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Dissipating Advertising Signals Quality Even Without Repeat Purchases*

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Abstract

Economists have emphasized the role of dissipative advertising and price as signals of quality. Most works, however, limit the number of types to two options: high and low quality. Yet, production costs and quality both result from R&D efforts and therefore are both uncertain. I characterize the optimal separating marketing mix (price and advertising) when quality and marginal cost are both subject to chance. In a static framework (no repeat purchases and no informed consumers), advertising appears to be necessary together with price to signal quality. Equilibrium profits depend on cost but not on quality: all rents are dissipated for signaling purpose.

Keywords: Quality, signaling, Dissipative Advertising.
***JEL* Classification: L120, L150, M370**

Résumé

Le rôle de la publicité dissipative (pure dépense observable par tous) dans le signal de la qualité d'un produit a été mis en avant à plusieurs reprises par les économistes. L'essentiel des travaux sur la question font l'hypothèse simplificatrice que la qualité à signaler ne peut prendre que deux valeurs: haute ou basse. L'analyse développée dans cet article tend à montrer qu'il s'agit d'une hypothèse restrictive qui tend à cacher le caractère optimal du recours à la publicité dans un contexte statique. A priori, les coûts de production, comme la qualité résultent d'effort en recherche et développement et ces efforts sont par nature même aléatoire. Il en résulte qu'à une qualité donnée plusieurs niveaux de coût sont possibles et non pas un seul comme cela est généralement supposé. Cet article caractérise la stratégie séparatrice optimale (un couple prix, dépense de publicité) lorsque la qualité et le coût marginal sont tous les deux aléatoires et que seule la firme (en situation de monopole) connaît leur valeurs. En l'absence d'achats répétés et de consommateurs informés, la publicité est nécessaire pour signaler la qualité.

Mots clefs: Qualité, signal, Publicité dissipative.
Classification du *JEL* : L120, L150, M370

1 Introduction

The producer of a new experience good of high quality has to signal its status to consumers.[1] For some products, direct and credible signals of quality as certification or warranties are available. Otherwise, a firm has to resort to indirect signals. Following Nelson (1974) and Milgrom and Roberts (1986) the economic literature emphasizes the role of dissipative advertising and price as signals of quality. Dissipative advertising is like burning money in public: it does not directly affect demand (neither persuasive nor informative content),[2] and it is observed by all potential buyers. Nelson, Milgrom and Roberts, and others use the generic example of a celebrity endorsing a brand in a commercial rather than an anonymous actress/actor. In Nelson's argument, advertising is a quality signal because only a higher quality firm finds it profitable to spend a large amount on advertising thanks to repeat purchases. In the short run, advertising attracts consumers whether quality is high or low but its cost, if large enough, cannot be recouped immediately. Only through repeat purchases can a high quality firm cash in on advertising while repeat purchases are not accessible to a flight by night lower quality firm.

In this paper, repeat purchases are absent or too far away.[3] In a static framework, a firm can still resort to advertising when a proportion of consumers is informed about quality as in Linnemer (2002) in a model à la Bagwell and Riordan (1991). However, without informed consumers, the analysis of the two type model (See Bagwell (2007a)) shows that dissipative advertising is not an effective signal.

The classical model postulates two quality levels (high and low) and one marginal cost of production for each level. That is, a one to one relationship between quality and cost is assumed.[4] Yet, even if marginal cost can, on average, be expected to increase with quality, a one to one relationship is a very restrictive assumption. In particular, for a new product (for which the problem of signaling quality is the most acute) the cost of production results of uncertain R&D efforts and for a given quality, marginal costs can be, *a priori*, more or less important. Even if the chances are small, it is possible that a higher quality good costs less than a lower quality one. Or the other way around: for a given cost, different qualities

[1] A good is said to be an experience good if its quality is known only after consumption (see Nelson (1970)). If quality is not even disclosed after consumption (some medical acts, or mechanical repairs) the good is referred to as a credence good (Darby and Karni (1973)), see Dulleck and Kerschbamer (2006).

[2] The bare fact that advertising contains little information is observed for many products besides experience goods. Anderson and Renault (2006) model explicitly advertising content in a search model where consumers are imperfectly informed about the characteristics (price and matching value) of a good unless they go to the shop (at a cost) or they receive informative enough advertising. They show that a monopolist prefers to convey only limited product information if possible.

[3] For example, for a durable good. But also when repeat purchases and future purchases in general are not linked to the number of consumers who buy in the introductory period. For instance, if information about the product quality spreads out very quickly through word of mouth or consumer reports. Then the problem of a high quality firm is only to signal itself in the short run. Finally, the static model applies to a credence good.

[4] Typically, the relationship is increasing: the higher the quality the higher the cost. However, as marginal cost can also be decreasing with quality (for example when a new technology is introduced) a decreasing relationship has also been considered. Wilson (1985) extends Milgrom and Roberts (1986) to a continuum of types but he still assumes an increasing (one to one) relationship between marginal cost and quality.

can be expected.

To take into account a variance in the distribution of marginal costs, for a given quality, more than two types are needed and types should be differentiated in both dimensions: quality and marginal cost. *A priori*, there is no reason to restrict the number of types and the most direct (if not natural) way is to assume a type to be a pair (c, q) composed of a marginal cost c and a quality level q. Both c and q are allowed to vary continuously. Consumers are able to observe neither c nor q but share *a priori* beliefs about their joint distribution and revise these beliefs after observing the price and advertising expenditures.

In such a static model, the profit function of a high quality type does not directly depends on its quality but only of consumers' beliefs about it. This makes imitation fierce. In comparison, in Milgrom and Roberts' model repeat purchases imply that the profit function depends both on consumers' beliefs and on the true quality (through the repeat purchases). In Bagwell and Riordan's model also profits depends on both beliefs and true quality due to the informed consumers. In the static framework only beliefs matter which makes imitation easier and separation more costly.[5]

The main result of the paper is that the additional uncertainty in terms of marginal costs makes dissipative advertising a necessary signal. That the monopoly has to resort to advertising contrasts with the two-type (but otherwise similar) model where (generically) advertising is not used. Therefore, it provides a new (and somehow simpler) explanation of Nelson's idea that advertising is a signal of quality.

