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Temporary Equilibrium

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Abstract :

This is the revised entry “Temporary equilibrium”, forthcoming in the new edition of *The New Palgrave*, Lawrence Blume and Steve Durlaub (Eds.), Palgrave Mc Millan, 2007. The entry surveys the developments of temporary general equilibrium theory and its contributions to a better understanding of the microeconomic foundations of macroeconomics, in particular to the analysis of monetary phenomena, non-clearing markets, imperfect competition and the foundations of Keynesian unemployment, as well as the study of economic dynamics and (in)stability of self-fulfilling expectations under various learning schemes, in relation in particular with “excess volatility” of financial markets.

JEL classification numbers : D50, D83, E12, E24, E30, E40.

Keywords : Temporary equilibrium, fiat money, non-clearing markets, new Keynesian macroeconomics, fix-price models, staggered price and wage setting, learning, excess volatility.

Résumé :

Ceci est la révision de la rubrique “Equilibre temporaire”, à paraître dans la nouvelle édition de *The New Palgrave*, Lawrence Blume et Steve Durlauf (Eds.), Palgrave McMillan, 2007. La rubrique passe en revue les développements de la théorie de l’équilibre général temporaire et ses apports à une meilleure compréhension des fondements microéconomiques de la macroéconomie, en particulier à l’analyse des phénomènes monétaires, des marchés en déséquilibre, de la concurrence imparfaite et des fondations du chômage Keynésien, ainsi qu’à l’étude de la dynamique économique et de l’(in)stabilité des anticipations auto-réalisatrices sous divers schémas d’apprentissage, en relation notamment avec la “volatilité excessive” des marchés financiers.

JEL numéros de classification : D50, D83, E12, E24, E30, E40.

Mots clés : Equilibre temporaire, monnaie fiduciaire, marchés en déséquilibre, nouvelle macroéconomie Keynésienne, modèles à prix fixes, formation échelonnée des prix et salaires, apprentissage, volatilité excessive.

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TEMPORARY EQUILIBRIUM

1. The Conceptual Framework

The fact that trade and markets take place sequentially over time in actual economies is a trivial observation. It has nevertheless far-reaching implications. At any moment, economic units have to make decisions that call for immediate action, in the face of a future that is as yet unknown. Expectations about the unknown future play therefore an essential role in the determination of current economic variables. On the other hand, the expectations that traders hold at any time are determined by the information that they have at that date on the economy, in particular on its current and past states. Observed economic processes are thus the result of a strong and complex interaction between expectations of the traders involved and the actual realizations of economic variables.

Economists have long recognized that such an interaction should be at the heart of any satisfactory theory of economic dynamics. The temporary equilibrium approach was indeed designed quite a while ago by the Swedish school (Lindahl, 1939) and J.R. Hicks (1939, 1965), with the intent to establish a general conceptual framework that would enable economists to cope with the study of dynamical economic systems, and in particular to incorporate in their models the subtle interplay between expectations and actual realizations of economic variables that seems factually so important. Economic theorists have employed this framework in a systematic way since then, using in particular the powerful techniques of modern equilibrium and/or game theory; this effort has yielded important improvements of our understanding of the microeconomic foundations of macroeconomics.

Before reviewing briefly a few of these important advances, it may be worthwhile to make clear what are the basic characteristics of the temporary equilibrium approach, and to compare it with others. To fix ideas, let us assume that time is divided into an infinite, discrete sequence of dates. We may envision first a specific institutional set-up, that was called a *futures economy* by Hicks (1946), and later generalized by Arrow and Debreu. Let us assume that markets for exchanging commodities are opened at a single date, say date 0; assume further that at that date, markets exist for contracts to deliver commodities at each and every future date $t \geq 0$. The specification of a 'commodity' will then involve not only the physical characteristics of the good or service to be delivered, but also the location and the circumstances ('state of nature') of the delivery. One gets then what has been called a 'complete' set of futures markets at the initial date $t = 0$ (Debreu, 1959, ch. 7).

It is clear that this framework is essentially timeless. Once an equilibrium is reached at date 0 (this equilibrium may be Walrasian or the result of any other game theoretic equilibrium notion), production and trade do take place sequentially in calendar time. But the coordination of the decisions of all traders is achieved at a single date through futures markets. There is no sequence of *markets* over time, and no role for expectations, money, financial assets, or stock markets.

