation rate. We also introduce some persistence in the changes of the monetary policy instrument:

$$r_t = \rho_r r_{t-1} + (1-\rho_r)\left(r + \phi_{r,\pi}\left(\pi^u_t - \pi^u\right)\right),\ 0 < \rho_r < 1,\ \phi_{r,\pi} > 1, \tag{34}$$

where $\pi^u_t = (\pi^h_t + \pi^f_t)/2$ is the aggregate CPI inflation rate.

2.6 Solution and parameter values

Preferences The utility function takes the following functional form

$$u\left(c^i_t, g^i_t, n^i_t\right) = \frac{1}{1-\rho}\left(\left(c^i_t\right)^{(1-\kappa)}\left(g^i_t\right)^{\kappa}\right)^{1-\rho} - \frac{\epsilon}{1+\psi}\left(n^i_t\right)^{1+\psi},\ \ \rho > 1,\ \kappa, \epsilon, \psi > 0. \tag{35}$$

In this expression, ρ is the constant degree of relative risk-aversion, κ is the share of public goods in the "aggregate" consumption bundle and $1/\psi$ is the (constant) Frisch elasticity of labor supply.

Shocks In the benchmark version of the model, only productivity shocks are considered.[6] Productivity innovations are purely country-specific (idiosyncratic) in the baseline case, but we allow for some cross-country correlation of innovations (labeled ν_a) as a robustness check. The asymmetry of shocks is a key element to generate our results. Combined with the assumption of home bias, different shocks imply that business cycles are different in each country.

[6] In Appendix C, we consider public spending shocks and monetary policy shocks as an extension.

As it reacts to aggregate variables in the monetary union, monetary policy is not well-suited to economic conditions in both economies simultaneously. For example, monetary policy is too tight for the country experiencing an asymmetric deflationary shock and too loose in the other country. The common monetary policy thus acts as an endogenous transmission mechanism, that enhances business cycle differences in the monetary union. In addition, these differences imply an adjustment of relative prices, i.e. terms of trade and the real exchange rate, introducing a trade-off for the central bank between closing the terms-of-trade gap and stabilizing national inflation rates (see Benigno (2004)).

Steady-state We assume that the decentralized steady-state is consistent with the steady-state that would be chosen by a central planner, so that all equilibria considered in the paper (including Ramsey equilibria) share the same steady-state. This is the case when $\tau = 1/(1-\theta)$ and $g = \kappa y$. The calculation of the decentralized symmetric steady-state of the model is then straightforward, as we further assume $b^h = b^f = 0$, normalize the steady-state value of productivity ($a = 1$), as well as the price level ($p = 1$). We get $w = 1$, $y = n$, $c = (1-\kappa) y$, and hours worked in the steady-state are

$$y = n = \left((1-\zeta)\,\epsilon^{-1} \left((1-\kappa)^{(1-\kappa)}\,\kappa^{\kappa} \right)^{(1-\rho)} \right)^{\frac{1}{\psi+\rho}}, \tag{36}$$

where the steady-state tax rate ζ is uniquely pinned down by the target level of public debt over GDP in the steady-state d_y, imposed exogenously by the government (see Equation (32)).

Preference parameters. The model is parameterized to be consistent with the situation of the EMU during its recent history (1999-2009). The discount factor is $\beta = 0.99$, implying an average 4.1 percent annual real interest rate. The preference parameter ϵ is adjusted so that $n = 1$. As in Smets and Wouters (2003), we set $\rho = 1.5$. The inverse of the Frisch elasticity is $\psi = 1$, which lies in the upper bound of the range put forth by Canzoneri, Cumby and Diba (2007). As our results are clearly sensitive to these parameters and since their value is highly debated in the literature, an extensive sensitivity analysis is also conducted. The parameter α is chosen to match the average share of imports in the Euro area (calculated using OECD Economic Outlook data for 2006), i.e. $\alpha = 0.35$. The value of μ is widely debated in the literature. The estimates of Harrigan (1993) range from 5 to 12, while the literature on international business cycles usually sets this parameter to much lower values – between 1 and 2.5 – to match the volatility of the trade balance (see Backus, Kehoe and Kydland (1993)). Following this literature, we set the value of this parameter to $\mu = 1.5$. For this parameter again, a sensitivity analysis is conducted. The elasticity of substitution between varieties is set according to Rotemberg and Woodford (1997) at $\theta = 7$. This parameter is of great importance as it shapes the magnitude of prices, output and hours dispersion implied by fluctuations in producer prices inflation rates, and affects the welfare losses from nominal rigidities, along with the degree of prices stickiness, as shown by Equation (30).

Invariant policy parameters. The steady-state share of public spending is chosen to match the average level of government expenditures (excluding transfers) in the EMU, i.e. $\kappa = 0.25$. The steady-state level of labor income taxes ζ is set to match the level of debt to GDP in the EMU. As our setting is quarterly, we impose $d_y = 2.4$, which corresponds to a 60 percent debt to annual GDP ratio. Given the chosen calibration, the corresponding steady-state tax rate on labor income is 44.09 percent.[7] This figure is in accordance with tax wedges on labor income reported by the OECD (Taxing wages 2007/2008: 2008 Edition). The value of $\phi_{\zeta,d}$, the sensitivity of labor income taxes to deviations of public debt to its steady-state value, is crucial in determining the stability of public debt dynamics. Up to a first-order approximation, a sufficient condition for stability and sustainability is $\phi_{\zeta,d} > \beta^{-1} - 1$. Tax policy is not our primary focus and tax rules are considered only to ensure stationarity under all circumstances. We thus calibrate $\phi_{\zeta,d}$ over a minimal value, namely $\phi_{\zeta,d} = 0.02$.

Other parameters. Nominal rigidities are symmetric within the monetary union. Building on recent New Keynesian Phillips curves estimates in the Euro area assessing that $\eta = 0.78$ in average (see Rumler (2007)), Calvo parameters are $\eta^h = \eta^f = 0.8$. These values imply that prices are reset every 5 quarters in average. The parameter governing portfolio costs is set to $\chi = 0.001$, close to Schmitt-Grohé and Uribe (2003), implying that transaction costs on international bonds represent an annual 0.4 percent interest rate premium. Lastly, we set parameters governing the dynamics of productivity shocks to $\rho_a = 0.9$, $\sigma\left(\varepsilon^i_{a,t}\right) = 0.01$. In the benchmark situation, the correlation of shocks is null, i.e. $\nu_a = 0$. Table 1 summarizes the baseline numerical values assigned to structural parameters of our economy.

Solution and implications for welfare. As the model does not admit a closed-form solution, it is solved by using a second-order approximation of equilibrium relations around the steady-state.[8] This ensures that our results are immune to the Kim and Kim (2003) critique concerning spurious welfare reversals.

3 Welfare reversals

This section addresses the implications of the structure of financial markets when public spending policies are passive. We thus consider constant public spending, $g^i_t = g$. In this case, stabilization is operated by monetary policy only. The latter evolves according to Equation (34). Following Smets and Wouters (2003), parameters of the monetary policy rule are set to $\rho_r = 0.9$, and $\phi_\pi = 1.5$. The case of active public spending policies is investigated as a robustness check in Section 5.

[7] The exact value is $\zeta = \kappa - 1/(1-\theta) - d_y\left(1-\beta^{-1}\right)$.

[8] Equilibrium relations are summarized in Appendix A.

Table 1: Benchmark parameter values

Discount factor	$\beta = 0.99$
Inverse of the Frisch elasticity	$\psi = 1$
Risk aversion	$\rho = 1.5$
Elasticity of substitution between goods	$\mu = 1.5$
Elasticity of substitution between varieties	$\theta = 7$
Home bias in private consumption	$1 - \alpha = 0.65$
Steady-state share of public spending in GDP	$\kappa = 0.25$
Steady-state stock of public debt over quarterly GDP	$d_y = 2.4$
Elasticity of labor income taxes to public debt	$\phi_{\zeta,d} = 0.02$
Average duration of prices, in quarters	$(1-\eta)^{-1} = 5$
Portfolio adjustment cost	$\chi = 0.001$
Persistence of productivity shocks	$\rho_a = 0.9$
Standard deviation of productivity shocks	$\sigma\left(\varepsilon_a^i\right) = 0.01$
Cross-country correlation of productivity shocks	$\nu_a = 0$

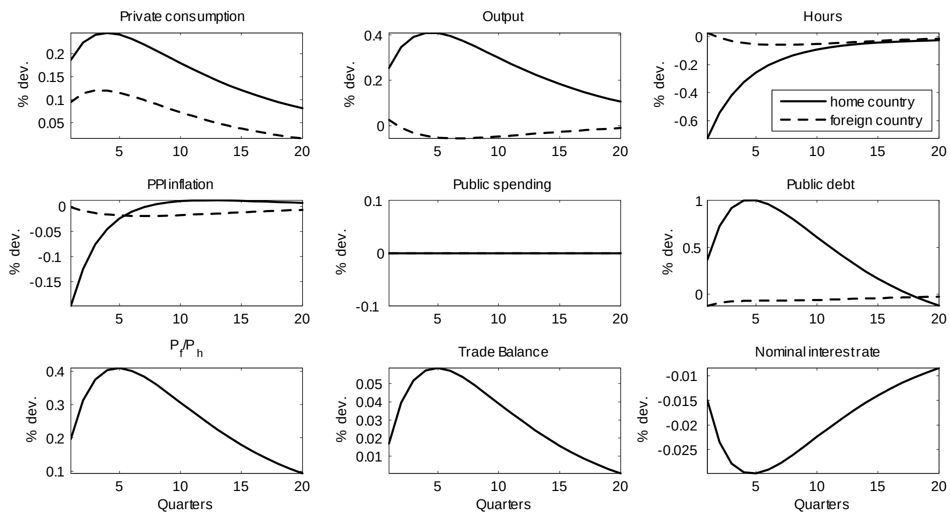
3.1 Dynamics

The dynamics implied by incomplete and complete markets are qualitatively very similar. For this reason and also to provide some intuition on our results, we first analyze the dynamics under incomplete financial markets, and second contrast the differences arising with the dynamics under complete financial markets.

Figure 1 displays the Impulse Response Functions (IRFs hereafter) of key macroeconomic variables in both countries after a one standard deviation purely asymmetric productivity shock in the domestic economy when financial markets are incomplete.

After the shock, domestic consumption and output increase, while hours fall because of the wealth effect. From a general equilibrium perspective, domination of the wealth effect over the substitution effect after an increase in the real wage critically depends on the form of preferences (separable or not in labor), on whether physical capital is considered or not and on the shape of the marginal utility of consumption. In our model, preferences are separable in labor, and capital accumulation is absent. After a positive productivity shock, the real wage increases, as well as consumption and the marginal utility of consumption decreases more than proportionally. According to Equation (6), the marginal disutility of hours worked has to decrease, which leads households to reduce their labor supply. Productivity gains increase the real wage but since production prices are sticky, a wedge between the real wage and productivity gains develops. The real marginal cost shrinks, implying a deflation. Public debt builds up since (i) the collection of labor income taxes increases less than sales subsidies on impact and (ii) the deflation magnifies the value of the stock of public debt over GDP.

Figure 1: IRFs after a one standard deviation purely asymmetric productivity shock in the domestic economy under incomplete financial markets.



After few periods, the tax rate on labor income increases, and hours worked return to their steady-state value. This induces a stabilization of real debt over GDP to its steady-state level.

Monetary policy consists of lowering the common nominal interest rate more than aggregate inflation. It further increases consumption and output in the domestic economy. In the foreign economy, consumption is boosted, while the response of output remains almost muted. Indeed, due to the domestic deflation, terms of trade worsens (competitiveness improves) in the domestic economy, leading both domestic and foreign households to shift temporarily their consumption toward goods produced in the domestic economy. This movement also generates a trade-balance surplus in the domestic economy. Noticeably, this expenditure-switching effect is strong enough to almost offset the boom in foreign output induced by lower interest rates. Therefore the overall effect of the domestic productivity shock on foreign output is close to zero, and the response of foreign hours worked exactly follows that of foreign output.

