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# Delivered Pricing and the Effect of Horizontal Differentiation on Optimal Collusion

**J. THAL**[1]



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[1] University of Toulouse (GREMAQ) and CRES-LEI. Email : jeaninethal@gmail.com

# Delivered Pricing and the Effect of Horizontal Differentiation on Optimal Collusion

**Jeanine Thal**


### Abstract

This paper analyzes the impact of horizontal differentiation on the sustainability of collusion when firms charge delivered prices. Gupta and Venkatu (2002) show that differentiation hinders collusion if firms employ standard grim trigger punishments. The reason is that competitive profits are higher the higher the degree of differentiation, which weakens deterrence. We show that the results change dramatically if collusion is sustained by optimal punishments instead, since these yield minmax profits irrespectively of the degree of differentiation.

A high degree of differentiation then tends to facilitate collusion by rendering deviations less profitable. Excessive differentiation sometimes hinders collusion, however, because it also implies high transportation costs for a successful cartel.



### Résumé

Cet article analyse l'impact de la différenciation horizontale sur la soutenabilité de la collusion quand les entreprises facturent en prix livré. Gupta et Venkatu (2002) montrent que la différenciation rend la collusion plus difficile quand les firmes utilisent des schémas standards de punition (retour à la concurrence). En effet, les profits concurrentiels sont croissants avec le degré de différentiation. Nous montrons que ce résultat est renversé quand la collusion est soutenue par des punitions optimales, parce que ces punitions conduisent au profit minmax indépendamment du degré de différenciation. Ainsi, un degré élevé de différenciation, en rendant les déviations moins profitables, tend à faciliter la collusion. Une différenciation excessive peut cependant parfois gêner la collusion à cause des forts coûts de transport qu'elle implique pour le cartel.


**JEL : D43 ; L13**

# 1 Introduction

Industries in which transportation services account for a substantial share of the total cost of selling a product, such as cement, steel, corn, milk or certain chemicals, frequently use delivered pricing schemes.[1] Instead of charging a uniform price to any consumer who travels to the "mill" to pick up the product, firms charge prices inclusive of freight that may vary as a function of the buyer's location. A number of recent cartel cases occurred in industries using delivered pricing policies,[2] e.g. the international lysine cartel, the international vitamin cartel, or the German cement cartel.

The purpose of this article is to examine the relationship between horizontal differentiation and collusion when firms use delivered pricing policies. In a simple duopoly model à la Hotelling (1929), we investigate whether differentiation facilitates or hinders collusion, and analyze the cartel members' incentives to differentiate. We assume that deviations from cartel behavior are deterred by optimal punishments, i.e. the harshest credible retaliation schemes firms can possibly agree on. It turns out that optimal punishments always yield minmax discounted future profits, which are zero here, for the deviator. These punishments may for example consist of a reversion to base-point pricing where the deviator's plant is adopted as the base point, or may have a dynamic stick-and-carrot structure à la Abreu (1986). In any case, punishment profits are independent of how differentiated firms are. Differentiation then affects collusion only via its impacts on the potential deviation gain and on aggregate cartel profits.

Our main result is that differentiation tends to facilitates collusion. The driving force is that deviation profits are lower if firms are more differentiated, since the deviator has to incur higher transportation costs to steal the other firm's consumers. If transportation costs are linear or convex, then collusion on any given pricing schedule is easiest for maximal differentiation. For concave transportation costs, a degree of differentiation exceeding that chosen by a multi-plant monopolist facilitates collusion most.

These results are different from those obtained in the small body of literature on collusion when firms use delivered pricing policies. Gupta and Venkatu (2002) analyze the sustainability of monopoly prices in a Hotelling model relying on grim trigger punishments that prescribe a reversion to the competitive equilibrium. In the case of linear demand and linear transportation costs, they find that collusion is easiest for minimal differentiation.[3] This result is driven by the

[1] For further examples and empirical evidence, see Stigler (1949), Greenhut, Greenhut & Li (1980), and Greenhut (1982).

[2] The idea that delivered pricing policies are a collusion-inducing practice has a long tradition in economics (see Fetter (1937), Machlup (1949), Stigler (1964), Carlton & Perloff (1994, p.418)). Espinosa (1992) and Thisse & Vives (1992) analyze this issue theoretically. We take the pricing strategy as given in order to focus on the relationship between horizontal differentiation and collusion.

[3] Matsumura and Matsushima (2005) extend this model to more than two firms. They find that the minimal

fact that competitive profits are increasing with differentiation. Deterrence is therefore weaker the more differentiated firms are. Since optimal punishments are maximal, this effect vanishes in our analysis, which explains the dramatic difference in results. Our analysis thus shows that the results obtained in the context of grim trigger punishments are not robust to a change in punishment schemes.

In the context of mill pricing, the kind of questions we address have been studied extensively.[4] Chang (1991) analyzes collusion in d'Aspremont et al.'s (1979) version of the Hotelling model. Relying on grim trigger strategies, he finds that differentiation facilitates collusion on prices. First, as differentiation increases, the critical discount factor to sustain monopoly pricing decreases. Second, whenever the discount factor is not sufficiently high to sustain monopoly profits, then the optimal collusive price is higher the more differentiated products are. The driving force is that differentiation renders deviations less profitable, as in our model. Häckner (1996) shows that Chang's (1991) findings continue to be true if collusion is sustained by optimal punishments. Unlike in the delivered pricing framework, the effect of differentiation hence seems to be robust with respect to changes in the punishment mechanism in the case of mill pricing.[5]

We also study the optimal cooperative choice of (irreversible) locations for the cartel members. We find that if firms are very patient (i.e. the discount factor is high) so that monopoly prices are always sustainable, then an intermediate degree of differentiation is optimal because it minimizes the average distance from consumers. For intermediate values of the discount factor, perfect collusion is still possible but differentiation must be larger to prevent deviations. If firms are very impatient, then a high degree of differentiation is not sufficient to prevent deviations and the cartel is obliged to charge below-monopoly prices.

In mill pricing models, a number of contributions deal with endogenous product design. Chang (1992) analyzes costly product redesign, and finds that low redesigning costs hinder collusion. This result is driven by the use of grim trigger punishments: if redesign is cheap, then firms can mitigate the severity of punishments by relocating their products and hence weakening competition. Häckner (1995) analyzes the optimal choice of locations by the cartel supposing that each firm can costlessly relocate at any time. Similar to us, Haan and Toolsema (2004) examine the cooperative choice of locations prior to a repeated game in which firms collude whenever possible.[6] In both models, the optimum depends on locations in the following way.

differentiation result of Gupta und Venkatu carries over to an industry with three firms.

[4]To facilitate the comparison, the following discussion focuses on spatial models of differentiation. See Martin (1993, p.116-117), Deneckere (1983) and Wernerfelt (1989) for other treatments of collusion between firms with horizontally differentiated products, sustained either by trigger or by optimal punishments.

[5]Raith (1996) shows that the presence of uncertainty alters these results. The idea is that demand functions are more correlated the less differentiated firms are. In an imperfect monitoring environment, heterogeneity then makes it more difficult for firms to distinguish between random shocks and small deviations.

[6]Haan and Toolsema (2004) also study the effect of anticipated collusion on the (non-cooperative) choice of

For sufficiently high discount factors, firms locate so as to maximize joint profits. For lower discount factors, they have to differentiate more to prevent the breakdown of collusion. Only if firms are very impatient, then they will also have to lower prices.

Our results on optimal locations are thus fully in line with those in the literature dealing with mill pricing policies. This is not surprising, since the same forces are at work whether firms do or do not discriminate between consumers on the basis of their locations. On the one hand, differentiation helps firms to sustain collusion because it renders deviations less profitable. On the other hand, profit maximization calls for an intermediate degree of differentiation. Depending on how patient firms are, this leads to an optimal degree of differentiation somewhere between an intermediate level and maximal differentiation. Our paper hence provides support for the "fairly general tendency within the Hotelling framework for differentiation to relax competition and facilitate collusive agreements" identified by Häckner (1995, p.293).

