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Leaving the Door Ajar : Nonlinear Pricing by a Dominant Firm

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Abstract

An incumbent firm and a buyer agree on a price-quantity schedule before the buyer negotiates with a rival firm. The rival's efficiency and the share of the buyer's demand he can address are unknown when the schedule is chosen. Incomplete information yields inefficient exclusion. We link the slope and the curvature of the optimal tariff to the distribution of the uncertainty, and investigate whether foreclosure is complete or partial. When the buyer's disposal costs are finite, she might buy more than needed with the sole purpose of qualifying for rebates, which limits the extent of inefficient exclusion. Conditional tariffs make it possible for the incumbent to overcome the opportunism problem and to exclude very efficient competitors.

1 Introduction

Nonlinear pricing, a ubiquitous business conduct, keeps attracting the attention of competition agencies.[1] Antitrust enforcers are concerned that quantitative rebates granted by firms with strong market power might discourage buyers from switching part of their requirements towards efficient competitors (for fear of losing the rebates). As a result, efficient competitors might be driven out of the market or marginalized, and, in any case, prevented from selling the efficient quantity.

The nonlinear pricing literature has investigated second-order price discrimination under monopoly and oligopoly, see Wilson (1993). The monopolist's pricing problem was first studied when consumers differ through a single unobserved characteristic (e.g. Mussa and Rosen (1978) and Maskin and Riley (1984), then extended in multidimensional

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[1] Among recent antitrust cases, Virgin/British Airways (Commission Decision 2000/74/EC of 14 July 1999), and in the U.S. Virgin v. British Airways, 69 F. Supp. 2d 571, 581, 582 (S.D.N.Y. 1999) as well as 257 F.3d 256 (2nd Cir. 2001), Concord Boat (United States Court of Appeals for the 8th Circuit - 207 F.3d 1039, 8th Cir. 2000), Michelin II (Commission decision of 20 June 2001 COMP/E-2/36.041/PO), Lepage's/3M (324 F.3d 141, 2003), Prokent-Tomra (COMP/E-1/38.113, 2006), and Intel (COMP/C-3 /37.990, 2009).

settings (Armstrong (1996), Rochet and Choné (1998) and Armstrong (1999)). Nonlinear pricing under oligopoly is studied in Armstrong and Vickers (2001), Martimort and Stole (2009), and Armstrong and Vickers (2010). The literature on competitive price discrimination has mainly explored simultaneous competition and often focused on symmetric equilibria.

In contrast, the present article considers a dominant firm in a market where a smaller firm is present and may challenge its position, at least to a certain extent. We adopt the basic incumbency setting with three players: an incumbent, a strategic competitor (or a competitive fringe), and a buyer.[2] We assume that the dominant firm and a large buyer agree on a price-quantity schedule prior to any negotiation with the competitor. As observed by Aghion and Bolton (1987) p.389, in the context of exclusive contracts, "when a buyer and a seller sign a contract, they have a monopoly power over the entrant. They can jointly determine what fee the entrant must pay in order to trade with the buyer." This insight applies under both complete and incomplete information. Nonlinear pricing by a dominant firm under complete information is now well understood.[3]

Our focus is on incomplete information.[4] We consider two dimensions of uncertainty. First, as in Aghion and Bolton (1987), the rival's cost –or more generally, the surplus he creates with the buyer– is unknown to the buyer and the incumbent when they agree on a price schedule.

Second, we assume that only a fraction of the buyer's requirements can be supplied by the competitor. As noticed by competition authorities, it is often unrealistic to assume that a buyer can shift all of her requirements within a relevant time period from the dominant supplier to a competitor. This can be due to demand-side or supply-side considerations. It may be the case that competitors are capacity constrained and cannot serve all of the demand of large customers. It may also be the case that the incumbent's product is a "must-stock" for retailers because only a fraction of final consumers are ready to experiment with competing products (regardless of their price). In both cases, within a relevant time horizon, the rival firm can address only a fraction of the buyer's demand, which constitutes the maximum scale of entry. This fraction constitutes the second characteristic of the rival that is unknown to the buyer-incumbent pair: ex ante, the size of the "captive market", and consequently that of the "contestable market," are uncertain.

The last important ingredient of our framework is the existence of disposal costs.

[2]Such a framework has been used to model incumbency and/or dominance at least since Spence (1977, 1979) and Dixit (1979, 1980).

[3]Marx and Shaffer (1999) look at two-part tariffs with a focus on below-cost pricing; Marx and Shaffer (2004) study how equilibrium is affected when certain classes of tariffs are forbidden; Marx and Shaffer (2007) focus on the order of negotiation (Is it better for the buyer to negotiate first with the incumbent or with the entrant?); Marx and Shaffer (2010) explain how bargaining powers affect profits and when break-up fees are used.

[4]Incomplete information differs from asymmetric information. For instance, in Majumdar and Shaffer (2009), a dominant firm resorts to nonlinear pricing to screen a buyer who is informed about the size of demand and who also sells a good provided by a competitive fringe –a situation with asymmetric information.

We allow the buyer to purchase more than her requirements and assume that she can dispose of excess units at some cost. This gives rise to a problem of buyer opportunism. The buyer and the incumbent agree on a price-quantity schedule that places competitive pressure on the rival and forces him to sell at a low price. We show that this "rent-shifting" strategy involves marginal prices below marginal costs, and may even involve negative marginal prices. A negative marginal price allows the buyer to extract rents from the rival, but also gives her an ex post incentive to buy more than she needs from the incumbent. While this opportunistic behavior is anticipated ex ante, it constrains the choice of the price schedule by the buyer-incumbent pair. The seriousness of the opportunism problem depends on the magnitude of disposal costs: buyer opportunism is maximal under free disposal and does not exist when disposal costs are infinite.

The contribution of our analysis is threefold. First, on the methodological side, we solve a multidimensional screening problem where the number of instruments is smaller than the dimension of unobserved heterogeneity, and we do so for general distributions of heterogeneity. Second, and more importantly, the generality of the analysis allows us to assess the robustness of the Aghion and Bolton framework and to reveal new properties, such as the curvature of optimal price schedules and the presence of partial foreclosure. Third, we explain how conditioning the tariff on the quantity purchased from the competing supplier (when feasible) makes it possible to eliminate buyer opportunism, and thus achieve the same outcome as if disposal costs were infinite. We now explain each of these contributions in more detail.

First, we contribute to the nonlinear pricing literature. In our framework, the competitor's type has two components: the surplus he creates with the buyer and the maximum scale of entry. However, to screen out the competitor's types, the incumbent has only one instrument, namely a price-quantity schedule. This configuration, which generates extensive bunching, has received little attention.[5] Here, the structure of the model makes it possible to characterize the set of implementable allocations, and to construct the solution with few restrictive assumptions on the distribution of the uncertainty. As explained below, the equilibrium pattern of the bunching regions reflects the barriers to entry and expansion created by the optimal tariff.

Second we link the curvature of optimal price schedules and the form of inefficient exclusion (partial versus full foreclosure) to two structural parameters of the model: the rival's bargaining power and the elasticity of entry. The former parameter is zero in the case of a competitive fringe and is, in general, positive in the case of a strategic competitor. The latter parameter expresses how entry at a given scale is sensitive to the competitive pressure exerted by the incumbent. It is a key statistic summarizing the two-dimensional distribution of the rival's characteristics. Our findings can be summarized as follows.

When the size of the contestable demand is known to the buyer and the incumbent

[5] A notable exception is Laffont, Maskin, and Rochet (1987) who solve an example with uniform distributions.

(only the rival surplus is unknown), the pricing problem involves a standard tradeoff between rent extraction and efficiency, and is a mere reformulation of the Aghion-Bolton analysis. Some efficient competitors are foreclosed, and the extent of inefficient exclusion increases with the rival's bargaining power vis-à-vis the buyer, and decreases with the elasticity of entry. The optimal schedule is a two-part tariff and inefficient exclusion arises in the form of full foreclosure only; no partial foreclosure is observed. These results readily extend under two-dimensional uncertainty, provided that the two unknown parameters are statistically independent or, equivalently, that the elasticity of entry remains constant with the size of the contestable market. We next turn to cases where the elasticity of entry varies with the size of the contestable market.

When the elasticity of entry increases with the size of the contestable share of the demand, the optimal policy of the buyer-incumbent pair is to reduce the competitive pressure exerted on the competitor as the scale of entry increases, which cannot be achieved with two-part tariffs. Optimal tariffs are shown to be concave for high quantities. Here again, inefficient exclusion arises, in the form of full foreclosure only. Solving the efficiency-rent tradeoff à la Aghion-Bolton separately for any given level of the contestable demand (i.e. solving the "relaxed problem") yields the optimal tariff.

When the elasticity of entry decreases in, or is non monotonic with, the contestable demand, the solution to the relaxed problem is not incentive-compatible, as a rival with a large contestable demand would mimic a smaller rival at the relaxed allocation. The incentive compatibility constraints translate into convex parts of the tariff, and into partial foreclosure at the optimum. Some efficient competitors sell a positive quantity but are prevented from achieving the maximum scale of entry and serving all of the contestable share of demand. When the elasticity of entry first decreases then increases, as the maximum scale of entry rises, the buyer and the incumbent want to be soft with competitors with small and large contestable markets, and to be aggressive with competitors with intermediate contestable markets. This tension generates highly nonlinear tariffs that induce competitors with very different types to choose the same quantity, as is the case with so-called "retroactive rebates" challenged by European competition agencies in recent cases.

Finally, we contribute to the literature on market-share rebates.[6] Specifically, we allow the buyer and the incumbent to condition the price schedule on the number of units purchased from the rival, and we show that this instrument allows them to overcome the buyer opportunism problem. When the price-quantity schedule depends only on the number of units purchased from the incumbent, the presence of finite disposal cost prevents the exclusion of very efficient competitors, because excluding them would require negative marginal prices and the incumbent must account for ex post buyer opportunism. In contrast, conditional tariffs make it possible to exert competitive pressure on the rival

[6] Inderst and Shaffer (2010) assume complete information and study a setting with a dominant firm, a competitive fringe and two retailers. They show that market-share rebates are used by the dominant firm to dampen (intra- and inter-brand) competition. Calzolari and Denicolo (2009) address the issue in a duopoly setting (simultaneous game, symmetric firms).

without resorting to negative marginal prices, i.e. without subsidizing units sold by the incumbent. Competitive pressure can instead come from the implicit price of marginal units sold by the competitor. Thus, conditional rebates, when feasible, make it possible for the buyer and the incumbent to exclude very efficient competitors as under infinite disposal costs.

The article is organized as follows. For ease of exposition, we assume first that disposal costs are infinite, thus abstracting away from the issue of buyer opportunism. Section 2 introduces the model. Section 3 explains how the negotiation between the buyer and the rival, which takes place under complete information, is affected by the incumbent's price schedule. Section 4 focuses on the negotiation between the buyer and the incumbent, which takes place under incomplete information. It introduces the notion of virtual surplus and of elasticity of entry, and presents the construction of the optimal price schedule. This section also relates the shape of the optimal price schedule to the primitives of the model. Section 5 explains the buyer opportunism problem under finite disposal costs and shows how a market-share tariff could overcome it. In Section 6 we discuss policy implications of our findings.

2 The model

A buyer, B, may purchase from an incumbent, dominant firm, I, and from a smaller competitor, E.[7] The firms are asymmetric in two ways: the incumbent is first to negotiate with the buyer; the incumbent can serve all the demand while the competitor can serve only a fraction of it. We call this fraction the "contestable" part of the demand.

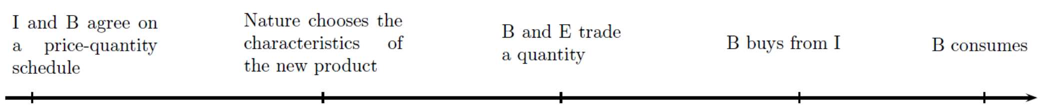

Figure 1: Timing of the game

The timing, sketched in Figure 1, reflects the incumbency advantage of the dominant firm. It is a four stage game which unfolds as follows. First, the buyer and the incumbent negotiate a price-quantity schedule. Formally, if the buyer eventually purchases q_I units from the incumbent, she will pay $T(q_I)$. In Section 5, we solve the game when the buyer and the incumbent can condition the tariff on the quantity, q_E, supplied from the competitor, i.e. they use a tariff of the form $T(q_E, q_I)$. The characteristics of the incumbent's good are common knowledge: its constant marginal cost of production is c_I and the buyer's gross benefit per unit is v_I. Next, the buyer and the competitor observe the characteristics of the new product: its marginal cost of production c_E, the size of s_E of the contestable demand and the gross (per unit) benefit, v_E, for the buyer. Then, the

[7]We sometimes refer to the rival firm as the "entrant", even though the game under study does not involve a genuine entry decision.

buyer and the competitor, both knowing the terms of the agreement between the buyer and the incumbent, agree on a price and a quantity.[8] This negotiation takes place under complete information and is assumed to be efficient. Finally, the buyer purchases from the incumbent.

We study how nonlinear prices affect the buyer's incentives to split her purchases between the dominant supplier and the smaller competitor. To this aim, we set aside the standard motives for nonlinear pricing, based on uncertainty or asymmetric information on the buyer's demand. Accordingly, we assume that the buyer's demand is known ex ante. Moreover, we assume, for simplicity, that the buyer's demand is bounded, and we normalize her requirements to one.

The assumption that the rival firm can address at most a fraction, s_E, of the buyer's demand embodies two interpretations: a supply-side variant in which the competitor has capacity s_E and a demand-side variant where the buyer does not value units of good E in excess of s_E. In both cases, the buyer never purchases more than s_E from the competitor: $q_E \leq s_E$.

Given v_E and v_I the buyer's gross benefit per unit of goods E and I, and the quantities $q_E \leq s_E$ and q_I purchased respectively from the competitor and the incumbent, the buyer's gross profit is:

$$V(q_E, q_I) = \begin{cases} v_E q_E + v_I q_I, & \text{if } q_E + q_I \leq 1 \\ -\infty & \text{otherwise.} \end{cases} \tag{1}$$

The above specification assumes infinitely large disposal costs: failing to consume all of the purchased units is infinitely costly. It follows that all the purchased units are indeed consumed and that the buyer does not purchase more than her requirements. This assumption is maintained in Section 3 and 4, and relaxed in Section 5, where finite disposal costs are introduced.

We note $\omega_E = v_E - c_E \geq 0$ the unit surplus generated by good E, and $\omega_I = v_I - c_I > 0$ the unitary surplus of good I. At the time of agreeing on the price schedule, the size of the contestable demand, s_E, and the surplus per unit of good E, ω_E, are uncertain. We denote by $[\underline{s}_E, \bar{s}_E]$ and by $[\underline{\omega}_E, \bar{\omega}_E]$ the supports of the random variables s_E and ω_E.

Assumption 1. *The rival may be more or less efficient than the incumbent:* $\underline{\omega}_E < \omega_I < \bar{\omega}_E$.

The cumulative distribution function of s_E, denoted by G, is assumed to admit a positive and continuous density function g on $[\underline{s}_E, \bar{s}_E]$. The distribution of ω_E conditional on s_E is denoted by $F(.|s_E)$ and is assumed to admit a positive and continuous density function $f(.|s_E)$ on $[\underline{\omega}_E, \bar{\omega}_E]$.

[8] We assume that the buyer and the incumbent cannot renegotiate their agreement once uncertainty is resolved. Otherwise they would agree on a tariff under complete information and appropriate all the surplus (see the end of this section). The contribution of the current paper is, on the contrary, to study the form of the price schedule negotiated under incomplete information.

Efficiency benchmark. The total surplus is $W(q_E, q_I) = \omega_E q_E + \omega_I q_I$ if $q_E + q_I \leq 1$ and $-\infty$ otherwise. The first best allocation maximizes W under the constraint $q_E \leq s_E$. Efficiency requires $q_E + q_I = 1$, because W increases with both quantities as long $q_E + q_I \leq 1$. Hence, at the the first best, the quantity purchased from the competitor satisfies

$$\omega_I + \max_{q_E \leq s_E} (\omega_E - \omega_I) q_E, \tag{2}$$

and hence is given by

$$q_E^*(s_E, \omega_E) = \begin{cases} s_E & \text{if } \omega_E \geq \omega_I \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

Entry, if efficient, should occur at maximum scale. Hence the maximal value of total surplus is $\omega_E s_E + \omega_I(1 - s_E)$ when $\omega_E \geq \omega_I$, and ω_I when $\omega_I \geq \omega_E$.

Second best. The negotiation between the buyer and the competitor, studied in Section 3, takes place under complete information and is assumed to be efficient. The buyer and the competitor maximize their joint surplus, which they share according to their outside options and relative bargaining power. Ex ante, the buyer and the incumbent design the price schedule to maximize their expected joint surplus, equal to the total surplus minus the profit left to the competitor, denoted by Π_E:

$$\mathbb{E}\Pi_{BI} = \mathbb{E}\left\{W(q_E, q_I) - \Pi_E\right\}. \tag{4}$$

The sharing of the expected joint surplus between the buyer and the incumbent, and hence the respective bargaining power of each party, play no role in the following analysis.