Besides advertising, separation entails a distortion in price from the perfect information monopoly price. In the two-types static model, the price is distorted upwards (resp. downwards) if the low-quality marginal cost is lower (resp. greater) than the high-quality marginal cost. The intuition being that a price increase (resp. decrease) is more costly for a low (resp. high) marginal cost firm. With a continuum of low-quality marginal costs this intuition fails as a high quality type needs to differentiate from below and from above. Yet, the direction of the price distortion can be determined. Under the (rather natural) assumption that (for a given marginal cost) the monopoly quantity increases with quality, the price is distorted upwards: The separating price is larger than the perfect information price.[6]

In any case, the separating price increases with both marginal cost and quality. The equilibrium amount of advertising increases with quality (for a given marginal cost) and decreases with marginal cost (for a given quality). More efficient types (in terms of both lower cost and higher quality) advertise more. Finally, at the unique separating equilibrium of the game, all rents due to a higher quality are dissipated: the equilibrium profit depends only on the marginal cost but not on the quality.

Even if for a given price, consumers care only about quality, they use their information about the potential value of the firm's marginal cost to infer quality and in equilibrium types are separating along both cost and quality dimensions. Hence the paper contributes to the small literature on signaling multiple dimensions with many signals. Quinzii and Rochet (1985) focus on a Spence signalling model with a discrete number of characteristics

[5] I am very grateful to Kyle Bagwell for pointing out this property.

[6] The reverse holds if the monopoly quantity decreases with quality.

and a corresponding number of investments in education. They characterize the separating equilibrium for signalling cost function linear and separable in the signals. Their analysis is extended in Engers (1987). More recently and in relation with the advertising, in Linnemer (1998) and Bagwell (2007b) limit-pricing games are studied with bi-dimensional incumbent's types. In Linnemer (1998) an incumbent can be of low or high quality and if of high quality of low or high cost. Two kind of hidden information have to be signalled to two different audiences: quality to the consumers and marginal cost to a potential entrant. Two signals are used: price and dissipative advertising. In Bagwell (2007b), the incumbent can be more or less patient (two possible values of the discount factor) and its marginal cost can be either low or high. A low-cost incumbent would like to signal it to a potential entrant. The entrant is not directly interested by the level of patience of the incumbent but this dimension of the private information plays an important role for the characterization of the equilibria. Two signals are also used: price and demand-enhancing advertising.

Literature related to Nelson (1974):[7] Schmalensee (1978) challenges Nelson's reasoning on the grounds that consumers rationality is bounded. Kihlstrom and Riordan (1984) and Milgrom and Roberts (1986) embed Nelson's idea in formal signaling games.[8] In particular, Milgrom and Roberts dissect how a monopoly uses both price and advertising to signal its quality while maximizing its profit. Milgrom and Roberts is a keystone paper in the literature on advertising as a signal as it shows how to select (quite generally) a unique separating equilibrium from many through the elimination of dominated strategies. Milgrom and Roberts (1986) (section II) analyses a specific example where the authors do not assume a particular ranking of the marginal costs. In the presence of repeat purchases, they show that if the high quality marginal cost is lower than the low quality one then a low price without any advertising signals quality. When the marginal cost increases with quality, however, advertising, in addition to price, can help to optimally signal quality.[9] On the dissenting side Hertzendorf (1993) argues that, if advertising is imperfectly observed by consumers, then price and advertising expenditure cannot be used in combination. Horstmann and MacDonald (1994) put forward an alternative model of quality signaling by assuming imperfect consumer learning and show that advertising signals quality rather for established products than newly introduced ones.

Fluet and Garella (2002) and Hertzendorf and Overgaard (2001) present a duopoly model where firms know each other's types and where price and advertising might be used in a separating equilibrium.[10]

[7] For a thorough survey of this literature see Bagwell (2007a), in particular section 6 on quality.

[8] In a monopolistic kind of competition model, Kihlstrom and Riordan (1984) distinguish two cases: lower or larger marginal cost for high quality relatively to low quality (overall due to fixed cost of production high quality is more costly). They find that if high quality increases fixed costs but not marginal costs (relative to low quality) then advertising can be part of a signal in equilibrium. If, however, higher quality raises marginal costs, then advertising is not used in equilibrium.

[9] Moreover, they also introduce the use of the Cho and Kreps (1987) criterion to eliminate pooling equilibria.

[10] In Daughety and Reinganum (2007b) firms signal their quality (low or high) in an oligopoly framework using only prices (the model is static). All low quality (resp. high quality) firms have the same marginal cost. Contrary to the papers of Fluet and Garella and Hertzendorf and Overgaard, a firm does not know the types of its rivals. See also, Daughety and Reinganum (2007a) where similar price strategies signal quality

On the empirical front, several old studies (see Bagwell (2007a), section 3.2.5 for more details) provide only mixed support for the role of price and advertising as signals of quality. This does not necessarily come as a surprise given that the quality-signaling theory is mainly relevant for new products (at least for products for which a large proportion of consumers are not aware of quality) and as pointed out by Horstmann and MacDonald (2003) because it is difficult to assess quality (that is the experience quality component of the quality of a product: a good can have very high consumer report rating, for example, because of high search quality. In most of the goods search and experience quality coexist.) Yet, at least three recent studies support the idea that dissipative advertising plays a role in signaling quality. Thomas, Shane, and Weigelt (1998) use data on the U.S. automobile industry and find that manufacturers resort to both price and advertising to signal the quality of their product. Horstmann and MacDonald (2003) examine data they have collected on compact disc players over 1983-1992. Their focus is on the dynamics of price and advertising through time. They find that price falls with the age of the product and that advertising follows an inverted U-shape (these results are in line with the theoretical prediction of Linnemer (2002)). Iizuka (2004) uses a panel data set containing more than 600 drug-year observations over 1996-1999. His purpose is not to test Nelson's theory but he finds that firms are more likely to advertise newer and higher quality drugs rather than older and lower quality ones.

The paper is organized as follows: Section 2 introduces the model and the structural assumptions. Section 3 shows the existence of a unique separating equilibrium which is characterized. A comparative static analysis is performed in Section 4. Conclusions are drawn in Section 5.

2 The model

A firm has just developed a new product for which it enjoys a monopoly situation. The quality of the good can be more or less important. Moreover, for each quality level, the marginal cost of production can take several values. The extensive form of the game is illustrated in Figure 1.

Nature chooses the type $t = (c, q)$ of the firm where q denote the quality of the good and c the marginal cost of production which is assumed to be constant. The type t is distributed on the set $T = [\underline{c}, \overline{c}] \times [0, 1]$ according to an *a priori* distribution function $F(.,.)$ such that any pair (c, q) has a positive probability to be chosen by Nature.

