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Contingent Auctions with Allocative Externalities : Vickrey Versus the Ausubel-Milgrom Proxy Auction*

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Contingent Auctions with Allocative Externalities: Vickrey versus the Ausubel-Milgrom Proxy Auction*

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Abstract

We introduce contingent auction mechanisms, which is a superset of combinatorial auctions, and where bidders submit bids on packages that are contingent on the whole final assignment. Without externalities, the Vickrey and the Ausubel-Milgrom Proxy Auction are both robust if items are perceived as substitutes. Such an equivalence between those formats may not hold with externalities and the analog of the substitute condition is a complex unexplored issue. We analyse those issues in the Negative Group-Dependent Externalities framework, a general structure with allocative externalities between joint-purchasers.

Keywords: Auctions, combinatorial bidding, contingent bidding, allocative externalities, identity-dependent externalities
JEL classification: D44, D45, D62, L90

Abstract

Nous introduisons la notion d'enchère contingente, une famille de mécanismes d'enchères englobant les enchères combinatoires, où les enchérisseurs soumettent des offres non seulement sur des combinaisons ou paquets de biens mais aussi en fonction des paquets acquis par leur concurrents. Sans exteranlités allocatives, les mécanismes de Vickrey et d'Ausubel-Milgrom sont robustes si les agents perçoivent les biens comme des substituts. Ce résultat d'équivalence n'est plus susceptible d'être vérifié avec des externalités et l'analogue de la condition de substituabilité est un problème complexe. Nous analysons ces questions dans le cadre du modèle avec des exteranlités négatives au sein de groupes d'agents, un modèle général avec des externalités allocatives entre co-acquéreurs.

Mots-clés: Enchères, Enchères combinatoires, Externalités allocatives
Classification JEL: D44, D45, D62, L90

1 Introduction

For the single item assignment problem in a pure private value framework without externalities, the Vickrey-Clarke-Groves mechanism, also briefly referred to as the Vickrey auction, corresponds to the second price auction. Nevertheless this format is not popular in 'real life' auctions and practitioners prefer the use of the English auction. However, from auction theory's perspective, those formats are roughly equivalent: the efficient allocation, with the price for the purchaser being the highest valuation of his opponents, is implemented in dominant strategy. Several arguments have been developed to explain why the Vickrey auction is so rare, e.g. due to future interactions, the winner may be reluctant to reveal truthfully his preferences as in Rothkopf et al [36].[1]

Similarly, the Vickrey auction is not used for multi-object assignment problems although a fully general ascending counterpart of the Vickrey auction (which implements the efficient equilibria in dominant strategy) is not available when bidders' valuations go beyond the gross substitutes condition.[2] Nevertheless, auction theorists are more reserved about the relevance of the Vickrey auction in this case insofar as it may fail to be robust against some joint deviations and more specifically against losers' deviations.[3] This failure is related to the fact that, in general, the Vickrey payoffs do not belong to the Core as it is formalized in Proposition 2.4 where the connection between the Core and the robustness against losers' deviations is established. On the contrary, spurred by the success of the Simultaneous Ascending Auction (see Milgrom [28]) used for FCC's spectrum auctions, new ascending multi-object auction formats have been proposed as new tools for practical market design. Among them, many Clock auctions, where the seller mimics a fictitious Walrasian auctioneer, have been proposed ([2], [3], [12], [14], [6]). In those mechanisms, bidders are explicitly restricted to report 'substitutes' preferences, i.e. decreasing marginal utilities if a homogenous good is auctioned.

Ausubel and Milgrom (hereafter AM) have recently proposed an Ascend-

[1] Kagel, Harstad and Levin [15] experimentations also show that unexperienced subjects converge quicker to the (dominant strategy) Nash Equilibrium in the English auction. Compte and Jehiel [8] argues that ascending auctions with a slow pace are beneficial according to an information acquisition perspective.

[2] De Vries et al [11] derive an impossibility result for an ascending auction to implement the Vikrey outcome when at least one buyer's preferences fail the gross substitutes condition. Relaxing the requirement that the final prices in the auction define the payments of the buyers, Mishra and Parkes [31]

[3] Another potential failure of the Vickrey auction, investigated by Yokoo et al. [39], is its lack of robustness against shill bidding: the use of multiple identifiers by a single bidder. In an IO perspective with non-anonymous mechanisms, this failure seems less relevant. In this paper, it is therefore not discussed. Nevertheless, as we characterize the set of preferences such that the bidder-submodularity holds, then Yokoo et al.'s results can be applied.

ing Auction with Package Bidding, where bidders can report general valuations including complementary preferences, which is converging to a Core outcome according to the reported preferences.[4] A question of primary interest is then to delimit the set of preferences which makes the desired efficient outcome an equilibrium. The Ausubel-Milgrom proxy auction implements the Vickrey payoffs in dominant strategy if the coalitional value function of the related coalitional game is satisfying a so-called bidder-submodularity condition. With this appropriate restriction on the whole set of preferences, Vickrey payoffs are in the Core and correspond to the bidder optimal frontier which is single-valued in that case. Also referred to as "Buyers are substitutes", this condition is central in the combinatorial auction literature, e.g. see the unifying approach of de Vries et al [11] and Mishra and Parkes [31]. However, in the perspective of applications, we are more interested on the restrictions that should be put on bidders' valuations, i.e. the primitives of our assignment model. Without externalities, AM links the abstract bidder-submodularity condition with individual bidders preferences, namely bidder-submodularity is linked with the mutual substitutes condition for goods. With allocative externalities, Ranger [34, 33] introduces a contingent version of the AM-proxy auction and exhibits an Industrial Organisation example, where the bidder-submodularity condition is satisfied: the allocation of capacities before a downstream Cournot market.

On the whole, this literature on multi-unit auctions has not derived a clear-cut argument against the Vickrey auction beside any dynamic substitute because the Vickrey auction is actually robust exactly in the case where those alternative formats implement the efficient allocation in dominant strategy. Without externalities, the Vickrey auction is robust against losers' deviation and shill bidding exactly in the case where items are perceived as substitutes.

The main contribution of this paper is to give somehow the analog of the 'substitutes' condition in a framework with externalities. For example, in the unit-demand case, this condition is straightforward without externalities: the bidder-submodularity condition is always satisfied, hence the substitutes condition always holds. But even in the unit-demand case, the characterization of the preferences such that the bidder-submodularity condition is satisfied is difficult if allocative externalities are allowed. In the so-called Negative Group-Dependent Externalities framework (briefly referred to as the NGDE framework), the set of bidders' preferences such the bidder-submodularity condition holds is characterized. At the same time, in this framework, this paper sheds some light on the differences between the Vickrey and Ausubel-Milgrom proxy auctions: the robustness of the former requires weaker condi-

[4] Closely related are the dynamic matching mechanisms following Gale and Shapley's algorithm, which also have important practical implications. Recently, Hatfield and Milgrom [16] unify those two literatures (without allocative externalities).

tions on the primitives of the model but it requires that the designer is able to restrict the set of the reported vectors of joint preferences. This point brings us to a point that has not been emphasized in the previous literature: for the Ausubel-Milgrom proxy auction to be robust, there is no need to restrict the set of possible reported valuations, whereas, in the Vickrey auction, it is a main issue which could undermine it.

The second contribution of this paper is to introduce a general framework with allocative externalities which could be a better description of many assignment problems, e.g. those that have been previously considered in the experimental literature on combinatorial auctions. In a nutshell, the bidders in the NGDE framework are partitioned into groups such that the more joint-purchasers into their group, the less they are valuing items. The terminology 'group-dependent' comes from the fact that the externalities from which a bidder suffers are confined to the final market structure of his own group. The terminology 'negative externalities' comes from the fact that a bidder prefers that few competing bidders of his group acquire items.

The paper is also linked to the auction literature with externalities. One strand, beginning with Milgrom and Weber [30], considers informational externalities. Efficient ex-post robust mechanisms have been derived in Dasgupta and Maskin [10] and Perry and Reny [32]. In generic cases with interdependent valuations and multidimensional signals, Jehiel and Moldovanu [21] and Jehiel et al [18] derive an impossibility result respectively for the Bayes-Nash implementation of the efficient allocation and for the ex post implementation of nontrivial mechanisms. Another strand considers allocative externalities. Identity-dependent externalities in single item environments have been analyzed at first (Jehiel and Moldovanu ([19],[21]), Jehiel, Moldovanu and Stacchetti ([22], [23]), Das Varma [9]). If bidders are not indifferent to the identity of the opponent that has purchased the item, standard auction designs may fail to be efficient and the Core may be empty. In an optimal design perspective with one-dimensional types, Figueroa and Skreta [13] consider a multi-unit assignment issue with both informational and allocative identity-dependent externalities. Here the focus is on dominant strategy implementation in a multi-unit environment with identity-dependent allocative externalities while keeping the private valuations assumption.

This paper is organized as follows: Section 2 introduces the general assignment problem with allocative externalities between joint-purchasers and the related contingent mechanisms. In the same way as standard auctions appear to be inadequate when a bidder values packages of objects and that combinatorial auctions, where bids are made on bundles of objects, may solve inefficiencies, contingent auctions are allowing bidders to express the full dimensions of their valuation vector. As for combinatorial auctions, the issue is the robustness of such mechanisms. We first adapt the results form the literature to our contingent environment. In Section 3, we introduce the NGDE framework and discusses its practical relevance. The main results of the paper appear in sections 4 and 5 which are respectively characterizing the conditions for the Vickrey auction and Ausubel-Milgrom proxy auctions to be robust in the NGDE framework. Section 6 considers the extension where the auctioneer is not only a pure revenue maximizer but also has preferences on the final allocation. Section 7 concludes with a policy perspective. The proofs of our two main results are gathered in the Appendix.

2 Contingent Auctions

2.1 The general assignment problem: definitions and notation

There are N risk neutral bidders (With a slight abuse of notation, N will represent the set as well as the number of bidders) indexed by $l = 1, \ldots, N$ and a seller designated by $l = 0$. There are $M > 1$ indivisible items to be auctioned. We define an allocation, denoted by $\mathcal{A} = \{\mathcal{A}_1, \cdots, \mathcal{A}_M\}$ where $\mathcal{A}_i \in N \cup \{0\}$ denotes the identity of the purchaser of item i, as an assignment of the M items to the bidders (some items could remain in the seller's hands). Denote by $\mathbf{A} := ([1, N] \cup \{0\})^M$ the set of feasible allocations. For a given allocation $\mathcal{A}$, denote by $\mathcal{A}^l$ the set of items assigned to bidder l and $\#\mathcal{A}$ the total number of items that are sold. For any couple of bidders (l, m) and an allocation $\mathcal{A}$, denote by $\mathcal{A}(l \curvearrowright m)$ the allocation resulting from allocation $\mathcal{A}$ by giving all the items assigned previously to bidder l to bidder m.[5]

We make some restrictions on the set of preferences. First we exclude any informational externalities: we consider a private values' framework where one's valuation depends solely on his private signal and not on his opponents' signals. Second, if a bidder does not acquire an item, his utility is normalized to zero. Note that this normalization is not innocuous: it states that a non-purchaser is indifferent to the final allocation. Thus we restrict our analysis to allocative externalities that are somehow orthogonal to the externalities in Jehiel et al papers ([19], [20], [22], [23]). Third, departing from the restricted set of preferences that AM considers, we allow for allocative externalities. The valuation of bidder l for an allocation does not depend solely on the bundle he acquires but also on the bundles his joint purchasers acquire. If bidder l acquires some items, i.e. $\mathcal{A}^l \neq \emptyset$, he derives the payoff $\Pi_l(\mathcal{A})$, where $\mathcal{A}$ is the final allocation. The function Π_l will also be qualified as bidder l's type. On the contrary to AM, we do not make any free disposal assumption. An item can worth less than zero: we may have $\Pi_l(\mathcal{A}) < 0$.[6] Finally, we

[5] Formally, $\mathcal{A}(l \curvearrowright m)$ is defined such that we have $\mathcal{A}(l \curvearrowright m)^k = \mathcal{A}^k$, if $k \neq l, m$; $\mathcal{A}(l \curvearrowright m)^l = \emptyset$; $\mathcal{A}(l \curvearrowright m)^m = \mathcal{A}^m \cup \mathcal{A}^l$.

[6] For example, free disposal can be a quite restrictive assumption for a capacity auction where a purchaser is contractually required to use the capacity.

make an additional assumption that makes sense only in a framework with allocative externalities: for any allocation, the valuation of a purchaser is never reduced if some of his joint-purchasers are replaced by the seller which then keeps the relating items in her hands. This is the following no positive externality assumption.