Complete information. Suppose the competitor is efficient and the buyer and the incumbent know the surplus per unit of good E, $\omega_E > \omega_I$. Then they agree on a two-part tariff with slope slightly above $v_I - \omega_E$, thus offering a surplus slightly below ω_E for each unit of good I. As units of good E create a slightly higher surplus and the negotiation between the buyer and the competitor is efficient, the buyer purchases all contestable units from the competitor. The incumbent sells the remaining units: the allocation is socially efficient. To sell units to the buyer, the competitor must match the incumbent's offer, and thus is left with negligible profit. The buyer and the incumbent therefore appropriate the entire surplus, $\omega_E s_E + \omega_I(1 - s_E)$.

As the slope $v_I - \omega_E$ does not depend on s_E, the above analysis holds when the buyer and the incumbent do not know the size of the contestable market.[9] The complete information environment is studied with more general demand functions in Marx and Shaffer (1999) and Marx and Shaffer (2004).

[9]The fixed part of the tariff determines the sharing of the surplus between the buyer and the incumbent. When s_E in unknown, the same is true for the expected surplus, $\omega_E \mathbb{E}(s_E) + \omega_I[1 - \mathbb{E}(s_E)]$.

3 Negotiation between the buyer and the competitor

In subsection 3.1, we describe the negotiation between the buyer and the competitor, which takes place under complete information: the parties maximize their joint surplus, knowing the incumbent's price-quantity schedule T, and share this surplus according to their relative bargaining power and outside options. In subsection 3.2, we provide a number of examples showing how the quantity purchased from the competitor depends on the shape of the tariff. Finally, in Subsection 3.3, we formally characterize the set of all implementable allocations.

3.1 Maximization of the joint surplus

After having purchased q_E units from the competitor, the buyer chooses q_I to solve

$$U_B(q_E) = \max_{q_I} V(q_E, q_I) - T(q_I). \tag{5}$$

Anticipating the above decision regarding q_I, the buyer and the competitor choose q_E to maximize their joint surplus

$$S_{BE}(c_E, s_E, v_E) = \max_{q_E \le s_E} U_B(q_E) - c_E q_E, \tag{6}$$

The price schedule $T(.)$ is key in the definition of this surplus as $U_B(q_E)$ depends on $T(.)$. The buyer and the competitor share S_{BE} according to their respective bargaining power and outside options. The competitor's outside option is normalized to zero. As to the buyer, she may source exclusively from the incumbent, so her outside option is $U_B(0)$. It follows that the surplus created by the relationship between B and E is given by

$$\Delta S_{BE}(c_E, s_E, v_E) = S_{BE}(c_E, s_E, v_E) - U_B(0).$$

Denoting by $\beta \in (0, 1)$ the competitor's bargaining power vis-à-vis the buyer, the competitor gets Π_E and the buyer gets Π_B given by

$$\begin{array}{lcccc} \Pi_E = & 0 & + & \beta & \Delta S_{BE} \\ \Pi_B = & U_B(0) & + & (1-\beta) & \Delta S_{BE}. \end{array}$$

If $\beta = 0$, the competitor has no bargaining power and may be seen as a competitive fringe from which the buyer can purchase any quantity at price c_E. On the contrary, the case $\beta = 1$ happens when the competitor has all the bargaining power vis-à-vis the buyer.

Now we observe that the quantity purchased from the incumbent, solution to (5), is ex post efficient, i.e. maximizes the joint surplus of the buyer-incumbent pair given q_E. In other words, the total quantity purchased by the buyer exactly meets her demand: $q_E + q_I = 1$. On the one hand, the buyer does not purchase more than her total requirements, because disposal costs are assumed to be infinite; hence the solution to problem (5) satisfies $q_I \le 1 - q_E$ for all q_E. On the other hand, buying less than $1 - q_E$

from the incumbent would destroy surplus as $v_I > c_I$. Lemma A.1 in appendix formally shows that the buyer and the incumbent, when choosing the tariff T, have both the ability and the incentive to make sure that, for any q_E, the buyer will purchase at least $1-q_E$ from the incumbent after having purchased q_E from the competitor. We may thus conclude that $q_E + q_I = 1$ at the second-best optimum.

Replacing q_I with $1 - q_E$ in (5) and noting that the joint surplus of the buyer and the competitor depends on c_E and v_E only through ω_E, we can write

$$S_{BE}(s_E, \omega_E) = v_I + \max_{q_E \le s_E} (\omega_E - v_I)q_E - T(1 - q_E).$$

As the buyer's outside option is $U_B(0) = v_I - T(1)$, the surplus from the trade between the buyer and the competitor is

$$\Delta S_{BE}(s_E, \omega_E) = \max_{q_E \le s_E} (\omega_E - v_I)q_E - T(1 - q_E) + T(1). \tag{7}$$

For any s_E, the function $\Delta S_E(s_E, .)$ is the upper bound of a family of affine functions of ω_E, and hence is convex in ω_E. It follows that $\Delta S_E(s_E, \omega_E)$ is differentiable with respect to ω_E, except possibly at countably many points. By the envelope theorem, its derivative with respect to ω_E is $q_E(s_E, \omega_E)$, solution to (7).[10] Hence, the function $q_E(s_E, \omega_E)$ is nondecreasing in ω_E. We have:

$$\Delta S_{BE}(s_E, \omega_E) = \int_{\underline{\omega}_E}^{\omega_E} q_E(s_E, x)\,\mathrm{d}x. \tag{8}$$

Moreover, it follows from (7) and (8) that q_E and ΔS_{BE} are nondecreasing in s_E. The buyer purchases more units from the competitor as the surplus per competitor's unit, ω_E, and the size of the contestable demand, s_E, rise.

3.2 Examples: Concave, linear, convex tariffs

The problem of the buyer-competitor pair's is not necessarily concave. Specifically, the objective in (7) is convex (concave) if and only if T is concave (convex). In any case, the price schedule is relevant only in the interval $[1 - \bar{s}_E, 1]$, because the competitor cannot sell more than $\bar{s}_E$. This section provides three illustrative examples.

We consider first the case where the tariff T is concave on the relevant range, $[1-\bar{s}_E, 1]$, and hence the objective in (7) is globally convex. The maximum is reached either at $q_E = 0$ or at $q_E = s_E$. The buyer purchases $q_E = s_E$ from the competitor if and only if

$$(\omega_E - v_I)s_E - T(1 - s_E) + T(1) \ge 0$$

[10]For any s_E, the set of solutions to problem (7) is included in the subgradient of the convex function $\Delta S_{BE}(s_E, .)$. At points where $\Delta S_{BE}(s_E, .)$ is differentiable, the subgradient consists of a single point, namely the derivative of ΔS_{BE} with respect to ω_E: the solution of (7) is unique. At points where $\Delta S_{BE}(s_E, .)$ has a convex kink, the subgradient is an interval, see Rockafellar (1997).

or $\omega_E - v_I \geq p^{\mathrm{e}}(s_E)$, where $p^{\mathrm{e}}(s_E)$ is the average price of the last s_E units sold by the incumbent:

$$p^{\mathrm{e}}(s_E) = \frac{T(1) - T(1 - s_E)}{s_E}. \tag{9}$$

Supplying all contestable units from the competitor ($q_E = s_E$) is efficient for the buyer-competitor's pair if and only if the joint surplus thus created, $\omega_E s_E$, exceeds the net surplus foregone by not purchasing the corresponding units from the incumbent, $(v_I - p^{\mathrm{e}}(s_E))s_E$. Geometrically, the effective price $p^{\mathrm{e}}(q_E)$ is the slope of the chord that connects the points $(1, T(1))$ and $(1 - q_E, T(1 - q_E))$, see the left panel of Figure 2.

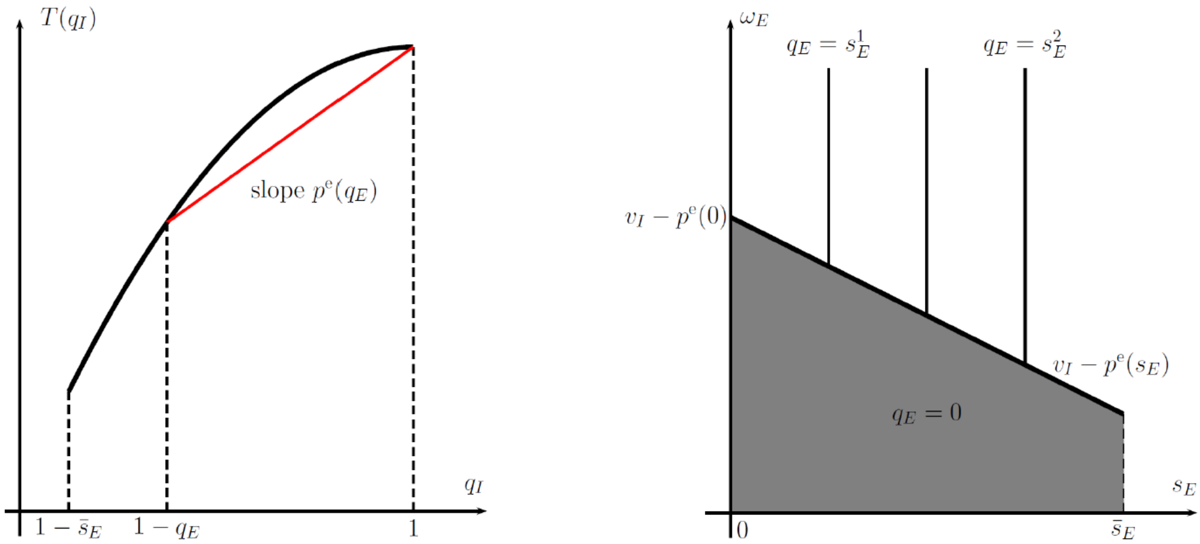

Figure 2: The buyer and the competitor choose q_E under a concave tariff

The right panel of Figure 2 represents the curve with equation $\omega_E = v_I - p^{\mathrm{e}}(s_E)$ in the (s_E, ω_E)-plan. This curve is decreasing by concavity of the tariff. Below the curve (shaded area), the competitor is inactive, $q_E = 0$. Above the curve, the buyer supplies all contestable units from the competitor, $q_E = s_E$, and hence the quantity isolines, i.e. the sets of types for which the quantity is constant, are vertical.

The above analysis holds in particular when the tariff is affine or, equivalently, when the incumbent's effective price $p^{\mathrm{e}}(q_E)$ is constant. This case is represented on Figure 3. Setting the effective price at p^{e} amounts to offering the surplus $v_I - p^{\mathrm{e}}$ per unit of good I. To serve the buyer, the competitor has to match this offer. Hence, competitors with ω_E above (below) $v_I - p^{\mathrm{e}}$ serve all of the contestable demand (are inactive). The efficient quantity, q_E^*, obtains when p^{e} is constant and equal to c_I.

When the price schedule T is strictly convex, the program (7) is concave and has a unique solution, which may or may not be interior. For ω_E higher than $v_I - T'(1 - s_E)$, the solution of (7) is $q_E = s_E$: the competitor serves all of the contestable demand. For ω_E lower than $v_I - T'(1)$, the solution is $q_E = 0$: the competitor is inactive. For ω_E between these two values, the solution is interior, and is given by the first-order

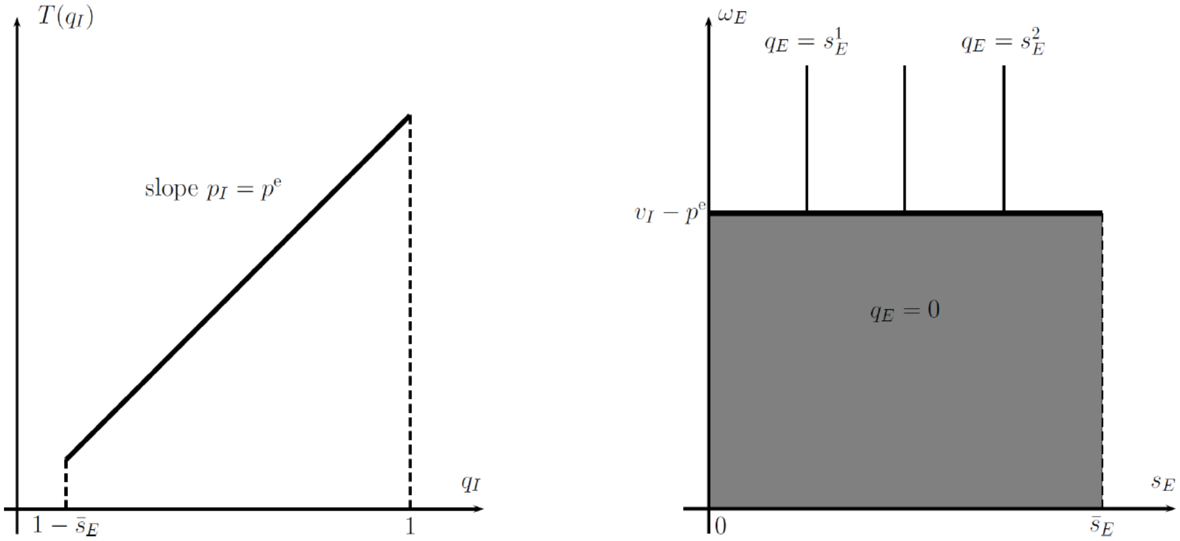

Figure 3: The buyer and the competitor choose q_E under a linear tariff

condition $\omega_E - v_I + T'(1 - s_E) = 0$: the competitor is active, but serves less than the contestable demand. The right panel of Figure 4 represents in the (s_E, ω_E)-plan the curve with equation $\omega_E = v_I - T'(1 - s_E)$, which is increasing by convexity of the tariff. The quantity isolines are "L"-shaped, with the vertical part above the curve and the horizontal part below.

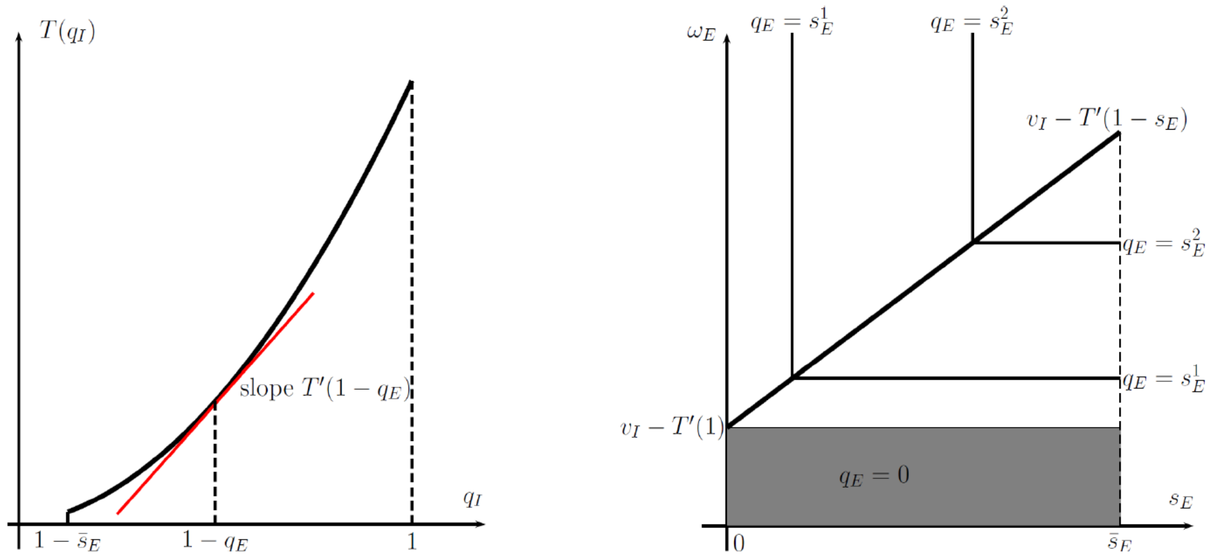

Figure 4: The buyer and the competitor choose q_E under a convex tariff

3.3 Implementable quantity functions

The buyer and the competitor negotiate under complete information and choose a quantity q_E that depends on the competitor's characteristics, (s_E, ω_E). A quantity function $q_E(s_E, \omega_E)$ from $[\underline{s}_E, \bar{s}_E] \times [\underline{\omega}_E, \bar{\omega}_E]$ to $[0, 1]$ is implementable if and only if there exists a tariff T such that q_E is solution to (7) for all (s_E, ω_E).

In this section, we show that any implementable quantity function $q_E(s_E, \omega_E)$ may be represented by a boundary line in the (s_E, ω_E)-plan such that $q_E = s_E$ above the boundary and q_E does not depend on s_E below the boundary. Such boundary lines have equations of the form $\omega_E = \Psi(s_E)$, where Ψ is called a boundary function. We demonstrate below the existence of a one-to-one map between quantity functions $q_E(s_E, \omega_E)$ and boundary functions $\Psi(s_E)$. To solve the two-dimensional problem, it turns out to be convenient to work with boundary functions rather than directly with quantity functions.