The origin of the studied problem lies in the asymmetric information about the type of the firm. The firm knows its type t while consumers are only aware of the *a priori* distribution of t. The firm strategy is to choose a price p and an advertising expenditure A. Consumers observe the pair (p, A) and revise in a Bayesian way their initial beliefs

in a duopoly (no dissipative advertising is allowed). In both Daughety and Reinganum's papers, the price increase needed to signal quality is like a collusive device as from the firms point of view, prices are too low under perfect information. Such an effect is absent in a monopoly setting. Yehezkel (2007) introduces informed consumers in a duopoly setting and identifies (though assuming that marginal cost do not depend on quality) the optimal price and dissipative advertising choices that ensure separation. Thus he combines a duopoly model *à la* Fluet and Garella/Hertzendorf and Overgaard with an informed consumer model *à la* Bagwell and Riordan/Linnemer.

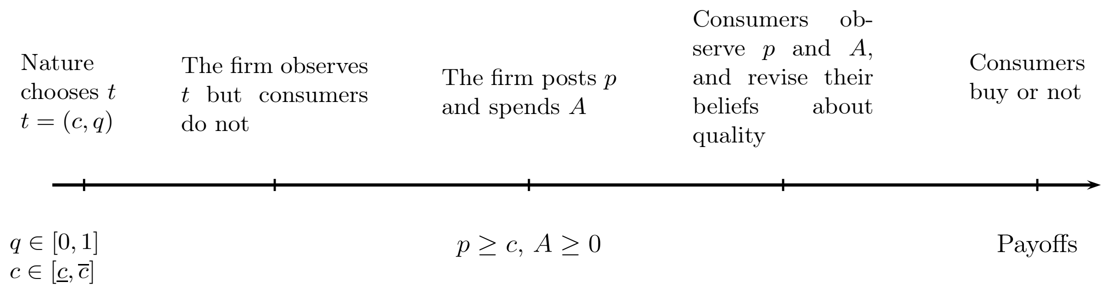

Figure 1: Timeline

about the type of the firm. Given the price and their beliefs, consumers take their buying decisions. There is only one period.

Let $D_q(p)$ denote the perfect information demand function if quality is q and price p.

Assumption 1. *Demand functions are continuous and non increasing with respect to price and not null everywhere. Moreover, demand is increasing with quality: if $q, q' \in [0, 1]$ with $q < q'$, then, for all p, $D_q(p) < D_{q'}(p)$ (wherever $D_{q'}(p) > 0$).*

Let

$$\Pi(p, c, q) - A = (p - c) D_q(p) - A$$

denote the perfect information profit function of type $t = (c, q)$. A key point of the model is that in a static framework, this profit function is also the profit function of any type $t = (c, .)$ when it is believed by consumers to produce a good of quality q with certainty.[11] That is, under asymmetric information the profit function of a type $(c, .)$ depends on its marginal cost and on the perceived quality but not on its true quality.

The following assumption ensures that the profits are maximum for a unique price.

Assumption 2. *For all c, q, the profit function $\Pi(p, c, q)$ is strictly quasi-concave in p.*

Let $P^m(c, q)$ denote the price that maximises $\Pi(p, c, q)$. For example, if information is perfect, $P^m(c, q)$ is the monopoly price set by a firm of type $t = (c, q)$. If, however, information is imperfect a firm of type $t = (c, q)$ which is perceived as producing quality q' would maximise its profits by setting $p = P^m(c, q')$. Moreover, let

$$\Pi^m(c, q) = \Pi(P^m(c, q), c, q)$$

denote the value of the maximum profit under perfect information. Notice that as the focus is on dissipative advertising and not on demand enhancing advertising, advertising is not used under perfect information.

To focus on the role of dissipative advertising as a signal of quality, the analysis concentrates on separating equilibria only. Formally, a separating equilibrium is a list:

[11]This property does not hold in a model with repeat purchases as they are conditioned on the real quality, nor when some consumers are informed about the true quality.

$\{(p_t, A_t)\}_{t\in T}$, such that: for all $t \neq t'$, $(p_t, A_t) \neq (p_{t'}, A_{t'})$, when (p_t, A_t) is observed consumers believe (Bayesian revision) with probability 1 that the firm is of type t. Outside the equilibrium path, that is, if a pair (p, A) is observed which corresponds to none of the equilibrium expected strategies, then (arbitrarily) consumers think that quality is the lowest one: $q = 0$. To form an equilibrium, each pair (p_t, A_t) maximizes type t profit function given consumers's beliefs.[12]

In a separating equilibrium, all lowest quality types select their perfect information prices and have no use of dissipative advertising. Indeed, in a separating equilibrium consumers rightly infer quality, their quality is $q = 0$ which mean they face the worst possible beliefs and a deviation to $P^m(c, 0)$ would always be profitable if it were not the equilibrium price. Consequently, if a separating equilibrium exists, then for all $t = (c, 0)$ $(p_t, A_t) = (P^m(c, 0), 0)$.

It remains to characterize the separating equilibrium strategies of all the other types. A usual assumption in such a signaling game is that the perfect information monopoly prices (and no advertising) are not separating. With a continuum of low-quality types, however, this assumption is automatically true. Indeed, a type $(c, 0)$ always gains to post $P^m(c, q)$ (and to spend nothing on advertising) if such a strategy makes consumers believe that quality is q as Assumption 1 implies that $\Pi^m(c, q) > \Pi^m(c, 0)$.[13]

3 Separating equilibrium

To characterize the separating equilibrium it is convenient to first study a game where the set of types is limited as follows. Let assume, as shown by Figure 2, a continuum of types at the lowest quality level: $t = (\gamma, 0)$ with $\gamma \in [\underline{c}, \overline{c}]$ but only one other type. The quality level of this "high" quality type is denoted $q > 0$ and its marginal cost is denoted c, $c \in [\underline{c}, \overline{c}]$.

This configuration is key to derive further results.

Proposition 1. *In a game with a continuum of lowest quality types and one high quality type (c, q), $\underline{c} < c < \overline{c}$ there exists a unique separating marketing mix (P^*, A^*) where $P^* = D_q^{-1}[D_0(P^m(c, 0))]$ and $A^* = \Pi(P^*, c, q) - \Pi^m(c, 0) > 0$. The equilibrium profit of the high quality type is $\Pi^m(c, 0)$.*

If $c = \underline{c}$ or $c = \overline{c}$ the separating equilibrium strategy described above is still valid but it is no longer unique.

Proof. The proof relies on the elimination of dominated strategies. For a type $(\gamma, 0)$, let $\underline{P}(\gamma, q) < \overline{P}(\gamma, q)$ be respectively the lowest and largest solution of the equation:

$$\Pi(p, \gamma, q) = \Pi^m(\gamma, 0)$$

and let

$$A(p, \gamma, q) = \max\{0\,;\, \Pi(p, \gamma, q) - \Pi^m(\gamma, 0)\}$$

[12] Note that as the separation of types in terms of marginal cost has no value for consumers, one could look at semi-separating equilibria where types are only separated in terms of quality. However, types of the same quality but different marginal cost will be separated at the unique separating equilibrium.