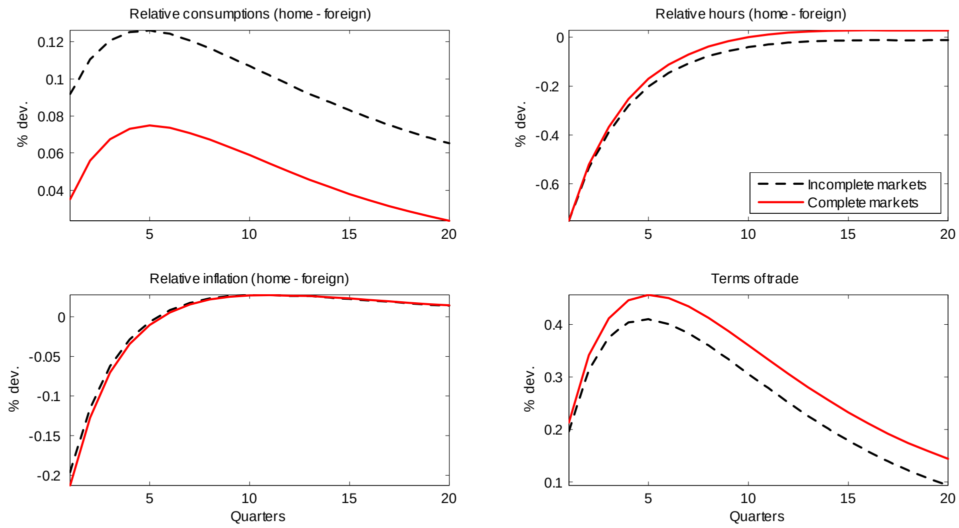
Responses when international financial markets are complete are qualitatively very similar. Quantitatively, the difference is clear, however. As shown by de Paoli (2009), incomplete asset markets provide partial insurance to households and the connection between income and consumption is tighter than under complete asset markets. While complete markets insulate households from income effects, incomplete markets do not. This difference is a source of quantitative differences in the response of terms of trade. After an asymmetric domestic productivity shock, domestic terms of trade worsen (domestic competitiveness improves), which increases the external purchasing power of foreign households. It allows them to increase their consumption level, but this increase is based on imports from the domestic country and associated with a muted response of foreign output. Under incomplete markets, imperfect risk-sharing renders foreign consumption more sensitive to fluctuations in foreign output. Therefore, due to income effects, foreign consumption increases less because the response of output is muted.

Ultimately, the way the structure of asset markets affects the response of terms of trade in equilibrium depends on how fluctuations of terms of trade affect the trade balance. If trade balance improves after a depreciation, imperfect risk-sharing lowers the equilibrium magnitude of the response of terms of trade. After an asymmetric domestic productivity shock, foreign terms of trade improve (foreign competitiveness is lower). It helps foreign households to increase their consumption but lowers foreign output, since both domestic and foreign households substitute domestic goods to foreign goods. If consumption is more closely tied to income (output) because of imperfect risk-sharing, less responsive terms of trade can help mitigate the income effects and help foreign consumption increase. Of course, if trade balance worsens after a depreciation, incomplete markets lead to the exact opposite effect. In this case, the response of terms of trade is larger under incomplete markets than under complete markets. In our framework, with productivity shocks only, a sufficient condition for trade balance to improve after a depreciation is $\mu > \mu^*$, a condition that is met in our baseline calibration (see Appendix B for a proof and Section 3.2 for an extensive discussion).

Figure 2 reports the IRFs of relative consumptions, hours, inflation rates, as well as terms of trade both under incomplete and complete asset markets. Clearly, after an asymmetric domestic productivity shock, an economy with complete markets features a larger adjustment in terms of trade than under incomplete markets. This relates to the absence of income effects under complete markets and to the fact that the condition $\mu > \mu^*$ is met in our basline calibration. These larger movements have two important consequences. First, domestic consumption increases less and domestic hours fall less due to the absence of income effects. This is expected to generate welfare gains for the households. Second, the domestic inflation rate falls more deeply because terms of trade adjust more. This effect is expected to produce welfare losses, because the associated price dispersion is larger.

Our analysis reveals that completeness of financial markets results in two competing effects, as compared to incompleteness. First, complete financial markets neutralize income effects and bring domestic and foreign consumptions closer to each other. Second, complete financial markets result in larger fluctuations in terms of trade, and therefore in PPI inflation rates. Complete financial markets are thus expected to affect welfare losses from fluctuations in two opposite directions: a better sharing of risks will increase households welfare in the monetary union, while more volatile national inflation rates will lead to additional welfare losses, as the

Figure 2: IRFs after a one standard deviation purely asymmetric productivity shock in the domestic economy under incomplete vs. complete financial markets.



dispersion of prices, output and hours will increase in both economies. Numerical simulations are conducted in the next section to determine which effect dominates and under which conditions.

3.2 Volatilities and welfare

Welfare losses are computed using the conditional mean of a welfare metric

$$W_0 = W_0^h + W_0^f, \tag{37}$$

that takes into account the impact of second-order moments on conditional first-order moments. As we use a second-order approximation of equilibrium conditions, the welfare measure differs from its steady-state (unconditional) value. We make use of this difference and convert welfare losses in an equivalent percentage of steady-state consumption Λ, that agents would be willing to give up to live in a world without fluctuations

$$W_0 = 2\frac{u\left(\left(1-\Lambda/100\right)c, g, n\right)}{(1-\beta)}, \tag{38}$$

where c, g, and n respectively denote steady-state consumption, public spending and hours worked.

Table 2 summarizes our main results. It reports the standard deviations of private consumption, public spending, hours worked, PPI inflation and terms of trade under alternative assumptions regarding international financial markets and the welfare losses from fluctuations. In the case of complete financial markets, it also reports the corresponding welfare loss with respect to the situation of incomplete financial markets. It is labeled Δ and expressed as the percentage increase in consumption that households would be ready to give up to live in an economy with incomplete financial markets. The results are presented for the benchmark parameter values.

Table 2: Standard deviations and welfare losses under passive public spending policies

	Standard deviations					Welfare Losses (%)	
	c_t^i	g_t^i	n_t^i	$\pi_{i,t}$	s_t	Λ	Δ
Incomplete markets	0.8946	0.0000	1.1730	0.2619	1.8073	0.1215	–
Complete markets	0.8343	0.0000	1.1517	0.2722	2.1144	0.1245	0.0035

Note: Shocks are purely asymmetric, i.e. $\nu_a = 0$. Simulations are carried out using a second-order approximation of equilibrium conditions.

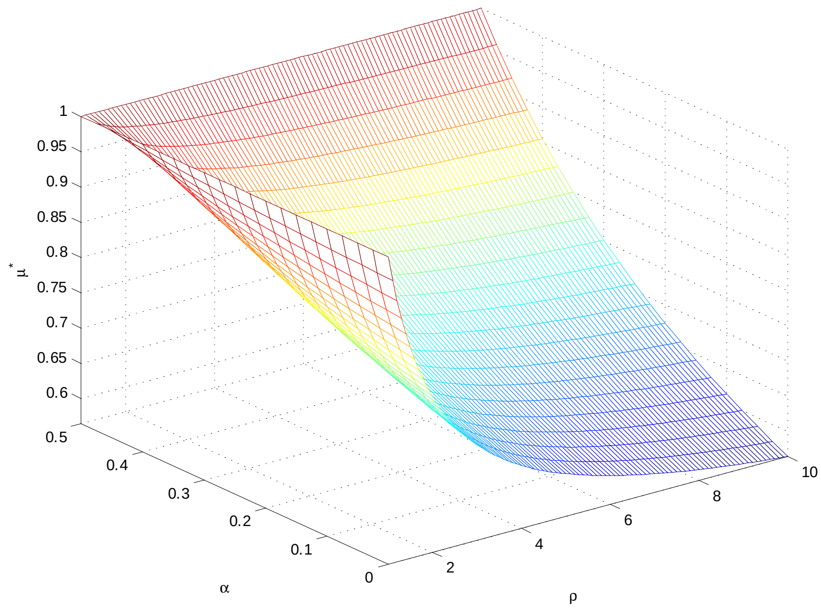
In terms of volatility, the main results can be summarized as follows: when international financial markets are incomplete, private consumptions and hours are more volatile, and PPI inflation rates and terms of trade are less volatile. In this case, both complete and incomplete markets deliver exactly similar volatilities. Completeness of financial markets implies that consumptions are less volatile because international wealth effects are perfectly offset. Therefore unexpected shocks do not generate wealth transfers. To achieve this smoother path of relative consumptions and hours, relative prices adjust more than when international financial markets are incomplete, as long as $\mu > \mu^*$. Due to price stickiness and to the lack of policy instruments to address this distortion to the efficient equilibrium, incomplete financial markets generate a positive externality on the volatility of PPI inflation rates. The latter are required to adjust less because terms of trade adjust less.

In our framework, a sufficient condition for terms of trade to be more volatile under complete markets is that the Marshall-Lerner condition is met (see de Paoli (2009)). As shown in Appendix B, the general condition for depreciations to generate trade surpluses writes

$$\mu > \mu^* = \frac{\rho + \kappa (1 - \rho) + 1 - 2\alpha}{2 (1 - \alpha) (\rho + \kappa (1 - \rho))}. \tag{39}$$

After productivity shocks, relative consumptions and terms of trade evolve in opposite directions, with opposite effects on the trade balance. A domestic productivity shock increases domestic consumption more than foreign consumption and terms of trade worsen (domestic firms are more competitive). The dynamics of relative consumptions deteriorates the trade balance, while that of terms of trade induces an expenditure-switching effect that improves the trade balance. For the latter to dominate, and the trade balance to improve, the condition is that $\mu > \mu^*$. When this condition is met, depreciations improve the trade balance and complete financial markets lead to more volatile terms of trade in equilibrium. Figure 3 plots the threshold value of the trade elasticity μ^* as a function of key parameters α (trade openness) and ρ (risk-aversion).

Figure 3: Threshold value of the trade elasticity (μ^*), as a function of risk-aversion (ρ) and trade openness (α). When $\mu > \mu^*$, depreciations generate trade balance surpluses.



The value of μ^* increases with trade-openness and falls with risk-aversion. Higher trade-openness implies that the trade balance is more sensitive to relative consumption levels. Therefore larger substitution effects are required for the trade balance to improve after depreciations. Risk-aversion exerts opposite effects. As it increases, households are more reluctant to tilt their consumption profile over time, and relative consumptions are less responsive. As a consequence, their influence on the trade balance is dampened, reducing the need for large substitution effects, i.e. for large values of the trade elasticity.

Lastly, Table 2 displays the welfare losses from fluctuations both under complete and incomplete financial markets. Since the seminal paper of Lucas (1987), many authors have measured the costs of business cycles in economies with price rigidities. As put by Walsh (2007): "Recent work in monetary theory (...) has demonstrated that focusing solely on consumption volatility misses important welfare costs associated with inflation". In the literature, depending on the modeling assumptions, the results are twenty to sixty times as large as the loss suggested by Lucas (1987, 2003). For instance, in Galí, Gertler and López-Salido (2007) welfare costs between 0.01 and 0.08 percent of steady-state consumption. Canzoneri et al. (2007) extend Galí et al. (2007) to allow for capital accumulation. They empirically estimate public spending and nominal interest rate rules and calibrate their model to US data. They calculate the welfare costs of nominal inertia and show that the average household is willing to forsake one to three per cent of consumption each period in order to avoid price and wage stickiness. The estimated welfare losses are substantial and come from the fact that the monetary authorities sub-optimally react to the deviations of output from the steady state (rather than to the output gap). This prominent role of the central bank is further highlighted by Ravenna and Walsh (2011), who show that "bad" policy rules lead to inefficient inflation and output gap stabilization, and result in large welfare losses. In addition, other contributions, such as Kiley (2002) or Paustian (2004), show that Calvo contracts can imply welfare costs that are 4 to 8 times higher than those implied by overlapping Taylor contracts, essentially due to the larger price dispersion induced by Calvo contracts. This statement is confirmed by Damjanovic and Nolan (2010), who find that even moderate price dispersion can imply significant welfare losses, between 0.5 and 2 percent of steady-state consumption.

