INSTITUT NATIONAL DE LA STATISTIQUE ET DES ETUDES ECONOMIQUES
Série des Documents de Travail du CREST
(Centre de Recherche en Economie et Statistique)

n° 2005-16

Decomposing Volume for VWAP Strategies

J. BIALKOWSKI[1]
S. DAROLLES[2]
G. LE FOL[3]

[1] Auckland University of Technology, New Zealand.
[2] CREST and Société Générale Asset Management AI, Paris.
[3] Angers University and CREST, Paris.
Financial support from the Europlace Institute of Finance is gratefully acknowledged.

Decomposing Volume for VWAP Strategies

Jędrzej Białkowski[*], Serge Darolles[**] et Gaëlle Le Fol[***]

[*] Auckland University of Technology, New Zealand.
[**] CREST and Société Générale Asset Management AI, Paris.
[***] Angers University and CREST, Paris.
Financial support from the Europlace Institute of Finance is gratfully acknowledged.

Abstract

In this paper, we present a new methodology for modeling intraday volume which allows for a significant reduction in the Volume Weighted Average Price (VWAP) on orders risk. The results are obtained for the all stocks included in the CAC40 index at the beginning of September 2004. The idea of considered models is based on the decomposition of traded volume into two parts: one reflects volume changes due to market evolutions, the second one describes the stock specific volume pattern. The dynamics of the specific part of volume is depicted by ARMA, and SETAR models.

Keywords : Intraday volume, VWAP strategies, Principal Component Analysis, Arbitrage

Résumé

Dans cet article, nous proposons une nouvelle méthode de modélisation du volume intra-journalier qui permet de réduire de façon significative du risque des ordres VWAP (Volume Weighted Average Price). L'application de cette méthode est effectuée sur tous les titres qui composent l'indice CAC40 au début de septembre 2004. L'idée des modèles considérés est basée sur la décomposition du volume en deux parties : la première traduit les variations de volume dues aux évolutions de marché, la seconde donne la forme intra-journalière du volume spécifique. La dynamique de la composante spécifique du volume est décrite par des modèles ARMA et SETAR.

Mots-Clés : Volume Intra-journalier, Stratégies VWAP, Analyse en Composantes Principales, Arbitrage

1 Introduction

While volume is often ignored in perfect markets financial literature, it is an important market characteristic for practitioners who aim to lower the market impact of their trades. This impact can be measured comparing the execution price of an order to a benchmark. The larger this price difference the higher the market impact. One of such benchmarks is known as Volume Weighted Average Price, or VWAP, trading. Informally, the VWAP of a stock, over a period of time, is just the average price paid per share, during that period. The VWAP benchmark is then the sum of every transaction price paid, weighted by its volume. The goal of any trader, tracking VWAP benchmark, is to find and define ex ante strategies, which ex post lead to an average trading price as close as possible to the VWAP price. Hence, VWAP strategies are defined as buying and selling a fix number of shares at an average price that tracks the VWAP.

VWAP execution orders represent around 50% of all the institutional investors' trading. The simplicity of such strategies explains its growing success. First, investors who ask for VWAP execution accept to postpone or to sequence their trades in order to reduce their trading cost when selling or buying large amounts of shares. Doing so, they reduce their market impact, and thus increase the profitability of their transactions by accepting a risk in time. Likewise, VWAP orders allows foreign investors to avoid the high risk related to the fact that their orders have to be placed before the market opening. Second, it is a common practice to evaluate the performance of trades based on their ability to execute the orders at a price better or equal to VWAP. In this case, VWAP can be seen as an optimal benchmark[1]. Finally, VWAP is a better benchmark than any price at a fixed time in the future as it cannot be manipulated. Consequently, it improves both market transparency and efficiency [see Cuching and Madhavan (2001)].

To implement VWAP strategies, we need to model the intraday evolution of the relative volume. It is now common knowledge that intraday volume move around a U-shape seasonal pattern [see for example Biais, al. (1995), Gouriéroux et al. (1999) for the French

[1]Berkowitz, Logue and Noser (1988) show that VWAP is a good proxy for the optimal price attainable by passive traders.

stock market.]. These high seasonalities have hampered volume modeling. One way to circumvent this problem is to work on a transaction or market time scale instead of calendar time scale [see Engle (2000), Gouriéroux and Le Fol (1998) for example]. However, this transformation are useless when working on strategies which are definitly defined on a calendar time scale [Le Fol and Mercier (1998) suppose that the time transformation is fixed and use this hypothesis to pass from one time scale to the other]. Other approaches correct volume by a stock by stock time varying average volume [Engle (1998), Easley, O'Hara (1987)], while others take the time varying, across stock, average volume [See Mc Culloch (2004)]. In all this work, seasonality is just a problem that they adequately and empirically dispose. However, we do not want to eliminate seasonality as we use it to get the common component and thus, to construct our volume benchmark for VWAP strategies. Here, we want to discriminate between the seasonal and static part of volume from the dynamic one. The identification of such components of volume comes from the observation that seasonality is common across stocks whereas dynamics is a stock by stock feature.

If volume has been analyzed in the financial market literature, it has often been used for a better understanding of other financial variables, like price [Easley, O'Hara (1987), Foster, Wisvanathan (1990) for example] or volatility [Tauchen, Pitts (1983), Karpoff (1987), Anderson (1996) and Manganelli (2002) for example]. Moreover most of these studies use daily or even lower data frequency [one exception is Darrat, Rahman, and Zhong (2003) who examine intraday data of stocks from Dow Jones index, and reported significant lead-lag relations between volume and volatility]. The rare papers that concentrate on volume are Kaastra and Boyd (1995), Darolles and Le Fol (2003).

If this paper is in the line of the methodology proposed by Darolles and Le Fol (2003) for the volume decomposition, the main contributions are first to work on intraday data, second to propose some predictions of volume and finally to use VWAP strategies to test the accuracy of the approach. Basically, volume is decomposed into two components : the first one describes the size of volume on ordinary days and is extracted from the forty stocks included in the CAC40 index. The second component measures the abnormal or unexpected changes of volume.

The CAPM is one of the most famous model for returns that is based on such technics. Lo and Wang (2000) are the first to transpose this model to volumes, also used by Darolles and Le Fol (2003). This study is a natural extension of this work on high frequency data to the problem of optimal executions of VWAP orders. Furthermore, it is worth highlighting that, by separating the market part from observed volume two additional goals were obtained. First, the specific component, as measure of liquidity for particular company, is a much more reliable indicator of arbitrage activity than the observed volume. Finally, this decomposition allows to accurately remove seasonality, without imposing any particular form.

The paper is organized as follows. Section 2 provides a description of the models for market component, and specific component of volume. Section 3 contains data description and summary statistics of the data. As well as in and out sample estimations results. Applications to VWAP strategies are presented in section 4. Finally, section 5 concludes the paper.

2 The *price*-volume trading model

In this section, we introduce the *price*-volume statistical model which include the decomposition of volume method and the intraday volume dynamics. As mentioned before, the major problem of intraday volume is its high intraday seasonality. Two approaches have been considered to deal with this problem. The first one takes an historical average of volume for any stock as its seasonal pattern or normal volume [Easley, O'Hara (1987)]. The second one takes the average volume across stocks to get this normal volume [McCulloch (2004)]. Here, we propose another method to extract the seasonal, or normal, volume based on principal component analysis. Such a method allows us to get a normal non stationary volume component, which is common across stocks, and a specific stationary component. Next, we propose to model the dynamics of the aforementioned components taken separately.

2.1 Intraday volume decomposition section

The chosen methodology comes from asset management practices, where any portfolio can be decomposed into a market and an arbitrage portfolios. Similar idea can be applied to intraday volume: the trading volume has a market and a specific components [Darolles and Le Fol (2003) propose a theoretical model to explain such a decomposition of volume as well as a link with market practices]. Any stock volume or stock turnover, at any date, depends on an average term and a deviation term. The average part corresponds to trading volume coming from market portfolio adjustments. Our interpretation is that the deviation part is due to the opening and closure of arbitrage positions. In order to get the two components of volume, we conduct a principal component analysis.

Let $x_{it} = \frac{V_{it}}{N_i}$, $i = 1, \ldots, I$, $t = 1, \ldots, T$ denote the turnover series for stock i at date t, i.e. the number of traded shares V_{it} divided by the number of float shares N_{it}. As shown in Darolles, Le Fol (2003), the market turnover x_t^I can be written as :

$$x_t^I = \frac{\sum_i P_{it} V_{it}}{\sum_k P_{kt} N_k} = \frac{\sum_i P_{it} N_i \frac{V_{in}}{N_i}}{\sum_k P_{kt} N_k} = \sum_i w_{it} x_{it}, \tag{1}$$

where P_{it} is the transaction price for stock i at date t, and w_{it} is the stock relative capitalization. In fact, all the series should also be indexed by day. It would become x_{it}^j, denoting the turnover for stock i and date t, and day j. However, we will ignore this last index, unless explicitly needed, for ease of the exhibition. Since the aim of principal component analysis is to explain the variance-covariance structure of the data through a few linear combinations of the original data, the first step is to calculate the $I \times I$ dimension variance-covariance matrix of the data. The spectral decomposition of this matrix leads to I orthogonal vectors, $C_t^k = x_{it}' u_k$, with dimension T, where u_k is the k^{th} eigenvector. Each eigenvector is associated with a positive eigenvalue λ_k such that:

$$Cov(C_t^k, C_t^l) = \lambda_k \delta_{kl}, \tag{2}$$

where δ_{kl} stands for Kroneker symbol. The standardized turnover times series can be de-

composed as:

$$\frac{x_{it} - \overline{x}_i}{\sigma_i} = \sum_k u_k^i C_t^k.$$

Since the correlation is $corr(x_{it}, C_t^k) = \sqrt{\lambda_k} u_k^i$, the previous equation can be rewritten as:

$$\begin{aligned} x_{it} - \overline{x}_i &= \sigma_i \sum_k \frac{corr(x_{it}, C_t^k)}{\sqrt{\lambda_k}} C_t^k, \\ &= \sigma_i \sum_k \frac{corr(x_{it}, C_t^k)}{\sqrt{var\left(C_t^k\right)}} C_t^k, \\ &= \sum_k \frac{Cov(x_{it}, C_t^k)}{var\left(C_t^k\right)} C_t^k. \end{aligned}$$

Finally, we get the centered turnovers :

$$\begin{aligned} x_{it} - \overline{x}_i &= \sum_k \frac{Cov(x_{it}, C_t^k)}{var\left(C_t^k\right)} C_t^k, & (3) \\ &= \sum_k \frac{1}{\lambda_k} Cov(x_{it}, C_t^k) C_t^k. & (4) \end{aligned}$$

Isolating the first factor, we get:

$$x_{it} - \overline{x}_i = \frac{1}{\lambda_1} Cov(x_{it}, C_t^1) C_t^1 + \sum_{k>1} \frac{1}{\lambda_k} Cov(x_{it}, C_t^k) C_t^k. \quad (5)$$

The first component is larger variance one and captures the seasonality. The others are stationary. In the following, we use this decomposition to predict future volume.

From equation (5), we get :

$$x_{it} = c_{i,t} + y_{i,t},$$

where

$$\begin{aligned} c_{i,t} &= \overline{x}_i + \frac{1}{\lambda_1} Cov(x_{it}, C_t^1) C_t^1, \\ y_{i,t} &= \sum_{k>1} \frac{1}{\lambda_k} Cov(x_{it}, C_t^k) C_t^k. \end{aligned}$$

The volume turnover $x_{i,t}$ at time t, is the sum of a common - or market - turnover $c_{i,t}$

and a specific turnover $y_{i,t}$. On the one hand, the market component of intraday volume is expected to capture all volume seasonalities and represents long term volume of the stock. On the other one, the specific component should feature no seasonal pattern and represents short term volume of the stock. It depends on the inflow of information about important events for the company's shareholders.

2.2 Intraday volume dynamics

In order to incorporate the features mentioned above into the model for intraday volume $x_{i,t}$ where $t = 1, \cdots, T$, we proposed the following framework:

$$\hat{x}_{i,t} = \bar{c}_{i,t} + y_{i,t}, \tag{6}$$

$\bar{c}_{i,t}$ represents the common component historical average of intraday volume over the last L-trading days. As said above, $c_{i,t}$ depends on the trading day and should be written as $c^j_{i,t}$ for day j. Hence, $\bar{c}_{i,t}$ is equal to:

$$\bar{c}_{i,t} = \frac{1}{L} \sum_{l=1}^{L} c^{j-l}_{i,t}. \tag{7}$$

This modeling choice seems accurate as the common component for short period (no longer than 3 months) is assumed to be static. Note that, in our empirical study, the size of the interval is equal to 20 minutes. The second term $y_{i,t}$ represents intraday specific volume for each equity and is modeled considering two specifications. The first on is an $ARMA(1,1)$ with white noise, defined as:

$$y_{t,i} = \psi_1 y_{t-1,i} + \psi_2 + \epsilon_{t,i}. \tag{8}$$

The alternative considered specification is a SETAR (self-extracting threshold autoregressive model) which allows for changes in regime in the dynamics. We get :

$$y_{t,i} = \big(\phi_{11} y_{t-1,i} + \phi_{12}\big)\mathbf{I}(y_{t-1,i}) + \big(\phi_{21} y_{t-1,i} + \phi_{22}\big)\big[1 - \mathbf{I}(y_{t-1,i})\big] + \epsilon_{t,i}. \tag{9}$$

where

$$\mathbf{I}(x) = \begin{cases} 1 & x \leq \tau, \\ 0 & x > \tau. \end{cases}$$

Therefore, we assume that when the specific part of intraday volume exceeds a threshold value of τ its dynamics is described by a different set of parameters.