[13] By continuity there exists $\varepsilon > 0$ such that for all $\gamma \in]c - \varepsilon, c + \varepsilon[$, a type $(\gamma, 0)$ would gain by using the strategy $(P^m(c, q), 0)$ instead of $(P^m(c, 0), 0)$.

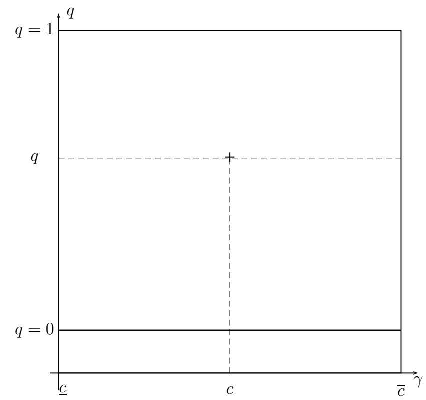

Figure 2: A continuum of types with $q = 0$ and one other type

Any strategy (p, A) such that $A > A(p, \gamma, q)$ is strictly dominated for type $(\gamma, 0)$ by the strategy $(P^m(\gamma, 0), 0)$. The construction of the function $A(p, \gamma, q)$ from which dominated strategies are inferred, is illustrated by Figure 3.

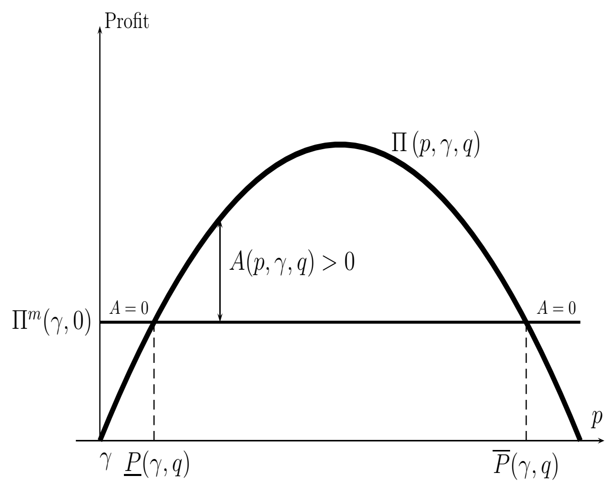

Figure 3: Construction of the function $A(p, \gamma, q)$

Let

$$A_q(p) = \sup_{\gamma} A(p, \gamma, q) \tag{1}$$

The elimination of the dominated strategies means that the type (c, q) can maximize its profit under the constraint that given its price p its advertising expenditure A is larger than $A_q(p)$. Indeed, if $A \geq A_q(p)$, then for all γ, $A \geq A(p, \gamma, q)$ and no type $(\gamma, 0)$ finds it more profitable to chose (p, A) rather than $(P^m(\gamma, 0), 0)$. Moreover, as advertising is costly, it is optimal for type (c, q) to choose $A = A_q(p)$. Using the envelop theorem, Appendix A shows that

$$A_q(p) = A\left(p, \min\left\{\max\left\{\underline{c}, \widehat{c}\right\}, \overline{c}\right\}, q\right)$$

where $\widehat{c}$ is the solution in γ of the equation $D_q(p) = D_0\left(P^m(\gamma, 0)\right)$. The advertising function $A_q(p)$ can be null if p is lower (respectively larger) than $\underline{P}(\underline{c}, q)$ (resp. $\overline{P}(\overline{c}, q)$).

Hence, type (c, q) maximizes:

$$\max_p \Pi\left(p, c, q\right) - A_q(p)$$

In Appendix B, it is shown that the solution P^* is such that $\widehat{c}(P^*) = c$ and is given by

$$D_q\left(P^*\right) = D_0\left(P^m(c, 0)\right)$$

It follows that

$$A^* = A_q(P^*) = \Pi\left(P^*, c, q\right) - \Pi^m(c, 0) > 0$$

and substituting A^* by its value, the equilibrium profit of the high quality type writes

$$\Pi\left(P^*, c, q\right) - A^* = \Pi^m(c, 0)$$

which proves that type (c, q) has no incentive to deviate from its equilibrium strategy, as a deviation would at most lead to a profit of $\Pi^m(c, 0)$. □

Appendix C details the other separating equilibria for $c = \underline{c}$ or $\overline{c}$. These additional separating strategies are neglected in the remaining of the paper without loss of generality.

Proposition 1 shows that a type (c, q) is able to signal itself but separation cost is maximal: its equilibrium profit is exactly what it would be if consumers believed quality is the lowest. The intuition is that with a continuum of types to separate from, the (c, q) type has to separate from its evil twin $(c, 0)$: same marginal cost but lowest quality. More precisely in equilibrium the incentive compatibility constraint of the evil twin has to be binding that is $\Pi\left(p, c, q\right) - A_q(p)$ has to be equal to $\Pi^m(c, 0)$ otherwise the $(c, 0)$ type would gain imitating the strategy of type (c, q). Therefore the separating price P^* has to be such that $(c, 0)$ is the low-quality type which gains the most by imitating (c, q). This happens (see Appendix A) when $D_q\left(P^*\right) = D_0\left(P^m(c, 0)\right)$ which characterizes the separating price P^*. Thus, at the (unique) separating equilibrium, type (c, q) and $(c, 0)$ produce the same quantity and consequently type (c, q) posts a greater price than $(c, 0)$. Finally, separation from the evil twin is the most costly and type (c, q) achieve the same profit as type $(c, 0)$.