Let us consider next another, more dynamic, type of organization, in which markets do open in every period. In this framework, traders would exchange at every date commodities immediately available on spot markets, promises to deliver specific commodities at later dates on futures markets, as well as money, financial assets and/or stocks (of course markets must be 'incomplete' in the sense of Arrow–Debreu at every date, otherwise reopening markets would serve no purpose). To convey the following discussion most simply, let us assume away all sources of uncertainty and consider the case where the state of the economy at any date can be described by a single real number. To simplify matters further, let us assume that the state of the economy at t, say x_t, is completely determined by the forecasts $x^e_{i,\,t+1}$ made by all traders $i = 1,\ldots,m$ at date t about the future state, through the relation

$$x_t = f\,(x^e_{1,\,t+1},\ldots x^e_{i,\,t+1},\ldots,\ x^e_{m,\,t+1}) \qquad (1)$$

The temporary equilibrium map f describes the result of the market equilibrating process at date t – be it Walrasian or not–for a given set of forecasts. Of course, in the study of any particular economy, the map f will be derived from the 'fundamental' characteristics of the economy: tastes, endowments, technologies, the rules of the game, the policies followed by the Government.

The foregoing formulation does seem to take into account the observed fact that markets unfold sequentially in calendar time. It is, however, incomplete since no specification of the way in which forecasts are made at each date has been offered at this stage.

We must first discuss a concept that was introduced by Hicks himself, that of an *intertemporal equilibrium*, with self-fulfilling expectations, and that has been extensively used recently in a variety of contexts. Such an intertemporal equilibrium is defined formally, in the present framework, as an infinite sequence of states $\{x_t\}$ and of forecasts $\{x^e_{i,\,t+1}\}$ satisfying (1) and

$$x^e_{i,\,t+1} = x_{t+1} \qquad (2)$$

for all dates. Although time appears explicitly in this formulation, it should be clear that this particular equilibrium concept is also intrinsically *timeless*. Indeed all elements of the sequences of equilibrium states $\{x_t\}$ and of equilibrium forecasts $\{x^e_{i,\,t+1}\}$ are determined simultaneously by an outside observer : present and future markets are equilibrated all at the same time.

The preceding discussion shows how we must proceed to describe a sequential adjustment of markets, in calendar time. We *must* add to the temporary equilibrium relationship (1) a specification of the way in which traders forecast the future at each date *as a function of their information on current and past states of the economy*. If we assume, for the simplicity of the exposition, that the information available to traders at date t is represented by the sequence $(x_t, x_{t-1},\ldots)$, that means that we have to add to (1), *m expectations functions* of the form

$$x^e_{i,\,t+1} = \psi_i\,(x_t, x_{t-1},\ldots) \qquad (3)$$

The equations (1) and (3) describe then in a consistent way a sequential adjustment of markets–a sequence of *temporary equilibria* – in which time goes forward, as it should. Given past history $(x_{t-1}, x_{t-2}, \ldots)$, (1) and (3) determine the current temporary equilibrium state and forecasts. Once such a temporary equilibrium is reached, production and exchange takes place at date *t*, and the economy can move forward to date *t* + 1, where the equilibrating process is repeated.

The temporary equilibrium approach, as sketched in the formulation (1) plus (3) is the general formulation, in fact the *only* sort of formulation that is allowed, if one wishes to describe the evolution of the economy as a *sequence* of markets that adjust one after each other. One should expect accordingly the approach to include self-fulfilling expectations as a special case. Indeed choose a particular intertemporal equilibrium.

Then the associated sequence of states, say $\{\bar{x}_t\}$, is a solution of the difference equation

$$\bar{x}_t = F(\bar{x}_{t+1}) \qquad (4)$$

in which $F(x) = f(x, \ldots, x)$ for all *x*. Consider now the economy at date *t*, and assume that past states have been $(\bar{x}_{t-1}, \bar{x}_{t-2}, \ldots)$. Assume that the traders know the characteristics of the economy, or at least the map *F*, and further that the map *F* is invertible (we are voluntarily vague about the domain of definitions of the functions under consideration, to simplify the present methodological discussion, but these technical details can be fixed up). The traders are then able to infer that the recurrence satisfied by current and past states, i.e. $x_t = F^{-1}(x_{t-1})$, will obtain in the future as well, their forecasting rule may be viewed as the result of iterating twice that relation, or of inverting $\bar{x}_{t+1} = F^2(\bar{x}_{t-1})$, for all *i = 1,…, m*

$$\psi_i(\bar{x}_t, \bar{x}_{t-1}, \ldots) = F^{-2}(\bar{x}_{t-1}) \qquad (5)$$

If this relation holds, $\bar{x}_t$ is indeed a temporary equilibrium state (i.e. it solves (1) and (3)) at date *t*, given past history $(\bar{x}_{t-1}, \bar{x}_{t-2}, \ldots)$.