Assumption 2.1 (No positive externality) *A bidder $l \geq 1$ is said to suffer from no positive externality if his type is such that for any allocation $\mathcal{A}$ and any bidder $m \neq 0, l$ we have*

$$\Pi_l(\mathcal{A}) \leq \Pi_l(\mathcal{A}(m \curvearrowright 0)).$$

Denote by $\Pi := (\Pi_0, \Pi_1, \dots, \Pi_N)$ the vector of types (With a slight abuse of notation Π will also represent the set of all types).

Eventually, we consider quasi-linear utility. If bidder l pays a bid $b_l(\mathcal{A})$ such that the allocation $\mathcal{A}$ is chosen, then he earns a net payoff of $\Pi_l(\mathcal{A}) - b_l(\mathcal{A})$. The payoff of the seller equals to her revenue $\sum_{l \in N} b_l(\mathcal{A})$ plus her private valuation for the chosen allocation $\Pi_0(\mathcal{A})$, which is assumed to be increasing in $\mathcal{A}^0$.

In such a general framework, bidders do not care solely on their own allocation, i.e. whether they obtain or not some bundles of items, but also on the identity and bundles of their joint purchasers. We will precise later, in section 3, practical examples where such externalities may intervene as first order in the valuations. Standard and also combinatorial auction mechanisms do not allow bidders to express this dimension in their valuations: bidders are not allowed to submit bids that are contingent on their joint-purchasers. Then full efficiency is not guaranteed. On the contrary, in a contingent mechanism, bidders can report contingent bids: the allocation rule and the associated transfers depend on the whole set of contingent bids Π. Thus we define a broader class of auction mechanisms: contingent auctions.

Definition 1 *A contingent auction mechanism $(A, p) : \Pi \to \mathbf{A} \times \mathbb{R}^N$ is a function mapping a vector of types Π into an allocation $A(\Pi)$ and a vector of transfers such that $p_l(\Pi)$ represents the transfer paid by bidder l to the seller.*[7]

[7]Contingent auctions should be viewed as a superset of combinatorial auctions. In the special case of unit-demand where a combinatorial auction reduces to a standard auction with a one-dimensional reported type, the dimension of a type could explode with the number of potential joint-purchasers. For example, with $N = 10$ potential bidders and $M = 6$ items, this dimension equals to $\sum_{i=0}^{M-1} \binom{N-1}{i} = 382$. In general, this raises a complexity issue and the computation of the Vickrey allocation is an NP-complete problem. However, in the following NGDE framework where items are homogenous and where externalities presents also a kind of homogeneity, this complexity is considerably reduced and we presume that the computation of the auction mechanism is not a practical issue anymore.

Throughout the analysis, our criteria is allocative efficiency. We are looking for auction mechanisms that are ex post efficient. A contingent mechanism is *ex post efficient* if for all Π, $A(\Pi)$, is an efficient allocation relative to the reported preferences, Π, namely:

$$\sum_{l\in\{0\}\cup N} \Pi_l(A(\Pi)) \geq \sum_{l\in\{0\}\cup N} \Pi_l(\mathcal{A}), \quad for\ all\ \mathcal{A} \in \mathbf{A}$$

Generically, the efficient allocation is unique and then denoted by $A^*(\Pi)$ or $\mathcal{A}^*$ (if the relative preferences are unambiguous). We mainly consider two kinds of ex post efficient contingent auction mechanisms: first the Vickrey Contingent Auction and then Ausubel-Milgrom contingent auctions that we define hereafter. Of course, ex post efficiency does not guarantee that the resulting allocation is efficient since the reported preferences may not fit with the real ones. The main issues are then the following ones. First what are the incentives for the bidders to report their true preferences? More precisely, we are interested in dominant strategy implementation. Second, how equilibria are robust out of the equilibrium's path? The dominant strategy equilibrium may not be convincing if some simple 'deviations' may be profitable. More precisely, we consider the deviations from coalitions of losing bidders with or without monetary transfers inside the coalition. Let us define formally those two kinds of coalitional deviations.

Definition 2 (Robustness to losers' deviation) *The outcome of a contingent auction is robust against losers' deviation (respectively strongly robust) at a given type vector* Π *if for any reported vector of types* Π^{dev} *such that* l *belongs to the set* S *of deviators, defined by* $S = \{k \in N | \Pi_k^{dev} \neq \Pi_k\}$*, implies that* l *is a loser in the allocation under truthful reporting, i.e.* $A^*(\Pi)^l = \emptyset$*, then this deviation is unprofitable for at least one bidder in* S*:*

$$\exists l \in S\ such\ that\ \Pi_l(A^*(\Pi_{dev})) < p_l(\Pi_{dev}).$$

(respectively then this deviation is unprofitable for the whole coalition S*:* $\sum_{l\in S} \Pi_l(A(\Pi_{dev})) < \sum_{l\in S} p_l(\Pi_{dev})$*.)*

The weak version considers coalitions of losers where monetary transfers are not available. This is the definition that will be used for characterizing the robustness of the Vickrey auction against losers' deviations in the NGDE framework. On the other hand, as will be shown in proposition 2.4, the outcome of AM-auctions (more generally of auctions that are leading to Core outcomes) are strongly robust against losers deviations, i.e. even if monetary transfers are allowed between losers. In AM framework without externalities, the subtlety between weak and strong robustness against losers' deviation does not matter as it will be clarified in section 4.

2.2 The coalitional form

A useful tool for the analysis of combinatorial auctions and similarly for contingent auctions is a coalitional form game $(N \cup \{0\}, w)$ that is associated with the assignement problem $\{\mathbf{A}, (\Pi_j)_{j \in N \cup \{0\}}\}$, where w is the coalitional value function. For any coalition of bidders $S \subset N \cup \{0\}$, w is defined by:

$$w(S) = \max_{\mathcal{A} \in \mathrm{A}} \sum_{l \in S} \Pi_l(\mathcal{A}), \quad if \quad 0 \in S \tag{1a}$$

$$w(S) = 0, \quad if \quad 0 \notin S \quad ^8 \tag{1b}$$

Denote by u_l the net payoff of bidder l. Then we define the set of core payoffs, denoted by $Core(N, w)$, related to this coalitional value function w:

$$Core(N, w) = \left\{ (u_l)_{0 \leq l \leq N} \mid (a): \sum_{l=0}^{N} u_l = w(N); \ (b): \ (\forall S \subset N \cup \{0\}) \ w(S) \leq \sum_{l \in S} u_l \right\}$$

(a) is the feasibility condition, whereas (b) means that the payoffs are not blocked by any coalition S.

A subset of the core plays a central role in the analysis of combinatorial auctions and similarly for contingent auctions: those are the Pareto-optima from the perspective of the bidders. This set is qualified as the bidder optimal frontier of the core.

Definition 3 *The bidder optimal frontier of the core is the set containing the elements $(u_l)_{0 \leq l \leq N} \in Core(N, w)$ such that there exists no $(u'_l)_{0 \leq l \leq N} \in Core(N, w)$ where $u'_l \geq u_l$ for all $l = 1 \ldots N$ and such that at least one inequality is strict.*

In the analysis of combinatorial auctions, a condition has emerged in the literature which renders truthful reporting an equilibrium of the Ausubel-Milgrom ascending proxy auction and also makes the Vickrey auction robust to shill bidding and against losers' deviations: this is bidder submodularity.

Bidder submodularity is a kind of 'substitutes' condition: the bidders should be viewed as substitutes insofar as the surplus associated with the presence of a bidder is decreasing with the set of competitors.

Definition 4 (Bidder submodularity) *The coalitional value function w is bidder-submodular if for any $l \in N$ and any coalitions S and S' satisfying $0, l \in S \subset S'$, we have*

$$w(S) - w(S \setminus \{l\}) \geq w(S') - w(S' \setminus \{l\})$$

[8] If the seller is not a member of the coalition, the coalition obtains no items. Due to our no positive externality assumption, this point is independent of any 'dumping' assumption, i.e. whether the auctioneer can dump objects on bidders who have not bid for them, if the seller is supposed to maximize her revenue among the remaining bidders outside the coalition.

The term $w(S) - w(S \setminus \{l\})$ represents the surplus associated with the presence of bidder l in the coalition S.

If the bidder-submodularity condition holds, then free-riding issues among bidders are avoided. For example, suppose that two bidders would like to acquire some items and that they have to block other potential bidders in order to win, then the exclusion of those competitors contains no public good between these two bidders. In a framework without externalities, AM [Theorem 12] characterizes in some way the set of individual preferences for w being bidder-submodular: items should be regarded as mutual substitutes by the bidders.

2.3 The Vickrey Contingent Auction

Definition 5 *The Vickrey Contingent Auction is the ex post efficient mechanism with the following pivotal prices:*

$$p_l^V(\Pi) = \max_{\mathcal{A} \in \mathbf{A}} \left\{ \sum_{k \neq l} \Pi_k(\mathcal{A}) \right\} - \sum_{k \neq l} \Pi_k(A^*(\Pi)), \quad \textit{for all } l \in N$$

In the most general framework with allocative externalities, the Vickrey Contingent Auction is such that a non-purchaser could pay and such that a purchaser may be paid to acquire an item that is valuable for him. Indeed, in the preceding restricted framework with externalities, those two peculiarities could not arise.

Proposition 2.1 *For any reported types in Π, then for all l, $p_l^V(\Pi) \geq 0$. If bidder l acquires no items, then $p_l^V(\Pi) = 0$.*

Proof 1

$$\max_{\mathcal{A} \in \mathbf{A}} \left\{ \sum_{k \neq l} \Pi_k(\mathcal{A}) \right\} \geq \sum_{k \neq l} \Pi_k(A^*(\Pi)(l \curvearrowright 0)) \geq \sum_{k \neq l} \Pi_k(A^*(\Pi))$$

The first inequality results from the definition of the maximization. The second inequality results from the no positive externality assumption for all $\Pi_k, k \neq l$. Those inequalities are equalities if bidder l acquires no item in $A^*(\Pi)$.

Note that the no positive externality assumption is crucial. In the Vickrey auction, for any reported vector of types for all bidders except two, we can prove easily that the two remaining bidders can report types suffering from positive externalities and then purchase all items and receive an arbitrary high transfer what makes the Vickrey contingent auction unattractive. Each of the two bidders declare that the other as joint purchaser is essential to obtain a very high profit. Thus it should be emphasized that for obvious robustness reasons, the auctioneer should imperatively restrict the set of preferences that can be reported to make it robust against losers' deviation. The no positive externality restriction can be easily implemented given that this restriction concerns each individual report and not the whole set of reports to the auctioneer.

2.4 The Ausubel-Milgrom Contingent Proxy Auction

AM have proposed an 'ascending auction with package bidding'. Indeed, their analysis focuses on the related proxy auction which constitutes a new sealed-bid combinatorial auction. Ranger [34] studies a generalization of this mechanism allowing bidders to express externalities, i.e. a specific contingent auction. Lamy [27] proposes a slight modification of the auction by adding a final stage giving strictly more incentives to report the truth while remaining a Core-selecting auction. The definitions of those specific auctions are not straightforward. However what matters in the following analysis is the property that the final outcome lies in the bidder optimal frontier of the Core given the reported preferences. Our results apply to this class of auctions that we will refer to as Ausubel-Milgrom-auctions (hereafter AM-auctions).[9]

Definition 6 *An AM-auction is a contingent auction mechanism such that the final payoff vector belongs to the bidder optimal frontier of the Core relative to the reported types.*

See Lamy [27] for an algorithm implementing a specific AM-auction. Note that the set of AM-auctions may be empty if the core corresponding to some possible types in Π is empty. Indeed in our framework, the core with respect to the true types is never empty. This is due to the assumption that non-purchasers suffer from no externalities. This crucial assumption is discussed in the supplementary material [26].

AM establishes a link between the structure of the Core and the Vickrey outcome.

Proposition 2.2 (AM Theorem 6) *The bidder optimal frontier is a singleton and then corresponds to the Vickrey outcome if and only if the Vickrey outcome is in the Core.*

If his competing bidders are truthful, we know from the definition of an AM-auction that a bidder can never obtain a payoff greater that his Vickrey

[9]Indeed, the differences between the mechanisms inside this class are completely unessential insofar all those mechanisms satisfy the same properties that we will recall below. Note that the original proxy auction proposed by AM is not really an AM-auction because the final outcome belongs only to the weak bidder optimal frontier of the Core. Refer to Lamy [27] for more details. Nevertheless, in this format, the bidder-submodularity condition is also a sufficient condition for truthful reporting to be an equilibrium.

payoff: this results from the fact that coalition $N \cup \{0\} \setminus \{l\}$ does not block. Thus if a bidder obtains his Vickrey payoff under truthful reporting, then it is a best response. Coupled with proposition 2.2, we have proved the following corollary.