The size of our welfare losses from fluctuations is in line with this literature. In the specific context of a two-country monetary union with sticky prices and productivity shocks, Lombardo (2006) reports welfare costs of business cycles of 0.84 percent of steady-state consumption. This is higher than our welfare losses (0.12 percent) but an important difference with Lombardo (2006) is that we cancel steady state mark-ups while he doesn't. Our welfare losses from fluctuations are 2 to 3 times higher than those reported by Lucas, and consistent with the literature reviewed in the last paragraph. They are also in line with the welfare losses reported by Galí and Monacelli (2005) in the case of an exchange rate peg, that range between 0.027 and 0.11 percent of steady-state consumption. The losses computed in our framework are somewhat larger because we consider additional distortions such as taxes and a rather inefficient monetary policy rule. When better stabilization policies are considered, such as in Section 5, welfare losses from fluctuations collapse and roughly match those obtained in flexible price models.

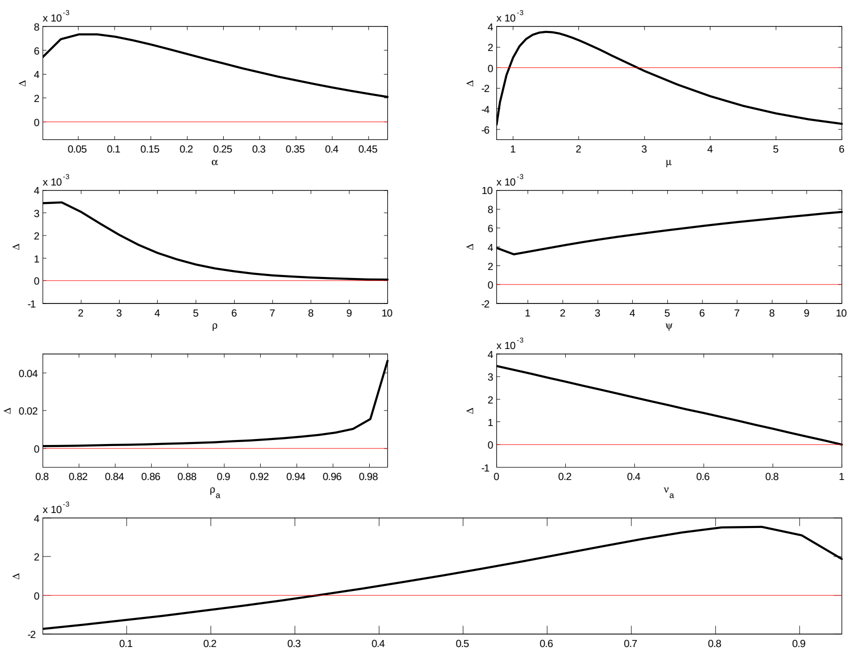
Differences between welfare losses arising under alternative financial market structures are smaller than the total welfare losses from fluctuations. They are in line with the figures reported by Benigno (2009), who finds that the welfare costs of incomplete financial markets range between 0.0075 and 0.1787 percent of steady-state consumption. Further, the overall welfare effect of having complete financial markets is negative, as equilibria under incomplete markets imply lower welfare losses from fluctuations with respect to equilibria under complete markets. Fluctuations are more costly under complete markets because this financial structure produces more volatile inflation rates. The latter cause welfare losses that overturn the welfare gains from a better sharing of risks within the monetary union.

4 Sensitivity to parameters

In this section, we proceed to a systematic robustness analysis of the welfare effects of complete financial markets in a monetary union with sticky prices and asymmetric shocks.

We contrast the welfare costs of fluctuations implied by complete and incomplete markets, as a function of key parameters. We span the whole range of plausible parameter values within the range of parameter domains defined in the model section.[9] We are particularly interested in determining what parameters are crucial to deliver the welfare reversals identified in the previous section. Figure 4 below reports Δ, the welfare loss implied by complete financial markets expressed in equivalent percentage of steady-state consumption. Therefore when Δ is negative, complete financial markets deliver lower welfare losses from fluctuations than incomplete financial markets. We consider preference parameters, such as risk-aversion (ρ), the inverse of the Frisch elasticity (ψ), home bias (α), and the trade elasticity (μ). We also consider nominal rigidities (η) and the characteristics of shocks, such as persistence (ρ_a) or cross-country correlation (ν_a).

Figure 4: Increase in welfare implied by incomplete markets (Δ). All parameters but the varying parameter are set to their baseline values.



[9]For example, we do not consider values of the risk-aversion parameter lower than one.

Figure 4 shows that varying trade openness (α), risk-aversion (ρ), the inverse of the Frisch elasticity (ψ), shocks persistence (ρ_a) and shocks cross-country correlation (ν_a) never reverses the result according to which having complete financial markets implies welfare losses as compared to a situation of incomplete financial markets.

Interestingly, larger values of trade openness, higher risk-aversion, and a greater sensitivity of hours worked to wages dampen the welfare losses from complete financial markets. Increasing openness (α) reduces the distance between allocations arising under complete and incomplete financial markets, as it makes the composition of consumption aggregates and CPI inflation rates more similar, and even perfectly similar in the limiting case of $\alpha = 1/2$. As risk-aversion (ρ) increases, agents are more reluctant to experience large variations of consumption and tend to smooth consumption more in the event of transitory shocks. As a corollary, higher risk-aversion increases the sensitivity of households to income effects implied by asymmetric shocks under incomplete markets. Therefore the welfare gains from enjoying perfect insurance through complete markets increase relative to the welfare costs from nominal rigidities. Overall, the welfare losses from having complete financial markets decrease steadily with risk-aversion. As the sensitivity of households labor supply to changes in the real wage increases (as ψ falls), households find it less costly to endure the additional volatility of hours implied by the effects of perfect risk-sharing on terms of trade, and on inflation rates.

Shocks persistence (ρ_a) tends to magnify the welfare losses from having complete markets while a higher cross-country correlation of shocks (ν_a) lowers these losses. On the one hand, persistence crucially affects the external adjustment mechanisms after unexpected shocks. Persistent shocks imply less smoothing through external channels (such as the current account) while temporary shocks imply more smoothing. Therefore, more persistent shocks are associated with more volatile terms of trade in equilibrium. It increases the welfare losses implied by complete markets, as the latter imply an additional volatility of terms of trade (and thus of national inflation rates) with respect to incomplete markets. On the other hand, cross-country shocks correlation affects the size of asymmetries, that are central to our result. More correlated shocks dampen asymmetries. In the limiting case when shocks are perfectly correlated, welfare losses from fluctuations are independent from the structure of financial markets, as the monetary union behaves as a single representative agent economy.

The results are more sensitive to the value of the trade elasticity (μ) and the magnitude of nominal rigidities (η).

The role of the elasticity of substitution is central. Indeed, the condition driving the relative volatility of terms of trade under alternative financial market structures crucially depends on μ. Figure 4 confirms that the volatility of terms of trade is the main driver of our result. When $\mu < \mu^*$, that is when μ is sufficiently small, complete financial markets imply *less* volatile terms of trade, as compared to incomplete financial markets, and unambiguously produce welfare gains. When $\mu > \mu^*$, welfare reversals arise and incomplete financial markets produce lower welfare losses from fluctuations with respect to complete financial markets, because terms of trade are *more* volatile under complete markets. According to our baseline calibration, $\mu^* = 0.9371$, and Figure 4 shows that the welfare reversal arises precisely when $\mu > \mu^*$.[10] However, when μ is larger than 3, complete markets welfare dominate again. This happens because larger substitution effects in international trade lower the equilibrium volatility of terms of trade under both financial markets structures, as smaller movements in relative prices produce larger expenditure-switching effects. Larger values of μ thus lead to smaller differences in the equilibrium volatility of terms of trade resulting from alternative financial market structures. The negative impact of complete financial markets on welfare – that acts through more volatile national inflation rates – is therefore reduced and, the welfare gains of having complete financial markets can overturn the implied welfare losses, producing net welfare gains. In our views, the sensitivity of our result to the value of the trade elasticity is central as the range of empirical estimates is typically very large and includes values as low as 0.43 (see Lubik and Schorfheide (2006)) as well as values as large as 4 or 6 (see Broda and Weinstein (2006)) or more (see Harrigan (1993)).

Lastly, nominal rigidities are also central to our result. When $\eta < 0.33$, we obtain the traditional result according to which complete financial markets yield welfare gains. When $\eta > 0.33$, welfare reversals arise for the reasons documented above. Given the empirical evidence for European countries put forth in Rumler (2007) this condition is rather loose and likely to be met. The degree of nominal rigidities crucially affects both the volatility of national inflation rates in equilibrium and the associated welfare costs. When prices are more flexible (η low), the welfare losses from nominal rigidities are lower, and the welfare gains from better risk-sharing are more likely to outweigh the welfare losses from more volatile national inflation rates.

5 Sensitivity to stabilization policies

We now investigate the welfare cost of business cycles under alternative financial market structures when public spending policies are active, i.e. when fiscal authorities use the level of public expenditure to affect fluctuations. Their ability to do so is however constrained by Equations (32) and (33). We are more particularly interested in determining whether active policies are able to stabilize national inflation rates enough to reduce the welfare losses from

[10] More precisely, when $\mu = \mu^*$, trade is always balanced and the equilibrium is isomorphic to that arising in a closed economy. Consequently agents are indifferent between alternative international financial markets structures. This case corresponds to parameterizations considered by Galí and Monacelli (2005), Galí and Monacelli (2008) and Pappa and Vassilatos (2007).

nominal rigidities, and therefore alter the ranking of welfare losses uncovered in the previous section.

Public spending policies may be an effective tool to stabilize the economy, and more specifically national inflation rates. For instance, an expansionary domestic public spending policy after an asymmetric domestic positive productivity shock will stabilize the inflation rate, and bring domestic output closer to its flexible price level (i.e. lower the output gap).

However, there is also good reason to believe that active public spending policies may not reverse the welfare ranking of alternative financial market structures. First, because public spending falls within the utility function of households, active policies may generate additional welfare losses. Second, because we make the realistic assumption that public expenditure is financed through debt or distortionary taxes, active public spending policies increase the volatility of real debts and therefore the volatility of tax rates, that are potentially costly in terms of welfare.

In what follows, we consider the following three policy configurations where public spending policies are active: (*i*) optimized public spending rules, (*ii*) optimal Ramsey public spending policies and (*iii*) a full Ramsey policy with optimal monetary and public spending policies. We contrast them with the equilibrium under passive public spending policies, and focus on cases where the economy is driven by productivity shocks only. Indeed, as we consider optimized or optimal public spending policies and/or an optimal monetary policy, considering additional public spending or monetary policy shocks is pointless as policymakers could offset those shocks via their policy decisions.