As we have just shown, the temporary equilibrium method includes self-fulfilling expectations as a special case. This shows incidentally that the opposition often made in the literature, between self-fulfilling expectations, that are claimed to be "forward looking", and "backward looking" expectations as in (3), is presumably misleading. The temporary equilibrium approach is indeed much more general, since it permits to incorporate in the analysis the fact that traders usually learn the dynamics laws of their environment only gradually, and thus to study in principle how convergence toward self-fulfilling expectations may or may not obtain in the long run.

The preceding discussion was carried out in a simple one-dimensional world operating under certainty. It should be clear nevertheless that the qualitative conclusions we obtained hold as well in a more complex, multidimensional world operating under uncertainty.

When the evolution of the economy is described as a sequence of temporary equilibria, at each date, the current equilibrium states are determined by past history. In this framework, a number of issues arise naturally. First, one has to find the conditions under which the dynamic evolution of the economy is well defined. In other words, when does a temporary equilibrium exist? Second, does the corresponding dynamical system have long run equilibrium states, such as deterministic stationary states or cycles, and/or stationary stochastic processes, along which expectations are self-fulfilling? Under which conditions, in particular on the formation of expectations, do the sequences of temporary equilibria so generated converge to such a long run equilibrium? This is precisely the sort of questions that have attracted the attention of modern economic theorists working in temporary equilibrium theory.

2. Overview

We turn now to a brief appraisal of this research effort, referring the interested reader to more extensive and more technical surveys that already exist in the literature, see for example Grandmont (1977, 1987, 1998).

Money and Assets in Competitive Markets

Considering a sequence of markets opens immediately the possibility for traders to hold money and more generally, assets of various kinds for saving, borrowing, transactions purposes and/or insurance motives. The application of the modern techniques of temporary equilibrium theory to the study of monetary phenomena has led to a major reappraisal, in the seventies, of classical and neoclassical monetary theories in competitive environments. It has permitted in particular to solve an old problem that had puzzled economic theorists for some time (Hahn, 1965), namely why fiat money, which has no intrinsic value, should have a positive value in exchange in competitive markets. The answer provided by traditional neoclassical theory relied essentially upon unit-elastic price expectations and the presence of real balance or wealth effects (Patinkin, 1965). Modern temporary equilibrium methods have shown that sort of answer to be surely incomplete and presumably mistaken: intertemporal substitution effects have to play an important role, and this can be only achieved by abandoning the hypothesis of unit-elastic expectations and by introducing some degree of inelasticity of expectations with respect to current observations (an example of such a condition was used in (5) above, where the forecast was made to depend on past states but not on the current state). The reappraisal of monetary theory by means of the temporary equilibrium method clarified greatly many confusing debates of the preceding literature: the relations between Walras's and Say's Law, the meaning and the validity of the Classical Dichotomy and the Quantity Theory of Money, the possibility of monetary authorities to manipulate the interest rates or the money supply, the existence of a 'liquidity trap' (Grandmont, 1983). The introduction of cash-in-advance constraints in temporary competitive equilibrium models of money (Grandmont and Younès, 1972, 1973) yielded important insights into the relations between its respective roles as a store of value and as a medium of exchange, and time preference, and permitted to make precise the microeconomic foundations of Milton Friedman's theory of optimum cash balances (1969, see also Woodford (1990)). Such models of money using cash-in-advance constraints have been popular in modern macroeconomics, following the contribution of R.E. Lucas, Jr. (1980). Capital markets imperfections that make explicit the essential role of money in exchange have since been central to the modern analysis of monetary theory and policy, see Wallace (2001).

The introduction of assets of various kinds in competitive markets leads also to the possibility of speculation and arbitrage in capital markets. Different persons with different tastes or expectations will then be willing to trade such assets. An important question is to study the conditions ensuring the existence of a temporary equilibrium in that context. A neat answer to that problem was provided by J.R. Green (1973) and O.D. Hart (1974): there must be some agreement between the traders' expectations about future prices.