Corollary 2.3 *If the Vickrey outcome is in the Core, then truthful reporting is a Nash Equilibrium in any AM-auction.*

Indeed, instead of investigating the class of preferences such that the Vickrey outcome is in the Core for the whole set of bidders N, we investigate preferences satisfying a stronger condition: for any subset of bidders $S \subset N$, the Vickrey outcome is a Core outcome (the Vickrey and Core outcome being defined relative to this subset S). According to Theorem 7 in AM, this is equivalent to the bidder-submodularity of w.

On the other hand, as it is depicted in Figure 2.4, if the Vickrey outcome is not in the Core, then truthful reporting is not an equilibrium: under truthful reporting, an outcome in the bidder-optimal frontier is implemented, then at least one bidder, say bidder 2, does not obtain his Vickrey payoff. If bidder 2 shades off his bids, he is able to reduce the bidder optimal frontier up to the point where the only bidder optima are such that he is guaranteed to obtain his Vickrey payoff. The truncation of the Core resulting from bid-shading is illustrated in Figure 2.4 where the Core is depicted under the shaded area. After the bid reduction Δb, the Core is truncated above by the snaked line. After the optimal bid reduction, Δb^{opt}, the Core becomes the singleton $(0, \pi_2^V)$. The dashed lines departing from π^{Max} the highest payoffs that each bidder can expect with a null transfer and ending in the bidder optimal frontier depicts possible dynamic of the highest expected payoff vector in a dynamic version of an AM-auction.

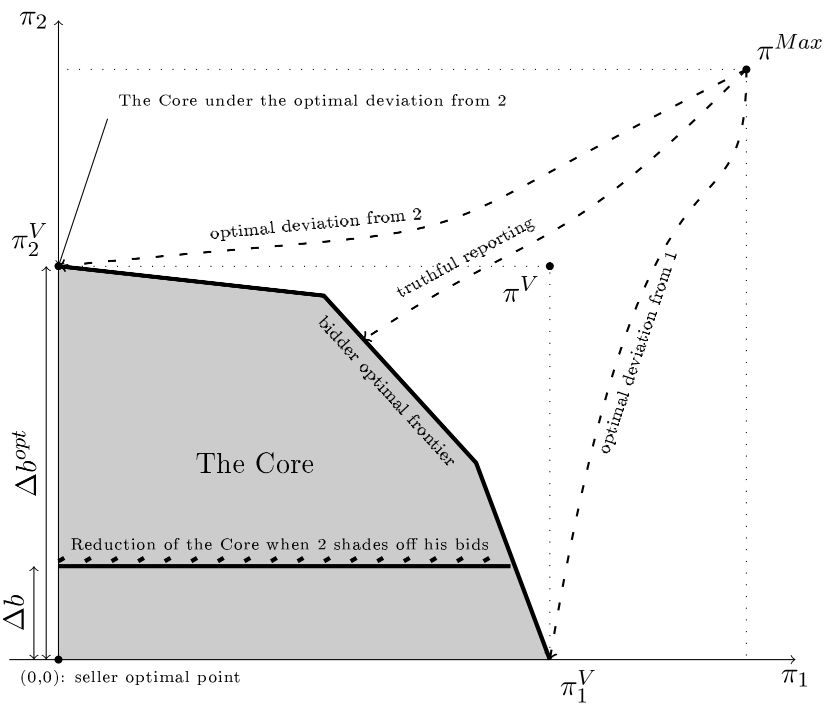

Fig 2.4: AM-auctions dynamics

2.5 Losers' Deviations and Core constraints

Intuitively the robustness against losers' deviations is very closely related to the Core constraints. However, AM do not make any formal link between those two types of constraints in a fully general framework. Indeed, without externalities, they establish a surprising equivalence result: the Vickrey auction is robust against losers' deviation if and only if bidders have 'substitutes' preferences, i.e. when the Vickrey outcome is in the Core. In the following proposition, we show that if a contingent mechanism outcome is in the Core given the reported preferences and also such that for any preferences reported by coalitions of losers the auction outcome remains a Core outcome, then this outcome is strongly robust against losers' deviations. Indeed, only a very restrictive set of constraints among the whole set of (b) constraints in the definition of the Core are used to obtain the result. It gives the intuition that in general, as we will actually show for the NGDE framework, the condition needed for the Vickrey outcome being in the Core is much stronger that the condition needed for the Vickrey outcome being strongly (and a fortiori weakly) robust against losers' deviation. Then it seems that the Vickrey contingent auction should be more robust than AM-auction beside those criteria. However, we should be careful in the interpretation of the result: it does not only require that the underneath true preferences are such that the mechanism outcome is in the Core, it also requires that if some losers jointly deviate, then the mechanism outcome still lies in the Core. As we will also illustrate for the NGDE framework, such conditions are not innocuous and on the whole the relative performance between the Vickrey and AM-auctions may depend on the ability of the auctioneer to restrict the set of joint preferences that may be reported by the bidders.

Proposition 2.4 (Core versus losers' deviation constraints) *Given a set of preferences Π and a contingent auction mechanism (A, p), if any outcome of the contingent mechanism is in the Core, then any truthful outcome is strongly robust against losers' deviation provided that the auctioneer is able to restrict the way losers can deviate such that the reported types always stay in Π.*

Proof 2 *In order to make the allocation switch from $\mathcal{A}^*$ to another allocation $\mathcal{A}'$, the set of losers S should make a total contribution P_S such that $P_S + \sum_{i \notin S} \Pi_i(\mathcal{A}') \geq \sum_{i=1}^{N} \Pi_i(\mathcal{A}^*)$ (otherwise the coalition $N \cup \{0\} \setminus S$, would block. Note that $w(N \cup \{0\} \setminus S) = \sum_{i=1}^{N} \Pi_i(\mathcal{A}^*)$ because bidders in S are losers). Because $\mathcal{A}^*$ maximizes over $\mathcal{A}$ the sum $\sum_{i \in N} \Pi_i(\mathcal{A})$, then it implies that $P_S > \sum_{i \in S} \Pi_i(\mathcal{A}')$ and that consequently the losers do not (jointly) find the deviation profitable.*

Corollary 2.5

1. *AM-auctions are strongly robust against losers' deviation.*
2. *If the set of preferences Π is such that the Vickrey outcome is in the Core and provided that the auctioneer is able to restrict the way preferences are reported such that they always stay in Π, then the Vickrey contingent auction is strongly robust against losers' deviation.*

An appealing feature of AM-auctions is that they are strongly robust against losers' deviations in a very general manner contrary to the Vickrey contingent auction. However, in the case where it is a dominant strategy to report truthfully its preferences in the AM-auctions, then the Vickrey auction is automatically robust to losers' deviation provided that the auctioneer is able to constraint the deviation of the losers in such a way that they can not jointly report preferences outside the set Π.

Remark 2.1 *Bernheim-Whinston first price menu auction [5] is a contingent auction such that any outcome is in the Core relative to the reported preferences: it selects the seller's optima among the Core outcomes, i.e. $\pi_0 = w(N)$, $\pi_l = 0$, for $l \geq 1$. Then proposition (2.4) implies that it is robust against losers' deviations. On the other hand, Bernheim-Whinston's outcome is the most distant outcome in the Core from the Vickrey outcome and gives then the weakest incentives to report the truth.*

2.6 An Example with allocative externalities

The following example is inspired from Hoppe, Jehiel and Moldovanu [17], where the authors report that 'a major investment bank estimated per-licenses values of Euro 14.75, 15.88, and 17.6 billion for a German UMTS market with 6, 5, and 4 firms, respectively'. It gives a foretaste of the Negative Group Dependent Externalities framework with only one group and unit-demand. It also illustrates our criteria for the performance of a mechanism: incentives to report the true preferences and robustness against losers' deviation. We consider a market for licences such that the more licenses are sold the less it worths for a licensee, whereas a non-licensee remains outside the market and is indifferent to the final market structure.

We consider 3 potential bidders and three identical licences. For each bidder, the value for a licence is supposed to depend only on the number of joint-licensees. We assume unit-demand: each bidder is indifferent between acquiring extra-licences or leaving them unsold in the seller's hands. The three bidders are designated by 1,2,3. Their corresponding valuations are given by the functions $x \to \pi_l(x)$ where x represents the total number of licensees in the final market structure. We discuss the numeric application where the reported preferences are: $\pi_l(1) = 11$, $\pi_l(2) = 7$, $\pi(3)_l = 6$ for each bidder l. The seller, designated by 0, is supposed to be a pure revenue maximizer. The numeric values we have chosen shares the common feature with Hoppe, Jehiel and Moldovanu's report that the functions $x \to \pi_l(x)$ are not concave. We will see later that the concavity of the function π_l will play a central role in our general result about the condition making contingent auctions robust in the NGDE framework. The efficient allocation given the reported preferences is the three-licensees structure $\{1,2,3\}$ and is thus the allocation implemented by the contingent auctions we consider.

According to the reported preferences, the coalitional value function w is not bidder-submodular because $w(N) - w(N \setminus \{l\}) = 4 > 3 = w(N \setminus \{m\}) - w(N \setminus \{l,m\})$ for $l,m \in 1,2,3$. For example, bidders 2 and 3 should be viewed as complement bidders: the additional value to the total surplus provided by bidder 2 grows with the mere presence of bidder 3.

This means that this example is similar to the one depicted in Figure 2.4: the Vickrey outcome lies outside the Core. The Core outcomes $(u_l)_{l=0,\cdots,3}$ are defined by the following constraints:

$$u_0 + u_1 + u_2 + u_3 = 18 \qquad \textit{Feasibility Constraint} \tag{2a}$$

$$u_0 + u_l + u_m \geq 14 \quad , l,m \in \{1,2,3\} \qquad \{0,l,m\} \textit{ do not block} \tag{2b}$$

$$u_0 + u_l \geq 11 \quad , l \in \{1,2,3\} \qquad \{0,l\} \textit{ do not block} \tag{2c}$$

$$u_l \geq 0 \quad , l \in \{0,1,2,3\} \quad \textit{Rationality Constraint} \quad \{l\} \textit{ do not block} \qquad (2d)$$

Independently of the true valuations, the reported preferences are never an equilibrium in AM-auctions since bidders can obtain the same allocation at a smaller price by reporting $\pi(1) = 7,\ \pi(2) = 3,\ \pi(3) = 2$, i.e. if he shades his bids by 4.

The Core outcomes can also be rewritten in term of constraints on the vector of prices $(p_l)_{l=1,2,3}$. The feasibility constraint sets that the revenue of the seller is the sum of those prices. The constraint (2b), which could also be qualified as the 'Vickrey constraint', states that each bidder must pay at least the externalities imposed on the remaining bidders. In this example, it corresponds to $p_l \geq 2$, 2 being the price set by the Vickrey contingent auction. The constraints (2c) (respectively (2d)) are equivalent to $p_l + p_m \geq 5$ (respectively $p_l \leq 6$, the individual rationality constraint). Note that for the Vickrey outcome, we have $\pi_2 + \pi_3 = 4 < 5$: that is the only kind of constraints that the Vickrey outcome fails to satisfy in order to be in the Core. It means that, whereas each bidder pays at least the externality that he imposes on the other bidders, a subset of bidders with strictly more than one bidder may possibly pay a total amount that is inferior to the externality they jointly impose. For example, suppose that the true preferences of bidders 2 and 3 are indeed $\pi(x) = 2.25$ for any x and suppose that bidder 1 has reported his true preferences, then the efficient allocation is to attribute one licence to bidder 1. On the other hand, bidder 2 and 3, as losers, find profitable to jointly deviate and report the same preferences as bidder 1 to obtain a strictly positive profit. Finally we have illustrated the losers' deviation issue which relies on the fact that $\pi(\cdot)$ fails to be concave for at least one bidder.

3 The Negative Group-Dependent Externalties Framework (NGDE)

Two preliminary definitions and some notation

Definition 7 *A partition of N is a set of subsets, denoted by $\{G_j\}_{j=1}^{g}$, such that $\bigcup_{j=1}^{g} G_j = N$ and $G_j \bigcap G_k = \emptyset$ for all G_j, G_k. A subset G_j is referred to a group. A partition will also be referred to as a group-dependent structure.*

For bidder l, denote by $G(l)$ his group and denote by $g(l)$ the index corresponding to his group and such that $G_{g(l)} = G(l)$. Denote by $n_l^{\mathcal{A}}$ the number of purchasers in bidder l's group in the allocation $\mathcal{A}$, i.e. the cardinality of the set $G(l) \cap \mathcal{A}$.