This paper is organized as follows. The next section presents the framework. Section 3 discusses the optimal punishments firms can use to sustain collusion in our model. Section 4 examines optimal symmetric collusive schemes and the impact of differentiation on the level of profits sustainable by the cartel. The last part of the section endogenizes locations by analyzing the optimal cooperative choices of both pricing schedules and locations. Section 5 discusses how the results would change if firms used standard grim trigger strategies instead of optimal punishments. Section 6 concludes.

## 2 Framework

We use a simple linear city model à la Hotelling to characterize horizontal differentiation. Two firms, $i = 1, 2$, are located at $y_1$ and $y_2$ in $[0, 1]$. They produce goods which are not differentiated in any dimension other than their locations. Consumers are uniformly distributed along the segment $[0, 1]$, and the total mass of consumers is normalized to one. The demand for the product at each location $x$ is given by a (downward-sloping) function $q(p)$ of the price $p$ a consumer at $x$ has to pay to consume "variety $x$" of the good. Arbitrage between consumers at different locations is infeasible.

The cost firm $i$ incurs to transport one unit of the good to a consumer at location $x$ is $t(|y_i - x|)$, where $t(\cdot)$ is an increasing and differentiable function of distance measured by the Euclidean norm. We suppose that $t(0) = 0$. Moreover, we will assume that the marginal transportation costs $t'(\cdot)$ are monotone; transportation costs are hence either (weakly) convex everywhere or (weakly) concave everywhere. We would expect this assumption to be satisfied if firms employ the same mode of transportation for different distances. The firms have identical

locations. Jehiel (1992) and Friedman and Thisse (1993) fall into the same strand of the literature.

and constant marginal costs of production, which are normalized to zero for simplicity. Firm $i$'s marginal cost of selling to a consumer located at $x$ is thus simply its transportation cost $t(|y_i - x|)$.

Firms can observe each consumer's location, and use *delivered pricing* schemes. Firm $i$ thus bears the transportation cost of serving each of its consumers and selects a price schedule $p_i(x)$, where $p_i(x)$ is the delivered price at which firm $i$ offers its product to consumers located at $x$. The effective mill price paid, i.e. $p_i(x) - t(|y_i - x|)$, may vary with the consumer's location $x$.

Delivered pricing is particularly pertinent in a geographical context, when sellers provide delivery to buyers who are differentiated only with respect to their geographical locations. Note that delivered pricing may also be interpreted in the context of product differentiation. It then amounts to firm $i$ bearing the cost $t(|y_i - x|)$ of redesigning its basic product $y_i$ and offering the whole band of varieties. This interpretation is not very natural, however, since it would require that firms observe each consumer's preferences and that consumers are restricted to buy the product that fits their preferences best, independently of the relative prices of different varieties of the good.

In the stage game we consider, firms simultaneously select price schedules. Each consumer then purchases from the firm offering the lower delivered price. In case of a price tie, the equilibrium we will focus on will be such that consumers behave socially optimally and buy from the firm with the lower marginal cost. If firms have symmetric production and transportation costs, this is the firm closer to the consumer.[7]

To analyze collusion, we use a standard supergame framework. Firms meet in every time period $t = 0, 1, 2, ..., \infty$. They simultaneously choose their delivered prices at the beginning of each period; these decisions can be conditioned on the full history of price schedules. Each firm aims to maximize its stream of discounted future profits, where the common discount rate is $\delta \in (0, 1)$.

# 3 Optimal Punishments

A credible retaliation or punishment mechanism is necessary to sustain any collusive agreement. The harsher the punishment for the deviator, the easier is collusion, since deterrence is improved. It is therefore ex ante optimal for colluding firms to agree on the fiercest sustainable (i.e., subgame perfect) punishments. As shown by Abreu (1988), any given path can be sustainable if and only if it can be sustained by reversion, in a case of a deviation from that path, to a punishment that is (one of) the deviator's worst possible subgame perfect equilibrium. An

[7] If the firms charge the same price and have the same marginal cost, they split the local demand. The set of locations for which the latter happens can however be generically neglected.

optimal punishment is a harshest credible punishment of the game.

The minmax of our supergame, that is the lowest continuation value a firm can force its rival down to, is zero for both firms. If any credible punishment mechanism achieves this lower bound on the deviator's continuation profits, then it is obviously optimal.

It is common in the literature on collusion to rely on trigger grim punishment (see Chang (1991), Häckner (1992), Thisse and Vives (1992), or Gupta and Venkatu (2002) in the context of horizontal differentiation). These prescribe a reversion to the static competitive equilibrium for ever once a deviation occurs. Such punishments are clearly subgame perfect. They also have the advantage of being simple. In our model, however, trigger punishments do not minmax firms, since static competitive profits are strictly positive for both firms. In fact, firms engage in localized Bertrand competition in each stage game. As Carlton (1983) puts it, "delivered pricing makes each consumer a competitive battleground". Since firms have different transportation costs, the standard static Nash equilibrium is then such that at each location the low-cost firm sells at the marginal cost of the high-cost firm. The competitive price schedule is hence

$$p^N(x) = \begin{cases} t(|x-y_{-i}|) \text{ where firm } i \text{ has a cost advantage,} \\ t(|x-y_i|) \text{ where firm } -i \text{ has a cost advantage.} \end{cases} \quad i \neq -i.$$

Each firm serves the consumers located closer to it than to its rival, earning its transportation cost advantage on each consumer.

Credible punishments that minmax the deviator indeed exist here for any discount factor $\delta \in (0,1)$. Previous works (Espinosa (1992), Thisse and Vives (1992)) have for example shown that an eternal switch to base-point pricing where the deviator's locations serves as a base point is an optimal punishment. Firms used similar kinds of punishment mechanisms in the cement industry (see Machlup (1949)). Upon a unilateral deviation either from the collusive or a punishment path, both firms charge the deviator's marginal costs at all locations for ever. This leaves the deviator with zero continuation profits, while the other firm earns as much as under competition. Such strategies indeed form a subgame perfect equilibrium: the punishing firm strictly wants to stick to its price strategy so as to earn positive profits instead of being punished itself, while the deviator is indifferent between deviating from its own punishment or complying with it. The disadvantage of such punishments is that they involve weakly dominated strategies. For any strategy of the punishing firm, the deviator would be at least as well off charging a price slightly above its own marginal cost in every period.[8] In particular, the deviator could gain if the

[8]Thisse and Vives (1992) point out that the one-shot equilibrium repeated in this punishment also involves the play of a weakly dominated strategy by the punishing firm, because it quotes prices below its own marginal costs to some consumers. The punisher could therefore weakly improve its position by charging the maximum of the two firms' marginal costs at every location. Note, however, that when the game is repeated the punisher no longer plays a weakly dominant strategy : it strictly prefers sticking to the punishment path, along which it earns

punishing firm ever "trembled" and charged a price above its own marginal cost. As illustrated in appendix A, however, it is easy to construct optimal stick-and-carrot punishments that are undominated.[9]

# 4 Optimal Collusion

## 4.1 Sustainability

For simplicity, we focus on the case of inelastic demand here. In section 5, we will also treat an example with elastic demand in order to compare our results with those of Gupta and Venkatu (2002). Formally, the demand at each location $x$ is

$$q(p) = \begin{cases} 1 \text{ if } p \leq v, \\ 0 \text{ if } p > v, \end{cases}$$

so that $v$ is the reservation price above which no consumer is willing to purchase the good.

Moreover, we assume that transportation costs are low enough such that a monopolist located at one edge of the city wants to cover the whole market:

**Assumption 1** $v \geq t(1)$.

This assumption allows us to rule out drastic cost advantages and focus on the interesting case of collusion in an otherwise competitive market.

Since the game is symmetric, we also suppose that locations are symmetric.