Temporary Equilibria with Nonclearing Markets and Imperfect Competition.

A temporary equilibrium need not be Walrasian. One may consider cases where prices and/or wages are set through monopolistic or oligopolistic competition at the beginning of each elementary period and remain temporarily fixed within that period. A temporary equilibrium corresponding to these prices is then achieved at each date by quantity rations that set upper or lower bounds on the traders' transactions.

It had been known for some time that traditional Keynesian macroeconomic models of unemployment involved, explicitly or implicitly, the assumption of temporarily fixed prices and/or wages, as noted by Hicks himself (1965). The choice-theoretic structure of these models was rather unclear, however, which was a source of some confusion. The systematic study of temporary equilibrium models with quantity rationing undertaken in the seventies produced deep insights on this issue, and unveiled the hidden but central role played by quantity signals, as perceived by the traders in addition to the price system, to achieve an equilibrium in such models.

One major outcome of this research programme was the discovery that different types of unemployment could obtain, and even co-exist. 'Keynesian unemployment' corresponds to a situation where there is an excess supply on the labour and the goods markets. In such a situation, firms perceive constraints on their sales because demand is too low. Keynesian policies aiming at increasing aggregate demand may work in such a case. But unemployment may co-exist with an excess demand on the goods markets. In such a regime, called 'Classical unemployment' by Malinvaud (1977), the source of unemployment is rather the low profitability of productive activities. Keynesian policies may not work in that case; one has to resort to policies that restore profits, such as lowering real wages. In that respect, these results achieved a remarkable synthesis, within a unified and clear conceptual framework, between two paradigms that appeared fundamentally distinct beforehand.

The research on this topic proceeded very early on to endogenize prices and wages and yielded numerous insights of the connections between Keynesian models of unemployment and price or wage making in monopolistic or oligopolistic models of competition (see Barro and Grossman (1976), Benassy (1982, 1986), Grandmont and Laroque (1976), Hart (1982), Malinvaud (1977), Negishi (1979)). It has since become a cornerstone building block of the modern reformulation of socalled "*New Keynesian Macroeconomics*" (Benassy (2002), Dreze (1991), Mankiw and Romer (1991)). While early formulations focused on temporary equilibria with nonclearing markets due to exogenously staggered price and wage setting (see Taylor (1999) for a survey), more recent research seeks to explain such staggering of price and wage changes as the rational reaction of agents under the gradual diffusion of "sticky information" (Mankiw and Reiss, 2003).

Learning and (In)stability

The temporary equilibrium approach includes self-fulfilling expectations as a particular case, and is in fact more general, since it can incorporate learning in the formation of the traders' expectations. An important issue, that has been early on the agenda of that research program (Fuchs and Laroque (1976)), is then to know whether the sequences of temporary equilibria that are associated to given learning processes or expectations functions converge eventually to a long run equilibrium along which forecasting mistakes vanish. The question arises of course for long run equilibria that are simple, such as steady states, or more complex, such as deterministic cycles (Grandmont (1985)). The general lesson that seems to come out the research works done on the topic appears to be some kind of "*uncertainty principle*" (Grandmont (1998)). Learning generates local instability of self-fulfilling expectations whenever agents are on average uncertain about the local stability of the system, and thus ready to extrapolate a wide range of regularities (trends) out of past deviations from equilibrium, and when the influence of expectations on the dynamics is significant. On the other hand, learning may generate locally stable dynamics when either expectations do not matter much, or traders extrapolate a restricted range of stable trends out of past deviations from equilibrium.

The above principle arises in a wide variety of learning processes, in particular in "error learning" models, least squares and Bayesian learning. Of course, if one is willing to restrict the range of learning schemes, one may be able to produce sharper stability criteria (see in particular the concept of "E-stability" developed by Evans and Honkapohja (1999) for a particular class of learning processes). Local learning instability due the above "uncertainty principle" may explain why markets in which expectations are thought to play a significant role, such as markets for financial assets, durable goods, capital or inventories, display more volatility than others. In a similar vein, Brock and Hommes (1997) have shown local instability in a cobweb model, and convergence to more or less complex cyclical or chaotic long run equilibria, when agents choose in variable proportions among different more or less efficient (and costly) learning schemes, and have seeked to apply this approach to explain "excess volatility" in financial markets.

See also ***general equilibrium; fiat money; non-clearing markets; new Keynesian macroeconomics; fix-price models; staggered price and wage setting; learning.***

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