In contrast to the above framework, the classical approach take the simple volume average over past l-trading days. Hence, intraday volume $x_{i,t}$ is approximated by:

$$\hat{x}_{i,t} = \frac{1}{L} \sum_{l=1}^{L} x_{i,t}^{j-l}. \tag{10}$$

The parameter n corresponds to the number of 20 minutes intervals in a one trading day. Undoubtedly, the advantage of classical approach is its simplicity. However, it ignores dynamics of intraday volume, what has a negative impact on quality of volume forecast.

3 Empirical analysis

3.1 The data

The empirical results are based on the analysis of the 40 securities included in CAC40 index at the beginning of September 2004. We use the turnover as a measure of (relative) volume. The turnover is defined as the traded volume divided by the outstanding number of shares. A similar measure was used by Lo and Wang (2000). Tick-by-tick volume and prices were obtained from the Euronext historical data base. We consider a one year sample, ranging from the beginning of September 2003 to the end of August 2004. The data are adjusted for stock's splits and dividends. The 24 and 31 of December 2003 were excluded from the sample. For any 20 minutes interval, volume is the sum of the traded volumes whereas the price is the average price, both over that period.

This study focuses on modeling volume during the day with continuous trading, therefore we consider transaction between $9:20$ and $17:20$, and exclude pre-opening trades. As the result, there are 25 20-minutes intervals per day. In addition to high-frequency data

from EURONEXT, volume weighted average prices with daily horizon for each company were used.

We give in table 1, intraday volume summary statistics for securities from the CAC40 index. The comparison of the mean with the 5% and 95% quintiles, gives clear indications of the large dispersion of volume stock by stock. For companies like Sodexho Alliance, Sanofi-Aventis, and Credit Agricole, the mean is around three times lower than the 95%-quantile. On average this ratio is equal to 2.7. In turn, 5%-quantiles are five to nine times smaller than the mean. This strong dispersion comes from the strong intraday seasonality. It is worth noting that the table also shows large dispersion across equities, where the average volume is ranging from 0.006 for Dexia up to 0.438 for Cap Gemini. The explanation comes from equities particular events such as earning announcements, dividend payments, changes in management board etc., which have direct influence on the price and volume of their stock. These observations encourage to apply a model as the one we propose, which is based on volume decomposition into market and specific components.

3.2 Estimation results

The first step of our methodology is to run a principal component analysis (PCA) on the intraday volumes for all companies included in CAC40. Table 2 shows that the longer the period the lower the dispersion explained by the first three components. For a one month period these components explain 48.5% of the dispersion. It falls to 35.6% when we extend the decomposition to a one year period. Since principal component analysis is a static method, it has to be applied to small periods of time. Over long periods PCA fails to capture the dynamical links which prevail. Therefore, we choose to work on a one month period to decompose volume. Next, we calculate the autocorrelation (ACF) and partial autocorrelation functions (PACF) for common and specific part which are plotted in Figure 1 for TOTAL equity. The upper graphs in the Figure show typical characteristics of the intraday volume, namely seasonality. From the middle figures, one recognizes the ability of common component to capture seasonality. The last graphs illustrate ACF and PACF for the specific part of volume. The fast decay of the autocorrelation suggests that the

ARMA type model is suitable to depict this time series. The results of stationarity tests are presented in Table 3. The null hypothesis of unit root is rejected by the Augmented Dickey-Fuller and Philips-Perron tests, for the specific volume. Finally, the inspection of residuals confirmed that ARMA and SETAR models are accurate to describe the dynamics of the specific volume. Figure 2 shows classical white noise properties. The conclusions drawn from these autocorrelation functions plots are confirmed by the results of Portmanteau tests.

Figure 3 shows the result of our decomposition for two succeeding days, for TOTAL company. The upper graphs give the intraday evolution of volume where we can see a stochastic evolution around a seasonal U-shape pattern. The middle graphs give the intraday evolution of the common component. This part of the volume is the same for any day of the sample. Finally, the lower graph represent the evolution of the specific component. This component is responsible for the stochastic behavior around the seasonal pattern and changes from day to day.

The final stage to evaluate the accuracy of the models is to use two error measures, such as the mean absolute percentage error (MAPE) and mean squared percentage error (MSPE) for daily horizon. Tables from 4 to 9 show the outcome of this analysis for all equities, for the classical approach, ARMA and SETAR model respectively. The results reported in the tables are obtained calculating the MAPE and MSPE for each day. Note that, the statistics has been computed over all trading days for the period from September, 2 to December 16, 2003. The summary for all examined companies is given in Table 10. The outcomes indicates that both models based on principal component decomposition outperform the classical approach to predict the daily U-shape of volume. Moreover, the SETAR model fits better the daily volume dynamics then the ARMA model. In fact, there are three over thirty nine companies for which ARMA slightly surpasses SETAR model. Further arguments in favor of the decomposition concept comes from the fact that the standard deviation for both models is significantly smaller than the one observed in the classical approach. The same applies to the maximum, and the 95%-quantile, what confirms the dominance of our approach.

To summarize, we have demonstrated that models based on decomposition are better in

modeling intraday volume than those assuming calculation simple averages from historical data. The importance of this outcome will be discussed in the next section which focus on the problem of reducing cost of VWAP orders.

4 Application to VWAP strategies

VWAP analysis works best under particular conditions. As we explain in the following section, VWAP analysis may be misleading and self-fulfilling under every day institutional trading conditions such, as rapidly changing market conditions, trades motivated by current news and recommendations, trade dominating daily volume, principal trades, and trades whose execution stretches out over several days. We detail below and argue, the set of assumptions ensuring the accuracy of VWAP benchmarking.

4.1 VWAP strategies : an overview

Trends in algorithmic trading An actual trend observed in financial markets is the increasing use of computer trading, or, shortly speaking, electronic trading versus a specific benchmark. Measurability is one of the more obvious benefits of benchmarking. Indeed, when trading performance is measured compare to a benchmark, by definition you easily get an execution quality measure. Two main factors explain this phenomena. On the one hand, computer trading offer is now easily accessible. If sell-side firms execution systems have been used internally by traders for years, these systems become recently available directly to clients via electronic platforms. A steady drop in transaction rates is forcing sell-side firms to become more efficient in processing trades and to lean on automation and computer power more than ever before to cut costs. At the same time, firms are looking to outsource their trading desks to increase their capacity to execute more volume. Major brokerage houses are then franchising their computer trading strategies to smaller firms which in term are pressured to offer the service. Small and midsize broker-dealers that lack resources and time to invest in developing VWAP engines and other quantitative strategies can then offer the proprietary benchmark trading to their buy-side customers. In return, the

source firms are paid a percentile per share based on the volume that's pumped through their models. Even if the franchisee broker puts it own name on the algorithm strategy, the execution occurs on major brokerage houses, virtually invisible to the institutional firm. The originating broker-dealer get credit for the volume since it represents the order at the exchange and still preserves the execution clearing relationship with the buy-side client.

On the other hand, buy-side customers are asking for the algorithms. They are numerous reasons for buy-side firms to ask for this type of trading. The buy side is being more closely monitored and scrutinized for its execution quality. Algorithmic trading offers a less expensive option to full service brokers, while providing a way to complete a complex order type. In general, pre trade analytic tools are readily and easily available. The execution environment allows clients to obtain analysis relevant to the context in which they make trades. Moreover, market fragmentation drives traders to use electronic tools to access the market in different ways. Quant fund traders began to be a bigger piece of liquidity and need flexible and easy access to the market. For small brokers, access to big brothers' algorithms is far from cheap. But if a customer feels that he's getting a better execution, it's incumbent on them to provide it.

VWAP benchmark Several benchmarks are proposed in the field of algorithmic trading (These prices are based on market on close, percentage of volume, opportunistic model for small-cap stocks, ...), but the most common and popular one is VWAP. The main reason is obvious: the computation of daily VWAP is straightforward for anyone with access to daily stock transactions records. Moreover the use of VWAP is simple in itself: if the price of a buy trade is lower than VWAP, it is a good trade; if the price is higher, it is a bad trade (and conversely for sell trades). In general, brokers propose several ways to reach VWAP benchmark. Agency and guaranteed VWAP execution services are the two main possibilities. In the guaranteed case, the execution is guaranteed at VWAP for a fixed commission per share, and the broker dealer ensures the entire risk of failing to meet the benchmark. In the agency trading case, the order is sent to a broker-dealer, to trade on an agency basis, with the aim of obtaining the VWAP or better. Obviously, the transaction costs are not the same depending on the chosen method and the larger the client residual

risk, the smaller the cost.

Timing dimension VWAP strategies introduce a time dimension in the order execution process. If the trader loses control of whether the trade will be executed during the day, VWAP strategies allows to dilute the impact of orders through the day. To understand the immediacy and *good* price trade-off, lets take the two examples of action and investor traders. Action traders go to where the action is, meaning that they don't care what about the firm stock they are trading. Investor traders lack that flexibility. Since their job represents the final task in a sequential decision process, they are expected to trade specific stocks, even if the action is over. Of course, trade information cannot remain proprietary for long and trade delays resulting in trade process that can defer greatly from the manager's original decision price. VWAP strategies ensure to investors tragers good participation during the day, and then trade completion at the closing time.

Size effect Under particular conditions VWAP evaluation may be misleading and even harmful to portfolio performance. Most institutional trading occurs in filling orders that exceed the daily volume. When large numbers of shares must be traded, liquidity concerns is against price goals. Then trade evaluation becomes more complicated. Action traders watch the market for this reason and try to benefit from those trades. A naive investor could indiscreetly reveal her interest for the market or a particular stock. Action traders can then cut themselves in by capturing available liquidity and reselling it to unskilled trader. On the contrary, skilled trader will deal amounts below or beyond the action trader's radar screen to avoid such behavior.. Using automatic participation strategies as VWAP may be dangerous in these cases. Since it pays no attention to the full size of the trade, trading cost are biased by VWAP benchmark since the benchmark itself depends on the trades.

For this reason, some firms offer multi-days VWAP strategies to respond to customers requests. To further reduce the market impact of large orders, customers can specify their own volume participation by limiting the volume of their orders on low expected volume days. Each order is first sliced into several days orders in a first step, and then sent to VWAP engine for the corresponding days.

To avoid this first limitation, we make the following first assumption concerning the order size send to VWAP engine. We assume that any considered VWAP execution order is low compared to the daily volume.

Trade motivation Most trading observed on the market, such as balancing or inflow trading, is not price sensitive and evaluation by a VWAP analysis will not be missleading. However, some trades and hence trading prices reflect objectives that cannot be captured by a VWAP analysis. To see this, we must look deeper into trading motivations to discriminate whether a particular price represents a good or bad execution. Let us consider two types of traders: value and growth managers. Value managers are looking for underpriced situations. They buy the stock and wait to sell it until a good news raises its price. Besides, growth managers react to good news, that hopefully portends more good news. Thus, while growth managers buy on good news value managers sell. Consequently growth managers have a clear trading disadvantage because they buy when the buying interest dominates the market. They are frequently lower ranked than value traders. If the skilled traders can understand the motivations beyond the decisions, they will try to adjust their strategy accordingly. Automatic participation algorithm cannot take into account such dimension in trading.

The second assumption we make in our empirical study is to only consider low motivation trading. In such case, VWAP benchmark can be used without bias.

Benchmarking arbitrage In the case of VWAP trading, any price is a good price if the size of the trade dominates the daily volume implying that the trading price dominates the VWAP. Trading dominating VWAP is evaluated as good trade no matter how expensive the price might be compared to manager decision targets. Hence, VWAP makes the trader insensitive to price since any price become as good as any other price. This denigrates traders skills and can destroy the value of research. Moreover, VWAP is very beneficial for screening people who don't know that it is used to evaluate them. Anyone who knew there were going to be evaluated by this measure would be some combination of dumb, impotent, or corrupt, depending on how they behaved. Even though you know you can game this method you don't game it.

As third assumption, we assume that traders have no strategic behavior.

4.2 Empirical results

In this section the question about usefulness of the above discussed models to the prediction of volume weight average price (VWAP) is addressed.

The answer has important meaning for brokers, who are supposed to execute VWAP orders, and whose trades are evaluated according to benchmarks based on VWAP.

This empirical study focuses on VWAP order with one day horizon. The examination is organized as follows: the proxy of volume weight price is computed based on twenty five time points during a trading day. The first point corresponds to the time $9:20$ and the last to the time $17:20$. The time interval between two succeeding time point is equal to 20 minutes. The price of equity for each of twenty five points were computed as arithmetic average over the transaction price which took place in the last twenty minutes. The prediction of volume is done by models based on principal component decomposition and by using the classical approach to describe daily pattern of intraday volume.