The literature on fiscal rules is amazingly vast both theoretically (see Beetsma and Jensen (2005), Ferrero (2009), Galí and Monacelli (2008), Kirsanova, Satchi, Vines and Wren-Lewis (2007), Kirsanova and Wren-Lewis (2012) and Pappa and Vassilatos (2007) among others) and empirically (see Candelon, Muysken and Vermeulen (2010), Fatás and Mihov (2003), Galí and Perotti (2003) and Wyplosz (2002) among others). Following Galí and Perotti (2003), we consider a simple public spending rule in which public spending reacts to output deviations from the steady-state and include a debt-to-GDP stabilization objective. The rule also includes a smoothing term, reflecting the persistence of past public spending decisions

$$g_t^i = \rho_g g_{t-1}^i + \left(1-\rho_g\right)\left(g + \phi_{g,y}\left(y_t^i - y\right) + \phi_{g,d}\left(d_{y,t-1}^i - d_y\right)\right). \tag{40}$$

First, under optimized rules, policymakers commit to the rule (40), and choose the coefficient values ρ_g, $\phi_{g,y}$ and $\phi_{g,d}$ so as to maximize aggregate welfare.[11] In addition, we consider that monetary policy is set according to Equation (34).

[11] We proceed to a welfare maximization on the following parameter domain $\rho_g \in [0,1[$, $\phi_{g,y} \in [-2,2]$ and $\phi_{g,d} \in [-0.1,0]$ to determine optimized rules. These domains systematically insure equilibrium determinacy.

Second, Ramsey public spending policies are determined optimally *conditionally* on the monetary policy rule (34). Monetary policymakers are thus the first movers and public spending policies are set cooperatively and simultaneously by both governments after monetary policy has been announced.

Lastly, under a full Ramsey policy, a central planner jointly determines and implements the optimal public spending and monetary policies so as to maximize the aggregate welfare within the monetary union subject to the set of equilibrium condition detailed in the model section. As Ramsey policies are typically known to be time-inconsistent, we adopt the timeless perspective (see Woodford (1999) and Giannoni and Woodford (2002)) both for the Ramsey public spending policies and for the fully optimal Ramsey policies.[12]

An intuition of how Ramsey policies can improve on simple rules is the following. As shown by Benigno (2004), a monetary union with sticky prices is a second best world in the sense that both domestic, foreign and relative prices are sticky. Therefore, authorities face a trade-off between stabilizing national prices (bringing inflation rates close to zero) and preserving enough adjustments of relative prices (bringing the terms-of-trade gap close to zero). Further, the linear-quadratic framework developed by Benigno (2004), Ferrero (2009) and Galí and Monacelli (2008) indicates that the weight placed on national inflation rates in the households loss function is very high as compared to other variables (such as the terms-of-trade gap or national output gaps). Therefore Ramsey policies could improve upon optimized rules by allowing public spending to react more directly to variables that are crucial for households welfare (such as inflation rates or terms-of-trade gaps), while optimized simple rules are restricted to react to output and deviations of the debt-to-GDP ratio.

Given the relative complexity of our model, an analytical derivation of the first-order conditions of the Ramsey problems described above is cumbersome. We thus solve for Ramsey policies using the "get_ramsey" procedure developed by López-Salido and Levin (2004) and used in Levin, Onatski, Williams and Williams (2006). We then compute a second-order approximation to the optimal conditions using Dynare's build-in routine and simulate the model.

Table 3 below summarizes the main statistics and welfare implications of the different equilibria considered under alternative financial market structures.

First, the welfare ranking of alternative fiscal policies yields the expected results. Optimized rules outperform passive policies in terms of welfare, Ramsey policies overturn optimized rules and full Ramsey policies deliver the highest level of welfare (the lowest welfare losses). Welfare improvements achieved by Ramsey policies are close to improvements brought by

[12] More precisely, we assume that optimal conditions at time zero are set consistently with a previous commitment, such as the one described by optimal conditions between period t and $t+1$.

Table 3: Standard deviations and welfare losses under active public spending policies

	Standard deviations					Welfare Losses (%)	
Incomplete markets	c_t^i	g_t^i	n_t^i	$\pi_{i,t}$	s_t	Λ	Δ
Passive G policies	0.8946	0.0000	1.1730	0.2619	1.8073	0.1215	–
Optimized G rules	0.6375	0.8209	1.0486	0.2713	1.3333	0.1139	–
Ramsey G policies	0.2456	0.5044	0.5423	0.1868	0.6377	0.0996	–
Full Ramsey Policy	0.5624	0.4127	0.4008	0.1292	0.6377	0.0638	–
Complete markets	c_t^i	g_t^i	n_t^i	$\pi_{i,t}$	s_t	Λ	
Passive G policies	0.8343	0.0000	1.1517	0.2722	2.1144	0.1245	0.0035
Optimized G rules	0.6006	0.8520	1.0122	0.2772	1.5941	0.1159	0.0023
Ramsey G policies	0.2511	0.5139	0.5443	0.1948	0.6942	0.1020	0.0028
Full Ramsey Policy	0.5649	0.4243	0.4035	0.1405	0.6942	0.0660	0.0026

Note: G policies denotes public spending policies. Passive G policies refers to the case with productivity shocks only. In all cases, the economy is driven by productivity shocks only and shocks are purely asymmetric, i.e. ν_a=0. Simulations are carried out using a second-order approximation of equilibrium conditions. The values of optimized coefficients are $\rho_g = 0$, $\phi_{g,y} = 0.60$ and $\phi_{g,d} = -0.035$ under incomplete markets and $\rho_g = 0$, $\phi_{g,y} = 0.61$ and $\phi_{g,d} = -0.039$ under complete markets.

optimized rules. However, the distance between optimized rules and Ramsey policies suggests that targets considered in Equation (40) are not the best targets in our framework. In particular, optimized rules increase welfare by achieving a more efficient stabilization of private consumptions and hours. Notwithstanding, they also result in more volatile inflation rates, while the latter are an important source of welfare losses for households. Ramsey policies, on the other hand, result in less volatile private consumption, hours, inflation rates, and terms of trade, together with less volatile policy instruments (public spending). The corresponding welfare gains with respect to both passive and optimized policies are sizeable. Lastly, optimized rules coefficients indicate a moderate stabilization of real debt and feature a positive reaction of public spending to output. While the latter could be interpreted as a pro-cyclical policy, recall that, in the model, the output gap is negative when output increases after productivity shocks. Therefore public spending are counter-cyclical with respect to the output gap, something that is more in accordance with common wisdom.

Second, equilibria under complete markets are systematically dominated in terms of welfare, at least for the chosen parameter values. Public spending policies, even when optimally designed and cooperatively implemented, are not able to reverse our main result. Indeed, the distortions implied by public spending policies, related to fluctuations of taxes rates, and the positive externality induced by incomplete markets on the volatility of these taxes prevent an economy with complete markets to outperform an economy with incomplete markets (see Appendix C for a detailled analysis of this effect).

Third, we check wether the financing scheme of public spending alters our results, and proceed to the analysis of active public spending policies when public expenditure are financed with lump-sum taxation. The results are presented in Table 4 below.

Table 4: Standard deviations and welfare losses under active public spending policies with lump-sum taxes

	Standard deviations					Welfare Losses (%)	
Incomplete markets	c_t^i	g_t^i	n_t^i	$\pi_{i,t}$	s_t	Λ	Δ
Passive G policies	0.9085	0.0000	1.1562	0.2683	1.8641	0.1241	–
Optimized G rules	0.6446	0.7680	0.8868	0.2334	1.4267	0.1048	–
Ramsey G policies	0.2351	0.5494	0.5069	0.1839	0.6382	0.0983	–
Full Ramsey Policy	0.5480	0.4413	0.3830	0.1276	0.6382	0.0678	–
Complete markets	c_t^i	g_t^i	n_t^i	$\pi_{i,t}$	s_t	Λ	
Passive G policies	0.8479	0.0000	1.1333	0.2792	2.1757	0.1281	0.0045
Optimized G rules	0.6144	0.7723	0.9105	0.2455	1.7225	0.1090	0.0048
Ramsey G policies	0.2416	0.5575	0.5125	0.1936	0.7075	0.1014	0.0035
Full Ramsey Policy	0.5508	0.4513	0.3905	0.1413	0.7075	0.0709	0.0035

Note: G policies denotes public spending policies. Passive G policies refers to the case with productivity shocks only. In all cases, the economy is driven by productivity shocks only. Shocks are purely asymmetric, i.e. $\nu_a = 0$. Simulations are carried out using a second-order approximation of equilibrium conditions. The values of optimized coefficients are $\rho_g = 0$, and $\phi_{g,y} = 0.44$ under incomplete markets and $\rho_g = 0$, and $\phi_{g,y} = 0.42$ under complete markets.

According to Table 4, considering an alternative financing scheme is not enough to reverse our result. Again, welfare losses from fluctuations are lower under incomplete markets than under complete markets. Similar volatility patterns characterize equilibria under incomplete and complete financial markets, pointing to the importance of our explanation in terms of wealth effects. Wealth effects under incomplete markets generate an additional volatility of consumption with negative welfare consequences, while they downsize the importance of terms-of-trade adjustments. This leads to less volatile national inflation rates with positive effects on households welfare. Importantly, public spending policies are more aggressive under complete markets. Indeed, optimized or optimal public spending policies are designed to reduce the volatility of inflation rates, that are more volatile under complete markets. As public spending enter the utility function, the larger corresponding deviations from the efficient provision of public goods prevent any reversal of our result.

6 Conclusion

A two-country monetary union with home bias, sticky prices and country-specific shocks is a second-best environment. A central bank cannot close all gaps and replicate the flexible prices equilibrium. National inflation rates are thus different from zero, and the dynamics of relative prices is different from its flexible prices dynamics. Alternative financial market structures, as they alter the dynamics of relative prices, also affect the dynamics of national inflation rates, and the welfare costs of nominal rigidities. In this paper, we have shown that complete financial markets, while they improve households welfare by enhancing risk-sharing, can also bring welfare losses by increasing the volatility of inflation rates. We have established the conditions under which the implied welfare losses from more volatile inflation rates overturn the welfare gains from a better sharing of risks. This is the case when the trade elasticity and the degree of price stickiness lie respectively in the range usually considered in international business cycle models and in the range considered in new Keynesian models. These values are consistent with most empirical estimates. Further, we have shown that this result is robust to cases where the number of policy instruments is extended. Allowing for active public spending policies, and/or considering various financing schemes for the provision of public goods and its potential variations along the business cycle, is not sufficient to reverse our result.

In terms of policy implications, our paper points to the importance of joint structural reforms concerning the integration of financial markets, the integration of goods markets and price flexibility. Indeed, we have shown that a decrease in home bias brings equilibria under alternative financial market structures closer, and dampens the associated welfare losses. Similarly, greater price flexibility substantially lowers the welfare costs from nominal rigidities and eventually yields results in accordance with previous studies about the welfare gains from risk-sharing. Alternatively, ignoring the interactions between financial market structures and the welfare costs of nominal rigidities can lead policymakers and governments to undertake reforms leading to unexpected effects.

