INSTITUT NATIONAL DE LA STATISTIQUE ET DES ETUDES ECONOMIQUES
Série des Documents de Travail du CREST
(Centre de Recherche en Economie et Statistique)

n° 2007-28

Individual Rationality Under Sequential Decentralized Participation Processes*

L. LAMY [1]

* I am grateful above all to my Ph. D. advisor Philippe Jehiel for his continuous support. I would like to thank Olivier Compte and seminar participants in Paris-PSE Theory Workshop and Nashville PET 2007 Conference. All errors are mine.

[1] Laboratoire d'Economie Industrielle, CREST-INSEE, 28 Rue des Saints-Pères, 75007 PARIS, France. laurent.lamy@ensae.fr

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†Laboratoire d'Economie Industrielle, CREST-INSEE, 28 rue des Saints-Pères 75007 Paris. e-mail: laurent.lamy@ensae.fr

Abstract

We consider the implementation of an economic outcome under complete information when the principal cannot commit to a simultaneous participation game. From a general class of sequential decentralized participation processes and without common knowledge on the details of the process, we introduce the concept of implementation under robust sequential individual rationality. We solve the optimal design program: the principal may fail to extract fully agents' surplus relative to the harsher threats but economic efficiency is not damaged.

Keywords: Mechanism Design, Individual Rationality, Imperfect Commitment, Surplus Extraction, Collusion on Participation

JEL classification: C72, D62

Abstract

Nous considérons la question de l'implémentation d'un mécanisme en information complète lorsque le principal ne peut pas s'engager sur un jeu simultané de participation. A partir d'une classe de jeux décentralisés de participation et en s'affranchissant d'hypothèses de connaissance commune relative à la forme spécifique du jeu, nous introduisons le concept d'implémentation robuste sous la contrainte d'agents séquentiellement individuellement rationnels. Nous résolvons le programme d'optimisation du principal: le surplus que le principal peut extraire est plus faible, néanmoins l'efficacité économique n'est pas altérée.

Mots-clés: Mechanism Design, Rationalité individuelle, Engagement, Extraction du surplus, Collusion sur la participation

Classification JEL: C72, D62

1 Introduction

The mechanism design paradigm considers that agents are taking their participation decisions simultaneously. However, the principal in many transactions lacks the ability to commit to close the participation at an exact deadline.[1] In corporate acquisitions and procurement auctions, it is common that the seller violates the announced rules to accept a subsequent better deal. McAdams and Schwarz [17] and Vartiainen [24] consider auction models where the seller is unable to commit not to solicit another round of offers after having publicly disclosed the previous offers. Similarly, in the corruption literature, e.g. Compte et al. [5], the auctioneer may also provide an opportunity for bid readjustments in exchange for a bribe.

We consider the implementation of an economic outcome under complete information relative to agents' preferences when the principal can not commit to any multi-stage mechanism and has no control on the participation process itself. Hence, the analysis is reduced to feasible participation games such that, sequentially, agents have the opportunity either to accept to participate in the mechanism or to delay their participation decisions, the final outcome depending only on the final set of participants. The general class of participation processes we consider relies on two ingredients that are common knowledge among agents: first, after each acceptation by a given agent, all the remaining agents will have the opportunity to participate. Second, agents that have already accepted the mechanism have the opportunity to secretly provide the evidence that they have accepted the mechanism to any nonparticipant before receiving his last opportunity to participate. Thus contrary to the aforementioned positive literature that assumes that participation decisions or offers are publicly observable, we consider the general case where participation decisions may not be observed but where participants have the opportunity to provide this evidence.

A mechanism is implementable if full participation is the only equilibrium outcome of any participation game. In the same vein as Moldovanu and Winter [20], we require implementation to be independent of the specific structure of the participation game, i.e. the order of the opportunities to participate. Moreover, in the same vein as Chung and Ely [4], we also require implementation to be robust to any kind of beliefs for the agents relative to the specific form of the process. This structure is not necessary for our main insight, i.e. the impossibility of full surplus extraction under decentralized sequential participation processes. But it allows a tractable characterization of the optimal mechanism.[2] The traditional individual rationality constraints

[1]In some auction design, as in the online version of the ascending auction used at Amazon and analysed by Ockenfels and Roth [21], the rule of the game explicitly involves an extension of the participation deadline after a submission.

[2]Relaxing the common knowledge assumptions on the trading game may seem at odd with the so-called Wilson Doctrine [25] whose agenda is to relax the common knowledge

are strengthened by requiring *robust sequential individual rationality.* Our implementation concept requires more than the traditional condition that participation is a best-response for agent i given that all the other agents participate. With perfect information, i.e. when participation decisions are publicly observed by all agents, Proposition 5.1 states that robust sequential individual rationality requires that there is no set of agents $S \subset N$ such that all agents in S prefer the outcome where only agents in $N \setminus S$ participate to the outcome where all agents accept the mechanism. Those additional constraints in the mechanism design program are non-linear and the set of implementable mechanisms is thus in general not convex. Nevertheless, the optimal design program can be simplified as done in Proposition 5.3, our main result: it allows us to separate the choice of the final allocation to the structure of the optimal threats. As under a simultaneous participation game, the coasian logic still applies and we obtain that the optimal mechanism is efficient. Nevertheless, full extraction relative to the harsher threats as in Jehiel et al [13] does not work anymore generally in presence of externalities. In an incomplete information setup, Heifetz and Neeman [11] show that generic priors on the universal type space do not allow for full surplus extraction. Their insight is that, generically, private information implies informational rents. Here, with the common concern for robust mechanism design, we relax the common knowledge assumptions on the details of the participation process and show that the principal may not be able to fully extract agents' surplus relative to their harsher threats in a complete information setup.

The paper is organized as follows. In section 2 we introduce the general allocation problem. Using a simple example, section 3 illustrates our critic of the traditional mechanism design approach and supplies intuition for our characterization of the optimal mechanism. In section 4 we describe a general class of noncooperative sequential participation games. In section 5 we define our main concept- *robust sequential individual rationality* -and prove the main results. In section 6 we provide two general examples where our alternative mechanism design approach may be relevant and change some insights. Concluding remarks are gathered in section 7.