The Tables from 11 to 16 are comparisons of in-sample performances for all models based on mean absolute percentage error (MAPE) and mean square percentage error (MSPE). The examination is carried out for the period ranging from September, 2 to December, 16 2003. In 26 cases the decomposition model with specific part described by an ARMA reduces the error measure MAPE by more than 25%. For ten equities the reduction exceeds 40% in comparison to the classical approach. The major decrease of the error measure is observed for SCHNEIDER ELECTRIC's VWAP, where it exceeds by 50%. The modelling of the specific part by a SETAR allows for further decline of the mean absolute prediction error in comparison to classical approach. A reduction of around 25% is observed for 34 equities. For sixteen equities the reduction exceeds 40%. The most substantial decrease of prediction error is obtained for VIVENDI UNIVERSAL equity. The application of decomposition model allows to improve the quality of VWAP forecast by around 60%.

According to the results we get with ARMA or SETAR all together, the decomposition

models outperform the classical approach. The only exception is TF1 equity where the classical approach beats both models. However, application of the ARMA model to the specific part of TF1's volume bring the error measures closer to those obtained by the classical approach. Nevertheless, the application of SETAR instead of ARMA in $60-70\%$ cases permit additional reduction of risk. The number of stocks for which SETAR model overwhelms the ARMA model depends on the error measure.

From broker's perspective the 95%-quantile contains important information about the risk of application of particular model. The 95% quantile has much smaller value for decompositions models than for classical approach. Furthermore, SETAR model seems to be better than ARMA to describe the specific part of the intraday volume. This is due the SETAR ability to discriminate between turbulent and flat periods of the market. The 95% quantiles for the classical approach and the model with an ARMA specific part are ranging from 0.19% to 0.78%, and from 0.1% to 0.49% respectively. In the SETAR case, the 95% quantiles for all companies range from 0.08% to 0.38%.

As result of in-sample performance comparisons, we show that decomposition models can be successfully used to predict the volume weight of average price (VWAP). Furthermore, a broker who exploit our approach to forecast VWAP, compared to the classical one, is lowering her risk.

Moreover, the in sample results are confirmed by out-of-sample ones. This analysis is carried out by application of a twenty days moving window. Thus, the decomposition is applied for the period of twenty trading days before the day during which execution of VWAP order take place. The average common part of intraday volume is computed and known in the evening of the day preceding VWAP trade. In turn, the specific part is forecasted with twenty minutes delay, on the considered day.

The out-of-sample performance of models under consideration for the period from September 2, to December 16, 2003 is summarized in tables form 17 to 22. The models based on decomposition outperform the classical approach. For all companies the approach based on direct computing averages result in higher risk of execution of VWAP orders. In the case of 20 equities, the application of the decomposition model with an ARMA spe-

cific part allows for a reduction of MAPE greater than 15%. The major decrease of the error measure is observed for THOMSON's VWAP, with almost 40%. The application of decomposition model with a SETAR specific part enables further decline of the mean absolute prediction error in comparison to the classical approach. The MAPE reduction above 15% is observed for 27 equities. For twenty one equities the reduction exceeds 20%. The most substantial decrease of the prediction error is obtained for THOMSON equity. The application of decomposition model allows to improve the quality of VWAP forecast around 46%.

Furthermore, on average, the application of either decomposition models is less risky than the use of the classical approach. The risk is associated with the 95% quantile. The large this quantity, the less reliable the VWAP prediction and thus the riskier the strategy.

The obtained results advocate to the approach based on principal component decomposition. In order to recapitulate the results, we estimate the cost of VWAP order execution when the subject of transaction are all stocks included in index CAC40. Therefore, we compute the VWAP for whole index as weighed average of VWAP over equities. We use the same weight to them used for constriction of index at the beginning of September 2004. Tables 23 and 24 present the summary of the model's performance comparison in case of VWAP order for whole index. The application of decomposition model with specific part describe by SETAR allows to reduce the risk around of 40% in case of in-sample comparison. Even, in out-sample case application of this model dimmish the risk of execution VWAP order by 24%. The 95%-quantile in case of classical approach is almost a double of that for decomposition models. Lastly, to present superiority of models based on principal component decomposition, the figures presenting the absolute and square percentage error for randomly selected equities is plotted. The Figure 4 and 5 correspond to in and out of sample case respectively.

The above outcomes show that using the decomposition of volume into market and specific parts reduce the cost of execution of VWAP orders. From the perspective of brokerage houses which are directly engaged in the process of VWAP orders execution, an additional issue of "beating the VWAP" seems crucial. It is clear, that the primary aim of a broker is

to the keep execution price of orders, as closely as possible to the VWAP price, and in this manner generates profit from the commissions paid by investors who asked for execution of VWAP orders. Nevertheless there is another potential source of profit. An additional gain can be made when brokers manage to execute a sale VWAP-order at a higher price higher than the observed end of the day volume weighed average price. The same apply to a buy VWAP-order at a lower price than the observed volume weighed average price. To verify the possibility of beating the VWAP applying our methodology, we present in table 25, separate statistics for situations, where the predicted VWAP is lower and higher than observed at end of the day. The results indicate that the difference between the predicted VWAP and the observed one can be either sign with the same probability. Roughly, the mean absolute percentage error average over the period ranging from September 2, to December 16, 2003, for the SETAR, the ARMA and the classical approach are equal to 0.07%, 0.08%, and 0.11% respectively.

5 Conclusion

This paper leans on a decomposition of intraday volume, measured by stock turnovers. This methodology allows us first to propose an accurate statistical method of volume predictions. These predictions are then used in a benchmark tracking price framework.

The following results are obtained through our analysis. Not only we get round the seasonality problem but we use it to propose a new price benchmark. We also show that some simple times series models give good volume predictions. Beside, applications of our methodology to VWAP strategies reduces the VWAP tracking error, and thus the execution risk due to the use of such order type and so the associated cost.

However, in order to beat the VWAP, our price adjusted-volume model is not sufficient and it is essential to derive a bivariate model for volume and price.

References

[1] Andersen, T., 1996, *Return volatility and trading volume: An information flow interpretation of stochatic volatility*, Journal of Finance 51(1), 169-204.

[2] Berkowitz, S., D., Logue, and E. Noser, 1988, *The Total Cost of Transactions on the NYSE*, Journal of Finance 41, 97-112.

[3] Biais, B., Hillion, P. and C. Spatt, 1995, *An empirical analysis of the limit order book and the order flow in the Paris bourse*, Journal of Finance 50(5), 1655-1689.

[4] Cuching, D., and Madhavan, A., 2000, *Stock Returns and Institutional Trading at the Close*, Journal of Financial Markets 3, 45-67.

[5] Darolles, S., and G., Le Fol,2003, *Trading volume and Arbitrage*, Working paper, CREST.

[6] Darat, A.,Rahman, S., and, M. Zhong, 2003, *Intraday Trading Volume and Return Volatility of the DJIA Stocks: A Note*, Journal of Banking and Finance 27(10), 2035-2043.

[7] Easley, D., and M., O'Hara, 1987, *Price; Trade Size, and Information in Securities Markets* , Journal of Financial Economics 19, 69-90.

[8] Engle, R. and J. Russel, 1998, *Autoregressive Conditional Duration: A New Model for Irregularly Spaced Transaction Data*, Econometrica (1998) 66: 1127-1162.

[9] Engle, R., 2000, *The Econometrics of Ultra High Frequency Data*, Econometrica, (2000) 68: 1-22.

[10] Foster, D., and S. Viswanathan, 1990, *A Theory of Intraday Variations in volume, variance and trading costs in securities market*, Review of Financial Studies 3, 593-624.

[11] Gouriéroux, C., Jasiak, J., and G. Le Fol, 1999, *Intra-day market activity*, Journal of Financial Markets 2, 193-226.

[12] Gouriéroux, C. And G. Le Fol, 1998, *Effet des Modes de Négociation sur les Echanges*, Revue Economique, Vol. 49, 3, 795-808.

[13] Kaastra, I., Boyd, M., 1995, *Forecasting Futures Trading Volume Using Neural Networks*, Journal of Futures Markets 15, 953-970.

[14] Karpoff, J., Boyd, M., 1987, *The Relationship between Price Changes and Trading volume: a Survey*, Journal of Financial and Quantitative Analysis 22(1), 109-126.

[15] Le Fol, G. and L. Mercier, 1998, *Time Deformation : Definition and Comparisons*, Journal of Computational Intelligence in Finance 6, 5, 19-33.

[16] Lo, A., Wang J., 2000, *Trading Volume: Definition, Data Analysis, and Implication of Portfolio Theory*, Review of Financial Studies 13, 257-300.

[17] Madhavan, A., 2002, *VWAP strategies*, Transaction Performance: The Changing Face of Trading Investment Guides Series, Institutional Investor Inc., 32-38.

[18] Manganelli, S., 2002, *Duration, Volume, and Volatility Impact of Trades*, European Central Bank Working Papers Series No. 125.

[19] McCulloch, J., 2004, *Relative Volume as a Doubly Stochastic Binomial Point Process*, Working Papers Series.

[20] Tauchen, G., Pitts, M.,1983,*The Price Variability-Volume Relationship on Speculative Markets*, Econometrica 51, 485-505.

Table 1: Summary statistics for the intraday aggregated volume over 20 minutes intervals.

Companies	*Mean*	*Std*	*Q5*	*Q95*
ACCOR	0.0191	0.0273	0.0028	0.0523
AGF-ASS.GEN.FRANCE	0.0076	0.0087	0.0010	0.0212
AIR LIQUIDE	0.0120	0.0182	0.0022	0.0314
ALCATEL	0.0381	0.0383	0.0062	0.1064
ARCELOR	0.0234	0.0241	0.0034	0.0648
AXA	0.0166	0.0220	0.0034	0.0404
BNP PARIBAS	0.0147	0.0350	0.0034	0.0338
BOUYGUES	0.0129	0.0264	0.0019	0.0344
CAP GEMINI	0.0438	0.0514	0.0058	0.1241
CARREFOUR	0.0132	0.0232	0.0025	0.0317
CASINO GUICHARD	0.0106	0.0118	0.0013	0.0312
CREDIT AGRICOLE	0.0083	0.0120	0.0012	0.0233
DANONE	0.0149	0.0310	0.0024	0.0381
DEXIA	0.0055	0.0069	0.0006	0.0164
EADS	0.0092	0.0092	0.0015	0.0265
FRANCE TELECOM	0.0123	0.0115	0.0025	0.0312
L'OREAL	0.0069	0.0120	0.0014	0.0177
LAFARGE	0.0188	0.0307	0.0035	0.0477
LAGARDERE S.C.A.	0.0163	0.0385	0.0020	0.0423
LVMH	0.0105	0.0185	0.0018	0.0276
MICHELIN	0.0167	0.0238	0.0024	0.0450
PERNOD-RICARD	0.0157	0.0303	0.0022	0.0427
PEUGEOT	0.0205	0.0454	0.0035	0.0515
PIN.-PRINT.REDOUTE	0.0149	0.0210	0.0020	0.0426
RENAULT	0.0165	0.0414	0.0024	0.0412
SAINT GOBAIN	0.0154	0.0332	0.0030	0.0382
SANOFI-AVENTIS	0.0151	0.0228	0.0020	0.0444
SCHNEIDER ELECTRIC	0.0145	0.0264	0.0021	0.0378
SOCIETE GENERALE	0.0155	0.0205	0.0031	0.0390
SODEXHO ALLIANCE	0.0172	0.0318	0.0016	0.0518
STMICROELECTRONICS	0.0223	0.0230	0.0030	0.0604
SUEZ	0.0162	0.0182	0.0032	0.0418
TF1	0.0198	0.0449	0.0026	0.0531
THALES	0.0120	0.0134	0.0016	0.0336
THOMSON (EX:TMM)	0.0270	0.0465	0.0035	0.0776
TOTAL	0.0150	0.0277	0.0031	0.0373
VEOLIA ENVIRON.	0.0120	0.0158	0.0017	0.0333
VINCI (EX.SGE)	0.0261	0.0687	0.0034	0.0689
VIVENDI UNIVERSAL	0.0215	0.0203	0.0044	0.0543
Overall	0.0166	0.0265	0.0026	0.0445

Table 2: Correlation matrix decomposition of intraday volume for CAC40 index stocks.

Est.period	*Rank*	*Eigenvalue*	*Difference*	*Proportion*	*Cumulative*
from 1 September to 30 September 2003	1	12.93	10.96	0.392	0.392
	2	1.967	0.281	0.050	0.442
	3	1.686	0.256	0.043	0.485
from 1 September to 31 October 2003	1	12.95	11.21	0.371	0.371
	2	1.740	0.197	0.044	0.411
	3	1.543	0.243	0.039	0.450
from 1 September to 30 November 2003	1	12.41	10.93	0.358	0.358
	2	1.484	0.151	0.038	0.396
	3	1.333	0.052	0.034	0.430
from 1 September to 28 February 2003	1	11.16	9.893	0.286	0.286
	2	1.267	0.126	0.032	0.318
	3	1.141	0.027	0.029	0.347
from 1 September 2003 to 31 August 2004	1	8.614	5.737	0.221	0.221
	2	2.877	0.502	0.074	0.295
	3	2.375	0.868	0.061	0.356

Table contains the highest eigenvalues of the correlation matrix, differences between successive eigenvalues, the portion of variance explained by each eigenvalue, and the cumulative proportion of the variance explained .

Table 3: Results of test on unit root for series defined as difference between intraday volume and its common component obtained from principal component analysis.