How does this result compare with the usual two type model? Assume that there are only two types denoted (without loss of generality) $(\gamma, 0)$ and (c, q) with $q > 0$. If $\gamma \neq c$ then a unique separating equilibrium exists: $p^{**} = \overline{P}(\gamma, q)$ and $a^{**} = 0$ if $\gamma < c$ and $p^{**} = \underline{P}(\gamma, q)$ and $a^{**} = 0$ if $\gamma > c$. At these unique equilibria, the high quality type makes more profit than $\Pi^m(c, 0)$. If $\gamma = c$ then a continuum of equilibria exists: $p^{**} \in \left[\underline{P}(c, q), \overline{P}(c, q)\right]$ and $a^{**} = A\left(p^{**}, c, q\right)$ but for all these equilibria the high quality type profit is $\Pi^m(c, 0)$. This case is illustrated by Figure 4a. Here, as in the more general model of the paper, advertising can be part of the separating strategy. Yet, to see that it is an mere artefact, consider the limit of the separating strategy (in the two-type model) when $\gamma \to c$. If $\gamma \to c^-$ then

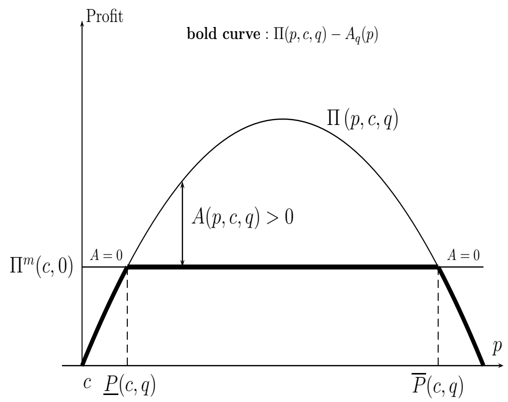

Figure 4a: Two types same marginal cost

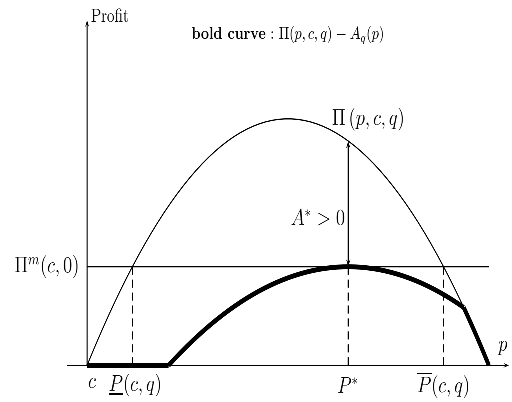

Figure 4b: Continuum of types

$(p^{**}, a^{**}) \to (\overline{P}(c,q), 0)$ while if $\gamma \to c^+$ then $(p^{**}, a^{**}) \to (\underline{P}(c,q), 0)$. In both cases, there is no dissipative advertising.

With a continuum of low quality types, the high quality type is no longer indifferent between a continuum of separating strategies. As shown by Figure 4b, only one price in $\left[\underline{P}(c,q), \overline{P}(c,q)\right]$ combined with the right advertising amount allows both separation and a profit of at least $\Pi^m(c,0)$.[14] Here, one can discretize the set of low-quality types by considering b low-quality types with a marginal cost strictly below c and a low-quality types with a marginal cost strictly above c (see Appendix D). For each discretization of the continuous game, the separating equilibrium strategy is unique, results of the elimination of the dominated strategies, and is characterized by a price inside $\left[\underline{P}(c,q), \overline{P}(c,q)\right]$ and a positive level of advertising. This unique separating equilibrium of the discrete low-quality type model converges to the strategy (P^*, A^*) of Proposition 1. That is, the equilibrium use of advertising is not an artefact of the presence of the evil twin (or to put it differently an artefact of the infinite set of types): advertising is positive at the limit but also along the sequence.

The results of Proposition 1 can be extended immediately to the case of a continuum of types at both levels: 0 and q. As stated in Corollary 1.

Corollary 1. *In a game with a continuum of lowest quality types* $(\gamma, 0)$ *and a continuum of higher quality type* (c,q)*, there exists a (unique) separating equilibrium. The marketing mix chosen by type* (c,q) *is* $(P^*(c,q), A^*(c,q))$ *where* $P^*(c,q) = D_q^{-1}\left[D_0\left(P^m(c,0)\right)\right]$ *and* $A^*(c,q) = \Pi\left(P^*, c, q\right) - \Pi^m(c,0) > 0$. *The equilibrium profit of a higher quality type is* $\Pi^m(c,0)$.

Proof. If types (c,q) play $(P^*(c,q), A^*(c,q))$ there is no incentive for any type $(\gamma, 0)$ to deviate from its equilibrium strategy. A particular type (c,q) as no incentive to chose a

[14] At first, it might be surprising that type (c,q) distorts its price and does not post its perfect information monopoly price $P^m(c,q)$. Indeed, once it has be recognized the incentive compatibility constraint of type $(c,0)$ is binding, it seems that as in the two-type model several combinations of price and advertising allow type (c,q) to reach the same profit. Yet, type (c,q) has also to separate itself from the other types, and for most prices it is not $(c,0)$ which is the most eager to imitate (c,q) but another low-quality type.

pair $(P^*(c',q), A^*(c',q))$. Indeed, it does not improve consumers' perception of its quality (they still believe quality is q) but it reduces its profits. To see why notice that the function $A_q(p)$ does only depend on q but not on c. It remains to check that for $c \neq c'$ then $(P^*(c,q), A^*(c,q)) \neq (P^*(c',q), A^*(c',q))$ which is true as $P^*(c,q) \neq P^*(c',q)$. $\square$

The intuition behind Corollary 1 is that the incentives for type (γ, q) to imitate a fellow quality type (c,q) are weaker than the incentives of its evil twin $(\gamma, 0)$ (in fact in equilibrium it has exactly the same incentives) and therefore once type (c,q) is separated from all low-quality types, it is also separated from its fellow quality types. What remains to be checked is that all quality types use different strategies (no partial pooling) and this is done in the proof of the Corollary.

Finally Proposition 1 and Corollary 1 are generalized for types distributed over an entire square.

Corollary 2. *In a game with types $(c,q) \in [\underline{c}, \overline{c}] \times [0,1]$, there exists a (unique) separating equilibrium. The marketing mix chosen by type (c,q) if $q > 0$ is $(P^*(c,q), A^*(c,q))$ where $P^*(c,q) = D_q^{-1}\left[D_0\left(P^m(c,0)\right)\right]$ and $A^*(c,q) = \Pi\left(P^*, c, q\right) - \Pi^m(c,0) > 0$. The equilibrium profit of any type (c,q) is $\Pi^m(c,0)$.*

Proof. From Corollary 1 no type $(\gamma, 0)$ has an incentive to deviate. Could a type (c,q), $q > 0$ gain by choosing a strategy $(P^*(c',q'), A^*(c',q'))$? In equilibrium type (c,q) profit is $\Pi^m(c,0)$ that is exactly the same profit as type $(c,0)$. Notice that if a strategy (p,a) makes consumers believe that the quality level is q' then by choosing this strategy, both types (c,q) and $(c,0)$ obtain the same profit, namely $\Pi\left(p,c,q'\right) - a$. Therefore if $(P^*(c',q'), A^*(c',q'))$ is not a profitable deviation for type $(c,0)$ (Proposition 1) it is also not profitable for type (c,q).

