

Série des Documents de Travail

n° 2012-11

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June 14, 2012

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Nonlinear pricing as exclusionary conduct*

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June 14, 2012

Abstract

We study the exclusionary properties of nonlinear pricing by dominant firms in a static environment. Optimal price schedules are nonlinear when the rivals' sensitivity to competitive pressure varies with the “contestable share” of the market. When buyers can dispose of unconsumed units at no cost, and thus might purchase units they do not need, dominant firms are prevented from placing too much pressure on rivals, which limits the extent of inefficient exclusion. When disposal costs are large and sensitivity to competitive pressure is not monotonic in the contestable share, optimal price schedules may be locally decreasing and highly nonlinear.

JEL codes: L12, L42, D82, D86

Keywords: Inefficient exclusion; buyer opportunism; disposal costs; quantity rebates; incomplete information.

*This article is a deeply revised version of an earlier draft entitled “Leaving the Door Ajar: Nonlinear Pricing by a Dominant Firm”.

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1 Introduction

In recent years, exclusionary conduct by firms with market power has become a high-priority issue on the agenda of antitrust agencies. For instance, the European Commission has made it clear that the emphasis of its enforcement activities is on “ensuring that undertakings which hold a dominant position do not exclude their competitors by other means than competing on the merits of the products or services they provide.” The U.S. Department of Justice concurs that “whether conduct has the potential to exclude, eliminate, or weaken the competitiveness of equally efficient competitors can be a useful inquiry”, and suggests that this inquiry “may be best suited to particular pricing practices.”¹

It is indeed in the area of pricing behavior that the so-called “equally efficient competitor test” most naturally applies. In essence, the test consists in checking that the “effective price” offered by the dominant firm, i.e. the price that competitors have to match, covers (some appropriate measure of) its costs. A violation of the test, therefore, is tantamount to a form of below-cost pricing. Such an outcome, however, says nothing about the precise channel by which the competitive process is harmed. The structure of the test –a price-cost comparison– might suggest a predatory scenario, whereby the dominant firm would incur a short-term sacrifice in the hope of later recoupment, but antitrust authorities are reluctant to engage in such a legally difficult route. As a general rule, they avoid being specific about possible “theories of harm”, for fear of weakening their cases in court. On the other hand, jurisprudence, in most countries, imposes a high standard of proof on defendants putting forward efficiency reasons for their conduct.

The purpose of the present article is to offer a static scenario of exclusion that accounts for the various, often highly nonlinear, price schedules observed in practice. The scenario is consistent with the as efficient competitor test and

¹See, respectively, [European Commission \(2009\)](#) and [U.S. Department of Justice \(2008\)](#). High-profile exclusionary cases involving pricing practices include *Virgin v British Airways* (S.D.N.Y. 1999 and 2nd Cir. 2001), *Concord Boat* (8th Cir. 2000), *Lepage’s v 3M* (3d Cir. 2003) in the United States; *Virgin/British Airways* (2000/74/EC of 14 July 1999), *Michelin* (COMP/E-2/36.041/PO of 20 June 2001), *Tomra* (COMP/E-1/38.113, 2006), and *Intel* (COMP/C-3 /37.990, 2009) in Europe.

provides a transparent interpretation of the shape and structure of the tariffs in terms of competitive pressure placed on rival firms. The scenario highlights a number of environmental parameters that influence the exclusionary power of price schedules.

Our anticompetitive scenario relies on a simple incumbency model whereby the dominant firm commits to a price schedule that is taken as given in the negotiations between the buyer and rival firms. As a starting point, we assume that the dominant firm can let the price charged to buyers depend not only on the quantity it supplies to them but also on the quantity they purchase from rivals. When the surplus created by rivals is uncertain, we find that the rivals sell less than the efficient quantity, that distortion increasing with the rivals' bargaining power vis-à-vis buyers. For a given level of the rivals' sales, however, the dominant firm does not need to distort its own quantity, because it can directly penalize buyers for supplying from rivals.

This preliminary analysis –a multi-unit extension of [Aghion and Bolton \(1987\)](#)– does not easily carry over to the more interesting case where the dominant firm's prices cannot explicitly depend on the quantity purchased from rivals. In this case, indeed, the incentives to purchase from the dominant firm are provided only through quantity rebates granted to buyers, and large rebates might induce the latter to opportunistically purchase units from the dominant firm with no intention to use them. To address buyer opportunism, we adopt a demand specification that involves two fundamental parameters: disposal costs and the “contestable share” of the market. This framework is sufficiently rich to reflect the notions that the dominant firm's product is a “must-have” good and that purchasing unneeded units is costly for the buyer.

Disposal costs are incurred by buyers when they fail to consume purchased units. Their magnitude depends on the particular industry and on the characteristics of the traded product. Buyer opportunism is unlikely under large disposal costs as it would then be costly for buyers to purchase units without consuming them. In the extreme case where disposal costs are infinite, the dominant firm achieves the same outcome as when the price schedule depends on the quantity purchased from rivals. On the other hand, low disposal costs, by making buyer opportunism possible, prevent the dominant firm from placing too much pressure

on rivals and thus limit the extent of inefficient exclusion. It follows that total welfare is lower under larger disposal costs. Antitrust agencies should therefore be particularly vigilant when disposal costs are high or when the price offered by the dominant firm depends on the quantity purchased from competitors.²

The notion of contestable share has been defined by the European Commission as “how much of a customer’s purchase requirements can realistically be switched to a competitor.” In practice, the contestable share is a critical ingredient of the the as-efficient competitor test as it defines the relevant quantity range over which the price and the cost should be computed. Not surprisingly, its determination has proved a very contentious issue in recent cases, reinforcing our view that contestable shares are fundamentally uncertain.

We show that optimal tariffs are nonlinear when the rivals’ sensitivity to competitive pressure depends on the contestable share. The sensitivity reflects the extent to which more pressure placed on rivals translates into more exclusion. With no a priori restriction on the shape of the tariffs, we are thus able to explain a wide variety of patterns. For instance, we find that optimal tariffs tend to be concave when sensitivity increases with the scale of entry, and hence the dominant firm wants to place less pressure on larger competitors. When disposal costs are sufficiently large and the sensitivity to competitive pressure is not monotonic in the contestable share, optimal schedules may exhibit highly nonlinear shapes and admit decreasing parts, as is the case under so-called “retroactive rebates”,³ a pattern that has received much attention from antitrust enforcers.

The article is organized as follows. Section 2 introduces the model. Section 3 assumes that the dominant firm’s tariff can depend on the quantity purchased from rivals, thus ruling out the buyer opportunism problem. Section 4 introduces the notions of contestable share and disposal costs. Section 5 solves the problem when both the surplus generated by rivals and the contestable share are uncertain, relates the shapes of optimal price schedules to the distribution of uncertainty, and explains how disposal costs affect the extent of inefficient exclusion.

²A similar policy recommendation, in the context of market-share contracts, is derived by [Inderst and Shaffer \(2010\)](#) and [Calzolari and Denicolo \(2012\)](#).

³This kind of rebates, granted for all purchased units once a quantity threshold is reached, induces downwards discontinuities in price schedules.

2 The model

A dominant firm, I , competes with a rival, E , to serve a buyer, B . The production costs are assumed to be constant and are denoted by c_E and c_I . We consider situations where the characteristics of the new, rival good are not yet known, and hence the cost c_E and the buyer's taste for the new, rival good are still uncertain. We introduce a multidimensional parameter θ_E in the buyer's utility to model uncertainty about her preference for good E . In contrast, we assume away any informational asymmetry as to the characteristics of the incumbent's good.

If the buyer purchases q_I units of good I and q_E units of good E , she earns a gross profit $V(q_E, q_I; \theta_E)$. The function V is assumed to be concave in (q_E, q_I) , with a negative cross-derivative, $\partial^2 V / \partial q_E \partial q_I < 0$, to reflect the imperfect substitutability of the two goods. The total surplus function is given by $W(q_E, q_I; c_E, \theta_E) = V(q_E, q_I; \theta_E) - c_E q_E - c_I q_I$. We denote by $q_E^*(c_E, \theta_E)$ and $q_I^*(c_E, \theta_E)$ the efficient quantities, i.e. the quantities that maximize W .

2.1 Timing of the game

The order of events reflects the incumbency advantage of the dominant firm and the uncertainty as to the characteristics of the rival good:⁴

- First, the buyer and the incumbent design a price schedule, $T(q_E, q_I)$, to maximize (and split) their joint expected surplus. At this stage, the buyer and the dominant firm know the production cost and the characteristics of good I , but do not know the production cost c_E and the characteristics θ_E of the new product.
- Next, the buyer and the competitor discover the cost and preference parameters, c_E and θ_E , relative to the rival good.
- Then, the buyer and the competitor, both knowing the terms of the agreement between the buyer and the incumbent, agree on a price and a quantity. This negotiation takes place under complete information and is as-

⁴The same timing has been studied in Marx and Shaffer (1999) and Marx and Shaffer (2004) under complete information.

sumed to be efficient. For example, B and E can use a two-part tariff with slope c_E . We denote by β the competitor's bargaining power, which determines the sharing of the surplus.

- Finally, the buyer purchases from the incumbent. At this stage, the buyer maximizes her net profit $V(q_E, q_I; \theta_E) - T(q_E, q_I)$, with no consideration for the incumbent's profit $T(q_E, q_I) - c_I q_I$. From an ex ante perspective, this behavior can be seen as opportunistic.

It may be unfeasible (e.g. for observational reasons) or legally prohibited to condition the price schedule T on the quantity purchased from the competitor. Accordingly, we also study the case where the tariff is independent from q_E , i.e. $T(q_E, q_I) = T(q_I)$. Hereafter, the term ‘‘marginal price’’ refers to the price of an extra unit of good I , i.e. $\partial T(q_E, q_I)/\partial q_I$ when the tariff is allowed to depend on q_E , $T'(q_I)$ for an unconditional tariff.

As regards the timing of negotiation, we assume that the buyer and the dominant firm cannot renegotiate once uncertainty is resolved. (If they could, they would simply agree on the optimal tariff under complete information.) The contribution of the current paper is, on the contrary, to study the shape of the price schedule under incomplete information. We also assume that the buyer and the dominant firm cannot renegotiate after the buyer has purchased from the competitor. In particular, they have a common incentive to renegotiate the quantity of good I whenever q_I does not maximize $W(q_E, q_I; c_E, \theta_E)$, where q_E is the quantity already purchased from the competitor. Hereafter, the ex post efficient, renegotiation-proof quantity of incumbent's good, which maximizes W given q_E , is denoted by $q_I^*(q_E; \theta_E)$.⁵

2.2 Deciding how many units to purchase

The last two stages of the game take place under perfect information, given the price schedule T and the known characteristics of the rival good. The buyer and

⁵With this slight abuse of notation, we observe that the efficient quantity of incumbent good, $q_I^*(c_E, \theta_E)$, coincides with $q_I^*(q_E^*(c_E, \theta_E); \theta_E)$.

the rival choose the quantities to maximize their joint surplus

$$S_{BE}(c_E, \theta_E) = \max_{q_E, q_I} V(q_E, q_I; \theta_E) - T(q_E, q_I) - c_E q_E. \quad (1)$$

It is worth, however, solving for the two quantities sequentially. Suppose the buyer has purchased q_E units from the competitor. Then the buyer picks $q_I(q_E; \theta_E)$ to maximize

$$\max_{q_I} V(q_E, q_I; \theta_E) - T(q_E, q_I). \quad (2)$$

Whenever the marginal price of an extra unit of good I , $\partial T/\partial q_I$, differs from c_I , the quantity $q_I(q_E; \theta_E)$ chosen by B does not maximize the joint surplus of B and I , i.e. differs from $q_I^*(q_E; \theta_E)$. The reason why this may happen is that the schedule T is also designed to put competitive pressure on the rival, which may involve setting the marginal price $\partial T/\partial q_I$ below the marginal cost c_I . This would give the buyer an ex post incentive to buy units of good I in excess of $q_I^*(q_E; \theta_E)$.

Anticipating the above decision regarding q_I , the buyer and the competitor choose q_E to maximize their joint surplus

$$S_{BE}(c_E, \theta_E) = \max_{q_E} V(q_E, q_I(q_E; \theta_E); \theta_E) - T(q_E, q_I(q_E; \theta_E)) - c_E q_E, \quad (3)$$

which is equivalent to (1). The buyer and the competitor share the surplus S_{BE} according to their respective bargaining power and outside options. The competitor's outside option is normalized to zero. As to the buyer, she may source exclusively from the incumbent, so her outside option is $V(0, q_I(0; \theta_E); \theta_E) - T(0, q_I(0; \theta_E))$. It follows that the surplus created by the relationship between B and E is given by

$$\Delta S_{BE}(c_E, \theta_E) = S_{BE}(c_E, \theta_E) - [V(0, q_I(0; \theta_E); \theta_E) - T(0, q_I(0; \theta_E))]. \quad (4)$$

Denoting by $\beta \in (0, 1)$ the competitor's bargaining power vis-à-vis the buyer, we derive the competitor's and buyer's profits:

$$\begin{aligned} \Pi_E &= \beta \Delta S_{BE} \\ \Pi_B &= (1 - \beta) \Delta S_{BE} + V(0, q_I(0; \theta_E); \theta_E) - T(0, q_I(0; \theta_E)). \end{aligned}$$

If $\beta = 0$, the competitor has no bargaining power and may be seen as a competitive fringe from which the buyer can purchase any quantity at price c_E . On the contrary, the case $\beta = 1$ happens when the competitor has all the bargaining power vis-à-vis the buyer.

2.3 Second-best equilibrium and inefficiencies

Ex ante, the buyer and the incumbent design the price schedule to maximize their expected joint surplus, equal to the total surplus minus the profit left to the competitor:

$$\mathbb{E}_{c_E, \theta_E} \Pi_{BI} = \mathbb{E}_{c_E, \theta_E} \{W(q_E, q_I; c_E, \theta_E) - \Pi_E\}. \quad (5)$$

The sharing of the expected joint surplus between the buyer and the incumbent, and hence the respective bargaining power of each party, play no role in the following analysis.

From the ex ante perspective, the tariff has two purposes: on the one hand, extracting rent from the rival, i.e. making ΔS_{BE} as small as possible; on the other, maximizing the expected welfare W . The rent extraction motive may generate two kinds of inefficiencies. First, after having purchased q_E from the rival, the buyer may not pick the efficient quantity $q_I^*(q_E; \theta_E)$, a phenomenon we call “ex post inefficiency”. Second, as pointed out by competition authorities, there might be “inefficient foreclosure”: the quantity purchased from the competitor may not be efficient, $q_E < q_E^*$. Inefficient foreclosure is complete when $q_E = 0 < q_E^*$, partial when $0 < q_E < q_E^*$. In both cases, the rival is prevented from selling the efficient number of units of good E .

2.4 Complete information

The complete information case has been studied in Marx and Shaffer (1999) and Marx and Shaffer (2004). We recall their results using our notations in Appendix E. The main point is that the second-best allocation is efficient. In particular, there is no inefficient exclusion.

When the price schedule is allowed to depend on both quantities, B and I commit on a two-part tariff in q_I , whose constant part depends on the quantity purchased from the rival: $T(q_E, q_I) = c_I q_I + P(q_E)$. The linear part induces the efficient choice $q_I^*(q_E, \theta_E)$ ex post, neutralizing buyer opportunism. The constant part, $P(q_E)$, is used to extract all the surplus created by the rival. Specifically, the incumbent imposes a “penalty” $P(q_E^*) - P(0)$ for supplying from the competitor. Setting this penalty at $W(q_E^*, q_I^*) - W(0, q_I^*(0))$ guarantees that $\Pi_E = \Delta S_{BE} = 0$.

When the price schedule depends on q_I only, there is a tension between buyer opportunism and rent extraction. Placing too much competitive pressure on the rival, in practice granting generous quantity rebates, may indeed attract the buyer, who is tempted to actually purchase the corresponding units from the incumbent. It may therefore be optimal to let the rival earn a positive profit at the second-best equilibrium under complete information (see Appendix E).

3 One-dimensional uncertainty

Hereafter, the buyer's preferences are described with a two-dimensional parameter $\theta_E = (s_E, v_E)$. In this and the next section, however, the component s_E is assumed to be known. The problem under two-dimensional uncertainty is studied in Section 5. The parameter v_E , which enters quasi-linearly in the utility function, represents the intensity of the taste for good E . The parameter s_E describes how fast the taste for good E declines with the quantity purchased. The buyer's gross utility is given by

$$V(q_E, q_I; \theta_E) = v_E q_E + v_I q_I - h(q_E, q_I; s_E),$$

where h is a convex function of (q_E, q_I) with first derivatives at $(0, 0)$ equal to zero and with nonnegative cross derivative. For instance, the function h may consist of quadratic terms in q_E and q_I . The preference for good I , represented by v_I , is known ex ante. We denote by $\omega_E = v_E - c_E \geq 0$ and $\omega_I = v_I - c_I > 0$ the unit surpluses generated by good E and good I respectively. We denote by $[\underline{s}_E, \bar{s}_E]$ and by $[\underline{\omega}_E, \bar{\omega}_E]$ the supports of the random variables s_E and ω_E . The cumulative distribution function of s_E , denoted by G , is assumed to admit a positive and continuous density function g on $[\underline{s}_E, \bar{s}_E]$. The distribution of ω_E conditional on s_E is denoted by $F(\cdot | s_E)$ and is assumed to admit a positive and continuous density function $f(\cdot | s_E)$ on $[\underline{\omega}_E, \bar{\omega}_E]$.