2 The Model

Let $N = \{1, 2, \dots, n\}$ be a set of agents and $A = \{a_1, a_2, \dots, a_K\}$ be a finite set of possible outcomes. Denote by $\Sigma(N)$ the set of the permutations over the set N. For a given permutation $\sigma : N \to N$, denote by T_i^σ the subset $\{\sigma(1), \sigma(2), \dots, \sigma(i-1)\}$, i.e. the $i-1$ first smallest agents according to the

assumption on agents' beliefs about another's preferences or information and not the ones about the trading process itself. We think that we remain coherent with the Wilson doctrine insofar as enforcement on the details of the participation process seems difficult.

implicit order defined by σ. We assume that the agents and the principal, characterized by the subscript 0, have quasilinear preferences over outcomes and (divisible) money. Preferences are assumed to be common knowledge. The utility of a player i over outcome $a \in A$ and the money transfer t_i is:

$$\mathcal{U}_i(a, t_i) = V_i^a - t_i.$$

We first describe the class of procedures among which the principal chooses an optimal mechanism. In step 1, the principal designs a mechanism. In a complete information setting, a mechanism, denoted by $(\mathbf{a}, \mathbf{t})$, specifies a final outcome $\mathbf{a}(S)$ and a vector of monetary transfers $\mathbf{t}(S)$ for each possible set of participants $S \subset N$. We emphasize that such a reduced form mechanism should not be viewed as resulting from some Revelation Principle, a logic that can not be invoked in our framework. On the contrary, we consider implicitly that the principal has a very limited commitment power: he can not commit to any multi-stage game, i.e. he can not commit not to change the rule of the game after observing some report, but rather only in one shot mechanism. We make the further restriction that the outcome depends only on the set of participants, a maxmin foundation for such an approach being given in section 5.3.[3] In step 2, the agents are playing a sequential decentralized participation process described in next section. In the previous mechanism design literature, the decisions whether to participate or not in the proposed mechanism are assumed to be taken simultaneously. Here we consider that the principal cannot commit to such a simultaneous participation game: an agent will always have at least one opportunity to participate in the mechanism after each decision to accept the mechanism by an agent. In step 3, the mechanism is implemented according to the participation set $S \subset N$. A mechanism is said to be *feasible* if:

- For each set of participants S, the final outcome belongs to $\mathcal{A}(S)$, the subset of A of accessible or feasible outcome with the consent of agents in S.
- If agent i decides not to participate the principal cannot extract a positive payement from that agent: $\mathbf{t}_i(S) \leq 0$, if $i \in N \setminus S$.
- Transfers are budget-balanced: $\sum_{i=0}^{n} \mathbf{t}_i(S) = 0$, for any $S \subset N$.

The second and third restrictions are standard. The first restriction means that some outcome in A may not be feasible if some agents refuse

[3]Implicitly, we also exclude any mechanism that depends on the precise order of the participation decisions of the agents. One argument is that the timing of the participation decisions is not verifiable. Note nevertheless that Jehiel et al [13]'s full extraction mechanism with the optimal threats for each agent can not always be reached with such richer mechanisms confirming thus our main insight that individual rationality constraints should be strengthened (it can be shown in our simple example).

to participate. For example, in the case of the sale of an indivisible good, Jehiel et al. [13] considers that one cannot 'dump' the object on a non-participating agent. We do not impose any specific structure on the feasibility sets $\{\mathcal{A}(S)\}_{S\subset N}$ except that:

Assumption 1 *$\mathcal{A}(S) \subset \mathcal{A}(T)$, whenever $S \subset T$.*

Assumption 1 states that if the consent of the agents in S is enough to implement a given final outcome a, then the extra consent of some agents outside S cannot make this outcome unfeasible. Then, there is no loss of generality to consider that $\mathcal{A}(N) = A$. To simplify the exposition, we assume that, for a given utility level, an agent strictly prefers to participate in the mechanism. With this trick, the set of implementable mechanisms - which is defined in section 5 -is a closed set and has thus an optimal element. Furthermore, in the sequential participation process, we also assume that agents prefers to accept the mechanism as soon as possible, for a given outcome. This additional trick is also innocuous, but allows us to consider a general universal belief space à la Mertens-Zamir [19] still avoiding related pathological phenomena.

For an agent i and a set of participants $S \subset N\setminus\{i\}$, denote by $a_i^*(S)$ the harsher feasible threat that the principal can inflict on i given that the agents in S have accepted the mechanism: $a_i^*(S) \in Arg\min_{a\in\mathcal{A}(S)} V_i^a$. Denote by $V_i^*(S) = V_i^{a_i^*(S)}$ the corresponding utility level. In mechanism design under simultaneous participation, only the threats $a_i^*(N\setminus\{i\})$ do matter. In the optimal design, if one agent refuses the mechanism, the remaining ones commit to this harsher threat also called 'minmax punishment' as in Jehiel et al. [13] or Dequiedt [8]. On the other hand, in mechanism design under robust sequential individual rationality, the whole set of the feasible threats $a_i^*(S)$ will play an active role in the computation of the optimal mechanism.

Finally, our framework is characterized by the 4-uple: $(N, A, \{V_i^a\}_{i\in N, a\in A}, \{\mathcal{A}(S)\}_{S\subset N})$. Let us define two special subsets among those frameworks: *externality-free* and *negative-externality-free* frameworks.

Definition 1
- *A framework is said to be* externality-free *if for any agent i, the map $a \to V_i^a$ is constant over the set $\mathcal{A}(N\setminus\{i\})$.*
- *A framework is said to be* negative-externality-free *if the optimal threat $V_i^*(S)$ for any agent i is independent of the set of participant $S \subset N\setminus\{i\}$: $V_i^*(S) = V_i^*(\emptyset)$ for any i.*

A framework is said to be externality-free if the agents do not care about the final outcome in the event where they do not participate in the mechanism. For the sale of some goods and under the assumption that a non-participant does not receive any good, it corresponds to the standard case where agents care only on the set of goods they obtain and in particular are indifferent to the final allocation when they are non-purchaser. Negative-externality-free is less restrictive: it only requires that the principal can credibly threat any agent with the minmax punishment independently to the other participants, i.e. by retaining all goods in the above example.

3 A Simple Example

The following example consider the sale of a single object involving identity-dependent externalities. It formalizes the starting examples of Jehiel and Moldovanu [12] and Das Varma [7] where two potential buyers suffering from important reciprocal negative externalities prefer not to participate in the bidding process for a single item and let a third buyer win at a low price. We emphasize that those previous modellings can not embrace our kind of strategic nonparticipation.

Let $n = 3$ and $A = \{0, 1, 2, 3\}$ where allocation i corresponds to the allocation of the item to player i. We consider that the seller is able to allocate the item only to participating agents: $\mathcal{A}(S) = \{i | i \in S \cup \{0\}\}$. Let V_i^i be equal respectively to V, v and 0 for $i \in \{1, 2\}$, $i = 3$ and $i = 0$. Let V_i^j, $i \neq j$ be equal to $-\alpha$ if $i, j \in \{1, 2\}$ and 0 otherwise. Assume that $V > v > V - \alpha > 0$. Thus the efficient allocation consists in allocating the item to agent 3. Nevertheless, agents 1 and 2 are valuing the item more than agent 3. They are also chosen symmetric only to simplify the exposition. The same kind of results holds in the neighborhood of the parameter values or even with huge asymmetries between agents 1 and 2 provided that the reciprocal negative-externalities between them are big enough.

