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Climatic Conditions and Productivity : An Impact Evaluation in Pre-industrial England

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Climatic Conditions and Productivity: An Impact Evaluation in Pre-industrial England*

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Abstract

In this paper, we bridge economic data and climatic time series to assess the vulnerability of a pre-industrial economy to changes in climatic conditions. We propose an economic model to extract a measure of total productivity from English data (real wages and land rents) in the pre-industrial period. This measure of total productivity is then related to temperatures and precipitations. We find that lower (respectively higher) than average precipitations (respectively temperatures) enhance productivity. Further, temperatures have non-linear effects on productivity: large temperature variations lower productivity. Quantitatively, a permanent two degree rise in temperatures lowers the level of productivity by more than 22%, and production by more than 26%. This historical impact evaluation may serve as an informative benchmark for currently under-developed economies in front of the upcoming climatic change.

Keywords: climatic conditions, TFP shocks, land rent.
JEL Classification: C22, N13, O41, O47, Q54.

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1 Introduction

Widespread poverty limits the ability of under-developed economies to adapt changes in climatic conditions. It is then highly probable that the poorest regions of the World (in particular in Africa and Asia) will be among the first to suffer from the upcoming climatic change. However, the shortage in data makes it difficult to evaluate the impact of changes in climatic conditions on these economies.

In this note, we use historical data in England before the Industrial Revolution to assess the economic vulnerability of emerging economies to changes in climatic conditions (temperatures and precipitations). We use real wages and real agricultural rents time series to obtain a measure of productivity, and investigate empirically the sensitivity of productivity to temperatures and precipitations. This impact evaluation may serve as a benchmark for the impact evaluation in currently under-developed economies.

First, we propose a simple growth model where economic activity depends on structural factors and exogenous shocks. The model establishes simple and testable relations between the prices of production factors (wages and rents) and the main driver of the economy (productivity). The use of such models to describe the dynamics of emerging economies is very frequent in the literature (see for instance Aguiar, M. and Gopinath, G. (2007) or Neumeyer, P. and Perri, F. (2005)). Second, we make use of these conditions to extract an empirical measure of productivity from the data. Third, we investigate the impact of two climatic factors (temperatures and precipitations) on productivity.

Our focus is on pre-industrial England, where this economy displayed patterns close to currently emerging or under-developed economies: a large agricultural sector, slow technological innovations, few ways to diversify individual risks, political instability, major impact of diseases and climatic calamities. We find that precipitations affect productivity negatively while temperatures play a positive role. However, in addition to these effects, temperatures have non-linear effects on productivity: large temperature variations (positive or negative) lower productivity. From a quantitative perspective, our simulations indicate that a permanent two degree rise in temperatures in pre-industrial England would have lowered the level of productivity by more than 12%, and production by more than 16 %. We see this historical impact evaluation as an informative benchmark for currently under-developed economies facing potentially large changes in climatic conditions in the near future.

The rest of the note is as follows. Section 2 contains a brief description of the data. Section 3 presents our main assumptions and the econometric results. A complete description of our model is provided in an Appendix Section.

2 Data

We collect three different types of data. The first one is an annual sequence of English real wages starting in 1264.[1]

The second source of data comes from Gregory Clark's website. We gather all the 4.983 rents of the Charity Commission Land Rents data set from 1502 to 1800. The full data set extends to 1912 but we concentrate on the pre-industrial period. The oldest record goes back to 1394, but there is no data from this date on to 1502. We use the estimated annual rental value of land in pounds (including land tax if paid by the tenant). Although the data set contains many details regarding the type of land – its usage, its owner – very few observations are directly comparable. We thus simply divide the estimated rent by the total surface to obtain a consistent proxy. An immediate consequence however is a considerable amount of unobserved heterogeneity. To mitigate this heterogeneity, we only consider observations for which we have a sufficiently large amount of data (namely 10 per year), which leaves us with 132 different observations from 1669 to 1800. Finally, we deflate these rents by the Retail Price Index, provided by measuringworth.org, to obtain a sequence of real rents.

The last source of data concerns climatic conditions.[2] The data set includes the

[1] This exceptionally long sequence is available at http://www.measuringworth.org.

[2] We would like to thank Juerg Luterbacher for providing this data set.

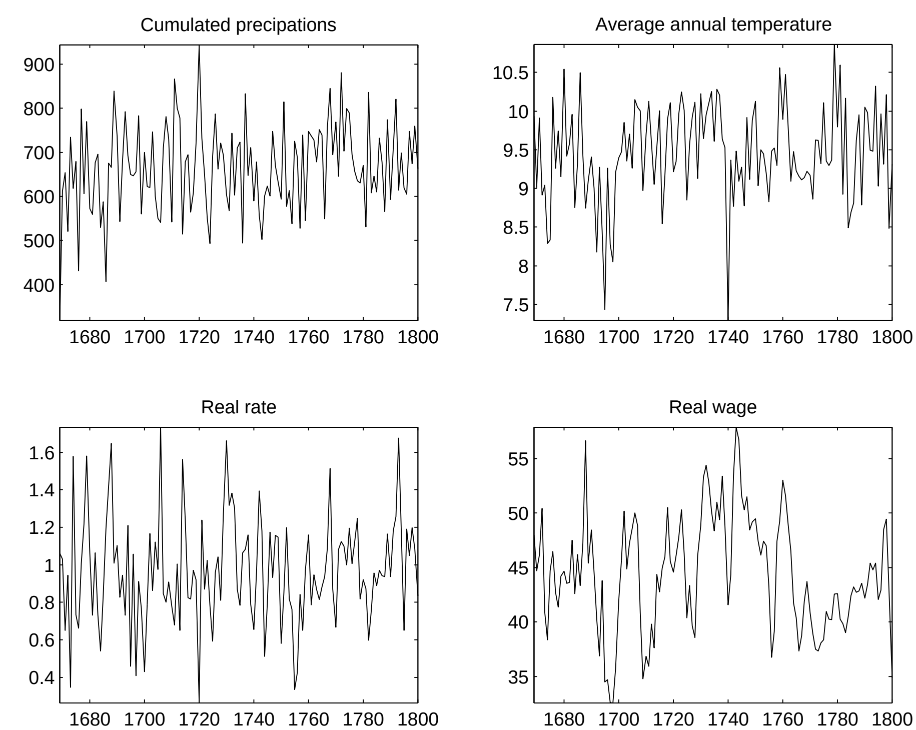
annual mean temperatures for an area around London (average of 4 grid points which is around 5000 km^2) and the annual cumulated precipitation from 1500 to 2000.