As the parameters c_E and the v_E intervene only through the unit surplus $\omega_E = v_E - c_E$ and the size of the contestable demand, s_E , is assumed to be known, we are left with a one-dimensional screening problem. The number of units of good I purchased by the buyer, $q_I(q_E; \theta_E)$ and the ex post efficient quantity, $q_I^*(q_E, \theta_E)$, do not depend on ω_E , so we simplify the notations into

$q_I(q_E)$ and $q_I^*(q_E)$. Moreover, we do not mention the known value of s_E in the arguments of q_E , Π_E and S_{BE} .

From the analysis of Section 2.2, we know that the rival firm earns $\Pi_E = \beta \Delta S_{BE}$, where ΔS_{BE} , given by equation (4), represents the surplus created by the trade between the buyer and the rival. Under the above quasi-linear specification, the surplus ΔS_{BE} can be rewritten as

$$\Delta S_{BE}(\omega_E) = \max_{q_E \geq 0} \left\{ \omega_E q_E + v_I q_I(q_E) - h(q_E, q_I(q_E)) - T(q_E, q_I(q_E)) - [\omega_I q_I(0) - h(0, q_I(0)) - T(0, q_I(0))] \right\}. \quad (6)$$

The surplus gain ΔS_{BE} being the upper bound of a family of functions that depend linearly on ω_E , is convex in ω_E , and hence almost everywhere differentiable. Its derivative is given by the envelope theorem:

$$\frac{\partial \Pi_E}{\partial \omega_E} = \beta q_E(\omega_E). \quad (7)$$

The convexity of Π_E thus implies that q_E is nondecreasing in ω_E . An integration by parts yields

$$\int_{\underline{\omega}_E}^{\bar{\omega}_E} \Pi_E(\omega_E) f(\omega_E | s_E) d\omega_E = \Pi_E(\underline{\omega}_E) + \beta \int_{\underline{\omega}_E}^{\bar{\omega}_E} q_E(\omega_E) [1 - F(\omega_E | s_E)] d\omega_E.$$

The buyer and the incumbent design the price schedule to maximize

$$\mathbb{E}_{\omega_E} \Pi_{BI} = \mathbb{E}_{\omega_E} W(q_E, q_I(q_E); \omega_E) - \beta \int_{\underline{\omega}_E}^{\bar{\omega}_E} q_E(\omega_E) [1 - F(\omega_E | s_E)] d\omega_E - \Pi_E(\underline{\omega}_E).$$

The above equation uncovers the general structure of the problem. The choice of q_I relates to ex post efficiency while the choice of q_E reflects the tradeoff between efficiency and rent extraction. By definition of $q_I^*(q_E)$, we have

$$\mathbb{E}_{\omega_E} \Pi_{BI} \leq \int_{\underline{\omega}_E}^{\bar{\omega}_E} S^v(q_E; \omega_E) f(\omega_E | s_E) d\omega_E - \Pi_E(\underline{\omega}_E),$$

where, following Jullien (2000), we have defined the ‘‘virtual surplus’’ S^v as

$$S^v(q_E; \omega_E) = W(q_E, q_I^*(q_E); \omega_E) - \beta q_E(\omega_E) \frac{1 - F(\omega_E | s_E)}{f(\omega_E | s_E)}. \quad (8)$$

The virtual surplus is the total surplus $W(q_E, q_I^*(q_E); \omega_E)$ adjusted for the informational rents $\beta q_E (1 - F(\omega_E | s_E)) / f(\omega_E | s_E)$ induced by the self-selection constraints.

If the tariff is allowed to depend on q_E , a two-part tariff with slope c_I , $T(q_E, q_I) = c_I q_I + P(q_E)$, ensures that the buyer picks the ex post efficient quantity, $q_I^*(q_E)$, for any prior choice of q_E . By (6), we see that the rival's profit, $\Pi_E = \beta \Delta S_{BE}$, depends only on the difference $P(q_E) - P(0)$, which thus governs the efficiency rent tradeoff.

Differentiating the virtual surplus with respect to q_E , we find that the second-best quantity is given by

$$\frac{d}{dq_E} W(q_E(\omega_E), q_I^*(q_E); \omega_E) \leq \beta \frac{1 - F(\omega_E | s_E)}{f(\omega_E | s_E)}, \quad (9)$$

with equality if $q_E > 0$. Given that $W_{q_I}(q_E, q_I^*(q_E); \omega_E) = 0$, the above total derivative in q_E is equal to the partial derivative $W_{q_E}(q_E, q_I^*(q_E); \omega_E)$, which, by the first-order condition of the buyer's and rival's problem (6), coincides with $P'(q_E)$. If $q_E(\omega_E) = 0$, then $\Pi_E(\omega_E) = \Delta S_{BE}(\omega_E) = 0$ by (6). If $q_E(\omega_E) > 0$, the less efficient rival's profit, $\Pi_E(\omega_E)$, is set to zero by an appropriate choice of $P(q_E(\omega_E)) - P(0)$. In Appendix A, we check that q_E is nondecreasing in ω_E under a standard condition regarding the distribution F , which yields the following result.

Proposition 1. *Suppose s_E is known ex ante and $(1 - F)/f$ is non-increasing in ω_E . The conditional tariff $T(q_E, q_I)$ that maximizes the buyer-incumbent pair's joint is profit is given by $c_I q_I + P(q_E)$, with*

$$P'(q_E(\omega_E)) = \beta \frac{1 - F(\omega_E | s_E)}{f(\omega_E | s_E)} \quad (10)$$

for positive values of $q_E(\omega_E)$. The quantity purchased from the dominant firm (the competitor) is ex post efficient (distorted downwards). The extent of inefficient exclusion increases with the rival's bargaining power vis-à-vis the buyer.

Proposition 1 extends the results of Aghion and Bolton (1987) in a multi-units setting. The difference $P(q_E) - P(0)$ can be interpreted as a "penalty" imposed to the buyer for supplying from the rival. Under the assumption of the proposition, the penalty is increasing and concave in q_E . The constant $P(0)$ serves to divide the expected surplus between the buyer and the dominant firm. As W_{q_E} is positive and W is concave, we find (complete or partial) inefficient

foreclosure: $q_E < q_E^*$. By concavity of the W , the higher the rival’s bargaining power vis-à-vis the buyer, β , the more severe the downward distortion of q_E .

When the price schedule is *not* allowed to depend on q_E , the tariff $T(q_I)$ governs the choice of the quantities purchased from both suppliers. Ex ante, the buyer and the dominant firm have only one instrument to manage buyer opportunism and solve the rent/efficiency tradeoff. For this reason, the analysis is more complex for unconditional tariffs $T(q_I)$ than for conditional tariffs $T(q_E, q_I)$, as already illustrated by the complete information case (Lemma E.1). Intuitively, the buyer and the dominant firm face two forces:

- the desire to neutralize buyer opportunism and to induce an ex post efficient choice of q_I , which pushes the marginal price towards the marginal cost c_I ;
- the rent/efficiency tradeoff, which may require to set the marginal price below c_I .

In the next section, we consider an environment where the first force translates into a simple constraint on the marginal price. Then the rent/efficiency tradeoff can be solved within the limits allowed by this constraint.

4 “Must-have” good and disposal costs

Hereafter, we specialize to a framework with inelastic buyer demand, meaning that the buyer’s total consumption will not exceed an exogenous level, even if prices become very low. This property entails no limitation given the purpose of our analysis because, as already mentioned, we are not interested in quantity distortions caused by inefficient bilateral bargaining,⁶ but rather in how nonlinear pricing by the dominant firm alters the split of the buyer’s purchase requirements between the two suppliers.

⁶Recall that we assume away any bilateral inefficiency (e.g. asymmetric information) between the buyer and each of the two suppliers. In particular, the buyer and the incumbent would, in the absence of a rival, have no reason to distort the traded quantity. Similarly, we assume throughout the article that the negotiation between the buyer and the rival takes place under perfect information and is efficient (see Section 2.1).

In Section 4.1, we specify the buyer preferences, introducing the size of the contestable demand and the level of disposal costs. The latter parameter measures the severity of the buyer opportunism problem, which we discuss in Section 4.2. The next two subsections are devoted to solving the problem when the size of the contestable demand is known ex ante. The analysis will then be used in Section 5 to solve the multidimensional problem with uncertain contestable demand.

4.1 Contestable market share and disposal costs

As explained in the introduction, it is often the case that the buyer is not ready to supply all of her requirements from the rival firm within a reasonable time horizon. Accordingly, we assume hereafter that the rival can address only a fraction of the buyer's demand within the relevant time period. We denote this fraction by s_E . The per unit utility derived from the rival good is v_E for the s_E first units and zero beyond.⁷ The characteristics of good E are thus summarized, as in Section 3, by a two-dimensional parameter $\theta_E = (s_E, v_E)$. So far, the preferences for the two goods are independent. The notion of substitutability comes from the assumption that the buyer purchase requirements are finite: she cannot consume more units than a known, fixed limit, which we normalize to one, and thus must split her requirements between the two goods.

We allow the buyer to purchase more units than needed, but assume that she incurs a cost if she fails to consume some of the purchased units. We denote by $\gamma > -c_I$ the disposal cost per unconsumed units. The magnitude of disposal costs may vary substantially across industries, as disposing of computer chips, tyres for trucks, or heavy pieces of machineries is likely to entail different costs. We allow disposal costs to be negative (as long as they remain above $-c_I$), in which case they represent in fact revenues from reselling unused quantities on a secondary market. Disposal costs also depend on the seller's ability to impose or to prevent particular uses of the purchased units and on the buyer's ability to avoid monitoring by the dominant firm. Throughout the paper, the level of

⁷The assumption that the rival firm can address at most a fraction of the buyer's demand may also reflect a competitor's capacity constraint. In both interpretations of the model, the buyer never purchases more than s_E from the competitor in equilibrium: $q_E \leq s_E$.

the disposal costs is assumed to be common knowledge.

Having purchased quantities q_E and q_I from the rival and the dominant firm, the buyer chooses consumption levels x_E and x_I so as to maximize

$$v_E \min(x_E, s_E) + v_I x_I - \gamma(q_E - x_E) - \gamma(q_I - x_I)$$

subject to the constraints $x_E \leq q_E$, $x_I \leq q_I$, and $x_E + x_I \leq 1$: the buyer cannot consume more than she has purchased nor more than her total requirement. We denote by $V(q_E, q_I; \theta_E)$ the buyer's indirect utility, i.e. the value of the utility at the solution to the above maximization program. It can be checked that the function V is concave and piecewise linear in (q_E, q_I) and has a non-positive cross-derivative, $\partial^2 V / \partial q_E \partial q_I \leq 0$, which reflects the substitutability between the incumbent and rival goods.

When $q_E \leq s_E$ and $q_I \leq 1 - q_E$, the functions V and W have simple expressions: $V(q_E, q_I; \theta_E) = v_E q_E + v_I q_I$ and $W(q_E, q_I; \theta_E) = \omega_E q_E + \omega_I q_I$.

Efficiency Suppose first that the buyer has purchased q_E units from the rival, with $0 < q_E \leq s_E$, and considers buying a unit of good I in excess of $1 - q_E$. If $v_E > v_I$, the buyer would dispose of this extra unit of good I , her utility would therefore decrease by γ , and the total surplus by $c_I + \gamma$. It follows that the ex post efficient quantity is $q_I^*(q_E; \theta_E) = 1 - q_E$ in this case. If $v_I > v_E$, the buyer would consume the extra unit of good I and dispose of a unit of good E instead, hence an effect on buyer's utility and total surplus given by $v_I - v_E - \gamma$ and $\omega_I - v_E - \gamma$ respectively. It follows that in both cases the ex post efficient quantity of good I given q_E is given by

$$q_I^*(q_E; \theta_E) = \begin{cases} 1 - q_E & \text{if } \omega_I \leq v_E + \gamma \\ 1 & \text{if } \omega_I \geq v_E + \gamma. \end{cases}$$

Next consider the efficient level of q_E . When $\omega_I \leq v_E + \gamma$, purchasing one extra unit from E and efficiently buying one unit less from I change the surplus W by $\omega_E - \omega_I$. When $\omega_I \geq v_E + \gamma$, we necessarily have $\omega_I > \omega_E$ and hence it is efficient to supply exclusively from I . The efficient quantities are therefore given by

$$(q_E^*(c_E, \theta_E), q_I^*(c_E, \theta_E)) = \begin{cases} (s_E, 1 - s_E) & \text{if } \omega_E > \omega_I \\ (0, 1) & \text{if } \omega_E < \omega_I. \end{cases} \quad (11)$$

Hence inefficient foreclosure occurs when $0 \leq q_E < s_E$ while $\omega_E > \omega_I$. Inefficient foreclosure is complete or partial according to whether the above inequality $0 \leq q_E$ holds as an equality or is strict.

4.2 The consequences of buyer opportunism

We explain in this section how the possibility of buyer ex post opportunism affects the design of an unconditional tariff $T(q_I)$. Assume that the incumbent subsidizes the purchase of good I to the point that the marginal price $T'(q_I)$ is below $-\gamma$ in some interval. The buyer would purchase the corresponding units from the dominant firm even if she does not need them. She would indeed find it optimal to dispose of the units at cost γ and to pocket the subsidy. Over-purchasing would be ex post profitable because the negative price would outweigh the disposal cost.

Yet the buyer and the dominant firm would soon realize that this outcome is suboptimal from an ex ante point of view. Anticipating the opportunistic behavior of the buyer, they would modify the above schedule, offering the buyer, together with the quantity \hat{q}_I at price $T(\hat{q}_I)$, the possibility to buy less units than \hat{q}_I , say $q_I \leq \hat{q}_I$, in return for a payment slightly below $T(\hat{q}_I) + \gamma(\hat{q}_I - q_I)$. This change would avoid useless production and disposal costs, without affecting the profit left to the competitor.

A symmetric reasoning shows that it is never optimal ex ante to sell units above the buyer's reservation price, v_I . The buyer and the dominant firm should always grant the buyer the opportunity to purchase as many units as she wants at a price slightly below v_I . The next proposition, proved in Appendix B, shows that the buyer and the dominant firm are better off committing to a price schedule with marginal price between $-\gamma$ and v_I . The main point to be checked is that this requirement does not raise the rent left to the rival.

Proposition 2. *The buyer and the dominant firm are better off using a tariff with marginal price between $-\gamma$ and v_I . The quantity purchased from the dominant firm is ex post efficient: $q_I = q_I^*(q_E; \theta_E) = 1 - q_E$, for any (c_E, θ_E) .*

Proposition 2 is an optimality result that holds whether or not the buyer and the dominant firm know good E 's characteristics when signing the contract and

whether or not the tariff is allowed to depend on q_E . This result guarantees that the buyer purchases the number units corresponding to her total demand.⁸ We are thus able to focus attention on the split of the buyer's requirements between the two suppliers.

Given ex post efficiency, the surplus in the buyer/rival relationship can be written as

$$\begin{aligned} S_{BE}(s_E, \omega_E) &= \max_{q_E} V(q_E, q_I^*(q_E; \theta_E); \theta_E) - T(q_I^*(q_E; \theta_E)) - c_E q_E \\ &= \max_{q_E} v_E \min(q_E, s_E) - c_E q_E + v_I(1 - q_E) - T(1 - q_E). \end{aligned} \quad (12)$$

Proposition 2 has strong implications for unconditional tariffs. To state this implications, we introduce the notion of super-efficiency. We say that the rival firm is *super-efficient* if and only if $\omega_E > v_I + \gamma$. When γ tends to $-c_I$, super-efficiency becomes equivalent to standard efficiency. When disposal costs are infinite, there are no super-efficient rivals.

Corollary 1. *When the tariff is not allowed to depend on q_E , super-efficient rivals serve all of the contestable demand.*

Proof. The maximand in (12) increases in q_E on the interval $[0, s_E]$. Indeed its derivative is given by

$$\omega_E - v_I + T'(1 - q_E) \geq \omega_E - v_I - \gamma,$$

which is positive if the rival firm is super-efficient. □

As γ tends to $-c_I$, the condition that all units of good I are sold at a price above $-\gamma$ represents a stronger constraint. At the limit $\gamma = -c_I$, the condition $T'(q_I) \geq -\gamma$ leaves no scope for anticompetitive exclusion. The desire to avoid buyer opportunism dominates the rent extraction motive. On the other hand, as γ tends to $+\infty$, buyer opportunism becomes a less severe problem. The buyer and the dominant firm can more easily exploit their incumbency advantage to extract rents from the rival, even when it is very efficient.