As q_E is nondecreasing in ω_E, there exists, for any $s_E > 0$, a threshold $\Psi(s_E)$ such that the buyer supplies all contestable units from the competitor, $q_E(s_E, \omega_E) = s_E$, if and only if $\omega_E > \Psi(s_E)$. We define the *boundary function* $\Psi(s_E)$ associated to the quantity function $q_E(s_E, \omega_E)$ by

$$\Psi(s_E) = \inf\{x \in [\underline{\omega}_E, \bar{\omega}_E] \mid q_E(x, s_E) = s_E\},$$

with the convention $\Psi(s_E) = \bar{\omega}_E$ when the above set is empty. Because the quantity function $q_E(s_E, \omega_E)$ is nondecreasing in s_E and constant below the boundary, we have:

$$q_E(s_E, \omega_E) = \begin{cases} \min\{\ x \leq s_E \mid \Psi(y) \geq \omega_E \text{ for all } y \in [x, s_E]\} & \text{if } \Psi(s_E) > \omega_E, \\ s_E & \text{if } \Psi(s_E) \leq \omega_E. \end{cases} \tag{10}$$

For type A (resp. B) on Figure 5, we have $\Psi(s_E) < \omega_E$ (resp. $\Psi(s_E) > \omega_E$) and the solution of the problem (7) is unique and equal to s_E^2. In contrast, type C is indifferent between s_E^2 and s_E^3 and, by convention, is assumed to choose s_E^3. In other words, when (7) has multiple solutions, equation (10) selects the highest.

The quantity q_E is continuous (discontinuous) when crossing increasing (decreasing) parts of the boundary $\omega_E = \Psi(s_E)$. Alternatively put, the constraint $q_E \leq s_E$ in problem (7) is binding (slack) on decreasing (nondecreasing) parts of the boundary. In Appendix B.1, we explain how to recover the price schedule T from the boundary function Ψ, thus proving the sufficient part, and thus prove next result.

Lemma 1. *A quantity function $q_E(.,.)$ is implementable if and only if there exists a boundary function $\Psi(.)$ defined on $[0, 1]$ such that (10) holds.*

Bunching areas and foreclosure (partial versus complete) The bunching sets, i.e. the sets on which the quantity $q_E(s_E, \omega_E)$ is constant, can be one- or two-dimensional. As shown on Figure 5, one-dimensional bunching sets can be of two types: (i) vertical lines above points on the boundary line where that line decreases; (ii) "L"-shaped unions of vertical lines above and horizontal lines at the right of points where the boundary

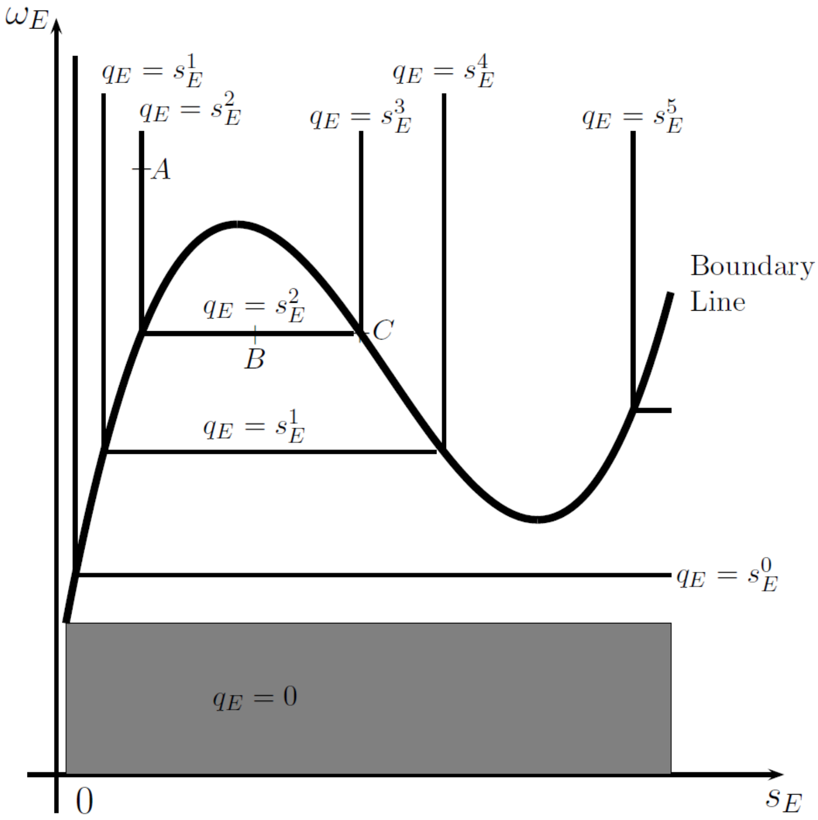

Figure 5: Implementable quantity function (isolines)

line increases. There always exists a two-dimensional bunching area, namely the region where the quantity is zero. Other two-dimensional bunching sets exist in regions where the boundary line increases and has a vertical part, see e.g. Figures 13a and 13b.

Increasing parts of the boundary function thus translate into horizontal bunching segments or two-dimensional bunching areas, and hence into partial foreclosure: $0 < q_E(s_E, \omega_E) < s_E$ for some types located below the boundary. In such regions, the constraint $q_E \leq s_E$ is slack: increasing s_E does not allow the competitor to enter at a larger scale and q_E does not depend on s_E.

Shape of the boundary line and curvature of the tariff As formally stated in Lemma B.1, flat parts of the boundary line correspond to linear parts of the tariff (see Figure 3) and increasing parts of the boundary line correspond to convex parts of the tariff (see Figure 4). In both cases, the constraint $q_E \leq s_E$ in the buyer-competitor pair's problem (7) is not binding.

In contrast, the curvature of the tariff may change along decreasing parts of the boundary: the tariff is concave near local maxima of the boundary line and convex near local minima, see equation (B.3) in appendix and Figures 8a, 8b, 9a, and 9b. Local maxima of the boundary line thus correspond to inflection points of the tariff. An example is the point $A3$ on Figures 12a and 12b.

Finally, it is worthwhile noticing that upward and downward discontinuities in the boundary line have different interpretations in terms of price schedule. Upward discontinuities of the boundary line correspond to convex kinks in the tariff, see Figures 13a and 13b. Downward discontinuities of the boundary correspond to upward discontinuities of the tariff, see Figures 14a and 14b.

4 Designing the price schedule

The buyer and the incumbent design the price schedule so as to maximize their joint surplus, given by (4). Using $q_I = 1 - q_E$ and replacing Π_E with the value derived in Section 3.1, we can rewrite their common objective as

$$\mathbb{E}\Pi_{BI} = \omega_I + \mathbb{E}\left\{(\omega_E - \omega_I)q_E - \beta\Delta S_{BE}\right\}. \tag{11}$$

To solve the buyer-incumbent pair's problem, we rely on the duality, exposed in Section 3, between the incumbent's price schedule, T, and the quantity purchased from the entrant, q_E. We look for the quantity function q_E, then we recover the price-quantity schedule T from this function.[11]

Section 4.1 expresses the problem in terms of the quantity purchased from the incumbent, q_E, introducing the notion of virtual surplus. The maximization of the virtual surplus, ignoring incentive compatibility, gives rises to a relaxed problem. Section 4.2 examines situations where the solution of the relaxed problem is implementable. Section 4.3 solves the complete problem in the general case. Sections 4.4 and 4.5 show that the optimal price-quantity schedule can have convex parts and hence that efficient competitors may be *partially* foreclosed in equilibrium.

4.1 Virtual surplus and elasticity of entry

Expanding (11), we write the joint expected surplus of the buyer-incumbent pair as:

$$\mathbb{E}\Pi_{BI} = \omega_I + \int_{s_E}\int_{\underline{\omega}_E}^{\bar{\omega}_E} \left\{(\omega_E - \omega_I)q_E - \beta\Delta S_{BE}\right\} \mathrm{d}F(\omega_E|s_E)\,\mathrm{d}G(s_E).$$

Using (8) and integrating the rent term $\beta\Delta S_{BE} f$ by parts with respect to ω_E, for each s_E, yields

$$\mathbb{E}\Pi_{BI} = \omega_I + \int_{s_E}\int_{\underline{\omega}_E}^{\bar{\omega}_E} S^{\mathrm{v}}(q_E; s_E, \omega_E)\,\mathrm{d}F(\omega_E|s_E)\,\mathrm{d}G(s_E), \tag{12}$$

where, following Jullien (2000), we have defined the "virtual surplus" S^{v} as

$$S^{\mathrm{v}}(q_E, s_E, \omega_E) = \left[\omega_E - \omega_I - \beta\,\frac{1 - F(\omega_E|s_E)}{f(\omega_E|s_E)}\right] q_E. \tag{13}$$

[11]In fact, the tariff will be determined only up to an additive constant, which reflects the sharing of the expected surplus between the buyer and the incumbent.

The virtual surplus is the total surplus $W(q_E, 1-q_E)$ adjusted for the informational rents $\beta q_E (1-F(\omega_E|s_E))/f(\omega_E|s_E)$ induced by the self-selection constraints. The virtual surplus depends linearly on the quantity q_E.

As observed in Section 3.2, setting a constant effective price p^{e} amounts to offering the surplus $v_I - p^{\text{e}}$ per unit of good I. Entrants with ω_E above (below) this value serve all of the contestable demand (are inactive). The fraction of active entrants, for a given size of the contestable demand, s_E, is thus $1 - F(v_I - p^{\text{e}}|s_E)$. Decreasing the effective price, i.e. increasing the offered surplus, places more competitive pressure on the entrant, and hence reduces the fraction of active entrants. This leads us to define the elasticity of entry by

$$\varepsilon(\omega_E|s_E) = \frac{\omega_E f(\omega_E|s_E)}{1 - F(\omega_E|s_E)}. \tag{14}$$

Hereafter, the bracketed term in (13) is called "virtual surplus per unit" and is denoted by $s^{\text{v}}(s_E, \omega_E)$. The virtual surplus per unit is negative for inefficient rivals ($\omega_E < \omega_I$), hence in particular, by Assumption 1, for $\omega_E = \underline{\omega}_E$. It is positive for $\omega_E = \bar{\omega}_E$, when $\bar{\omega}_E$ is finite. It can be rewritten as

$$s^{\text{v}}(s_E, \omega_E) = \omega_E[1 - \beta/\varepsilon(\omega_E|s_E)] - \omega_I.$$

Throughout the paper, we maintain the following assumption.

Assumption 2. *For any given size of the contestable demand s_E, the elasticity of entry, $\varepsilon(\omega_E|s_E)$, is nondecreasing in ω_E. Moreover, if $\bar{\omega}_E = \infty$, the upper bound of $\varepsilon(\omega_E|s_E)$ as ω_E rises is greater than one, for all s_E.*

Under Assumption 2, the virtual surplus per unit, s^{v}, is nondecreasing in ω_E provided that it is positive. Assumption 2 holds in particular when the hazard rate $f/(1-F)$ is nondecreasing in ω_E, a usual assumption in the nonlinear pricing literature. It is also true in the limit case where the elasticity does not depend on ω_E; this happens when ω_E, conditionally on s_E, follows a Pareto distribution, given by $1 - F(\omega_E|s_E) = (\omega_E/\underline{\omega}_E)^{-\varepsilon(s_E)}$; the elasticity of entry is then constant in ω_E and equal to $\varepsilon(s_E)$. Hereafter, we denote by $\underline{\varepsilon}(s_E)$ and $\bar{\varepsilon}(s_E)$ the minimum and maximum of $\varepsilon(\omega_E|s_E)$ for a given value of s_E, and by $\underline{\varepsilon}$ and $\bar{\varepsilon}$ the global minimum and maximum of $\varepsilon(\omega_E|s_E)$.

The variations of the elasticity of entry with s_E are related to the statistical link between the random variables s_E and ω_E. The relationship is stated in Lemma 2, proved in Appendix C.

Lemma 2. *The elasticity of entry, $\varepsilon(\omega_E|s_E)$, does not depend on s_E if and only if the random variables s_E and ω_E are independent.*

If the elasticity of entry increases (decreases) with s_E, then ω_E first-order stochastically decreases (increases) with s_E.

The buyer and the incumbent maximize the expected virtual surplus, given by (12), over all implementable quantity function q_E. To solve this problem, we first ignore the implementability conditions derived in Section 3.3 and maximize (12) over all quantity functions. This is what we call the "relaxed problem". We denote by q_E^{r} its solution. If q_E^{r} is implementable, then it is the solution of the complete problem.

Proposition 1. *The solution of the relaxed problem is given by*

$$q_E^{\mathrm{r}}(s_E, \omega_E) = \begin{cases} 0 & \text{if } \omega_E \leq \hat{\omega}_E(s_E) \\ s_E & \text{otherwise,} \end{cases}$$

where $\hat{\omega}_E(s_E) \in (\omega_I, \bar{\omega}_E)$ is the unique solution to

$$\frac{\hat{\omega}_E(s_E) - \omega_I}{\hat{\omega}_E(s_E)} = \frac{\beta}{\varepsilon(\hat{\omega}_E(s_E)|s_E)}. \tag{15}$$

The efficiency-rent tradeoff leads to more inefficient exclusion as the rival's bargaining power, β, rises and the elasticity of entry, ε, falls.

Proof. By linearity, the solution to the relaxed problem is s_E (zero) when the virtual surplus per unit, s^{v}, is positive (negative). The equation $s^{\mathrm{v}} = 0$ is equivalent to (15). We already know that the virtual surplus per unit is negative for $\omega_E = \omega_I$ and positive for $\omega_E = \bar{\omega}_E$, when $\bar{\omega}_E < \infty$. If $\bar{\omega}_E = \infty$, the second-part of Assumption (2) guarantees that s^{v} is positive for high values of ω_E. Hence the existence of a solution to equation (15) lying between ω_I and $\bar{\omega}_E$. The left-hand side of (15) increases in $\hat{\omega}_E$, and the right-hand side is nonincreasing in $\hat{\omega}_E$ by the first part of Assumption 2, which yields uniqueness. □

The threshold $\hat{\omega}_E(s_E)$ summarizes the tradeoff between efficiency and rent extraction at a given level of s_E. Equation (15) shows an analogy with the textbook monopoly pricing formula. The buyer-incumbent pair indeed has a monopoly power over entry, or more precisely over the quantity produced by the smaller rival. The buyer and the incumbent jointly act like a monopoly towards the rival, setting $\hat{\omega}_E$ to extract rent at the cost of reducing the probability of entry. When the threshold $\hat{\omega}_E$ is higher, the efficiency-rent tradeoff pushes towards less entry. The higher ε, the more reactive the entrant: the buyer and the incumbent cannot easily extract rents and cannot place strong competitive pressure on the entrant, hence a lower $\hat{\omega}_E$, and more entry.

The buyer has two tools to extract surplus from the entrant. First, her bargaining power $1-\beta$. Second, the tariff negotiated with the incumbent which determine both the size of the surplus created by the entry and the outside option of the buyer. They are very different in nature. First, the former is exogenous and the latter is endogenous. Second, whereas β does not directly impact the efficiency (if entry creates a positive surplus its sharing is irrelevant), the price schedule can deter efficient entry. Equation (15) shows that they are related. The larger the bargaining power of the buyer (i.e. the lower β) and the lower the threshold $\hat{\omega}_E(s_E)$; the efficiency-rent tradeoff pushes towards more entry. In the limit case where the buyer has all the bargaining power vis-à-vis the entrant ($\beta = 0$), there is no tradeoff, and hence no inefficient exclusion: $\hat{\omega}_E(s_E)$ coincides with the efficient threshold ω_I. On the contrary, the lower the bargaining power of the buyer and the higher $\hat{\omega}_E(s_E)$; the efficiency-rent tradeoff pushes towards less entry.

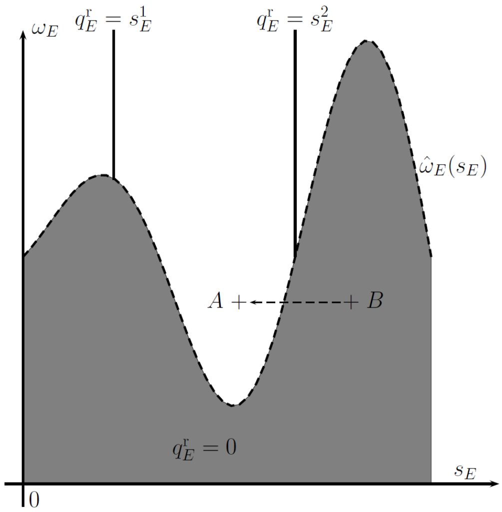

Figure 6: ERT line (dashed). Here, the the relaxed solution is not implementable.

Hereafter, we call the curve with equation $\omega_E = \hat{\omega}_E(s_E)$ in the (s_E, ω_E)-plan the ERT line.[12] As shown on Figure 6, the solution to the relaxed problem is zero below this line and s_E above. In the represented case, the quantity function q^{r}_E is not implementable, because implementable functions are nondecreasing in s_E and q^{r}_E decreases from s_E to zero when crossing increasing parts as the ERT line. For example, the type represented at point B, who sells $q^{\mathrm{r}}_E = 0$ and earns zero rent, would have an incentive to mimic type A, who sells all of the contestable demand and earns a positive rent. The relaxed quantity function, shown on Figure 6, is not consistent with the pattern of implementable quantity allocations, represented on Figure 5.