Table 1 and Figure 1 give a brief description of the data.

Table 1: Descriptive Statistics

Series	Min	Q_1	Med.	Q_3	Max	Aver.	Std. Err.	Kurt.
Wages	32.58	40.75	44.36	48.38	57.9	44.39	5.22	-0.25
Rents (in %)	0.26	0.79	0.95	1.16	1.73	0.97	0.28	0.45
Precipitations	318.93	601.15	672.11	734.45	943.99	664.87	101.13	0.50
Temperatures	7.29	9.11	9.44	9.94	10.86	9.45	0.62	0.9

Figure 1: Raw time series



Two interesting features of the economic time series can be stressed. First, they do not display any clear trend. Second, annual variations are quite large. Both features are strikingly different from what we observe in currently developed countries. In currently developed countries, rents and wages are much more stable and display a clear upward trend. The trend is usually linked to the presence of economic growth, while private and public insurance systems largely explain the low variability. Several centuries ago, growth was much lower and insurance systems were much less efficient or did not exist at all. From this point of view, our data show that pre-industrial England resembled the currently poorest areas of the World.[3]

In addition, Table 2 shows that the correlations between economic and climatic variations is rather strong.

Table 2: Correlation coefficients

	Temperatures	Precipitations
Wages	0.30	-0.23
Rents	0.17	-0.10

However, this first-pass analysis is hardly interpretable and remains silent on any kind of causality pattern. Further, it cannot be used directly for our purpose since non-linear phenomenons may play an important role.[4] Hence, the potential (negative) relation between annual cumulative precipitations and factor prices (wages and rents) must be further investigated. Finally, the economic literature suggests that the reaction of economic factors to exogenous factors generally follows complex dynamic patterns. For instance, the reaction of economic agents to good or bad states of the economy implies the reallocation of resources over time, with consequence on rents, wages, savings and investment. In the next section, we propose a model based on agents decisions to capture these complex interactions and to guide us in our impact evaluation.

[3]Notice that these wage and rent time series are real prices. Research in economic history has made clear that most of the variation in real prices is due to large fluctuations of nominal prices, in particular frequent episodes of large inflation. This feature is also shared with less developed economies.

[4]For instance, it may be noted wages averaged over ten years show a 5% difference before and after the worst flooding episode, which occurred in 1760.

3 Economic fluctuations and impact of climate

We first describe a model with infinitely lived price-taker agents (a variation of the well-known Solow growth model). We then use our data to infer the impact of climatic variations. The model belongs to the Dynamic Stochastic General Equilibrium (DSGE) framework. This approach emerged in late XX-th century as a cornerstone in the short term analysis of macroeconomic time series. A detailed account and analytical derivation of the model is provided in the Appendix section, while the main text contains only the most relevant features.

3.1 Economic model

A DSGE model contains simple building blocks. First, an infinitely-lived agent, representative of a dynastic sequence of short-lived agents, chooses optimally his/her consumption and labor supply plans. The remaining share of income is left for investment and contributes to the dynamics of the capital stock. The production of goods by the representative firm requires physical capital and labor. Second, these plans must be repeatedly reconsidered because of exogenous random shocks. Good (bad) shocks push (lower) investment, production and consumption. Third, observable economic times series such as production, consumption or wages, follow stochastic dynamic equations that are explicitly linked to the parameters of the model (agents' preferences, production and capital accumulation technologies).

We assume that the representative agent maximizes a time-separable Cobb-Douglas utility function

$$E_0 \left[\sum_{t=0}^{+\infty} \beta^t \log(C_t) - \chi \log(1 - N_t) \right] \tag{1}$$

with respect to consumption (C_t) and labor (N_t) paths subject to the following constraints

$$C_t + I_t = W_t N_t + R_t K_{t-1} + \pi_t, \tag{2}$$

$$K_t = A_K K_{t-1}^{\delta} I_t^{1-\delta}. \tag{3}$$

Equation (3) is the budget constraint, where W_t and R_t stand respectively for the real wage and the real interest rate, and π_t is the representative firms' profit. Equation (3) reflects the law of motion of the capital stock K_t. This equation which is a slight variation of the usual linear case has been proposed by Lucas and Prescott (1971) (see also Hercowitz and Sampson (1991)). The parameter $0 < \delta < 1$ may be interpreted as a quality of installed capital (see above references for details).[5] From now on, we assimilate capital and land, hence the real rate and real rents coincide.

The profits of the representative firm are $\pi_t = Y_t - W_t N_t - R_t K_{t-1}$ where Y_t the production level. We model the production process as $Y_t = A_t K_{t-1}^{\alpha} N_t^{1-\alpha}$ where A_t is the Total Factor Productivity level (TFP hereafter) affected by uninsurable random shocks and K_{t-1} is the installed capital available for production at time t. It is labeled with one lag since it depends on decisions and random events up to date $t-1$. This Cobb-Douglas functional form is standard. We assume constant returns to scale with respect to optimizing production factors.