Definition 8 *A subpartition of a partition $\{G_j\}_{j=1}^{g}$ of a set N is a partition of N, denoted by $\{B_j\}_{j=1}^{b}$, such that for all $j = 1 \dots b$, there exists a group G_k, such that $B_j \subset G_k$. A subset $B_j \subset G_k$ is referred to a 'bundle' of bidders inside the group G_k.*

Before the formal definition of the NGDE frameworks, let us present some motivating examples.

3.1 Some motivations for the NGDE framework

If the German UMTS auctions were a starting point because the number of licensed firms was endogenous and that the valuation for a license was a function of this number, firms do not suffer from identity-dependant externalities at first glance: they do not care whether a competing licence is sold to either firm A or firm B. Such a kind of externalities is already present in the literature and especially in Segal [37] and Segal and Whinston [38]. Refer to [37] for a survey of the motivating examples for such 'level of trades' externalities. Note that those works are considering bilateral trading excluding de facto contingent bids. Ranger [33] considers both 'level of trade' externalities through a unique downstream Cournot market that link the bidders and contingent bids in AM proxy auction. In some applications, it is more reasonable to model the downstream competition through several different Cournot markets. In particular, in the problem of allocating airport slots (studied by the pioneering work of Rassenti, Smith and Bulfin [35]), it is clear that some companies do not compete in the same area. The valuation of a North-American company for a slot in London will not depend on the total number of slots that are sold in London's airport but rather on the total number of slots purchased by her direct competitors. She probably prefers that a slot is allocated to a Russian company rather than to one of her most closest competitors, e.g. another North-American company. The NGDE framework will broaden the analysis in this direction and also in a more abstract perspective allowing other forms of downstream interaction between joint-purchasers. Instead of a linkage through the demand function in the downstream market as in Ranger [33], the following example considers the downstream linkage between the producers through their cost functions. In power markets, the final price of electricity is often strongly regulated or/and hedged by forward contracts, whereas the prices of the various inputs may be very volatile: uncertainties about the producers' profits may therefore rely more on the costs than the demand.

Example 3.1 (Capacity in Power Markets) *Investment incentives is a great issue in power markets and a large number of non-market mechanism* *have been imposed to avoid shortages.*[10] *Either a transmission system operator (ISO) or a Load Serving Entity (LSE) may be willing to run a procurement auction respectively in order to achieve a suitable level of reliability or to meet their capacity obligations.*[11] *Here we consider the case where the inputs (the energy source) is not specified in the procurement. Consider that different kinds of fossil fuel projects are competing. For example, some projects concern generations that use natural gas, whereas some other generators use fuel oil as their primary fuel. Then the production costs suffer from identity-dependent externalities: a generator is valuing differently such a procurement contract according to the nature of the fuel used by the generators that have signed the other contacts. For example, a generator that uses natural gas will prefer that few gas users emerge insofar as a large amount of gas users will induce extra costs, e.g. due to the congestion costs on interconnectors. In such a framework, it seems a reasonable first order approximation to assume that the production costs of a given unit depend on the amount of capacity provided by joint-procurers' units that are using the same kind of fuel, whereas, inside a group, generators differ only beside their output efficiency. The group-structure of the related NGDE framework corresponds here to the partition of the units according to the kinds of inputs. Note that the production costs of a generator are increasing with the number of joint-procurers inside his group: the value of a contract is then decreasing with the number of joint-procurers inside his group. This point brings us to refine the model in considering* negative group-dependent externalities*.*

Previous theoretical works about allocations with externalities do not fully include the issues raised by those examples. Jehiel, Moldovanu and Stacchetti's externalities are limited to single-item assignements. Pure level of trade externalities as in Segal [37] correspond to a structure with only one group.[12] Consider the allocation of a scarce resource to a given group such that from a welfare analysis, the optimum corresponds to assign all items, then the problem is straightforward and a standard English auction achieve the optimum since it puts the items in the hand to those who value them at most. On the other hand, with indentity-dependent externalities, the allocation problem of a scarce resource such that the optimum corresponds to assign all items is not obvious and a standard English auction does not achieve efficiency.[13] A notable exception in the literature about identity-

[10] See Joskow and Tirole [24] for more details on the inability of competitive market to achieve the optimal level of reliability and the relevant regulatory instruments.

[11] Reliability-Must-Run contracts, long-term contracts for delivered energy at fixed prices, capacity contracts are signed usually under bilateral negotiations. Procurement auctions for capacity are however common for renewable energies.

[12] Nevertheless, we give up a degree of generality relative to the literature on level of trade externalities by restricting the analysis to 'negative' externalities.

[13] In Lamy [25], various standard auction mechanisms are shown to implement a 'stable' allocation in the NGDE framework.

dependent externalities, which considers positive externalities between joint-purchasers, is Aseff and Chade [1] who characterize the optimal auction when the informational asymmetry is reduced to a one-dimensional parameter.

The rest of this section is devoted to the formal definition of the NGDE framework: first we define the unit demand framework where bidders are interested in at most one item, second we define the multi-unit demand framework where each bidder demand is viewed as a 'bundle' of some unit demand bidders. From now on, except in section 6, we implicitly assume under the NGDE terminology that the seller is indifferent to the final allocation: $\Pi_0(\mathcal{A}) = 0$, for any $\mathcal{A} \in \mathbf{A}$.

3.2 The NGDE framework with unit demand

Definition 9 (NGDE with unit-demand) *An assignment problem* $\{\mathbf{A}, (\Pi_j)_{j\in N\cup\{0\}}\}$ *is said to satisfy the Negative Group Dependent Externalities framework with unit demand if there exists a N+1-uple* $(\{G_j\}_{j=1}^{g}, \pi_1, \cdots, \pi_N)$ *where* $\{G_j\}_{j=1}^{g}$ *is a partition of* N *and where* π_l*,* $l \in N$*, are nonincreasing functions mapping an integer in* $[1, N]$ *into a real number such that for any* $l \in N$ *and for any allocation* $\mathcal{A}$*:*

$$\Pi_l(\mathcal{A}) = \pi_l(n_l^{\mathcal{A}}),\ if\ \mathcal{A}^l \neq \emptyset \tag{3a}$$

$$\Pi_l(\mathcal{A}) = 0,\ if\ \mathcal{A}^l = \emptyset. \tag{3b}$$

The N+1-uple $(\{G_j\}_{j=1}^{g}, \pi_1, \cdots, \pi_N)$ will also be referred as a NGDE framework with unit-demand.

The terminology group-dependent comes from the fact that the valuation of a purchaser depends solely on the assignment of his joint-purchasers inside his group and more specifically on the number of those joint-purchasers, which introduces some symmetry in the model insofar as inside a group there are no identity-dependent externalities. Then we can abusively qualify π_l as bidder's l type. We assume also that π_l is nonincreasing. It means that a purchaser is suffering from negative externalities from the joint-purchasers of his group.

The remaining assumption to complete the NGDE framework is technical and standard in economic analysis.

Assumption 3.1 *Non-Crossing (NC) assumption*
For any group G*, for any* $i, j \in G$ *such that* $i < j$*, then*

$$\pi_i(x) > \pi_j(x)$$

In a nutshell, it says that inside a group, bidders can be unambiguously ranked in term of efficiency. Nevertheless, this assumption is not much restrictive insofar as that in the following multi-unit demand framework where different unit-demand bidders are 'bundled', then for a given number of acquired items and making vary the number of joint-purchasers, then bidders are not ranked unambiguously.

3.3 The NGDE framework with multi-unit demand

Definition 10 (NGDE with multi-unit demand) *A Negative Group Dependent Externalities framework with multi-unit demand (due to bundling) is a N+2-uple* $(\{G_j\}_{j=1}^g, \{B_j\}_{j=1}^g, \pi_1, \cdots, \pi_N)$ *such that the N+1-uple* $(\{G_j\}_{j=1}^g, \pi_1, \cdots, \pi_N)$ *is a NGDE framework with unit demand and* $\{B_j\}_{j=1}^g$ *is a subpartition of* $\{G_j\}_{j=1}^g$*.*

Let us connect this definition with the previous general assignment problem with externalities and the NGDE framework with unit-demand. Consider the NGDE framework with unit-demand and consider that inside a group, some of the unit-demand identities could be bundled together under a same common bidder that bids under a unique identity and on the behalf of their mutual interests. Those 'bundled' bidders bid in order to maximize the sum of the profit of its identities, who after the first allocation step (the contingent auction) allocate the items to the identities in their bundle who are valuing the items at most. Due to the non crossing condition (3.1) and because we consider that a bundle B of individual entities covers only entities of the same group, then it is a dominant strategy to allocate the acquired items to the most 'efficient' bidders in B, i.e. those with the smallest index in the NGDE framework. A bidder may possibly prefer not to use an acquired item and then obtain the payoff as if he leaves it in the seller's hand. However, such an event will never happen for any best response strategy in the auctions we consider such that there is no loss of generality to exclude those events.

By an appropriate indexation, we can write $B_j := \{j_1, \dots j_{B_j}\}$. Finally, the valuation of the bundle B_j for an allocation $\mathcal{A}$ depends on two parameters: the number of items purchased by the bidder (bundle) j and the total number of items purchased in j's group $n_{g(j)}^{\mathcal{A}}$. This valuation function is then denoted by $\widetilde{\pi}_{B_j}(x, y)$, a function of x the number of items that the bundle purchases and $y \geq x$ the number of acquired items in his group and can be derived from the original valuations $(\pi_k)_{k \in N}$:

$$\widetilde{\pi}_{B_j}(x, y) = \sum_{l=1}^{x} \pi_{j_l}(y).$$

Note that $(x, y) \to \widetilde{\pi}_{B_j}(x, y)$ is concave over x for all y because $l \to \pi_l(y)$ is decreasing for all y. Thus in the NGDE framework with multi-unit demand bidders exhibit diminishing marginal utilities as expected since it corresponds to the 'substitutes' condition of AM in the case on homogeneous items. Conversely, for any function $(x, y) \to \widetilde{\pi}_{B_j}(x, y)$ which is concave over x for all

y and such that $(x, y) \to \widetilde{\pi}_{B_j}(x, y) - \widetilde{\pi}_{B_j}(x-1, y)$ is decreasing over y, then it can be written as the sum of valuations satisfying the NGDE-framework. The 'virtual' valuation of the identities composing the bundle can be derived as:

$$\forall l, \pi_{j_l}(y) = \widetilde{\pi}_{B_j}(l, y) - \widetilde{\pi}_{B_j}(l-1, y)$$

The following lemma will allow us to extend our bidder-submodularity characterization from unit-demand to multi-unit demand in the NGDE framework.

Lemma 3.1 *If a NGDE framework with unit-demand* $(\{G_j\}_{j=1}^g, \pi_1, \cdots, \pi_N)$ *is such that the related coalitional function* w *is bidder-submodular, then for any subpartition* $\{B_j\}_{j=1}^g$ *of* $\{G_j\}_{j=1}^g$ *the NGDE framework with multi-unit demand* $(\{G_j\}_{j=1}^g, \{B_j\}_{j=1}^g, \pi_1, \cdots, \pi_N)$ *is such that the related coalitional function* w *is bidder-submodular.*

Proof 3 *Consider two subsets* B_i, B_j *of* N *and let* $B_j = \{j_1, \cdots, j_{B_j}\}$*.*

$$w(N) - w(N \setminus B_i) \leq w(N \setminus \{j_1\}) - w(N \setminus \{B_i, j_1\}) \leq \cdots$$

$$\cdots \leq w(N\setminus\{j_1, \cdots, j_{k-1}\}) - w(N\setminus\{B_i, j_1, \cdots, j_{B_{j-1}}\}) \leq w(N\setminus B_j) - w(N\setminus\{B_i, B_j\})$$

Each inequality results from the definition of the bidder-submodularity of the unit-demand framework. The resulting inequality between the two extremes implies the bidder-submodularity of the multi-unit demand framework.

The argument is true for any bundle of bidders and so not only for a subpartition of $\{G_j\}_{j=1}^g$. Nevertheless, the NGDE framework with multi-unit demand does not make sense if a bidder is a bundle of some unit-demand bidders belonging to different groups because there is then an ambiguity about the usage of an item by that bidder and even a contingent auction is unable to internalize those externalities.