It remains to check that for $(c,q) \neq (c',q')$ then $(P^*(c,q), A^*(c,q)) \neq (P^*(c',q'), A^*(c',q'))$. In the proof of Corollary 1, it has been shown that two types (c,q) and (c',q) (with $c \neq c'$) do not have the same prices. Assume that two types (c,q) and (c',q') (with $c \neq c'$ and $q \neq q'$) have the same price: $P^*(c,q) = P^*(c',q')$, then their advertising expenditures would be different as for a given price $A_q(p)$, which does not depend on c, is increasing with q. $\square$

Corollary 2 describes the unique separating equilibrium (once dominated strategies are eliminated and if one neglect that for $c = \underline{c}$ or $c = \overline{c}$ other equilibrium strategies are possible). The rent dissipation result of Proposition 1 extends to the most general set up where types are distributed over a square. The intuition is that once a type (c,q) separates itself from all low-quality types it *de facto* separates itself from other (γ, q') types. Formally, it is obvious that the separating equilibrium of Corollary 2 is intuitive in the sense of Cho-Kreps as in this equilibrium, all types $(\gamma, .)$ achieve the same profit (irrespectively of their true quality). Therefore one cannot find a deviation that would be equilibrium dominated by type $(\gamma, 0)$ (for example) but not for type (γ, q).

An interesting question not addressed in this paper is the shape of pooling equilibria if they exist. This question is, however, different from the one of the paper which is: does dissipative advertising signals quality in a static framework? Indeed, to study pooling or semi-pooling equilibrium one would typically assumed away dissipative advertising. Notice that in non-separating equilibria, the property that profit functions do not directly depend on the true quality still holds. A semi-pooling equilibrium where all types $(c, .)$ post the same price, if it exists (existence would depend on specific assumptions on the distribution of the types over the square), should be intuitive in the sense of Cho-Kreps. Indeed, if a deviation is equilibrium dominated for a type (c, q) it is also equilibrium dominated by any other type (c, q').

4 Comparative statics

In this section, comparative static analysis of the separating equilibrium are derived. First, does signaling entails an upward or a downward price distortion? With only two types of quality, the answer depends on marginal cost being increasing or decreasing with quality. If it is increasing (resp. decreasing), then the price is distorted upwards (resp. downwards). With types heterogeneous in terms of both quality and marginal costs, the price distortion rather depends on quite a different property as shown in Corollary 3.

Corollary 3. *The separating equilibrium price $P^*(c, q)$ is greater than the perfect information monopoly price $P^m(c, q)$ if and only if (for a given marginal cost) the perfect information monopoly quantity increases with quality (formally if and only if $D_0(P^m(c, 0)) < D_q(P^m(c, q))$).*

Proof. From $D_q(P^*) = D_0(P^m(c, 0))$ it is clear that $D_0(P^m(c, 0)) < D_q(P^m(c, q))$ implies $D_q(P^*) < D_q(P^m(c, q))$ and therefore $P^*(c, q) > P^m(c, q)$. And conversely if $D_0(P^m(c, 0)) > D_q(P^m(c, q))$. □

The most natural assumption is that (given that marginal cost remains the same) the monopoly quantity increases with quality.[15] In that case, the price distortion is always upward whereas low quality types produce with both lower and greater marginal cost than type (c, q). That is, even if on average the distribution of types is such that marginal cost is decreasing with quality, the signaling price remains above the perfect information price. This result is consistent with the empirical findings of Thomas, Shane, and Weigelt (1998): "manufacturers of high quality autos set price above the full information price".

Next, how do the equilibrium price and advertising expenditure vary with quality and marginal cost?

Corollary 4. *The separating equilibrium price $P^*(c, q)$ is increasing with c and q. The separating equilibrium advertising expenditure $A^*(c, q)$ is increasing with q, decreasing with c if $D_0(P^m(c, 0)) < D_q(P^m(c, q))$ and increasing otherwise.*

Proof. See Appendix E □

[15] Let $P(x, q)$ denote the inverse demand function and let $MR(x, q) = P(x, q) + x\frac{\partial P}{\partial x}$ denote the marginal revenue. The monopoly quantity x^m is given by the first order condition: $MR(x, q) = c$. Therefore $\frac{\partial x^m}{\partial q}\frac{\partial MR}{\partial x} + \frac{\partial MR}{\partial q} = 0$. As $\frac{\partial MR}{\partial x} < 0$ (by the second order condition), then $\frac{\partial x^m}{\partial q}$ is of the same sign as $\frac{\partial MR}{\partial q}$. If the marginal revenue is increasing with quality, then also the monopoly quantity. For a Mussa-Rosen demand function, $P = q(1 - x)$ one can directly compute $x^m = \frac{1}{2}(1 - c/q)$ which is increasing with q.

The separating price follow a pattern similar to the perfect information monopoly price as both increase with marginal cost. If it is assumed that the monopoly price increases with quality, then both prices also vary similarly with quality.

Advertising increases with quality. This property is not obvious as the advertising function $A_q(p)$ (the minimal separating advertising amount) follows an inverted U-shape. Therefore as price vary, advertising could increase or decrease. Yet, in equilibrium higher quality means larger advertising expenditures. Again this is in line with Thomas, Shane, and Weigelt (1998) who find that manufacturers of high quality cars spend more on advertising than manufacturers of low quality products. They present this result as broadly consistent with Milgrom and Roberts (1986) but with only two quality levels only the high quality should advertise. This is also compatible with the study of Iizuka (2004) who finds that firms are more likely to advertise newer and higher quality drugs rather than older and lower quality ones. Finally, if the monopoly quantity increases (resp. decreases) with quality, then advertising decreases (resp. increases) with the marginal cost.

From an empirical point of view, once no one to one relationship between costs and quality is assumed, one has to be careful when estimating how advertising (resp. price) varies with quality (resp. cost) to control for cost level (resp. quality level). This makes empirical studies difficult to interpret when cost data are not available.

5 Conclusion

This paper shows that the usual assumption made in the quality-signaling literature that quality can only be of two types (low or high) is restrictive. The main result is that dissipative advertising has to be part of the optimal marketing mix in presence of types heterogeneous in terms of both quality and marginal costs. Previous literature focused on whether repeat purchases or a proportion of informed consumers to explain the rational use of dissipative advertising.