As explained in the Appendix, a possible solution to the optimization problem is given by $Y_t = SI_t$ where $S = \alpha\beta(1-\delta)/(1-\delta/\beta)$. Using the convention $x_t = \log(X_t)$ for every almost surely positive sequence X_t, the law of motion for (the logarithm of) the capital stock is

$$k_t = a_K + (1-\delta)s + \rho k_{t-1} + (1-\delta)a_t, \tag{4}$$

where $\rho = \delta + (1-\delta)\alpha$.

If we assume that the random shocks A_t are such that a_t admits an ARMA(p,q) representation, Equation (4) shows that the installed capital stock admits an ARMA(p + 1,q) representation. As $\rho < 1$ since both α, δ belong to $[0, 1]$, k_t converges to a stationary random process whenever a_t is stationary.

[5]This formulation may account for adjustment costs, the capital stock at time t being a concave function of investment I_t.

3.2 TFP shocks extraction

Our model can be used to compute the impact of an episodic or long-lasting shock on economic time series and welfare. It is known that climatic conditions are a major source of shocks in pre-industrial as well as less developed economies. However, these economies are also affected by other sources of randomness (conflicts, diseases, political instability,...). In addition, the TFP process may include a trend due to technological progress. We must therefore identify the part of randomness attributed to changes in climatic conditions and then study its impact on the economy.

The first statistical problem is that a_t is not directly observable. Nevertheless, our model allows us to derive the TFP process as an explicit function of the bivariate observable stochastic process r_t, w_t, i.e. as a function of wages and rents. The relation arises from the model itself. Indeed, agents are assumed to react optimally to unobservable shocks, and these reactions affect observable variables, such as prices. This extraction strategy solely relies on wages and rents and not on the climatic time series. It also avoids any "forced" relationship between climatic conditions and economic variables, and may thus be qualified as "agnostic".[6]

As explained in the Appendix, the model implies the following system of equations

$$r_t = \log(\alpha) + y_t - k_{t-1}, \tag{5}$$

$$w_t = \log(1-\alpha) + y_t - n, \tag{6}$$

$$y_t = \alpha k_{t-1} + a_t + (1-\alpha)n, \tag{7}$$

where we have used the fact that labor is constant in equilibrium, *i.e.* $n_t = n$. Equations (5) and (6) derive from the maximization of private profits. Equation (7) is the production function expressed in logs, where we use the fact that n_t is a constant term that may be computed explicitly. Substituting Equation (5) in (7) gives

$$y_t = \alpha(\log(\alpha) + y_t - r_t) + a_t + (1-\alpha)n \tag{8}$$

[6] Of course, the strategy relies heavily on specific assumptions about economic behaviors.

and using Equation (6) we get

$$(1-\alpha)(w_t - \log(1-\alpha)) + \alpha(r_t - \log(\alpha)) = a_t. \tag{9}$$

This equation shows that some affine function of w_t and r_t with positive slopes is equal to the (logarithm) of the TFP process.

3.3 Statistical inference

Before we can extract the TFP process, we still need to estimate the parameter α. Direct regression of w_t on r_t (or the other way around) would lead to biased estimates, since r_t and a_t are correlated. A common solution is to rely on Generalized Method of Moments.

Assume a_t is a strong ARMA(1,q) process

$$a_t = (1-\rho_a)a_\infty + \rho_a a_{t-1} + \epsilon_t + \sum_{i=1}^{q} \theta_i \epsilon_{t-i}, \tag{10}$$

where $(\epsilon_t)_{t>0}$ is a strong white noise. The process $(\epsilon_t)_{t>0}$ is the genuine – unobserved – sequence of exogenous shocks. In particular, the random variable ϵ_t is independent from – observed – quantities w_s, r_s if $s < t$ since agents are not able forecast perfectly these shocks.

The processes $(1-\alpha)(w_t - \rho_a w_{t-1}) + \alpha(r_t - \rho_a r_{t-1})$ and $\epsilon_t + \sum_{i=1}^{q} \theta_i \epsilon_{t-i}$ differ only by some constant term. It implies that the following moment equations

$$Cov[(1-\alpha)(w_t - \rho_a w_{t-1}) + \alpha(r_t - \rho_a r_{t-1}); w_{t-j}] = 0, \tag{11}$$

$$Cov[(1-\alpha)(w_t - \rho_a w_{t-1}) + \alpha(r_t - \rho_a r_{t-1}); r_{t-j}] = 0, \tag{12}$$

must hold for all $j > q$. They may be used to estimate (α, ρ_a). The statistical device amounts to compute the solution of the following program

$$\min v^{\mathrm{T}}(\hat{\alpha}, \hat{\rho_a}) \Omega v(\hat{\alpha}', \hat{\rho_a}) \tag{13}$$

$$\hat{\alpha}', \hat{\rho_a} \tag{14}$$

where Ω is a positive definite matrix of size $2h \geq 2$ and

$$v(\hat{\alpha}', \hat{\rho_a}) = \begin{pmatrix} Cov[(1-\alpha)(w_t - \rho_a w_{t-1}) + \alpha(r_t - \rho_a r_{t-1}); w_{t-q-1}] \\ Cov[(1-\alpha)(w_t - \rho_a w_{t-1}) + \alpha(r_t - \rho_a r_{t-1}); w_{t-q-2}] \\ \dots \\ Cov[(1-\alpha)(w_t - \rho_a w_{t-1}) + \alpha(r_t - \rho_a r_{t-1}); w_{t-q-h}] \\ Cov[(1-\alpha)(w_t - \rho_a w_{t-1}) + \alpha(r_t - \rho_a r_{t-1}); r_{t-q-1}] \\ \dots \\ Cov[(1-\alpha)(w_t - \rho_a w_{t-1}) + \alpha(r_t - \rho_a r_{t-1}); r_{t-q-h}] \end{pmatrix} \tag{15}$$

An asymptotically optimal choice of Ω then provides the so-called GMM estimates of our parameters. If $2h > 2$, we have more constraints than we strictly need to perform the estimation. Hausman (1978) shows that extra constraints may be used to test whether data reject the model or not.