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A Summary of equilibrium conditions

Euler equations and interest rates on government bonds when the international financial market is incomplete are given by

$$\left(\beta r_t / \left(1 + \chi \left(b^h_{r,t} - b^h_r\right)\right)\right) E_t \left[\left(u_{c^h,t+1}/u_{c^h,t}\right)/\pi^h_{t+1}\right] = 1, \tag{41}$$

$$\left(\beta r_t / \left(1 - \chi \left(b^h_{r,t}/q_t - b^h_r/q\right)\right)\right) E_t \left[\left(u_{c^f,t+1}/u_{c^f,t}\right)/\pi^f_{t+1}\right] = 1, \tag{42}$$

$$r^h_{g,t} = \frac{r_t}{1 + \chi \left(b^h_{r,t} - b^h_r\right)}, \tag{43}$$

$$r^i_{g,t} = \frac{r_t}{1 - \chi \left(b^h_{r,t}/q_t - b^h_r/q\right)}, \tag{44}$$

where

$$b^h_{r,t} = b^h_t / p^h_t \text{ and } q_t = p^f_t / p^h_t = \left(\frac{\alpha + (1-\alpha) {s_t}^{1-\mu}}{1 - \alpha + \alpha {s_t}^{1-\mu}}\right)^{\frac{1}{1-\mu}}. \tag{45}$$

Alternatively, Euler equations and interest rates on government bonds when the international financial market is complete are

$$\beta r_t E_t \left[\left(u_{c^h,t+1}/u_{c^h,t}\right)/\pi^h_{t+1}\right] = 1, \tag{46}$$

$$\frac{u_{c^f,t}}{u_{c^h,t}} = \Gamma_0 q_t, \tag{47}$$

$$r^i_{g,t} = r_t, \quad i = \{h, f\}. \tag{48}$$

The dynamics of CPI inflation rates and terms of trade are determined by

$$\left(\pi^h_t\right)^{1-\mu} = (1-\alpha)\left(\pi_{h,t}\right)^{1-\mu} + \alpha \left(\pi_{f,t}\right)^{1-\mu}, \tag{49}$$

$$\left(\pi^f_t\right)^{1-\mu} = (1-\alpha)\left(\pi_{f,t}\right)^{1-\mu} + \alpha \left(\pi_{h,t}\right)^{1-\mu}, \tag{50}$$

$$\frac{s_t}{s_{t-1}} = \frac{\pi_{f,t}}{\pi_{h,t}}. \tag{51}$$

The dynamics of PPI inflation rates is given by

$$\eta^i \pi_t^{\theta-1} + \left(1 - \eta^i\right)\left(\Phi x^i_{1,t}/x^i_{2,t}\right)^{1-\theta} = 1, \quad i = \{h, f\}, \tag{52}$$

$$x^i_{1,t} - \eta^i \beta E_t \left[x^i_{1,t+1} \left(\pi_{i,t+1}\right)^{1+\theta}/\pi^i_{t+1}\right] = y^i_t u_{c^i,t} \omega^i_t / a^i_t, \quad i = \{h, f\}, \tag{53}$$

$$x^i_{2,t} - \eta^i \beta E_t \left[x^i_{2,t+1} \left(\pi_{i,t+1}\right)^{\theta}/\pi^i_{t+1}\right] = y^i_t u_{c^i,t}, \quad i = \{h, f\}, \tag{54}$$

where $\Phi = \theta / ((\theta - 1)(1 - \tau))$, while the dispersion of production prices writes

$$\Upsilon_{i,t} = \eta^i \Upsilon_{i,t-1} \pi^\theta_{i,t} + \left(1 - \eta^i\right)\left(\Phi x^i_{1,t}/x^i_{2,t}\right)^{-\theta}, \quad i = \{h, f\}. \tag{55}$$

Labor supplies conditions follow from the first-order conditions and give

$$-\epsilon \frac{u_{n^h_t}}{u_{c^h_t}} = \left(1 - \zeta^h_t\right) \omega^h_t \left(1 - \alpha + \alpha s_t^{1-\mu}\right)^{\frac{1}{\mu-1}}, \tag{56}$$

$$-\epsilon \frac{u_{n^f_t}}{u_{c^f_t}} = \left(1 - \zeta^f_t\right) \omega^f_t \left(1 - \alpha + \alpha s_t^{\mu-1}\right)^{\frac{1}{\mu-1}}. \tag{57}$$

The dynamics of real public debt is the following:

$$d^i_{y,t} - r^i_{g,t-1}\left(y^i_{t-1}/y^i_t\right)\left(d^i_{y,t-1}/\pi_{i,t}\right) = g^i_t/y^i_t - \tau - \zeta^i_t \omega^i_t n^i_t / y^i_t, \quad i = \{h, f\}. \tag{58}$$

where labor income tax rates evolve as follows:

$$\zeta^i_t = \zeta + \phi_{\zeta,d}\left(d^i_{y,t-1} - d_y\right), \quad i = \{h, f\}. \tag{59}$$

Production functions and markets clearing conditions write

$$y^i_t \Upsilon_{i,t} = a^i_t n^i_t, \quad i = \{h, f\}, \tag{60}$$

$$y^h_t = (1-\alpha)\left(1-\alpha+\alpha s_t^{1-\mu}\right)^{\frac{\mu}{1-\mu}}\left(c^h_t + ac^h_t\right) + \alpha\left((1-\alpha)s_t^{1-\mu}+\alpha\right)^{\frac{\mu}{1-\mu}}\left(c^f_t + ac^f_t\right) + g^h_t, \tag{61}$$

$$y^f_t = (1-\alpha)\left(1-\alpha+\alpha s_t^{\mu-1}\right)^{\frac{\mu}{1-\mu}}\left(c^f_t + ac^f_t\right) + \alpha\left((1-\alpha)s_t^{\mu-1}+\alpha\right)^{\frac{\mu}{1-\mu}}\left(c^h_t + ac^h_t\right) + g^f_t. \tag{62}$$

Finally, the dynamics of real net foreign assets is given by

$$b^h_{r,t} - r_{t-1} b^h_{r,t-1}/\pi^h_t = tb^h_t, \tag{63}$$

where

$$tb^h_t = \left(1-\alpha+\alpha s_t^{1-\mu}\right)^{\frac{1}{\mu-1}}\left(y^h_t - g^h_t\right) - c^h_t. \tag{64}$$

while the monetary policy rule followed by the central bank is

$$r_t = \rho_r r_{t-1} + (1-\rho_r)\left(r + \phi_{r,\pi}\left(\pi^u_t - \pi^u\right)\right).$$

B Trade balance and the Marshall-Lerner condition

We provide a characterization of the Marshall-Lerner (ML) condition under complete financial markets. We determine the combinations of parameter values that imply a positive relationship between terms-of-trade movements and the domestic trade balance. If this condition is met, complete financial markets induce more volatile terms of trade than incomplete financial markets in equilibrium. To do that, we proceed with a first-order approximation of equilibrium conditions. A first-order approximation of the domestic trade balance yields

$$\frac{tb_t^h}{c} = \widehat{tb}_t^h \simeq \frac{1}{1-\kappa}\widehat{y}_t^h - \frac{\kappa}{1-\kappa}\widehat{g}_t^h - \widehat{c}_t^h - \alpha\widehat{s}_t, \tag{65}$$

where hats denote logdeviations from the steady-state. A first-order approximation of the domestic goods market clearing condition is

$$\widehat{y}_t^h = (1-\kappa)\left((1-\alpha)\,\widehat{c}_t^h + \alpha\widehat{c}_t^f + 2\mu\alpha\,(1-\alpha)\,\widehat{s}_t\right) + \kappa\widehat{g}_t^h, \tag{66}$$

which plugged in the trade balance equation gives

$$\widehat{tb}_t^h \simeq \alpha\left(\widehat{c}_t^f - \widehat{c}_t^h\right) + \alpha\,(2\mu\,(1-\alpha) - 1)\,\widehat{s}_t. \tag{67}$$

A zero trade balance in the steady-state requires $\Gamma_0 = 1$. Consequently, the risk-sharing condition gives

$$\left(c_t^h/c_t^f\right)^{\rho+\kappa(1-\rho)}\left(g_t^f/g_t^h\right)^{\kappa(1-\rho)} = q_t. \tag{68}$$

Taking logs, this condition writes

$$(\rho + \kappa\,(1-\rho))\left(\widehat{c}_t^h - \widehat{c}_t^f\right) + \kappa\,(1-\rho)\left(\widehat{g}_t^f - \widehat{g}_t^h\right) = \widehat{q}_t^f = \widehat{p}_t^f - \widehat{p}_t^h = (1-2\alpha)\,\widehat{s}_t, \tag{69}$$

or

$$\widehat{c}_t^f - \widehat{c}_t^h = \frac{\kappa\,(\rho-1)}{\rho+\kappa\,(1-\rho)}\left(\widehat{g}_t^h - \widehat{g}_t^f\right) - \frac{1-2\alpha}{\rho+\kappa\,(1-\rho)}\widehat{s}_t, \tag{70}$$

which, plugged into the trade balance equation finally gives

$$\widehat{tb}_t^h \simeq \frac{\alpha}{\rho+\kappa\,(1-\rho)}\left(\kappa\,(\rho-1)\left(\widehat{g}_t^h - \widehat{g}_t^f\right) + \left[(2\mu\,(1-\alpha)-1)\,(\rho+\kappa\,(1-\rho)) - (1-2\alpha)\right]\widehat{s}_t\right). \tag{71}$$

Finally, under passive public spending policies, $\widehat{g}_t^i = 0$ so that

$$\widehat{tb}_t^h \simeq \frac{\alpha}{\rho+\kappa\,(1-\rho)}\left((2\mu\,(1-\alpha)-1)\,(\rho+\kappa\,(1-\rho)) - (1-2\alpha)\right)\widehat{s}_t. \tag{72}$$

Obviously when $\alpha = 0$, trade balance is always zero. In other cases, as $\frac{\alpha}{\rho+\kappa(1-\rho)} > 0$, the ML condition is

$$(2\mu\,(1-\alpha)-1)\,(\rho+\kappa\,(1-\rho)) - (1-2\alpha) > 0. \tag{73}$$

Assume that κ is fixed. As $\alpha \in [0, 1/2]$, $1-2\alpha \leq 1$ always and $1-\alpha \geq 1/2$ always. Further, as $\rho + \kappa\,(1-\rho) \geq 1$ since $\rho > 1$, the condition critically depends on the trade elasticity. From equation (73), the condition on μ for trade balance to improve after depreciations writes

$$\mu > \mu^* = \frac{\rho+\kappa\,(1-\rho)+1-2\alpha}{2\,(1-\alpha)\,(\rho+\kappa\,(1-\rho))}. \tag{74}$$

C Robustness

C.1 Adding public spending and monetary policy shocks

As an extension of our main results, we investigate the robustness of our results to the introduction of public spending and monetary policy shocks when public spending policies are passive. In this case, public spending evolve according to

$$g_t^i = \left(1 - \rho_g\right) g + \rho_g g_{t-1}^i + \varepsilon_{g,t}^i. \tag{75}$$

where $\varepsilon_{g,t}^i$ is a mean-zero iid innovation with constant variance, and the monetary policy rule is augmented in the following way

$$r_t = \rho_r r_{t-1} + (1 - \rho_r) \left(r + \phi_{r,\pi} \left(\pi_t^u - \pi^u\right)\right) + \varepsilon_{r,t}, \; 0 < \rho_r < 1, \; \phi_{r,\pi} > 1, \tag{76}$$

where $\varepsilon_{r,t}$ is a mean-zero iid innovation with constant variance.

These shocks are purely neutral with respect to the steady state and the latter remains unchanged. For the calibration of public spending shocks, empirical studies typically report high values of the persistence parameter, almost never below 0.5 on annual time series (see Candelon et al. (2010) and Galí and Perotti (2003)). Because our set-up is quarterly, we impose $\rho_g = 0.85$. In addition, the standard deviation of public spending and monetary policy shocks is $\sigma\left(\varepsilon_{g,t}^i\right) = 0.01$ and $\sigma\left(\varepsilon_{r,t}\right) = 0.0025$. As for productivity shocks, the correlation of shocks is null, i.e. $\nu_g = 0$.

C.1.1 Dynamics

Before analyzing the welfare implications of alternative financial structures when considering public spending and monetary policy shocks in addition to productivity shocks, we contrast the dynamics of the model after these new shocks.

Figure 5 below plots the IRFs after an asymmetric public spending shock in the domestic economy under incomplete financial markets.

A public spending shock in the domestic economy depresses consumption due to a negative wealth effect, boosts domestic output and hours, thereby pressuring production capacities. It generates a demand-driven inflation in the domestic economy. This inflationary stance temporarily alleviates the real debt burden even though the stock of nominal debt rises to finance public expenditure. Once inflation returns to the steady-state, the real debt stock then increases significantly. This rise is further reinforced by the increase in the nominal interest rate by the common central bank to stabilize aggregate inflation.