Standard Auctions Consider first a simultaneous participation game as in [12]: the buyers have first the opportunity to decide whether or not they want to participate in the auction. Those decisions are made simultaneously and are publicly revealed before the auction takes place. We consider the first price auction, but the results are similar for any other standard auction as the English button auction considered in [7]. In any equilibrium, the item is sold either to agent 1 or to agent 2. In the unique symmetric equilibrium, agents 1 and 2 both participate with probability 1 and are submitting the bid $V + \alpha$. They are both suffering from a loss of α compared to their profit in the case where they could jointly coordinate themselves not to participate. In our example, non-participation from agent 1 is vain and cannot prevent the purchase by agent 2 in the auction because $V > v$.[4]
Now consider a sequential participation game with agents 1 and 2 such that potential buyers are always eligible to participate after one has decided to participate and such that participation decisions are publicly observable

[4]Strategic non-participation as in [12] emerges only if $v > V$ and thus not here.

among agents (the proper formalization is done in section 4). Now it is a subgame perfect equilibrium for agents 1 and 2 not to participate if and only if his 'feared' opponent did so. Under sequential participation, we obtain the paradox that seems to correspond to the stories reported in [12, 7] and that cannot emerge in previous models with simultaneous participation: an agent may prefer not to submit a bid though his intrinsic value for the good, i.e. excluding the motivations to outbid resulting from the fear of negative externalities, is greater than the final bid.

The Optimal Mechanism Under a simultaneous participation game, Jehiel et al. [13] presents an optimal mechanism where participation is a strictly dominant strategy. The optimal mechanism is efficient and the seller can extract surplus from agents who do not obtain the object by using the optimal threats $a_i^*(N \setminus \{i\})$ for each agent i. Here the efficient allocation is to allocate the object to agent 3 and the optimal mechanism raises the revenue $v + 2 \cdot \alpha$: each non-purchaser has to pay α in order to avoid that the seller gives the object to his most feared opponent. But what can be implemented if agents 1 and 2 can coordinate their participation decisions thanks to a sequential participation process? Then the seller can not allocate the object to agent 3 and extract a strictly positive surplus from both agents 1 and 2. In particular, she cannot threat simultaneously agents 1 and 2 with their respective tougher threat. Otherwise, they could jointly not participate and obtain a null payoff since the seller is assumed to be unable to 'dump' the object. To maximize her revenue, the seller should use a *divide and conquer* strategy: it consists in giving the incentive to participate for one agent, say 1, independently of the participation decision of agent 2. Then given that agent 1 participates, she could really threat agent 2 to allocate the object to agent 1 in case of non-participation. Indeed we will show that it is the optimal mechanism and it raises the revenue $v + \alpha$. It illustrates several features that are generalized in section 5: first, the optimal selling procedure is still efficient under the *robust sequential individual rationality* constraints; second, those constraints reduce the revenue. Finally, we find surprisingly that although agents 1 and 2 are symmetric, they should not be treated in a symmetric way in an optimal mechanism. That is the reason why standard auctions that are intrinsically symmetric were leading to joint non-participation.

4 Sequential Participation Processes

We describe a simple sequential participation procedure based on a given mechanism $(\mathbf{a}, \mathbf{t})$. Suppose that the agents in S have already accepted the mechanism, then the remaining agents are playing a participation game where each agent has once the possibility to accept the mechanism or to delay his decision. If all those agents do not accept the mechanism, then the participation game stops and the outcome $(\mathbf{a}(S), \mathbf{t}(S))$ is implemented. Otherwise, if at least one agent i accepts the mechanism, then his acceptance is followed by a participation game given the consent of $S \cup \{i\}$. Observe that, in this informal described situation, the order according to which agents have the possibility to accept or not the mechanism has not been specified. Indeed, each order generates a different extensive form game. Moreover, we should also specify the structure of the information sets. We wish to compare final outcomes in these different games, and therefore we proceed to a formal description of the participation games.

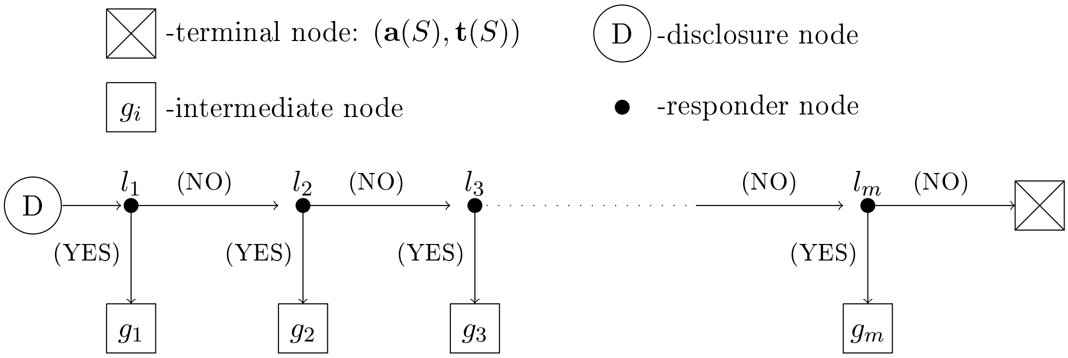

Figure 1

For a given mechanism $(\mathbf{a}, \mathbf{t})$, we define recursively the set of participation games as a function of the cardinality of the set of the agents that have already accepted the mechanism. We denote by $\mathcal{G}(\mathbf{a}, \mathbf{t}, S, \beta)$ the set of participation games if the agents in S have accepted the mechanism and where $\beta = \{\beta_i\}_{i \in N \setminus S}$ represents the vector of the initial priors about the acceptance of the other agents. If $S = N$, this set corresponds to the (unique) degenerate game where agents make no choice and the final outcome $(\mathbf{a}(N), \mathbf{t}(N))$ is implemented. If $S \subsetneq N$, we consider a participation game $g = ((\mathbf{a}, \mathbf{t}), S, \{l_i\}_{i=1,\ldots,m}, \{g_i\}_{i=1,\ldots,m}, \{\beta_i\}_{i \in N \setminus S})$ where $(\mathbf{a}, \mathbf{t})$ is a feasible mechanism, S is the set of the agents that have previously accepted the mechanism, $\{l_i\}_{1,\ldots,m}$, where $m = \# N \setminus S$, is an ordered list of the agents in $N \setminus S$ and g_i is a participation game in $\mathcal{G}(\mathbf{a}, \mathbf{t}, S \cup \{l_i\}, \{\beta_i\}_{i \in N \setminus S \cup \{l_i\}})$ which is properly defined by the induction hypothesis. See Figure 1.