⁸As mentioned at the end of Section 2.1, it also implies that the buyer and the incumbent have no joint incentive to renegotiate the quantity q_I once the buyer has purchased q_E from the rival.

In contrast, when the tariff is allowed to depend on q_E , the surplus $\omega_E q_E + v_I(1 - q_E) - T(q_E, 1 - q_E)$ does not necessarily increase with q_E , even for super-efficient rivals, because of the first argument in the tariff. Using a conditional tariff, the buyer and the dominant firm can induce ex post efficiency while keeping latitude to extract rent from the rival. For instance, under complete information, we have seen that the rent is fully extracted (see Proposition E.1). To illustrate, consider the complete information case.

Complete information We know from Lemma E.1 that a conditional tariff $T(q_E, q_I)$ makes it possible to extract all the surplus generated by the rival. Consider now the case of an unconditional tariff $T(q_I)$ and suppose first that the rival is efficient but not super-efficient: $\omega_I < \omega_E < v_I + \gamma$, the value of ω_E being known to the buyer and the incumbent. Then these two players agree on a two-part tariff with marginal price slightly above $v_I - \omega_E$, thus offering a surplus slightly below ω_E per unit of good I . To sell units to the buyer, the rival must match this offer, and hence give up all the surplus to the buyer. He therefore serves all of the contestable demand, earning negligible profit. The incumbent sells the remaining units.⁹ The allocation is efficient. The buyer and the incumbent appropriate the entire surplus even if they do not know the size of the contestable market.

On the other hand, if the rival is super-efficient, $\omega_E > v_I + \gamma$, the buyer and the incumbent cannot exert the competitive pressure reflected in the marginal price $v_I - \omega_E$. Such a subsidy would indeed induce the buyer to purchase too many units of good I and to incur disposal costs, which is suboptimal. The best the buyer and the incumbent can do is to set the marginal price at $-\gamma$, thus offering a surplus of $v_I + \gamma$ per unit of good I . The rival keeps $\omega_E - v_I - \gamma$ per unit of good E sold to the buyer.

We conclude that under complete information and an unconditional tariff $T(q_I)$, the rival's profit is $\max [0, \beta(\omega_E - v_I - \gamma)s_E]$ at the optimum. (This expression of the rival's profit is a particular case of (E.1).) Only super-efficient rivals earn a positive profit. Extracting all the surplus created by super-efficient

⁹The fixed part of the tariff determines the sharing of the surplus between the buyer and the incumbent.

rivals is impossible because the competitive pressure that can be placed on them is limited by buyer opportunism.

4.3 Virtual surplus and elasticity of entry

As in Section 3, we assume that the buyer and the incumbent know the size of the contestable demand, s_E , when agreeing on the price schedule T .¹⁰ With no loss of generality, we can assume ex post efficiency (Proposition 2), and replace q_I with $1 - q_E$ in the expression of the buyer-rival surplus:

$$S_{BE}(s_E, \omega_E) = \max_{q_E} v_E \max(q_E, s_E) + v_I(1 - q_E) - c_E q_E - T(1 - q_E).$$

The rival's profit is given by $\Pi_E = \beta \Delta S_{BE}$, where ΔS_{BE} is the surplus created by the buyer-rival relationship:

$$\Delta S_{BE}(s_E, \omega_E) = \max_{q_E} v_E \max(q_E, s_E) - c_E q_E - v_I q_E - T(1 - q_E) + T(0, 1).$$

Relying on Proposition 2, we restrict attention to tariffs satisfying $-\gamma \leq T'(q_I) \leq v_I$, for which the function to be maximized in q_E decreases beyond s_E . We may therefore assume that q_E never exceeds s_E and rewrite the surplus gain ΔS_{BE} as

$$\Delta S_{BE}(s_E, \omega_E) = \max_{q_E \leq s_E} (\omega_E - v_I) q_E - T(1 - q_E) + T(0, 1). \quad (13)$$

Following the same as analysis as in Section 3, we compute the virtual surplus, i.e. the total surplus corrected for the informational rent left to the rival:

$$S^v(q_E; s_E, v_E, c_E) = \omega_I + \left[\omega_E - \omega_I - \beta \frac{1 - F(\omega_E | s_E)}{f(\omega_E | s_E)} \right] q_E. \quad (14)$$

Hereafter, the bracketed term in (14) is denoted by $s^v(s_E, \omega_E)$ and called the “virtual surplus per unit”. We now introduce the notion of elasticity of entry, which expresses the sensitivity of entry to competitive pressure:

$$\varepsilon(\omega_E | s_E) = \frac{\omega_E f(\omega_E | s_E)}{1 - F(\omega_E | s_E)}. \quad (15)$$

This quantity is interpreted as follows. Setting a constant marginal price $T' = p$ amounts to offering the surplus $v_I - p$ per unit of good I . Rivals with ω_E

¹⁰This assumption will be dropped in Section 5.

above (below) this value serve all of the contestable demand (are inactive). The fraction of active rivals, for a given size of the contestable demand, s_E , is thus $1 - F(v_I - p|s_E)$. Decreasing the price p , or equivalently raising the offered surplus, places more competitive pressure on the rival, and hence reduces the fraction of active rivals, in the proportion given by the above elasticity. In the remainder of the paper, we maintain the following assumption regarding the distribution of ω_E given s_E .

Assumption 1. *For any given size of the contestable demand s_E , the elasticity of entry, $\varepsilon(\omega_E|s_E)$, is nondecreasing in ω_E . Moreover, if $\bar{\omega}_E = \infty$, the upper bound of $\varepsilon(\omega_E|s_E)$ as ω_E rises is greater than one, for all s_E . Finally, ω_I belongs to the support of F : $\underline{\omega}_E < \omega_I < \bar{\omega}_E$.*

The monotonicity of the elasticity of entry holds in particular when the hazard rate $f/(1-F)$ is nondecreasing in ω_E , a usual assumption in the nonlinear pricing literature. It is also true in the limit case where the elasticity does not depend on ω_E ; this happens when ω_E , conditionally on s_E , follows a Pareto distribution, given by $1 - F(\omega_E|s_E) = (\omega_E/\underline{\omega}_E)^{-\varepsilon(s_E)}$; the elasticity of entry is then constant in ω_E and equal to $\varepsilon(s_E)$.

4.4 Efficiency-rent tradeoff and disposal costs

When $\beta = 0$, the virtual surplus coincides with the total welfare and is thus maximum at the efficient quantity, namely s_E if $\omega_E > \omega_I$, zero otherwise. For positive β , however, there is a tradeoff between inducing efficient entry and extracting the rival's rent, as the expression of the virtual surplus shows. The buyer and the incumbent want to exert pressure on the rival to extract rent, which distorts the entry decision.

Hereafter, the maximization of the virtual surplus for a given size of the contestable market is called the "relaxed problem". We denote by q_E^r the solution of this problem.

Lemma 1. *The virtual surplus $S^v(q_E; s_E, v_E, c_E)$ attains its maximum at q_E^r given by*

$$q_E^r(s_E, \omega_E) = \begin{cases} 0 & \text{if } \omega_E \leq \hat{\omega}_E(s_E) \\ s_E & \text{otherwise,} \end{cases}$$

where $\hat{\omega}_E(s_E) \in (\omega_I, \bar{\omega}_E)$ is the unique solution to

$$\frac{\hat{\omega}_E(s_E) - \omega_I}{\hat{\omega}_E(s_E)} = \frac{\beta}{\varepsilon(\hat{\omega}_E(s_E)|s_E)}. \quad (16)$$

The efficiency-rent tradeoff leads to more inefficient exclusion as the rival's bargaining power, β , rises and the elasticity of entry, ε , falls.

Proof. The virtual surplus attains its maximum at $q_E = s_E$ if $s^v > 0$ and at $q_E = 0$ if $s^v < 0$. The unit virtual surplus s^v , which can be expressed as

$$s^v(s_E, \omega_E) = \omega_E[1 - \beta/\varepsilon(\omega_E|s_E)] - \omega_I,$$

is positive if and only if

$$\frac{\omega_E - \omega_I}{\omega_E} > \frac{\beta}{\varepsilon(\omega_E|s_E)}.$$

The left-hand side increases in ω_E , and the right-hand side is nonincreasing in ω_E by the first part of Assumption 1, which yields the uniqueness of a solution (16). Moreover, the virtual surplus per unit is negative for $\omega_E = \omega_I$ and positive for $\omega_E = \bar{\omega}_E$ when $\bar{\omega}_E < \infty$. If $\bar{\omega}_E = \infty$, the second-part of Assumption 1 guarantees that s^v is positive for high values of ω_E . Hence the existence of a solution to equation (16) lying between ω_I and $\bar{\omega}_E$.

The above observations also show that $\hat{\omega}_E$ increases with β and decreases with ε . \square

The threshold $\hat{\omega}_E(s_E)$ summarizes the efficiency-rent tradeoff (henceforth abbreviated as ERT) at a given level of s_E . Equation (16) shows an analogy with the textbook monopoly pricing formula. The buyer-incumbent pair indeed has a monopoly power over entry, or more precisely over the quantity produced by the smaller rival. The buyer and the incumbent jointly act like a monopoly towards the rival, setting $\hat{\omega}_E$ to extract rent at the cost of reducing the probability of entry. When the threshold $\hat{\omega}_E$ is higher, the tradeoff pushes towards less entry. The higher ε , the more reactive the rival: the buyer and the incumbent cannot easily extract rents and cannot place strong competitive pressure on the rival, hence a lower $\hat{\omega}_E$, and more entry.

A high value of β means that the bargaining power of the buyer vis-à-vis the competitor is low, and hence the latter will get a higher rent, which makes rent extraction a more serious issue and pushes towards less entry, i.e. a higher

threshold $\hat{\omega}_E(s_E)$. On the contrary, in the limit case where the buyer has all the bargaining power vis-à-vis the rival ($\beta = 0$), there is no tradeoff, and hence no inefficient exclusion: $\hat{\omega}_E(s_E)$ coincides with the efficient threshold ω_I .

Proposition 3. *If the tariff cannot be made contingent upon q_E , the equilibrium outcome depends on the level of the disposal costs:*

- *When $\hat{\omega}_E < v_I + \gamma$, the rival serves all of the contestable demand if $\omega_E > \hat{\omega}_E$ and is inactive otherwise;*
- *When $v_I + \gamma < \hat{\omega}_E$, the rival serves all of the contestable demand if it is super-efficient ($\omega_E > v_I + \gamma$), and is inactive otherwise, as under complete information.*

By letting the price schedule depend on q_E , the buyer and the incumbent can achieve the same outcome as under infinite disposal cost.

Proof. Considering an unconditional tariff $T(q_I)$ and ignoring first the issue of buyer opportunism, we maximize the virtual surplus as explained in Lemma 1. Rivals with $\omega_E < \hat{\omega}_E$ are not active, and hence earn zero profit; this includes inefficient rivals ($\underline{\omega}_E < \omega_E < \omega_I$), but also some efficient ones ($\omega_I < \omega_E < \hat{\omega}_E$). Rivals with $\omega_E > \hat{\omega}_E$ serve all of the contestable demand and, from (7), earn a positive profit, $\beta(\omega_E - \hat{\omega}_E)s_E$. The rival with marginal type $\omega_E = \hat{\omega}_E$ earns zero profit:

$$\Delta S_{BE}(s_E, \hat{\omega}_E) = (\hat{\omega}_E - v_I)s_E + T(1) - T(1 - s_E) = 0,$$

implying that the average incremental price $[T(1) - T(1 - s_E)]/s_E$ is equal to $v_I - \hat{\omega}_E$. When $\hat{\omega}_E < v_I + \gamma$, the buyer and the incumbent may set the price of contestable units at $v_I - \hat{\omega}_E$, without generating buyer opportunism because this price is above $-\gamma$.

When $v_I + \gamma < \hat{\omega}_E$, the above marginal price would induce the buyer to purchase too many units from the incumbent. We know from Corollary 1 that it is optimal for the buyer and the incumbent to let super-efficient rivals serve all of the contestable demand. This is done by setting the marginal price at $-\gamma$. Only super-efficient rivals are active, earning $\beta(\omega_E - v_I - \gamma)s_E$, as under perfect information (see Appendix E).

The above conflict between managing buyer opportunism and extracting optimally the surplus created by the competitor disappears under a conditional tariff. The condition for full surplus appropriation, $\Delta S_{BE}(s_E, \hat{\omega}_E) = 0$, is indeed that

$$\frac{T(0, 1) - T(s_E, 1 - s_E)}{s_E} = v_I - \hat{\omega}_E \quad (17)$$

This condition is met when $T(q_E, q_I)$ is equal to $(\hat{\omega}_E - \omega_I)q_E + c_I q_I$ up to an additive constant. The marginal price $\partial T / \partial q_I = c_I$ induces the efficient choice of q_I given q_E . The penalty $\partial T / \partial q_E = \hat{\omega}_E - \omega_I$ serves to extract the surplus generated by the rival of type $\hat{\omega}_E$.¹¹ \square

Effective price and market foreclosure The price that “the competitor will have to match” to serve the contestable units is called the “effective price” by the European Commission (European Commission (2009), para. 41). This price reflects the competitive pressure placed on the rival. When the tariff only depends on the quantity purchased from the incumbent, the effective price is simply the average incremental price of the contestable units. When the tariff also depends on the quantity purchased from the rival, buying more from the rival (as opposed to buying less from the incumbent) may trigger a penalty that affects the effective price of the contestable units, so the effective price is given by the left-hand side of (17).

To make sure that the competitor serves all of the contestable demand if $\omega_E \geq \hat{\omega}_E$ and is inactive otherwise, the buyer and the incumbent want to set the effective price at $v_I - \hat{\omega}_E$, thus offering the surplus $\hat{\omega}_E$ per unit of good I and forcing the rival with type $\hat{\omega}_E$ to give up all the surplus to the buyer. With an unconditional price schedule, this is possible only if when $v_I - \hat{\omega}_E \geq -\gamma$. In general the effective price is set to $\max(v_I - \hat{\omega}_E, -\gamma)$ to avoid buyer opportunism.

The tradeoff between efficiency and rent extraction results in some efficient rivals being fully foreclosed in equilibrium, namely rivals with $\omega_I < \omega_E < \hat{\omega}_E$. Inefficient foreclosure arises due to incomplete information as in Aghion and Bolton (1987). The fraction of efficient types that are inactive increases with the rival’s bargaining power vis-à-vis the buyer ($\hat{\omega}_E$ increases with β).

¹¹Notice that the above expression of $\partial T / \partial q_E$ is consistent with (10) as $\hat{\omega}_E - \omega_I = \beta[1 - F(\hat{\omega}_E | s_E)] / f(\hat{\omega}_E | s_E)$.

5 Two-dimensional uncertainty

Building on the one-dimensional analysis of Section 4, we now introduce uncertainty about the size of the contestable demand, s_E . This parameter, as the utility v_E , depends on the characteristics of the rival good, which are not yet known when the buyer and the incumbent agree on the price schedule. Facing uncertainty about both s_E and v_E , they maximize the expected virtual surplus

$$\iint s^v(s_E, \omega_E) q_E(s_E, \omega_E) dF(\omega_E | s_E) dG(s_E)$$

over all implementable quantity functions q_E , where s^v is the unit virtual surplus defined in Section 4.3. Recall that the virtual surplus is positive for $\omega_E > \hat{\omega}_E(s_E)$ and negative for $\omega_E < \hat{\omega}_E(s_E)$, where $\hat{\omega}_E(s_E)$ is the ERT threshold given by (16). Hereafter, we call the curve with equation $\omega_E = \hat{\omega}_E(s_E)$ in the (s_E, ω_E) -plan the ERT line. The next lemma, proved in Appendix C, relates the shape of this line to the primitives of the model.

Lemma 2. *When the elasticity of entry, $\varepsilon(\omega_E | s_E)$, does not depend on s_E , the ERT-line is flat and the random variables s_E and ω_E are independent. When the elasticity of entry increases (decreases) with s_E , the ERT-line is decreasing (increasing) and ω_E first-order stochastically decreases (increases) with s_E .*

Assuming first infinite disposal costs, we explain how the shape of the optimal unconditional price schedule depends on the variations of the elasticity of entry. When the elasticity of entry is nondecreasing in the size of the contestable demand, the buyer and the incumbent want to exert less competitive pressure on larger rivals, which essentially results in concave tariffs (Section 5.1). The other cases require a more careful analysis because the relaxed solution is not implementable (Section 5.2). We then consider decreasing and non-monotonic elasticity of entry (Sections 5.3 and 5.4). Finally, we introduce finite disposal costs and show that conditioning the tariff on the quantity purchased from the rival makes it possible for the buyer and the incumbent to achieve the same outcome as under infinite disposal costs (Section 5.5).