**Assumption 2** $y_2 = 1 - y_1$.

This assumption, which is also made by Chang (1991), Häckner (1995, 1996), as well as Gupta and Venkatu (2002), simplifies the analysis. In particular, it allows us to focus on equilibria in which each firm serves the consumers closest to it, instead of considering a myriad of possible market sharing rules. Thanks to the symmetry assumption, we can drop the subscripts and simply denote $y_1$ by $y$. We then suppose, without loss of generality, that $y \leq \frac{1}{2}$, so that firm 1 is located to the left of firm 2. Spatial differentiation is then decreasing in $y$; firms are not differentiated at all for $y = \frac{1}{2}$, and differentiated maximally for $y = 0$.

Our focus will be on collusive schemes in which both firms charge identical price schedules:

$$p_1^C(x) = p_2^C(x) = p^C(x). \tag{1}$$

positive profits, in order to avoid its own punishment being started.

[9]Thal (2005) characterizes such punishments in a non-spatial model of price competition with asymmetric linear production costs. Since the spatial model considered here corresponds to localized Bertrand competition between cost asymmetric firms, the insights of that previous analysis can be applied here.

To see why this does not limit the scope of the analysis, suppose (in negation) that at some location firm $i$ charges a strictly higher price than firm $-i \neq i$. Then, firm $-i$ serves the whole demand at this location. The firms' collusive profits would not change if firm $i$ decreased its price down to that of firm $-i$ (plus an arbitrarily small amount). Yet, the latter change in pricing strategy would limit firm $-i$'s deviation possibilities by ruling out upward deviations at the location in question.[10] Firms thus never gain by charging different prices at the same location.

Furthermore, we will focus on collusive price schedules that lie between the competitive price and the monopoly price at all locations:

$$p^C(x) \in [p^N(x), v] \text{ for any } x. \tag{2}$$

It is obvious that firms never want to collude on delivered prices above the reservation price: higher prices would reduce cartel profits but not deviation incentives. Firms never have a strict incentive to set collusive price below $p^N(x)$ either. Charging the competitive price instead would strictly increase collusive profits but leave deviation profits unchanged, since no firm ever has an incentive to undercut a price equal to or below its own marginal cost.

Finally, we will restrict attention to price schedules that are symmetric around the centre of the linear city, i.e.

$$p^C(x) = p^C(1-x) \text{ for all } x, \tag{3}$$

and agreements such that each firm serves the consumers who are closer to its own location than to that of its rival. Since firms are completely symmetric here, collusion cannot be facilitated by opting for an asymmetric scheme. We also have no reason to assume that one firms has more bargaining power than the other one.

Under our symmetry assumption, each firm's collusive profits are equal to

$$\pi^C(p^C(x), y) = R(p(x)) - T^C(y), \tag{4}$$

where its revenues are

$$R(p^C(x)) = \int_0^{\frac{1}{2}} p^C(x)dx = \int_{\frac{1}{2}}^1 p^C(x)dx,$$

and its total (transportation) costs are

$$T^C(y) = \int_0^{\frac{1}{2}} t\left(|y-x|\right) dx = \int_{\frac{1}{2}}^1 t\left(|(1-y)-x|\right) dx.$$

Since demand is inelastic, the monopoly price schedule is simply $p^C(x) = v$; monopoly prices are hence independent of the degree of differentiation in our simple model, and uniform for all

[10] If firms are able to collude on charging the monopoly price $v$ everywhere, this price decrease does not have any impact. The high-price firm can quote any price above the reservation price without facilitating collusion, since no firm ever wants to deviate to a price above $v$.

consumers. We will say that collusion is *perfect* if the cartel charges the monopoly price $v$ to all consumers.

Firm $i$'s best one-shot deviation is to

- keep charging $p^C(\cdot)$ in its own market area to preserve its collusive profits, and
- slightly undercut $p^C(\cdot)$ in $-i$'s market area to steal $-i$'s business; not that since $p^C(x) \geq p^N(x)$, business stealing is weakly profitable everywhere.

The short-term profit *gain* a firm can achieve is hence

$$\pi^D(p^C(x), y) = R(p^C(x)) - T^D(y),$$

where

$$R(p^C(x)) = \int_0^{\frac{1}{2}} p^C(x)dx = \int_{\frac{1}{2}}^1 p^C(x)dx$$

is the gain in revenues that can be attained, while

$$T^D(y) = \int_0^{\frac{1}{2}} t\left((1-y)-x\right)dx = \int_{\frac{1}{2}}^1 t\left(x-y\right)dx$$

are the total transportation cost that must be incurred to serve the additional consumers.

A price schedule $p^C(x)$ is then sustainable by minmax punishments, which yield zero continuation profits as shown in the previous section, if and only if

$$\delta\left[\pi^C(p^C(x), y)\right] \geq (1-\delta)\left[\pi^D(p^C(x), y)\right]$$

These constraints yields the following condition on the discount factor:

$$\delta \geq \delta^*(p^C(x), y), \tag{5}$$

where

$$\begin{aligned} \delta^*(p^C(x), y) &= \frac{\pi^D(p^C(x), y)}{\pi^D(p^C(x), y) + \pi^C(p^C(x), y)} && (6) \\ &= \frac{R(p^C(x)) - T^D(y)}{2R(p^C(x)) - T^D(y) - T^C(y)}. && (7) \end{aligned}$$

**The Impact of Differentiation - Exogenous Prices** From the expression for the critical discount factor in (7), one can immediately see that some differentiation helps collusion in the presence of transportation costs. If firms are not differentiated (i.e. $y = \frac{1}{2}$), then serving consumers in either "half" of the linear city is equally costly:

$$\pi^C\left(\frac{1}{2}\right) = \pi^D\left(\frac{1}{2}\right).$$

Thus,

$$\delta^*(p^C(x), \frac{1}{2}) = \frac{1}{2} \text{ for any } p^C(x).$$

As firms differentiate, however, a wedge is driven between the transportation costs associated with serving assigned customers as opposed to stealing the rival's business:

$$t((1-y)-x) > t(|x-y|) \text{ for } y, x < \frac{1}{2}.$$

Hence,

$$\delta^*(p^C(x), y) < \frac{1}{2} \text{ for } y < \frac{1}{2}.$$

Indeed, as we will show formally in the proof of proposition 1, the profit gain from a deviation decreases as firms are more differentiated from one another for any given pricing schedule. This is true because the transportation costs associated with "stealing" the other firm's consumers is higher the more differentiated firms are:

$$\frac{\partial T^D}{\partial y}(y) < 0,$$

Differentiation thus has a *deviation effect*, which predicts that ceteris paribus differentiation helps collusion by rendering deviations less profitable.

Differentiation also affects the critical discount factor via its impact on cartel profits. Given pricing schedules, the effect of differentiation on collusive profits depends solely on whether total transportation costs increase or decrease as firm locate further apart. In the simple Hotelling model we consider, the aggregate transportation costs incurred to serve all consumers are minimized for $y$ equal to

$$\frac{1}{4} = \min_y 2T^C(y).$$

Differentiation has hence a *cartel transportation cost effect*, which predicts that locations that minimize the transportation costs incurred to serve all consumers help collusion by increasing collusive profits.

The total impact of the degree of differentiation on the critical discount factor, treating prices as given, then results from the combination of the "deviation effect" and the "cartel transportation cost effect": (the proof of the following proposition is relegated to the appendix)

**Proposition 1** *There exists a unique* $\underline{y} \in [0, \frac{1}{4})$ *such that*

$$\begin{aligned} \frac{\partial \delta^*}{\partial y}(\cdot, y) &< 0 \text{ for } y \in [0, \underline{y}), \\ \frac{\partial \delta^*}{\partial y}(\cdot, y) &> 0 \text{ for } y \in \left(\underline{y}, \frac{1}{2}\right). \end{aligned}$$

*Moreover, if* $t(\cdot)$ *is linear or convex, then* $\underline{y} = 0$.

For $y > \frac{1}{4}$, further differentiation (i) decreases cartel transportation costs, and (ii) increases the transportation costs associated with a deviation. Both effects go in the direction of facilitating collusion. Therefore, given any exogenously fixed pricing schedule, the critical discount factor must be minimal for some $\underline{y} \leq \frac{1}{4}$.