4 Robustness of the Vickrey Contingent Auction

Our starting example in section 2.6 illustrates the necessity of the condition that the negative externality is concave relative to the number of joint-purchasers in one's group for the Vickrey auction to be robust against losers' deviation. Indeed, it can be shown more generally that, for any number of items, if a bidder's preferences, say 1, fail to be concave, then there exists some preferences for his opponents that are exempt of any allocative externalities and such that bidder 1 wins under truthful reporting but such that this outcome is not (weakly) robust against losers' deviation. The failure for bidder 1 of the concavity means the existence of a number of joint-purchasers

$x > 0$ such that: $\pi_1(x) - \pi_1(x+1) > \pi_1(x+1) - \pi_1(x+2)$, which is also equivalent to $\pi_1(x) - \pi_1(x+2) > 2 \cdot (\pi_1(x+1) - \pi_1(x+2))$. Suppose that the participation of two bidders (in bidder 1's group) modifies the number of items assigned in this group from x to $x+2$ and that the presence of a single of those bidders modifies this number of items from x to $x+1$ whereas bidder 1 always acquires one item. In the Vickrey contingent auction, each bidder internalizes through his payment at least the externality that he imposes on bidder 1: each should pay at least $\pi_1(x+1) - \pi_1(x+2)$. On the whole, they both contribute at least $2 \cdot (\pi_1(x+1) - \pi_1(x+2))$ and exactly this amount if bidder 1 is the only bidder suffering from allocative externalities and if the resource is not scarce. But this amount is smaller than the amount of the externality that they both impose on bidder 1: $\pi_1(x) - \pi_1(x+2)$. Actually, those two bidders may obtain a positive profit from their participation though they would be losers under truthful reporting.

On the other hand, next proposition shows that if the reported mappings π_i of the winners are concave, then the outcome of the Vickrey auction is (weakly) robust against losers' deviation. The proof is relegated in the appendix.

Proposition 4.1 *In the NGDE frameworks with unit or multi-unit demand, if the functions $x \to \pi_l(x)$ are concave for any l such that $\mathcal{A}^l \neq \emptyset$, then any joint deviation by losing bidders, such that the whole set of reported types still fits with the NGDE framework*[14]*, is unprofitable for at least one deviator in the Vickrey contingent auction.*

Remark that contrary to AM-auctions which are strongly robust against losers' deviation, we obtain here only the weak form of the robustness against losers' deviation.[15] This point is illustrated in the following example in the unit-demand framework with two groups.

Example 4.1 *Consider two items and four unit-demand bidders without allocative externalities. Bidders 1 and 2 are valuing the item 100. Bidders 3 and 4 are valuing the item respectively 99 and 10. Under truthful reporting, the Vickrey outcome is to give the items to 1 and 2 and to make each pay 99. However, this outcome is not strongly robust against the joint losers' deviation where bidders 3 and 4 are reporting types according to the group structure $\{G_1 = \{1, 2, 3\}, G_2 = \{4\}\}$ and where the reported valuations are $\pi_3(1) = 200,\ \pi_3(x) = 0,\ x > 1$ and $\pi_4(x) = 100,\ x > 0$. Note that this joint*

[14]Note that we do not require the maps $x \to \pi_l(x)$ reported by losing bidders to be concave neither the non-crossing assumption.

[15]If we consider only joint deviations by losing bidders which are belonging to the same group, then the Vickrey auction is strongly robust against losers' deviation. See the proof of proposition 4.1 for more details. Hence, if bidders are restricted to bid according to the NGDE framework with only one group, then proposition (4.1) is valid with the strong robustness against losers' deviation concept.

deviation fits with the NGDE framework since bidders 1 *and* 2 *do not report any externalities. Under this deviation, the items are allocated to* 3 *and* 4*: bidder* 4 *pays nothing, whereas bidder* 3 *pays* 100*. If they can transfer at least a unit of money, then the deviation may be profitable for both deviators.*

Proposition [4.1] should be put in parallel with Theorem 13 in AM. Without externalities, AM characterizes the set of preferences such that losers' deviation is unprofitable in the Vickrey auction: the items should be viewed as substitutes. This result depends on the crucial point that bidders are restricted in the way they could bid: they are not allowed to report contingent valuations or equivalently to submit contingent bids. However, if bidders are allowed to report in a more general valuation set with contingent bids, then the Vickrey auction is not guaranteed to be immune from losers' joint deviation as been emphasized in section 2.3 even if the underneath valuations satisfy AM's theorem. Nevertheless, in AM, it is not an issue to restrict the way bids are reported. On the contrary, in proposition 4.1 bidders should be constrained in the way they could report their types insofar as the reported types should fit with the NGDE framework. This restriction corresponds to preclude any report such that the whole set of reported valuations does not satisfy the NGDE framework. But this is not easily done because the fact that a bid is coherent with this framework depends on the bids of the other participants. Those kind of deviations are illustrated in the following example where the Vickrey contingent auction is not immune to a losers' joint deviation with each loser reporting valuations that could fit (independently of each other) with the NGDE framework but such that the whole set of reported valuations does not fit with this framework.

Example 4.2 *Consider two items and four unit-demand bidders without externalities. Buyers* 1 *and* 2 *are valuing the item* 10*, whereas bidders* 3 *and* 4 *are valuing the item* 1*. The efficient allocation is* $\{1, 2\}$*. However, bidders* 3 *and* 4 *could jointly and strictly profitably deviate with the following bidding scheme. Buyers* 3 *and* 4 *report that: with a joint purchaser in* $\{1, 2\}$*, they are valuing the item* 0*, else they are valuing the item* 30*. Then bidders* 3 *and* 4 *both obtain the items for a null transfer. Independently of the other reported valuation, bidder* 3 *(respectively* 4*) could fit with the NGDE framework. However, in the NGDE framework, no group structure can fit with the reported valuation of* 3 *and* 4 *which implies that they both belong to bidders* 1 *and* 2*'s group but that they do not belong to the same group which raises a contradiction.*

To summarize, the Vickrey contingent auction requires a special monitoring ability of the seller: the possibility to exclude any report that lies outside the NGDE framework. On the one hand, if the designer can credibly commit to the null allocation, it can be implemented with the augmented grand-mechanism such that the auctioneer cancels the auction if the reported joint preferences do not fit the NGDE framework and such that the same Vickrey contingent auction is then proposed. On the other hand, if the mechanism designer is able ex ante to attribute to each participant its right group, then he can propose the simplified Vickrey mechanism where each bidder is asked to report a mapping $x \to \pi_l(x)$.

5 Robustness of AM-auctions

Our starting example in section 2.6 illustrates the necessity of the condition that the negative externality is concave relative to the number of joint-purchasers in one's group for the coalitional value function w to be bidder-submodular. However, this condition is not sufficient as it will be illustrated by the two next examples. A much stronger condition is needed which considerably reduces the dimensionality of bidder's preferences. Inside a group, bidders valuations must be equal up to a translation: for any $i, j \in N$ such that $G(i) = G(j)$, equalities $\pi_i(x) - \pi_i(x+1) = \pi_j(x) - \pi_j(x+1)$ must be satisfied for any $x > 0$. This is somehow a restriction of measure null, which could nevertheless have some relevance in some applications.

Now we deliver two examples that illustrate why, for any $x > 0$ and $i < j$ (i.e. bidder i is more efficient than j), both conditions

$$\pi_i(x) - \pi_i(x+1) \leq \pi_j(x) - \pi_j(x+1) \tag{4a}$$

$$\pi_i(x) - \pi_i(x+1) \geq \pi_j(x) - \pi_j(x+1) \tag{4b}$$

are necessary when there is only one group (and a fortiori with several groups). Inequality (4a) (respectively (4b)) means that the externality imposed on a bidder by an additional joint-purchaser in a given group is nonincreasing (nondecreasing) according to the efficiency of that bidder. Those two inequalities do not play a fully symmetrical role in the proof of proposition (5.1). Inequality (4a) implies that the number of purchaser in i's group does not increase if bidder i is removed. We give the intuition below.[16] Suppose that two additional bidders in i's group, say $l1$, $l2$, $l1 < l2$, win an item after bidder i's removal, then the externality imposed by $l2$ on the other winners of his group is greater than the externality imposed by the addition of bidder $l1$ relative to the original efficient allocation. Moreover, the gross contribution of $l2$ to the surplus is smaller than $l1$'s. Finally, the net contribution (incorporating the externalities) of bidder $l2$ (after i's removal) is smaller than $l1$'s in the original allocation problem, which is negative. Then the second additional winner in bidder i's group makes a negative contribution to the surplus, which raises a contradiction. Similarly, inequality

[16] The complete argument is more complex due to the linkage with the other groups. Refer to the proof of proposition [5.1] and footnote (23) for more details.

(4b) implies than the joint-purchasers of a given bidder i under the optimal allocation are still purchasers under the optimal allocation without bidder i.

Example 5.1 (Failure of condition 4a) *Consider three items and one group. There are five bidders denoted by $1, 2, 3, 4, 5$ whose valuations are defined such that: $\pi_i(x) = 7$ for $i = 1, 2$ and $x = 1, 2$ and $\pi_i(x) = 3$ in any other case. The coalitional value function w is not bidder-submodular because $w(N) - w(N \setminus \{1\})(= w(N) - w(N \setminus \{2\})) = 4 > 1 = w(N \setminus \{2\}) - w(N \setminus \{1, 2\})$. Buyer 1 and 2 should be viewed as complement bidders who have to share non-cooperatively the surplus of 2.*

Example 5.2 (Failure of condition 4b) *Consider two items and one group. There are three bidders denoted by $1, 2, 3$ whose valuations are defined such that: $\pi_i(1) = 10$ for any bidder i, $\pi_1(2) = \pi_2(2) = 8$ and $\pi_3(2) = 0$. The coalitional value function w is not bidder-submodular because $w(N) - w(N \setminus \{1\}) = 6 > 0 = w(N \setminus \{2\}) - w(N \setminus \{1, 2\})$. Buyer 1 and 2 should be viewed as complement bidders who have to share non-cooperatively the surplus of 6.*

As illustrated in the above representative examples and as clarified later in the proof of proposition [5.1], the failure of proposition (4a) (respectively 4b) leaves the scope for complementarity between some bidders in a group against an alternative allocation with more (respectively less) bidders in their group.

The following proposition establishes that if both conditions (4a) and (4b) and the previous concavity assumption are satisfied, then w is bidder-submodular in the NGDE framework.

Proposition 5.1 *In the NGDE framework with unit demand or with multi-unit demand, if the functions mapping $x \to \pi_i(x)$ are concave and if for any $i, j \in N$ belonging to the same group we have $\pi_i(x) - \pi_i(x + 1) = \pi_j(x) - \pi_j(x + 1)$ for any $x > 0$, then w is bidder-submodular.*

From lemma [3.1], bidder-submodularity in the multi-unit demand framework is a immediate corollary of the proposition under unit-demand. The proof for the unit-demand framework which is relegated in the appendix contains two part. First, we show that the optimal allocation when one bidder is removed is closely related to the optimal allocation with that bidder included: the purchasers in the latter allocation are still purchasers in the former. Indeed, when a bidder i is removed, three possibilities may arise: the final allocation is unchanged except that the item previously allocated to bidder i goes in the hand of either another bidder in i's group or another bidder in another group, or finally remains in the seller's hands. The second part of the proof is very tedious: the inequalities $w(N) - w(N \setminus \{i\}) \leq w(N \setminus \{j\}) - w(N \setminus \{i, j\})$ are carefully checked according to the different possibilities about the identity (-ies) of the purchaser(s) of the item after bidder i's removal (j and $\{i, j\}$ removals).

Proposition [5.1] requires much stronger conditions on the form of the preferences than proposition [4.1]. In that perspective, it can be viewed as a critic about the relevance of AM-contingent auctions relative to the Vickrey contingent auction. In the perspective of generalizing the substituability condition with allocative externalities, the proposition and the related counterexamples show that a nongeneric congruence relation need to be satisfied such that externalies are aligned in each group. The congruence relation that $\pi_j(x) - \pi_j(x+1)$ is independent of j seems very restrictive, but it could be a reasonable first order approximation in some applications. This point is discussed in the following application studied by Ranger [33] which corresponds to the NGDE framework with multi-unit demand and a single group.

5.1 An application: capacity auctions

Suppose that bidders are competing in an auction for capacity prior to a Cournot market interaction with a concave demand function denoted by $D(p)$. In the whole game, [33] considers the possibility to acquire capacity and not to use it. Here let us consider that a capacity that is purchased must be used.[17] While Ranger's analysis covers a scope without indivisibilies, we find the analog of Ranger's main result, i.e. the vector of valuations of the capacity game satisfies the bidder-submodularity condition, with indivisible good of the same size, say q. The analog result without indivisibilities is obtained by taking the limit $q \to 0$.