4.2 Nondecreasing elasticity of entry

In this section we assume that the elasticity of entry does not decrease with the size of the contestable demand, s_E. We consider first the case where $\varepsilon(\omega_E|s_E)$ does not depend on s_E, i.e. s_E and ω_E are independent. Then we examine the case where $\varepsilon(\omega_E|s_E)$ increases with s_E, i.e. ω_E first-order stochastically decreases with s_E. In both cases, the solution of the relaxed problem is incentive compatible and is therefore the solution of the buyer-incumbent pair's problem.

Proposition 2. *When the elasticity of entry, $\varepsilon(\omega_E|s_E)$, does not depend on s_E, the second best can be achieved through a two-part tariff with slope: $v_I - \hat{\omega}_E$. The equilibrium features inefficient exclusion. Partial foreclosure is not present.*

Proof. The ERT threshold given by (15) does not depend on s_E, because the elasticity ε does not. The solution of the relaxed problem, given by Proposition 1, is implementable with a constant boundary function $\Psi(s_E) = \hat{\omega}_E$, see Figure 7a.

[12]The acronym ERT stands for Efficiency Rent Tradeoff.

The second best tariff is obtained as follows. From (8), the gain from trade between the buyer and the entrant is given by $\Delta S_{BE}(s_E, \omega_E) = (\omega_E - \hat{\omega}_E)s_E$ for $\omega_E > \hat{\omega}_E$. By definition of ΔS_{BE}, we have: $\Delta S_{BE}(s_E, \omega_E) = (\omega_E - v_I)s_E + T^{\text{SB}}(1) - T^{\text{SB}}(1 - s_E)$, hence

$$T^{\text{SB}}(1) - T^{\text{SB}}(1 - s_E) = (v_I - \hat{\omega}_E)s_E.$$

The effective price, defined by (9), is constant and equal to $v_I - \hat{\omega}_E$. The second best allocation is achieved by a two-part tariff, see Figure 7b. □

To make sure that the competitor serves all of the contestable demand if $\omega_E \geq \hat{\omega}_E$ and is inactive otherwise, the buyer and the incumbent set the effective price at $v_I - \hat{\omega}_E$. The smaller the elasticity of entry, ε, the larger the ERT threshold, $\hat{\omega}_E$, the smaller the slope of the two-part tariff, the stronger the competitive pressure put on the entrant. The slope of the optimal price schedule is negative whenever v_I is lower than $\hat{\omega}_E$. In such a case, the buyer would be better off purchasing more than $1 - s_E$ units from the incumbent. Yet the buyer cannot take advantage of the negative marginal price offered by the incumbent because doing so would leave her with unconsumed units and disposal costs are assumed to be infinite (see Section 5 for finite disposal costs).

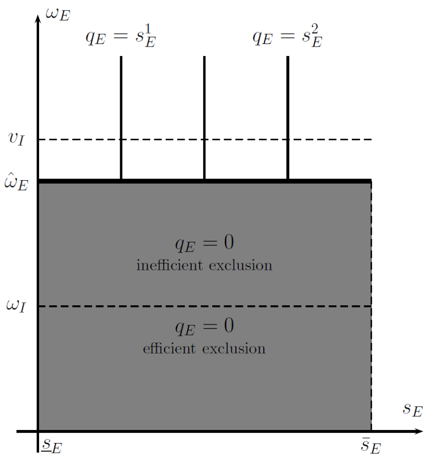

Figure 7a: Second best with $\varepsilon(\omega_E|s_E)$ constant in s_E

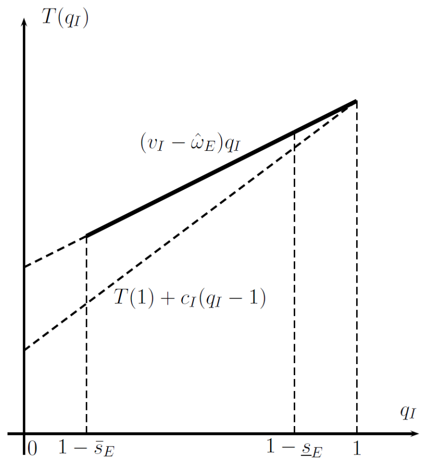

Figure 7b: Optimal price schedule (case $v_I > \hat{\omega}_E$)

As pictured in Figure 7a, the tradeoff between efficiency and rent extraction results in some efficient entrants being fully foreclosed in equilibrium. Inefficient foreclosure arises due to incomplete information as in Aghion and Bolton (1987). The fraction of efficient types that are inactive increases with the entrant's bargaining power vis-à-vis the buyer as $\hat{\omega}_E$ increases with β.

From now on, we consider cases where the elasticity of entry is not constant with s_E and show that two-part tariffs are no longer optimal. We start with the case where the elasticity increases with s_E: larger competitors, i.e. competitors with a larger contestable demand, are more sensitive to competitive pressure. Under this circumstance, the efficiency-rent tradeoff leads the buyer and the incumbent to place *less* competitive pressure on larger competitors.

Proposition 3. *When the elasticity of entry $\varepsilon(\omega_E|s_E)$ increases with s_E, the effective price, $p^{\mathrm{e}}(q_E)$, increases with q_E. The price schedule is concave in a neighborhood of $q_I = 1$. It is globally concave if $\hat{\omega}_E$ is concave or moderately convex in s_E. The equilibrium features inefficient exclusion. Partial foreclosure is not present.*

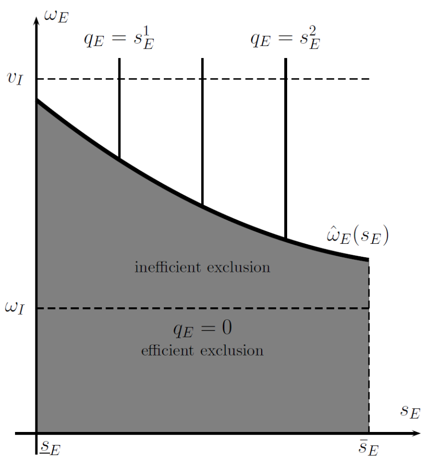

Figure 8a: Second best with $\varepsilon(\omega_E|s_E)$ increasing in s_E

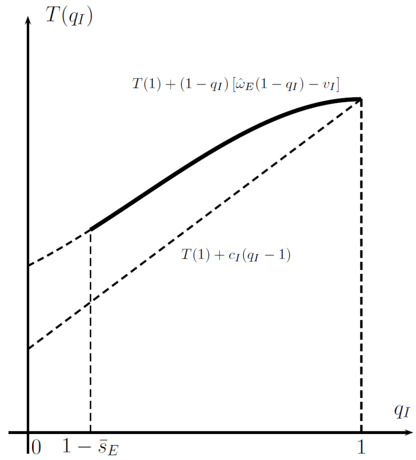

Figure 8b: Optimal price schedule (case $\underline{s}_E = 0$ and $v_I > \hat{\omega}_E(0)$)

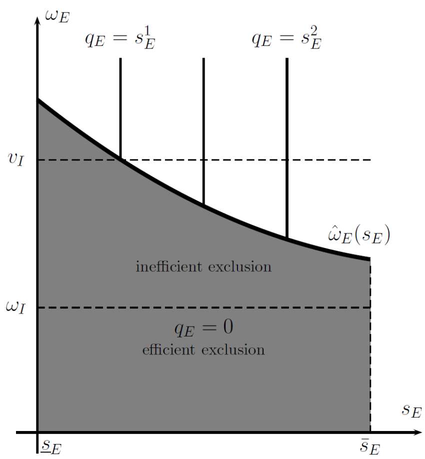

Figure 9a: Second best with $\varepsilon(\omega_E|s_E)$ increasing in s_E

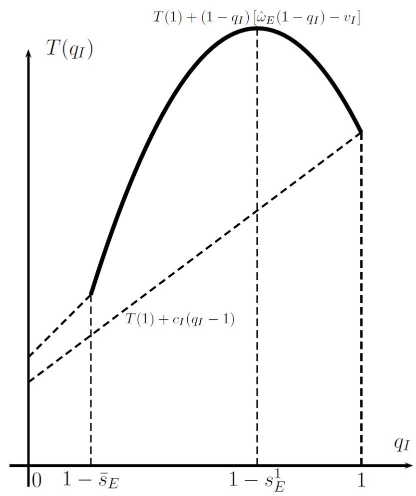

Figure 9b: Optimal price schedule (case $\underline{s}_E = 0$ and $v_I < \hat{\omega}_E(0)$)

Proof. When $\varepsilon(\omega_E|s_E)$ increases with s_E, the ERT threshold, $\hat{\omega}_E$, given by (15), decreases with s_E, and the solution of the relaxed problem is implementable. Its associated boundary function has equation $\Psi(s_E) = \hat{\omega}_E(s_E)$, see Figure 8a.

By the same reasoning as in Section 4.2, the surplus gain from the trade between the buyer end the entrant, $\Delta S_{BE}(s_E, \omega_E)$, equals $(\omega - \hat{\omega}_E(s_E))s_E$ above the ERT line and zero below, and the second best tariff is given by

$$T(1) - T(1 - s_E) = (v_I - \hat{\omega}_E(s_E))s_E.$$

In other words, the effective price $p^{\mathrm{e}}(s_E)$ is set at $v_I - \hat{\omega}_E(s_E)$, and is thus increasing in s_E. To prove that T is concave in a neighborhood of $q_I = 1$, we compute $T(q_I) = T(1) + (v_I - \hat{\omega}_E(1 - q_I))(q_I - 1)$, then $T'(q_I) = (v_I - \hat{\omega}_E(1 - q_I)) + \hat{\omega}'_E(1 - q_I)(q_I - 1)$ and $T''(q_I) = 2\hat{\omega}'_E(1 - q_I) - \hat{\omega}''_E(1 - q_I)(q_I - 1)$. The term $\hat{\omega}'_E$, which is negative for any q_I, tends to make the tariff concave. Assuming that $\hat{\omega}''_E(0)$ is not infinite, we get $T''(1) = 2\hat{\omega}'_E(0) < 0$, hence the concavity at the top. □

As shown on Figures 8a and 9a, the entrant is either inactive ($q_E = 0$) or serves all the contestable demand ($q_E = s_E$). For a given ω_E the jump from zero to s_E can never occur if ω_E is not large enough. The jump occurs when s_E is large enough for intermediate values of ω_E. Finally, if ω_E is large enough, $q_E = s_E$ for any s_E. On the other hand, for a given s_E, $q_E = 0$ if ω_E is small (below $\hat{\omega}_E(s_E)$) while $q_E = s_E$ when ω_E is large (above $\hat{\omega}_E(s_E)$).

Some efficient entrants are foreclosed. As the elasticity of entry increases with s_E, the ERT results in a lower $\hat{\omega}_E$ as s_E increases. Consequently, the optimal effective price $p^{\mathrm{e}}(q_E) = v_I - \hat{\omega}_E(q_E)$ increases with q_E: the larger the contestable market-share, the lower the competitive pressure. If $v_I \geq \hat{\omega}_E(\underline{s}_E)$, the effective price is positive for any quantity, as shown on Figures 8a and 8b. If $v_I < \hat{\omega}_E(\underline{s}_E)$, the effective price is negative for small values of q_E, as represented on Figures 9a and 9b.

4.3 The general case

We now consider the complete problem, which consists in maximizing the expected virtual surplus

$$\iint s^{\mathrm{v}}(s_E, \omega_E) q_E(s_E, \omega_E)\, \mathrm{d}F(\omega_E|s_E)\, \mathrm{d}G(s_E)$$

over all implementable quantity functions q_E. As explained at the end of Section 4.1, solving the problem separately for each s_E generally yields non implementable quantity functions, see Figure 10a. Our strategy consists in solving the problem separately for each ω_E and checking that the obtained quantity function is implementable.

In Appendix D, we consider the problem of maximizing

$$\int_{s_E} s^{\mathrm{v}}(s_E, \omega_E) q_E(s_E, \omega_E)\, \mathrm{d}F(\omega_E|s_E)\, \mathrm{d}G(s_E)$$

for each ω_E. This leads to construct, for each ω_E, horizontal segments where the quantity q_E is constant. Let $[AB]$ be such a segment, see Figure 10b. We show in the appendix that the virtual surplus must be positive at A and zero at B at the optimum. In other words, the point B belongs to the ERT line. In fact, we show that B must belong to a decreasing part of the ERT line. We also show that the expected virtual surplus on the segment $[AB]$ is zero

$$\mathbb{E}(\ s^{\mathrm{v}} \mid [AB]\ ) = 0. \tag{16}$$

The analysis presented in Appendix D shows that the quantity function obtained by the above method is implementable if and only if the left extremities of the constructed intervals (e.g. the point A on Figure 10b) are nondecreasing in ω_E. In Appendix D.4, we provide sufficient conditions for this monotonicity condition to hold.

Proposition 4. *Assume that one of the sufficient conditions stated in Appendix D.4 holds. Then the complete problem can be solved separately for each* ω_E. *The optimal boundary line* Ψ *lies above the ERT line,* $\Psi \geq \hat{\omega}_E$, *and can be constructed from the following properties:*

1. $\Psi(1) = \hat{\omega}_E(1)$;
2. *Its non-increasing parts coincide with the ERT line;*
3. *Its increasing parts are defined by equation (16).*

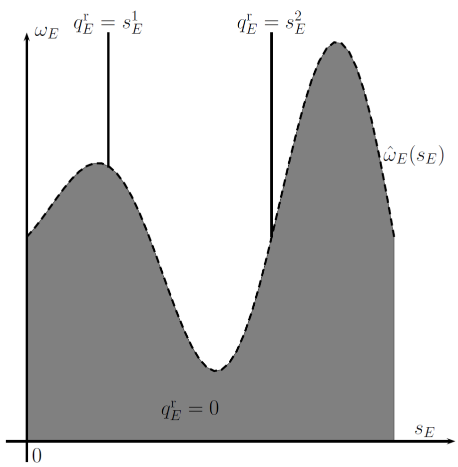

Figure 10a: The relaxed solution locally decreases with s_E.

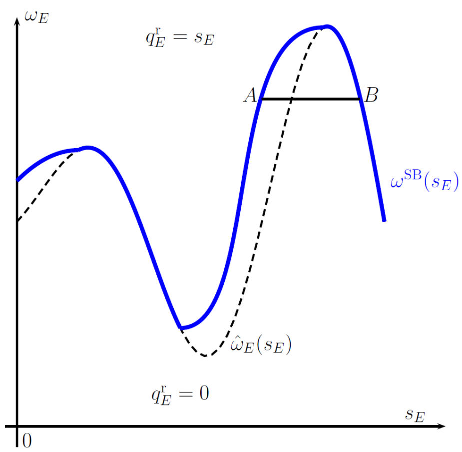

Figure 10b: ERT line $\hat{\omega}_E(s_E)$ (dashed). Optimal boundary ω^{SB}_E (solid)

The sufficient conditions mentioned in Proposition 4 are fairly mild. A first sufficient condition is f being nondecreasing in ω_E. A second set of sufficient conditions is the hazard rate $f/(1-F)$ being nondecreasing in ω_E and the range of the entry elasticity being not too wide (condition (D.3)). A third set of sufficient conditions consists of the elasticity of entry being nondecreasing in ω_E, as stated in Assumption 2,[13] and of

[13]Assumption 2 is weaker than f or $f/(1-F)$ being nondecreasing in ω_E.

another condition on the range of ε, (D.4), more restrictive than (D.3). Technically, the conditions (D.3) and (D.4) involve the rival's bargaining power, β, and the minimum and maximum values of ε in the whole distribution of types, $\underline{\varepsilon}$ and $\bar{\varepsilon}$. Even the stronger condition (D.4) is not very restrictive, in the sense that it allows for a wide range $[\underline{\varepsilon}, \bar{\varepsilon}]$. For instance, if the rival's bargaining power, β, equals one, the elasticity of entry may vary freely between $\underline{\varepsilon} = 1.2$ and $\bar{\varepsilon} = 3.98$, or between $\underline{\varepsilon} = 5$ and $\bar{\varepsilon} = 26.64$. If β equals .75, then the elasticity of entry may vary freely between $\underline{\varepsilon} = 1.2$ and $\bar{\varepsilon} = 5.99$, or between $\underline{\varepsilon} = 5$ and $\bar{\varepsilon} = 33.59$.

To construct the optimal boundary Ψ under the sufficient conditions of Appendix D.4, we proceed as follows. We first draw the ERT line $\omega_E = \hat{\omega}_E(s_E)$. We start with $s_E = 1$ and then consider lower and lower values of s_E. For $s_E = 1$, we know that $\Psi(1) = \hat{\omega}_E(1)$. If the ERT line decreases at $s_E = 1$, the boundary coincides with the ERT line, as long as it remains decreasing. When the ERT line starts increasing (possibly at $s_E = 1$), we know that there is horizontal bunching. Equation (16) provides a unique value for $\Psi(s_E)$. If the candidate boundary hits the ERT line at some value of s_E, it must be on a decreasing part of that line and, from that value on, the optimal boundary coincides with the ERT line (as long as $\hat{\omega}_E$ remains decreasing).