For higher degrees of differentiation (i.e. $y \leq \frac{1}{4}$), an increases in differentiation has two opposing effects. On the one hand, it increases the cartel's transportation costs. On the other hand, it renders deviations less attractive. The overall effects then depends on the transportation cost technology. If $t(\cdot)$ is convex, then the deviation effect always dominates the cartel transportation cost effect, since the marginal decrease of deviation profits exceeds the marginal increase of the cartel's transportation costs. If $t(\cdot)$ is concave instead, then the total effect of differentiation is ambiguous and $\underline{y}$ may exceed 0.

This analysis allows us to draw conclusion about the impact of differentiation on the discount factor threshold for perfect collusion, i.e. collusion on the monopoly price $v$ at all locations. Since the monopoly price is independent of $y$, the impact of a change in the firms' locations is then fully captured by the partial derivative $\frac{\partial \delta^*}{\partial y}(\cdot, y)$. Perfect collusion is therefore "easiest" for maximal differentiation in the case of convex or linear transportation costs. In the case of concave transportation costs, $\delta^*(v, y)$ attains its minimum somewhere in the range $[0, \frac{1}{4})$; the degree of differentiation for which collusion is easiest thus always exceeds the cost minimizing or "socially optimal" degree of differentiation $y = \frac{1}{4}$.

## 4.2 Optimal Cartel Pricing

We now characterize the price schedules that maximize collusive profits subject to being sustainable by optimal punishments, treating the degree of differentiation as given. This will permit us to add to our analysis the indirect impact via prices of differentiation on collusive sustainability. Denote by $\widetilde{p}(x; \delta, y)$ an optimal collusive price schedule, defined as follows

$$\begin{aligned} \widetilde{p}(x;\delta,y) \quad &\in \quad \arg \max_{p^C(x) \in [p^N(x), v]} R(p(x)) - T^C(y) \\ s.t. \qquad \delta \quad &\geq \quad \delta^*(p(x), y). \end{aligned} \tag{P1}$$

The delivered prices $\widetilde{p}(x; \delta, y)$ hence maximize joint profits under the constraint that collusion on these prices be sustainable by maximal punishments.

**Proposition 2** *The solution of the cartel profit maximization problem $(P1)$ has the following form:*

*(i) For $\delta \geq \delta^*(v, y)$, firms charge the monopoly price everywhere:*

$$\widetilde{p}(x; \delta, y) = v.$$

*(ii) For $\delta < \delta^*(v,y)$, any price schedule $\widetilde{p}(x;\delta,y)$ such that*

$$R(\widetilde{p}(x;\delta,y)) - T^C(y) = \frac{1-\delta}{1-2\delta}\left(T^D(y) - T^C(y)\right). \tag{8}$$

*is optimal.*

**Proof.** The unconstrained solution of $(P1)$ is $p(x) = v$, since charging the monopoly price $v$ everywhere maximizes revenues $R(p(x))$ and thus cartel profits (given that serving all consumers is optimal by assumption).

If $\delta < \delta^*(v,y)$, the unconstrained solution violates the sustainability condition. To characterize the constrained solutions in this case, we rewrite the cartel problem $(P1)$ as

$$\max_{p(x)} R^C(p(x)) \tag{9}$$

$$s.t.\ R^C(p(x)) \leq \frac{1-\delta}{1-2\delta}\left(T^D(y) - \frac{\delta}{1-\delta}T^C(y)\right). \tag{10}$$

Since $\delta^*(v,y) < \frac{1}{2}$ for any $y < \frac{1}{2}$, $\delta < \delta^*(v,y)$ implies that $\delta < \frac{1}{2}$. Thus, $1-2\delta > 0$, and $\frac{\delta}{1-\delta} < 1$. Since moreover $T^D(y) \geq T^C(y)$ for any $y \leq \frac{1}{2}$, the right-hand-side of (10) is positive. Hence, the sustainability condition (10) imposes an upper bound on cartel revenues. It is then obvious that for $\delta < \delta^*(v,y)$ any price schedule $p(x)$ such that (10) is binding is optimal, as it generates the highest possible sustainable profits. At any such solution, the cartel profits $R^C(p(x)) - T^C(y)$ are equal to $\frac{1-\delta}{1-2\delta}\left(T^D(y) - T^C(y)\right)$. ■

Hence, whenever perfect collusion is not feasible, cartel members have to set prices strictly below the monopoly price $v$ at some locations, so that (8) is satisfied. One possibility is that both firms use a base point system, where the base point is $\frac{1}{2}$, customers located sufficiently close to the centre pay according to some price schedule increasing in distance (e.g. standard mill prices), but all customers whose distance from the base point exceeds a certain threshold are charged the monopoly price $v$.

Taking into account both the effect of differentiation on transportation costs as well as its effect on optimal cartel prices, we can analyze how cartel profits depend on differentiation (the formal proof of the following proposition is relegated to the appendix):

**Proposition 3** *(i) If perfect collusion is sustainable (i.e. $\delta^*(v,y) < \delta$), then*

$$\frac{d\pi^C(\widetilde{p}(x;\delta,y),y)}{dy} \mid_{\delta^*(v,y)<\delta} > 0 \text{ if } y \in \left[0,\frac{1}{4}\right),$$

$$\frac{d\pi^C(\widetilde{p}(x;\delta,y),y)}{dy} \mid_{\delta^*(v,y)<\delta} = 0 \text{ for } y = \frac{1}{4},$$

$$\frac{d\pi^C(\widetilde{p}(x;\delta,y),y)}{dy} \mid_{\delta^*(v,y)<\delta} < 0 \text{ for } y \in \left(\frac{1}{4},\frac{1}{2}\right).$$

*(ii) If perfect collusion is not sustainable, then there exists a unique $\widehat{y} \in \left[0, \frac{1}{4}\right)$ such that*

$$\frac{d\pi^C(\widetilde{p}(x;\delta,y),y)}{dy} \mid_{\delta^*(v,y)>\delta} > 0 \text{ for } y \in [0,\widehat{y}),$$
$$\frac{d\pi^C(\widetilde{p}(x;\delta,y),y)}{dy} \mid_{\delta^*(v,y)>\delta} < 0 \text{ for } y \in \left(\widehat{y}, \frac{1}{2}\right).$$

*Moreover, if $t(\cdot)$ is linear or convex, then $\widehat{y} = 0$.*

To understand the first part of the lemma, consider some $y$ such that perfect collusion is sustainable at the prevailing discount factor, i.e. $\delta^*(v,y) < \delta$. Firms can then earn the same revenues for degrees of differentiation close to $y$, but whenever $y \neq \frac{1}{4}$ a small move in the direction that allows a transportation cost reduction leads to a profit increase. Obviously, if $\delta^*(v,y) < \delta$ for all $y \in [0, \frac{1}{2}]$, then cartel profits are highest for the locations $\left(\frac{1}{4}, \frac{3}{4}\right)$.

If perfect collusion is not sustainable at $y$, however, then a change in the degree of differentiation has an effect not only on transportation costs but also on the cartel's revenues via the sustainability condition. At $y = \frac{1}{4}$, for example, an increase of differentiation always increases constrained cartel profits: due to the deviation effect, the discount factor required to sustain any given price schedule decreases as $y$ is reduced marginally. The induced slack in the sustainability condition in turn allows firms to increase prices and earn higher collusive profits

In the case of convex transportation costs, this effect is so strong that constrained cartel profits are always increasing in the degree of differentiation. If transportation costs are concave, then, as shown before, there may exists some threshold $\underline{y} \in \left(0, \frac{1}{4}\right)$ such that the direct effect of differentiation on the critical discount factor is positive for $y$ below $\underline{y}$. We can compare this threshold $\underline{y}$ to the the threshold $\widehat{y}$ of part (ii) of lemma 2:

**Lemma 1** *$\underline{y}$ defined in lemma 1 and $\widehat{y}$ defined in lemma 2 compare as follows:*

*(a) If $\widehat{y} = 0$, then $\underline{y} = \widehat{y} = 0$.*

*(b) If $\widehat{y} > 0$, then $\underline{y} < \widehat{y}$.*

The formal proof of this lemma is relegated to the appendix, but the intuition for the result is straightforward. As $y$ approaches $\underline{y}$, the critical discount factor to sustain any given price schedule decreases, so that it becomes more likely that the cartel is able to sustain monopoly prices. As soon as perfect collusion is sustainable, however, the cartel members gain if the degree of differentiation is close to the social optimum $\frac{1}{4} > \underline{y}$. Therefore, $\underline{y} \leq \widehat{y}$.