Using the GMM method with $q = 2$ and $h = 6$ we get the results reported in Table 3.[7]

Table 3: GMM estimates

	coef.	std. err.	p. value
α	0.1124	0.0053	0.0000
ρ_a	0.5978	0.0054	0.0000
J-stat			0.0904

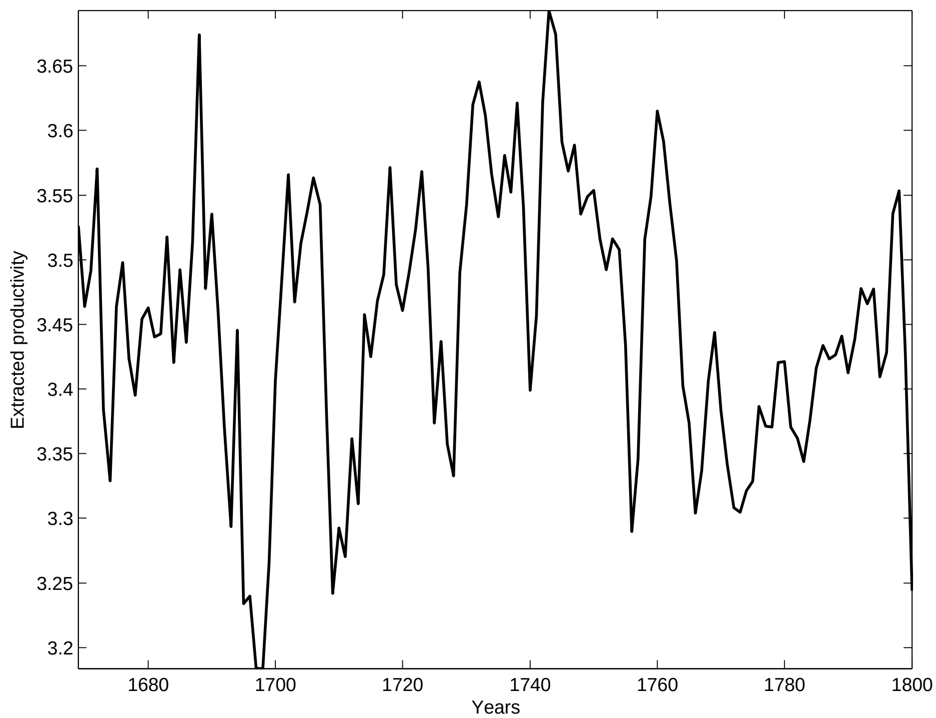
The coefficient α is significant and has the correct sign. The above estimation is consistent with a low level for the (relative) total productivity of the capital stock (with respect to estimates derived using data on developed economies) and a small amount of technological progress. It is also consistent with the fact that production is much more related to labor resources in pre-industrial economies. In addition, the autocorrelation parameter of the TFP process is somehow lower than those usually estimated for currently developed economies. Finally the model appears well specified, as the p-value of the Hausman specification test is $0.0904 > 0.05$, which means that the model is not-rejected by the data at the 5% confidence level.

[7]The optimal weighting matrix Ω is the inverse of the long-run variance-covariance matrix of moment conditions. We correct it from its dynamic heteroskedasticity using a Bartlett Kernel to insure that estimates are unbiased.

3.4 Impact evaluation

The estimates of α, ρ_a may now be used to compute $\hat{a}_t$, an extraction of the TFP process. The extracted process is reported in Figure 2.

Figure 2: Extracted productivity process ($\hat{a}_t$)



We are able to evaluate the magnitude of the shocks that may be attributed to climatic variations. Remember that the above estimates of $\hat{a}_t$ did *not* make any use of the climatic time series. In particular, if economic variables and climatic time series were independent, $\hat{a}_t$ should be independent of changes in climatic conditions, as it was derived as a function of real prices only.

We now perform an estimation of an ARMA model for $\hat{a}_t$ with temperature and precipitations as additional potential exogenous explanatory effects. We estimate the following equation:

$$\hat{a}_t = \phi_0 + \phi_1 \hat{a}_{t-1} + \gamma_1 \text{Prec}_t + \gamma_2 \text{Prec}_t^2 + \beta_1 \text{Temp}_t + \beta_2 \text{Temp}_t^2 + \xi_t + \theta_1 \xi_{t-1}, \qquad (16)$$

where the exogenous variables are expressed in relative deviation from their means.

More precisely, $\text{Temp}_t = 0.01$ whenever the average annual temperature for year t is 1% larger than the overall average (which is 9.45 degree Celsius, see Table 1). The variable Temp_t^2 is the square of Temp_t, and the variables Prec_t and Prec_t^2 are defined accordingly. We report the results in Table 4.