When the monotonicity constraints on the left extremities of horizontal bunching intervals is violated (hence the sufficient conditions do not hold), the increasing parts of the optimal boundary line have vertical portions, generating two-dimensional bunching areas. An example of such an area is the shaded region, D, represented on Figure 15b.

Whether or not the monotonicity constraints are binding, the above construction shows that the optimal boundary is located below the maximal value of $\hat{\omega}_E(s_E)$, hence below $\bar{\omega}_E$ (see Proposition 1). For high values of ω_E, lying between the maximum of $\hat{\omega}_E(s_E)$ and $\bar{\omega}_E$, the second-best quantity is efficient: $q_E(s_E, \omega_E) = s_E = q_E^*(s_E)$ for all s_E. There is no distortion at the top of the distribution of ω_E.

4.4 Decreasing elasticity of entry

We now turn to the case where the elasticity of entry is decreasing with s_E: larger competitors, i.e. competitors with a larger contestable demand, are less sensitive to competitive pressure. Under this circumstance, the efficiency-rent tradeoff leads the buyer and the incumbent to place *more* competitive pressure on larger competitors. Larger competitors, however, can mimic smaller ones by producing less, implying that the optimal quantity function derived from the tradeoff is not implementable.

Proposition 5. *Assume that $\varepsilon(\omega_E|s_E)$ decreases with s_E and the assumptions of Proposition 4 hold. Then the optimal tariff is convex. The equilibrium outcome exhibits inefficient exclusion, in the form of both full and partial foreclosure.*

Proof. When $\varepsilon(\omega_E|s_E)$ decreases with s_E, the ERT line $\hat{\omega}_E$ is monotonically increasing and cannot be the optimal boundary line, as this would violate incentive compatibility. Hence the presence of horizontal pooling segments. As explained in Section 4.3, the expected virtual surplus on these horizontal segments must be zero, which yields a candidate boundary line. Under the sufficient assumptions of Appendix D.4, the candidate line is nondecreasing and hence determines the optimal quantity function, see the solid line labeled ω_E^{SB} on Figure 11a.

The light shaded area on the figure represents the set of types for which the competitor is partially foreclosed. For all $s_E \in [\underline{s}_E, \bar{s}_E]$, $\omega_E = \omega_E^{\text{SB}}(s_E)$ and $s'_E > s_E$, the solution of the buyer-competitor problem (7) is interior for (s'_E, ω_E), and the solution, $q_E = s_E$, is given by the first-order condition $T'(1 - s_E) = v_I - \omega_E^{\text{SB}}(s_E)$ or

$$T'(q_I) = v_I - \omega_E^{\text{SB}}(1 - q_I),$$

which increases in q_I as ω_E^{SB} is increasing. We conclude that the price-quantity schedule T is convex. $\square$

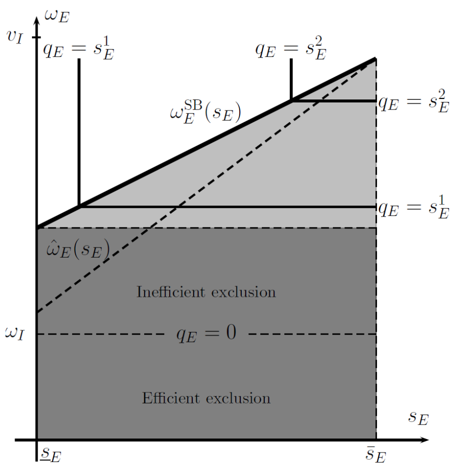

Figure 11a: ERT line (dashed), optimal boundary line (solid) with $\varepsilon(\omega_E|s_E)$ decreasing in s_E.

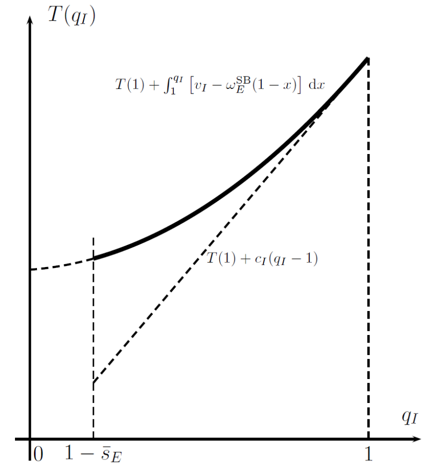

Figure 11b: Optimal price schedule with $\varepsilon(\omega_E|s_E)$ decreasing in s_E ($\underline{s}_E = 0$ and $v_I > \hat{\omega}_E(1)$).

As depicted in Figure 11a, when the entrant's type lies in the light shaded triangle below the boundary line and above the horizontal line $\omega_E^{\text{SB}}(0)$, the entrant produces a quantity strictly lower than s_E; entry is partially foreclosed. Here, the price schedule plays the role of a barrier to expansion. Some efficient competitor types are active but prevented to serve all the contestable demand

A small market share of the competitor, therefore, reflect either a small contestable demand or a large one with partial foreclosure: this is the case when E is sufficiently efficient to enter but not enough to break the ω_E^{SB} line and sell at full capacity. These situations are qualitatively very different. In the first one, the competitor is frustrated because he had to abandon a fraction of his surplus to the buyer. However, depending on the interpretation of s_E, either he cannot produce more or the buyer is not interesting in buying more from the entrant. In the second case (partial foreclosure), the competitor is similarly deprived of some surplus, but in addition he is also frustrated because he cannot sell all the units that the buyer would like to acquire in the absence of price schedule T.

The optimal price-quantity schedule represented on Figure 11b is increasing because the picture is drawn under the assumption that v_I is larger than $\omega_E^{SB}(s_E)$ for all s_E. If, however, ω_E^{SB} becomes larger than v_I for s_E large enough, then the slope of T is negative for the small q_I (as $q_I = 1 - s_E$).

4.5 Non monotonic elasticity of entry

We now turn to a case where the elasticity of entry is non monotonic with the size of the contestable demand, s_E. We assume that the elasticity of entry is first decreasing then increasing as the size of the contestable demand rises: competitors with intermediate size are less sensitive to competitive pressure than competitors with small or large size. Under this circumstance, the efficiency-rent tradeoff induces the buyer and the incumbent to place strong competitive pressure on competitors with intermediate size and less on small or large competitors. In other words, the ERT line has an inverted U-shape, see the dashed line on Figure 12a.

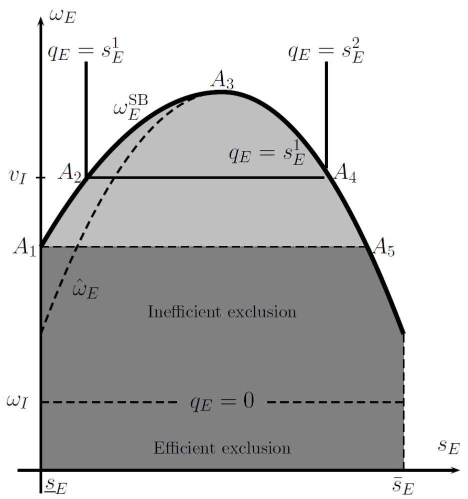

Figure 12a: ERT line (dashed), optimal boundary line (solid) with $\varepsilon(\omega_E|s_E)$ U-shaped in s_E

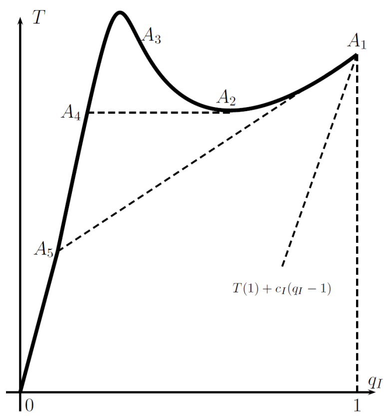

Figure 12b: Optimal price schedule with $\varepsilon(\omega_E|s_E)$ U-shaped in s_E ($\underline{s}_E = 0$ and $\bar{s}_E = 1$)

We rely on Figures 12a and 12b to explain the shape of the optimal price schedule in this instance. The picture is drawn under the assumption that $\hat{\omega}_E(0) < v_I < \max \hat{\omega}_E$.

Between A_1 and A_3, the optimal boundary line, ω_E^{SB}, is increasing. As already seen in Section 4.4, the quantity negotiated between the buyer and the entrant is given by the first-order condition: $T'(1-s_E) = v_I - \omega_E^{\mathrm{SB}}(s_E)$; the price-quantity schedule T is convex in this region. In particular, at point A_2, we have $v_I = \omega_E^{\mathrm{SB}}(s_E)$ and $T' = 0$. Between A_3 and A_5, we recover T by using the bunching condition: in the grey area q_E is constant along horizontal lines. For example, if the entrant is at A_4 the buyer-entrant pair is indifferent between buying s_E^1 or s_E^2:

$$(\omega_E - v_I)s_E^1 - T(1-s_E^1) = (\omega_E(s_E^2) - v_I)s_E^2 - T(1-s_E^2).$$

As $T(1-s_E^1)$ is known, one can infer $T(1-s_E^2)$. Rewriting the above expression yields

$$T(1-s_E^2) = T(1-s_E^1) + (\omega_E(s_E^2) - v_I)(s_E^2 - s_E^1)$$

At these particular points A_2 and A_4, we have $\omega_E = v_I$, and hence $T(1-s_E^1) = T(1-s_E^2)$, as shown on Figure 12b. It is readily confirmed that $T'' = 0$ at A_3, i.e. T has an inflexion point. After A_5, the same indifference condition applies but $s_E^1 = 0$. Therefore $T(1-s_E^2) = T(1) + (\omega_E(s_E^2) - v_I)s_E^2$. Thus, an inverted U-shape ω_E^{SB} is associated to a price schedule which is convex at the end (small values of s_E), concave in the middle and either concave or convex for the small values of q_I.

In addition to characterizing the shape of T, Figures 12a and 12b are also helpful to show what happens when ω_E^{SB} is first below then above and finally below v_I as s_E rises. Under this circumstance, the shape of the price-quantity schedule T is reminiscent to that of "retroactive rebates". (Such rebates are granted for all units once a quantity threshold is reached. They induce downwards discontinuities in price-quantity schedules.) The buyer has a strong incentive to buy up to A_2 as T is decreasing. In this section, however, disposal costs are infinite by assumption, so the buyer cannot purchase more than her requirements.[14] We now turn to the case where the buyer can get rid of unconsumed units at a finite cost and might thus opportunistically buy more than her requirements.

5 Disposal costs, buyer opportunism, and conditional tariffs

We have assumed so far that the buyer incurs an infinite cost if she does not consume all of the purchased units. Yet in practice the buyer may have the ability to get rid of unnecessary items at some cost. The magnitude of disposal costs may vary substantially across industries, as disposing of computer chips, tyres for trucks, or heavy pieces of machineries[15] is likely to entail different costs. Disposal costs also depend on the existence of a second-hand market and on the seller's ability to monitor the buyer's behavior.[16] In Section 5.1, we extend the previous analysis to the case of finite disposal costs, taking into account possible buyer opportunism. In Section 5.2, we show how conditionnal tariffs make it possible to overcome the opportunism problem.

[14] We have assumed that the distribution of types is continuous, so each point of the tariff, even in regions where it is decreasing, is chosen by a competitor. If instead the size of the contestable demand took only a finite number of values, a true retroactive rebate could be optimal.

[15] These products are involved in some of the antitrust cases mentioned in footnote 1.

[16] Disposal costs can also be seen as costs to avoid the monitoring of the incumbent.

5.1 Finite disposal costs and buyer opportunism

We denote by γ the exogenous cost incurred by the buyer if she fails to consume some of the purchased units. We distinguish units that are purchased, q_E and q_I from units that are consumed, x_E and x_I. As previously, the buyer's consumption is bounded, and normalized to one: $x_E + x_I \leq 1$. Having purchased quantities $q_E \leq s_E$ and q_I from the rival and the incumbent, the buyer chooses consumption levels so as to maximize

$$V(q_E, q_I) = \max_{(x_E,x_I)\in X} v_E x_E + v_I x_I - \gamma(q_E - x_E) - \gamma(q_I - x_I), \tag{17}$$

where the set X is defined by the constraints $x_E \leq q_E$, $x_I \leq q_I$, and $x_E + x_I \leq 1$: the buyer cannot consume more than she has purchased nor more than her total requirement. In appendix E.1, we extend Lemma A.1 to the case of finite disposal costs, showing that the buyer's total purchases are not lower than her total demand in equilibrium.

The buyer, however, could purchase *more* than her requirement, as disposal costs are now assumed to be finite, with the sole purpose of benefiting from a rebate offered by the incumbent. We call such a behavior opportunistic. The expression of V given in (17) shows that marginal prices below $-\gamma$ create buyer opportunism. Indeed, suppose that the buyer already purchased q_E from the competitor and that $T'(1 - q_E) < -\gamma$. Then she would purchase more than $1 - q_E$ from the incumbent to benefit from a lower total price while consuming only $x_I = 1 - q_E$. These excess purchases are costly for the incumbent because of production costs.

We first establish an optimality result that holds irrespective of the informational structure, i.e. whether or not the buyer and the incumbent know the competitor's characteristics when signing the contract. The proof can be found in Appendix E.2.

Proposition 6. *The buyer and the incumbent are better off using a tariff with slope greater than or equal to $-\gamma$. There is no buyer opportunism in equilibrium: the buyer does not buy more than her total requirements, $q_E + q_I \leq 1$.*

Combining the extended version of Lemma A.1 with Proposition 6 shows that the buyer purchases the exact quantity necessary to meet her requirements: $q_E + q_I = 1$, and consumes all purchased units: $x_E = q_E$, $x_I = q_I$. Hence, the buyer actually incurs no disposal costs. It follows that the expressions (7) and (11), respectively for the surplus created by the trade between the buyer and the competitor, ΔS_{BE}, and the expected profit of the buyer-incumbent pair, $\mathbb{E}\Pi_{BI}$, still hold.

It follows that the maximal expected profit of the buyer-incumbent pair is obtained by solving the same problem as above, under the extra constraint that $T'(q_I) \geq -\gamma$. Consequently, and regardless of the informational structure, the optimal expected profit of the buyer-incumbent pair is nondecreasing in the magnitude of the disposal costs, γ.

Throughout this section, we say that the rival firm is *super-efficient* if $\omega_E \geq v_I + \gamma$. It follows from Proposition 6 and from the buyer-competitor problem (7) that, in equilibrium, whatever the informational structure, a super-efficient competitor serves all the contestable demand: $q_E(s_E, \omega_E) = s_E$ for all $\omega_E > v_I + \gamma$. Indeed, for such a competitor, the function $(\omega_E - v_I)q_E - T(1 - q_E)$ is nondecreasing on $(0, s_E)$.

Proposition 6 prompts us to extend the notion of implementability to the case with finite disposal costs. We say that a quantity function is implementable if it can be obtained as solution to (7), where T is a tariff satisfying $T' \geq -\gamma$ for all q. Lemma 1 must be adapted as follows.

Lemma 3. *A quantity function $q_E(.,.)$ is implementable if and only if there exists a boundary function $\Psi(.)$ defined on $[0,1]$, with $\Psi \leq v_I + \gamma$, such that (10) holds.*

The new condition on the boundary line, $\Psi(s_E) \leq v_I + \gamma$, expresses that super-efficient competitors serve all of the contestable demand: $q_E(s_E, \omega_E) = s_E$ for all $\omega_E \geq v_I + \gamma$ and all s_E. The sufficient part of the lemma is proved in Appendix E.3. It follows from Lemma 3 that the only change due to the presence of finite disposal costs concerns super-efficient competitors. To explain this point in more detail, we slightly change the notations, denoting by $q_E(s_E, \omega_E; \gamma)$ the optimal quantity function and by $\Psi(s_E; \gamma)$ the optimal boundary function when the magnitude of the disposal costs is given by the parameter γ.

Proposition 7. *Under the assumptions of Proposition 4, the optimal quantity function is given by*

$$q_E(s_E, \omega_E; \gamma) = \begin{cases} q_E(s_E, \omega_E; \infty) & \text{if } \omega_E < v_I + \gamma \\ s_E & \text{if } \omega_E \geq v_I + \gamma. \end{cases}$$

The existence of finite disposal costs matters if and only if the efficiency-rent tradeoff induces the exclusion of some super-efficient competitors, i.e. if and only if $\hat{\omega}_E(s_E) > v_I + \gamma$ for some values of s_E.

Proof. We solve the problem separately for each ω_E. For $\omega_E \geq v_I + \gamma$, we must have $q_E = s_E$: super-efficient competitors serve all of the contestable demand. For $\omega_E < v_I + \gamma$, we use the same method as under $\gamma = \infty$, which, under the assumptions of Proposition 4, yields an implementable quantity functions. The above construction amounts to truncating the optimal boundary function as follows:

$$\Psi(s_E; \gamma) = \min(\Psi(s_E; \infty), v_I + \gamma).$$

If there are no super-efficient competitors, $\bar{\omega}_E \leq v_I + \gamma$, the optimal quantity function is the same as under $\gamma = \infty$. This is also true when the efficiency-rent tradeoff leads any super-efficient competitor to serve all of the contestable demand, i.e. $\hat{\omega}_E(s_E) \leq v_I + \gamma$ for all s_E. Indeed, we know from Section 4.3 that $\Psi(s_E; \infty) \leq \max_{s_E} \hat{\omega}_E(s_E)$, implying that $\Psi(s_E; \infty) \leq v_I + \gamma$, and hence $\Psi(s_E; \gamma) = \Psi(s_E; \infty)$.