## 4.3 The Cooperative Choice of Locations

We now use the previous results to study the optimal cooperative choice of locations by cartel members. In other words, we find the degree of differentiation for which cartel profits are highest.

This corresponds to a game in which firms (with equal bargaining power) cooperatively agree on locations that cannot be changed anymore thereafter, anticipating optimal collusion in the future.[11] Under the symmetry assumption, the cartel optimally chooses

$$y^* = \arg\max_y \pi^C\left(\widetilde{p}(x;\delta,y),y\right). \tag{P2}$$

Note that the simultaneous cooperative choice of (symmetric) locations would yield the same outcome as analyzing $(P1)$ and $(P2)$ separately. The solution of this problem follows from the previous results (the proof of the following proposition is relegated to the appendix):

**Proposition 4** *Let $\widehat{y}$ be as defined in proposition 2. Then, the solution of the cartel problem $(P2)$ is as follows:*

*(i) If $\delta \geq \delta^*(v,\frac{1}{4})$, then*

$$y^* = \frac{1}{4}.$$

*(ii) If $\delta^*(v,\widehat{y}) \leq \delta < \delta^*(v,\frac{1}{4})$, then there exists a unique solution $y^* \in [\widehat{y},\frac{1}{4})$ defined by the condition*

$$\delta^*(v,y^*) = \delta. \tag{11}$$

*(iii) If $\delta^*(v,\widehat{y}) > \delta$, then*

$$y^* = \widehat{y}.$$

If the cartel members are able to sustain the prices and the locations $\left(v,\frac{1}{4}\right)$ that a monopoly with two production sites would choose, then it will obviously be in their joint interest to do so. If $\delta < \delta^*(v,\frac{1}{4})$ however, then the firms have to either locate further apart, thus incurring higher transportation costs, or/and decrease the price charged to some consumers below the monopoly price $v$. The proposition show that as long as perfect collusion is still sustainable for some locations above $\widehat{y}$, it is optimal for the cartel to insist on monopoly prices but increase differentiation by as much as necessary to achieve such perfect collusion. Only if the critical discount factor is too low to sustain perfect collusion for $y = \widehat{y}$, then firms prefer to decrease prices rather than locate further apart. This result follows from the finding of lemma 2 that constrained cartel profits are decreasing with differentiation for $y < \widehat{y}$.

In the special case of linear or convex transportation costs, $\widehat{y} = 0$. The cooperative choice of prices and locations is then simple. As long as the cartel is able to sustain monopoly prices for some $y$, it will choose the $y$ closest to $\frac{1}{4}$ at which monopoly pricing is possible. If the discount factor is too low to sustain monopoly prices, then the firms will choose to locate as far apart as possible, and set prices so as to maximize profits subject to their agreement being self-enforcing.

[11] If firms could change locations costlessly at any time, then a deviation would always involve a relocation to the centre of the line. Our deviation effect wouldhence vanish in that case, and the analysis would boil down to minimizing the cartel's transportation costs.

## 5 Comparison with Grim Trigger Strategies

We now compare collusion sustained by Nash reversion trigger strategies with optimal collusion. Strikingly, the qualitative result with respect to the impact of differentiation on the sustainability are very different in the two situations. In particular, minimal differentiation may minimize the discount factor threshold for collusion sustained by standard trigger punishments. The reason is straightforward: since competitive profits are increasing with the degree of differentiation, the retaliation threat is less severe and collusion thus less stable if firms locate far away from each other. This *punishment effect* hence acts counter to the previously identified *deviation effect*, which predicts that differentiation facilitates collusion by lowering short-term deviation profits.

As also argued by Gupta and Venkatu (2002), if demand is inelastic, then spatial differentiation does not affect the scope for collusion sustained by Nash reversion. It is easy to show this. The competitive profits of firm 1 in our model are

$$\pi^N(y) = \int_0^{\frac{1}{2}} (t(1-y-x) - t(\mid x-y \mid))\, dx.$$

The deviation gain as defined previously is therefore equal to the difference between collusive and competitive profits:

$$\pi^D(p^C(x), y) = \pi^C(p^C(x), y) - \pi^N(y). \tag{12}$$

Collusion is sustainable by grim trigger strategies if and only if

$$\delta\pi^C(p^C(x), y) \geq (1-\delta)\pi^D(p^C(x), y) + \delta\pi^N(y)$$

The implied discount factor threshold is

$$\delta_N^*(p^C(x), y) \equiv \frac{\pi^D(p^C(x), y)}{\pi^D(p^C(x), y) + \pi^C(p^C(x), y) - \pi^N(y)}.$$

By the equality identified in (12), this threshold is independent of $y$:

$$\delta_N^*(p^C(x), y) = \frac{1}{2}.$$

In the case of elastic demand, Gupta and Venkatu (2002) consider a simple model with linear (inverse) demand of the form

$$p(q) = \alpha - q$$

for all $x$, and linear transportation costs

$$t(x, y) = \mid x - y \mid .$$

They show that the discount factor threshold for collusion on monopoly prices (which depend both on $x$ and on $y$ now) is decreasing in $y$, so that minimal differentiation makes collusion

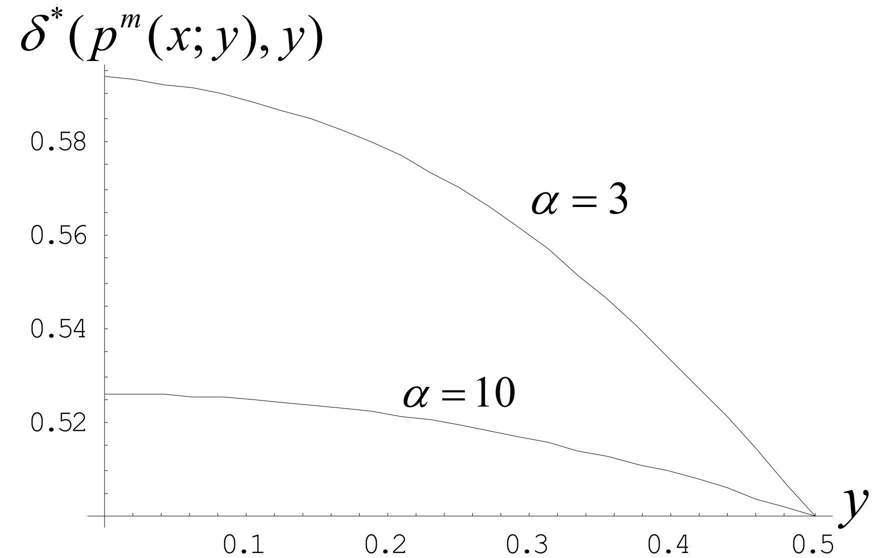


Figure 1: Critical discount factor for perfect collusion sustained by Nash reversion trigger punishments