First, transmission to the foreign economy is driven by the increase in the common nominal interest rate, which depresses foreign consumption. Second, foreign output is also affected by the drop in domestic imports, induced by the dynamics of domestic consumption. This effect is slightly attenuated by the deterioration of foreign terms of trade (foreign firms are more competitive), which leads both domestic and foreign households to substitute foreign goods to domestic goods in their total consumption expenditure. The overall effect of the shock in the domestic economy is to depress foreign output and hours, which leads to a slight drop in the foreign inflation rate.

Figure 5: IRFs after a one standard deviation purely asymmetric public spending shock in the domestic economy under incomplete financial markets.

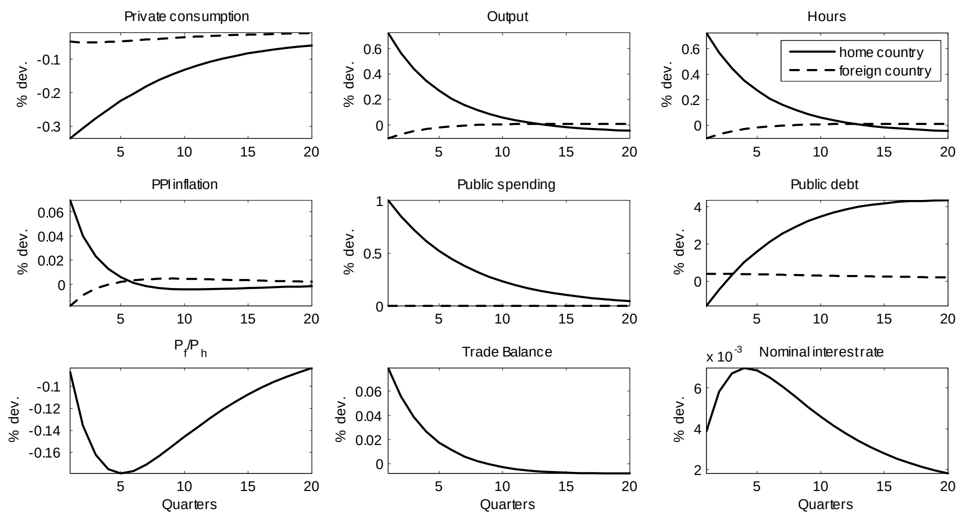


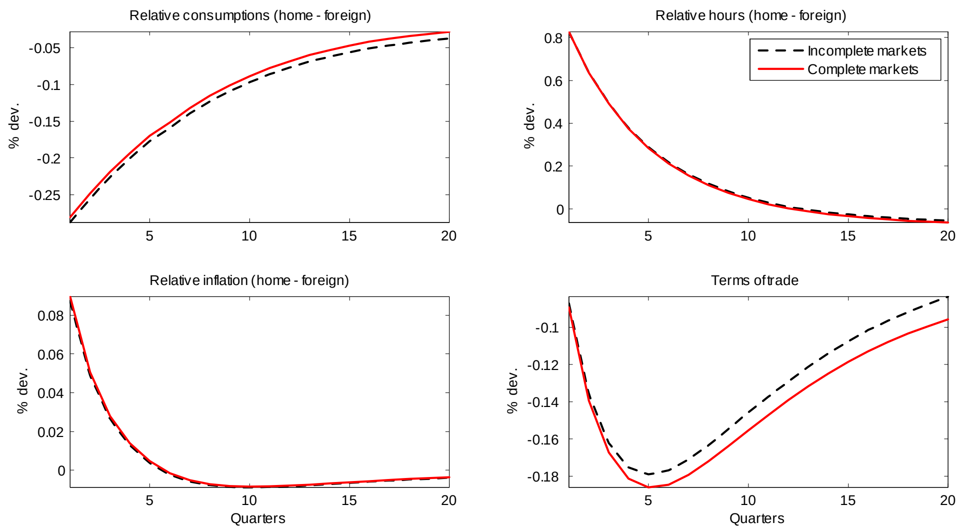
Figure 6 plots the IRFs of relative consumptions, hours, inflation rates, as well as terms of trade both under incomplete and complete asset markets.

In comparison to what happens after productivity shocks, the same mechanisms differentiate the incomplete markets economy from the complete markets economy after public spending shocks. The absence of income effects in the complete markets economy leads relative consumptions to be less responsive, while terms of trade are more responsive. It leads national inflation rates to be more responsive, as compared to the incomplete markets economy. However, in the case of public spending shocks, there is an additional channel through which the financial structure affects the dynamics of the equilibrium and thus the welfare losses from fluctuations.

The effect derives from wealth effects but also relies on the assumptions made about the financing scheme that lead both debt and labor income taxes to be very persistent over time. In the short run, domestic hours increase after the positive domestic public spending shock. Under incomplete markets, because of wealth effects, domestic hours as well as output increase more, leading public spending over GDP to increase less than under complete markets. Still in the short run, the rise in public expenditure is financed through an increase in public debt. Therefore, under incomplete markets, debt over GDP increases less than under complete markets. In the medium and long run, public spending returns to its steady-state values, as does the public spending over GDP ratio. However, inherited debt is still present and labor income taxes need to adjust. This persistent increase in taxes leads domestic hours to fall under their steady-state level persistently as well, after 14-15 quarters. The rise in taxes is lower under incomplete markets because inherited debt over GDP is smaller than under complete markets, and domestic hours drop less under incomplete markets in the long run.

Summarizing, under incomplete markets, the presence of wealth effects in the short run generates effects that are similar to those identified after productivity shocks. Additionally,

Figure 6: IRFs after a one standard deviation purely asymmetric public spending shock in the domestic economy under incomplete vs. complete financial markets.



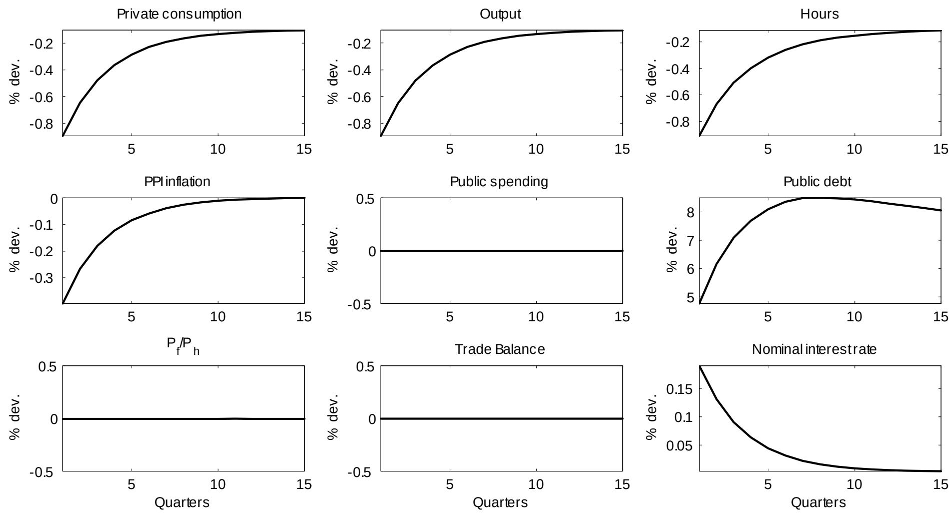
wealth effects arising under incomplete markets lead to less responsive public debt-to-GDP ratios, and translate into less responsive debt-to-GDP ratios and labor income tax rates in the long run. Thus, deviations from the steady-state in hours worked are larger in the short run and smaller in the long run, as compared to the case of complete financial markets. The overall effect of alternative financial market structures on the volatility of hours – and thus on welfare losses from fluctuations – when the economy is driven by public spending shocks only is therefore uncertain, at least when analyzing IRFs. More details about these effects are offered in the next section.

Lastly, when financial markets are incomplete or complete, Figure 7 shows that a monetary policy shock leads to purely symmetric IRFs in both economies.

First and as expected, a restrictive monetary policy shock depresses consumption, hours, output and inflation in both economies. Notice that real debt over GDP increases significantly. First, as output shrinks, fiscal revenue falls, and debt increases on impact. Second, the movement is amplified (i) by the fall in output, that increases the debt-to-GDP ratio mechanically and (ii) by the rise in interest rates, that magnifies the burden of inherited debt. Second, an important conclusion arising from Figure 7 is that, as long as nominal rigidities and monetary transmission mechanisms are symmetric within the monetary union, domestic and foreign variables exhibit perfectly similar responses. In this case, the monetary union behaves as a single representative agent economy. As a consequence, the structure of international financial markets has no effects on the equilibrium and on the resulting welfare losses from fluctuations.

As in the reference case, our analysis reveals that completeness of financial markets results in two competing effects, as compared to incompleteness. First, complete financial markets neutralize income effects and bring domestic and foreign consumptions closer to each other.

Figure 7: IRFs after a one standard deviation monetary policy shock.



Second, complete financial markets result in larger fluctuations in terms of trade, and therefore in PPI inflation rates. After public spending shocks, wealth effects arising under incomplete markets also generate a positive outcome as they lead to less volatile labor income tax rates.

C.1.2 Volatilities and welfare

Table 5 summarizes our main results with various shocks. It reports the standard deviations of private consumption, public spending, hours worked, PPI inflation and terms of trade under alternative assumptions regarding international financial markets and the welfare losses from fluctuations. In the case of complete financial markets, it also reports the corresponding welfare loss with respect to the situation of incomplete financial markets. It is labeled Δ and expressed as the percentage increase in consumption that households would be ready to give up to live in an economy with incomplete financial markets. The results are presented for the benchmark parameter values and for various combinations of shocks.

In terms of volatility, most results arising with productivity shocks only are preserved when new shocks are introduced. Incomplete markets increase the volatility of private consumption, and lower the volatility of terms of trade and national inflation rates. However, the volatility of hours exhibits different patterns depending on the types of shocks driving the economy.

With productivity shocks only, income effects under incomplete markets clearly result in more volatile hours worked, as compared to the complete markets economy.

With public spending shocks only, the incomplete markets economy generates less volatile hours worked. This result arises because wealth effects in the short run imply less volatile public spending-to-GDP and debt-to-GDP ratios, that in turn translate into less volatile labor income tax rates and hours in the long run.

Table 5: Standard deviations and welfare losses under passive public spending policies

	Standard deviations					Welfare Losses (%)	
Incomplete markets	c_t^i	g_t^i	n_t^i	$\pi_{i,t}$	s_t	Λ	Δ
PS	0.8946	–	1.1730	0.2619	1.8073	0.1215	–
GS	0.9325	1.8983	1.2912	0.0888	1.1370	0.1253	–
MS	1.6382	–	1.6382	0.5388	0.0000	0.3237	–
PS+GS	1.2923	1.8983	1.7444	0.2766	2.1352	0.2466	–
PS+MS	1.8666	–	2.0148	0.5991	1.8073	0.4446	–
GS+MS	1.8850	1.8983	2.0858	0.5460	1.1370	0.4484	–
All shocks	2.0865	1.8983	2.3930	0.6056	2.1352	0.5692	–
Complete markets	c_t^i	g_t^i	n_t^i	$\pi_{i,t}$	s_t	Λ	
PS	0.8343	–	1.1517	0.2722	2.1144	0.1245	0.0035
GS	0.9037	1.8983	1.3553	0.0903	1.4592	0.1275	0.0024
MS	1.6382	–	1.6382	0.5388	0.0000	0.3237	0.0000
PS+GS	1.2300	1.8983	1.7785	0.2868	2.5691	0.2518	0.0059
PS+MS	1.8384	–	2.0025	0.6036	2.1144	0.4477	0.0035
GS+MS	1.8709	1.8983	2.1261	0.5463	1.4592	0.4506	0.0024
All shocks	2.0485	1.8983	2.4180	0.6104	2.5691	0.5744	0.0059

Note: PS denotes productivity shocks, GS denotes public spending shocks and MS denotes monetary policy shocks. Shocks are purely asymmetric, i.e. $\nu_a = \nu_g = 0$. Simulations are carried out using a second-order approximation of equilibrium conditions.