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Appendix

A Characterization of $A_q(p)$

By definition

$$A_q(p) = \sup_{\gamma} \left[\max\left\{0\,;\, \Pi\left(p,\gamma,q\right) - \Pi^m(\gamma,0)\right\}\right]$$

Let $H(\gamma) = \Pi\left(p,\gamma,q\right) - \Pi^m(\gamma,0) = \Pi\left(p,\gamma,q\right) - \max_z\left[(z-\gamma)D_0(z)\right]$, using the envelop theorem, it comes that

$$H'(\gamma) = D_0\left(P^m(\gamma,0)\right) - D_q(p) \text{ and that } H''(\gamma) = \frac{\partial P^m(\gamma,0)}{\partial\gamma} D_0'\left(P^m(\gamma,0)\right)$$

As the perfect information monopoly price increases with γ and demand decreases in price $H'' < 0$. Therefore if the equation $H'(\gamma) = 0$ has a solution it characterizes a maximum. Let c^* denote the value of the marginal cost such that $A_q(p) = A\left(p, c^*, q\right)$.

Let $p_{\min}$ and $p_{\max}$ be such that

$$D_q(p_{\min}) = D_0\left(P^m(\underline{c},0)\right) \text{ and } D_q(p_{\max}) = D_0\left(P^m(\overline{c},0)\right) \tag{2}$$

If $p_{\min} \leq p \leq p_{\max}$, then $c^* = \widehat{c}$ where $\widehat{c}$ is the unique γ solution of

$$D_0\left(P^m(\gamma,0)\right) = D_q(p)$$

while if $p < p_{\min}$ (resp.$p_{\max} < p$) , then $c^* = \underline{c}$ (resp. $c^* = \overline{c}$) as in that case $H'(\gamma) < 0$ (resp. $H'(\gamma) > 0$) for all γ which shows that

$$A_q(p) = A\left(p, \min\left\{\max\left\{\underline{c}, \widehat{c}\right\}, \overline{c}\right\}, q\right)$$

Notice that when $c^* = \widehat{c}$, then

$$\begin{aligned} H(\widehat{c}) &= (p - \widehat{c})\, D_q(p) - \Pi^m(\widehat{c}, 0) \\ &= (p - \widehat{c})\, D_q(p) - (P^m(\widehat{c}, 0) - \widehat{c})\, D_0\left(P^m(\widehat{c}, 0)\right) \\ &= (p - \widehat{c})\, D_q(p) - (P^m(\widehat{c}, 0) - \widehat{c})\, D_q(p) \\ &= (p - P^m(\widehat{c}, 0))\, D_q(p) \end{aligned}$$

B Characterization of the separating price

To find the solution of

$$\max_p \Pi(p, c, q) - A_q(p)$$

Note that

$$\Pi(p, c, q) - A_q(p) = \begin{cases} -(c - \underline{c})D_q(p) + \Pi^m(\underline{c}, 0) & \text{if } p < p_{\min} \\ (P^m(\widehat{c}, 0) - c)\, D_q(p) & \text{if } p_{\min} \le p \le p_{\max} \\ (\overline{c} - c)D_q(p) + \Pi^m(\overline{c}, 0) & \text{if } p_{\max} < p \end{cases}$$

where $p_{\min}$ and $p_{\max}$ are defined in Appendix A.

For $c > \underline{c}$ (resp. $c < \overline{c}$) , $-(c - \underline{c})D_q(p)$ (resp. $(\overline{c} - c)D_q(p)$)is an increasing (resp. decreasing) function of p. Therefore the maximum has to be found for $p_{\min} \le p \le p_{\max}$.

Let $G(p) = (P^m(\widehat{c}, 0) - c)\, D_q(p)$, where $\widehat{c}$ is a function of p defined by $D_0\left(P^m(\widehat{c}, 0)\right) = D_q(p)$. Differentiating this equation with respect to p, it comes

$$\frac{\partial \widehat{c}}{\partial p} \frac{\partial P^m(\widehat{c}, 0)}{\partial \gamma} D_0'\left(P^m(\widehat{c}, 0)\right) = D_q'(p)$$

on the other hand by definition of $P^m(\widehat{c}, 0)$ one has

$$(P^m(\widehat{c}, 0) - \widehat{c})\, D_0'\left(P^m(\widehat{c}, 0)\right) + D_0\left(P^m(\widehat{c}, 0)\right) = 0$$

therefore

$$\begin{aligned} G'(p) &= \left(\frac{\partial \widehat{c}}{\partial p} \frac{\partial P^m(\widehat{c}, 0)}{\partial \gamma}\right) D_q(p) + (P^m(\widehat{c}, 0) - c)\, D_q'(p) \\ &= \left(\frac{D_q'(p)}{D_0'\left(P^m(\widehat{c}, 0)\right)}\right) D_q(p) + (P^m(\widehat{c}, 0) - c)\, D_q'(p) \\ &= -\left((P^m(\widehat{c}, 0) - \widehat{c})\, \frac{D_q'(p)}{D_0\left(P^m(\widehat{c}, 0)\right)}\right) D_q(p) + (P^m(\widehat{c}, 0) - c)\, D_q'(p) \\ &= -\left((P^m(\widehat{c}, 0) - \widehat{c})\, D_q'(p)\right) + (P^m(\widehat{c}, 0) - c)\, D_q'(p) \\ &= -(c - \widehat{c})\, D_q'(p) \end{aligned}$$

that is (as p is such that $D_q' < 0$)

$$G'(p) = 0 \Leftrightarrow \widehat{c} = c \Leftrightarrow D_0\left(P^m(c, 0)\right) = D_q(p)$$

C Separating equilibria for extreme cost values

If the high quality type is $(\underline{c}, q)$ (resp. $(\overline{c}, q)$) then it has no low quality type with a lower (resp. higher) marginal cost and equation 2 shows that $p_{\min} = P^*$ (resp. $p_{\max} = P^*$. Therefore for all $p < P^*$ (resp. $p > P^*$) a type $(\underline{c}, q)$ (resp. $(\overline{c}, q)$) has to spend $A(p, \underline{c}, q)$ (resp. $A(p, \overline{c}, q)$) on advertising which means its profit is constant on $[\underline{P}(\underline{c}, q), P^*]$ (resp. $\left[P^*, \overline{P}(\overline{c}, q)\right]$). Figure 5a (resp. 5b) show type $(\underline{c}, q)$ (resp. $(\overline{c}, q)$) profit as a function of price.