Consider the NGDE framework with only one group: a bidder suffers from negative externalites from all his joint-purchasers insofar as he prefers other items to stay in the seller's hands rather to be sold to his potential opponents. Denote by K the total capacity that is auctioned and $K^T \in [0, K]$ the total capacity purchased by all bidders at the end of the auction. Each bidder i corresponds to the bundle of K identical unit-demand bidders with the cost functions $\{c_l^i(K^T)\}_{l \in [1,K]}$. The profit of the unit-demand entity l of bidder i when it acquires an item equals to: $\pi_l^i(K^T) = D^{-1}(K^T) \times 1 - c_l^i(K^T)$. Then the profit of bidder i when he purchases x_i items equals to: $D^{-1}(K^T) \times x_i - \sum_{l=1}^{x_i} c_l^i(K^T)$. This expression matches without loss of generality [33]'s expression for the profit of a firm since he restricts the analysis to convex cost functions[18]: any cost function $C_i(x_i, K^T)$ which is convex relative to the number x_i of items purchased and zero-valued at the origin can be expressed

[17] In equilibrium, capacities are always used and this restriction point does not modify the analysis. Indeed, preemption and acquisition motivation are disconnected in contingent auctions contrary to standard auctions.

[18] In [33], for the cost function of bidder i, Ranger considers the general form $C_i(x_i)$ such that $\frac{\partial C_i(x_i)}{\partial x_i} \geq 0$ and $\frac{\partial^2 C_i(x_i)}{\partial^2 x_i} \geq 0$. So he excludes any dependence on K^T the total capacity sold. Nevertheless, he does not exclude $C_i(0) > 0$, that is a fixed cost introducing a non-convexity. Indeed with such a non-convexity it is possible to construct an example similar to our starting example where w is not bidder-submodular. Thus we fix $C_i(0) = 0$ in the following discussion and then assume that the cost function is convex.

as the sum $\sum_{l=1}^{x_i} c_l^i(K^T)$. Then to apply Proposition 5.1 on the primitives $\{\pi_l^i\}_{l\in[1,K],i\in N}$ of the model, we have to check the following points in [33]:

- The non-crossing assumption (3.1) is satisfied since profit functions of any unit-demand identities are equals up to a translation.
- The map $\pi_l^i(K^T)$ is concave over K^T, which is satisfied since the demand is concave.
- The condition $\pi_l^i(K^T) - \pi_l^i(K^T+1) = \pi_k^j(K^T) - \pi_k^j(K^T+1)$ for any i, j, l, k is satisfied. Those differences are independent of the couples (l, i) and (k, j) in this application because $\pi_l^i(K^T)$ is additively separable in K^T and the couple (l, i).

First, our analysis shows that most of [33]'s assumptions are binding: in particular the various concavity assumptions. Second the additive separability between the dependance in i and in K^T is also crucial. Nevertheless, a slightly more general form for the cost function could be suitable: $C_i(x_i, K^T) = C_1(K^T)\cdot x_i + C_2^i(x_i)$, where $C_1(\cdot)$ and $C_2^i(\cdot)$ are both convex. It corresponds to the situation where the other producers impose also a negative externality to the cost function due to congestion costs for the supplying of the inputs as an example. The bidder-submodularity condition requires that this congestion cost is independent of the identity of the purchaser: the function $C_1(\cdot)$ does not depend on i (Note also that consequently assumption 3.1 is then automatically satisfied). On the other hand, this separability condition is not required for Vickrey to be robust: only the convexity of $C_i(x_i, K^T)$ over the variable K^T is required.[19]

Note that we can also apply proposition [4.1], which means that the Vickrey contingent auction is also robust in this framework. Indeed, there is no monitoring issue in the Vickrey contingent auction in this application because the seller can restrict easily the way bidder could bid because the group structure is clearly common knowledge: bidders should be constrained to report a valuation $(x, y) \to \pi(x, y)$ which is decreasing relative to y the number of capacity units sold to guarantee the robustness to losers' deviation. Thus in this particular application the benefit of AM-contingent auction relative to the Vickrey auction is not clear.

Anyway, we should stay modest relative to the relevance of this analysis for many capacity markets: it assumes that the bidders do not own any capacity prior to the market, i.e. non purchasers do not suffer from identity-dependant externalities. In particular, non-purchasers are indifferent to the

[19] Provided that assumption (3.1) is also satisfied, i.e. the most efficient producers without congestion are remaining the most efficient under congestion. As a example, this is satisfied for the general form $C_i(x_i, K^T) = C_1(K^T)\cdot\sum_{l=1}^{x_i}\lambda_l^i + C_2^i(x_i)$, where λ_l^i is the productivity factor of the unit l of producer i and $C_1(K^T)$ corresponds to the price of the input.

total amount of capacity sold at the auction. Otherwise, the analysis does not hold anymore and the classical free-riding issues between incumbents in order to preempt entry arise. Thus this application is relevant only if the scarce resource is in possession of a monopoly who sold it once.

6 Extension with seller preferences

Our preceding results rely on the fact that the auctioneer is reporting non-strategically its true preferences $\Pi_0(\mathcal{A}) = 0$. Nevertheless, the contingent auctions we have defined in a general way take as input the preferences of the auctioneer Π_0. If the auctioneer can increase at some costs the total amount of items available, then it is worthwhile to investigate the scope of validity of our previous results if we allow the auctioneer to report (non strategically) her preferences over the number of items that are sold. The question is how should we restrict the seller's preferences to extend proposition [4.1] and [5.1]. The answer is that the seller's cost function should be convex.

Definition 11 (Convex production function) *The seller's cost function is convex if there exists an increasing and convex production function $c : \mathbb{N} \to \mathbb{R}^+$ such that:*

$$\Pi_0(\mathcal{A}) = -c(\#\mathcal{A}), \; for\ all \quad \mathcal{A} \in \mathbf{A}$$

The following example illustrates the standard point that the bidder-submodularity condition may fail with a production function exhibiting increasing return to scale.

Example 6.1 *Consider the allocation of two items to two (unit-demand) bidders which are valuing an item 2. We suppose that the items are costly to produce for the auctioneer such that to produce one item costs 2, whereas the production of two items costs only 3 (in case of no production this costs is normalized to zero). Then there is a free-riding issue between the two bidders in order to make the seller produce. The bidder optimal frontier is not a singleton (The bidders have to share a surplus of 1) and truthful bidding is not an equilibrium in the AM-auction. To report the valuation 1 is a best response if the other bidder is truthful.*

We show that if the seller's cost function is convex then both propositions [4.1] and [5.1] still hold. We have restricted the analysis to a framework where the seller only cares about the number of items she sells and so does not care about the identity of the related purchasers. The proof relies on the fact that the seller can be viewed as additional bidders in an additional group with a neutral seller. Nevertheless, the seller does not have the same incentives to report the truth as any other bidders since she also captures the revenue. Therefore, we have to assume that she is non-strategic.

Proposition 6.1 *In the NGDE frameworks with unit-demand or with multi-unit demand, propositions [4.1] and [5.1] extend if the seller's cost function is convex and if the seller reports her true preferences nonstragically.*

Proof 4 *The assignment problem with the seller's revealed preferences derived from a convex production function is equivalent from the bidders' point of view to the assignment problem where the seller is neutral but where another group of M bidders has been added. Denote by w^* the coalitional value function of the 'new' assignment problem that we construct. Index by $N+1, \ldots, N+1+M$ the M bidders that we add and who belong to the 'new' group G_{g+1}. Consider that they have unit-demand preferences suffering from no identity dependant externalities such that:*

$$\Pi_j = c(N+M+1-j) - c(N+M-j), \quad \forall N+1 \leq j \leq N+M,.$$

Then we obtain that:

$$w(S) = w^*(S \cup \{N+1, \ldots, N+1+M\}), \; for \; all \; S \subset N \; and \; 0 \in S$$

The bidder-submodularity of the coalitional value w^* implies that w is also bidder-submodular. Then both propositions 4.1 and 5.1 can be applied to this new framework.

7 Conclusion: some policy perspectives and limits of the model

A practical issue for policy makers facing an assignment problem is to choose between a centralized procedure and a decentralized market-based procedure as auction mechanisms. This is a very old question in the combinatorial auction literature. In particular, Rassenti, Smith and Bulfin [35] studied the problem of allocating airport slots. Banks, Ledyard and Porter [4]'s experimentations have been motivated by the allocation of the Space Transportation System (sometimes called Space Shuttle) which is a scarce resource for which very different kind of bidders are competing (commercial satellites or scientific experiments for example) for the same homogenous good. Brewer and Plott [7] designed an auction to allocate use of a railroad track. This last allocation issue is even more difficult because allocated goods are determined endogenously by the market. Those papers experiment different combinatorial auction designs in order to argue that those complex assignments may be tackled by a decentralized auction process instead of the prevailing complex systems of hierarchical committees and detailed administrative rules.[20]

This paper is a critic of this literature which endows bidders with very simplified preferences, whereas many technical aspects of the application field have been very finely modelled as in Brewer and Plott [7].

In the Space Transportation System, the value to do a scientific experiment would be considerably reduced if some competing laboratories use the shuttle for the same experiment. Added to the insurance motives, those possible inefficiencies may explain why the competing institutions, i.e. the bidders inside a same group in our framework, may prefer to 'bid' jointly as a single bidder and avoid the inefficiencies due to the impossibility to express those externalities. Indeed, our work suggests that such a level of decentralization may be conceivable. The risk of coordination failure and the resulting duplication of capacities for similar outputs should not be an argument for a joint-bid in a contingent auction or for the persistence of centralized procedures.[21] Between full centralization and full decentralization, there is midway: the planner determines the number of items that are to be auctioned for each sub-market and then entrusts to the market the assignment among each sub-market. This work suggests that the efficient allocation of the scarce resource among the different usage is achievable with contingent auctions which endogenize efficiently the number of items for each sub-markets.

In the slot allocation issue, the groups could be viewed as the different destinations each representing a specific market. In this case, some diversified airlines may use a slot for different destinations, which is not captured by our model where a bidder is exogenously attached to a group and only one group. The problem at hand is then more complex and is still characterized by externalities. In that case, an airline may be not only concerned about the identity of its joint-purchasers but also on the way its joint-purchasers uses their slots. It suggests that it could be appropriate in practical auction design to consider contingent auctions where bids are conditional not only on the identity of the purchasers but that are also contingent on the way

[20] There is much doubt that those committees are able to allocate efficiently the relating scarce resources. The allocation of airport slots in Europe is a good example. Following the International Air Transport Association's (the airlines' international trade association) guidelines, the European Commission, through the EU Slot allocation directive (95/93), confirms the grandfathering rights in case of disagreement in the committee as it is usually the case with new entrants. The possibility of resale markets (article 8.4) may solve crude inefficiencies in the allocation but the directive leaves each states free to regulate the secondary markets. Anyway, as emphasized in our companion paper, there are few chances that bilateral markets lead to the efficient allocation if externalities intervene at first order. The 'baby-sitting' of slots, i.e. the use of slots for unprofitable markets to prempt entry, is an obvious evidence that the assignment procedure is inefficient.

[21] Such joint-bids could nevertheless be justified by budget-constraints or complementarities.

the goods are used: therefore, as an example, an airport slot right should then be defined as a function of its destination. For railroad track use, bids could depend on the use (freight shipment versus different kind of passenger transportation). Such bids that are contingent on the usage of the good may be also relevant if the seller values some external effects that depend on the usage. Then it means that for an assignment problem the right derived from the allocation should depend on the specific use of the scarce resource.

Finally two open questions should be examined before opting for a decentralized assignment procedure. First, what is the practical relevance of the restriction imposed in propositions [4.1] and [5.1]? Second, does AM-auctions perform so bad when truthful reporting is not a dominant strategy? The second question suggests further research in economic experimentation, in particular to test the performance of AM-auctions with allocative externalities and without bidder-submodularity. Although experimental work seems desirable for testing the validity of auction design and determinate the most efficient auction for some families of preferences, such works deserve a lot of caution insofar as it could be difficult to replicate in experimental design the coalitional dimension of real life design. As our analysis, in line with AM, points out, combinatorial designs present some failures relative to joint deviations strategies.