Table 4: ARMAX model

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
ϕ_0	1.1358^a (0.0178)	1.1146^a (0.0133)	1.1376^a (0.0060)	1.1180^a (0.0111)	1.1206^a (0.0067)	1.1006^a (0.0065)	1.0234^a (0.0148)
ϕ_1	0.6704^a (0.0084)	0.6766^a (0.0071)	0.6707^a (0.0068)	0.6756^a (0.0065)	0.6773^a (0.0064)	0.6837^a (0.0050)	0.7061^a (0.0087)
Precipitations	–	-0.0861^b (0.0413)	-0.0836^b (0.0412)	–	–	-0.0989^b (0.0410)	-0.1050^b (0.0410)
$(\text{Precipitations})^2$	–	–	-0.1200 (0.2064)	–	–	–	–
Temperature	–	–	–	0.2513^a (0.0939)	0.1765^c (0.0959)	0.1395 (0.0961)	0.1551 (0.0952)
$(\text{Temperature})^2$	–	–	–	–	-2.0037^b (0.8463)	-2.3943^a (0.8547)	-2.3690^a (0.8678)
θ_1	0.1712^a (0.0561)	0.1309^b (0.0551)	0.1454^a (0.0556)	0.0966^c (0.0532)	0.1178^b (0.0533)	0.0731 (0.0523)	–

Note: Standard errors in parantheses, with a, b, and c respectively denoting significance at the 1%, 5% and 10% levels

Model (1) is a simple ARMA(1,1) where the X vector does not play any role. The result tells us that both the MA(1) and the AR(1) coefficient are significant. The AR(1) coefficient is in line with our GMM-estimated value of ρ_a.

Results for models (2) and (3) show that the level of precipitations significantly affects productivity but the non-linear effect is not significant. Higher-than-average precipitations actually reduce productivity.

Results for models (4) and (5) indicate that temperatures significantly affect productivity both linearly and with a non-linear effect. While the linear effect is positive, rather small and baerly significant at the 10% level, the non-linear effect is negative, large and significant at the 5% level. This means that a small rise in temperature above the average has positive effects on productivity while larger changes (positive or negative) lead productivity to fall.

Results for model (6) show that the level effect of temperatures on productivity is not very robust, as the introduction of precipitations in the vector of exogenous variables turns it statistically non-significant. Further, the MA(1) coefficient becomes non-significant.

Finally, model (7) delivers the best fit with the data. It differs from model (6) only in that it imposes that the MA(1) coefficient is zero. This model combines a linear effect of precipitations and temperatures and a non-linear effect of temperatures. All exogenous variables are significant at the 10% level (the level of temperatures is significant at the 10.32% level) and the values of coefficients are quite stable with respect to the other constrained versions of the model.

We now use model (7) to assess the impact of a two degree Celsius rise above the average temperature.[8] We contrast the impact on productivity (TFP) as well as the impact on wages, output and welfare. The impact on TFP can be computed directly from our estimation. Further, as explained in the Appendix, because our model is quite simple, output and welfare correspond exactly to real wages (up to some constant terms). Therefore deviations from the mean are identical. In addition, the way the dynamics of wages depends on the TFP can be described explicitly, as the logarithm of real wages is an ARMA(1,1) transformation of the TFP process:

$$w_t = (\log(1-\alpha) - \alpha n)(1-\delta)(1-\alpha) + (\delta + (1-\delta)\alpha)w_{t-1} + a_t - \delta a_{t-1}. \tag{17}$$

Consequently, if a_t admits a strong ARMA(1,q) representation, w_t admits a strong ARMA(2,q+1) representation with the same shocks. This representation can be used to derive an estimate of δ, the only relevant unknown parameter in the above equation, using the observed dynamics of w_t. We derive an estimate of δ using the ARMA(2,3) estimates performed on w_t and the following equations:

$$w_t = \beta_w + \rho_w w_{t-1} + a_t - \delta a_{t-1}, \tag{18}$$

$$a_t = \rho_a a_{t-1} + \epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2}. \tag{19}$$

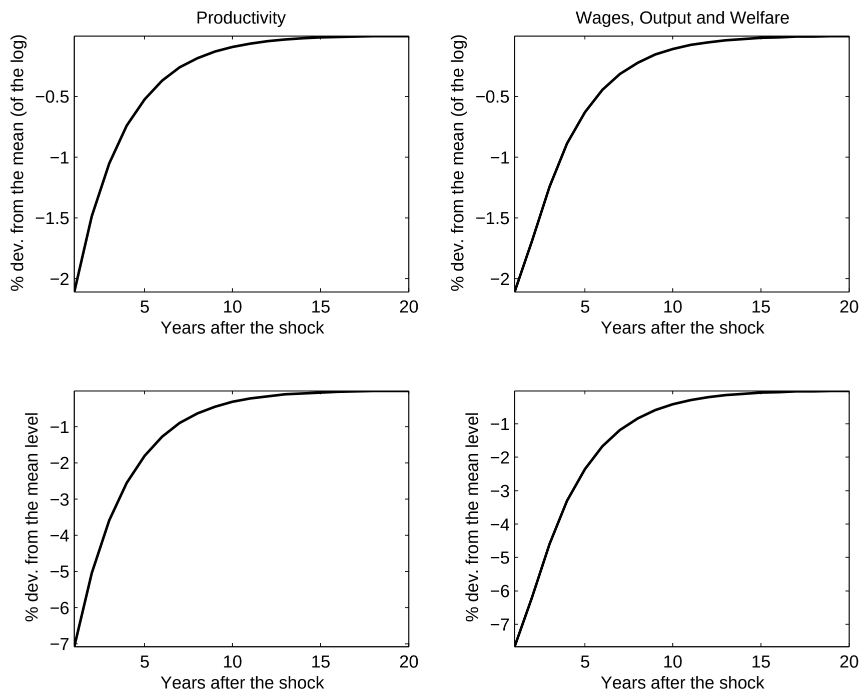
We obtain $\hat{\delta} = 0.15$.[9] Using this figure, we can now report the simulation results.