	ADF			PP		
	Mean	Min	Max	Mean	Min	Max
Zero mean	-7.98	-11.14	-5.18	-10.83	-16.37	-6.53
Single mean	-15.92	-19.66	-11.37	-22.28	-28.59	-14.93
Trend	-16.14	-19.71	-11.45	-22.57	-28.66	-15.80

Outcomes of Augmented Dickey-Fuller (ADF), Philips-Perron (PP). For all examined time series the null hypothesis was rejected at 1% significant level was rejected.

Figure 1: Autocorrelation and partial autocorrelation functions of the two components, TOTAL stock.

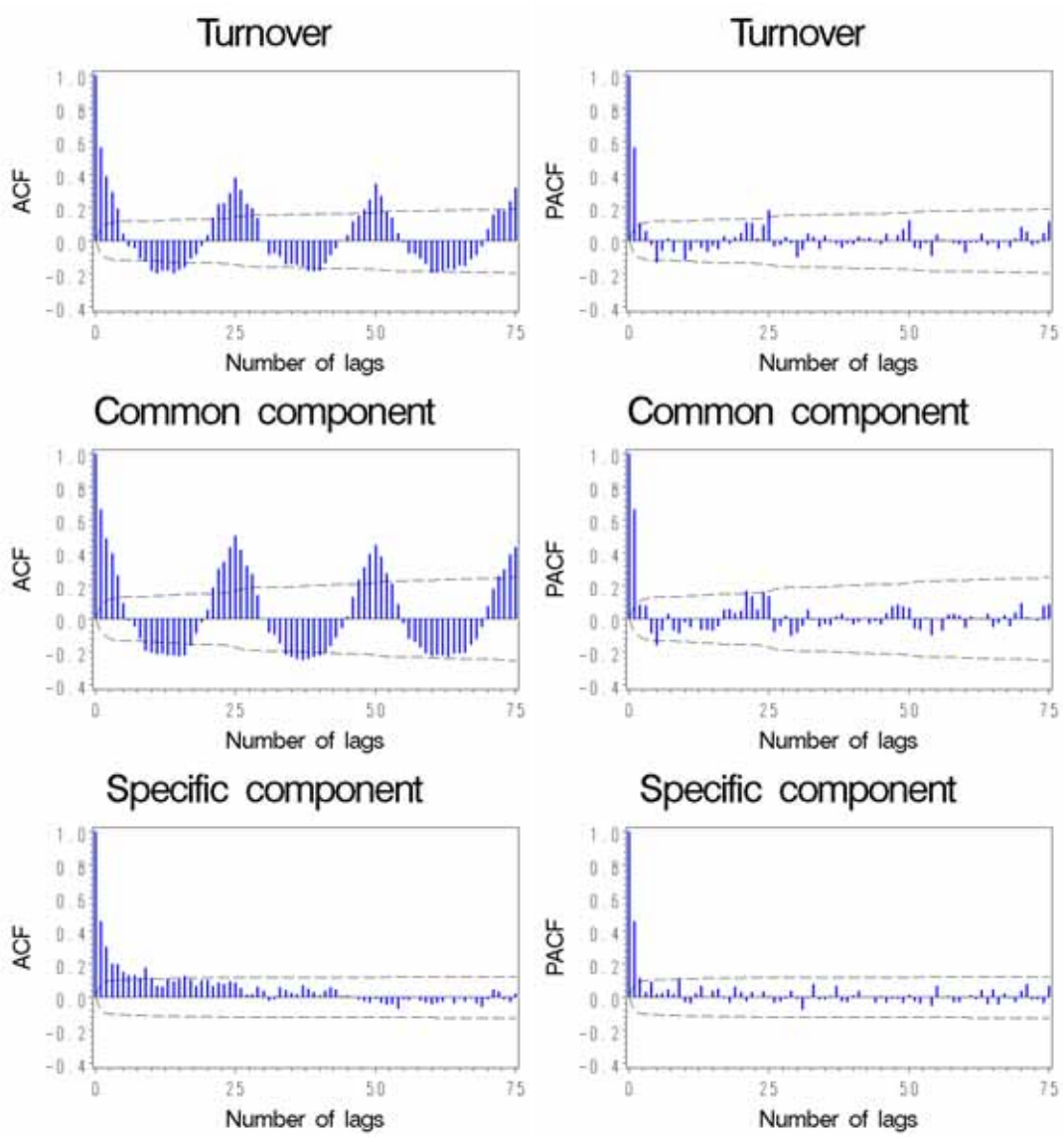

Figure 2: Autocorrelation functions of ARMA (left graph) and SETAR (right graph) residuals for specific component of EADS, SANOFI-AVENTIS and TOTAL stock.

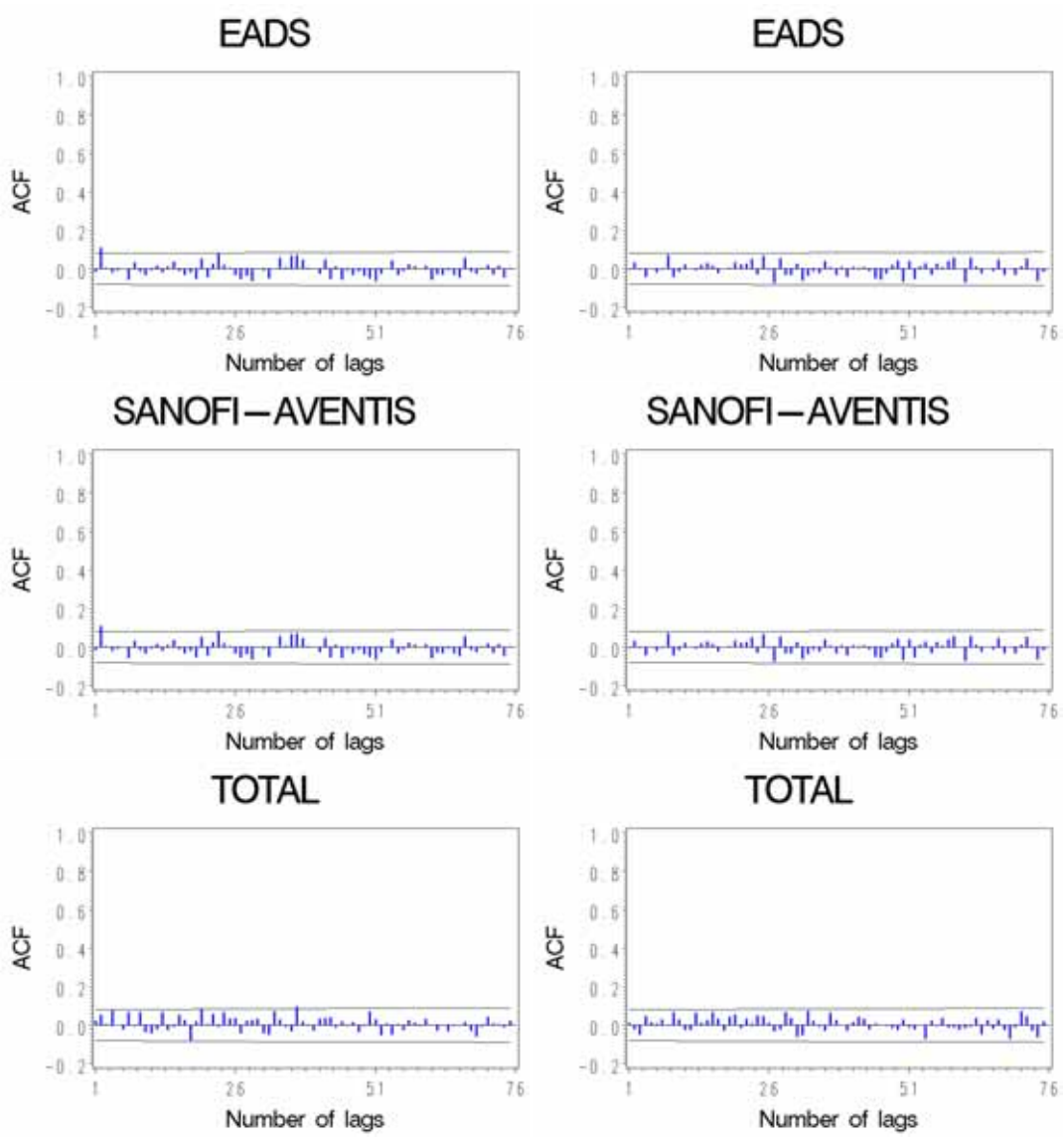

Figure 3: TOTAL stock daily volume patterns on September 9 and 10, 2003, left and right respectively. The first two graphs represent the intraday turnover evolution. The two next, give the common component evolution and last two ones, the specific component evolution.

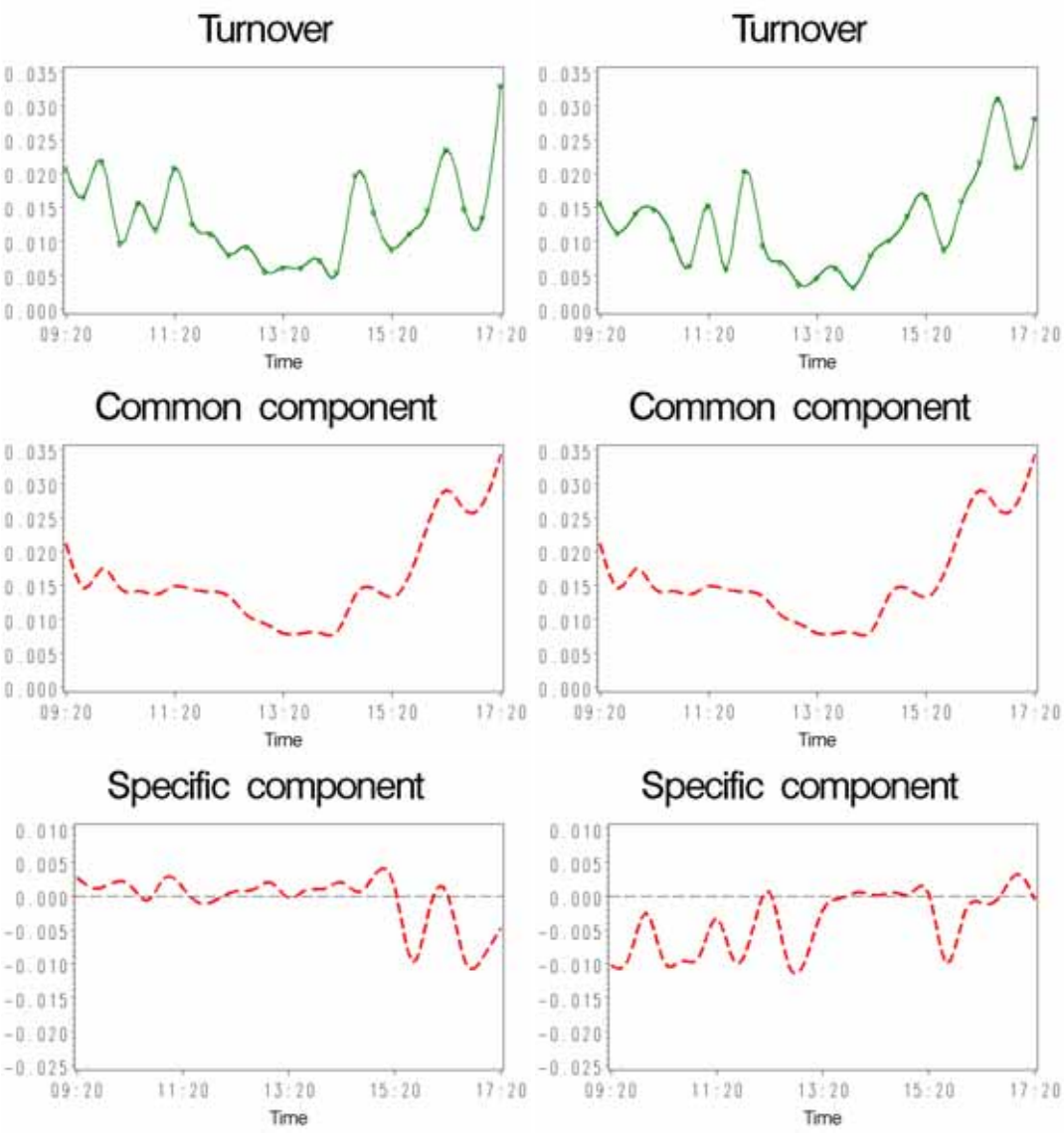

Table 4: Comparison of intraday volume models performance. The result obtained in case when intraday volume was approximated by classical approach.