There are four kinds of positions in $g \in \mathcal{G}(\mathbf{a}, \mathbf{t}, S, \beta)$:

1. Disclosure nodes of the form $S_{current}$ where $S_{current}$ is the current set of the agents that have previously accepted the mechanism $(\mathbf{a}, \mathbf{t})$.

2. Responder nodes of the form (l_i, S), where $S \subset N$ is the set of the agents that have previously accepted the mechanism and $l_i \in N \setminus S$ is the identity of the agent with the initiative.

3. Intermediate nodes of the form g_i, where g_i is a participation game in $\mathcal{G}(\mathbf{a}, \mathbf{t}, S \cup \{l_i\}, \beta)$.

4. Terminal nodes of the form $(\mathbf{a}, \mathbf{t}, S)$ where S is the set of the agents that have previously accepted the mechanism $(\mathbf{a}, \mathbf{t})$.

At an intermediate node g_i, agents have no choice and the game moves to the disclosure node of the game g_i or moves to the terminal node $(\mathbf{a}, \mathbf{t}, N)$ if all agents give their consent. At a terminal node $(\mathbf{a}, \mathbf{t}, S)$, the game ends and the outcome $(\mathbf{a}(S), \mathbf{t}(S))$ is implemented.

At any responder position (l_i, S) there is the choice:

1. (l_{i+1}, S) if $i < m$ which means that agent l_i delays participation and l_{i+1} becomes the new responder. It corresponds to the two first arrays (NO) at the left of Fig. 1.

2. $(\mathbf{a}, \mathbf{t}, S)$ if $i = m$ which means that agent l_i refuses participation and the game ends at this terminal node. It corresponds to the array (NO) at the extreme right of Fig. 1.

3. g_i which means that agent l_i accepts the mechanism and the game moves to the intermediate node g_i. It corresponds to the arrays (YES) in Fig. 1.

At a disclosure node $S_{current}$, each agent in $S_{current}$ can secretly give the evidence that he has accepted the mechanism to any subset of his opponents. This evidence can also be revealed by the nature in a purely exogenous way.

The disclosure structure in the participation game has been incorporated to go beyond participation games with perfect information, where participation decisions are publicly observed by the agents. The precise structure of the information states will not matter: the crucial ingredient is that each participant has the opportunity at least once to prove to any of the remaining potential participants that he has accepted the mechanism.

The closure of the participation game after a finite number of delays may seem incoherent with our paradigm that agents do not decide whether they accept or reject the mechanism but rather that they either accept the mechanism or delay their acceptance decision. Indeed, apart from technical details, our following analysis is unchanged with the related infinite participation games, i.e. if $\{l_i\}_{i=1,\ldots,m}$ with $m = \infty$ is an infinite ordered list of agents, where an infinite delay for the agents in $N \setminus S$, given that agents in S gave their consent, results in the implementation of the outcome $(\mathbf{a}(S), \mathbf{t}(S))$. Participation games with a finite number of nodes and a unique opportunity to accept the mechanism at a given stage have been chosen to ease the backward induction argument and the presentation.

Let $\mathcal{G} = \bigcup_{S \subset N} \mathcal{G}(\mathbf{a}, \mathbf{t}, S, \beta)$ be the set of all participation games and $\mathcal{G}_{PI} \subsetneq \mathcal{G}$ the subset of the participation games with perfect information, i.e. the set of current participants is publicly revealed at the disclosure node. Since the number of participation games is finite, the parameter-space of the game they are playing about which they are uncertain is compact. Then we can build a universal type space Ω à la Mertens-Zamir [19] to represent agents' beliefs in the participation game (respectively Ω_{PI} with perfect information). Hence, our analysis does not hinge on any specific bidders' beliefs about irrelevant details of the participation process.

Then we assume that, whenever possible, beliefs are updated according to Bayes' rule. When an agent (or the Nature) provides the evidence to some agents that he has accepted the mechanism then their first-order beliefs are then stuck to the probability one that he has accepted the mechanism.

5 Optimal Design under robust sequential individual rationality

We now define a rationality constraint that removes the dependence of the participation decisions on the exact structure of the participation game and the corresponding beliefs of the agents.

Definition 2
- *A mechanism* $(\mathbf{a}, \mathbf{t})$ *is* robustly sequentially individually rational *(respectivelly robustly sequentially individually rational with perfect information) if* $(\mathbf{a}(N), \mathbf{t}(N))$ *is the final outcome in any subgame perfect equilibrium of any participation game (with perfect information)* $g \in \mathcal{G}(\mathbf{a}, \mathbf{t}, S, \beta)$.

- *A mechanism is* implementable *(with perfect information) if it is feasible and robustly sequentially individually rational (with perfect information).*

- *A mechanism* $(\mathbf{a}, \mathbf{t})$ *is* implementable under simultaneous participation *if it is feasible and if full participation is an equilibrium of the simulatenous participation game, i.e. if participation is a best response conditionally on the participation of all the other agents.*

The sets of implementable, implementable with perfect information and implementable under simultaneous participation are respectively denoted by $\beth$, $\beth_{PI}$ and $\beth_{sim}$. Note that our implementation concept under simultaneous participation is milder than the dominant strategy implementation concept according to which Jehiel et al. [13] shows that the principal obtains full extraction of the optimal threats under complete information.

Note the difference of the 'order independent' nature of our implementation concept with the 'order independent equilibrium' concept of Moldovanu and Winter [20]. In a nutshell, [20] considers *strategy profiles* that are *an* equilibrium independently of the specific structure of their coalition formation game. Here we consider *final outcomes*, more precisely the outcome

$(\mathbf{a}(N), \mathbf{t}(N))$ derived from full participation, that are the equilibrium outcome of *any* equilibrium independently of the specific structure of the participation game and agents' initial beliefs.