5.1 Nondecreasing elasticity of entry

When s_E and ω_E are independent, the ERT line is flat, as represented on Figure 1a. For each size of the contestable market, the problem is the same form as in Section 4.4. It follows that the optimal tariff is affine, with the same effective price as above, namely $v_I - \hat{\omega}_E$, see Figure 1b.

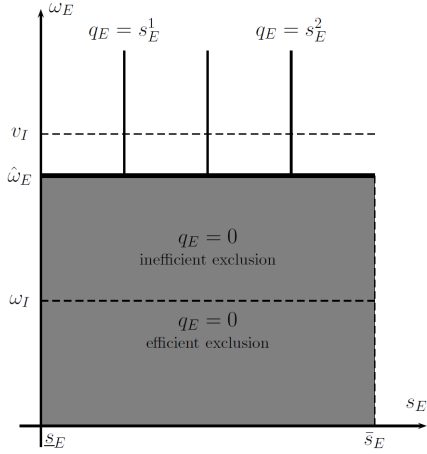


Figure 1a: Quantity purchased from the rival in the (s_E, ω_E) plan

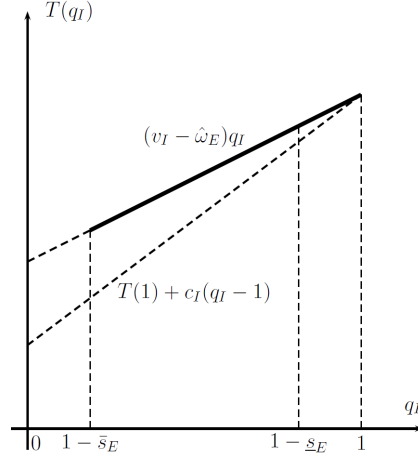


Figure 1b: Optimal price schedule (case $v_I > \hat{\omega}_E$)

From now on, we consider cases where the elasticity of entry is not constant with s_E and show that two-part tariffs are no longer optimal: the optimal tariff must exhibit some curvature. We start with the case where the elasticity increases with s_E : larger competitors, i.e. competitors with a larger contestable demand, are more sensitive to competitive pressure. Under this circumstance, the efficiency-rent tradeoff leads the buyer and the incumbent to place *less* competitive pressure on larger competitors.

Proposition 4. *When the elasticity of entry $\varepsilon(\omega_E|s_E)$ increases with s_E , the effective price, $p^e(q_E)$, increases with q_E . The price schedule is concave in a neighborhood of $q_I = 1$. It is globally concave if $\hat{\omega}_E$ is concave or moderately convex in s_E . The equilibrium features inefficient exclusion. Partial foreclosure is not present.*

Proof. When $\varepsilon(\omega_E|s_E)$ increases with s_E , the ERT threshold, $\hat{\omega}_E$, given by (16), decreases with s_E , see Figure 2a. Solving the problem separately for each s_E ,

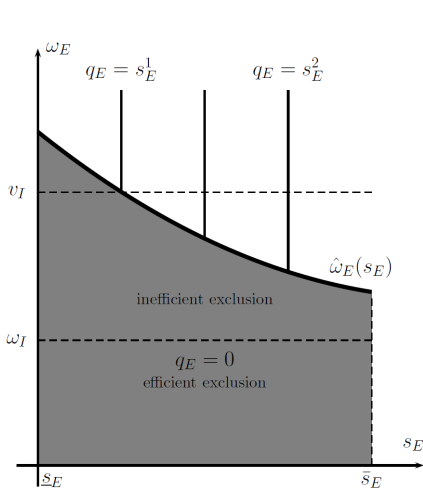


Figure 2a: Second best with $\varepsilon(\omega_E|s_E)$ increasing in s_E

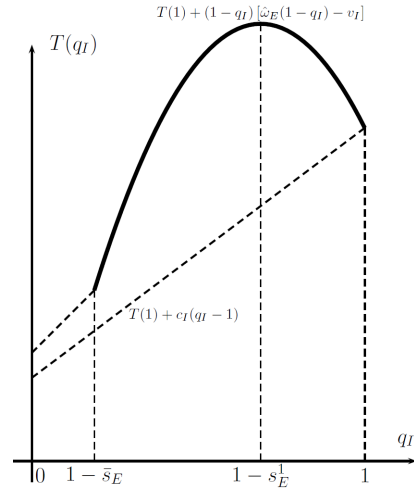


Figure 2b: Optimal price schedule with $\underline{s}_E = 0$ and $v_I < \hat{\omega}_E(0)$

the buyer and the incumbent set the effective price $p^e(s_E)$ at $v_I - \hat{\omega}_E(s_E)$, which increases in s_E . According to equation (13), the buyer and the rival then maximize

$$(\omega_E - v_I)q_E + p^e(q_E)q_E = (\omega_E - v_I)q_E + [v_I - \hat{\omega}_E(q_E)]q_E = [\omega_E - \hat{\omega}_E(q_E)]q_E$$

over $q_E \leq s_E$. As $\hat{\omega}_E(q_E)$ decreases with q_E , the rival with type (s_E, ω_E) is either inactive, $q_E = 0$ if $\omega_E < \hat{\omega}_E(s_E)$, or serves all the contestable demand, $q_E = s_E$ if $\omega_E > \hat{\omega}_E(s_E)$, see Figure 2a.

To prove that the price schedule is concave in a neighborhood of $q_I = 1$, we differentiate

$$T(q_I) = T(1) + (v_I - \hat{\omega}_E(1 - q_I))(q_I - 1)$$

with respect to q_I , which yields $T'(q_I) = (v_I - \hat{\omega}_E(1 - q_I)) + \hat{\omega}'_E(1 - q_I)(q_I - 1)$ and $T''(q_I) = 2\hat{\omega}'_E(1 - q_I) - \hat{\omega}''_E(1 - q_I)(q_I - 1)$. The term $\hat{\omega}'_E$, which is negative for any q_I , tends to make the tariff concave. Assuming that $\hat{\omega}''_E(0)$ is not infinite, we get $T''(1) = 2\hat{\omega}'_E(0) < 0$, hence the concavity at the top. \square

Proposition 4 assumes that the rival becomes more sensitive to competitive pressure as the size of the contestable market rises. Under this assumption, the buyer and the dominant firm want to exert less pressure on larger rivals, and the optimal effective price $p^e(q_E) = [T(1) - T(1 - q_E)]/q_E$ increases with q_E .

It is worthwhile noticing that the effective price is negative for small values of q_E whenever $\hat{\omega}_E(\underline{s}_E)$ is larger than v_I —the case shown on Figures 2a and 2b. The price schedule thus gives the buyer strong incentives to supply exclusively from the dominant firm.

5.2 General case

In the above section, the optimal price schedule has been obtained by solving the rent-efficiency tradeoff separately for each size of the contestable market. This method, however, does not in general yield an incentive compatible allocation. To illustrate, suppose that the ERT line is as shown on Figure 3. The solution to the relaxed problem, which is zero below this line and s_E above, is not incentive compatible. Indeed, the rival of type $B = (\omega_E, s_E)$ is inactive and earns zero profit, while the rival $A = (\omega_E, s'_E)$, $s'_E < s_E$, serves all of the contestable demand. It follows that rival B has an incentive to sell s'_E and mimic rival A . Hence, in this example, solving the relaxed problem does not yield the second-best allocation.

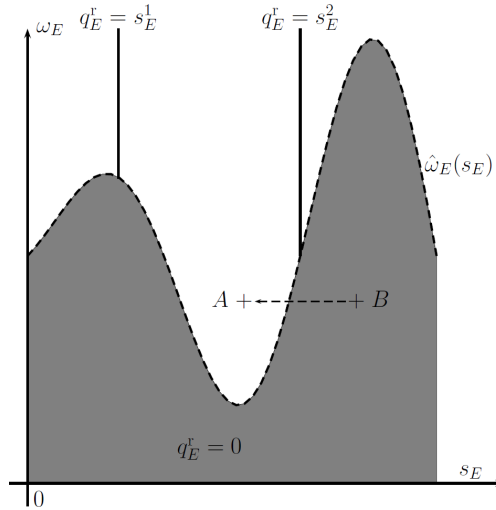


Figure 3: ERT line (dashed). Here, the relaxed solution is not implementable.

We now characterize implementable quantity functions and offer a heuristic derivation of second-best allocations. The main idea is that configurations like that of Figure 3 give rise to partial foreclosure, for which an appropriate first-order condition must be derived. We do not insist on the mathematical resolution

of the problem, which is relegated in a technical appendix available from the authors.¹² The readers interested only by the qualitative results regarding the shape of optimal price schedules should proceed directly to Section 5.3.

Implementable quantity functions The buyer and the competitor maximize their joint surplus, knowing the unconditional price schedule $T(q_I)$ agreed upon by the buyer and the incumbent. They choose a quantity q_E that depends on the competitor's characteristics, (s_E, ω_E) , which gives rise to a "quantity function" $q_E(s_E, \omega_E)$. Assume infinite disposal costs ($\gamma = +\infty$) and relying on Proposition 2, we can restrict attention to price schedules whose marginal does not exceed v_I . A quantity function $q_E(s_E, \omega_E)$ is implementable with an unconditional price schedule if and only if there exists a function $T(q_I)$ satisfying $T'(q_I) \leq v_I$ such that $q_E(s_E, \omega_E)$ is solution to (13) for all (s_E, ω_E) .

As q_E is nondecreasing in ω_E , there exists, for any $s_E > 0$, a threshold $\Psi(s_E)$ such that the buyer supplies all contestable units from the competitor, $q_E(s_E, \omega_E) = s_E$, if and only if $\omega_E > \Psi(s_E)$. We define the *boundary line* $\omega_E = \Psi(s_E)$ associated to the quantity function $q_E(s_E, \omega_E)$ by

$$\Psi(s_E) = \inf\{x \in [\underline{\omega}_E, \bar{\omega}_E] \mid q_E(x, s_E) = s_E\},$$

with the convention $\Psi(s_E) = \bar{\omega}_E$ when the above set is empty. Above the boundary line, $q_E(s_E, \omega_E)$ equals s_E ; below that line, it is independent on s_E .

Boundary line and quantity function As shown on Figure 4, an implementable quantity function $q_E(., .)$ is entirely described by the associated boundary line. The bunching sets, i.e. the sets on which the quantity $q_E(s_E, \omega_E)$ is constant, are determined by the boundary line. They can be of three types: (i) vertical lines above points on the boundary line where that line decreases (e.g. $q_E = s_E^3$ and $q_E = s_E^4$ on the Figure); (ii) "L"-shaped unions of vertical lines above and horizontal lines above and at the right of points where the boundary line increases (e.g. $q_E = s_E^1$, $q_E = s_E^2$ and $q_E = s_E^5$); (iii) two-dimensional areas whose left border is vertical, being included either in the ω_E -axis (then

¹²Deneckere and Severinov (2009) propose a method for solving a more general class of problems, which relies on a characterization of "isoquants". We exploit here the particular shape of these curves, see in particular Figure 4 below.

$q_E = 0$, see the shaded area on Figure 4) or in a vertical part of the boundary line (see the light shaded area on Figure 9b).

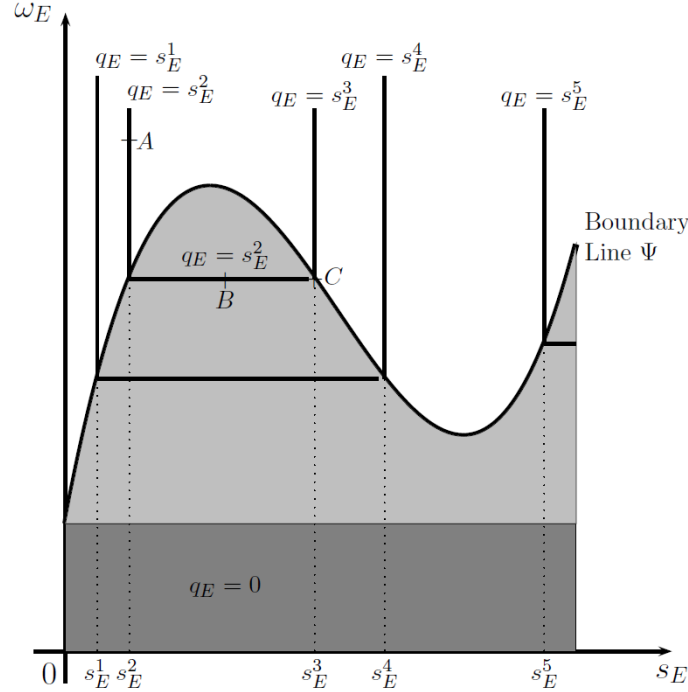


Figure 4: Implementable quantity function (isolines)

Partial foreclosure Increasing parts of the boundary function thus translate into horizontal bunching segments or into two-dimensional bunching areas, and hence into partial foreclosure: $0 < q_E(s_E, \omega_E) < s_E$ for some types located below the boundary. (For instance, type B on Figure 4 sells $q_E = s_E^2$, which is lower than the size of its contestable market.) In such regions, the constraint $q_E \leq s_E$ is slack: increasing s_E does not allow the competitor to enter at a larger scale and q_E does not depend on s_E .

Shape of the boundary line and curvature of the tariff In Appendix G, we explain how to recover the price schedule T from the boundary function Ψ and we link the shape of the price schedule to that of the boundary line. Flat parts of the boundary line correspond to linear parts of the tariff (see Figure 1a and 1b) and increasing parts of the boundary line correspond to convex parts of the tariff (see Figures 6a and 6b, or the interval A_1A_3 on Figures 7a and 7b). In

both cases, the constraint $q_E \leq s_E$ in the buyer-competitor pair's problem (13) is not binding.

In contrast, the curvature of the tariff may change along decreasing parts of the boundary: the tariff is concave near local maxima of the boundary line and convex near local minima. Local maxima of the boundary line thus correspond to inflection points of the tariff. An example is the point $A3$ on Figures 7a and 7b.

Construction of the optimal allocation We now explain intuitively how to derive the optimal boundary line $\omega_E = \Psi(s_E)$ from the ERT line $\omega_E = \hat{\omega}_E(s_E)$.

Consider a point (s_E, ω_E) above the ERT line. If the virtual surplus is always positive at the right of this point, there is no objection to setting $q_E = s_E$. In contrast, if the virtual surplus is negative at the right of this point, setting $q_E = s_E$ implies that q_E will have to be positive in an area where the virtual surplus is negative. By a standard ironing procedure, we show that the expected virtual surplus on horizontal bunching segments is zero. Denoting by $[AB]$ such a segment (see Figure 5b), we get

$$\mathbb{E}(s^v \mid [AB]) = 0, \quad (18)$$

with the boundary conditions that the virtual surplus is positive at A and zero at B . This leads to the following construction of the optimal boundary line $\omega_E = \Psi(s_E)$. We first draw the ERT line $\omega_E = \hat{\omega}_E(s_E)$. For $s_E = \bar{s}_E$, we set $\Psi(\bar{s}_E) = \hat{\omega}_E(\bar{s}_E)$. Then we consider lower values of s_E . If the ERT line decreases at \bar{s}_E , the boundary coincides with the ERT line, as long as it remains decreasing. When the ERT line starts increasing (possibly at \bar{s}_E), we know that there is horizontal bunching. Equation (18) provides a unique value for $\Psi(s_E)$. If the candidate boundary hits the ERT line at some value of s_E , it must be on a decreasing part of that line and, from that value on, the optimal boundary coincides with the ERT line (as long as $\hat{\omega}_E$ remains decreasing). Proposition D.1 in Appendix D presents three different sets of sufficient conditions under which the above construction indeed yields the optimal allocation.¹³

¹³When none of the three sufficient conditions holds, the increasing parts of the optimal boundary line may have vertical portions, generating two-dimensional bunching areas. A vertical ironing procedure is thus needed (see Appendix H.5).

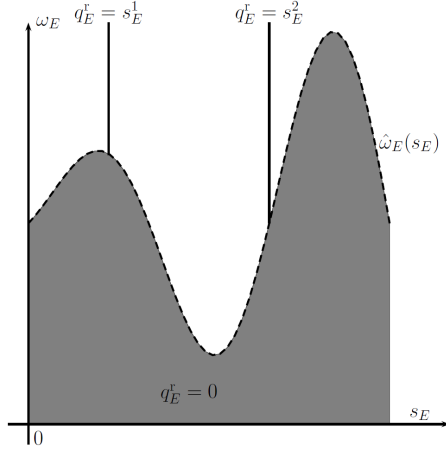


Figure 5a: The relaxed solution locally decreases with s_E .