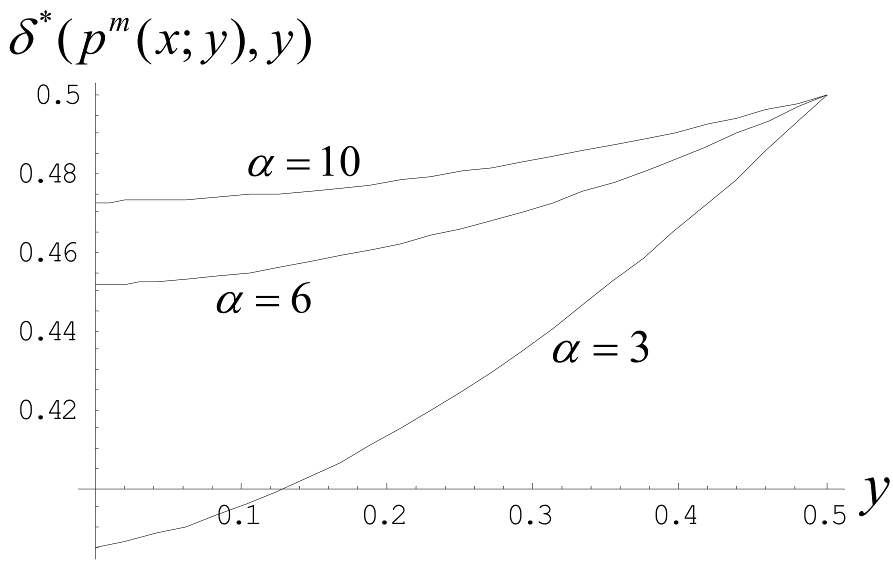


Figure 2: Critical discount factor for perfect collusion sustained by optimal punishments

easiest.[12] They also prove that, for given locations, collusion is easier to sustain the larger the "size of the market" $\alpha$. Figure 1 illustrates these results: (i) the discount factor thresholds for profit-maximizing collusion are decreasing as differentiation diminishes, and (ii) as the market size $\alpha$ increases, the thresholds for collusion are lower for any given degree of differentiation.

With optimal punishments, these results are reversed. In figure 2, we plot the discount factor thresholds for optimal punishments. Maximal differentiation now facilitates collusion because it minimizes deviation incentives. Moreover, increasing $\alpha$ now raises the critical discount factor.

The different results concerning the size of potential profits $\alpha$ are intuitive: as $\alpha$ increases, transportation costs become relatively less important with respect to revenues. Therefore, in both cases, the critical discount factor approaches $\frac{1}{2}$, the threshold if transportation were costless or firms were not differentiated at all. With grim trigger punishments this translates into a decrease of the discount factor threshold. With optimal punishments however, this "loss" of heterogeneity raises the critical discount factor for perfect collusion.

# 6 Concluding Remarks

We have analyzed the impact of horizontal differentiation on optimal collusion in a model where firms charge delivered prices to their customers. Our main contribution is to show that a high degree of differentiation stabilizes a cartel by rendering deviations less profitable. This result is in stark contrast with previous finding in the literature (see Gupta and Venkatu (2002)) that suggest that minimal differentiation facilitates collusion. This difference is due to the fact that we allow firms to "collude" on optimal punishment schemes, which yield minmax profits irrespectively of the degree of differentiation. Gupta and Venkatu, on the other hand, rely on reversion to Nash punishments, which are harsher the lower the degree of differentiation.

Our findings are in line with those obtained in models of mill pricing. There, differentiation facilitates collusion both if firms use grim trigger punishments (see Chang (1991) and Häckner (1995)) and if firms use optimal punishments (see Häckner (1996)).

Our result that some differentiation always facilitates collusion does not rely on firms using maximal punishments but rather on the fact that punishment profits do not decrease with differentiation. Differentiation would have similar effects on cartel sustainability if firms fixed a self-binding agreement which determines punishment profits proportional to collusive profits for example. In such a scheme, a deviation could be punished by a price reduction for a number of periods necessary to achieve the required profit loss, the punishment prices possibly remaining even above the competitive level.

Our results are easily extended to a model of product differentiation in a circular city à

[12] A sufficient condition for this result is $\alpha > 3$.

la Salop (1979). Maximal differentiation would always be optimal in that framework. First, maximal differentiation combined with a symmetric market sharing rule where each firm serves the consumers closest to its location allows firms to minimize transportation costs. Second, maximal differentiation also renders deviations the least profitable, as it maximizes the distance each firms has to travel to steal its rival's clients.

# 7 Appendix A: Optimal stick-and-carrot punishments

In this appendix, we construct credible stick-and-carrot punishments that yield minmax profits to all firms, and argue why these punishments are undominated. Suppose that some collusion is indeed sustainable by maximal punishments. Then consider the following punishment for firm $i$ ($\neq -i$), which is started whenever $i$ unilaterally deviates from either the collusive or a punishments path already in place:

- For the first $T$ periods following a deviation, both firms charge the price schedule

$$p^P(x) = \begin{cases} \varepsilon & \text{if } t(|x-y_i|) \leq t(|x-y_{-i}|) \\ t(|x-y_i|) & \text{if } t(|x-y_i|) > t(|x-y_{-i}|) \end{cases}$$

- After compliance with the first $T$ punishment periods, the firms return to the collusive outcome.

- $T$ and $\varepsilon \geq 0$ are such that firm $i$'s discounted profits along its punishment are zero, i.e. firm $i$ earns negative profits in the first periods but is rewarded for compliance by a return to collusive pricing.[13]

This punishment is clearly maximal, since the deviator is minmaxed by construction. It is also credible if and only if no firm has an incentive for a one-shot deviation from it at any stage. Given the collusive agreement is indeed sustainable with minmax punishments, this is definitely true for the "carrot" stages of the punishments. In the first "stick" period, firm $i$ has no (strict) incentive to deviate from its own punishment: a deviation to higher prices charged to those consumers buying from $i$ yields at most zero profits in the current period, but would relaunch the punishment of $i$ with continuation profits zero. Firm $i$ is thus indifferent between compliance with or deviating from the stick phase of its own punishment. In later "stick" periods, firm $i$ has a strict incentive to comply, as its discounted future profits are positive, whereas a deviation would at most yields zero continuation profits. Firm $-i$ strictly prefers compliance with $i$'s punishment even in the first period: there are no short term gains to be made from deviating as firm $i$'s profits are already as high as possible given firm $-i$'s prices, but a deviation would trigger firm $-i$'s own punishment and thus drastically reduce its future profits.[14]

To show that a punishment is not weakly dominated, it suffices that it is the best responses to some strategy of the rival firm. This is not difficult here. Since the punisher $-i$ strictly

[13] We allow for strictly positive punishment prices $\varepsilon > 0$ to avoid the integer problems that may arise for $\varepsilon = 0$. To make the stick phase of the punishment as short as possible, $\varepsilon$ should be chosen as close to zero as possible.

[14] Espinosa (1992) also considers maximal stick-and-carrot punishments, and shows that they are indeed subgame perfect. The main difference with our analysis is that her punishments are symmetric in the sense that both firms, the deviator and the punisher, get the worst continuation profits.

prefers compliance, we only need to show that firm $i$'s compliance in all periods, in particular the first one, is not dominated by any other strategy. To show this, simply consider a strategy of firm $-i$ that consists in undercutting firm $i$ on its "home" market during the stick phase of $i$'s punishment, but then returning to collusion if and only if firm $i$ previously complied. For firm $i$, compliance with its own punishment is a strictly dominant reply to this strategy of firm $-i$. At the "carrot" stages of the punishment, a similar reasoning can be used.

Note that the described stick-and-carrots punishments also have some other attractive features as compared to the base point pricing trigger strategies considered earlier. First, they are more "targeted" at the deviator. For both punishment mechanisms, the deviator is minmaxed, so that retaliation is optimal. However, the punisher receives competitive profits for ever with base point pricing; with the proposed stick-and-carrot punishment, it earns competitive profits for $T$ periods but collusive profits thereafter. Second, the stick-and-carrot punishments are no longer prone to renegotiation once the stick phase is over. Although punishments are never played in equilibrium, firms may want to agree on punishments with such characteristics in an explicit collusive agreement.