[8] A two degree rise corresponds to the lower bound of the rise induced by the actual change in climatic conditions according to the IPCC.

[9] Again we remark that this figure is low compared to contemporary estimates.

We simulate the effects of a one-time increase in temperatures (Figure 3) and the effects of a permanent two degree increase in temperatures (Figure 4).

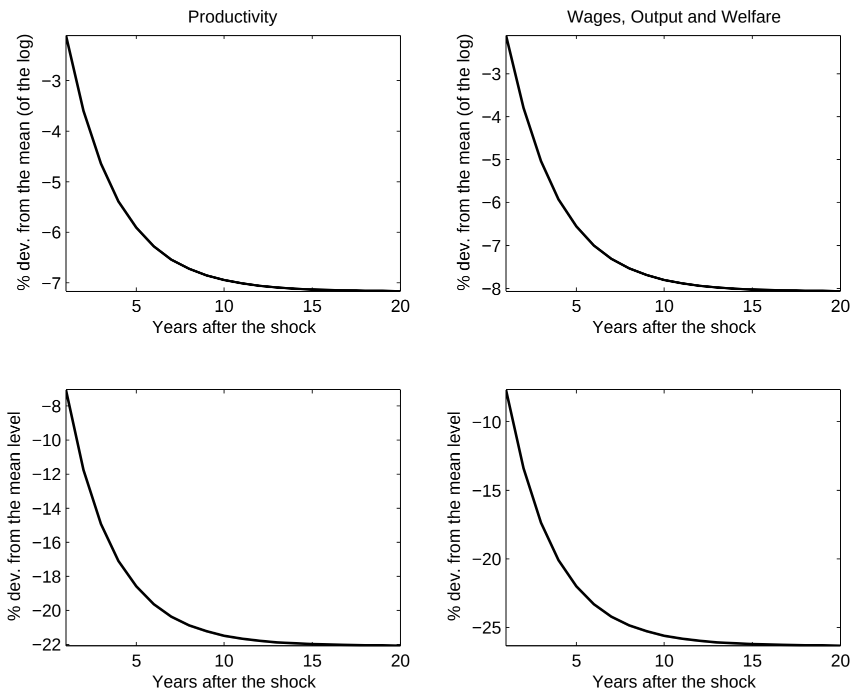
Figure 3: Impact on TFP of a one-time 2 degree rise



The model forecasts that a temporary increase in temperatures would induce a 2.11% decrease in the log of TFP with respect to its mean loglevel. In levels, its means that the TFP would experience a 7.1% drop. The corresponding fall in wages, output and welfare is the same on impact in logdeviation from the mean. However, in levels, the fall in wages, output and welfare reaches 7.67%, which is already quite large. It should be remembered however that a two degree Celsius rise represents a 21% deviation from the mean temperatures, which is also very large.

Now if the rise in temperatures is permanent, as the rise in temperatures is expected to be, figures are much larger. The overall drop in productivity in levels is now around 22%. The positive autocorrelation of the TFP process magnifies the total impact of shocks on economy, and the corresponding fall in wages, output, and welfare is around 26%.

Figure 4: Impact on TFP of a permanent 2 degree rise



4 Conclusion

In this paper, we quantified the impact of temperatures and precipitations in England over the period 1669-1800. Using a standard growth model and historical data on real wages and real rents, we extracted the variations of productivity that could be due to the reallocation of labor and land. The remaining source of variations was then related to climatic factors. Large deviations of temperatures from their mean level affected TFP negatively in this pre-industrial economy. A temporary two degree rise in the temperature above the mean level induced a 4% decrease in the level of TFP. A permanent two degree increase in temperatures led to a 12% decrease in the level of TFP, and to a 16% fall in wages, output and welfare. These results could serve as a useful benchmark to assess the vulnerability of currently underdeveloped economies to upcoming climatic changes.

References

Aguiar, M. and Gopinath, G. (2007). "Emerging markets business cycles: the cycle is the trend", *Journal of Political Economy*, **115**, (1), 69–102.

Arrow, K. (1962). "Economic Welfare and the Allocation of Resources for Invention", NBER Chapters, in: The Rate and Direction of Inventive Activity: Economic and Social Factors, 609–626 National Bureau of Economic Research, Inc.

Cooley, Thomas F. (1995) *Frontiers of Business Cycle Research*, Princeton University Press.

Hausman, J. A. (1978), "Specification Tests in Econometrics", *Econometrica*, **46** (6), 1251–1271.

Hercowitz, Z. and M. Sampson (1991), "Output Growth, the Real Wage, and Employment Fluctuations", *American Economic Review*, **81** (5), 1215–1237.

Kydland, Finn E.; Prescott, Edward C. (1982). "Time to Build and Aggregate Fluctuations", *Econometrica* **50** (6), 1345–1370.

Lucas, R. and E. Prescott (1971), "Investment under uncertainty", *Econometrica*, **39** (5), 659–681.

Neumeyer, P. and Perri, F. (2005) "Business cycles in emerging economies: the role of interest rates", *Journal of Monetary Economics*, **52** (2), 345–380.

Solow, R. (2010) "Building a Science of Economics for the Real World", House Committee on Science and Technology, Subcommittee on Investigations and Oversight, July 20.

Uzawa, H. (1965) "Optimum Technical Change in An Aggregative Model of Economic Growth", *International Economic Review* **6** (1), 18–31.

Appendix : Model

Our model is a variant of a benchmark model in the Real Business Cycle (RBC hereafter) literature. The first subsection below is a brief account on RBC modeling. It is dedicated to readers not familiar with the current state of art of macro-economic models. The readers aware of RBC model may skip this first subsection below. The second subsection provides the analytical derivation we use to forecast the impacts of climate change as performed in Section 3 above.