Appendix

A Preliminary remark

In all this appendix, in order to alleviate notation, the seller is associated to a group g_s such that if some items are remaining in the sellers' hand, it is considered that there are allocated in group g_s where all purchasers receive a null payoff. The reader can have in mind that there is a reserve of outside bidders which are valuing the item 0 and that do not impose any externality. Thus the number of items that are sold under those notations is a constant. If an item is removed from a group, it should go in the hands of another group.

B Proof Proposition 4.1

Note that any losing bidder receives a null transfer in the Vickrey auction because he suffers from no allocative externality when he receives no item and that consequently his presence do not modify the efficient assignment. Therefore, a deviation is profitable for a losing bidder if and only if he acquires a bundle of items at a smaller price than his valuation in this assignement.

We first derive the proof in the unit-demand framework: it consists in assuming that a profitable deviation of losing bidders exists and then raising a contradiction by proving that at least one deviant bidder makes a strictly negative profit. More precisely, we show that there is a group such that all deviant losers in that group obtain a negative profit. The proof in the multi-demand framework is similar and sketched very briefly.

B.1 Unit-demand

Suppose that a profitable joint-deviation of some losing bidders exists. For each group g, denote by N_g^V (N_g) the set of winning bidders in the Vickrey auction under truthful reporting (under the profitable joint deviation by some losing bidders) and n_g^V (n_g) the cardinality of this set. For each bidder l, denote respectively by π_l^V and p_l^V (π_l and p_l) the type reported by bidder l and the price paid this bidder in the Vickrey auction under truthful reporting (under the profitable joint deviation by some losing bidders) and $\mathcal{A}^V$ ($\mathcal{A}^{dev}$) the Vickrey, i.e. efficient allocation given the reported preferences. Note that throughout the proof we implicitly assume that the deviant losers can not report a type group than their own. Otherwise the report preferences would not fit with the NGDE framework.

Consider first the case where there exists a group g and two bidders l and k such that $l \in N_g \setminus N_g^V$ and $k \in N_g^V \setminus N_g$. It means that through the deviation bidder l has managed to obtain an item whereas a previous winner k under truthful reporting is not a purchaser anymore. Then, from the definition of Vickrey's transfer, $p_i \geq \pi_k(n_g) > \pi_l(n_g)$. The last inequality holds because bidder k is more efficient than bidder l. Finally, bidder k makes a strictly negative profit.

On the hand, if such case do not happen, then two groups g and g' exist such that: $N_g^V \subsetneq N_g$ and $N_{g'} \subsetneq N_{g'}^V$. Then consider $l \in N_g \setminus N_g^V$ and $k \in N_{g'}^V \setminus N_{g'}$. First we derive a condition resulting from $l \notin \mathcal{A}^V$ and $k \in \mathcal{A}^V$. Under the true preferences, the allocation $\mathcal{A}^V(k \curvearrowright l)$ is less efficient that $\mathcal{A}^V$:

$$\sum_{s \in N_g^V} \pi_s^V(n_g^V) + \sum_{s \in N_{g'}^V} \pi_s^V(n_{g'}^V) \geq \sum_{s \in N_g^V} \pi_s^V(n_g^V + 1) + \sum_{s \in N_{g'}^V \setminus \{k\}} \pi_s^V(n_{g'}^V - 1) + \pi_l^V(n_g^V + 1)$$

This expression can be rewritten in the following more suitable form to apply the concavity assumption on the function π_s where s is a winning bidder under truthful reporting.

$$\pi_l^V(n_g^V + 1) < \pi_k^V(n_{g'}^V) + \sum_{s \in N_g^V} \left(\pi_s^V(n_g^V) - \pi_s^V(n_g^V + 1)\right) - \sum_{s \in N_{g'}^V \setminus \{k\}} \left(\pi_s^V(n_{g'}^V - 1) - \pi_s^V(n_{g'}^V)\right) \tag{5}$$

Second we derive a similar condition resulting from $i \in \mathcal{A}^{dev}$ and $j \notin$

$\mathcal{A}^{dev}$.

$$p_l \geq \pi_k(n_{g'}+1)+\sum_{s\in N_g\setminus\{l\}}(\pi_s(n_g-1)-\pi_s(n_g))-\sum_{s\in N_{g'}}(\pi_s(n_{g'})-\pi_s(n_{g'}+1)) \quad (6)$$

The price p_l equals to the externality imposed by bidder l on the remaining bidders, whereas the second term is a lower bound for this externality.

Note that $\pi_s^V = \pi_s$ for any winner and thus for $s = j$, $s \in N_g^V$ or $s \in N_{g'}^V$ Then we can compare each term in the right hand of equations 5 and 6:

- The first terms satisfy $\pi_k(n_{g'}+1) \geq \pi_k(n_{g'}^V)$ because $n_{g'} < n_{g'}^V$ and the mapping $x \to \pi_k(x)$ is decreasing.
- The second terms satisfy

$$\sum_{s\in N_g\setminus\{l\}}\left(\pi_s^V(n_g-1)-\pi_s^V(n_g)\right) \geq \sum_{s\in N_g^V}\left(\pi_s^V(n_g^V)-\pi_s^V(n_g^V+1)\right)$$

  because $N_g\setminus\{l\} \subset N_g^V$, $\pi_s^V(n_g-1)-\pi_s^V(n_g) \geq \pi_s^V(n_g^V)-\pi_s^V(n_g^V+1)$ for $s \in N_g^V$ due to our suitable concavity assumption for the types reported by winners and finally $\pi_s^V(n_g-1)-\pi_s^V(n_g) \geq 0$ for any reported preferences (reported should suffer from no-positive externalities).
- It is proved similarly that the third term satisfies

$$\sum_{s\in N_{g'}}(\pi_s(n_{g'})-\pi_s(n_{g'}+1)) \geq \sum_{s\in N_{g'}^V\setminus\{k\}}\left(\pi_s^V(n_{g'}^V-1)-\pi_s^V(n_{g'}^V)\right).$$

Finally, we have established that $p_l > \pi_l(n_g^V+1) \geq \pi_l(n_g)$ which raises a contradiction with bidder l making a profitable deviation.

B.2 Multi-unit

Suppose that proposition [4.1] is false under multi-unit demand. Similarly, we can distinguish two events. First there exists one 'bundled' agents $B_i := \{l_1, \cdots, l_{B_i}\}$ who manages to purchase m items and such that the winners in bidder B_i's group now obtain $m' > m$ items less than they would under truthful reporting. In that event, it is rather immediate that B_i makes a strictly negative profit. Otherwise, there exist one 'bundled' agents B_i and some group $\{g_b\}_{b=1\dots B_i}$ where $N_{g_b} \subsetneq N_{g_b}^V$ and such that the m items that B_i has purchased with the joint deviation can be decomposed in m' items taken to some winners in his group and $m-m'$ items taken to some winners in a group g_b. Finally, we prove that B_i makes a strictly negative profit in a similar way than in the case with unit-demand. This latter case was simpler because there was only one group g_b. On the other hand, with multi-unit demand, the analogs of condition 5 and 6 involve a double indexed sum.

C Proof Proposition 5.1

From proposition 4, it is sufficient to prove proposition 5.1 in the NGDE framework with unit demand. Thus we restrict the analysis to this case.

Note first that, as shown in Milgrom [29] [Theorem 8.2], bidder-submodularity is equivalent to:

$$w(S \setminus \{l\}) - w(S \setminus \{l,k\}) \geq w(S) - w(S \setminus \{k\}),\ \forall l,k,S \textit{ such that } l,k \in S \subset N$$

Note also that if $\{N, (\Pi_l)_{l \in N}\}$ is an allocation problem which satisfies the conditions of proposition 5.1, then for any $S \subset N$, the allocation problem $\{S, (\Pi_l)_{l \in S}\}$ also satisfies those conditions.

Consequently, to prove proposition 5.1, it is sufficient to prove that the following inequalities are true:

$$w(N \setminus \{l\}) - w(N \setminus \{l,k\}) \geq w(N) - w(N \setminus \{k\}), \quad \forall l,k \in N \tag{7}$$

for any allocation problem under proposition 5.1's assumptions.

For any allocation problem $\{S, (\Pi_l)_{l \in S}\}$, denote by S^* the optimal allocation.

Note that if either $l \notin N^*$ or $k \notin N^*$, then equation [7] is trivially satisfied because if $s \notin S^*, w(S \setminus \{s\}) = w(S)$ and w is an increasing function (we use the assumption that non-purchasers suffer from no allocative externalities). Thus, it is sufficient to prove equations [7] for $l,k \in N^*$.

Then our proof contains two steps. First, we establish a useful lemma which states that, under suitable conditions, if a bidder is a winner in the optimal allocation for a given set of competitors then if the set of competitors is reduced, this bidder remains a winner in the corresponding optimal allocation. Second, we check carefully equation [7], l and k belonging to either the same or different groups, each of this cases being divided in different events according to the groups of the 'new' winners for the allocation problems $N \setminus \{i\}$,$N \setminus \{j\}$ and $N \setminus \{i,j\}$.[22]

Lemma C.1 *In the NGDE framework, if $x \to \pi_i(x)$ is concave and if the equalities $\pi_i(x) - \pi_i(x+1) = \pi_j(x) - \pi_j(x+1)$ are fulfilled for any $x > 0$, any $i,j \in N$ such that $G(i) = G(j)$, then for any i, $(N \setminus \{i\})^* \supset N^* \setminus \{i\}$.*

Thus it states that if some winners are removed then the original winners are still winners.

[22]This last part of the proof, enumerating the different events, is a bit fastidious and it might be fulfilled with some general formulas covering different events, but we do not believe that it will help to clarify the proof.

Proof 5 *The optimal allocation is characterized by a vector* $(x_s)_{s=1,\ldots,g}$ *where* x_s *represents the number of bidders chosen in group* g*. Immediately, the optimal allocation given* x_s *is to allocate the item to the more efficient bidders in* g *which is unambiguous in the NGDE framework with assumption 3.1. For* $s = 1,\ldots,g$*, denote by* H_s *(respectively for all* $l \in N$*,* H_s^{-l}*) the function mapping* x_s *into a real number which represents the payoff of the group* s *(which is independent of the allocation of the other groups in the NGDE framework)(respectively of the group* s *when bidder* l *is removed). Denote by* $\Gamma_s(x)$ *(respectively* $\Gamma_s^{-l}(x)$*) the set of the* x *more efficient bidders in group* s *(respectively in group* s *when bidder* l *is removed). Then,* $H_s(x) = \sum_{j\in\Gamma_s(x)} \pi_j(x)$ *(respectively* $H_s^{-l}(x) = \sum_{j\in\Gamma_s^{-l}(x)} \pi_j(x)$*). Note that* H_s *and* H_s^{-l} *differ only if bidder* l *belongs to the group indexed by* s*.*

Lemma C.2 *For all* s*,* H_s *is concave on* $\mathbb{N}$*.*

Proof 6 *First, we can check easily than* $2\cdot H_s(1) \geq H_s(2)$*. Second for* $x > 0$*, we derive:* $H_s(x+2) - H_s(x+1) = \pi_{\Gamma_s(x+2)\setminus\Gamma_s(x+1)}(x+2) + \sum_{i\in\Gamma_s(x+1)} (\pi_i(x+2) - \pi_i(x+1))$ *and* $H_s(x+1) - H_s(x) = \pi_{\Gamma_s(x+1)\setminus\Gamma_s(x)}(x) + \sum_{i\in\Gamma_s(x+1)} (\pi_i(x+1) - \pi_i(x))$*. The concavity assumption for the function* π_i *implies that the sum in the latter expression is superior than the sum in the former. We also have* $\pi_{\Gamma_s(x+1)\setminus\Gamma_s(x)}(x) \geq \pi_{\Gamma_s(x+2)\setminus\Gamma_s(x+1)}(x) \geq \pi_{\Gamma_s(x+2)\setminus\Gamma_s(x+1)}(x+2)$*. Thus the concavity holds.*

The optimal allocation is the solution of the following optimization program:

$$(x_s)_{s=1,\ldots,g} \in Arg \max_{\sum_{s=1}^{g} y_s = M} \sum_{s=1}^{g} H_s(y_s)$$

Now suppose that a bidder l *exists such that* $(N\setminus\{l\})^* \supset N^*\setminus\{l\}$ *is false. Four cases may happen: first,* $n_{g(l)}^{(N\setminus\{l\})^*} = n_{g(l)}^{N^*}$*, second* $n_{g(l)}^{(N\setminus\{l\})^*} = n_{g(l)}^{N^*} - 1$*, third* $n_{g(l)}^{(N\setminus\{l\})^*} > n_{g(l)}^{N^*}$ *and fourth* $n_{g(l)}^{(N\setminus\{l\})^*} < n_{g(l)}^{N^*} - 1$*.*