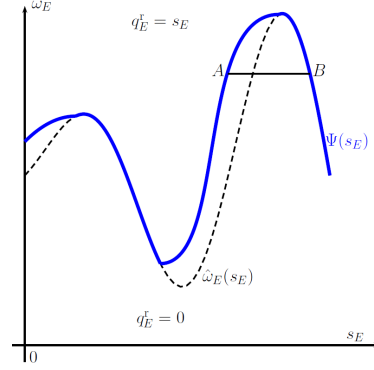


Figure 5b: ERT line $\hat{\omega}_E$ (dashed). Optimal boundary Ψ (solid)

The sufficient conditions of Proposition D.1 are fairly mild. A first sufficient condition is f being nondecreasing in ω_E . A second set of sufficient conditions is the hazard rate $f/(1 - F)$ being nondecreasing in ω_E and the range of the entry elasticity being not too wide (condition (D.1)). A third set of sufficient conditions consists of the elasticity of entry being nondecreasing in ω_E , as stated in Assumption 1,¹⁴ and of another condition on the range of ε , (D.2), more restrictive than (D.1). Technically, the conditions (D.1) and (D.2) involve the rival's bargaining power, β , and the minimum and maximum values of ε in the whole distribution of types, $\underline{\varepsilon}$ and $\bar{\varepsilon}$. Even the stronger condition (D.2) is not very restrictive, in the sense that it allows for a wide range $[\underline{\varepsilon}, \bar{\varepsilon}]$.¹⁵

5.3 Decreasing elasticity of entry

We now turn to the case where the elasticity of entry is decreasing with s_E : larger competitors, i.e. competitors with a larger contestable demand, are less sensitive to competitive pressure. Under this circumstance, the efficiency-rent tradeoff

¹⁴Assumption 1 is weaker than f or $f/(1 - F)$ being nondecreasing in ω_E .

¹⁵For instance, if the rival's bargaining power, β , equals one, the elasticity of entry may vary freely between $\underline{\varepsilon} = 1.2$ and $\bar{\varepsilon} = 3.98$, or between $\underline{\varepsilon} = 5$ and $\bar{\varepsilon} = 26.64$. If β equals .75, then the elasticity of entry may vary freely between $\underline{\varepsilon} = 1.2$ and $\bar{\varepsilon} = 5.99$, or between $\underline{\varepsilon} = 5$ and $\bar{\varepsilon} = 33.59$.

leads the buyer and the incumbent to place *more* competitive pressure on larger competitors, and the optimal price schedule is convex. Although convex price schedules are rarely seen in practice, this mechanism is important to understand the more general shapes exhibited in the next section.

Proposition 5. *Assume that $\varepsilon(\omega_E|s_E)$ decreases with s_E and the assumptions of Proposition D.1 hold. Then the optimal tariff is convex. The equilibrium outcome exhibits inefficient exclusion, in the form of both full and partial foreclosure.*

Proof. When $\varepsilon(\omega_E|s_E)$ decreases with s_E , the ERT line $\hat{\omega}_E$ is monotonically increasing and cannot be the optimal boundary line, as this would violate incentive compatibility. Hence the presence of horizontal pooling segments. As explained in Section 5.2 and expressed by equation (18), the expected virtual surplus on these horizontal segments must be zero. Under each of the sufficient assumptions presented in Proposition D.1, the boundary line $\omega_E = \Psi(s_E)$ given by

$$\int_{\Psi^{-1}(\omega_E)}^{\bar{s}_E} s^V(s, \omega_E) f(\omega_E|s) g(s) ds = 0.$$

is nondecreasing, and hence determines the optimal quantity function, see Figure 6a. The light shaded area on the figure represents the set of types for which the competitor is partially foreclosed. For all $s_E \in [\underline{s}_E, \bar{s}_E]$, $\omega_E = \Psi(s_E)$ and $s'_E > s_E$, the solution of the buyer-competitor problem (13) is interior for (s'_E, ω_E) , and the solution, $q_E = s_E$, is given by the first-order condition $T'(1 - s_E) = v_I - \Psi(s_E)$ or $T'(q_I) = v_I - \Psi(1 - q_I)$, which is nondecreasing in q_I as Ψ is nondecreasing. We conclude that the price-quantity schedule T is convex. \square

The price schedule plays the role of a barrier to expansion. Some efficient competitor types are active but prevented to serve all the contestable demand. When his type lies in the light shaded triangle represented on Figure 6a, the efficient rival sells only part of the contestable demand, i.e. is partially foreclosed from the market.

A small observed market share of the competitor reflects either a small contestable demand or the presence of partial foreclosure. These situations are

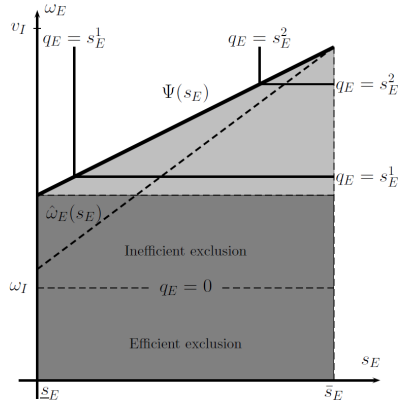


Figure 6a: ERT line (dashed), optimal boundary line (solid) with $\varepsilon(\omega_E|s_E) \searrow$ in s_E .

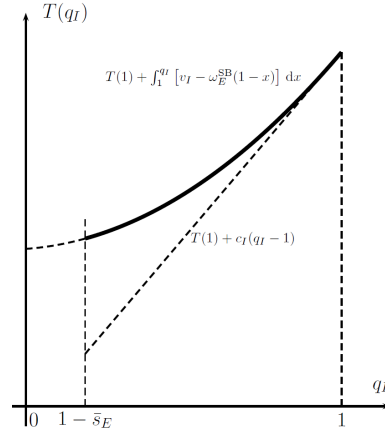


Figure 6b: Optimal price schedule with $\underline{s}_E = 0$ and $v_I > \hat{\omega}_E(1)$.

qualitatively very different. In the first one, the competitor is frustrated because he had to abandon a fraction of his surplus to the buyer. However, depending on the interpretation of s_E , either he cannot produce more or the buyer is not interesting in buying more from the rival. In the second case (partial foreclosure), the competitor is not only deprived of some surplus, but is also frustrated because he cannot sell all the units that the buyer would like to acquire in the absence of price schedule.

5.4 Non-monotonic elasticity of entry

We now turn to a case where the elasticity of entry is non monotonic with the size of the contestable demand, s_E . We assume that the elasticity of entry is first decreasing then increasing as the size of the contestable demand rises: competitors with intermediate size are less sensitive to competitive pressure than competitors with small or large size. Under this circumstance, the efficiency-rent tradeoff induces the buyer and the incumbent to place strong competitive pressure on competitors with intermediate size and less on small or large competitors. In other words, the ERT line is inverted U-shaped.

We rely on Figures 7a and 7b to explain the shape of the optimal price schedule in this instance. The pictures are drawn under the assumption that

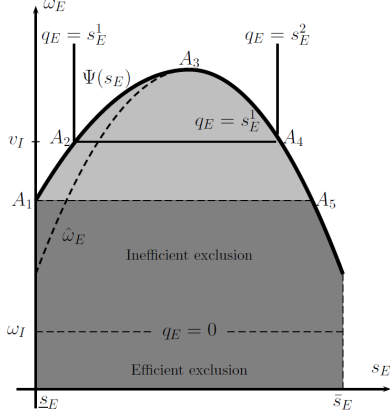


Figure 7a: ERT line (dashed), optimal boundary line (solid) with $\varepsilon(\omega_E|s_E)$ U-shaped in s_E

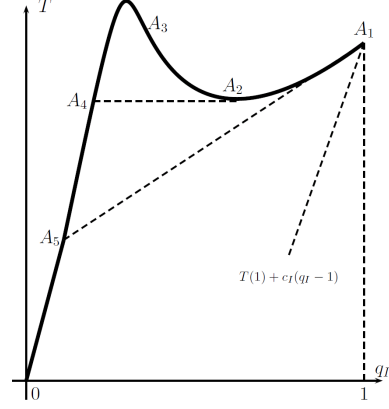


Figure 7b: Optimal price schedule with $\underline{s}_E = 0$ and $\bar{s}_E = 1$

$\hat{\omega}_E(0) < v_I < \max \hat{\omega}_E$. The optimal boundary line $w_E = \Psi(s_E)$, which is increasing up to A_3 and decreasing beyond that point, is represented by the solid curve on Figure 7a. The ERT line is first increasing up to the point A_3 (dashed line on Figure 7a), then decreasing and coincident with the boundary line. The corresponding price schedule is represented on Figure 7b (smooth thin line).

The equation of the optimal boundary line between A_1 and A_3 follows from the bunching condition that the expected surplus on horizontal bunching segments is zero. For instance, to find the point A_2 , we write the condition for the segment $[A_2A_4]$, with A_4 being on the ERT line. As seen in Section 5.3, the quantity negotiated between the buyer and the competitor is given by the first-order condition: $T'(1 - s_E) = v_I - \Psi(s_E)$; the price-quantity schedule T is convex in this region.

Between A_3 and A_5 , we recover the tariff by expressing that the quantity purchased from the rival is constant on the bunching segments. For example, if the rival is at A_4 , the buyer-rival pair is indifferent between buying s_E^1 or s_E^2 : $(\omega_E - v_I)s_E^1 - T(1 - s_E^1) = (\omega_E - v_I)s_E^2 - T(1 - s_E^2)$. As $T(1 - s_E^1)$ is known, one can infer $T(1 - s_E^2)$. At points A_2 and A_4 , we have $\omega_E = v_I$, and hence $T(1 - s_E^1) = T(1 - s_E^2)$, as shown on Figure 7b. It is readily confirmed that $T'' = 0$ at A_3 , i.e. T has an inflexion point.

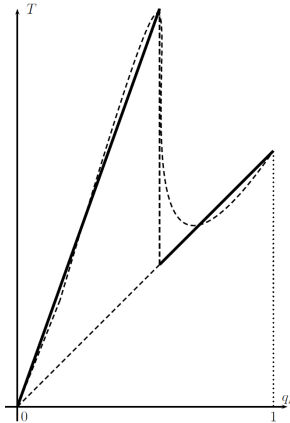


Figure 8: Retroactive rebate approximating an optimal price schedule

Thus, an inverted U-shaped boundary line corresponds to a price schedule that is neither globally concave nor globally convex. The decreasing part of the price schedule gives the buyer a strong incentive to supply from the incumbent beyond the point A_2 . This kind of incentives is exacerbated in so-called “retroactive rebates”, see Figure 8, that are granted for all units once a quantity threshold is reached. Such rebates induce downwards discontinuities in price-quantity schedules. When the distribution of types is continuous, the optimal tariff is continuous. If instead the size of the contestable demand took only a finite number of values, a retroactive rebate, such as the one superimposed on Figure 8, would be optimal.

5.5 Disposal costs and conditional tariffs

In the above example with infinite disposal costs (and a continuous distribution of types), each point in the price schedule, even in regions where it is decreasing, is chosen by some rival type. That is no longer the case if the buyer can get rid of unconsumed units at a finite cost and opportunistically purchase more than her requirements.

According to Proposition 2, the possibility of buyer opportunism just adds the extra constraint that the marginal price is above $-\gamma$. As stated in Corollary 1, under an unconditional price schedule, super-efficient rivals serve all of the contestable demand, which, in terms of boundary line, simply means:

$$\Psi(s_E) \leq v_I + \gamma$$

for all s_E . Conversely, we show in Appendix I that if the boundary line remains below $v_I + \gamma$, the marginal price T' remains above $-\gamma$. This property implies that the only change due to the presence of finite disposal costs concerns super-efficient competitors. To state this point formally, we slightly change the notations, denoting by $q_E(s_E, \omega_E; \gamma)$ the optimal quantity function and by $\Psi(s_E; \gamma)$ the optimal boundary function when the magnitude of the disposal costs is given by the parameter γ .

Proposition 6. *Assume that one of the sufficient conditions of Proposition D.1 holds. Then, relative to the situation with infinite disposal costs, the existence of finite disposal costs*

- *does not affect the optimal quantity purchased from the rival, except possibly for super-efficient rivals;*
- *lowers (raises) or leaves unchanged the buyer-incumbent pair's expected profit (the total welfare);*
- *alters the equilibrium if and only if the efficiency-rent tradeoff requires the exclusion of some super-efficient competitors.*

Proof. The proof follows from the construction of the optimal boundary line. For $\omega_E < v_I + \gamma$, we use the same method as under $\gamma = \infty$, which, under the assumptions of Proposition D.1, yields an implementable quantity functions. For $\omega_E \geq v_I + \gamma$, we know that $q_E = s_E$: super-efficient competitors serve all of the contestable demand. Hence, to obtain the optimal boundary line under $\gamma < \infty$, one has to truncate the corresponding line when $\gamma = \infty$ as follows:¹⁶

$$\Psi(s_E; \gamma) = \min(\Psi(s_E; \infty), v_I + \gamma).$$

The truncation does not change the boundary line when there are no super-efficient competitors, $\bar{\omega}_E \leq v_I + \gamma$, or when the efficiency-rent tradeoff requires that any super-efficient competitor serve all of the contestable demand, i.e. $\hat{\omega}_E(s_E) \leq v_I + \gamma$ for all s_E .

¹⁶The truncation of the boundary line generalizes the formula for the effective price when s_E is known, $p^e = \max(v_I - \hat{\omega}_E, -\gamma)$, see Section 4.4.

On the other hand, suppose that $\hat{\omega}_E(s_E) > v_I + \gamma$ for some value of s_E . Then the construction of Ψ under $\gamma = +\infty$ shows that the maximum of Ψ is larger than $v_I + \gamma$: the constraint $T' \geq -\gamma$ is binding and the possibility of buyer opportunism under finite disposal costs lowers the buyer-incumbent pair's profit. \square

The possibility of ex post buyer opportunism under finite disposal costs prevents the buyer and the incumbent from placing too strong a competitive pressure on the rival, thus protecting super-efficient competitors from exclusion (but not against rent-shifting). The presence of finite disposal costs therefore limits the extent of inefficient foreclosure and hence improves the welfare compared to the case $\gamma = \infty$. We now turn to conditional price schedules.

Proposition 7. *Conditioning the tariff on the quantity purchased from the competitor allows the buyer and the incumbent to earn the same profit as if disposal costs were infinite.*

Proof. When the price schedule does not depend on q_E , it can be recovered from the quantity function $q_E(\cdot, \cdot)$ by

$$T(1) - T(1 - q) = (v_I - \omega_E)q + \Delta S_{BE}(s_E, \omega_E) = (v_I - \omega_E)q + \int_{\underline{\omega}_E}^{\omega_E} q_E(x, s_E) dx.$$

with $q = q_E(s_E, \omega_E)$, see Appendix G. When the price schedule depends on q_E , the expression (13) for the surplus from the trade between the buyer and the competitor must be replaced with

$$\Delta S_{BE}(s_E, \omega_E) = \max_{q_E \leq s_E} (\omega_E - v_I)q_E - T(q_E, 1 - q_E) + T(0, 1),$$

and the above method allows to recover the function $T(q_E, 1 - q_E)$ instead of $T(q_I)$ from the quantity function. In other words, the whole schedule $T(q_E, q_I)$ is not identified; only its values for $(q_E, 1 - q_E)$ are. This implies that the constraint on the marginal price, $\partial T / \partial q_I \geq -\gamma$, has no bite for conditional tariffs. For instance, tariffs of the form $T(q_E, q_I) = P(q_E) + c_I q_I$ induce the efficient choice of q_I given q_E , i.e. are not subject to buyer opportunism, and generate any effective price function $[T(0, 1) - T(q_E, 1 - q_E)] / q_E$. It follows that the set of implementable quantity functions with conditional tariffs does not depend on $\gamma \in [0, +\infty]$, and coincides with the set of quantity functions implementable with unconditional tariffs for $\gamma = \infty$. \square

When the price-quantity schedule depends only on q_I , the presence of finite disposal cost prevents the exclusion of super-efficient competitors, because the incumbent must account for ex post buyer opportunism. Conditional tariffs make it possible for the buyer and the incumbent to overcome the buyer opportunism problem and to exclude super-efficient competitors.

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Appendix

A Proof of Proposition 1

From (9), we get

$$W_{q_E}(q_E(\omega_E), q_I^*(q_E(\omega_E)); \omega_E) \leq \beta \frac{1 - F(\omega_E)}{f(\omega_E)}, \quad (\text{A.1})$$

where $q_I^*(q_E)$ is given by $W_{q_I}(q_E, q_I^*(q_E); \omega_E) = 0$. Differentiating the latter equation with respect to q_E yields

$$W_{q_E q_I} + (q_I^*)'(q_E) W_{q_I q_I} = 0.$$

By concavity of W , the left-hand side of (A.1) is non-increasing in q_E :

$$\frac{d}{dq_E} W_{q_E}(q_E, q_I^*(q_E); \omega_E) = W_{q_E q_E} + W_{q_E q_I} (q_I^*)' = W_{q_E q_E} - \frac{(W_{q_E q_I})^2}{W_{q_I q_I}} \leq 0.$$

The left-hand side of (A.1) is increasing in ω_E . By assumption, the right-hand side is non-increasing in ω_E . It follows that q_E increases with ω_E .