5.1 Participation processes with perfect information

Proposition 5.1 *A mechanism* $(\mathbf{a}, \mathbf{t})$ *is implementable with perfect information if and only if it is feasible and for any* $S \subset N$

$$\max_{i \in N \setminus S} \{V_i^{\mathbf{a}(N)} - \mathbf{t}_i(N) - V_i^{\mathbf{a}(S)}\} \geq 0 \tag{1}$$

Proof 1 *We first prove the 'Only if' part. Suppose that* $(\mathbf{a}, \mathbf{t})$ *is implementable and that there exists a subset* S *such that* $V_i^{\mathbf{a}(N)} - \mathbf{t}_i(N) - V_i^{\mathbf{a}(S)} < 0$ *for any agent* $i \in N \setminus S$. *Then consider a participation game* $g \in \mathcal{G}(\mathbf{a}, \mathbf{t}, N, \beta)$ *where* β *is such that it is common knowledge among the agents in* S *believe that the agents in* $N \setminus S$ *have accepted the mechanism. Hence, agents in* S *are playing as in a participation game in* $\mathcal{G}(\mathbf{a}, \mathbf{t}, S, \beta)$ *where it is common knowledge that the agents in* $N \setminus S$ *have accepted the mechanism. At any node where he is the last responder, the best response of an agent in* S *is to refuse the mechanism (if he accepts, the only equilibrium outcome is full participation since we have assumed that* $(\mathbf{a}, \mathbf{t})$ *is implementable). By backward induction, the best response of an agent in* S *at any node is to delay. Consequently, any subgame perfect equilibrium of the game* g *leads to the non-participation of the agents in* $N \setminus S$ *which raises a contradiction.*

The sufficiency part is proved by induction on the cardinality of the set of the agents that have already accepted the mechanism. The initial step where this set has the cardinality n *is immediate. Now consider that all agents in* $S \subsetneq N$ *have accepted the mechanism and suppose that* $\max_{i \in N \setminus S} \{V_i^{\mathbf{a}(N)} - \mathbf{t}_i(N) - V_i^{\mathbf{a}(S)}\} \geq 0$. *By the induction hypothesis, we obtain that every agents accept the mechanism in any subgame perfect equilibrium of any subgame* $\{g_i\}_{i=1,\ldots,m}$ *of the participation game* $g \in \mathcal{G}(\mathbf{a}, \mathbf{t}, S, \beta)$. *It remains to show that, for any game* $g \in \mathcal{G}(\mathbf{a}, \mathbf{t}, S, \beta)$, *it cannot belong to any equilibrium path that all agents refuse the mechanism at the responder nodes* $\{l_i\}_{i=1,\ldots,m}$. *In such a case, the agent* i *such that* $V_i^{\mathbf{a}(N)} - \mathbf{t}_i(N) - V_i^{\mathbf{a}(S)} \geq 0$ *has a profitable deviation: he accepts (with probability one) the mechanism when he is the responder, i.e. for a responder node such that* $l_k = i$, *which exists from the structure of the participation game. Note that in the case where he believes that the equilibrium outcome is still full participation if he delays, then he prefers strictly to accept immediately the mechanism as it has been assumed.*

The inequality (1) with $S = N \setminus \{i\}$ corresponds to the standard individual rationality constraint of agent i in the standard mechanism design approach under a simultaneous participation game. Thus the lack of commitment in the participation game results in a limitation of the set of implementable mechanisms. On the other hand, in an externality-free framework, the standard individual rationality constraints $V_i^{\mathbf{a}(N)} + \mathbf{t}_i(N) \geq V_i^{\mathbf{a}(N\setminus\{i\})}$ imply that $V_i^{\mathbf{a}(N)} + \mathbf{t}_i(N) - V_i^{\mathbf{a}(S)} \geq 0$ for any S and any $i \in N \setminus S$, the inequalities (1) are thus satisfied. Those points are summed up in the following corollary.

Corollary 5.2 *Any implementable mechanism with perfect information is implementable under simultaneous participation. In an externaltity-free framework, the converse holds: a mechanism that is implementable under simultaneous participation is implementable with perfect information.*

In the previous literature on mechanism design (with possibly incomplete information), the set of constraints that makes a mechanism implementable, i.e. feasibility, incentive compatibility and individual rationality constraints, results from inequalities that are linear according to the mechanisms $(\mathbf{a}, \mathbf{t})$.[5] Thus the set of the mechanisms that are implementable is a convex set. Moreover, the payoff of the principal depends linearly on the mechanism. From an optimal design perspective, there is thus no loss of generality to consider mechanisms that are symmetric if agents are symmetric. Suppose that a given asymmetric mechanism m is optimal. Then consider the permutations m_σ of this mechanism where $\sigma \in \Sigma(N)$. By symmetry, those mechanisms *implement* the same revenue for the principal. Finally, the mechanism $\frac{1}{n!}\sum_{\sigma\in\Sigma(N)} m_\sigma$ implements the same revenue in a symmetric way. On the contrary, the robust sequential individual rationality constraint results from inequalities involving the maximum of some linear maps and is thus not linear. Let us reconsider our simple example to illustrate the possible non-convexity of the set of implementable mechanisms.

Example 5.1 ***A simple example (suite)*** *Let $\mathbf{a}(S) = 1$, $\mathbf{t}_1(S) = V$ and $\mathbf{t}_i(S) = 0$ if $i \neq 1$ in the event where $1 \in S$ and $2 \notin S$. Let $\mathbf{a}(S) = 0$, $\mathbf{t}_i(S) = 0$ for any $i \in N$ in the event where $1 \notin S$. Let $\mathbf{a}(S) = 3$, $\mathbf{t}_1(S) = 0$, $\mathbf{t}_2(S) = \alpha$ and let $\mathbf{t}_3(S) = v$, if $S = \{1, 2, 3\}$ and $\mathbf{a}(S) = 0$, $\mathbf{t}_1(S) = 0$, $\mathbf{t}_i(S) = \alpha$ for any $i \in N$ in the event where $S = \{1, 2\}$. It is easily checked that this mechanism is feasible. Agents 1 and 3 obtain the same utility level (zero) independently of the final set of participants. Thus the inequalities (1) are satisfied if either 1 or 3 belongs to $N \setminus S$. Thus, it remains to check that the inequality (1) is satisfied if $S = \{1, 3\}$. Finally, the mechanism $(\mathbf{a}, \mathbf{t})$ is implementable. The mechanism $(\mathbf{a}', \mathbf{t}')$ where the roles of 1 and 2 have been*

[5] The implicit space structure according to which linearity applies is the following. For two mechanisms, $(\mathbf{a}, \mathbf{t})$ and $(\mathbf{a}', \mathbf{t}')$ and a real number $\lambda \in [0, 1]$, the mechanism $\lambda \cdot (\mathbf{a}, \mathbf{t}) + (1 - \lambda) \cdot (\mathbf{a}', \mathbf{t}')$ is the mechanism that implements the mechanism $(\mathbf{a}, \mathbf{t})$ (respectively $(\mathbf{a}', \mathbf{t}')$) with probability λ (resp. $(1 - \lambda)$).