When all shocks are considered together, incomplete financial markets produce less volatile hours, which means that the positive spillover of less volatile labor income tax rates debts on the volatility of hours overturn the influence of income effects.

In terms of welfare losses, fluctuations are more costly under complete markets because this financial structure produces more volatile inflation rates and more volatile labor income tax rates (with public spending shocks). The latter cause welfare losses that overturn the welfare gains from a better sharing of risks within the monetary union. Our main result thus holds true for all the proposed combinations of shocks.

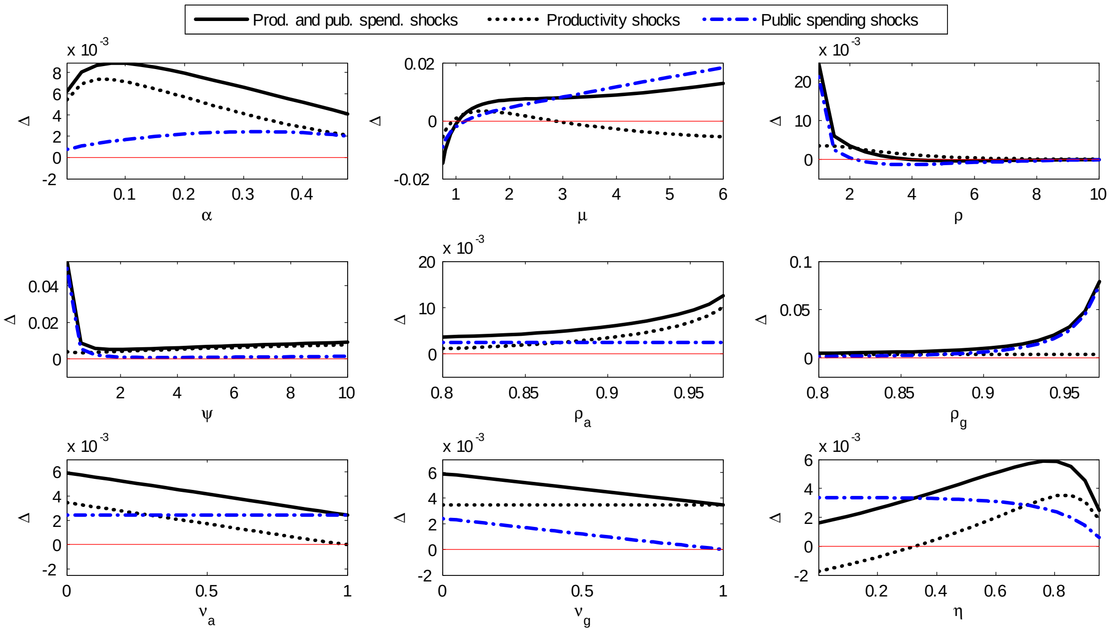
C.1.3 Sensitivity to parameters

We now proceed to the same robustness analysis of the welfare effects of complete financial markets in a monetary union with sticky prices and asymmetric shocks, when the additional shocks are taken into account. The simulations abstract from monetary policy shocks, as when both countries are symmetric in terms of nominal rigidities, monetary policy shocks do not generate asymmetries, and are irrelevant in determining the ranking of welfare losses under alternative financial market structures.

Two new parameters related to the dynamics of public spending shocks are investigated in addition to those analyzed in the main article: the persistence of public spending shocks (ρ_g) and their cross-country correlation (ν_g).

Just as for productivity shocks, the persistence (ρ_g) and cross-country correlation of public spending shocks (ν_g) do not lead our results to reverse. Shocks persistence (ρ_g) tends to magnify the welfare losses from having complete markets while higher cross-country correlations of shocks (ν_g) lower these losses. Similarly, our results are robust to changes in trade openness (α) and the inverse of the Frisch elasticity on labor supply (ψ).

Figure 8: Increase in welfare implied by incomplete markets (Δ). All parameters but the varying parameter are set to their baseline values.



We get different results when the risk-aversion parameter (ρ), the trade elasticity (μ) and the price rigidity parameter (η) vary.

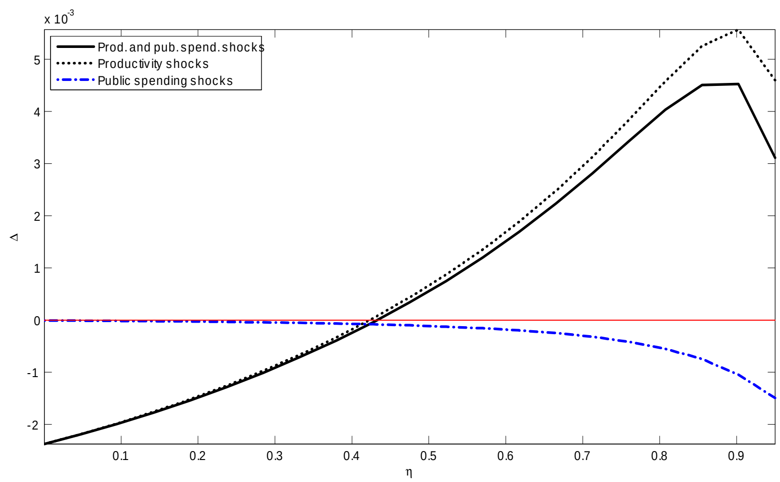
When the economy features public spending shocks, our result now reverses as long as $\rho > 2$ (with public spending shocks only) or when $\rho > 4$ (with combined productivity and public spending shocks). This sensitivity is important because the empirical literature reveals great heterogeneity in the estimation of risk-aversion, and reversal areas clearly belong to the range of plausible estimates. However, because our result does not reverse when the economy is driven by productivity shocks, having public spending in the utility function is clearly the main driver of the reversal when public spending shocks are considered, and not the risk-aversion parameter in itself.

When varying the trade elasticity (μ), the condition $\mu > \mu^*$ remains valid when considering public spending shocks (alone or combined with productivity shocks). However, our result does not reverse anymore when μ is large. This results from the presence of relative public spending in the ML condition (see Appendix B), that alter the dynamics of the trade balance (at least under complete markets) due to their presence in the risk-sharing condition.

Lastly, nominal rigidities play a less important role in the ranking when public spending shocks are considered. In this case, incomplete financial markets always lead to lower welfare losses (positive Δ), regardless of the degree of nominal rigidities. As explained before, income effects implied by incomplete markets produce an additional positive spillover in the economy. They lower the volatility of the public spending-to-GDP ratio, that of the debt-to-GDP ratio, thereby reducing the need for debt stabilization, the volatility of distortionary taxes and the volatility of hours worked. With less volatile distortionary taxes, the equilibrium under incomplete financial markets when the economy features public spending shocks produces additional welfare gains with respect to the equilibrium under complete financial markets. Those welfare gains are large enough to prevent any reversal of our result.

One way of shutting down this channel is to run the same sensitivity analysis with respect to nominal rigidities in a simplified model with lump-sum taxation only. Therefore the additional positive spillover from having incomplete markets with public spending shocks vanishes (see Figure 9).

Figure 9: Increase in welfare implied by incomplete markets (Δ) with lump-sum taxation. All parameters but the varying parameter are set to their baseline values.



In the case of lump-sum taxation, public spending shocks are a source of welfare losses since public spending enters the utility function but their financing scheme does not spillover positively on the dynamics of hours. Therefore, alternative financial market structures do not imply the additional difference in the patterns of hours described in the case with distortionary taxation and public debt, and the sensitivity of welfare reversals to nominal rigidities is only marginally affected in comparison to the case with productivity shocks only.

C.2 Welfare effects

In this section we provide more details about how the welfare losses breakdown. As we use a second-order approximation of the model, the conditional expectation of welfare differs from its steady state level. We go a step further than in the paper and report the contributions of the different components of conditional welfare: mean effects and volatility effects. Eventhough Benigno and Woodford (2005) have shown that mean (level) effects can always be recasted in terms of volatilities, we believe it might be important to provide this information to the reader.

C.2.1 A second-order approximation of the utility function

We start the decomposition by approximating the utility function up to a second-order. The latter is

$$u(c_t^i, g_t^i, n_t^i) = \frac{1}{1-\rho}\left(\left(c_t^i\right)^{1-\kappa}\left(g_t^i\right)^{\kappa}\right)^{1-\rho} - \frac{\epsilon}{1+\psi}\left(n_t^i\right)^{1+\psi}, \tag{77}$$

and a second-order Taylor expansion around the steady state gives

$$\begin{aligned} u(c_t^i, g_t^i, n_t^i) &\approx u + u_c(c,g)\left(c_t^i - c\right) + u_g(c,g)\left(g_t^i - g\right) + u_n(n)\left(n_t^i - n\right) \\ &+ \frac{1}{2}\left(\begin{array}{c} u_{cc}(c,g)\left(c_t^i - c\right)^2 + u_{gg}(c,g)\left(g_t^i - g\right)^2 \\ +2u_{cg}(c,g)\left(c_t^i - c\right)\left(g_t^i - g\right) + u_{nn}(n)\left(n_t^i - n\right)^2 \end{array}\right), \end{aligned} \tag{78}$$

where

$$u_c(c,g) = \frac{(1-\kappa)}{c}\Phi,\ u_g(c,g) = \frac{\kappa}{g}\Phi,\ u_n(n) = -\frac{\epsilon n^{1+\psi}}{n}, \tag{79}$$

$$u_{cc}(c,g) = \frac{((1-\kappa)(1-\rho)-1)(1-\kappa)}{c^2}\Phi,\quad u_{gg}(c,g) = \frac{(\kappa(1-\rho)-1)\kappa}{g^2}\Phi, \tag{80}$$

$$u_{cg}(c,g) = \frac{(1-\kappa)\kappa(1-\rho)}{cg}\Phi,\quad u_{nn}(n) = -\frac{\psi\epsilon n^{1+\psi}}{n^2}, \tag{81}$$

where $\Phi = \left(c^{1-\kappa}g^{\kappa}\right)^{1-\rho}$. Using the simplifying steady state relations $c = (1-\kappa)$, $g = \kappa$, $n = 1$, we get

$$\begin{aligned} u(c_t^i, g_t^i, n_t^i) &\approx u + \Phi\left(c_t^i - c\right) + \Phi\left(g_t^i - g\right) - \epsilon\left(n_t^i - n\right) \\ &+ \Phi_{cc}\left(c_t^i - c\right)^2 + \Phi_{gg}\left(g_t^i - g\right)^2 \\ &+ (1-\rho)\Phi\left(c_t^i - c\right)\left(g_t^i - g\right) - \Phi_{nn}\left(n_t^i - n\right)^2, \end{aligned} \tag{82}$$

where $\Phi_{cc} = \frac{((1-\kappa)(1-\rho)-1)}{2(1-\kappa)}\Phi$, $\Phi_{gg} = \frac{(\kappa(1-\rho)-1)}{2\kappa}\Phi$and $\Phi_{nn} = \frac{\psi\epsilon}{2}$.