Conversely, suppose that $\hat{\omega}_E(s_E) > v_I + \gamma$ for some value of s_E. Consider a super-efficient competitor (s_E, ω_E) who would be excluded under the rent-efficiency tradeoff: $v_I + \gamma \leq \omega_E \leq \hat{\omega}_E(s_E)$. Since the boundary line under $\gamma = \infty$ lies above the ERT line (see Proposition 4), we have: $q_E(s_E, \omega_E; \infty) < s_E = q_E(s_E, \omega_E; \gamma)$. The constraint $T' \geq -\gamma$ is therefore binding. □

Disposal costs and ex post buyer opportunism prevent the buyer and the incumbent from placing too strong a competitive pressure on the rival, thus protecting super-efficient competitors from exclusion (but not against rent-shifting). The presence of finite disposal costs therefore limits the extent of inefficient foreclosure. As the disposal costs are not incurred in equilibrium, their presence enhances the welfare compared to the case $\gamma = \infty$.

5.2 Conditional tariffs

In this section, we assume that the buyer and the incumbent are able to condition the price paid for q_I units to the number of units of purchased from the competitor, i.e. they are able to enforce a tariff $T(q_E, q_I)$. Applying the same reasoning as in Appendices E.1 and E.2 for each value of q_E, one can show that the buyer and the incumbent are better off using a tariff $T(q_E, q_I)$ such that the marginal price of an extra unit of good I satisfies

$$-\gamma \leq \frac{\partial T}{\partial q_I} \leq v_I,$$

and consequently that the buyer purchases the quantity necessary to meet her requirement: $q_E + q_I = 1$.[17]

The left inequality, however, does no longer imply that the effective price is greater than $-\gamma$ and that super-efficient competitors serve all of the contestable demand. Indeed, given that $q_E + q_I = 1$, the effective price of the last q_I units sold by the incumbent is now given by

$$p^{\mathrm{e}}(q_I) = \frac{T(0,1) - T(q_I, 1 - q_I)}{q_I}, \tag{18}$$

instead of $[T(1) - T(1 - q_I)]/q_I$. which can be lower than $-\gamma$. Moreover, the objective of the buyer-competitor coalition, $(\omega_E - v_I)q_E - T(q_E, 1 - q_E)$, may decrease in q_E even for super-efficient competitors. All this can happen if T increases with its first argument, i.e. if the price paid for q_I units increases with the number of units purchased from the rival.

Proposition 8. *Conditioning the tariff on the quantity purchased from the competitor allows the buyer and the incumbent to earn the same profit as if disposal costs were infinite.*

Proof. The expression (7) for the surplus from the trade between the buyer and the competitor must be replaced with

$$\Delta S_{BE}(s_E, \omega_E) = \max_{q_E \leq s_E} (\omega_E - v_I)q_E - T(q_E, 1 - q_E) + T(0, 1).$$

Lemma 1 characterizes implementable quantity functions when the price-quantity schedule depends only on q_I. The proof in Appendix B.1 consists in recovering $T(q_I)$ from the the functions $q_E(.,.)$ and $\Delta S_{BE}(.,.)$. The proof extends when the schedule depends also

[17] When T depends on q_E, the tariffs $\tilde{T}$ and $\hat{T}$ can be constructed in the same manner as in Appendices E.1 and E.2. The equalities $U = \tilde{U} = \hat{U}$ are proved similarly.

on q_E, leading to recover $T(q_E, 1-q_E)$. In other words, the whole schedule $T(q_E, q_I)$ is not identified; only its values for $(q_E, 1-q_E)$ are. This implies that the constraint on the marginal price, $\partial T/\partial q_I \geq -\gamma$, has no bite: only $T(q_E, 1-q_E)$ matters, and any such function may (for instance) be obtained from a schedule $T(q_E, q_I)$ that is independent from q_I. It follows that the set of implementable quantity functions with conditional tariffs does not depend on $\gamma \in [0, +\infty]$. Moreover, with $\gamma = +\infty$, this set is the set of quantity functions implementable with unconditional tariffs. □

When the price-quantity schedule depends only on q_I, the presence of finite disposal cost prevents the exclusion of super-efficient competitors, because the incumbent must account for ex post buyer opportunism. Conditional tariffs make it possible for the buyer and the incumbent to overcome the buyer opportunism problem and to exclude super-efficient competitors.

6 Discussion

The chief concern of antitrust enforcers as regards abuses of dominant position is inefficient exclusion. In its guidelines on exclusionary conducts by dominant undertakings, the European Commission advocates the "as-efficient competitor test", which consists in checking that efficient rivals are not foreclosed. This test is presented as a first step in the legal assessment: if the test is violated, the dominant firm may have the burden of justifying its pricing policy, for instance by putting forward efficiency considerations.

We study nonlinear pricing by a dominant firm which competes with a smaller rival, focusing on exclusionary effects. We exclude any efficiency reasons for the dominant firm to use nonlinear pricing as well as any predation purposes. We examine the consequences of the incumbent's monopoly power over the rival, in the spirit of Aghion and Bolton (1987). In our model, the common distinction in the antitrust doctrine between *exploitative* and *exclusionary* abuses is blurred because it is the exploitation of the incumbency advantage, combined with incomplete information, that yields inefficient exclusion.[18] The two aspects are intertwined in the tradeoff between rent extraction and efficiency.

The exploitative part of the mechanism is sometimes called "rent-shifting": the existence of the tariff enhances the buyer's bargaining position vis-à-vis the entrant by altering her outside option in the negotiation. Under incomplete information, the buyer and the incumbent leave the door ajar: they adjust the competitive pressure placed on the entrant to solve the efficiency-rent tradeoff, allowing very efficient rivals to enter the market while driving some efficient rivals out (complete foreclosure) or preventing them from selling at full capacity (partial foreclosure).[19] In any case, those who enter are

[18] Under complete information, only exploitative abuse is involved, as the second-best allocation is efficient (see the end of Section 2).

[19] In several cases, Virgin/British Airways, Michelin, and Intel (See references in footnote 1), the defendant argued that his market share had declined during the year under scrutiny. This could happen in our model, for a given s_E, if ω_E increases but remains below ω_E^{SB}. The rival remains partially foreclosed but less and less as ω_E rises.

forced to grant favorable conditions to attract buyers.

Our analysis places no *a priori* restriction on the shape of the incumbent's tariff. Depending on the distribution of the uncertainty, optimal tariffs may be locally increasing or decreasing, and locally convex, linear or concave. The competitive pressure placed on the rival firm translates into the amount of rebates that the buyer gives up by supplying units from the rival. Hence the importance of the incumbent's "effective price" emphasized by the European Commission.[20] At the second best, the effective price is always below the incremental cost, because the buyer and the incumbent only want to shift rents from a rival who is more efficient than the incumbent. If the Commission could enforce its "as-efficient competitor test", then any exploitative attempt would be eliminated and hence there would be no exclusion of efficient rivals.

Yet enforcing the as-efficient competitor test is by no means trivial, as cost measurements and contestable market shares are imprecise in nature. To our knowledge, the *Intel* decision contains the first and, to date, the sole attempt to implement the test in an antitrust case.[21]

Incumbents can take advantage of their position only if they can commit to the price schedule, whether information is complete or incomplete. It is therefore crucial to check this point in practice. In two recent cases,[22] the European Commission stressed that the dominant firm used a long period to calculate the rebates, and in both cases, the dominant firm was able to commit to a price schedule for the whole reference period. In the case of markets where firms interact for long periods of time, the commitment ability may come from repeated interactions.

Finally, our analysis explains how the presence of low disposal costs limits the ability of dominant firms to exploit their position. When some rivals are expected to be much more efficient than incumbents, dominant firms may want to use negative marginal prices, with the result of excluding very efficient rivals. Negative prices, however, would induce buyers to purchase more than needed simply to cash in on the tariff.[23] It follows that low disposal costs and ex post buyer opportunism prevent the exclusion of very efficient competitors.

To counter buyer opportunism, the incumbent has two strategies: first he could monitor the buyer, making sure that she purchases only up to her needs. Second, the dominant firm may want to condition his prices on quantities purchased from rivals. We show that resorting to a conditional tariff is equivalent to imposing an infinite disposal

[20]When the tariff only depends on the quantity purchased from the incumbent, the effective price is simply the average price of the last units offered by the incumbent. The computation must be adapted when the tariff also depends on the quantity purchased from the entrant (see (18)), as buying more from the entrant (as opposed to buying less from the incumbent) can in itself affect the effective price.

[21]Admittedly, the Commission run the test for a single value of q, namely $q = 1 - s_E$, where s_E was a "realistic" value for the size of the contestable demand. This method requires the determination, *ex post*, of the magnitude of the contestable demand, which proved highly contentious in *Intel*.

[22]Virgin/British Airways and Michelin. See references in footnote 1.

[23]The price schedule agreed upon between the incumbent and the buyer can be seen (in broad terms) as a specific investment by the incumbent. As often with specific investment, one party can in some state of nature take advantage of the fact that the other party is committed to this investment.

cost on the buyer. Under such a tariff, excluding very efficient competitors is possible even when the buyer can dispose of unconsumed units at a low cost.

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Appendix

A The buyer's total purchases are not lower than her total demand

This section presents an optimality result that holds irrespective of the information structure, i.e. whether the buyer and the incumbent know the entrant's characteristics at the time of contracting. It is in the buyer's and incumbent's common interest to agree on a price schedule that induces the former to purchase at least $1 - q_E$ units from the latter, once she has purchased q_E from the entrant, for any value of q_E. Hence, in equilibrium, the buyer's total purchases are equal to, or exceed, her total demand.[24]

[24] Lemma A.1 is stated and proved under infinite disposal costs. Appendix E.1 extends the result to the case of finite disposal costs.

Lemma A.1. *The buyer and the incumbent are better off using a tariff with slope T' smaller than or equal to v_I. Consequently, we may assume, with no loss of generality, that the buyer does not buy less than its total requirements: $q_E + q_I \geq 1$.*

Proof: We start from any price schedule T. Let $\tilde{T}$ be defined by

$$\tilde{T}(q_I) = \inf_{q \leq q_I} T(q) + v_I(q_I - q). \tag{A.1}$$

The tariff $\tilde{T}$ is derived from the tariff T as follows. When the incumbent offer q units at price $T(q)$, he also offers to sell more units than q, say $q_I > q$, at price $T(q) + v_I(q_I - q)$. The additional units are offered at the monopoly price v_I. By construction, the slope of $\tilde{T}$ is lower than or equal to v_I.

Let $\tilde{U}_B(q_E)$ denote the buyer's net utility after she has purchased q_E units from the entrant under the price schedule $\tilde{T}$

$$\tilde{U}_B(q_E) = \max_{q_I} V(q_E, q_I) - \tilde{T}(q_I). \tag{A.2}$$

As $\tilde{T} \leq T$, we have: $\tilde{U}_B \geq U_B$. Suppose that, under $\tilde{T}$, it is optimal for the buyer to purchase $\tilde{q}_I$ from the incumbent if she has purchased q_E from the entrant. By construction of $\tilde{T}$, there exists $q_I \leq \tilde{q}_I$ such that $\tilde{T}(\tilde{q}_I)$ equals or is arbitrarily close to $T(q_I) + v_I(\tilde{q}_I - q_I)$. We have:

$$\begin{aligned} \tilde{U}_B(q_E) = V(q_E, \tilde{q}_I) - \tilde{T}(\tilde{q}_I) &= V(q_E, \tilde{q}_I) - T(q_I) - v_I(\tilde{q}_I - q_I) \\ &= V(q_E, q_I) - T(q_I), \end{aligned} \tag{A.3}$$

which implies $\tilde{U}_B(q_E) \leq U_B(q_E)$, and hence $\tilde{U}_B(q_E) = U_B(q_E)$ for all q_E. As the problem of the buyer-entrant pair depends only on the functions $U_B(.)$ and $\tilde{U}_B(.)$, they agree on the same quantity q_E and the entrant earns the same profit, $\beta \Delta S_{BE}$, under T and $\tilde{T}$ for all (c_E, s_E, v_E).

We now examine the quantity purchased from the incumbent. Suppose that the buyer, having purchased q_E from the entrant, chooses to purchase q_I from the incumbent under the original price schedule T. As $\tilde{T}(q_I) \leq T(q_I)$, the buyer may choose to purchase the same quantity from the incumbent under the new tariff $\tilde{T}$:

$$U_B(q_E) = \tilde{U}_B(q_E) = V(q_E, q_I) - T(q_I) \leq V(q_E, q_I) - \tilde{T}(q_I).$$

Yet, under the tariff $\tilde{T}$, if $q_I < 1 - q_E$, the buyer may as well choose to purchase $1 - q_E$ from the incumbent. Indeed, by definition of $\tilde{T}$, we have $\tilde{T}(1 - q_E) \leq T(q_I) + v_I(1 - q_E - q_I)$ and hence

$$\begin{aligned} U_B(q_E) = \tilde{U}_B(q_E) &= V(q_E, q_I) - T(q_I) \\ &\leq V(q_E, q_I) + v_I(1 - q_E - q_I) - \tilde{T}(1 - q_E) \\ &= V(q_E, 1 - q_E) - \tilde{T}(1 - q_E). \end{aligned} \tag{A.4}$$

As $v_I > c_I$, the change from q_I to $1 - q_E > q_I$ increases the total surplus:

$$\begin{aligned} W(q_E, 1-q_E) &= V(q_E, 1-q_E) - c_E q_E - c_I(1-q_E) \\ &= V(q_E, q_I) - c_E q_E - c_I q_I + (v_I - c_I)(1 - q_E - q_I) \\ &\geq W(q_E, q_I). \end{aligned} \tag{A.5}$$

In sum, the change from T to $\tilde{T}$ does not alter the entrant's profit and does not decrease the total surplus. We conclude from (4) that the change does not decrease the expected payoff of the buyer-incumbent coalition.

B Implementation

B.1 Recovering the tariff from the boundary line

We prove here the sufficient part of Lemma 1. Starting from any boundary function Ψ defined on $[0,1]$, we define the quantity function $q_E(s_E, \omega_E)$ by equation (10), and the profit function $\Delta S_{BE}(s_E, \omega_E)$ by equation (8). We observe that the functions thus defined $q_E(s_E, \omega_E)$ and $\Delta S_{BE}(s_E, \omega_E)$, are nondecreasing in both arguments, and the latter function is convex in ω_E. Next, we notice that the expression

$$(\omega_E - v_I) q_E(s_E, \omega_E) - \Delta S_{BE}(s_E, \omega_E)$$

is constant on q_E-isolines. Indeed, both $q_E(., \omega_E)$ and $\Delta S_{BE}(., \omega_E)$ are constant on horizontal isolines (located below the boundary Ψ). On vertical isolines (above the boundary), $\Delta S_{BE}(s_E, .)$ is linear with slope s_E, guaranteing, again, that the above expression is constant. We may therefore define $T(q)$, up to an additive constant, by

$$T(1) - T(1-q) = (v_I - \omega_E)q + \Delta S_{BE}(s_E, \omega_E), \tag{B.1}$$

for any (s_E, ω_E) such that $q = q_E(s_E, \omega_E)$. Equation (B.1) unambiguously defines $T(1) - T(1-q)$ on the range of the quantity function $q_E(.,.)$. This range contains zero, but may have holes when $\bar{\omega}_E$ is finite and Ψ is above $\bar{\omega}_E$ on some intervals. Specifically, if Ψ is above $\bar{\omega}_E$ on the interval $I = [s_E^1, s_E^2]$, then q_E does not take any value between s_E^1 and s_E^2. In this case, we define T by imposing that it is linear with slope $v_I - \bar{\omega}_E$ on the corresponding interval: $T(1 - s_E^1) - T(1-q) = (v_I - \bar{\omega}_E)(q - s_E^1)$ for $q \in I$.

We now prove that the buyer and the entrant, facing the above defined tariff T, agree on the quantity $q_E(s_E, \omega_E)$. We thus have to check that

$$\Delta S_{BE}(s_E, \omega_E) \geq (\omega_E - v_I)q' + T(1) - T(1-q') \tag{B.2}$$

for any $q' \leq s_E$. When q' is the range of the quantity function, we can write $q' = q_E(s'_E, \omega'_E)$ for some (s'_E, ω'_E), with $q' \leq s'_E$. Observing that $q' = q_E(q', \omega'_E)$ and using successively the monotonicity of ΔS_{BE} in s_E and its convexity in ω_E, we get:

$$\begin{aligned} \Delta S_{BE}(s_E, \omega_E) &\geq \Delta S_{BE}(q', \omega_E) \\ &\geq \Delta S_{BE}(q', \omega'_E) + (\omega_E - \omega'_E)q', \end{aligned}$$

which, after replacing $T(1) - T(1-q')$ with its value from (B.1), yields (B.2). To check (B.2) when q' is not in the range of the quantity function (q' belongs to a hole $[s_E^1, s_E^2]$ as explained above), use (B.2) at s_E^1 and the linearity of the tariff between s_E^1 and q'.