	MAPE					MSPE				
Company	Mean	Std	Min	Max	Q95	Mean	Std	Min	Max	Q95
ACCOR	1.15E-02	1.64E-02	3.69E-06	1.98E-01	2.88E-02	4.01E-04	2.40E-03	1.36E-11	3.93E-02	8.27E-04
AGF-ASS.GEN.FRANCE	3.78E-03	4.35E-03	1.07E-06	5.09E-02	1.01E-02	3.32E-05	1.45E-04	1.15E-12	2.59E-03	1.02E-04
AIR LIQUIDE	6.99E-03	7.75E-03	4.80E-06	6.66E-02	1.86E-02	1.09E-04	3.60E-04	2.31E-11	4.44E-03	3.47E-04
ALCATEL	2.30E-02	2.39E-02	2.85E-05	2.88E-01	6.13E-02	1.10E-03	4.01E-03	8.14E-10	8.32E-02	3.76E-03
ARCELOR	1.18E-02	1.18E-02	2.54E-05	8.71E-02	3.39E-02	2.78E-04	7.10E-04	6.48E-10	7.59E-03	1.15E-03
AXA	9.97E-03	9.97E-03	3.14E-05	8.33E-02	2.69E-02	1.99E-04	5.35E-04	9.86E-10	6.94E-03	7.23E-04
BNP PARIBAS	6.59E-03	7.06E-03	1.98E-06	6.65E-02	1.68E-02	9.32E-05	3.22E-04	3.90E-12	4.42E-03	2.83E-04
BOUYGUES	5.50E-03	6.67E-03	1.33E-05	9.57E-02	1.63E-02	7.46E-05	4.11E-04	1.77E-10	9.15E-03	2.65E-04
CAP GEMINI	2.40E-02	2.40E-02	4.35E-05	2.55E-01	6.56E-02	1.16E-03	3.62E-03	1.89E-09	6.51E-02	4.31E-03
CARREFOUR	4.70E-03	5.36E-03	1.58E-06	6.17E-02	1.29E-02	5.08E-05	2.15E-04	2.49E-12	3.81E-03	1.65E-04
CASINO GUICHARD	8.25E-03	8.69E-03	7.70E-06	9.99E-02	2.12E-02	1.43E-04	5.08E-04	5.93E-11	9.97E-03	4.50E-04
CREDIT AGRICOLE	5.35E-03	5.20E-03	1.26E-05	4.39E-02	1.44E-02	5.56E-05	1.49E-04	1.60E-10	1.93E-03	2.06E-04
DANONE	1.16E-02	1.45E-02	1.67E-06	1.25E-01	3.66E-02	3.43E-04	1.27E-03	2.78E-12	1.55E-02	1.34E-03
DEXIA	4.88E-03	8.25E-03	2.15E-05	9.77E-02	1.23E-02	9.18E-05	6.52E-04	4.64E-10	9.54E-03	1.52E-04
EADS	4.82E-03	5.02E-03	3.33E-05	5.77E-02	1.29E-02	4.84E-05	1.75E-04	1.11E-09	3.32E-03	1.66E-04
FRANCE TELECOM	7.81E-03	7.93E-03	2.60E-05	6.53E-02	2.07E-02	1.24E-04	3.33E-04	6.76E-10	4.27E-03	4.27E-04
L'OREAL	3.73E-03	5.10E-03	2.88E-06	6.79E-02	1.03E-02	3.99E-05	2.18E-04	8.30E-12	4.61E-03	1.07E-04
LAFARGE	1.16E-02	1.33E-02	3.49E-06	1.40E-01	2.93E-02	3.11E-04	1.20E-03	1.22E-11	1.95E-02	8.57E-04
LAGARDERE S.C.A.	1.05E-02	1.34E-02	1.97E-05	1.34E-01	2.68E-02	2.89E-04	1.35E-03	3.87E-10	1.79E-02	7.20E-04
LVMH	6.17E-03	7.55E-03	4.19E-06	9.06E-02	1.62E-02	9.49E-05	4.39E-04	1.75E-11	8.22E-03	2.61E-04
MICHELIN	9.35E-03	1.02E-02	2.17E-05	1.31E-01	2.76E-02	1.92E-04	8.00E-04	4.71E-10	1.71E-02	7.62E-04
PERNOD-RICARD	9.15E-03	1.16E-02	6.30E-06	1.81E-01	2.37E-02	2.19E-04	1.39E-03	3.97E-11	3.26E-02	5.60E-04

Table 5: (Continued) Comparison of intraday volume models performance. The result obtained in case when intraday volume was approximated by classical approach.

	MAPE					MSPE				
Company	Mean	Std	Min	Max	Q95	Mean	Std	Min	Max	Q95
PEUGEOT	1.28E-02	1.51E-02	3.28E-05	1.66E-01	3.71E-02	3.89E-04	1.55E-03	1.08E-09	2.76E-02	1.38E-03
PIN.-PRINT.REDOUTE	1.09E-02	1.16E-02	2.94E-06	1.14E-01	3.03E-02	2.53E-04	8.26E-04	8.65E-12	1.29E-02	9.15E-04
RENAULT	1.14E-02	1.36E-02	5.40E-06	1.31E-01	3.31E-02	3.16E-04	1.21E-03	2.92E-11	1.73E-02	1.09E-03
SAINT GOBAIN	8.29E-03	9.26E-03	1.51E-06	9.11E-02	2.42E-02	1.54E-04	5.28E-04	2.27E-12	8.30E-03	5.84E-04
SANOFI-AVENTIS	4.95E-03	6.21E-03	1.32E-06	7.96E-02	1.43E-02	6.29E-05	3.12E-04	1.75E-12	6.33E-03	2.04E-04
SCHNEIDER ELECTRIC	7.43E-03	1.02E-02	9.83E-06	1.56E-01	1.84E-02	1.59E-04	1.12E-03	9.66E-11	2.43E-02	3.38E-04
SOCIETE GENERALE	7.33E-03	7.52E-03	2.65E-05	6.03E-02	2.10E-02	1.10E-04	3.15E-04	7.05E-10	3.63E-03	4.42E-04
SODEXHO ALLIANCE	9.11E-03	1.89E-02	8.60E-06	3.89E-01	2.29E-02	4.40E-04	6.25E-03	7.40E-11	1.51E-01	5.23E-04
STMICROELECTRONICS	1.23E-02	1.34E-02	1.87E-06	1.61E-01	3.40E-02	3.31E-04	1.30E-03	3.49E-12	2.61E-02	1.16E-03
SUEZ	8.87E-03	1.05E-02	3.08E-05	1.41E-01	2.74E-02	1.89E-04	9.28E-04	9.51E-10	2.00E-02	7.48E-04
TF1	1.12E-02	1.41E-02	1.29E-05	2.13E-01	3.05E-02	3.23E-04	1.98E-03	1.68E-10	4.52E-02	9.33E-04
THALES	9.45E-03	1.28E-02	3.07E-06	1.18E-01	2.75E-02	2.54E-04	1.09E-03	9.44E-12	1.40E-02	7.57E-04
THOMSON (EX:TMM)	1.13E-02	1.33E-02	1.06E-05	1.97E-01	3.21E-02	3.04E-04	1.69E-03	1.12E-10	3.87E-02	1.03E-03
TOTAL	6.12E-03	7.17E-03	4.71E-06	8.42E-02	1.75E-02	8.88E-05	3.70E-04	2.22E-11	7.09E-03	3.06E-04
VEOLIA ENVIRON.	1.19E-02	1.95E-02	1.22E-05	2.83E-01	3.45E-02	5.20E-04	3.61E-03	1.49E-10	8.01E-02	1.19E-03
VINCI (EX.SGE)	1.38E-02	1.57E-02	1.87E-05	1.71E-01	3.65E-02	4.34E-04	1.71E-03	3.50E-10	2.93E-02	1.33E-03
VIVENDI UNIVERSAL	1.26E-02	1.89E-02	2.27E-05	2.14E-01	3.20E-02	5.13E-04	2.92E-03	5.16E-10	4.59E-02	1.03E-03

Table 6: Comparison of intraday volume models performance. The result obtained in case when intraday volume was approximated by model based on principal component decomposition, where the dynamics of specific component was described by ARMA model.

	MAPE					MSPE				
Company	Mean	Std	Min	Max	Q95	Mean	Std	Min	Max	Q95
ACCOR	1.08E-02	1.61E-02	1.97E-05	1.99E-01	2.83E-02	3.76E-04	2.40E-03	3.87E-10	3.94E-02	8.01E-04
AGF-ASS.GEN.FRANCE	3.59E-03	3.93E-03	4.73E-07	4.17E-02	9.79E-03	2.83E-05	1.06E-04	2.24E-13	1.73E-03	9.58E-05
AIR LIQUIDE	6.62E-03	7.49E-03	1.1E-05	6.33E-02	2.05E-02	1.00E-04	3.21E-04	1.21E-10	4.01E-03	4.19E-04
ALCATEL	2.10E-02	2.31E-02	9.71E-05	2.84E-01	6.19E-02	9.72E-04	3.85E-03	9.43E-09	8.05E-02	3.83E-03
ARCELOR	1.08E-02	1.10E-02	1.51E-06	9.74E-02	3.26E-02	2.38E-04	6.43E-04	2.28E-12	9.49E-03	1.06E-03
AXA	8.97E-03	9.29E-03	7.87E-06	7.48E-02	2.38E-02	1.67E-04	4.75E-04	6.19E-11	5.59E-03	5.68E-04
BNP PARIBAS	6.35E-03	7.04E-03	2.69E-05	6.87E-02	1.67E-02	8.97E-05	3.39E-04	7.24E-10	4.72E-03	2.80E-04
BOUYGUES	5.23E-03	6.91E-03	4.13E-06	1.02E-01	1.43E-02	7.49E-05	4.65E-04	1.71E-11	1.04E-02	2.05E-04
CAP GEMINI	2.17E-02	2.19E-02	3.7E-05	2.49E-01	5.86E-02	9.51E-04	3.20E-03	1.37E-09	6.20E-02	3.43E-03
CARREFOUR	4.48E-03	5.48E-03	3.61E-05	6.14E-02	1.21E-02	5.00E-05	2.20E-04	1.31E-09	3.77E-03	1.46E-04
CASINO GUICHARD	7.63E-03	8.46E-03	9.75E-06	9.57E-02	2.31E-02	1.30E-04	4.64E-04	9.50E-11	9.16E-03	5.33E-04
CREDIT AGRICOLE	4.89E-03	4.99E-03	5.7E-06	3.90E-02	1.40E-02	4.88E-05	1.33E-04	3.25E-11	1.52E-03	1.95E-04
DANONE	1.01E-02	1.24E-02	1.79E-05	1.08E-01	3.00E-02	2.55E-04	8.85E-04	3.19E-10	1.17E-02	8.97E-04
DEXIA	4.18E-03	5.72E-03	4.48E-06	6.69E-02	1.11E-02	5.01E-05	2.62E-04	2.00E-11	4.48E-03	1.23E-04
EADS	4.63E-03	4.97E-03	1.82E-05	5.95E-02	1.21E-02	4.60E-05	1.78E-04	3.31E-10	3.54E-03	1.45E-04
FRANCE TELECOM	6.76E-03	6.98E-03	2.36E-05	6.55E-02	1.71E-02	9.43E-05	2.88E-04	5.55E-10	4.29E-03	2.92E-04
L'OREAL	3.38E-03	4.84E-03	1E-05	6.88E-02	9.20E-03	3.48E-05	2.14E-04	1.01E-10	4.74E-03	8.46E-05
LAFARGE	1.04E-02	1.27E-02	4.05E-05	1.35E-01	2.95E-02	2.70E-04	1.08E-03	1.64E-09	1.81E-02	8.71E-04
LAGARDERE S.C.A.	9.64E-03	1.33E-02	2.6E-05	1.42E-01	2.86E-02	2.71E-04	1.39E-03	6.75E-10	2.01E-02	8.18E-04
LVMH	5.80E-03	7.30E-03	3.94E-06	9.12E-02	1.63E-02	8.69E-05	4.18E-04	1.55E-11	8.32E-03	2.67E-04
MICHELIN	8.69E-03	1.02E-02	9.86E-05	1.37E-01	2.53E-02	1.80E-04	8.63E-04	9.72E-09	1.88E-02	6.42E-04
PERNOD-RICARD	8.49E-03	1.07E-02	2.02E-05	1.66E-01	2.40E-02	1.87E-04	1.17E-03	4.09E-10	2.74E-02	5.75E-04

Table 7: (Continued) Comparison of intraday volume models performance. The result obtained in case when intraday volume was approximated by model based on principal component decomposition, where the dynamics of specific component was described by ARMA model.