Independently of the assumption on the differences $\pi_i(x) - \pi_i(x+1) = \pi_j(x) - \pi_j(x+1)$*, the two first cases can be excluded. It is immediate that if the same number of items is sold in group* g_l*, then the optimal allocation of the remaining items among the bidders in the other group is independent of bidder* l*'s presence. Now suppose that* $n_{g(l)}^{N^*} - 1$ *items are sold in bidder* l*'s group after his removal, then the assumption that* $(N\setminus\{l\})^* \supset N^*\setminus\{l\}$ *is false implies that there exist two groups* $s, s' \neq g_l$ *such that* $n_s^{N^*} > n_s^{(N\setminus\{l\})^*}$ *and* $n_{s'}^{N^*} < n_{s'}^{(N\setminus\{l\})^*}$*. We have:*

$$\begin{cases} H_s(n_s^{(N\setminus\{i\})^*}) - H_s(n_s^{(N\setminus\{i\})^*}+1) \leq H_s(n_s^{N^*}-1) - H_s(n_s^{N^*}) < \\ < H_{s'}(n_{s'}^{N^*}) - H_{s'}(n_{s'}^{N^*}+1) \leq H_{s'}(n_{s'}^{(N\setminus\{i\})^*}-1) - H_{s'}(n_{s'}^{(N\setminus\{i\})^*}) \end{cases}$$

The first inequality (respectively the third) holds from the concavity of H_s ($H_{s'}$) and $n_s^{N^*} > n_s^{(N\setminus\{i\})^*}$ ($n_{s'}^{N^*} < n_{s'}^{(N\setminus\{l\})^*}$). The second inequality holds from the optimality of the allocation such that $x_s = n_s^{N^*}, x_{s'} = n_{s'}^{N^*}$ against $x_s = n_s^{N^*} - 1, x_{s'} = n_{s'}^{N^*} + 1$ everything else being equal.

The strict inequality between the two extreme term implies that

$$H_s^{-l}(n_s^{(N\setminus\{l\})^*}) - H_s^{-l}(n_s^{(N\setminus\{l\})^*}+1) < H_{s'}^{-l}(n_{s'}^{(N\setminus\{l\})^*}-1) - H_{s'}^{-l}(n_{s'}^{(N\setminus\{l\})^*}) \tag{8}$$

because H_k and H_k^{-l} are equal for $k = s, s'$ since bidder l does not belong to s and s'. This inequality raises a contradiction with the optimality of $x_s = n_s^{(N\setminus\{l\})^*}, x_{s'} = n_{s'}^{(N\setminus\{l\})^*}$ against $x_s = n_s^{(N\setminus\{l\})^*} + 1, x_{s'} = n_{s'}^{(N\setminus\{l\})^*} - 1$. Then it raises a contradiction with $(N \setminus \{i\})^*$ been optimal.

Now consider the third case: $n_{g(l)}^{(N\setminus\{l\})^*} > n_{g(l)}^{N^*}$. Then there exists a group s such that $n_s^{N^*} > n_s^{(N\setminus\{l\})^*}$. Denote g_l by s'. As in the above calculation, we have:

$$H_s(n_s^{(N\setminus\{l\})^*}) - H_s(n_s^{(N\setminus\{l\})^*}+1) < H_{s'}(n_{s'}^{(N\setminus\{l\})^*}-1) - H_{s'}(n_{s'}^{(N\setminus\{l\})^*})$$

Since $H_s = H_s^{-l}$ (l does not belong to s') and due to the assumption (4a) which implies that $H_{s'}(x) - H_{s'}(x+1) \leq H_{s'}^{-l}(x) - H_{s'}^{-l}(x+1)$, then we have that the inequality (8) is satisfied raising the same contradiction as above. The fourth case $n_{g(l)}^{(N\setminus\{l\})^*} < n_{g(l)}^{N^*} - 1$ is treated similarly by using assumption (4b).[23]

The lemma allows to define[24] k_i and k_j such that: $(N \setminus \{i\})^* = N^* \cup \{k_i\} \setminus \{i\}$ and $(N \setminus \{j\})^* = N^* \cup \{k_j\} \setminus \{j\}$. Then, noting that $N \setminus \{i,j\} = (N \setminus \{i\}) \setminus \{j\}$, lemma C.1 implies that k_i and k_j are belonging to the set $(N \setminus \{i,j\})^*$. Then, if $k_i \neq k_j$, $(N \setminus \{i,j\})^* = N^* \cup \{k_i, k_j\} \setminus \{i,j\}$. On the contrary, if $k_i = k_j = k_*$, then the lemma allows us to define k_{ij} such that $(N \setminus \{i,j\})^* = N^* \cup \{k_*, k_{ij}\} \setminus \{i,j\}$.

First event: $G(i) = G(j) = G^*$ Consider first the case $k_i \neq k_j$. Then either $k_i \in G^*$ and $k_j \notin G^*$ or $k_j \in G^*$ and $k_i \notin G^*$. By symmetry of equation 7, it is sufficient to prove it in one this case, say $k_i \in G^*$ and $k_j \notin G^*$. Now,

[23] Remark the difference between assumption (4a) and (4b). For example, if assumption (4b) is satisfied, then the number of purchasers should not decrease more than 1 in bidder l's group after his removal.

[24] In what follows, k_i, k_j, k_{ij} are not empty due to the preliminary remark of the appendix.

$w(N)-w(N\setminus\{i\}) = \pi_i(n_G^{N^*})-\pi_{k_i}(n_G^{N^*})$ whereas $w(N\setminus\{j\})-w(N\setminus\{i,j\}) = \pi_i(n_G^{N^*}-1) - \pi_{k_i}(n_G^{N^*}-1)$. We conclude with the convexity of π_j over j noting that $n_G^{N^*} > 1$.

Now consider the case $k_i = k_j = k_*$. This case is divided in four cases either $k_* \in G^*$ or $k_* \notin G^*$ and $k_{ij} \in G^*$ or $k_{ij} \notin G^*$.

1. $k_*, k_{ij} \in G^*$
   $w(N) - w(N \setminus \{i\}) = \pi_i(n_G^{N^*}) - \pi_{k_*}(n_G^{N^*})$
   $w(N \setminus \{j\}) - w(N \setminus \{i,j\}) = \pi_i(n_G^{N^*}-1) - \pi_{k_{ij}}(n_G^{N^*}-1)$.
   The conclusion is straightforward because $i < k_* < k_{ij}$ and that $G(i) = G(j)$ $\pi_i(x) - \pi_i(x+1) = \pi_j(x) - \pi_j(x+1)$.

2. $k_*, k_{ij} \notin G^*$

$$\begin{cases} w(N) - w(N \setminus \{i\}) = \pi_i(n_{G^*}^{N^*}) - \pi_{k_*}(n_{G(k_*)}^{N^*}+1) \\ + \sum_{h\in\Gamma_{G^*}(n_{G^*}^{N^*})\setminus\{i\}} \left\{\pi_h(n_{G^*}^{N^*}) - \pi_h(n_{G^*}^{N^*}-1)\right\} \\ + \sum_{h\in\Gamma_{G(k_*)}(n_{G(k_*)}^{N^*})} \left\{\pi_h(n_{G(k_*)}^{N^*}) - \pi_h(n_{G(k_*)}^{N^*}+1)\right\} \end{cases}$$

The expression of $w(N\setminus\{j\})-w(N\setminus\{i,j\})$ depends on either $G(k_*) = G(k_{ij})$ or not. In the first case $G(k_*) = G(k_{ij})$,

$$\begin{cases} w(N \setminus \{j\}) - w(N \setminus \{i,j\}) = \pi_i(n_{G^*}^{N^*}-1) - \pi_{k_{ij}}(n_{G(k_*)}^{N^*}+2)+ \\ \sum_{h\in\Gamma_{G^*}(n_{G^*}^{N^*})\setminus\{i,j\}} \left\{\pi_h(n_{G^*}^{N^*}-1) - \pi_h(n_{G^*}^{N^*}-2)\right\}+ \\ \sum_{h\in\Gamma_{G(k_{ij})}(n_{G(k_{ij})}^{N^*})\cup\{k_*\}} \left\{\pi_h(n_{G(k_{ij})}^{N^*}+1) - \pi_h(n_{G(k_{ij})}^{N^*}+2)\right\} \end{cases}$$

Then, making the difference and using the fact that π_h is concave, we have the following lower bound for $(w(N \setminus \{j\}) - w(N \setminus \{i,j\})) - (w(N) - w(N \setminus \{i\}))$:

$$\begin{cases} \pi_i(n_{G^*}^{N^*}-1) - \pi_i(n_{G^*}^{N^*}) \geq 0 \\ +\pi_{k_*}(n_{G(k_*)}^{N^*}+1) - \pi_{k_*}(n_{G(k_*)}^{N^*}+2) \geq 0 \\ +\pi_{k_*}(n_{G(k_*)}^{N^*}+1) - \pi_{k_{ij}}(n_{G(k_{ij})}^{N^*}+2) \geq 0 \\ +\pi_j(n_{G^*}^{N^*}-1) - \pi_j(n_{G^*}^{N^*}) \geq 0 \end{cases}$$

Then, inegality 7 holds.

In the second case $G(k_*) \neq G(k_{ij})$,

$$\begin{cases} w(N\setminus\{j\}) - w(N\setminus\{i,j\}) = \pi_i(n_{G^*}^{N^*}-1) - \pi_{k_{ij}}(n_{G(k_*)}^{N^*}+1) + \\ \sum_{h\in\Gamma_{G^*}(n_{G^*}^{N^*})\setminus\{i,j\}} \left\{\pi_h(n_{G^*}^{N^*}-1) - \pi_h(n_{G^*}^{N^*}-2)\right\} + \\ \sum_{h\in\Gamma_{G(k_{ij})}(n_{G(k_{ij})}^{N^*})} \left\{\pi_h(n_{G(k_{ij})}^{N^*}) - \pi_h(n_{G(k_{ij})}^{N^*}+1)\right\} \end{cases}$$

Denote by $w(N\setminus\{i\}, k_* \curvearrowright k_{ij})$ the surplus associated with allocation $(N\setminus\{i\})^* \cup \{k_{ij}\} \setminus \{k_*\}$. Then, we derive:

$$w(N) - w(N\setminus\{i\}) = (w(N) - w(N\setminus\{i\}, k_* \curvearrowright k_{ij})) + (w(N\setminus\{i\}, k_* \curvearrowright k_{ij}) - w(N\setminus\{i\}))$$

The second term is negative, whereas we can derive the first term as an expression easily comparable with $w(N\setminus\{j\}) - w(N\setminus\{i,j\})$:

$$\begin{cases} w(N) - w(N\setminus\{i\}, k_* \curvearrowright k_{ij}) = \pi_i(n_{G^*}^{N^*}) - \pi_{k_{ij}}(n_{G(k_*)}^{N^*}+1) + \\ \sum_{h\in\Gamma_{G^*}(n_{G^*}^{N^*})\setminus\{i\}} \left\{\pi_h(n_{G^*}^{N^*}) - \pi_h(n_{G^*}^{N^*}-1)\right\} + \\ \sum_{h\in\Gamma_{G(k_{ij})}(n_{G(k_{ij})}^{N^*})} \left\{\pi_h(n_{G(k_{ij})}^{N^*}) - \pi_h(n_{G(k_{ij})}^{N^*}+1)\right\} \end{cases}$$

Then we can conclude.

The next two remaining cases are solved similarly.

3. $k_* \in G^*$ and $k_{ij} \notin G^*$
4. $k_* \notin G^*$ and $k_{ij} \in G^*$

Second event: $G(i) \neq G(j)$ The proof for this case is completely analogous. Note that there are three possibilities for k_i: it could be either the best remaining bidder of $G(i)$ (denoted by k_1), the best remaining bidder of $G(j)$ (denoted by k_2) or the best remaining bidder of an another group say G' (denoted by k_3). For k_j, it is also only these three possibilities that matters. Then we write the couple (k_i, k_j) as $(k_h, k_{h'}), h, h' \in \{1,2,3\}$. Among those nine cases, we can first rule out (k_2,k_1), (k_3,k_1) and (k_2,k_3). Then, it is also quite straightforward that for (k_1,k_2), (k_1,k_3) and (k_3,k_2), $w(N\setminus\{j\}) - w(N\setminus\{i,j\}) = w(N) - w(N\setminus\{i\})$. The three remaining cases are treated similarly as before so we do not detail the proof.

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