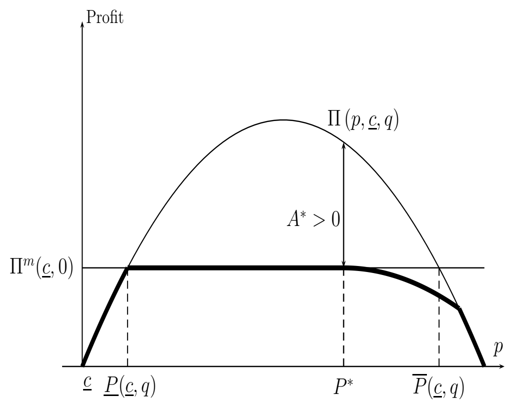

Figure 5a: Equilibria when $c = \underline{c}$

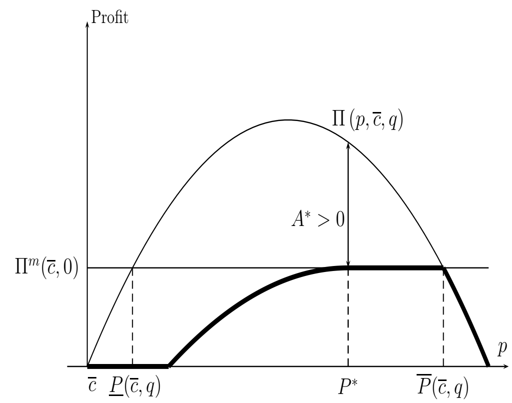

Figure 5b: Equilibria when $c = \overline{c}$

D Model with a discrete number of low-quality types

Only one high-quality type (c, q) is assumed. Let $\mathcal{B} = \{l_1, \ldots, l_b\}$ denote the set of low-quality types with a marginal cost below c and $\mathcal{A} = \{L_1, \ldots, L_a\}$ the set of low-quality types with a marginal cost above c. Without loss of generality it is assumed that $c_{l_1} < c_{l_2} < \cdots < c_{l_b} < c < c_{L_1} < \cdots < c_{L_a}$.

The elimination of the dominated strategies for each low quality type remains identical as the one described in the proof of Proposition 1. The high-quality type can after the elimination of the dominated strategies maximize its profit under the constraint of spending at least the following on dissipative advertising:

$$\sup_{\gamma = \left\{ \substack{c_{l_1}, \cdots, c_{l_b} \\ c_{L_1}, \cdots, c_{L_a}} \right.} A(p, \gamma, q)$$

This elimination of dominated strategies leads to a unique separating marketing mix $\left(P^*(c_{l_b}, c_{L_1}), A^*(c_{l_b}, c_{L_1})\right)$ such that $P^*(c_{l_b}, c_{L_1})$ is the unique solution of $A(p, c_{l_b}, q) = A(p, c_{L_1}, q)$ and $A(c_{l_b}, c_{L_1})^* = A(P^*, c_{l_b}, q) = A(P^*, c_{L_1}, q) > 0$.

That is, the separating equilibrium strategy depends only on the largest marginal cost in $\mathcal{B}$ and on the lowest marginal cost in $\mathcal{A}$. Indeed, as long as the minimal separating advertising level $A(p,\gamma,q)$ corresponds to a marginal cost lower than c the objective of the high quality type is

$$\Pi(p,c,q) - A(p,\gamma,q) = -(c-\gamma)D_q(p) + \Pi^m(\gamma,0)$$

which is increasing with the price as $c > \gamma$. Conversely when the minimal advertising function corresponds to marginal cost larger than c the objective of the high quality type is

$$\Pi(p,c,q) - A(p,\gamma,q) = (\gamma - c)D_q(p) + \Pi^m(\gamma,0)$$

which is decreasing with the price as $\gamma > c$. The optimal price is necessary at the (unique) intersection of $A(p,c_{l_b},q)$ and $A(p,c_{L_1},q)$. That is, it is given by

$$(c_{L_1} - c_{l_a})\, D_q(P^*) = \Pi^m(c_{l_a},0) - \Pi^m(c_{L_1},0)$$

One can check that the separating equilibrium profit of the high quality type is strictly larger than $\Pi^m(c,0)$.

Finally with more and more low quality types, both c_{l_a} and c_{L_1} tend to c. The above equation being continuous in c_{L_1} one can make $c_{L_1} = c$. Making c_{l_a} tend to c the above equation writes:

$$D_q(P^*) = \lim_{c_{l_a}\to c}\left[\frac{\Pi^m(c_{l_a},0) - \Pi^m(c,0)}{c - c_{l_a}}\right]$$

Let $\Psi(\gamma) = \max_p[(p-\gamma)D_0(p)]$. The above equation writes:

$$D_q(P^*) = -\lim_{\gamma\to c}\left[\frac{\Psi(c)-\Psi(\gamma)}{c-\gamma}\right] = -\Psi'(c) = D_0(P^m(c,0))$$

as in Proposition 1.

E Proof of Corollary 4

From

$$D_q(P^*) = D_0(P^m(c,0))$$

It follows that

$$\frac{\partial P^*}{\partial c}\underbrace{D'_q}_{<0} = \underbrace{\frac{\partial P^m}{\partial c}}_{>0}\underbrace{D'_0}_{<0} \Rightarrow \frac{\partial P^*}{\partial c} > 0 \text{ and } \frac{\partial P^*}{\partial q}\underbrace{D'_q}_{<0} + \underbrace{\frac{\partial D_q}{\partial q}}_{>0} = 0 \Rightarrow \frac{\partial P^*}{\partial q} > 0$$

From

$$A^* = \Pi(P^*,c,q) - \Pi^m(c,0)$$

It follows that

$$\begin{aligned}
\frac{\partial A^*}{\partial q} &= \frac{\partial P^*}{\partial q}\frac{\partial \Pi}{\partial p} + \frac{\partial \Pi}{\partial q} \\
&= \frac{\partial P^*}{\partial q}\left[(P^* - c)D'_q + D_q\right] + (P^* - c)\frac{\partial D}{\partial q} \\
&= \frac{\partial P^*}{\partial q}\left[(P^* - c)D'_q + D_q\right] - (P^* - c)\frac{\partial P^*}{\partial q}D'_q \\
&= \frac{\partial P^*}{\partial q}D_q > 0
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial A^*}{\partial c} &= \frac{\partial P^*}{\partial c}\frac{\partial \Pi}{\partial p} + \frac{\partial \Pi}{\partial c} + D_0\left(P^m(c,0)\right) \\
&= \frac{\partial P^*}{\partial c}\frac{\partial \Pi}{\partial p} - D_q(P^*) + D_0\left(P^m(c,0)\right) \\
&= \frac{\partial P^*}{\partial c}\frac{\partial \Pi}{\partial p}
\end{aligned}$$

therefore the sign of $\frac{\partial A^*}{\partial c}$ is the same as the sign of $\frac{\partial \Pi}{\partial p}$ evaluated at P^*. If $P^*(c,q) > P^m(c,q)$ then $\frac{\partial \Pi}{\partial p} < 0$ while it is positive otherwise.