B Proof of Proposition 2

The proof follows from two lemmas.

Lemma B.1. *Starting from any tariff T , we can find a tariff \hat{T} such that the marginal price \hat{T}' is greater than or equal to $-\gamma$ and the surplus of the buyer-incumbent pair is not lower under \hat{T} than under T . The buyer never purchases more than her total requirements: $q_I(q_E; \theta_E) \leq 1 - q_E$ for any q_E .*

Proof. Starting from any tariff T , we define \hat{T} as

$$\hat{T}(q_I) = \inf_{q \geq q_I} T(q) + \gamma(q - q_I). \quad (\text{B.1})$$

The tariff \hat{T} is affine with slope $-\gamma$ in regions where the lower bound in (B.1) is reached at $q > q_I$. Formally, we have: $\hat{T}(q_I) = T(q_I) + (\gamma - \lambda)(q - q_I)$, where q

is a solution to the above problem and λ is the Lagrange multiplier associated to the constraint $q \geq q_I$. The envelope theorem yields $\hat{T}'(q_I) = \lambda - \gamma \geq -\gamma$.

First we check that the buyer and the rival choose the same quantity q_E under the tariffs T and \hat{T} . Let $U(q_E)$ and $\hat{U}(q_E)$ denote the buyer's net utility if she has purchased units q_E units from the competitor under T and \hat{T} :

$$U(q_E) = \max_{q_I} V(q_E, q_I) - T(q_I) \quad \text{and} \quad \hat{U}(q_E) = \max_{q_I} V(q_E, q_I) - \hat{T}(q_I).$$

As $\hat{T} \leq T$, we have: $\hat{U} \geq U$. Suppose that, under \hat{T} , it is optimal for the buyer to purchase \hat{q}_I from the incumbent if she has purchased q_E from the competitor. By construction of \hat{T} , there exists $q_I \geq \hat{q}_I$ such that $\hat{T}(q_I)$ equals or is arbitrarily close to $T(q_I) + \gamma(q_I - \hat{q}_I)$. Observing that buying an extra unit of good I cannot deteriorate the buyer's utility by more than $-\gamma$, i.e. $\partial V / \partial q_I \geq -\gamma$, we get:

$$\hat{U}(q_E) = V(\hat{q}_I, q_E) - \hat{T}(\hat{q}_I) = V(\hat{q}_I, q_E) - \gamma(q_I - \hat{q}_I) - T(q_I) \leq V(q_I, q_E) - T(q_I).$$

It follows that $\hat{U}(q_E) \leq U(q_E)$, and hence $\hat{U}(q_E) = U(q_E)$ for any q_E . To decide on the quantity q_E , the buyer and the rival maximize $U(q_E) - c_E q_E$ under tariff T and $\hat{U}(q_E) - c_E q_E$ under tariff \hat{T} . As the two objectives coincide, they agree on the same quantity under the two tariffs: $q_E(c_E, \theta_E) = \hat{q}_E(c_E, \theta_E)$ for any c_E, θ_E . For the same reason, the rival's profit, $\beta \Delta S_{BE} = \beta[U(q_E) - U(0) - c_E q_E]$, is the same under T and \hat{T} .

Second, we check that under tariff \hat{T} the buyer may purchase less than $1 - q_E$ from the incumbent and that the total welfare is not lower under \hat{T} than under T . Let q_E and q_I denote the purchased quantities under tariff T . As $\hat{T}(q_I) \leq T(q_I)$, the buyer may always choose to purchase the same quantity from the incumbent ($\hat{q}_I = q_I$) under the tariffs T and \hat{T} :

$$U(q_E) = \tilde{U}(q_E) = V(q_E, q_I) - T(q_I) \leq V(q_E, q_I) - \hat{T}(q_I).$$

Now consider the special case where $q_I > 1 - q_E$. As explained at the end of Section 4.1, purchasing one extra unit of good I in the region where $q_I > 1 - q_E$ decreases the buyer utility by γ if $v_E > v_I$ or if $q_E = 0$ and by $v_I - v_E - \gamma$ if $v_I > v_E$ and $q_E > 0$. In the latter case, the buyer would indeed consume the extra unit of good I and dispose of a unit of the rival good instead. Yet this

latter case is impossible here because the buyer and the rival would reduce q_E in the first place, thus improving their joint surplus $V(q_E, q_I) - T(q_I) - c_E q_E$. We conclude that $\partial V / \partial q_I = -\gamma$ in this region. Under tariff \hat{T} , the buyer is better off purchasing $\hat{q}_I = 1 - q_E$ rather than $q_I > 1 - q_E$ from the incumbent. This is because she saves $\gamma(q_E + q_I - 1)$ in terms of disposal costs and loses no more than the same amount in terms of price subsidy.¹⁷ The change from q_I to \hat{q}_I does not decrease the total surplus. On the contrary, it avoids inefficient production and disposal costs:

$$W(q_E, \hat{q}_I) = V(q_E, \hat{q}_I) - c_E q_E - c_I \hat{q}_I \geq V(q_E, q_I) - c_E q_E - c_I q_I = W(q_E, q_I).$$

In sum, the change from T to \hat{T} does not alter the competitor's profit and does not decrease the total surplus. We conclude from (5) that the change does not decrease the expected payoff of the buyer-incumbent coalition, and that $q_I \leq 1 - q_E$ for any (c_E, θ_E) . \square

Lemma B.2. *Starting from any tariff T , we can find a tariff \tilde{T} such that the marginal price \tilde{T}' is lower than or equal to v_I and the surplus of the buyer-incumbent pair is not lower under \tilde{T} than under T . We conclude that the buyer never purchases less than her total requirements: $q_I(q_E; \theta_E) \geq 1 - q_E$ for any q_E .*

Proof. The proof is very similar to that of Lemma B.1. See Appendix F. \square

Taken together, Lemmas B.1 and B.2 yield ex post efficiency: $q_I = q_I^*(q_E; \theta_E) = 1 - q_E$ for all c_E, θ_E .

C Elasticity of entry and distribution of uncertainty

In this section, we prove Lemma 2. The elasticity of entry varies with s_E in the same way as the hazard rate h given by

$$h(\omega_E | s_E) = \frac{f(\omega_E | s_E)}{1 - F(\omega_E | s_E)}.$$

¹⁷If the tariff is affine with slope $-\gamma$ in the corresponding region, the buyer is actually indifferent between purchasing q_I and $1 - q_E$ from the incumbent. To break the indifference, we use $\hat{T}(q_I) = \inf_{q \geq q_I} T(q) + \gamma'(q - q_I)$, for γ' slightly lower than γ . The buyer then strictly prefers $1 - q_E$ to $q_I > 1 - q_E$, for any (c_E, θ_E) .

We have

$$\int_{\underline{\omega}_E}^{\omega_E} h(x|s_E) dx = -\ln[1 - F(\omega_E|s_E)].$$

If the elasticity of entry does not depend on (increases with, decreases with) s_E , the same is true for the hazard rate, and hence also for the cdf $F(\omega_E|s_E)$, which yields the results.¹⁸

D Derivation of the optimal quantity function

Proposition D.1. *Assume that one of the following conditions hold:*

1. *The conditional density $f(\omega_E|s_E)$ is nondecreasing in ω_E ;*
2. *The hazard rate, $f/(1 - F)$, is nondecreasing in ω_E and β , $\underline{\varepsilon}$ and $\bar{\varepsilon}$ satisfy*

$$\beta \leq 4\underline{\varepsilon}\bar{\varepsilon}/(\Delta\varepsilon)^2; \tag{D.1}$$

3. *The elasticity of entry is nondecreasing in ω_E (Assumption 1) and and β , $\underline{\varepsilon}$ and $\bar{\varepsilon}$ satisfy*

$$\beta \leq \frac{\bar{\varepsilon}}{1 + (1 + \Delta\varepsilon)^2/4\underline{\varepsilon}}. \tag{D.2}$$

Then the complete problem can be solved separately for each ω_E . The optimal boundary line Ψ lies above the ERT line, $\Psi \geq \hat{\omega}_E$, and can be constructed from the following properties:

1. $\Psi(1) = \hat{\omega}_E(1)$;
2. *Its non-increasing parts coincide with the ERT line;*
3. *Its increasing parts are defined by equation (18).*

Proof. See Appendix H. □

¹⁸The variable ω_E first-order stochastically decreases (increases) with s_E if and only if $F(\omega_E|s_E)$ increases (decreases) with s_E .

Supplementary material, for online publication

E Complete information

Lemma E.1 (Marx and Shaffer, 2004). *Under complete information, the buyer purchases the efficient quantities q_E^* and q_I^* from both suppliers. If the tariff is allowed to depend on both quantities, then all the surplus is extracted from the rival: $\Pi_E = 0$. If the tariff is function of q_I only, then the rival earns*

$$\Pi_E = \beta \lim_{\hat{q}_I \rightarrow +\infty} \max_{q_E} [V(q_E, \hat{q}_I) - c_E q_E - V(0, \hat{q}_I)] \geq 0. \quad (\text{E.1})$$

As a preliminary observation, notice that, for any \hat{q}_I , the value of the maximum term in (E.1) is nonnegative by construction. Moreover, this maximum value is non-increasing in \hat{q}_I . Indeed its derivative, given by the envelope theorem, satisfies:

$$\frac{d}{d\hat{q}_I} \max_{q_E} [V(q_E, \hat{q}_I) - c_E q_E - V(0, \hat{q}_I)] = V_{q_I}(q_E, \hat{q}_I) - V_{q_I}(0, \hat{q}_I) \leq 0. \quad (\text{E.2})$$

The limit in (E.1) is therefore the lower bound of the maximum term as \hat{q}_I varies. The rival earns a positive profit under complete information if and only if this lower bound is positive. This is the case with the utility function introduced in Section 4.1 and a super-efficient rival. Indeed, in this circumstance, the maximum term in (E.1) is constant and equal to $(\omega_E - v_I - \gamma)s_E$ for any $\hat{q}_I \leq 1$. It follows that super-efficient rivals earn a positive profit, $\Pi_E = \beta(\omega_E - v_I - \gamma)s_E > 0$, at the second-best optimum under complete information.

The proof of Lemma E.1 proceeds in two steps. First, we derive a lower bound for the rival's profit. Second, we find a tariff such that the chosen quantities are efficient and the lower bound for the rival's profit is attained.

Step 1.- Let T be any price schedule. Let (q_E, q_I) be the chosen quantities under tariff T , as defined in (1). We also set $\hat{q}_I = q_I(0)$, where the function $q_I(\cdot)$ is defined by (2), and $\hat{T} = T(0)$. For any \hat{q}_E , we have

$$S_{BE}(\hat{q}_E) = V(\hat{q}_E, q_I(\hat{q}_E)) - c_E \hat{q}_E - T(q_I(\hat{q}_E)) \geq V(\hat{q}_E, \hat{q}_I) - c_E \hat{q}_E - \hat{T},$$

hence, using the definition of $S_{BE}(q_E)$:

$$S_{BE}(q_E) \geq \max_{\hat{q}_E} V(\hat{q}_E, \hat{q}_I) - c_E \hat{q}_E - \hat{T}.$$

and

$$\Pi_E = \beta(S_{BE}(q_E) - S_{BE}(0)) \geq \beta \max_{\hat{q}_E} V(\hat{q}_E, \hat{q}_I) - c_E \hat{q}_E - V(0, \hat{q}_I).$$

We have seen above that the value of the maximum term is non-increasing in \hat{q}_I . We conclude that: $\Pi_E \geq \beta L$. We have thus found an upper bound for the buyer-incumbent pair's profit:

$$\Pi_{BI} \leq W(q_E, q_I) - \Pi_E \leq W(q_E^*, q_I^*) - \beta L.$$

Step 2.- We now show that we can find a tariff such that the chosen quantities are q_E^* and q_I^* and the rival's profit equals or is arbitrarily close to βL .

Let \hat{q}_I be such that the maximum term in (E.1) equals or is arbitrarily close to L . Let \hat{q}_E be such that $V(\hat{q}_E, \hat{q}_I) - c_E \hat{q}_E$ is maximal and $V(\hat{q}_E, \hat{q}_I) - c_E \hat{q}_E - V(0, \hat{q}_I)$ equals or is arbitrarily close to L . We have: $V_{q_E}(\hat{q}_E, \hat{q}_I) - c_E \leq 0$, with equality when $\hat{q}_E > 0$.

First we observe that $\hat{q}_I > q_I^*$ and $\hat{q}_E \leq q_E^*$. Indeed, the derivative in (E.2) evaluated at q_I^* is given by $V_{q_I}(q_E^*, q_I^*) - V_{q_I}(0, q_I^*)$ which is negative because $q_E^* > 0$ by assumption. This shows that $\hat{q}_I > q_I^*$. It follows that

$$0 \leq V_{q_E}(\hat{q}_E, \hat{q}_I) - c_E \leq V_{q_E}(\hat{q}_E, q_I^*) - c_E,$$

which yields $\hat{q}_E \leq q_E^*$.

We now define a tariff T up to an additive constant by the following properties: T is linear on the interval $[0, \hat{q}_I]$ with slope c_I and the difference $T(\hat{q}_I) - T(q_I^*)$ is given by

$$V(q_E^*, q_I^*) - T(q_I^*) - c_E q_E^* = V(\hat{q}_E, \hat{q}_I) - T(\hat{q}_I) - c_E \hat{q}_E. \quad (\text{E.3})$$

Using the definition of (q_E^*, q_I^*) and $\hat{q}_I > q_I^*$, we get

$$\begin{aligned} T(\hat{q}_I) - T(q_I^*) &= c_I(\hat{q}_I - q_I^*) \\ &\quad + \{[V(\hat{q}_E, \hat{q}_I) - c_I \hat{q}_I - c_E \hat{q}_E] - [V(q_E^*, q_I^*) - c_E q_E^* - c_I q_I^*]\} \\ &< c_I(\hat{q}_I - q_I^*). \end{aligned}$$

We conclude that the above-defined tariff T jumps downwards at \hat{q}_I .

Now we check that the buyer, having purchased q_E^* from the rival, strictly prefers purchasing q_I^* than \hat{q}_I from the incumbent:

$$V(q_E^*, q_I^*) - T(q_I^*) > V(q_E^*, \hat{q}_I) - T(\hat{q}_I). \quad (\text{E.4})$$

Indeed, the inequality (E.4) is equivalent, after replacing $T(\hat{q}_I) - T(q_I^*)$ with its value from (E.3), to

$$V(q_E^*, \hat{q}_I) - V(\hat{q}_E, \hat{q}_I) < c_E(q_E^* - \hat{q}_E),$$

which follows from the concavity of V in q_E and $V_{q_E}(\hat{q}_E, \hat{q}_I) \leq c_E$. Next, we check that the buyer, having purchased \hat{q}_E from the rival, strictly prefers purchasing \hat{q}_I than q_I^* from the incumbent:

$$V(\hat{q}_E, \hat{q}_I) - T(\hat{q}_I) > V(\hat{q}_E, q_I^*) - T(q_I^*). \quad (\text{E.5})$$

Indeed the inequality (E.5) is equivalent to

$$V(q_E^*, q_I^*) - V(\hat{q}_E, q_I^*) > c_E(q_E^* - \hat{q}_E),$$

which follows from the concavity of V in q_E and $V_{q_E}(q_E^*, q_I^*) = c_E$. It follows that there exists $\bar{q}_E \in (\hat{q}_E, q_E^*)$ such that the buyer, having purchased q_E from the rival, purchases q_I from the incumbent with

$$q_I = q_I(q_E) = \begin{cases} \hat{q}_I & \text{if } q_E \leq \bar{q}_E \\ q_I^*(q_E) & \text{if } q_E \geq \bar{q}_E, \end{cases}$$

where $q_I^*(q_E)$ is given by

$$V_{q_I}(q_E, q_I^*(q_E)) = c_I. \quad (\text{E.6})$$

The surplus function $S_{BE}(q_E) = V(q_E, \hat{q}_I) - c_E q_E - T(\hat{q}_I)$ is concave on $[0, \bar{q}_E]$ with a local maximum at \hat{q}_E . It has a local minimum and a convex kink at \bar{q}_E , because

$$S'_{BE}((\bar{q}_E)^-) = V_{q_E}(\bar{q}_E, \hat{q}_I) - c_E < V_{q_E}(\bar{q}_E, q_I^*) - c_E = S'_{BE}((\bar{q}_E)^+).$$

For $q_E > \bar{q}_E$, the surplus function is given by $S_{BE}(q_E) = V(q_E, q_I^*(q_E)) - c_E q_E - T(q_I^*(q_E))$. Its first derivative is given by the envelope theorem:

$$S'_{BE}(q_E) = V_{q_E}(q_E, q_I^*(q_E)) - c_E.$$

Differentiating (E.6) yields the first derivative of $q_I^*(q_E)$. We then derive the second derivative of S_{BE} for $q_E > \hat{q}_E$

$$S''_{BE}(q_E) = V_{q_E, q_E} - (V_{q_E, q_I})^2 / V_{q_I, q_I},$$

which is negative by concavity of V . It follows that S_{BE} is concave for $q_E > \bar{q}_E$. The function has another local maximum at q_E^* . Thanks to (E.3), the values of the two local maxima of the function $S_{BE}(\cdot)$ are equal. The difference between this common maximal value and $S_{BE}(0)$ is equal to L by construction, which achieves the proof of the lemma.