	MAPE					MSPE				
Company	Mean	Std	Min	Max	Q95	Mean	Std	Min	Max	Q95
PEUGEOT	1.18E-02	1.40E-02	7.18E-06	1.66E-01	3.27E-02	3.34E-04	1.48E-03	5.15E-11	2.77E-02	1.07E-03
PIN.-PRINT.REDOUTE	8.76E-03	9.68E-03	2.08E-05	9.08E-02	2.62E-02	1.70E-04	5.48E-04	4.34E-10	8.24E-03	6.89E-04
RENAULT	1.03E-02	1.23E-02	6.54E-05	1.39E-01	3.07E-02	2.56E-04	1.05E-03	4.27E-09	1.92E-02	9.44E-04
SAINT GOBAIN	7.77E-03	9.11E-03	2.92E-05	9.39E-02	2.42E-02	1.43E-04	5.27E-04	8.55E-10	8.81E-03	5.88E-04
SANOFI-AVENTIS	4.73E-03	6.13E-03	5.43E-06	8.27E-02	1.35E-02	5.99E-05	3.23E-04	2.95E-11	6.85E-03	1.82E-04
SCHNEIDER ELECTRIC	7.24E-03	1.02E-02	5.27E-06	1.55E-01	1.90E-02	1.56E-04	1.12E-03	2.78E-11	2.39E-02	3.59E-04
SOCIETE GENERALE	7.05E-03	7.45E-03	2.38E-05	6.29E-02	2.13E-02	1.05E-04	3.22E-04	5.64E-10	3.96E-03	4.56E-04
SODEXHO ALLIANCE	7.90E-03	1.75E-02	1.17E-05	4.01E-01	2.06E-02	3.67E-04	6.45E-03	1.38E-10	1.61E-01	4.25E-04
STMICROELECTRONICS	1.18E-02	1.26E-02	1E-05	1.23E-01	3.24E-02	2.98E-04	1.01E-03	1.00E-10	1.52E-02	1.05E-03
SUEZ	7.88E-03	1.00E-02	8.84E-06	1.31E-01	2.05E-02	1.63E-04	8.47E-04	7.82E-11	1.72E-02	4.21E-04
TF1	1.09E-02	1.43E-02	5.99E-07	2.23E-01	3.04E-02	3.23E-04	2.17E-03	3.58E-13	4.99E-02	9.23E-04
THALES	8.85E-03	1.27E-02	1.59E-05	1.23E-01	2.69E-02	2.40E-04	1.11E-03	2.51E-10	1.50E-02	7.25E-04
THOMSON (EX:TMM)	1.06E-02	1.30E-02	2.09E-05	1.84E-01	2.91E-02	2.83E-04	1.51E-03	4.37E-10	3.38E-02	8.49E-04
TOTAL	5.93E-03	7.01E-03	1.82E-05	8.51E-02	1.67E-02	8.42E-05	3.73E-04	3.32E-10	7.25E-03	2.78E-04
VEOLIA ENVIRON.	8.15E-03	1.40E-02	5.2E-06	2.12E-01	2.66E-02	2.62E-04	2.03E-03	2.70E-11	4.51E-02	7.07E-04
VINCI (EX.SGE)	1.22E-02	1.42E-02	3.68E-06	1.42E-01	3.46E-02	3.49E-04	1.32E-03	1.36E-11	2.01E-02	1.20E-03
VIVENDI UNIVERSAL	1.11E-02	1.51E-02	1.49E-05	1.51E-01	2.88E-02	3.50E-04	1.74E-03	2.23E-10	2.29E-02	8.30E-04

Table 8: Comparison of intraday volume models performance. The result obtained in case when intraday volume was approximated by model based on principal component decomposition, where the dynamics of specific component was described by SETAR model.

	MAPE					MSPE				
Company	Mean	Std	Min	Max	Q95	Mean	Std	Min	Max	Q95
ACCOR	1.04E-02	1.46E-02	5.08E-05	1.96E-01	2.63E-02	3.21E-04	2.22E-03	2.58E-09	3.85E-02	6.91E-04
AGF-ASS.GEN.FRANCE	2.77E-03	3.21E-03	1.39E-05	3.85E-02	7.44E-03	1.80E-05	8.21E-05	1.93E-10	1.48E-03	5.53E-05
AIR LIQUIDE	6.57E-03	7.25E-03	7.41E-06	6.40E-02	2.00E-02	9.56E-05	3.04E-04	5.49E-11	4.10E-03	4.00E-04
ALCATEL	1.85E-02	2.13E-02	4.30E-05	2.70E-01	5.35E-02	7.94E-04	3.41E-03	1.85E-09	7.29E-02	2.86E-03
ARCELOR	7.32E-03	7.14E-03	5.44E-06	6.70E-02	1.98E-02	1.04E-04	2.77E-04	2.96E-11	4.49E-03	3.91E-04
AXA	9.12E-03	9.08E-03	1.97E-05	7.54E-02	2.41E-02	1.66E-04	4.66E-04	3.88E-10	5.69E-03	5.81E-04
BNP PARIBAS	5.17E-03	6.24E-03	2.00E-07	6.11E-02	1.41E-02	6.55E-05	2.61E-04	3.99E-14	3.74E-03	1.99E-04
BOUYGUES	4.18E-03	5.84E-03	7.67E-06	9.43E-02	1.10E-02	5.16E-05	3.83E-04	5.88E-11	8.88E-03	1.22E-04
CAP GEMINI	2.08E-02	2.14E-02	4.99E-05	2.43E-01	5.48E-02	8.87E-04	3.06E-03	2.49E-09	5.93E-02	3.00E-03
CARREFOUR	3.85E-03	5.07E-03	3.32E-07	6.23E-02	9.98E-03	4.05E-05	2.10E-04	1.10E-13	3.89E-03	9.96E-05
CASINO GUICHARD	5.36E-03	5.79E-03	7.21E-06	7.45E-02	1.43E-02	6.22E-05	2.57E-04	5.19E-11	5.56E-03	2.04E-04
CREDIT AGRICOLE	3.64E-03	4.14E-03	1.77E-06	3.48E-02	1.07E-02	3.04E-05	9.31E-05	3.15E-12	1.21E-03	1.15E-04
DANONE	7.17E-03	8.60E-03	6.18E-06	8.67E-02	2.28E-02	1.25E-04	4.80E-04	3.82E-11	7.52E-03	5.20E-04
DEXIA	3.85E-03	4.73E-03	1.06E-06	5.57E-02	1.05E-02	3.72E-05	1.86E-04	1.12E-12	3.10E-03	1.09E-04
EADS	3.29E-03	4.07E-03	5.04E-06	5.49E-02	9.23E-03	2.73E-05	1.40E-04	2.54E-11	3.01E-03	8.51E-05
FRANCE TELECOM	6.53E-03	6.43E-03	1.27E-05	6.49E-02	1.59E-02	8.38E-05	2.56E-04	1.60E-10	4.22E-03	2.53E-04
L'OREAL	3.30E-03	4.03E-03	1.59E-06	3.25E-02	9.13E-03	2.71E-05	9.88E-05	2.54E-12	1.06E-03	8.34E-05
LAFARGE	8.34E-03	9.96E-03	1.93E-06	1.12E-01	2.18E-02	1.69E-04	7.23E-04	3.74E-12	1.26E-02	4.75E-04
LAGARDERE S.C.A.	7.14E-03	1.07E-02	3.30E-05	1.23E-01	1.99E-02	1.64E-04	9.90E-04	1.09E-09	1.51E-02	3.98E-04
LVMH	4.72E-03	5.60E-03	7.24E-07	7.43E-02	1.23E-02	5.35E-05	2.59E-04	5.24E-13	5.52E-03	1.52E-04
MICHELIN	8.34E-03	9.82E-03	2.72E-06	1.34E-01	2.36E-02	1.66E-04	8.13E-04	7.40E-12	1.79E-02	5.55E-04
PERNOD-RICARD	7.48E-03	1.03E-02	1.54E-05	1.63E-01	2.09E-02	1.62E-04	1.13E-03	2.36E-10	2.67E-02	4.36E-04

Table 9: (Continued) Comparison of intraday volume models performance. The result obtained in case when intraday volume was approximated by model based on principal component decomposition, where the dynamics of specific component was described by SETAR model.

	MAPE					MSPE				
Company	Mean	Std	Min	Max	Q95	Mean	Std	Min	Max	Q95
PEUGEOT	8.80E-03	1.13E-02	9.78E-06	1.42E-01	2.30E-02	2.05E-04	1.04E-03	9.57E-11	2.03E-02	5.27E-04
PIN.-PRINT.REDOUTE	6.67E-03	7.83E-03	2.17E-05	8.96E-02	1.82E-02	1.06E-04	4.31E-04	4.71E-10	8.03E-03	3.30E-04
RENAULT	8.18E-03	7.64E-03	4.01E-05	8.32E-02	2.19E-02	1.25E-04	3.55E-04	1.61E-09	6.93E-03	4.81E-04
SAINT GOBAIN	7.74E-03	8.73E-03	1.03E-05	9.21E-02	2.34E-02	1.36E-04	4.99E-04	1.05E-10	8.49E-03	5.47E-04
SANOFI-AVENTIS	4.21E-03	5.53E-03	6.97E-07	7.89E-02	1.10E-02	4.83E-05	2.86E-04	4.85E-13	6.23E-03	1.22E-04
SCHNEIDER ELECTRIC	6.89E-03	9.80E-03	1.61E-05	1.56E-01	1.62E-02	1.43E-04	1.12E-03	2.59E-10	2.45E-02	2.64E-04
SOCIETE GENERALE	6.99E-03	7.24E-03	4.30E-06	6.28E-02	2.02E-02	1.01E-04	3.12E-04	1.85E-11	3.94E-03	4.07E-04
SODEXHO ALLIANCE	7.55E-03	1.00E-02	2.83E-05	1.47E-01	2.08E-02	1.58E-04	9.88E-04	8.02E-10	2.15E-02	4.31E-04
STMICROELECTRONICS	1.12E-02	1.19E-02	5.85E-05	1.20E-01	2.82E-02	2.67E-04	9.17E-04	3.42E-09	1.44E-02	7.97E-04
SUEZ	7.93E-03	1.00E-02	5.01E-05	1.40E-01	2.00E-02	1.64E-04	9.11E-04	2.51E-09	1.96E-02	4.01E-04
TF1	8.10E-03	1.23E-02	5.71E-06	2.07E-01	2.05E-02	2.16E-04	1.84E-03	3.26E-11	4.29E-02	4.21E-04
THALES	6.46E-03	8.03E-03	2.89E-06	8.61E-02	1.87E-02	1.06E-04	4.58E-04	8.38E-12	7.42E-03	3.49E-04
THOMSON (EX:TMM)	8.04E-03	8.70E-03	2.38E-06	1.37E-01	2.02E-02	1.40E-04	7.93E-04	5.66E-12	1.88E-02	4.10E-04
TOTAL	6.21E-03	6.93E-03	1.19E-05	8.47E-02	1.58E-02	8.65E-05	3.65E-04	1.41E-10	7.18E-03	2.51E-04
VEOLIA ENVIRON.	7.90E-03	1.44E-02	2.72E-05	2.27E-01	2.18E-02	2.69E-04	2.28E-03	7.37E-10	5.15E-02	4.75E-04
VINCI (EX.SGE)	9.35E-03	1.28E-02	1.70E-06	1.48E-01	2.66E-02	2.51E-04	1.22E-03	2.90E-12	2.19E-02	7.09E-04
VIVENDI UNIVERSAL	1.10E-02	1.47E-02	1.02E-06	1.48E-01	3.00E-02	3.37E-04	1.69E-03	1.04E-12	2.19E-02	9.00E-04

Table 10: Summary of comparison for intraday volume models performance.

	MAPE					MSPE				
	Mean	Std	Min	Max	Q95	Mean	Std	Min	Max	Q95
SETAR	7.52E-3	8.69E-3	1.43E-6	1.00E-1	2.01E-2	1.59E-4	6.88E-4	4.26E-10	1.25E-2	4.95E-4
ARMA	8.29E-3	9.73E-3	1.78E-6	1.08E-1	2.33E-2	1.94E-4	8.17E-4	5.56E-10	1.46E-2	6.53E-4
Classical approach	9.05E-3	1.05E-2	1.45E-5	1.14E-1	2.49E-2	2.32E-4	9.48E-4	3.69E-10	1.66E-2	7.58E-4

Note:The volume is defined as percentage of total number of shares on the stock market.

Table 11: Summary of in-sample estimated costs of execution VWAP order for period from September 2, 2003 to December 16, 2003. The dynamics of specific component was modeled by classical approach basing on calculating means.

	MAPE					MSPE				
Company	Mean	Std	Min	Max	Q95	Mean	Std	Min	Max	Q95
ACCOR	0.1161	0.1024	2.23E-03	0.4429	0.3618	0.0080	0.0134	1.72E-06	0.0656	0.0447
AGF-ASS.GEN.FRANCE	0.1305	0.1270	1.78E-03	0.5670	0.387263	0.0144	0.0272	1.27E-06	0.1417	0.0693
AIR LIQUIDE	0.0878	0.0973	8.95E-04	0.4966	0.301276	0.0214	0.0506	1.06E-06	0.3028	0.1142
ALCATEL	0.1800	0.1813	1.28E-03	0.7605	0.546576	0.0071	0.0134	1.87E-07	0.0659	0.0309
ARCELOR	0.1443	0.1545	1.22E-03	0.6068	0.531083	0.0051	0.0099	1.85E-07	0.0421	0.0341
AXA	0.1425	0.2487	1.38E-03	1.5999	0.513325	0.0132	0.0606	3.00E-07	0.4251	0.0411
BNP PARIBAS	0.0952	0.1138	1.57E-03	0.5683	0.32196	0.0096	0.0235	1.11E-06	0.1366	0.0472
BOUYGUES	0.1767	0.1454	5.23E-03	0.7857	0.486319	0.0126	0.0239	6.56E-06	0.1470	0.0570
CAP GEMINI	0.1964	0.2767	2.21E-03	1.2944	0.775938	0.0444	0.1295	2.06E-06	0.6666	0.2291
CARREFOUR	0.0876	0.1119	8.65E-04	0.6629	0.252594	0.0088	0.0275	3.36E-07	0.1906	0.0290
CASINO GUICHARD	0.1023	0.0849	3.31E-03	0.4390	0.241523	0.0137	0.0255	8.48E-06	0.1520	0.0450
CREDIT AGRICOLE	0.1650	0.1843	3.70E-04	1.1034	0.453169	0.0106	0.0314	2.37E-08	0.2167	0.0347
DANONE	0.0763	0.0657	3.52E-03	0.3286	0.190152	0.0133	0.0247	1.60E-05	0.1439	0.0484
DEXIA	0.1291	0.2939	5.02E-04	2.0266	0.388591	0.0124	0.0701	3.30E-08	0.4956	0.0201
EADS	0.1745	0.1858	1.33E-04	1.0620	0.507217	0.0097	0.0263	2.72E-09	0.1760	0.0359
FRANCE TELECOM	0.1139	0.1657	1.91E-03	0.9721	0.345296	0.0084	0.0300	7.85E-07	0.2011	0.0254
L'OREAL	0.0980	0.1044	1.90E-03	0.4776	0.31271	0.0124	0.0269	2.27E-06	0.1375	0.0632
LAFARGE	0.1461	0.1767	1.61E-04	0.7172	0.665107	0.0306	0.0730	1.60E-08	0.3159	0.2544
LAGARDERE S.C.A.	0.1263	0.1245	6.82E-03	0.7049	0.348792	0.0133	0.0329	1.97E-05	0.2181	0.0498
LVMH	0.0893	0.1075	2.41E-03	0.4778	0.319032	0.0110	0.0259	3.23E-06	0.1320	0.0579
MICHELIN	0.1401	0.1266	4.09E-03	0.5544	0.442438	0.0118	0.0208	5.55E-06	0.0991	0.0653
PERNOD-RICARD	0.0920	0.1151	2.44E-03	0.6983	0.291567	0.0177	0.0584	4.82E-06	0.4041	0.0688

Table 12: (Continued) Summary of in-sample estimated costs of execution VWAP order for period from September 2, 2003 to December 16, 2003. The dynamics of specific component was modeled by classical approach basing on calculating means.