# 8 Appendix B: Proofs

**Proof of proposition 1:** The partial derivative of the critical discount factor $\delta^*(p^C(x;y),y)$ with respect to $y$ is

$$\frac{\partial\delta^*}{\partial y} = \frac{\left[R(\cdot)-T^C(y)\right]\left(-\frac{\partial T^D}{\partial y}(y)\right)-\left[R(\cdot)-T^D(y)\right]\left(-\frac{\partial T^C}{\partial y}\right)}{\left[2R(\cdot)-T^C(y)-T^D(y)\right]^2}. \tag{13}$$

The sign of this derivative is equal to the sign of its numerator. Whenever firms are indeed differentiated ($y<\frac{1}{2}$) so that $T^D(y)>T^C(y)$, collusive profits strictly exceed the positive profit gain from a deviation:

$$R(\cdot)-T^C(y)>R(\cdot)-T^D(y)>0. \tag{14}$$

This implies that if $\left(-\frac{\partial T^D}{\partial y}\right)\geq\left(-\frac{\partial T^C}{\partial y}\right)$, then $\frac{\partial\delta^*}{\partial y}>0$. The partial derivatives of transportation costs are

$$\begin{aligned}\frac{\partial T^C}{\partial y} &= \int_0^y t'(y-x)dx-\int_y^{\frac{1}{2}} t'(x-y)dx = t(y)-t\left(\frac{1}{2}-y\right). &(15)\\ \frac{\partial T^D}{\partial y} &= -\int_0^{\frac{1}{2}} t'(1-y-x)dx = t\left(1-y-\frac{1}{2}\right)-t(1-y). &(16)\end{aligned}$$

Note that the transportation costs associated with a stealing the rival's customers always decrease as firms locate closer to each other:

$$\frac{\partial T^D}{\partial y}<0. \tag{17}$$

Let us compare $\left(-\frac{\partial T^D}{\partial y}\right)$ with $\left(-\frac{\partial T^C}{\partial y}\right)$ now:

$$\begin{aligned}
-\frac{\partial T^D}{\partial y} &\geq -\frac{\partial T^C}{\partial y} && (18)\\
&\leftrightarrow && (19)\\
\frac{1}{2}\left[t(1-y)+t(y)\right] &\geq t\left(\frac{1}{2}-y\right). && (20)
\end{aligned}$$

Recall that $t'(\cdot)$ is assumed to be monotone, so that transportation costs are either convex or concave everywhere. First (weakly) convex transportation costs. Weak convexity of $T(\cdot)$ implies that

$$\frac{1}{2}t(1-y)+\frac{1}{2}t(y) \geq t\left(\frac{1}{2}\right). \tag{21}$$

Since transportation costs are increasing with distance, $t\left(\frac{1}{2}\right) > t\left(\frac{1}{2}-y\right)$. Hence, (21) implies that (20) is satisfied, which, given (14), implies that $\frac{\partial\delta^*}{\partial y} > 0$ for any $y$.

Now consider strictly concave transportation costs. In this case, it is easy to see that $\frac{\partial\delta^*}{\partial y} > 0$ for $y \geq \frac{1}{4}$, since $\frac{\partial T^C}{\partial y} > 0$ in that case. To find out more in case $y < \frac{1}{4}$, we examine the curvature of the critical discount factor given by

$$\delta^* = \frac{R(p^C(x)) - T^D(y)}{2R(p^C(x)) - T^D(y) - T^C(y)}.$$

The second derivative of the deviation profits in the numerator of $\delta^*$ is:

$$\frac{\partial^2\left[R(p^C(x)) - T^D(y)\right]}{\partial y^2} = -\frac{\partial^2 T^D}{\partial y^2} = -\left(t'(1-y) - t'(\frac{1}{2}-y)\right)$$

If $t(\cdot)$ is strictly concave, then $t'(1-y) < t'(\frac{1}{2}-y)$ since $1-y > \frac{1}{2}-y$, which implies that the deviation profits $\left[R(p^C(x)) - T^D(y)\right]$ are convex in $y$. We now examine the curvature of the denominator of $\delta^*$:

$$\begin{aligned}
\frac{\partial^2\left[2R(p^C(x)) - T^D(y) - T^C(y)\right]}{\partial y^2} &= -\frac{\partial^2 T^D}{\partial y^2} - \frac{\partial^2 T^C}{\partial y^2}\\
&= -\left(t'(1-y) - t'(\frac{1}{2}-y)\right) - \left(t'(y) + t'(\frac{1}{2}-y)\right)\\
&= -t'(1-y) - t'(y) < 0.
\end{aligned}$$

Hence, the denominator of $\delta^*$ is (strictly) concave, while the numerator is (strictly) convex. This implies that, if $t(\cdot)$ is concave, then $\delta^*$ is (strictly) convex in $y$. We also know that $\delta^*(p^C(x), y) = \frac{1}{2}$ for $y = \frac{1}{2}$, and decreasing in the range $\left[\frac{1}{4}, \frac{1}{2}\right]$. Hence, the problem $\min_y \delta^*(\cdot, y)$ either has a unique interior solution somewhere in the interval $\left(0, \frac{1}{4}\right)$, or it has a corner solution at $y = 0$. We will denote the solution by $\underline{y}$. The critical discount factor is then decreasing in $y$ for $y < \underline{y}$ (if such $y$ exist), but increasing in $y$ for $y > \underline{y}$. Moreover, if $\underline{y} > 0$, then $\frac{\partial\delta^*}{\partial y} \mid_{y=\underline{y}} = 0$.

Q.E.D.

**Proof of proposition 3:** Suppose the degree of differentiation $y$ and the discount factor $\delta$ are such that $\delta > \delta^*(v, y)$. Then perfect collusion on the monopoly price $v$ is sustainable. Moreover, the effect of a marginal change in the degree of differentiation on revenues is zero: since the critical discount factor $\delta^*(\cdot, \cdot)$ is smooth in the degree of differentiation, the sustainability condition for perfect collusion is satisfied in the neighborhood of $\widehat{y}$. Hence, for any $y$ such that $\delta > \delta^*(v, y)$

$$\frac{dR}{dy}(\widetilde{p}(x; \delta, y)) \mid_{\delta^*(v,y)<\delta} = \frac{dR}{dy}(v) = 0.$$

The marginal effect of differentiation on cartel profits then only depends on its effect on aggregate transportation costs:

$$\frac{d\pi^C(\widetilde{p}(x; \delta, y), y)}{dy} \mid_{\delta^*(v,y)<\delta} = 0 - \frac{\partial T^C}{\partial y}(y).$$

Since $T^c(y)$ is decreasing for $y < \frac{1}{4}$, i.e. for "socially" excessive differentiation, but increasing for $y > \frac{1}{4}$,

Now consider $y$ and $\delta$ such that $\delta < \delta^*(v, y)$. In this case, $\widetilde{p}(x, \delta, y) \neq v$ for at least some $x$, and the highest sustainable per-firm profits are

$$\pi^C(\widetilde{p}(x; \delta, y), y) = R(\widetilde{p}(x; \delta, y)) - T^C(y) = \frac{1-\delta}{1-2\delta}\left(T^D(y) - T^C(y)\right).$$

The derivative of profits with respect to $y$ is then

$$\frac{d\pi^C(\widetilde{p}(x; \delta, y), y)}{dy} \mid_{\delta^*(v,y)>\delta} = \frac{dR}{dy}(\widetilde{p}(x; \delta, y)) \mid_{\delta^*(v,y)>\delta} - \frac{\partial T^C}{\partial y} = \frac{1-\delta}{1-2\delta}\left(\frac{\partial T^D}{\partial y} - \frac{\partial T^C}{\partial y}\right). \quad (22)$$

Since $\delta^*(v, y) < \frac{1}{2}$ for all $y < \frac{1}{2}$, we can infer that $\delta < \frac{1}{2}$ whenever $\delta < \delta^*(v, y)$. Therefore, $\frac{1-\delta}{1-2\delta} > 0$ so that the sign of the derivative in (22) are determined by the sign of the differences in brackets. Hence, whenever $\frac{\partial T^D}{\partial y} - \frac{\partial T^C}{\partial y} < 0$, which is equivalent to

$$\left(-\frac{\partial T^C}{\partial y}\right) < \left(-\frac{\partial T^D}{\partial y}\right), \quad (23)$$

then

$$\frac{d\pi^C(\widetilde{p}(x; \delta, y), y)}{dy} \mid_{\delta^*(v,y)>\delta} < 0. \quad (24)$$