switched is implementable by symmetry. Now consider the mechanism where, at a terminal node, each mechanisms $(\mathbf{a}, \mathbf{t})$ *and* $(\mathbf{a}', \mathbf{t}')$ *are implemented with probability one half. This mechanism is of course feasible. Nevertheless, it is not robustly sequentially individually rational. The constraint (1) with* $S = \{3\}$ *is violated. If agents 1 and 2 do not jointly participate, they obtain a null payoff. On the contrary, under full participation, their expected payoff is equal to* $-\frac{\alpha}{2}$. *Indeed the (efficient) mechanism* $(\mathbf{a}, \mathbf{t})$ *is the optimal design as it will appear as an application of proposition 5.3.*

There is no loss of generality to invite all agents to the mechanism since the set of feasible allocations does not shrink when some participants are added (Assumption 1). It suffices to extent the mechanism to the additional agents such that they do not modify the final outcomes and that they receive no transfer. The optimal design program is thus:

$$\max_{(\mathbf{a},\mathbf{t})} V_0^{\mathbf{a}(N)} + \sum_{i=1}^{n} \mathbf{t}_i(N)$$

subject to

$$\forall S \subset N, \max_{i \in N \setminus S} \{V_i^{\mathbf{a}(N)} - \mathbf{t}_i(N) - V_i^{\mathbf{a}(S)}\} \geq 0,$$

where $(\mathbf{a}, \mathbf{t})$ is a feasible mechanism.

Nevertheless, in this form, the program is hardly tractable and it is unclear whether the optimal design is efficient. We simplify the program by showing that there is no loss of generality to restrict the maximisation to a subclass of implementable mechanisms which are fully characterized by a couple $(\alpha, \sigma) \in A \times \Sigma(N)$. Let us introduce a last useful notation: for a given set $S \subset N$ and a permutation $\sigma \in \Sigma(N)$, denote by $j(S, \sigma)$ the smallest agent according to the order σ that is not belonging to S. Formally, $j(S, \sigma) = \max\{j \in N | T_j^\sigma \subset S\}$. This agent plays a key role in the subclass that we define below and such that if the set of participants is S, the principal will inflict the minmax punishment to the agent $j(S, \sigma)$.

Definition 3 *For* $(\alpha, \sigma) \in A \times \sigma(N)$, *we define the* (α, σ)*- optimal threat mechanism as the mechanism* $(\mathbf{a}, \mathbf{t})$ *defined in the following way:*

- $\mathbf{a}(N) = \alpha$
- $\mathbf{a}(S) = a^*_{j(S,\sigma)}(S)$, *if* $S \subsetneq N$
- $\mathbf{t}_i(N) = V_i^\alpha - V_i^*(T^\sigma_{\sigma^{-1}(i)})$
- $\mathbf{t}_i(S) = 0$, *if* $S \subsetneq N$

Those mechanisms can be interpreted in the following way: take one agent, $\sigma(1)$, and give him the incentive to participate independently to the participation decision of the other agents by using the optimal threat among $\mathcal{A}(\emptyset)$; then take another agent, $\sigma(2)$, and give him the incentive to participate taken as given that $\sigma(1)$ surely participates and independently to the participation decisions of the other agents in $N \setminus \{\sigma(1)\}$ by using the optimal threat among $\mathcal{A}(\{\sigma(1)\})$; and so on. In particular, for the last agent, $\sigma(N)$, in this new order σ, the principal uses the optimal threat in $\mathcal{A}(N \setminus \{\sigma(N)\})$ as in the standard literature with simultaneous participation.

We first show that this restricted class of mechanisms is a subset of the implementable mechanisms.

Lemma 5.1 *Any (α, σ)- optimal threat mechanism is implementable.*

Proof 2 *It is immediately feasible by definition of $a^*_{j(S,\sigma)}(S)$ which is the minmax punishment for agent $j(S,\sigma)$ given the participation set S. Consider $S \subset N$ and the agent $j(S,\sigma)$ who does not belong to S. We have:*

$$V_{j(S,\sigma)}^{\mathbf{a}(N)} - \mathbf{t}_{j(S,\sigma)}(N) - V_{j(S,\sigma)}^{\mathbf{a}(S)} = V^*_{j(S,\sigma)}(T^{\sigma}_{\sigma^{-1}(j(S,\sigma))}) - V^*_{j(S,\sigma)}(S) \geq 0$$

The equality comes from the definition of $\mathbf{t}_{j(S,\sigma)}(N)$ and because $\mathbf{a}(S) = a^*_{j(S,\sigma)}(S)$. The inequality is satisfied because $T^{\sigma}_{\sigma^{-1}(j(S,\sigma))} = \{\sigma(1), \ldots, \sigma(j(S,\sigma) - 1)\} \subset S$ (the inclusion comes from the definition of $j(S,\sigma)$). Thus we have proved that the inequality (1) holds for any $S \subset N$.

Then we show that there is no loss of generality to look for an (α, σ)-optimal threat mechanism to solve the optimal design program.

Proposition 5.3 *For any implementable mechanism $(\mathbf{a}, \mathbf{t})$, there exists an implementable mechanism that belongs to the class of (α, σ)- optimal threat mechanisms and that raises at least the same utility level for the principal. The optimal design program becomes:*

$$\max_{(\alpha,\sigma)\in A\times\sigma(N)} \left\{ \sum_{i=0}^{n} V_i^{\alpha} - \sum_{i=1}^{n} V_i^*(\{\sigma(1), \ldots, \sigma(\sigma^{-1}(i) - 1)\}) \right\} \tag{2}$$

Proof 3 *For a given mechanism $(\mathbf{a}, \mathbf{t})$, we define a corresponding (α, σ)- optimal threat mechanism in the following way: $\alpha = \mathbf{a}(N)$, σ is defined by induction such that $\sigma(1) = Arg\max_{i\in N} \{V_i^{\mathbf{a}(N)} - \mathbf{t}_i(N) - V_i^{\mathbf{a}(\emptyset)}\}$ (initial step) and $\sigma(i) = Arg\max_{i\in N\setminus\{\sigma(1),\ldots,\sigma(i-1)\}} \{V_i^{\mathbf{a}(N)} - \mathbf{t}_i(N) - V_i^{\mathbf{a}(\{\sigma(1),\ldots,\sigma(i-1)\})}\}$ (inductive step). The map σ is by definition a permutation. From lemma 5.1, the (α, σ)- optimal threat mechanism is implementable. It remains to* *show that it raises a greater utility for the principal than the original mechanism* $(\mathbf{a}, \mathbf{t})$. *More precisely, the principal implements the same economic outcome and extracts more surplus from each agent. Let* $\mathbf{t}_i^{(\alpha,\sigma)}(N)$ *be the transfer for agent* i *in the* (α, σ)*- optimal threat mechanism at equilibrium. We have:*

$$\mathbf{t}_i^{(\alpha,\sigma)}(N) = V_i^{\mathbf{a}(N)} - V_i^*(T^\sigma_{\sigma^{-1}(i)}) \geq V_i^{\mathbf{a}(N)} - V_i^{\mathbf{a}(T^\sigma_{\sigma^{-1}(i)})} \geq \mathbf{t}_i(N)$$