	MAPE					MSPE				
Company	Mean	Std	Min	Max	Q95	Mean	Std	Min	Max	Q95
PEUGEOT	0.1035	0.1084	9.21E-04	0.5013	0.287562	0.0083	0.0171	3.13E-07	0.0914	0.0300
PIN.-PRINT.REDOUTE	0.1373	0.1394	2.99E-03	0.7602	0.424165	0.0295	0.0676	6.55E-06	0.4099	0.1397
RENAULT	0.1497	0.1301	1.63E-03	0.5811	0.39367	0.0213	0.0366	1.43E-06	0.1921	0.0832
SAINT GOBAIN	0.1238	0.1338	1.83E-03	0.7419	0.319529	0.0110	0.0276	1.14E-06	0.1745	0.0361
SANOFI-AVENTIS	0.1063	0.1420	1.31E-04	0.8494	0.352434	0.0165	0.0563	8.88E-09	0.3864	0.0657
SCHNEIDER ELECTRIC	0.0991	0.0943	3.00E-03	0.5345	0.234314	0.0088	0.0196	4.41E-06	0.1293	0.0281
SOCIETE GENERALE	0.0939	0.0981	2.61E-03	0.4271	0.393898	0.0112	0.0250	4.17E-06	0.1100	0.0897
SODEXHO ALLIANCE	0.1386	0.1733	6.83E-04	0.9847	0.472744	0.0117	0.0339	1.17E-07	0.2283	0.0522
STMICROELECTRONICS	0.0989	0.1176	2.00E-03	0.5661	0.350686	0.0052	0.0123	8.79E-07	0.0674	0.0286
SUEZ	0.1365	0.1143	9.01E-04	0.5169	0.338701	0.0045	0.0068	1.22E-07	0.0369	0.0172
TF1	0.1070	0.1009	1.53E-03	0.5220	0.272837	0.0058	0.0129	6.06E-07	0.0792	0.0197
THALES	0.1320	0.1724	1.62E-03	0.7725	0.621652	0.0115	0.0285	6.62E-07	0.1432	0.0942
THOMSON (EX:TMM)	0.1762	0.2763	9.81E-04	1.6851	0.562888	0.0172	0.0671	1.65E-07	0.4518	0.0510
TOTAL	0.0683	0.0753	6.82E-04	0.3208	0.208425	0.0137	0.0284	6.22E-07	0.1380	0.0588
VEOLIA ENVIRON.	0.1071	0.1001	1.43E-04	0.4116	0.297758	0.0040	0.0067	3.88E-09	0.0321	0.0162
VINCI (EX.SGE)	0.0720	0.0744	2.38E-03	0.3527	0.219507	0.0066	0.0136	3.59E-06	0.0777	0.0298
VIVENDI UNIVERSAL	0.1529	0.1523	6.51E-04	0.7945	0.448559	0.0076	0.0161	6.69E-08	0.1015	0.0315

Table 13: Summary of in-sample estimated costs of execution VWAP order for period from September 2, 2003 to December 16, 2003. The dynamics of specific component was described by ARMA model.

	MAPE					MSPE				
Company	Mean	Std	Min	Max	Q95	Mean	Std	Min	Max	Q95
ACCOR	0.0952	0.0871	1.78E-03	0.3792	0.2903	0.0056	0.0103	1.06E-06	0.0522	0.0282
AGF-ASS.GEN.FRANCE	0.0985	0.0922	6.84E-04	0.5101	0.2351	0.0078	0.0168	2.05E-07	0.1126	0.0244
AIR LIQUIDE	0.0668	0.0706	7.07E-05	0.3700	0.2162	0.0118	0.0285	6.34E-09	0.1681	0.0596
ALCATEL	0.0919	0.0939	4.56E-04	0.4378	0.3043	0.0019	0.0042	2.18E-08	0.0204	0.0100
ARCELOR	0.1142	0.1261	1.49E-03	0.4826	0.4629	0.0033	0.0066	2.73E-07	0.0276	0.0245
AXA	0.1014	0.2357	1.98E-03	1.6533	0.2720	0.0107	0.0640	6.51E-07	0.4540	0.0121
BNP PARIBAS	0.0599	0.0487	4.06E-04	0.2168	0.1553	0.0026	0.0040	7.40E-08	0.0209	0.0105
BOUYGUES	0.1296	0.1026	9.17E-03	0.5991	0.3062	0.0067	0.0135	2.03E-05	0.0920	0.0228
CAP GEMINI	0.1403	0.1833	4.80E-03	1.1443	0.3913	0.0205	0.0714	9.94E-06	0.4989	0.0564
CARREFOUR	0.0639	0.0605	8.30E-04	0.2468	0.2276	0.0034	0.0062	3.05E-07	0.0274	0.0224
CASINO GUICHARD	0.0732	0.0483	4.38E-05	0.2164	0.1646	0.0060	0.0078	1.39E-09	0.0370	0.0217
CREDIT AGRICOLE	0.1059	0.1300	6.26E-03	0.8630	0.2361	0.0049	0.0187	6.95E-06	0.1326	0.0102
DANONE	0.0700	0.0712	1.45E-03	0.4283	0.1724	0.0130	0.0354	2.77E-06	0.2432	0.0393
DEXIA	0.0810	0.0677	4.23E-03	0.3215	0.2471	0.0014	0.0026	2.46E-06	0.0133	0.0082
EADS	0.1433	0.1478	4.94E-03	0.7029	0.4665	0.0062	0.0133	3.83E-06	0.0710	0.0304
FRANCE TELECOM	0.0781	0.1430	7.28E-03	0.9949	0.2501	0.0055	0.0297	1.16E-05	0.2107	0.0126
L'OREAL	0.0553	0.0399	2.87E-03	0.1985	0.1214	0.0028	0.0041	5.17E-06	0.0234	0.0089
LAFARGE	0.0863	0.0788	1.38E-03	0.3153	0.2573	0.0080	0.0126	1.11E-06	0.0585	0.0381
LAGARDERE S.C.A.	0.0956	0.0876	4.11E-04	0.4197	0.2609	0.0071	0.0135	7.36E-08	0.0773	0.0292
LVMH	0.0518	0.0538	2.26E-03	0.2684	0.1527	0.0031	0.0069	2.82E-06	0.0416	0.0126
MICHELIN	0.1172	0.0816	1.33E-03	0.4158	0.2416	0.0069	0.0095	6.28E-07	0.0576	0.0194
PERNOD-RICARD	0.0784	0.0809	2.54E-03	0.4079	0.2096	0.0104	0.0233	5.35E-06	0.1379	0.0356

Table 14: (Continued) Summary of in-sample estimated costs of execution VWAP order for period from September 2, 2003 to December 16, 2003. The dynamics of specific component was described by ARMA model.

	MAPE					MSPE				
Company	Mean	Std	Min	Max	Q95	Mean	Std	Min	Max	Q95
PEUGEOT	0.0731	0.0652	2.42E-03	0.2749	0.2085	0.0036	0.0059	2.15E-06	0.0274	0.0169
PIN.-PRINT.REDOUTE	0.0793	0.0813	3.62E-03	0.3642	0.2589	0.0102	0.0209	1.08E-05	0.1163	0.0543
RENAULT	0.0753	0.0597	1.29E-03	0.2633	0.2319	0.0051	0.0082	8.95E-07	0.0394	0.0312
SAINT GOBAIN	0.1002	0.0815	4.90E-04	0.3259	0.2651	0.0057	0.0081	8.37E-08	0.0337	0.0234
SANOFI-AVENTIS	0.0749	0.0877	1.02E-03	0.5433	0.1974	0.0070	0.0226	5.67E-07	0.1581	0.0204
SCHNEIDER ELECTRIC	0.0877	0.0721	4.73E-04	0.2982	0.2661	0.0062	0.0098	1.00E-07	0.0402	0.0362
SOCIETE GENERALE	0.0466	0.0367	1.31E-03	0.1637	0.1023	0.0022	0.0032	1.16E-06	0.0177	0.0066
SODEXHO ALLIANCE	0.0953	0.1012	6.09E-04	0.5330	0.3103	0.0047	0.0115	9.01E-08	0.0743	0.0216
STMICROELECTRONICS	0.0612	0.0540	9.73E-04	0.2394	0.1562	0.0015	0.0024	2.22E-07	0.0121	0.0054
SUEZ	0.0911	0.0765	3.57E-03	0.3406	0.2312	0.0020	0.0032	1.92E-06	0.0162	0.0081
TF1	0.1011	0.0967	2.38E-03	0.6163	0.2082	0.0053	0.0156	1.67E-06	0.1104	0.0112
THALES	0.1175	0.1395	8.50E-03	0.5471	0.4870	0.0082	0.0177	1.82E-05	0.0718	0.0586
THOMSON (EX:TMM)	0.0908	0.0990	2.79E-04	0.6205	0.2116	0.0029	0.0087	1.31E-08	0.0613	0.0075
TOTAL	0.0388	0.0368	8.22E-04	0.1533	0.1138	0.0038	0.0063	8.99E-07	0.0308	0.0176
VEOLIA ENVIRON.	0.0772	0.0707	9.98E-04	0.2832	0.2202	0.0020	0.0033	1.91E-07	0.0146	0.0091
VINCI (EX.SGE)	0.0492	0.0390	1.20E-04	0.1833	0.1233	0.0024	0.0037	9.12E-09	0.0210	0.0094
VIVENDI UNIVERSAL	0.0818	0.0708	3.17E-03	0.3719	0.1927	0.0019	0.0036	1.66E-06	0.0228	0.0069

Table 15: Summary of in-sample estimated costs of execution VWAP order for period from September 2, 2003 to December 16, 2003. The dynamics of specific component was described by SETAR model.

	MAPE					MSPE				
Company	Mean	Std	Min	Max	Q95	Mean	Std	Min	Max	Q95
ACCOR	0.0777	0.0823	4.36E-03	0.4442	0.2359	0.0042	0.0104	6.79E-06	0.0660	0.0180
AGF-ASS.GEN.FRANCE	0.0841	0.0977	8.45E-06	0.5449	0.2490	0.0072	0.0197	3.06E-11	0.1285	0.0288
AIR LIQUIDE	0.0666	0.0705	1.49E-04	0.4144	0.1771	0.0117	0.0316	2.85E-08	0.2108	0.0393
ALCATEL	0.0811	0.0980	3.48E-03	0.5847	0.2432	0.0018	0.0059	1.39E-06	0.0390	0.0064
ARCELOR	0.0900	0.0797	1.43E-04	0.3147	0.2459	0.0017	0.0026	2.45E-09	0.0113	0.0074
AXA	0.1030	0.2367	5.16E-04	1.6533	0.2823	0.0108	0.0641	4.19E-08	0.4540	0.0131
BNP PARIBAS	0.0601	0.0534	1.04E-03	0.2265	0.1663	0.0029	0.0045	4.86E-07	0.0228	0.0125
BOUYGUES	0.1240	0.0970	5.16E-03	0.4038	0.2876	0.0060	0.0086	6.20E-06	0.0393	0.0201
CAP GEMINI	0.1047	0.1732	1.59E-03	1.0626	0.3224	0.0157	0.0643	9.84E-07	0.4302	0.0383
CARREFOUR	0.0492	0.0464	4.39E-05	0.2017	0.1352	0.0020	0.0035	8.93E-10	0.0176	0.0084
CASINO GUICHARD	0.0646	0.0439	3.38E-04	0.1611	0.1530	0.0047	0.0055	8.33E-08	0.0200	0.0181
CREDIT AGRICOLE	0.0939	0.1032	9.31E-04	0.6922	0.1817	0.0034	0.0120	1.47E-07	0.0853	0.0056
DANONE	0.0536	0.0336	1.22E-03	0.1683	0.1135	0.0052	0.0066	1.91E-06	0.0373	0.0172
DEXIA	0.0763	0.0784	8.44E-04	0.3936	0.2565	0.0015	0.0035	9.00E-08	0.0205	0.0087
EADS	0.1265	0.1324	3.12E-03	0.6988	0.3869	0.0049	0.0110	1.44E-06	0.0701	0.0234
FRANCE TELECOM	0.0792	0.1502	6.01E-03	1.0195	0.2927	0.0060	0.0313	7.80E-06	0.2212	0.0172
L'OREAL	0.0463	0.0432	2.51E-04	0.2178	0.1425	0.0025	0.0051	3.91E-08	0.0305	0.0127
LAFARGE	0.1001	0.1033	6.04E-03	0.4688	0.2875	0.0120	0.0257	2.10E-05	0.1293	0.0467
LAGARDERE S.C.A.	0.0765	0.0647	2.03E-03	0.3079	0.2157	0.0042	0.0073	1.66E-06	0.0407	0.0197
LVMH	0.0517	0.0473	8.40E-04	0.2438	0.1348	0.0027	0.0053	4.03E-07	0.0344	0.0101
MICHELIN	0.1101	0.0830	2.75E-03	0.3588	0.2658	0.0064	0.0084	2.75E-06	0.0429	0.0235
PERNOD-RICARD	0.0707	0.0678	8.53E-04	0.3101	0.2397	0.0078	0.0151	6.02E-07	0.0797	0.0465

Table 16: (Continued)Summary of in-sample estimated costs of execution VWAP order for period from September 2, 2003 to December 2, 2003. The dynamics of specific component was described by SETAR model.