Main building blocks of the model

The arguments below cannot be considered as a full account of this stream of research. It is intended to provide the amount of knowledge necessary to understand the main mechanisms. We postpone the modeling of growth and first describe a model without endogenous growth.

These models are part of Dynamic Stochastic General Equilibrium approach to macro-economics. In the last decade, DSGE has become the central block of research in academic world. DSGE models are also used by central bankers in short term forecasting and simulations exercises. The paper of Kydland and Prescott (1982) is often considered as the starting point of RBC theory A detailed presentation may be found in Cooley (1995). A common feature of the many variants of the DSGE models is to provide a complete description of the economy as optimal, feasible response to some non anticipated stochastic shock. In the previous statement, "optimal" refers to explicit assumption about agents preferences and beliefs, whereas "feasible" to explicit production and resources accumulation constraints. The main goal is to provide complete explicit microeconomic foundations to macro-economic modeling.[10]

According to Solow (2010) RBC methodology may be presented as follow: "a single, consistent person or dynasty carr[ies] out a rationally designed, long-term plan, occa-

[10]This methodology comes with a price. The models are somewhat more complicated than usual Keynesian ones. Also, the micro-economic assumptions may be too stringent to handle the dynamic specificities of macro-economic times-series.

sionally disturbed by unexpected shocks, but adapting to them in a rational, consistent way."[11]

More precisely, in a discrete time environment, $t = 0, 1, 2, \ldots$ a single, immortal decision maker is suppose to perform the following tasks when t increases

1. Observe (perfectly) the new state of the world (that is to say, the levels of resources, available technology, current and forecast budget constraints)
2. Derive the best forecasts about future states of the world
3. Compute the optimal levels of decisions regarding usage of current and forecast resources

In the above statement, "best forecasts" refers to the usual L^2 sense, hence forecasts are the values taken by conditional expectations of the stochastic processes at various horizons. Moreover, "optimal levels" refers to the maximization under budget and technological constraints of some explicitly defined utility function. In most cases, utility is assumed to increase with current as well as future consumption (C) and leisure (L) levels.

More formally, let $E_t[X_s]$ be the expectation of the value of the random process $(X_r)_{r \geq 0}$ at date s conditionally on the available information set at date t. The agents' goal is to maximize at date t

$$E_t \left[\sum_{s>t} \beta^s U(C_s, L_s) \right],$$

where $U(C_s, L_s)$ is a measure of agent welfare at date s and $0 < \beta < 1$. The parameter β is often interpreted as a measure of "patience". For instance higher β will lead to higher (optimal) level of savings. [12]

[11] The above quotation is part of a skeptical statement about DSGE modeling. We shall not elaborate here on the various variants and controversies raised in this literature.

[12] Though very usual, the above formulation corresponds a specific modeling choice coined as "separable utility hypothesis". More elaborate variants may take into account reluctance to drastic changes in the consumption and/or leisure levels, or more empirically sounded models of discounting.

The decision maker chooses at each date t, the levels C_t and L_t as to maximize the above objective subject to three constraints.

First, the budget must be balanced. As in any General Equilibrium model, the unit of account may be arbitrarily chosen. The most common choice is that the price of one unit of consumption is always set to one.[13] The budget constraint asserts that the total resources provided by labor, capital rent and firm ownership must be either consumed or saved.

$$C_t + S_t = W_t N_t + R_t K_t + \pi_t,$$

where W_t is the real wage, $N_t = 1 - L_t$ is the level of hours worked (total amount of time per period is set to 1 without loss of generality), S_t is the saving flow and π_t is the profit realized by the unit of production. This profit is defined as $Y_t - W_t N_t - R_t K_t$ that is the total cash flow – Y_t in real terms – minus (real) cost of production factors – labor and capital K_t and labor N_t – the real price of unit of capital being R_t and the real price of labor being W_t. It should be stressed that "capital" is a shortcut for several factors of production (machine, land,...).

Second, the production of Y_t depends on the amount of labor and installed capital. In most applied works, a constant return-to-scale Cobb-Douglas production function is assumed so that

$$Y_t = A_t K_{t-1}^{\alpha} N_t^{1-\alpha}$$

In the above equation, the parameter α measures the relative productivity of labor and capital. Technically, we must have $0 < \alpha < 1$ but its calibrated values are most of the time set between 0.25 and 0.45 to account for observed proportions of cost of factors. The process $A_t > 0$ is the Total Factor Productivity. When A_t increases, a larger amount of production is feasible with the same amount of factors. In such a case, we observe an economic boom. We assume A_t is stationary, as it is usually done in the literature dealing with response to transitory shocks. However, we shall stress

[13] Again, this statement is a simplification. For instance, if monetary reserves an/or inflation play an important role, budget constraints in nominal terms may be better suited.

the fact that A_t is not an iid random sequence. Indeed, if we compute the TFP process compatible with most real data, it displays a significant positive autocorrelation.

Third, the capital process is subject to some dynamical constraints. In our model, the constraint is

$$K_t = A_K K_{t-1}^{\delta} I_t^{1-\delta}.$$

This is – slightly – unusual. Most applied works use a linear formulation in which K_t is some fixed proportion of its previous value K_{t-1} augmented by the real investment I_t. The above choice is made so that exact computation of the model is feasible.

Finally, as in any General Equilibrium model, prices must induce full market clearing. Hence the demand for labor by the unit of production must match the quantity supplied and the same must be true for savings.