The first equality results from the definition of $\mathbf{t}_i^{(\alpha,\sigma)}(N)$ *and that* $\alpha = \mathbf{a}(N)$. *The first inequality comes from the definition of the map* $V_i^*(.)$ *and since* $a(T^\sigma_{\sigma^{-1}(i)}) \in \mathcal{A}(T^\sigma_{\sigma^{-1}(i)})$. *The last inequality results from our subtle construction of* σ *and the inequality (1) for the set* $T^\sigma_{\sigma^{-1}(i)}$. *This latter inequality states that* $\max_{j \in N \setminus \{\sigma(1),\ldots,\sigma(\sigma^{-1}(i)-1)\}} \{V_j^{\mathbf{a}(N)} - t_j(N) - V_j^{\mathbf{a}(\{\sigma(1),\ldots,\sigma(\sigma^{-1}(i)-1)\})}\} \geq 0$ *if* $(\mathbf{a}, \mathbf{t})$ *is implementable. The construction of* $\sigma(i)$ *guarantees that the expression in the 'max' is positive for* $j = \sigma(i)$, *i.e.* $V_{\sigma(i)}^{\mathbf{a}(N)} - V_{\sigma(i)}^{\mathbf{a}(T^\sigma_{\sigma^{-1}(i)})} \geq \mathbf{t}_{\sigma(i)}(N)$. *To sum up, we have proved that* $\alpha = \mathbf{a}(N)$ *and* $\mathbf{t}_i^{(\alpha,\sigma)}(N) \geq \mathbf{t}_i(N)$ *for all agents. The utility level of the principal is thus higher in the* (α, σ)*-optimal threat mechanism we have constructed than in* $(\mathbf{a}, \mathbf{t})$.

The optimal program (2) allows us to separate the choice of the final outcome α to the choice of the optimal threat structure, which is indeed reduced to the choice of a permutation that specifies the order according to which agents will be threat taken as given the participation decision of the agents that are lower in this order. The optimal choice of α thus coincides with the maximisation of the allocative efficiency.

Corollary 5.4 *Optimal robustly sequentially individually rational feasible mechanisms are efficient.*

The expression (2) of the utility level of the principal should be compared with the standard expression under simultaneous participation:

$$\max_{\alpha \in A} \left\{ \sum_{i=0}^{n} V_i^\alpha \right\} - \sum_{i=1}^{n} V_i^*(N \setminus \{i\}) \tag{3}$$

In general, the possibility to commit to a simultaneous participation game leads to a greater payoff for the principal since $V_i^*(S)$ is decreasing in S. Under robust sequential individual rationality and in an (α, σ)-optimal threat mechanism, the set of implementable threats is reduced to $V_{\sigma(i)}^*(\{\sigma(1), \ldots, \sigma(i-1)\})$ for the agent $\sigma(i)$. Nevertheless, in a negative-externality-free framework, the optimal threat $V_i^*(N \setminus \{i\})$ against agent i requires economic an outcome a that is always feasible independently to the set of participant, i.e. $a \in \mathcal{A}(\emptyset)$, and is thus always equal to $V_i^*(\{\sigma(1), \ldots, \sigma(\sigma^{-1}(i)-1)\})$. We obtain the following corollary:

Corollary 5.5 *In a negative-externality-free framework, the optimal revenue under simultaneous participation can be implemented under robustly sequentially individually rationality.*

5.2 Participation processes with imperfect information

The optimal design derived in Proposition 5.3 extends to a general information states structure with partial observability of participations decisions but where participants can strategically disclose evidence that they have accepted the mechanism. We exclude any strategic disclosure by the seller of evidence on participation decisions to remain coherent with our assumption that the seller can not control the participation process. It would not alter the following result.

Proposition 5.6 *The optimal outcome $(\mathbf{a}(N), \mathbf{t}(N))$ implementable with perfect information is implementable for any participation process with possibly imperfect information.*

Proof 4 *The construction is exactly the same as with perfect information with only an additional care on the transfers $\mathbf{t}_i(S)$ when S is a strict subset of N. Remind that those transfers did not play any role with perfect information. A mechanism that is implementable with perfect information may not be implementable in general because some agents may prefer to participate without disclosing this information. There may exist some subgame perfect equilibria where agents uses mixed strategies leading in some cases to incomplete participation. Nevertheless, if the transfers with incomplete participation are sufficiently high, e.g. $\mathbf{t}_i(S) > \mathbf{t}_i(N) - V_i^{\mathbf{a}(N)} + V_i^{\mathbf{a}(S)}$ for $S \subsetneq N$, then it guarantees that any participant would disclose that he has accepted the mechanism, if there were a terminal node with incomplete participation in the equilibrium path.*

5.3 A maxmin foundation

In the same way as a maxmin principal which is uncertain about agents' beliefs lays the foundation of dominant strategy implementation (see Chung and Ely [4]), our implementation concept requires that full acceptance is the only equilibrium for any kind of beliefs for the agents. Moreover, we made the somehow *ad hoc* assumption that the mechanism's outcomes depend only on the set of participants and so that agents can not report their beliefs. With a maxmin decision maker, that maximizes the worst-case performance, there is no loss of generality to search for a detail-free optimal design where a participant does not report any message.

Proposition 5.7 *The use of implementable mechanisms has a maxmin foundation; i.e.,*

$$\sup_{\substack{(a,t):\Omega^*\to\mathcal{A}\times\mathbb{R}^N \\ (a,t) feasible}} \inf_{\mu\in\mathcal{M}(\Omega^*)} E_\omega[V_0^{\mathbf{a}(\omega)} + \sum_{i=1}^n \mathbf{t}_i(\omega)] = \sup_{(a,t)\in\beth} V_0^{\mathbf{a}(N}) + \sum_{i=1}^n \mathbf{t}_i(N),$$

where $\mathcal{M}(\Omega^*)$ is the set of all probability measures on Ω^*. The same equality holds for by replacing Ω and $\beth$ by respectively Ω^*_{PI} and $\beth_{PI}$.

Proof 5 *Suppose that we can implement a higher revenue with a more complex mechanism with a arbitrary set of messages. Hence, by proposition 5.1, there is a set S such that the constraint (1) is violated. If it is common knowledge among the agents in $N \setminus S$ that there are playing a given participation process where the agents in S have already accepted the mechanism, then any subgame perfect outcome involves nonparticipation of the agents in $N \setminus S$. Thus we have raised a contradiction.*

6 Examples

As illustrated by our starting example, an important class of applications where our new rationality constraints are binding is auctions with negative externalities as in [12, 13, 14, 7, 9]. The scope of application may seem quite limited since the optimal design is unchanged in a negative-externality-free framework. Our two following examples show how robust sequential individual rationality may be fruitful first to model general collusion mechanisms and second contracting in dynamic environments when long-term contracts are not available.