F Proof of Lemma B.2

We start from any price schedule T . Let \tilde{T} be defined by

$$\tilde{T}(q_I) = \inf_{q \leq q_I} T(q) + v_I(q_I - q). \quad (\text{F.1})$$

The tariff \tilde{T} is derived from the tariff T as follows. When the incumbent offer q units at price $T(q)$, he also offers to sell more units than q , say $q_I > q$, at price $T(q) + v_I(q_I - q)$. The additional units are offered at the monopoly price v_I . By construction, the slope of \tilde{T} is lower than or equal to v_I .

Let $\tilde{U}_B(q_E)$ denote the buyer's net utility after she has purchased q_E units from the competitor under the price schedule \tilde{T}

$$\tilde{U}_B(q_E) = \max_{q_I} V(q_E, q_I) - \tilde{T}(q_I). \quad (\text{F.2})$$

As $\tilde{T} \leq T$, we have: $\tilde{U}_B \geq U_B$. Suppose that, under \tilde{T} , it is optimal for the buyer to purchase \tilde{q}_I from the incumbent if she has purchased q_E from the competitor. By construction of \tilde{T} , there exists $q_I \leq \tilde{q}_I$ such that $\tilde{T}(q_I)$ equals or is arbitrarily close to $T(q_I) + v_I(\tilde{q}_I - q_I)$. We have:

$$\begin{aligned} \tilde{U}_B(q_E) = V(q_E, \tilde{q}_I) - \tilde{T}(q_I) &= V(q_E, \tilde{q}_I) - T(q_I) - v_I(\tilde{q}_I - q_I) \\ &= V(q_E, q_I) - T(q_I), \end{aligned} \quad (\text{F.3})$$

which implies $\tilde{U}_B(q_E) \leq U_B(q_E)$, and hence $\tilde{U}_B(q_E) = U_B(q_E)$ for all q_E . As the problem of the buyer-competitor pair depends only on the functions $U_B(\cdot)$ and

$\tilde{U}_B(\cdot)$, they agree on the same quantity q_E and the competitor earns the same profit, $\beta\Delta S_{BE}$, under T and \tilde{T} for all (c_E, s_E, v_E) .

We now examine the quantity purchased from the incumbent. Suppose that the buyer, having purchased q_E from the competitor, chooses to purchase q_I from the incumbent under the original price schedule T . As $\tilde{T}(q_I) \leq T(q_I)$, the buyer may choose to purchase the same quantity from the incumbent under the new tariff \tilde{T} :

$$U_B(q_E) = \tilde{U}_B(q_E) = V(q_E, q_I) - T(q_I) \leq V(q_E, q_I) - \tilde{T}(q_I).$$

Yet, under the tariff \tilde{T} , if $q_I < 1 - q_E$, the buyer may as well choose to purchase $1 - q_E$ from the incumbent. Indeed, by definition of \tilde{T} , we have $\tilde{T}(1 - q_E) \leq T(q_I) + v_I(1 - q_E - q_I)$ and hence

$$\begin{aligned} U_B(q_E) = \tilde{U}_B(q_E) &= V(q_E, q_I) - T(q_I) \\ &\leq V(q_E, q_I) + v_I(1 - q_E - q_I) - \tilde{T}(1 - q_E) \\ &= V(q_E, 1 - q_E) - \tilde{T}(1 - q_E). \end{aligned} \quad (\text{F.4})$$

As $v_I > c_I$, the change from q_I to $1 - q_E > q_I$ increases the total surplus:

$$\begin{aligned} W(q_E, 1 - q_E) &= V(q_E, 1 - q_E) - c_E q_E - c_I(1 - q_E) \\ &= V(q_E, q_I) - c_E q_E - c_I q_I + (v_I - c_I)(1 - q_E - q_I) \quad (\text{F.5}) \\ &\geq W(q_E, q_I). \end{aligned}$$

In sum, the change from T to \tilde{T} does not alter the rival's profit and does not decrease the total surplus. We conclude from (5) that the change does not decrease the expected payoff of the buyer-incumbent coalition.

G Implementable quantity functions

G.1 From the boundary line to the quantity function

Because the quantity function $q_E(s_E, \omega_E)$ is nondecreasing in s_E and constant below the boundary, we have:

$$q_E(s_E, \omega_E) = \begin{cases} \min\{x \leq s_E \mid \Psi(y) \geq \omega_E \text{ for all } y \in [x, s_E]\} & \text{if } \Psi(s_E) > \omega_E, \\ s_E & \text{if } \Psi(s_E) \leq \omega_E. \end{cases} \quad (\text{G.1})$$

For type A (resp. B) on Figure 4, we have $\Psi(s_E) < \omega_E$ (resp. $\Psi(s_E) > \omega_E$) and the solution of the problem (13) is unique and equal to s_E^2 . In contrast, type C is indifferent between s_E^2 and s_E^3 and, by convention, is assumed to choose s_E^3 . In other words, when (13) has multiple solutions, equation (G.1) selects the highest.

Lemma G.1. *A quantity function $q_E(.,.)$ is implementable if and only if there exists a boundary function $\Psi(.)$ defined on $[0, 1]$ such that (G.1) holds.*

We prove here the sufficient part of Lemma G.1. Starting from any boundary function Ψ defined on $[0, 1]$, we define the quantity function $q_E(s_E, \omega_E)$ by equation (G.1), and the surplus gain $\Delta S_{BE}(s_E, \omega_E)$ by

$$\Delta S_{BE}(s_E, \omega_E) = \int_{\omega_E}^{\omega_E} q_E(s_E, x) dx.$$

We observe that the functions thus defined $q_E(s_E, \omega_E)$ and $\Delta S_{BE}(s_E, \omega_E)$, are nondecreasing in both arguments, and the latter function is convex in ω_E . Next, we notice that the expression $(\omega_E - v_I)q_E(s_E, \omega_E) - \Delta S_{BE}(s_E, \omega_E)$ is constant on q_E -isolines. Indeed, both $q_E(., \omega_E)$ and $\Delta S_{BE}(., \omega_E)$ are constant on horizontal isolines (located below the boundary Ψ). On vertical isolines (above the boundary), $\Delta S_{BE}(s_E, .)$ is linear with slope s_E , guaranteeing, again, that the above expression is constant. We may therefore define $T(q)$, up to an additive constant, by

$$T(1) - T(1 - q) = (v_I - \omega_E)q + \Delta S_{BE}(s_E, \omega_E), \quad (\text{G.2})$$

for any (s_E, ω_E) such that $q = q_E(s_E, \omega_E)$. Equation (G.2) unambiguously defines $T(1) - T(1 - q)$ on the range of the quantity function $q_E(.,.)$. This range contains zero, but may have holes when $\bar{\omega}_E$ is finite and Ψ is above $\bar{\omega}_E$ on some intervals. Specifically, if Ψ is above $\bar{\omega}_E$ on the interval $I = [s_E^1, s_E^2]$, then q_E does not take any value between s_E^1 and s_E^2 . In this case, we define T by imposing that it is linear with slope $v_I - \bar{\omega}_E$ on the corresponding interval: $T(1 - s_E^1) - T(1 - q) = (v_I - \bar{\omega}_E)(q - s_E^1)$ for $q \in I$.

We now prove that the buyer and the competitor, facing the above defined tariff T , agree on the quantity $q_E(s_E, \omega_E)$. We thus have to check that

$$\Delta S_{BE}(s_E, \omega_E) \geq (\omega_E - v_I)q' + T(1) - T(1 - q') \quad (\text{G.3})$$

for any $q' \leq s_E$. When q' is the range of the quantity function, we can write $q' = q_E(s'_E, \omega'_E)$ for some (s'_E, ω'_E) , with $q' \leq s'_E$. Observing that $q' = q_E(q', \omega'_E)$ and using successively the monotonicity of ΔS_{BE} in s_E and its convexity in ω_E , we get:

$$\begin{aligned} \Delta S_{BE}(s_E, \omega_E) &\geq \Delta S_{BE}(q', \omega_E) \\ &\geq \Delta S_{BE}(q', \omega'_E) + (\omega_E - \omega'_E)q', \end{aligned}$$

which, after replacing $T(1) - T(1 - q')$ with its value from (G.2), yields (G.3). To check (G.3) when q' is not in the range of the quantity function (q' belongs to a hole $[s_E^1, s_E^2]$ as explained above), use (G.3) at s_E^1 and the linearity of the tariff between s_E^1 and q' .

G.2 From the boundary function to the price schedule

Lemma G.2. *The shape of the boundary function Ψ and the curvature of the price schedule T are linked in the following way:*

1. *If Ψ is increasing (resp. constant) around s_E , then the tariff is strictly convex (resp. linear) around $1 - s_E$.*
2. *If Ψ decreases and is concave around s_E , then the tariff is concave around $1 - s_E$.*
3. *If Ψ decreases and is convex around s_E and s_E is close to a local minimum of Ψ , then the tariff is convex around $1 - s_E$.*
4. *If Ψ has a local maximum at s_E , then the tariff has an inflection point at $1 - s_E$.*

Proof. First, suppose that Ψ is nondecreasing on a neighborhood of s_E . Let s'_E slightly above s_E . Then $q_E = s_E$ is an interior solution of the buyer-rival pair's problem (13) for s'_E and $\omega_E = \Psi(s_E)$. It follows that the first order condition $\Psi(s_E) - v_I + T'(1 - s_E) = 0$ holds, implying property 1 of the lemma. The property holds when Ψ has an upward discontinuity at s_E , in which case the tariff has a convex kink at $1 - s_E$. To illustrate, Figures 9a and 9b consider the case where the boundary line is a nondecreasing step function with two pieces.

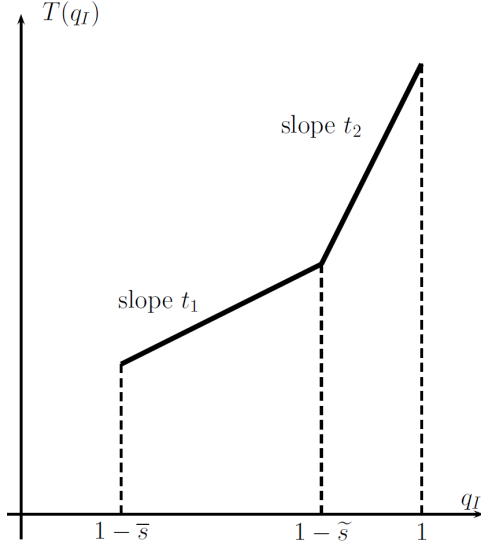


Figure 9a: Convex kink in the price schedule

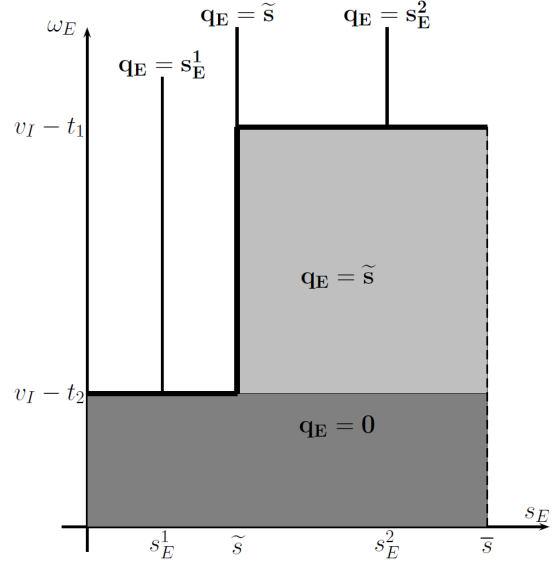


Figure 9b: Two-step increasing boundary line

Next, suppose that the boundary line decreases around s_E . Here we assume that Ψ is twice differentiable. We denote by $[\sigma(s_E), s_E]$ the set of value s'_E such that $q_E(s'_E, \omega_E) = \sigma(s_E)$, where $\omega_E = \Psi(s_E)$. The buyer-rival surplus $\Delta S_{BE}(s_E, \omega_E)$ is convex and hence continuous in ω_E . It can be computed slightly below or above $\Psi(s_E)$. At $(s_E, \Psi(s_E))$, the buyer and the rival are indifferent between quantities s_E and $\sigma(s_E)$:

$$\Delta S_{BE}(s_E, \Psi(s_E)) = [\Psi(s_E) - v_I] \sigma(s_E) - T(1 - \sigma(s_E)) = [\Psi(s_E) - v_I] s_E - T(1 - s_E).$$

Differentiating and using the first-order condition at $\sigma(s_E)$ yields

$$T'(1 - s_E) = -\Psi'(s_E)[s_E - \sigma(s_E)] - \Psi(s_E) + v_I.$$

Differentiating again yields

$$T''(1 - s_E) = \Psi''(s_E)[s_E - \sigma(s_E)] + \Psi'(s_E)[2 - \sigma'(s_E)]. \quad (\text{G.4})$$

In the above equation, the two bracketed terms are nonnegative (use $\sigma' \leq 0$), and the slope Ψ' is negative by assumption, which yields item 2 of the lemma. Around a local minimum of Ψ , Ψ' is small, and the first term is positive, hence property 3. Property 4 follows from items 1 and 2. \square

H Proof of Proposition D.1

In Section H.1, we offer a convenient parametrization of horizontal bunching intervals. In Section H.2, we state and prove a one-dimensional optimization result, which serves to maximize the expected virtual surplus for a given level of ω_E . In Section H.3, we rewrite the complete problem as the maximization of the expected virtual surplus under monotonicity constraints. In Section H.4, we show that these constraints are not binding under fairly mild conditions. In Section H.5, we address the case where the monotonicity constraint are binding and two-dimensional bunching occurs.

H.1 Parameterizing horizontal bunching intervals

Consider an implementable quantity function q_E . For any ω_E , the function of one variable $q_E(\cdot, \omega_E)$ is nondecreasing on $[0, 1]$, being either constant or equal to the identity map: $q_E = s_E$. By convention, we call regions where it is constant “odd intervals”, and regions where $q_E = s_E$ “even intervals”.

We are thus led to consider any partition of the interval $[0, 1]$ into “even intervals” $[s_{2i}, s_{2i+1})$ and “odd intervals” $[s_{2i+1}, s_{2i+2})$, where (s_i) is a finite, increasing sequence with first term zero and last term one.¹⁹ We associate to any such partition the function of one variable that coincides with the identity map on even intervals, is constant on odd intervals, and is continuous at odd extremities. We denote by K the set of the functions thus obtained.

For any implementable quantity function q_E , the functions of one variable, $q_E(\cdot, \omega_E)$, belong to K for all ω_E . Conversely, any quantity function such that $q_E(\cdot, \omega_E)$ belong to K for all ω_E is implementable if and only if even (odd) extremities do not increase (decrease) as ω_E rises. Hereafter, we call the conditions on the extremities the “monotonicity constraints”.

Even (odd) extremities constitute decreasing (increasing) parts of the boundary line. Odd intervals, $[s_{2i+1}, s_{2i+2})$, constitute horizontal bunching segments, or, more precisely, the horizontal portions of the L-shaped bunching regions.

¹⁹ For notational consistency, we denote the first term of the sequence by $s_0 = 0$ if the first interval is even and by $s_1 = 0$ if the first interval is odd. Similarly, we denote the last term by $s_{2n} = 1$ if the last interval is odd and by $s_{2n+1} = 1$ if the last interval is even.

H.2 A one-dimensional optimization result

In this section, we maximize a linear integral functional on the above-defined set K .

Lemma H.1. *Let $a(\cdot)$ be a continuous function on $[0, 1]$. Then the problem*

$$\max_{r \in K} \int_0^1 a(s)r(s) \, ds$$

admits a unique solution r^ characterized as follows. For any interior even extremity s_E^{2i} , the function a equals zero at s_E^{2i} and is negative (positive) at the left (right) of s_E^{2i} . For any interior odd extremity s_E^{2i+1} , the function a is positive at s_E^{2i+1} and satisfies*

$$\int_{s_E^{2i+1}}^{s_E^{2i+2}} a(s) \, ds = 0. \quad (\text{H.1})$$

If $a(1) > 0$, then $r^(s) = s$ at the top of the interval $[0, 1]$. If $a(1) < 0$, then r^* is constant at the top of the interval.*

Proof. Letting $I(r) = \int_0^1 a(x)r(x) \, dx$, we have

$$I(r) = \sum_i \int_{x_{2i}}^{x_{2i+1}} xa(x) \, dx + \sum_i x_{2i+1} \int_{x_{2i+1}}^{x_{2i+2}} a(x) \, dx,$$

where the index i in the two sums goes from either $i = 0$ or $i = 1$ to either $i = n - 1$ or $i = n$, in accordance with the conventions exposed in Footnote 19. Differentiating with respect to an interior even extremity yields

$$\frac{\partial I}{\partial x_{2i}} = a(x_{2i}) \cdot [x_{2i-1} - x_{2i}].$$

The first-order condition therefore imposes $a(x_{2i}^*) = 0$. The second-order condition for a maximum shows that a must be negative (positive) at the left (right) of x_{2i}^* .