The above model may be modified as to deal for growth in two different ways. In the so-called "exogenous growth" model, one departs from the stationarity assumption. The TFP process A_t incorporates a (stochastic or deterministic) trend. One common issue is to assume for instance

$$log(A_t) = log(A_{t-1}) + \epsilon_t$$

with ϵ_t an iid sequence of shocks. Another path pioneered by Arrow (1962), or Uzawa (1965) tried to link the rate of technological progress to some endogenous properties of the economy but we do not elaborate more on this.

Complete analytical derivation

In our model, we assume a Cobb-Douglas utility function

$$U(C_s, L_s) = \log(C_s) + \chi \log(L_s)$$

Again this choice is driven by computational considerations (namely the fact that exact derivation is feasible). We first solve for the quantities as real prices may be derived using market clearing conditions.

First observe the budget constraint may be written as

$$C_t + I_t = Y_t = A_t K_{t-1}^{\alpha} N_t^{1-\alpha}$$

and the capital accumulation constraints may then be written as

$$K_t = A_K K_{t-1}^{\delta} \left(A_t K_{t-1}^{\alpha} N_t^{1-\alpha} - C_t \right)^{1-\delta}$$

At date 0, we then end up with the following problem

$$\begin{array}{l} \max_{C_{t\geq 0}, N_{t\geq 0}} \; E_0 \left[\sum_{t>0} \beta^s \log(C_s) + \chi \log(1 - N_s) \right] \\ K_t = A_K K_{t-1}^{\delta} \left(A_t K_{t-1}^{\alpha} N_t^{\alpha} - C_t \right)^{1-\delta} \; \forall \, t > 0 \end{array}$$

As a matter of fact, this complicated optimization problem may be simplified further. Indeed, future plans at date t depend only on the amount of installed capital we intend to have at the end of period t. This leaves us at date t with three control variables (namely C_t, N_t, K_t) and one constraint. Denote λ_t the Lagrange multiplier associated with the constraint, the First Order Conditions may be written as

$$\begin{array}{ll} \frac{1}{C_t} = \frac{\lambda_t (1-\delta) K_t}{I_t} & (C_t) \\ \frac{\chi N_t}{1-N_t} = \frac{\lambda_t (1-\delta) K_t}{I_t} (1-\alpha) Y_t & (N_t) \\ \lambda_t K_t = \beta E_t \left[\lambda_{t+1} K_{t+1} \left(\delta + \alpha (1-\delta) \frac{Y_{t+1}}{I_{t+1}} \right) \right] & (K_t) \\ K_t = A_K K_{t-1}^{\delta} \left(A_t K_{t-1}^{\alpha} N_t^{\alpha} - C_t \right)^{1-\delta} & (\lambda_t) \end{array}$$

Now let us look for a particular solution to the above system such that $S_t = S Y_t$, and $\lambda_t K_t = X$. The proportionality between savings and GDP is not arbitrarily chosen as it is a key assumption in the famous Solow model. It is also a key assumption in the growth literature. The main purpose of the above model is to encompass these models in the DSGE framework. Using the above constraints, the above system becomes

$$\begin{array}{ll} S = \frac{X(1-\delta)}{1+X(1-\delta)} & (C_t) \\ \frac{\chi N_t}{1-N_t} = \frac{X(1-\delta)(1-\alpha)}{S} & (N_t) \\ 1 = \beta E_t \left[\delta + (1-\alpha)(1-\delta) \frac{1}{S} \right] & (K_t) \\ K_t = A_K K_{t-1}^{\delta} \left(A_t S N^{1-\alpha} \right)^{1-\delta} & (\lambda_t) \end{array}$$

The (K_t) equation provides S as a function of preference, production and capital accumulation parameters. Using the (C_t) equation we also get X as a function of these parameters. The (N_t) equation shows that labor corresponds to a fixed proportion of the available time, hence $N_t = N$. Equation (λ_t) is Equation (4) in the main body of the text. Finally, maximization of the individual profit of the firm implies real prices of factors must equal their respective marginal productivity and this gives us Equations (5) and (6). In our setup, the link between welfare and reals wages derives from the following argument. The utility function is

$$U(C_s, L_s) = \log(C_s) + \chi \log(L_s) = \log((1-S)Y_t) + \chi \log(N) = \log((1-S) + \chi \log(N) + y_t$$

Hence the (logarithm of the) total output equal the welfare up to some insignificant constant. As $w_t = \log(1-\alpha) + y_t + n$ both y_t and/or w_t may be used as a measure of welfare.

The dynamic link between welfare (or, as we just claimed, the logarithm of real wages) and TFP may then be derived explicitly. We have

$$\begin{aligned} k_t &= a_k + (1-\delta)s + \delta k_{t-1} + (1-\delta)y_t, && (20) \\ y_t &= \alpha k_{t-1} + a_t + (1-\alpha)n, && (21) \\ w_t &= \log(1-\alpha) + y_t - n. && (22) \end{aligned}$$

Using the production function to substitute for y_t, we get

$$\begin{aligned} k_t &= a_k + (1-\delta)s + \delta k_{t-1} + (1-\delta)(\alpha k_{t-1} + a_t + (1-\alpha)n) && (23) \\ &= a_k + (1-\delta)(s + (1-\alpha)n + a_t) + (\delta + (1-\delta)\alpha k_{t-1}, && (24) \\ w_t &= \log(1-\alpha) + \alpha k_{t-1} + a_t - \alpha n. && (25) \end{aligned}$$

As $\alpha > 0$ both equations combine to yield

$$w_{t+1} = (\log(1-\alpha) - \alpha n)(1-\delta)(1-\alpha) + (\delta + (1-\delta)\alpha)w_t + a_{t+1} - \delta a_t.$$