C.2.2 Welfare as a function of means and variances

We can now express conditional welfare in country i as

$$\begin{aligned} W_0^i &= E_0\sum_{t=0}^{\infty}\beta^t u(c_t^i, g_t^i, n_t^i) \\ &\simeq \frac{1}{1-\beta}(u + \Phi E_t\left(c_t^i - c\right) + \Phi E_t\left(g_t^i - g\right) - \epsilon E_t\left(n_t^i - n\right) \\ &+ \Phi_{cc} var\left(c_t^i\right) + \Phi_{gg} var\left(g_t^i\right)^2 - \Phi_{nn} var\left(n_t^i\right)^2 \\ &+ (1-\rho)\Phi cov\left(c_t^i, g_t^i\right)). \end{aligned} \tag{83}$$

Notice that variances and covariances must be computed with respect to the steady state, not with respect to the expectation of variables. This particularity requires an additional procedure to recover unconditional moments while Dynare reports conditional moments.[13]

[13] The relation between both variances is

$$var_{uncond}(x_t) = var_{cond}(x_t) + (E_t(x_t) - x)^2 \tag{84}$$

The total conditional welfare is thus $W_0 = W_0^h + W_0^f$ and, because both countries feature the same conditional expectations, variances and covariances

$$W_0 \simeq \underbrace{\frac{2u}{1-\beta}}_{\omega_1} + \underbrace{\frac{2\Phi}{1-\beta}\left(Ec_t - c\right)}_{\omega_2} + \underbrace{\frac{2\Phi}{1-\beta}\left(Eg_t - g\right)}_{\omega_3} + \underbrace{-\frac{2\epsilon}{1-\beta}\left(En_t - n\right)}_{\omega_4}$$
$$+ \underbrace{\frac{2\Phi_{cc}}{1-\beta} var\left(c_t\right)}_{\omega_5} + \underbrace{\frac{2\Phi_{gg}}{1-\beta} var\left(g_t\right)}_{\omega_6} + \underbrace{\frac{2\left(1-\rho\right)}{1-\beta} cov\left(c_t, g_t\right)}_{\omega_7} + \underbrace{-\frac{2\Phi_{nn}}{1-\beta} var\left(n_t^i\right)}_{\omega_8}.$$

C.2.3 Welfare decomposition

For the baseline model with passive public spending policies, we get the following results. We report the different components of welfare losses, and the sum of components ($\sum \omega$) for the various combinations of shocks considered:

Decomposition of welfare effects

	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8	$\sum \omega$
	Incomplete markets								
PS	-603.9316	-0.2224	0	0.0105	-0.0196	0	0	-0.0102	-604.1734
GS	-603.9316	-0.0223	0	0.0018	-0.0211	-0.1622	0.0195	-0.0123	-604.1282
MS	-603.9316	-0.5865	0	0.0269	-0.0664	0	0	-0.0199	-604.5774
PS+GS	-603.9316	-0.2447	0	0.0123	-0.0408	-0.1622	0.0195	-0.0225	-604.3699
PS+MS	-603.9316	-0.8089	0	0.0374	-0.0869	0	0	-0.0301	-604.8201
GS+MS	-603.9316	-0.6088	0	0.0288	-0.0876	-0.1622	0.0195	-0.0322	-604.7741
All	-603.9316	-0.8312	0	0.0393	-0.1081	-0.1622	0.0195	-0.0424	-605.0167
	Complete markets								
PS	-603.9316	-0.2297	0	0.0088	-0.0171	0	0	-0.0098	-604.1795
GS	-603.9316	-0.0272	0	0.0027	-0.0198	-0.1622	0.0192	-0.0136	-604.1325
MS	-603.9316	-0.5865	0	0.0269	-0.0664	0	0	-0.0199	-604.5774
PS+GS	-603.9316	-0.2569	0	0.0115	-0.0370	-0.1622	0.0192	-0.0234	-604.3804
PS+MS	-603.9316	-0.8163	0	0.0357	-0.0844	0	0	-0.0297	-604.8262
GS+MS	-603.9316	-0.6137	0	0.0296	-0.0863	-0.1622	0.0192	-0.0335	-604.7784
All	-603.9316	-0.8434	0	0.0384	-0.1044	-0.1622	0.0192	-0.0433	-605.0272

$u = -6.0393$ / $\Phi = 1.3247$ / $\Phi_{cc} = -1.2143$ / $\Phi_{gg} = -2.2500$ / $\Phi_{nn} = -0.3703$

By definition, ω_1 is invariant. Other components make the welfare losses/gains.

ω_5, ω_6 and ω_8 are the weighted contributions of variances. All these components are small and affect the welfare negatively. ω_7 is the weighted contribution of the covariance between public spending and consumption. If this covariance is positive, both components of the "aggregate consumption bundle" move together, which yields welfare losses. In the model, this covariance is negative, i.e. when public spending increase (fall) private consumption falls (increases), which yields welfare gains. However, these gains are low relative to the losses related to "mean effects".

ω_2, ω_3 and ω_4 are the contributions of expected consumption, public spending and hours. Public spending move only because of unexpected shocks, so that expected public spending exactly equate their steady state value, and $\omega_3 = 0$. Further, the above table shows that expected private consumption is always lower than steady-state consumption ($\omega_2 < 0$), due to consumption risks, i.e. the effect of variances on means. This term is quite large and plays an essential role in the overall size of welfare losses from fluctuations. As shown above, the effect is the largest when monetary policy shocks are considered, which suggests that variations in the real interest rate and sticky prices are the main drivers of this consumption risk. However, one could argue that heterogeneity is an important driver of this effect, as shocks are asymmetric, consumption bundles have different compositions and terms-of-trade movements imply different expenditure-switching and wealth effects in both countries.[14] This argument is inspected in the next subsection.

Finally, expected hours are slightly below their steady state value, which yields (relatively low) welfare gains ($\omega_4 > 0$). This positive effect relates to the so-called positive terms-of-trade spillovers on labor supply. In the model, terms of trade are a device of "automatic stabilization" of hours: when domestic hours increase because of an increase in public spending (or because of a negative productivity shock), terms of trade appreciate and the corresponding wealth effect dampens the required increase in hours. Therefore the impact of uncertainty on the conditional mean of hours is negative, i.e. hours are lower in the stochastic steady state than in the deterministic steady state.

Importantly, according to this exercise, the ranking of welfare losses is identical to the ranking reported in the paper, as incomplete financial markets always lead to higher values of $\sum \omega$ with respect to complete financial markets. In addition, eventhough the welfare effects seem to be mainly driven buy the level effects, remember that the latter can always be expressed as a function of volatilities, the favored interpretation in the paper.

C.2.4 Heterogeneity, sticky prices and the size of welfare losses

Now we perform this exercise in three particular cases to assess the role of heterogeneity and sticky prices in size of welfare losses. We run simulations with all shocks when $\alpha = 1/2$, when the correlation of shocks is perfect, and when prices are flexible, and report the corresponding results in the Table below. In the last column we also report the consumption equivalent welfare loss implied by fluctuations (Λ).

Decomposition of welfare effects (all shocks special parameter values)

	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8	$\sum \omega$	Λ (%)
	Incomplete markets									
$\alpha = 1/2$	-603.9316	-0.8381	0	0.0396	-0.1033	-0.1622	0.0179	-0.0435	-605.0211	0.5462
$\nu_a = \nu_g = 1$	-603.9316	-0.8258	0	0.0404	-0.1302	-0.1622	0.023	-0.0409	-605.0270	0.5492
$\eta = 0$	-603.9316	-0.0944	0	0.0219	-0.0759	-0.1622	0.0229	-0.0117	-604.2309	0.1505
	Complete markets									
$\alpha = 1/2$	-603.9316	-0.8452	0	0.0395	-0.1023	-0.1622	0.0182	-0.0442	-605.0278	0.5496
$\nu_a = \nu_g = 1$	-603.9316	-0.8258	0	0.0404	-0.1302	-0.1622	0.023	-0.0409	-605.0270	0.5492
$\eta = 0$	-603.9316	-0.1018	0	0.0217	-0.0678	-0.1622	0.0222	-0.0142	-604.2338	0.1519

$u = -6.0393$ / $\Phi = 1.3247$ / $\Phi_{cc} = -1.2143$ / $\Phi_{gg} = -2.2500$ / $\Phi_{nn} = -0.3703$

This Table shows that the size of welfare effects essentially relies on the presence of sticky prices, not on cross-country heterogeneity. This point is also made clear in the paper, as welfare reversals are very sensitive to the degree of nominal rigidities (see for instance the case with productivity shocks only in the paper or Figure 8 and 9 in this Appendix)

[14] Because sticky prices are introduced à la Calvo, an inherent heterogeneity is also present in the sense that there is price and labor demand dispersion in the model.

C.2.5 Models solved in log-levels

Finally, we perform the same exercise using models in log-levels.

Decomposition of welfare effects (model solved using log-levels)

	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8	$\sum \omega$	Λ (%)
	Incomplete markets									
PS	-603.9316	-0.1502	0	0.0091	-0.0195	0	0	-0.0102	-604.1024	0.0859
GS	-603.9316	-0.0223	0	0.0018	-0.0211	-0.1622	0.0195	-0.0123	-604.1282	0.0989
MS	-603.9316	-0.5865	0	0.0269	-0.0664	0	0	-0.0199	-604.5774	0.3243
PS+GS	-603.9316	-0.1725	0	0.0109	-0.0407	-0.1622	0.0195	-0.0225	-604.2990	0.1847
PS+MS	-603.9316	-0.7367	0	0.0360	-0.0865	0	0	-0.0301	-604.7488	0.4101
GS+MS	-603.9316	-0.6088	0	0.0288	-0.0876	-0.1622	0.0195	-0.0322	-604.7741	0.4227
All	-603.9316	-0.7590	0	0.0379	-0.1077	-0.1622	0.0195	-0.0424	-604.9455	0.5085
	Complete markets									
PS	-603.9316	-0.1575	0	0.0074	-0.0170	0	0	-0.0098	-604.1086	0.0890
GS	-603.9316	-0.0272	0	0.0027	-0.0198	-0.1622	0.0192	-0.0136	-604.1325	0.1010
MS	-603.9316	-0.5865	0	0.0269	-0.0664	0	0	-0.0199	-604.5774	0.3243
PS+GS	-603.9316	-0.1847	0	0.0101	-0.0369	-0.1622	0.0192	-0.0234	-604.3094	0.1899
PS+MS	-603.9316	-0.7440	0	0.0343	-0.0840	0	0	-0.0297	-604.7550	0.4132
GS+MS	-603.9316	-0.6137	0	0.0296	-0.0863	-0.1622	0.0192	-0.0335	-604.7784	0.4249
All	-603.9316	-0.7712	0	0.0370	-0.1040	-0.1622	0.0192	-0.0433	-604.9560	0.5137

$u = -6.0393$ / $\Phi = 1.3247$ / $\Phi_{cc} = -1.2143$ / $\Phi_{gg} = -2.2500$ / $\Phi_{nn} = -0.3703$

Obviously the number reported in this Table are slightly different than those obtained when solving models in levels, but the magnitude of welfare effects is the same. In addition our results are robust to this solving procedure of the models.

C.2.6 High-order approximations

As an additional check, we also solve and simulate the models using a third and fourth order approximation. The results with all shocks are reported in below.

Welfare effects for high order approximations

	$\sum \omega$	Λ (%)
	Incomplete markets	
$3rd$ order	-605.2103	0.6407
$4th$ order	-604.7783	0.4249
	Complete markets	
$3rd$ order	-605.2202	0.6456
$4th$ order	-604.7862	0.4288

These results must be compared with previous results with much care, as they rely on simulated moments (80 replications of 100 periods), and not on theoretical moments. However, they indicate welfare effects that are of the same order of magnitude than those obtained with lower order approximations, and confirm that welfare losses from fluctuations are higher under complete markets.

C.3 Conclusion

In this Appendix, we have shown that our results are quite robust to the inclusion of additional shocks, such as public spending and monetary policy shocks. Further, we have shown how the welfare effects of fluctuations breakdown between level and variance effects, eventhough both can be expressed as a function of volatilities only. Finally, we have checked that welfare reversals were robust to alternative solution methods (in log-levels) or high-order approximations.