B.2 Shape of the boundary function and curvature of the tariff

Lemma B.1 relates the shape of the boundary function Ψ to the curvature of the price schedule T.

Lemma B.1. *The following properties hold:*

1. *If Ψ is increasing (resp. constant) around s_E, then the tariff is strictly convex (resp. linear) around $1 - s_E$.*

2. *If Ψ decreases and is concave around s_E, then the tariff is concave around $1 - s_E$.*

3. *If Ψ decreases and is convex around s_E and s_E is close to a local minimum of Ψ, then the tariff is convex around $1 - s_E$.*

4. *If Ψ has a local maximum at s_E, then the tariff has an inflection point at $1 - s_E$.*

Proof. First, suppose that Ψ is nondecreasing on a neighborhood of s_E. Let s'_E slightly above s_E. Then $q_E = s_E$ is an interior solution of the buyer-entrant pair's problem (7) for s'_E and $\omega_E = \Psi(s_E)$. It follows that the first order condition $\Psi(s_E) - v_I + T'(1-s_E) = 0$ holds, implying property 1 of the lemma. The property holds when Ψ has an upward discontinuity at s_E, in which case the tariff has a convex kink at $1 - s_E$. To illustrate, Figures 13a and 13b consider the case where the boundary line is a nondecreasing step function with two pieces.

Next, suppose that the boundary line decreases around s_E. Here we assume that Ψ is twice differentiable. We denote by $[\sigma(s_E), s_E]$ the set of value s'_E such that $q_E(s'_E, \omega_E) = \sigma(s_E)$, where $\omega_E = \Psi(s_E)$. The buyer-entrant surplus $\Delta S_{BE}(s_E, \omega_E)$ is convex and hence continuous in ω_E. It can be computed slightly below or above $\Psi(s_E)$. At $(s_E, \Psi(s_E))$, the buyer and the entrant are indifferent between quantities s_E and $\sigma(s_E)$:

$$\Delta S_{BE}(s_E, \Psi(s_E)) = [\Psi(s_E) - v_I]\sigma(s_E) - T(1 - \sigma(s_E)) = [\Psi(s_E) - v_I]s_E - T(1 - s_E).$$

Differentiating and using the first-order condition at $\sigma(s_E)$ yields

$$T'(1 - s_E) = -\Psi'(s_E)[s_E - \sigma(s_E)] - \Psi(s_E) + v_I.$$

Differentiating again yields

$$T''(1 - s_E) = \Psi''(s_E)[s_E - \sigma(s_E)] + \Psi'(s_E)[2 - \sigma'(s_E)]. \tag{B.3}$$

In the above equation, the two bracketed terms are nonnegative (use $\sigma' \leq 0$), and the slope Ψ' is negative by assumption, which yields item 2 of the lemma. Around a local minimum of Ψ, Ψ' is small, and the first term is positive, hence property 3. Property 4 follows from items 1 and 2. □

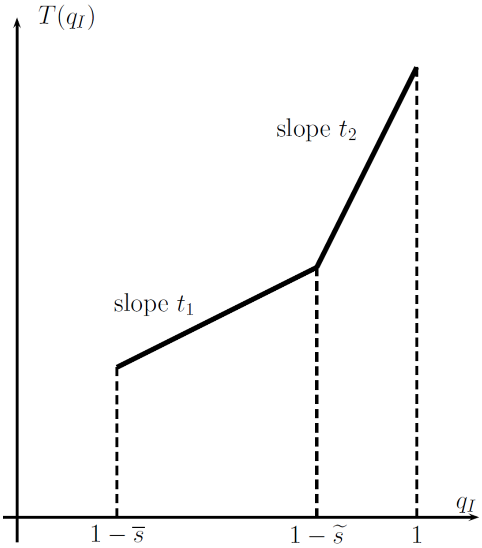

Figure 13a: Convex kink in the price schedule

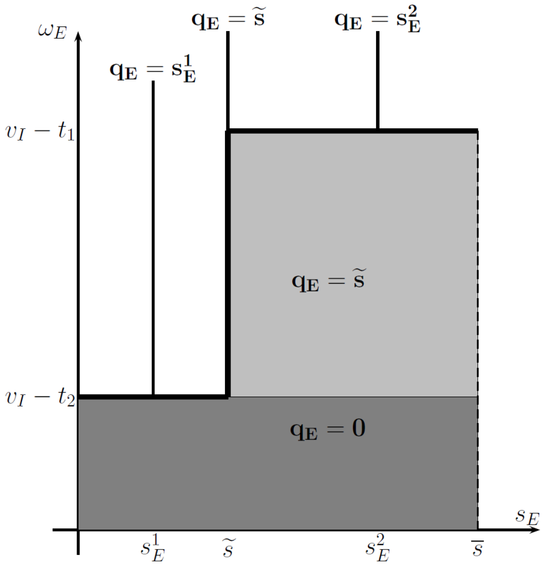

Figure 13b: Two-step increasing boundary line

Finally note that when Ψ has a downward discontinuity at s_E, the tariff has an upward discontinuity at $1 - s_E$. To illustrate, Figures 14a and 14b consider the case where the boundary line is a nonincreasing step function with two pieces.

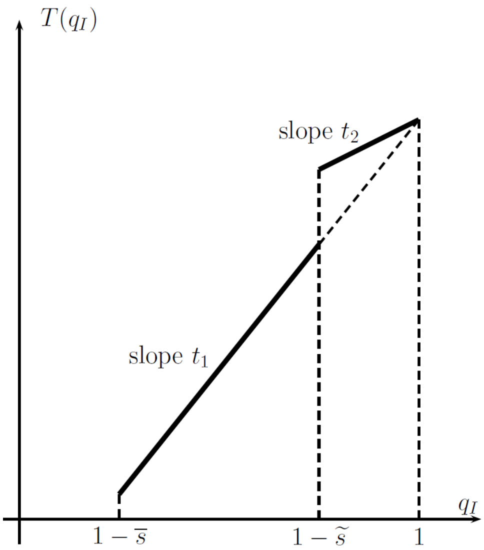

Figure 14a: Upward jump in the price schedule

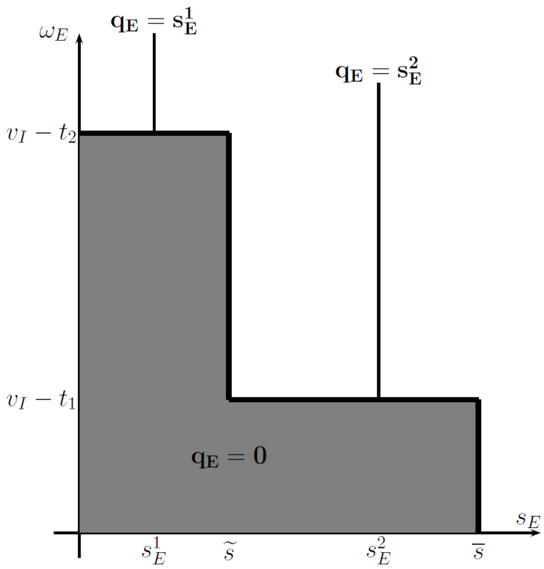

Figure 14b: Two-step decreasing boundary line

C Elasticity of entry and distribution of uncertainty

In this section, we prove Lemma 2. The elasticity of entry varies with s_E in the same way as the hazard rate h given by

$$h(\omega_E|s_E) = \frac{f(\omega_E|s_E)}{1 - F(\omega_E|s_E)}.$$

We have

$$\int_{\underline{\omega}_E}^{\omega_E} h(x|s_E)\,\mathrm{d}x = -\ln[1 - F(\omega_E|s_E)].$$

If the elasticity of entry does not depend on (increases with, decreases with) s_E, the same is true for the hazard rate, and hence also for the cdf $F(\omega_E|s_E)$, which yields the results.[25]

D Derivation of the optimal quantity function

In this section, we formally prove Proposition 4. In Section D.1, we offer a convenient parametrization of horizontal bunching intervals. In Section D.2, we state and prove a one-dimensional optimization result, which serves to maximize the expected virtual surplus for a given level of ω_E. In Section D.3, we rewrite the complete problem as the maximization of the expected virtual surplus under monotonicity constraints. In Section D.4, we show that these constraints are not binding under fairly mild conditions. In Section D.5, we address the case where the monotonicity constraint are binding and two-dimensional bunching occurs.

D.1 Parameterizing horizontal bunching intervals

In this section, we revisit the characterization of implementable function presented in Section 3.3. Consider an implementable quantity function q_E. For any ω_E, the function of one variable $q_E(., \omega_E)$ is nondecreasing on $[0, 1]$, being either constant or equal to the identity map: $q_E = s_E$. By convention, we call regions where it is constant "odd intervals", and regions where $q_E = s_E$ "even intervals".

We are thus led to consider any partition of the interval $[0, 1]$ into "even intervals" $[s_{2i}, s_{2i+1})$ and "odd intervals" $[s_{2i+1}, s_{2i+2})$, where (s_i) is a finite, increasing sequence with first term zero and last term one.[26] We associate to any such partition the function of one variable that coincides with the identity map on even intervals, is constant on odd intervals, and is continuous at odd extremities. We denote by K the set of the functions thus obtained.

[25] The variable ω_E first-order stochastically decreases (increases) with s_E if and only if $F(\omega_E|s_E)$ increases (decreases) with s_E.

[26] For notational consistency, we denote the first term of the sequence by $s_0 = 0$ if the first interval is even and by $s_1 = 0$ if the first interval is odd. Similarly, we denote the last term by $s_{2n} = 1$ if the last interval is odd and by $s_{2n+1} = 1$ if the last interval is even.

For any implementable quantity function q_E, the functions of one variable, $q_E(., \omega_E)$, belong to K for all ω_E. Conversely, any quantity function such that $q_E(., \omega_E)$ belong to K for all ω_E is implementable if and only if even (odd) extremities do not increase (decrease) as ω_E rises. Hereafter, we call the conditions on the extremities the "monotonicity constraints".

Even (odd) extremities constitute decreasing (increasing) parts of the boundary line. Odd intervals, $[s_{2i+1}, s_{2i+2})$, constitute horizontal bunching segments, or, more precisely, the horizontal portions of the L-shaped bunching regions.

D.2 A one-dimensional optimization result

In this section, we maximize a linear integral functional on the above-defined set K.

Lemma 4. *Let $a(.)$ be a continuous function on $[0, 1]$. Then the problem*

$$\max_{r \in K} \int_0^1 a(s) r(s) \, \mathrm{d}s$$

admits a unique solution r^* characterized as follows. For any interior even extremity s_E^{2i}, the function a equals zero at s_E^{2i} and is negative (positive) at the left (right) of s_E^{2i}. For any interior odd extremity s_E^{2i+1}, the function a is positive at s_E^{2i+1} and satisfies

$$\int_{s_E^{2i+1}}^{s_E^{2i+2}} a(s) \, \mathrm{d}s = 0. \tag{D.1}$$

If $a(1) > 0$, then $r^*(s) = s$ at the top of the interval $[0, 1]$. If $a(1) < 0$, then r^* is constant at the top of the interval.

Proof. Letting $I(r) = \int_0^1 a(x) r(x) \, \mathrm{d}x$, we have

$$I(r) = \sum_i \int_{x_{2i}}^{x_{2i+1}} x a(x) \, \mathrm{d}x + \sum_i x_{2i+1} \int_{x_{2i+1}}^{x_{2i+2}} a(x) \, \mathrm{d}x,$$

where the index i in the two sums goes from either $i = 0$ or $i = 1$ to either $i = n - 1$ or $i = n$, in accordance with the conventions exposed in Footnote 26. Differentiating with respect to an interior even extremity yields

$$\frac{\partial I}{\partial x_{2i}} = a(x_{2i}).[x_{2i-1} - x_{2i}].$$

The first-order condition therefore imposes $a(x_{2i}^*) = 0$. The second-order condition for a maximum shows that a must be negative (positive) at the left (right) of x_{2i}^*.

Differentiating with respect to an interior odd extremity yields

$$\frac{\partial I}{\partial x_{2i+1}} = \int_{x_{2i+1}}^{x_{2i+2}} a(x) \, \mathrm{d}x.$$

The first-order condition therefore imposes $\int_{x_{2i+1}^*}^{x_{2i+2}^*} a(x) \, \mathrm{d}x$. The second-order condition for a maximum imposes that a is nonnegative at x_{2i+1}^*.

If $a(1) > 0$, then it is easy to check that $r^*(x) = x$ at the top, namely on the interval $[x^*_{2n}, x^*_{2n+1}]$ with x^*_{2n} being the highest zero of the function a and $x^*_{2n+1} = 1$. If the function a admits no zero, it is everywhere positive and hence $r^*(x) = x$ on the whole interval $[0, 1]$.

If $a(1) < 0$, then r^* is constant at the top, namely on the interval $[x^*_{2n-1}, x^*_{2n}]$, with $x^*_{2n} = 1$ and $\int_{x^*_{2n-1}}^1 a(x)\,\mathrm{d}x = 0$. If the integral $\int_y^1 a(x)\,\mathrm{d}x$ remains negative for all y, then r^* is constant and equal to zero on the whole interval $[0, 1]$. $\square$

D.3 Solving the complete problem

The complete problem consists in maximizing the expected virtual surplus subject to the even (odd) extremities being nonincreasing (nondecreasing). The latter conditions are called hereafter the "monotonicity constraints".

Applying Lemma 4 with $a(s_E) = s^{\mathrm{v}}(s_E, \omega_E)$ for any given ω_E, we find that the virtual surplus is zero at candidate even extremities: $s^{\mathrm{v}}(x_{2i}(\omega_E), \omega_E) = 0$ and is negative (positive) at the left (right) of these extremities. In other words, candidate even extremities belong to decreasing parts of the ERT line. Thus, as regards even extremities, the monotonicity constraints are never binding.

Lemma 4 also implies that the virtual surplus is positive at odd extremities. These extremities therefore lie above the ERT line. By the first-order condition (D.1), the expected virtual surplus is zero on horizontal bunching intervals:

$$\mathbb{E}(s^{\mathrm{v}}|H) = 0, \tag{D.2}$$

where H is a horizontal bunching interval with extremities s_E^{2i+1} and s_E^{2i+2}. The virtual surplus on a bunching interval is first positive, then negative as s_E rises, and its mean on the interval is zero. The segment $[AB]$ on Figure 10b is an example of horizontal bunching interval (in fact the horizontal part of an "L"-shaped bunching set). Unfortunately, the first-order condition (D.2) does not imply that candidate odd extremities $x_{2i+1}(\omega_E)$ are nondecreasing in ω_E: odd extremities might decrease with ω_E in some regions, generating two-dimensional bunching.

D.4 Sufficient conditions for nondecreasing odd extremities

We introduce three sets of conditions:

1. The conditional density $f(\omega_E|s_E)$ is nondecreasing in ω_E;
2. The hazard rate, $f/(1-F)$, is nondecreasing in ω_E and β, $\underline{\varepsilon}$ and $\bar{\varepsilon}$ satisfy
$$\beta \leq 4\underline{\varepsilon}\bar{\varepsilon}/(\Delta\varepsilon)^2; \tag{D.3}$$
3. The elasticity of entry is nondecreasing in ω_E (Assumption 2) and and β, $\underline{\varepsilon}$ and $\bar{\varepsilon}$ satisfy
$$\beta \leq \frac{\bar{\varepsilon}}{1 + (1+\Delta\varepsilon)^2/4\underline{\varepsilon}}. \tag{D.4}$$

We now check that the odd extremities, $s_E^{2i+1}(\omega_E)$, are nondecreasing in ω_E when one of the above set of conditions holds.

We can restrict attention to efficient rivals, $\omega_E \geq \omega_I$.[27] We rewrite equation (D.2) as $A(s_E^{2i+1}, \omega_E) = 0$ with

$$\begin{aligned} A(s_E^{2i+1}, \omega_E) &= \int_{s_E^{2i+1}}^{s_E^{2i+2}} s^{\mathrm{v}}(s, \omega_E) f(\omega_E|s) g(s)\, \mathrm{d}s \\ &= \int_{s_E^{2i+1}}^{s_E^{2i+2}} \left[(\omega_E - \omega_I) f(\omega_E|s) - \beta(1 - F(\omega_E|s))\right] g(s)\, \mathrm{d}s. \end{aligned}$$

The function A is nonincreasing in s_E^{2i+1}, as the virtual surplus is nonnegative at this point:

$$\frac{\partial A}{\partial s_E^{2i+1}}(s_E^{2i+1}, \omega_E) = -s^{\mathrm{v}}(s_E^{2i+1}, \omega_E) f(\omega_E|s_E^{2i+1}) g(s_E^{2i+1}) \leq 0.$$

Differentiating with respect to ω_E, we get

$$\frac{\partial A}{\partial \omega_E}(s_E^{2i+1}, \omega_E) = \int_{s_E^{2i+1}}^{s_E^{2i+2}} \left[(\omega_E - \omega_I) f'(\omega_E|s) + f(\omega_E|s) + \beta f(\omega_E|s)\right] g(s)\, \mathrm{d}s,$$

where we denote by f' the derivative of f in ω_E.