Differentiating with respect to an interior odd extremity yields

$$\frac{\partial I}{\partial x_{2i+1}} = \int_{x_{2i+1}}^{x_{2i+2}} a(x) \, dx.$$

The first-order condition therefore imposes $\int_{x_{2i+1}^*}^{x_{2i+2}^*} a(x) \, dx$. The second-order condition for a maximum imposes that a is nonnegative at x_{2i+1}^* .

If $a(1) > 0$, then it is easy to check that $r^*(x) = x$ at the top, namely on the interval $[x_{2n}^*, x_{2n+1}^*]$ with x_{2n}^* being the highest zero of the function a and $x_{2n+1}^* = 1$. If the function a admits no zero, it is everywhere positive and hence $r^*(x) = x$ on the whole interval $[0, 1]$.

If $a(1) < 0$, then r^* is constant at the top, namely on the interval $[x_{2n-1}^*, x_{2n}^*]$, with $x_{2n}^* = 1$ and $\int_{x_{2n-1}^*}^1 a(x) dx = 0$. If the integral $\int_y^1 a(x) dx$ remains negative for all y , then r^* is constant and equal to zero on the whole interval $[0, 1]$. \square

H.3 Solving the complete problem

The complete problem consists in maximizing the expected virtual surplus subject to the even (odd) extremities being nonincreasing (nondecreasing). The latter conditions are called hereafter the “monotonicity constraints”.

Applying Lemma H.1 with $a(s_E) = s^v(s_E, \omega_E)$ for any given ω_E , we find that the virtual surplus is zero at candidate even extremities: $s^v(x_{2i}(\omega_E), \omega_E) = 0$ and is negative (positive) at the left (right) of these extremities. In other words, candidate even extremities belong to decreasing parts of the ERT line. Thus, as regards even extremities, the monotonicity constraints are never binding.

Lemma H.1 also implies that the virtual surplus is positive at odd extremities. These extremities therefore lie above the ERT line. By the first-order condition (H.1), the expected virtual surplus is zero on horizontal bunching intervals:

$$\mathbb{E}(s^v|H) = 0, \tag{H.2}$$

where H is a horizontal bunching interval with extremities s_E^{2i+1} and s_E^{2i+2} . The virtual surplus on a bunching interval is first positive, then negative as s_E rises, and its mean on the interval is zero. The segment $[AB]$ on Figure 5b is an example of horizontal bunching interval (in fact the horizontal part of an “L”-shaped bunching set). Unfortunately, the first-order condition (H.2) does not imply that candidate odd extremities $x_{2i+1}(\omega_E)$ are nondecreasing in ω_E : odd extremities might decrease with ω_E in some regions, generating two-dimensional bunching.

H.4 Sufficient conditions

We now check that each of the three conditions mentioned in Proposition D.1 is sufficient for the odd extremities $s_E^{2i+1}(\omega_E)$ to be nondecreasing in ω_E .

We can restrict attention to efficient rivals, $\omega_E \geq \omega_I$.²⁰ We rewrite equation (H.2) as $A(s_E^{2i+1}, \omega_E) = 0$ with

$$\begin{aligned} A(s_E^{2i+1}, \omega_E) &= \int_{s_E^{2i+1}}^{s_E^{2i+2}} s^v(s, \omega_E) f(\omega_E|s) g(s) ds \\ &= \int_{s_E^{2i+1}}^{s_E^{2i+2}} [(\omega_E - \omega_I) f(\omega_E|s) - \beta(1 - F(\omega_E|s))] g(s) ds. \end{aligned}$$

The function A is nonincreasing in s_E^{2i+1} , as the virtual surplus is nonnegative at this point:

$$\frac{\partial A}{\partial s_E^{2i+1}}(s_E^{2i+1}, \omega_E) = -s^v(s_E^{2i+1}, \omega_E) f(\omega_E|s_E^{2i+1}) g(s_E^{2i+1}) \leq 0.$$

Differentiating with respect to ω_E , we get

$$\frac{\partial A}{\partial \omega_E}(s_E^{2i+1}, \omega_E) = \int_{s_E^{2i+1}}^{s_E^{2i+2}} [(\omega_E - \omega_I) f'(\omega_E|s) + f(\omega_E|s) + \beta f(\omega_E|s)] g(s) ds,$$

where we denote by f' the derivative of f in ω_E .

When f is nondecreasing in ω_E , or $f' \geq 0$, we have $\partial A/\partial \omega_E \geq 0$, and hence the odd extremities are nondecreasing in ω_E . We now examine successively the cases where the hazard rate is nondecreasing in ω_E (a weaker condition than $f' \geq 0$) and the elasticity of entry is nondecreasing in ω_E (an even weaker condition).

H.4.1 Assuming that the hazard rate does not decrease in ω_E

We now assume that the hazard rate, $f/(1 - F)$, is nondecreasing in ω_E , which can be expressed as $f' \geq -\varepsilon f/\omega_E$. Using $\omega_E \geq \omega_I$, we find that

$$\begin{aligned} \frac{\partial A}{\partial \omega_E} &\geq \int_{s_E^{2i+1}}^{s_E^{2i+2}} \left[-(\omega_E - \omega_I) \frac{\varepsilon}{\omega_E} + 1 + \beta \right] f(\omega_E|s) g(s) ds \\ &= \int_{s_E^{2i+1}}^{s_E^{2i+2}} \left\{ \varepsilon \left[\frac{\omega_I}{\omega_E} - 1 + \frac{\beta}{\varepsilon} \right] + 1 \right\} f(\omega_E|s) g(s) ds. \end{aligned}$$

²⁰For $\omega_E < \omega_I$, the virtual surplus is negative for all s_E and the solution is $q_E = 0$ for all s_E .

On a horizontal interval H , the variable ω_E is constant, and only the elasticity ε may vary. Hence, the first order condition (H.2) yields: $\mathbb{E}(1 - \beta/\varepsilon | H) = \omega_I/\omega_E$. The right-hand side of the above inequality is equal, up to a positive multiplicative constant, to

$$1 - \text{cov} \left(\varepsilon, 1 - \frac{\beta}{\varepsilon} \middle| H \right).$$

We now look for a sufficient condition for this expression to be nonnegative for any distribution of ε . Noting $m = \mathbb{E}(\varepsilon|H)$ the expectation of ε on H , the condition can be rewritten as

$$\mathbb{E} \left[(\varepsilon - m) \left(1 - \frac{\beta}{\varepsilon} \right) \middle| H \right] \leq 1.$$

The function $(\varepsilon - m)(1 - \beta/\varepsilon)$ is convex in ε . We denote by $[\underline{\varepsilon}, \bar{\varepsilon}]$ the support of the distribution of ε . For given values of $\underline{\varepsilon}$, $\bar{\varepsilon}$ and $m = \mathbb{E}(\varepsilon|H)$, the expectation of this convex function is maximal when the distribution of ε has two mass points at $\underline{\varepsilon}$ and $\bar{\varepsilon}$, associated with the respective weights $\frac{\bar{\varepsilon}-m}{\bar{\varepsilon}-\underline{\varepsilon}}$ and $\frac{m-\underline{\varepsilon}}{\bar{\varepsilon}-\underline{\varepsilon}}$. We thus need to make sure that

$$(\bar{\varepsilon} - m)(\underline{\varepsilon} - m) \left(1 - \frac{\beta}{\underline{\varepsilon}} \right) + (m - \underline{\varepsilon})(\bar{\varepsilon} - m) \left(1 - \frac{\beta}{\bar{\varepsilon}} \right) \leq \bar{\varepsilon} - \underline{\varepsilon},$$

for any $m \in [\underline{\varepsilon}, \bar{\varepsilon}]$. The left-hand side of the above inequality is maximal for $m = (\underline{\varepsilon} + \bar{\varepsilon})/2$. It follows that the inequality holds for all $m \in [\underline{\varepsilon}, \bar{\varepsilon}]$ if and only if the condition (D.1) is satisfied.

H.4.2 Assuming that the elasticity of entry does not decrease in ω_E

We now assume that the $\varepsilon(\omega_E|s_E)$ is nondecreasing in ω_E , as stated in Assumption 1. We have:

$$\frac{\partial \varepsilon(\omega_E|s_E)}{\partial \omega_E}(s_E^{2i+1}, \omega_E) = \frac{\partial}{\partial \omega_E} \left[\frac{\omega_E f(\omega_E|s_E)}{1 - F(\omega_E|s_E)} \right] \geq 0$$

which can be rewritten as $f' \geq -(1 + \varepsilon)f/\omega_E$. Using $\omega_E \geq \omega_I$, we find that

$$\frac{\partial A}{\partial \omega_E} \geq \int_{s_E^{2i+1}}^{s_E^{2i+2}} \left[\frac{\omega_I}{\omega_E} - \varepsilon \left(1 - \frac{\beta}{\varepsilon} - \frac{\omega_I}{\omega_E} \right) \right] f(\omega_E|s)g(s) ds.$$

On a horizontal interval H , the variable ω_E is constant, and only the elasticity ε may vary. Hence, the first order condition (H.2) yields: $\mathbb{E}(1 - \beta/\varepsilon | H) =$

ω_I/ω_E . The right-hand side of the above inequality is equal, up to a positive multiplicative constant, to

$$\mathbb{E} \left(1 - \frac{\beta}{\varepsilon} \middle| H \right) - \text{cov} \left(\varepsilon, 1 - \frac{\beta}{\varepsilon} \middle| H \right).$$

We now look for a sufficient condition for this expression to be nonnegative for any distribution of ε . Noting $m = \mathbb{E}(\varepsilon|H)$ the expectation of ε on H , the condition can be rewritten as

$$\mathbb{E} \left[(\varepsilon - m - 1) \left(1 - \frac{\beta}{\varepsilon} \right) \middle| H \right] \leq 0.$$

The function $(\varepsilon - m - 1)(1 - \beta/\varepsilon)$ is convex in ε . We denote by $[\underline{\varepsilon}, \bar{\varepsilon}]$ the support of the distribution of ε . For given values of $\underline{\varepsilon}$, $\bar{\varepsilon}$ and $m = \mathbb{E}(\varepsilon|H)$, the expectation of this convex function is maximal when the distribution of ε has two mass points at $\underline{\varepsilon}$ and $\bar{\varepsilon}$, associated with the respective weights $\frac{\bar{\varepsilon}-m}{\bar{\varepsilon}-\underline{\varepsilon}}$ and $\frac{m-\underline{\varepsilon}}{\bar{\varepsilon}-\underline{\varepsilon}}$. We thus need to make sure that

$$(\bar{\varepsilon} - m)(\underline{\varepsilon} - m - 1) \left(1 - \frac{\beta}{\underline{\varepsilon}} \right) + (m - \underline{\varepsilon})(\bar{\varepsilon} - m - 1) \left(1 - \frac{\beta}{\bar{\varepsilon}} \right) \leq 0, \quad (\text{H.3})$$

for any $m \in [\underline{\varepsilon}, \bar{\varepsilon}]$. The above function is the sum of two quadratic functions of m . The first is convex with roots $\underline{\varepsilon} - 1$ and $\bar{\varepsilon}$; the second is concave with roots $\underline{\varepsilon}$ and $\bar{\varepsilon} - 1$. Both quadratic functions have zero derivative at $m = (\underline{\varepsilon} + \bar{\varepsilon} - 1)/2$. The sum of the two functions is concave as $\underline{\varepsilon} < \bar{\varepsilon}$.

When $\bar{\varepsilon} \leq \underline{\varepsilon} + 1$, the concave quadratic function is negative on the interval $[\underline{\varepsilon}, \bar{\varepsilon}]$, and hence the inequality (H.3) holds on that interval. When $\bar{\varepsilon} > \underline{\varepsilon} + 1$, we need to make sure that the maximum value of the concave quadratic function is lower than the minimum value of the convex quadratic function. This is the case if and only if

$$\left(1 - \frac{\beta}{\bar{\varepsilon}} \right) (\Delta\varepsilon - 1)^2 \leq \left(1 - \frac{\beta}{\underline{\varepsilon}} \right) (\Delta\varepsilon + 1)^2.$$

which is equivalent to (D.2).

H.5 Two-dimensional bunching

When none of the above sufficient conditions holds, it may happen that solving the problem separately for each ω_E yields odd extremities (left extremities

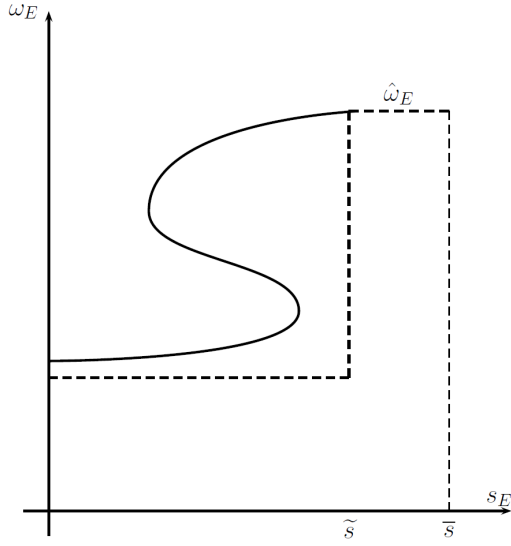


Figure 10a: ERT line (dashed). Non-monotonic odd extremities (solid line)

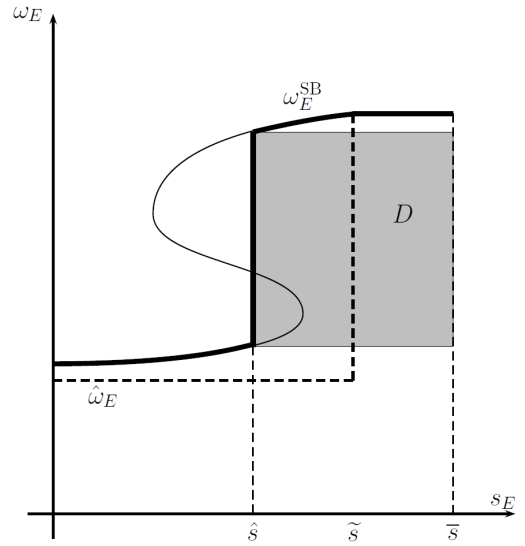


Figure 10b: Two-dimensional bunching area: $q_E = \hat{s}$ on D .

of horizontal bunching segments) that are non-monotonic with ω_E , as represented on Figure 10a. Such a line does not define a boundary function $\Psi(s_E)$. This means that the monotonicity constraints are binding and that the optimal boundary line has an increasing vertical portion, generating a two-dimensional pooling area. An example of such an area is the shaded region D pictured on Figure 10b, on which the quantity is constant. The value of the constant (\hat{s} on the picture) is determined by the first-order condition

$$\mathbb{E}(s^v|D) = 0.$$

This example has been constructed by assuming that (i) ω_E follows a Pareto distribution conditionally on s_E , for all s_E ; (ii) the elasticity of entry takes two values, $\underline{\varepsilon}$ and $\bar{\varepsilon}$, with a large difference $\bar{\varepsilon} - \underline{\varepsilon}$; (iii) small rivals are very sensitive to the competitive pressure placed by the incumbent (their elasticity is $\bar{\varepsilon}$) and large rivals are much less sensitive (their elasticity is $\underline{\varepsilon}$). Hence the increasing ERT line with two pieces.

I Finite disposal costs

Assume that $\Psi(s_E) \leq v_I + \gamma$, and define the quantity function by (G.1), the surplus gain from trade between the buyer and the rival by

$$\Delta S_{BE}(s_E, \omega_E) = \int_{\underline{\omega}_E}^{\omega_E} q_E(x, s_E) dx,$$

and the tariff T by (G.2). Differentiating the latter equation with respect to ω_E below the boundary line, a region where q_E increases with ω_E , yields

$$T'(q) \frac{\partial q}{\partial \omega_E} = (v_I - \omega_E) \frac{\partial q}{\partial \omega_E} - q + q,$$

and hence $T'(q) = v_I - \omega_E \geq -\gamma$. Differentiating (G.2) with respect to s_E above the boundary line but below $v_I + \gamma$, a region where $q_E = s_E$, yields

$$T'(s_E) = v_I - \omega_E + \frac{\partial \Delta S_{BE}}{\partial s_E} \geq v_I - \omega_E \geq -\gamma,$$

because ΔS_{BE} is nondecreasing in s_E .