6.1 Example 1: The design of collusion mechanisms

In the recent mechanism design literature on collusion as in Che and Kim [3], one agent or a third party proposes a mechanism that can be vetoed by each agent. When an agent breaks the collusion process, the game is played in a non-cooperative way under passive-beliefs. Thus contrary to the mainstream mechanism design literature, the principal is significantly limited in the way she can punish non-participants. In an auction framework, Caillaud and Jehiel [1] relax slightly this veto power assumption by also considering the case where a defection leads to a collusive report from the agents that are remaining in the collusion process. Dequiedt [8] considers that the remaining agent can commit to the harsher punishment if the other agent refuses the collusion mechanism. The reluctance to adopt the standard mechanism design approach to model collusion may come from the seemingly excessive commitment power that it requires and which is slightly softened under our approach.

Let us discuss those differences in a simple example under complete information: a symmetric triopoly under Cournot competition. Each firm has a constant null marginal cost and a maximum capacity $q_{max} = 0.5$. Inverse demand is given by $P = 1 - Q$, where Q denotes the total quantity supplied. Without collusion, the quantity supplied by each firm in equilibrium is equal to $1/4$ and the corresponding total profit of the triopoly is $\Pi_{nc} = 3/16$. The collusive outcome corresponds to the total production $Q = 1/2$ and the joint profit $\Pi_{col} = 1/4$. Suppose that a collusion mechanism, which specifies the quantities produced by each participant and balanced monetary transfers among participants for each possible set of participant, is proposed by one firm, say 1. Under complete information, all the different models, leads to the collusive outcome in the optimal mechanism. Nevertheless, the distribution of the profits from collusion that can be implemented are very different according to the model for collusion. Under veto power, an assumption that is often made, each firm is guaranteed to obtain her non-cooperative profit $1/16$. The proposer is able to capture all the rents from collusion $\Pi_{col} - \Pi_{nc} = 1/16$. At the other extreme as in Dequiedt [8], a non-participant can be punished by the minmax punishment which leads here to a null payoff: the two remaining participants commit to produce $q = 0.5$ which leads to a null price. Nevertheless, this mechanism may seem poorly convincing since firm 1 manages to extract all the surplus from trade $(1/4)$ from both firms by threatening each to flood the market with the help of the other one. With our model, the maximal surplus that firm 1 can extract is intermediate: she can extract the full surplus only to one firm and has to leave the surplus $1/36$ to the other one, the profit corresponding to the Cournot outcome after the commitment to produce $q = 0.5$ by firm 1. Thus she should use a divide and conquer strategy.

6.2 Example 2: Dynamic processes of social and economic interactions

Gomes and Jehiel [10] consider a model of dynamic interactions in complete information where, at each period, an agent is selected to make an offer to a subset of the other agents to move the state of the economy. They do not only assume that long-term contracts are not available but also restrict the analysis to simple-offer contracts where each approached agent can veto the proposed move. Indeed, as they emphasize, this restriction is with no loss of generality if a third party can coordinate the approached agents by means of a 'strong' collusion contract with transfers. With general contracts -i.e. without any form of collusion- the economy moves immediately to the efficient state. On the contrary, with simple-offer contracts, efficiency is no longer guaranteed. This negative result compared with the Coasian intuition depends critically on the model for collusion. If collusion is modeled without any monetary tranfers only by means of robust sequential individual rationality, interpreted here as a mild collusion device, then the transposition of corollary 5.4 in their framework restores efficiency: all Markov Perfect Equilibria of the economy with general spot contract that are robustly sequentially individually rational are efficient, entailing an immediate move to the efficient state, where it remains forever. Note however that, under our milder collusion device, the expected payoff of the selected proposer is lower than with general contracts. At the other extreme, under a mildly stronger form of collusion where the third party can also contract with non-approached agents and where collusion is not observable by the proposer, the economy also moves immediately to the efficient state.

7 Concluding Remarks

We relax the commitment ability of the principal in some minimal way and give some theoretical foundations for such a refinement of the standard mechanism design approach. The scope of application may seem relatively restricted since the optimal design is unchanged in a negative-externality-free framework. Nevertheless, jointed with other commitment failures as the unability to commit not to propose a new mechanism if the first one fails to work, e.g. the unability to commit never to attempt to resell the good if she fails to sell it as in McAfee and Vincent [18] and Skreta [23], robust sequential individual rationality may have some bite even in pure private value trading framework without externality. For example, in a procurement auction, the designer may be unable to set a high reserve price since this would trigger a joint boycott of the main market participants that will force the designer to propose a new mechanism.

Our sequential participation game can be also interpreted as a minimal collusive device for the agents. The main contributions on collusion-proof implementation [15, 16, 3] preclude any collusion on the participation decisions themselves and restrict the collusive activity to the reports. In this literature, the collusion technologies allow agents to fully contract (with monetary transfers) their reports to the principal. Surprisingly, Che and Kim [3] show that optimal noncollusive mechanism can be made collusion-proof in a broad class of circumstances including economic environment with (allocative) externalities. Here our collusive device is much weaker: neither monetary transfers nor binding agreements on the reports are available. Nevertheless, it consists in a form of collusion that includes the participation decisions. We show that in general, except when the framework is negative-externality free, the principal may raise a lower revenue at the optimal design under this device. It contrasts with the insights of Pavlov [22] and Che and Kim [2], where the collusion mechanism proposed by a third party takes place before the participation decisions, and where the second best is still implementable with collusion.[6] Those papers consider the auction of a single item in the independent private value framework and thus exclude any kind of externality. We thus shed some light on the impact of collusion on participation -any stronger collusive device as the ones in [8, 22, 2] above would only strengthen our results- independently of any informational asymmetry.

Finally, we have restricted attention to a complete information setup. It is left for further research how to extend the notion of implementation under sequential participation processes in incomplete information in order to analyse the interactions with the incentive compatibility constraints and ask whether this constraint is beneficial or not to the welfare. As for the concept of ratifiability introduced by Cramton and Palfrey [6], incomplete information requires a careful treatment of how agents revise their beliefs relative to the participation decisions of their opponents.

[6] In this line, Dequiedt [8] is an exception: in a binary type environment, he shows that asymmetric information do not prevent bidders to collude efficiently, i.e. to act as a single agent when the third party can manipule the participation decisions.

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