In the proof of the previous lemma, we have already analyzed condition (23), and found that for (weakly) convex transportation costs $t(\cdot)$, (23) is satisfied for any $y \in [0, \frac{1}{2})$, which proves part (iia) of the lemma. If transportation costs $t(\cdot)$ are strictly concave, then we already know that $\frac{\partial T^D}{\partial y} - \frac{\partial T^C}{\partial y} < 0$ for any $y \in [\frac{1}{4}, \frac{1}{2})$. Moreover,

$$\frac{\partial^2\left(T^D(y) - T^C(y)\right)}{\partial y^2} = t'(1-y) - t'(y) - 2t'\left(\frac{1}{2} - y\right) \quad (25)$$

If $t(\cdot)$ is strictly concave, then $t'(1-y) < t'(y)$ since $1-y > y$. Since transportation costs are increasing, this implies that

$$\frac{\partial^2\left(T^D(y) - T^C(y)\right)}{\partial y^2} < 0,$$

so that the (positive) difference $\left(T^D(y) - T^C(y)\right)$ is concave in $y$. Hence, either $\max_y \frac{1-\delta}{1-2\delta}\left(T^D(y) - T^C(y)\right)$ has a unique interior solution in the interval $\left(0, \frac{1}{4}\right)$, or it has a corner solution at 0. We will denote the solution by $\widehat{y}$. Concavity then implies that the (constrained) cartel profits are increasing in $y$ if $y < \widehat{y}$, but decreasing if $y > \widehat{y}$.

Q.E.D.

**Proof of lemma 1:** In the case of linear or convex transportation costs, it is always the case that $\underline{y} = \widehat{y} = 0$. We hence focus on concave transportation costs here.

$\underline{y}$ minimizes the critical discount factor $\delta^*(y)$. From expression (13) in the proof of lemma 1, it is clear that

$$sign\left(\frac{\partial \delta^*}{\partial y}(y)\right) = sign\left(\left[R(\cdot) - T^C(y)\right]\left(-\frac{\partial T^D}{\partial y}(y)\right) - \left[R(\cdot) - T^D(y)\right]\left(-\frac{\partial T^C}{\partial y}(y)\right)\right). \quad (26)$$

Rearranging terms on the right-hand side yields

$$sign\left(\frac{\partial \delta^*}{\partial y}(y)\right) = sign\left(-\left[R(\cdot) - T^C(y)\right]\left(\frac{\partial T^D}{\partial y}(y) - \frac{\partial T^C}{\partial y}(y)\right) + \left[T^D(y) - T^C(y)\right]\left(-\frac{\partial T^C}{\partial y}(y)\right)\right). \quad (27)$$

Since $T^D(y) > T^C(y)$, and $\frac{\partial T^C}{\partial y} < 0$ for $y < \frac{1}{4}$,

$$\left[T^D(y) - T^C(y)\right]\left(-\frac{\partial T^C}{\partial y}\right) > 0 \text{ for } y < \frac{1}{4}. \quad (28)$$

As argued in the proof of lemma 3,

$$sign\left(\frac{d\pi^C(\widetilde{p}(x;\delta,y),y)}{dy}\,|_{\delta^*(v,y)>\delta}\right) = sign\left(\frac{\partial T^D}{\partial y}(y) - \frac{\partial T^C}{\partial y}(y)\right).$$

Since $\widehat{y}$ maximizes $\pi^C(\widetilde{p}(x;\delta,y),y)$ if $\delta^*(v,y) > \delta$, and the concavity of $t(\cdot)$ implies that the objective function of this problem is concave in $y$,[15] it must be that

$$\frac{\partial T^D}{\partial y}(\widehat{y}) - \frac{\partial T^C}{\partial y}(\widehat{y}) \leq 0.$$

Jointly with (28), this implies that

$$\frac{\partial \delta^*}{\partial y}(\widehat{y}) > 0.$$

Hence, at $\widehat{y}$ decreasing $y$ lowers the critical discount factor. This implies that $\underline{y} < \widehat{y}$ whenever $\widehat{y} > 0$. If $\widehat{y} = 0$, then the discount factor minimization problem has a corner solution at $\underline{y} = 0$.

Q.E.D.

[15]See the proof of lemma 2.

**Proof of Proposition 4:** By proposition 1, $\frac{d\pi^C(\widetilde{p}(x,\delta,y),y)}{dy} < 0$ for $y \in \left(\frac{1}{4},\frac{1}{2}\right)$ no matter whether the cartel is able to earn monopoly profits or not, and independently of the curvature of $t(\cdot)$. Hence, it is always optimal for the cartel to choose locations at least as distant as socially optimal, i.e.

$$y^* \leq \frac{1}{4}.$$

We now consider the three different subcases spelled out in the proposition.

(i) $\delta^*(v,\frac{1}{4}) \leq \delta$

Proposition 1 implies that in this case $\delta > \delta^*(v,y)$ for any $y \in \left[\underline{y},\frac{1}{4}\right)$. By proposition 2(i), it is then true that $\frac{d\pi^C(\widetilde{p}(x;\delta,y),y)}{dy} > 0$ for $y \in \left[\underline{y},\frac{1}{4}\right)$. Moreover, since $\underline{y} \leq \widehat{y}$, proposition 2 also implies that $\frac{d\pi^C(\widetilde{p}(x;\delta,y),y)}{dy} > 0$ for $y < \underline{y}$, no matter whether $\delta > \delta^*(v,y)$ or not. The optimal location is therefore $y^* = \frac{1}{4}$.

(ii) $\delta^*(v,\widehat{y}) \leq \delta < \delta^*(v,\frac{1}{4})$

In this case, since $\delta^*(v,y)$ is strictly increasing in the range $\left[\underline{y},\frac{1}{4}\right)$ by proposition 1, and $\underline{y} \leq \widehat{y}$, there exists a unique $\widetilde{y} \in [\widehat{y},\frac{1}{4})$ such that

$$\delta = \delta^*(v,\widetilde{y}).$$

For $\widetilde{y} < y < \frac{1}{4}$, $\delta < \delta^*(v,\widetilde{y})$. Since $\widehat{y} \leq \widetilde{y}$, proposition 2(ii) implies that in this range $\frac{d\pi^C(\widetilde{p}(x,\delta,y),y)}{dy} < 0$. For $\underline{y} \leq y < \widetilde{y}$, on the other hand, $\delta > \delta^*(v,\widetilde{y})$. By proposition 2(i), $\frac{d\pi^C(\widetilde{p}(x,\delta,y),y)}{dy} > 0$ in this range. Finally, for $y < \underline{y}$, the fact that $\underline{y} \leq \widehat{y}$ implies that $\frac{d\pi^C(\widetilde{p}(x,\delta,y),y)}{dy} > 0$ no matter whether $\delta > \delta^*(v,y)$ or not. Hence cartel profits are increasing in $y$ up to the point where $y = \widetilde{y}$, and decreasing thereafter. We can conclude that $y^* = \widetilde{y}$.

(iii) $\delta^*(v,\widehat{y}) > \delta$

By proposition 1, the fact that $\widehat{y} \geq \underline{y}$ implies that $\delta^*(v,y) > \delta$ for $y > \widehat{y}$ here. Hence, by proposition 2(ii), $\frac{d\pi^C(\widetilde{p}(x,\delta,y),y)}{dy} < 0$ for $y \geq \widehat{y}$. For $y < \widehat{y}$, proposition 2 moreover implies that $\frac{d\pi^C(\widetilde{p}(x,\delta,y),y)}{dy} > 0$, no matter whether $\delta^*(v,y) > \delta$ or not. The cartel profits hence reach their maximum at $y^* = \widehat{y}$.

Q.E.D.

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