When f is nondecreasing in ω_E, or $f' \geq 0$, we have $\partial A/\partial \omega_E \geq 0$, and hence the odd extremities are nondecreasing in ω_E. We now examine successively the cases where the hazard rate is nondecreasing in ω_E (a weaker condition than $f' \geq 0$) and the elasticity of entry is nondecreasing in ω_E (an even weaker condition).

D.4.1 Assuming that the hazard rate does not decrease in ω_E

We now assume that the hazard rate, $f/(1-F)$, is nondecreasing in ω_E, which can be expressed as $f' \geq -\varepsilon f/\omega_E$. Using $\omega_E \geq \omega_I$, we find that

$$\begin{aligned} \frac{\partial A}{\partial \omega_E} &\geq \int_{s_E^{2i+1}}^{s_E^{2i+2}} \left[-(\omega_E - \omega_I)\frac{\varepsilon}{\omega_E} + 1 + \beta\right] f(\omega_E|s) g(s)\, \mathrm{d}s \\ &= \int_{s_E^{2i+1}}^{s_E^{2i+2}} \left\{\varepsilon \left[\frac{\omega_I}{\omega_E} - 1 + \frac{\beta}{\varepsilon}\right] + 1\right\} f(\omega_E|s) g(s)\, \mathrm{d}s. \end{aligned}$$

On a horizontal interval H, the variable ω_E is constant, and only the elasticity ε may vary. Hence, the first order condition (D.2) yields: $\mathbb{E}(1 - \beta/\varepsilon \,|\, H) = \omega_I/\omega_E$. The right-hand side of the above inequality is equal, up to a positive multiplicative constant, to

$$1 - \operatorname{cov}\left(\varepsilon, 1 - \frac{\beta}{\varepsilon}\,\middle|\, H\right).$$

[27] For $\omega_E < \omega_I$, the virtual surplus is negative for all s_E and the solution is $q_E = 0$ for all s_E.

We now look for a sufficient condition for this expression to be nonnegative for any distribution of ε. Noting $m = \mathbb{E}(\varepsilon|H)$ the expectation of ε on H, the condition can be rewritten as

$$\mathbb{E}\left[(\varepsilon - m)\left(1 - \frac{\beta}{\varepsilon}\right)\middle| H\right] \leq 1.$$

The function $(\varepsilon - m)(1 - \beta/\varepsilon)$ is convex in ε. We denote by $[\underline{\varepsilon}, \bar{\varepsilon}]$ the support of the distribution of ε. For given values of $\underline{\varepsilon}$, $\bar{\varepsilon}$ and $m = \mathbb{E}(\varepsilon|H)$, the expectation of this convex function is maximal when the distribution of ε has two mass points at $\underline{\varepsilon}$ and $\bar{\varepsilon}$, associated with the respective weights $\frac{\bar{\varepsilon}-m}{\bar{\varepsilon}-\underline{\varepsilon}}$ and $\frac{m-\underline{\varepsilon}}{\bar{\varepsilon}-\underline{\varepsilon}}$. We thus need to make sure that

$$(\bar{\varepsilon} - m)(\underline{\varepsilon} - m)\left(1 - \frac{\beta}{\underline{\varepsilon}}\right) + (m - \underline{\varepsilon})(\bar{\varepsilon} - m)\left(1 - \frac{\beta}{\bar{\varepsilon}}\right) \leq \bar{\varepsilon} - \underline{\varepsilon},$$

for any $m \in [\underline{\varepsilon}, \bar{\varepsilon}]$. The left-hand side of the above inequality is maximal for $m = (\underline{\varepsilon}+\bar{\varepsilon})/2$. It follows that the inequality holds for all $m \in [\underline{\varepsilon}, \bar{\varepsilon}]$ if and only if the condition (D.3) is satisfied.

D.4.2 Assuming that the elasticity of entry does not decrease in ω_E

We now assume that the $\varepsilon(\omega_E|s_E)$ is nondecreasing in ω_E, as stated in Assumption 2. We have:

$$\frac{\partial\varepsilon(\omega_E|s_E)}{\partial\omega_E}(s_E^{2i+1}, \omega_E) = \frac{\partial}{\partial\omega_E}\left[\frac{\omega_E f(\omega_E|s_E)}{1 - F(\omega_E|s_E)}\right] \geq 0$$

which can be rewritten as $f' \geq -(1+\varepsilon)f/\omega_E$. Using $\omega_E \geq \omega_I$, we find that

$$\frac{\partial A}{\partial\omega_E} \geq \int_{s_E^{2i+1}}^{s_E^{2i+2}} \left[\frac{\omega_I}{\omega_E} - \varepsilon\left(1 - \frac{\beta}{\varepsilon} - \frac{\omega_I}{\omega_E}\right)\right] f(\omega_E|s)g(s)\,\mathrm{d}s.$$

On a horizontal interval H, the variable ω_E is constant, and only the elasticity ε may vary. Hence, the first order condition (D.2) yields: $\mathbb{E}(1 - \beta/\varepsilon \,|\, H) = \omega_I/\omega_E$. The right-hand side of the above inequality is equal, up to a positive multiplicative constant, to

$$\mathbb{E}\left(1 - \frac{\beta}{\varepsilon}\middle| H\right) - \mathrm{cov}\left(\varepsilon, 1 - \frac{\beta}{\varepsilon}\middle| H\right).$$

We now look for a sufficient condition for this expression to be nonnegative for any distribution of ε. Noting $m = \mathbb{E}(\varepsilon|H)$ the expectation of ε on H, the condition can be rewritten as

$$\mathbb{E}\left[(\varepsilon - m - 1)\left(1 - \frac{\beta}{\varepsilon}\right)\middle| H\right] \leq 0.$$

The function $(\varepsilon - m - 1)(1 - \beta/\varepsilon)$ is convex in ε. We denote by $[\underline{\varepsilon}, \bar{\varepsilon}]$ the support of the distribution of ε. For given values of $\underline{\varepsilon}$, $\bar{\varepsilon}$ and $m = \mathbb{E}(\varepsilon|H)$, the expectation of this convex function is maximal when the distribution of ε has two mass points at $\underline{\varepsilon}$ and $\bar{\varepsilon}$, associated with the respective weights $\frac{\bar{\varepsilon}-m}{\bar{\varepsilon}-\underline{\varepsilon}}$ and $\frac{m-\underline{\varepsilon}}{\bar{\varepsilon}-\underline{\varepsilon}}$. We thus need to make sure that

$$(\bar{\varepsilon} - m)(\underline{\varepsilon} - m - 1)\left(1 - \frac{\beta}{\underline{\varepsilon}}\right) + (m - \underline{\varepsilon})(\bar{\varepsilon} - m - 1)\left(1 - \frac{\beta}{\bar{\varepsilon}}\right) \leq 0, \tag{D.5}$$

for any $m \in [\underline{\varepsilon}, \bar{\varepsilon}]$. The above function is the sum of two quadratic functions of m. The first is convex with roots $\underline{\varepsilon} - 1$ and $\bar{\varepsilon}$; the second is concave with roots $\underline{\varepsilon}$ and $\bar{\varepsilon} - 1$. Both quadratic functions have zero derivative at $m = (\underline{\varepsilon} + \bar{\varepsilon} - 1)/2$. The sum of the two functions is concave as $\underline{\varepsilon} < \bar{\varepsilon}$.

When $\bar{\varepsilon} \leq \underline{\varepsilon} + 1$, the concave quadratic function is negative on the interval $[\underline{\varepsilon}, \bar{\varepsilon}]$, and hence the inequality (D.5) holds on that interval. When $\bar{\varepsilon} > \underline{\varepsilon} + 1$, we need to make sure that the maximum value of the concave quadratic function is lower than the minimum value of the convex quadratic function. This is is the case if and only if

$$\left(1 - \frac{\beta}{\bar{\varepsilon}}\right) (\Delta\varepsilon - 1)^2 \leq \left(1 - \frac{\beta}{\underline{\varepsilon}}\right) (\Delta\varepsilon + 1)^2.$$

which is equivalent to (D.4).

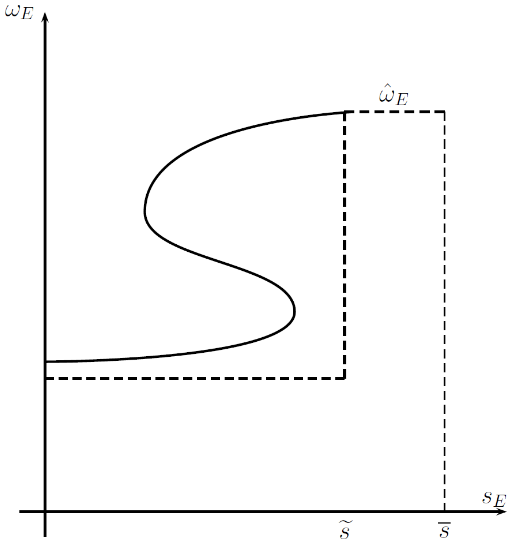

Figure 15a: ERT line (dashed). Non-monotonic odd extremities (solid line)

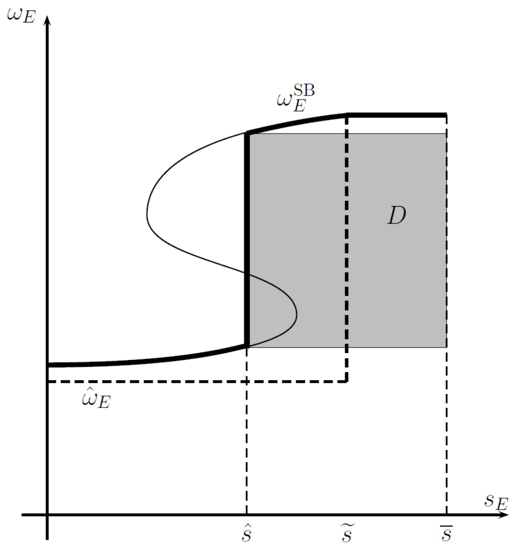

Figure 15b: Two-dimensional bunching area: $q_E = \hat{s}$ on D.

D.5 Two-dimensional bunching

When none of the above sufficient conditions holds, it may happen that solving the problem separately for each ω_E yields odd extremities (left extremities of horizontal bunching segments) that are non-monotonic with ω_E, as represented on Figure 15a. Such a line does not define a boundary function $\Psi(s_E)$. This means that the monotonicity constraints are binding and that the optimal boundary line has an increasing vertical portion, generating a two-dimensional pooling area. An example of such an area is the shaded region D pictured on Figure 15b, on which the quantity is constant. The value of the constant ($\hat{s}$ on the picture) is determined by the first-order condition

$$\mathbb{E}(s^{\mathrm{v}}|D) = 0.$$

This example has been constructed by assuming that (i) ω_E follows a Pareto distribution conditionally on s_E, for all s_E; (ii) the elasticity of entry takes two values, $\underline{\varepsilon}$ and $\bar{\varepsilon}$, with a large difference $\bar{\varepsilon} - \underline{\varepsilon}$; (iii) small rivals are very sensitive to the competitive pressure placed by the incumbent (their elasticity is $\bar{\varepsilon}$) and large rivals are much less sensitive (their elasticity is $\underline{\varepsilon}$). Hence the increasing ERT line with two pieces.

E Finite disposal costs

E.1 The buyer's total purchases are not lower than her total demand

In this section, we extend Lemma A.1 to the case of finite disposal costs. Using the definition of V, equation (17), and applying the envelope theorem, we get

$$\frac{\partial V}{\partial q_I}(q_E, q_I) = \mu - \gamma = v_I - \nu,$$

where $\mu \geq 0$ and $\nu \geq 0$ are the respective Lagrange multipliers for the constraints $x_I \leq q_I$ and $x_E + x_I \leq 1$ in the buyer's problem (17).

The proof of Lemma A.1 follows the same route as in Appendix A. The only needed modification in the proof consists in replacing equality (A.3) with the inequality

$$V(q_E, \tilde{q}_I) - T(q_I) - v_I(\tilde{q}_I - q_I) \leq V(q_E, q_I) - T(q_I),$$

where we have used $\partial V/\partial q_I \leq v_I$. This inequality is enough to guarantee $U_B(q_E) = \tilde{U}_B(q_E)$ for all q_E. Equations (A.4) and (A.5) continue to hold as equalities because $\partial V/\partial q_I = v_I$ in the region where q_I is below $1 - q_E$.

E.2 The slope of the tariff is above $-\gamma$

In this section, we prove Proposition 6. Starting from any tariff T, we define $\hat{T}$ as

$$\hat{T}(q_I) = \inf_{q \geq q_I} T(q) + \gamma(q - q_I).$$

Starting from any quantity level q, the incumbent offers the incumbent the opportunity to buy less units than q, $q_I \leq q$, in return for the payment $T(q) + \gamma(q - q_I)$. This option allows the buyer to avoid disposal costs, and is relevant only if γ is finite. The slope of the new tariff is larger than or equal to $-\gamma$.

Let $\hat{U}(q_E)$ the buyer's net utility after she has purchased units q_E units from the entrant under the price schedule $\hat{T}$:

$$\hat{U}_B(q_E) = \max_{q_I} V(q_E, q_I) - \hat{T}(q_I). \tag{E.1}$$

As $\hat{T} \leq T$, we have: $\hat{U}_B \geq U_B$. Suppose that, under $\hat{T}$, it is optimal for the buyer to purchase $\hat{q}_I$ from the incumbent if she has purchased q_E from the entrant. By construction of $\hat{T}$, there exists $q_I \geq \hat{q}_I$ such that $\hat{T}(\hat{q}_I)$ equals or is arbitrarily close to $T(q_I)+\gamma(q_I-\hat{q}_I)$. Using the definition of V, we get:

$$\begin{aligned}\hat{U}_B(q_E) &= V(\hat{q}_I, q_E) - \hat{T}(\hat{q}_I)\\ &= V(\hat{q}_I, q_E) - \gamma(q_I - \hat{q}_I) - T(q_I)\\ &\leq V(q_I, q_E) - T(q_I).\end{aligned}$$

It follows that $\hat{U}_B(q_E) \leq U_B(q_E)$, and hence $\hat{U}_B(q_E) = U_B(q_E)$. The buyer and the entrant agree on the same quantity q_E as their choice only depends on U_B and $\hat{U}_B$, which coincide. The entrant's profit, $\beta\Delta S_{BE}$, is the same under T and $\hat{T}$.

Suppose that the buyer has purchased q_E from the entrant and let q_I be the optimal quantity purchased from the incumbent under tariff T. As $\hat{T}(q_I) \leq T(q_I)$, the buyer may always choose to purchase the same quantity from the incumbent ($\hat{q}_I = q_I$) under the tariffs $\hat{T}$ and T:

$$U_B(q_E) = \tilde{U}_B(q_E) = V(q_E, q_I) - T(q_I) \leq V(q_E, q_I) - \hat{T}(q_I).$$

Now consider the case where $q_I > 1 - q_E$. Since, by assumption, the buyer has purchased q_E from the rival, it must be the case that $x_E = q_E$, otherwise reducing q_E would increase the joint surplus of the buyer an the rival. It follows from $q_I > 1 - q_E$ and $x_E + x_I \leq 1$ that x_I must be smaller than q_I, hence $\mu = 0$, implying that the derivative of V with respect to q_I equals and $-\gamma$. By definition of $\hat{T}(1 - q_E)$, we get

$$V(q_E, q_I) - V(q_E, 1 - q_E) = -\gamma[q_I - (1 - q_E)] \leq T(q_I) - \hat{T}(1 - q_E)$$

or

$$U_B(q_E) = \hat{U}(q_E) = V(q_E, q_I) - T(q_I) \leq V(q_E, 1 - q_E) - \hat{T}(1 - q_E).$$

It follows that the buyer may purchase $\hat{q}_I = 1 - q_E$ from the incumbent. The change from q_I to $\hat{q}_I$ does not decrease the total surplus. On the contrary, it avoids production and disposal costs:

$$V(q_E, \hat{q}_I) - c_E q_E - C_I(\hat{q}_I) \geq V(q_E, q_I) - c_E q_E - C_I(q_I).$$

In sum, the change from T to $\hat{T}$ does not alter the entrant's profit and does not decrease the total surplus. We conclude from (4) that the change does not decrease the expected payoff of the buyer-incumbent coalition.

E.3 Implementability under finite disposal costs

In this section, we prove the sufficient part of lemma 3. Assume that $\Psi(s_E) \leq v_I + \gamma$, and define the quantity function by (10), the profit function $\Delta S_{BE}(s_E, \omega_E)$ by equation (8), and the tariff by (B.1).

Differentiating (B.1) with respect to ω_E below the boundary line, a region where q_E increases with ω_E, yields

$$T'(q)\frac{\partial q}{\partial \omega_E} = (v_I - \omega_E)\frac{\partial q}{\partial \omega_E} - q + q,$$

and hence $T'(q) = v_I - \omega_E \geq -\gamma$. Differentiating (B.1) with respect to s_E above the boundary line, a region where $q_E = s_E$, yields

$$T'(s_E) = (v_I - \omega_E) + \frac{\partial \Delta S_{BE}}{\partial s_E} \geq v_I - \omega_E \geq -\gamma,$$

because ΔS_{BE} is nondecreasing in s_E.

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