	MAPE					MSPE				
Company	Mean	Std	Min	Max	Q95	Mean	Std	Min	Max	Q95
PEUGEOT	0.0610	0.0500	1.79E-04	0.2504	0.1407	0.0023	0.0038	1.16E-08	0.0236	0.0072
PIN.-PRINT.REDOUTE	0.0782	0.0785	2.76E-03	0.3978	0.2471	0.0096	0.0209	6.65E-06	0.1283	0.0433
RENAULT	0.0808	0.0616	2.21E-03	0.2286	0.2133	0.0057	0.0076	2.65E-06	0.0297	0.0240
SAINT GOBAIN	0.0701	0.0521	2.33E-03	0.2509	0.1623	0.0026	0.0038	1.90E-06	0.0215	0.0095
SANOFI-AVENTIS	0.0640	0.0697	5.67E-04	0.3293	0.2291	0.0047	0.0110	1.71E-07	0.0562	0.0278
SCHNEIDER ELECTRIC	0.0844	0.0790	3.19E-03	0.3975	0.2287	0.0064	0.0124	4.80E-06	0.0715	0.0262
SOCIETE GENERALE	0.0526	0.0451	3.12E-03	0.1870	0.1459	0.0029	0.0044	6.38E-06	0.0203	0.0127
SODEXHO ALLIANCE	0.0852	0.1020	3.29E-03	0.5126	0.3249	0.0042	0.0115	2.80E-06	0.0687	0.0237
STMICROELECTRONICS	0.0597	0.0546	3.59E-04	0.2058	0.1929	0.0015	0.0024	2.94E-08	0.0091	0.0086
SUEZ	0.0976	0.0827	4.54E-03	0.3293	0.2602	0.0023	0.0036	2.90E-06	0.0150	0.0095
TF1	0.1109	0.1204	1.09E-02	0.7697	0.2645	0.0073	0.0244	3.51E-05	0.1723	0.0185
THALES	0.0961	0.1098	2.34E-03	0.4640	0.3503	0.0053	0.0114	1.30E-06	0.0517	0.0305
THOMSON (EX:TMM)	0.0941	0.0899	5.36E-03	0.4480	0.2728	0.0028	0.0054	5.38E-06	0.0319	0.0126
TOTAL	0.0397	0.0371	4.12E-04	0.1627	0.1164	0.0039	0.0067	2.35E-07	0.0347	0.0179
VEOLIA ENVIRON.	0.0723	0.0656	1.26E-04	0.3145	0.2007	0.0018	0.0032	2.84E-09	0.0180	0.0079
VINCI (EX.SGE)	0.0355	0.0259	2.49E-03	0.1209	0.0830	0.0012	0.0018	3.87E-06	0.0093	0.0043
VIVENDI UNIVERSAL	0.0661	0.0685	2.85E-03	0.3587	0.1749	0.0015	0.0033	1.27E-06	0.0212	0.0059

Table 17: Summary of out-sample estimated costs of execution VWAP order for period from September 2, 2003 to December 16, 2003. The dynamics of specific component was modeled by classical approach basing on calculating means.

	MAPE					MSPE				
Company	Mean	Std	Min	Max	Q95	Mean	Std	Min	Max	Q95
ACCOR	0.10473	0.120887	9.63E-05	0.551475	0.364007	0.008667	0.019298	3.31E-09	0.106835	0.047337
AGF-ASS.GEN.FRANCE	0.131635	0.143419	0.004299	0.780154	0.420724	0.016356	0.040833	7.97E-06	0.257852	0.081774
AIR LIQUIDE	0.080098	0.07862	0.000213	0.347809	0.267794	0.016298	0.030737	6.04E-08	0.151459	0.096209
ALCATEL	0.13355	0.121244	0.00334	0.470753	0.436743	0.003573	0.005914	1.17E-06	0.024805	0.020877
ARCELOR	0.117065	0.133359	0.001586	0.613585	0.351661	0.003782	0.008209	2.95E-07	0.042996	0.014443
AXA	0.092991	0.134509	0.00521	0.672537	0.386335	0.004239	0.013041	4.34E-06	0.073352	0.024652
BNP PARIBAS	0.078249	0.06503	0.000919	0.312203	0.208593	0.004732	0.007629	3.81E-07	0.04441	0.019698
BOUYGUES	0.171501	0.100734	0.000489	0.503438	0.308118	0.009813	0.011386	5.6E-08	0.061117	0.025105
CAP GEMINI	0.23231	0.295336	0.003557	1.30021	1.138396	0.054896	0.141549	4.96E-06	0.672608	0.479306
CARREFOUR	0.062836	0.059824	0.000457	0.249051	0.191973	0.003346	0.005899	8.99E-08	0.028191	0.016761
CASINO GUICHARD	0.146471	0.219818	0.005503	1.472293	0.439929	0.053252	0.236859	2.41E-05	1.675549	0.152666
CREDIT AGRICOLE	0.138865	0.197163	0.002775	1.078061	0.542391	0.010325	0.032196	1.44E-06	0.206887	0.050779
DANONE	0.054773	0.049043	1.06E-05	0.200395	0.156687	0.006984	0.011398	1.45E-10	0.051469	0.032055
DEXIA	0.109919	0.224349	0.001054	1.443772	0.536146	0.008178	0.039745	1.45E-07	0.277381	0.038698
EADS	0.194675	0.23971	0.007875	1.266802	0.593885	0.016546	0.04886	9.58E-06	0.29084	0.068889
FRANCE TELECOM	0.139777	0.211803	0.001373	1.138462	0.502473	0.013507	0.047594	4.05E-07	0.27759	0.051672
L'OREAL	0.086626	0.092238	0.004831	0.460046	0.276497	0.009811	0.0224	1.5E-05	0.130733	0.045345
LAFARGE	0.107592	0.137086	0.001946	0.659896	0.425454	0.018543	0.050934	2.36E-06	0.26746	0.120901
LAGARDERE S.C.A.	0.100285	0.083665	0.008624	0.394801	0.275215	0.00736	0.013494	3.25E-05	0.068864	0.033642
LVMH	0.113146	0.115475	0.001387	0.571628	0.325866	0.01497	0.034401	1.11E-06	0.18723	0.060437
MICHELIN	0.154091	0.206228	0.004786	1.109848	0.556597	0.022093	0.067374	7.74E-06	0.40964	0.103156
PERNOD-RICARD	0.077535	0.074666	0.003623	0.292375	0.245627	0.009856	0.016423	1.09E-05	0.072868	0.051538

Table 18: (Continued) Summary of out-sample estimated costs of execution VWAP order for period from September 2, 2003 to December 16, 2003. The dynamics of specific component was modeled by classical approach basing on calculating means.

	MAPE					MSPE				
Company	Mean	Std	Min	Max	Q95	Mean	Std	Min	Max	Q95
PEUGEOT	0.07616	0.091633	0.001125	0.448031	0.292859	0.00534	0.013486	5.03E-07	0.074993	0.031624
PIN.-PRINT.REDOUTE	0.138913	0.127963	0.000861	0.455642	0.416876	0.029041	0.045141	6.11E-07	0.174501	0.14263
RENAULT	0.140574	0.12552	0.001126	0.561398	0.364709	0.019575	0.031702	7.29E-07	0.179263	0.075486
SAINT GOBAIN	0.097856	0.085823	0.000143	0.291448	0.252967	0.006039	0.008351	7.75E-09	0.03071	0.023002
SANOFI-AVENTIS	0.099893	0.108369	0.000635	0.477165	0.379653	0.011763	0.025023	2.26E-07	0.128397	0.080006
SCHNEIDER ELECTRIC	0.086546	0.131574	0.003445	0.873692	0.225114	0.012561	0.055929	6.17E-06	0.394043	0.025897
SOCIETE GENERALE	0.069867	0.068663	0.000897	0.41165	0.208138	0.006245	0.016314	5.25E-07	0.108959	0.030047
SODEXHO ALLIANCE	0.123303	0.13401	0.001181	0.589764	0.414888	0.007619	0.016537	3.19E-07	0.08303	0.039809
STMICROELECTRONICS	0.090614	0.090484	0.001886	0.381984	0.258905	0.003741	0.006951	7.77E-07	0.033773	0.015468
SUEZ	0.096785	0.098772	0.004347	0.530998	0.28356	0.002744	0.006571	2.63E-06	0.043443	0.01122
TF1	0.110343	0.100563	0.002831	0.475552	0.290891	0.005941	0.010768	2.2E-06	0.060747	0.022388
THALES	0.09586	0.139949	0.000712	0.67571	0.45149	0.007064	0.02128	1.26E-07	0.112826	0.051969
THOMSON (EX:TMM)	0.145994	0.158754	0.013218	0.790626	0.424288	0.007884	0.016793	3.22E-05	0.104628	0.031645
TOTAL	0.052786	0.053217	0.001484	0.211851	0.163156	0.00753	0.013916	2.9E-06	0.060592	0.037718
VEOLIA ENVIRON.	0.12997	0.162395	0.000275	0.822174	0.408336	0.00836	0.023107	1.46E-08	0.138172	0.032558
VINCI (EX.SGE)	0.077354	0.108837	0.000834	0.595414	0.250262	0.011363	0.035806	4.46E-07	0.229369	0.039174
VIVENDI UNIVERSAL	0.109508	0.101303	0.00145	0.491998	0.288309	0.004099	0.007506	4.02E-07	0.047049	0.013952

Table 19: Summary of out-sample estimated costs of execution VWAP order for period from September 2, 2003 to December 16, 2003. The dynamics of specific component was described by ARMA model.

	MAPE					MSPE				
Company	Mean	Std	Min	Max	Q95	Mean	Std	Min	Max	Q95
ACCOR	0.101032	0.125263	3.65E-05	0.656141	0.406081	0.008812	0.023944	4.76E-10	0.151236	0.055117
AGF-ASS.GEN.FRANCE	0.100311	0.133961	7.15E-05	0.780385	0.260686	0.012014	0.038906	2.13E-09	0.258005	0.030916
AIR LIQUIDE	0.073266	0.073724	0.003349	0.339292	0.256931	0.01399	0.027697	1.4E-05	0.144132	0.087253
ALCATEL	0.098674	0.096642	0.001016	0.435166	0.25912	0.002094	0.004222	1.19E-07	0.020726	0.007399
ARCELOR	0.090767	0.090891	0.000691	0.382858	0.30878	0.00199	0.003707	5.71E-08	0.017965	0.011092
AXA	0.069464	0.091932	0.001602	0.472202	0.250792	0.002106	0.006311	4.1E-07	0.036161	0.010481
BNP PARIBAS	0.070067	0.059323	0.000462	0.297555	0.187281	0.003848	0.006829	9.54E-08	0.040341	0.015806
BOUYGUES	0.165915	0.082428	0.023921	0.41475	0.318636	0.008515	0.008201	0.000152	0.041817	0.025001
CAP GEMINI	0.178651	0.219114	0.002489	1.174109	0.561131	0.031315	0.081067	2.41E-06	0.52526	0.116454
CARREFOUR	0.06174	0.059977	0.000472	0.245889	0.193687	0.003294	0.00584	1.02E-07	0.027479	0.017062
CASINO GUICHARD	0.123652	0.179234	0.008428	1.247152	0.240706	0.036171	0.169248	5.37E-05	1.202284	0.043829
CREDIT AGRICOLE	0.104677	0.15018	0.001983	0.937612	0.327376	0.005932	0.022765	7.2E-07	0.156492	0.019785
DANONE	0.051885	0.04278	0.001511	0.184403	0.141413	0.005841	0.009211	2.94E-06	0.044261	0.02611
DEXIA	0.094336	0.199633	0.003382	1.415669	0.237838	0.006381	0.03761	1.51E-06	0.266688	0.007577
EADS	0.157575	0.206163	0.004654	1.316431	0.430686	0.011847	0.044937	3.2E-06	0.314075	0.033562
FRANCE TELECOM	0.107563	0.18937	0.002274	1.114558	0.252288	0.009981	0.041812	1.09E-06	0.266055	0.013434
L'OREAL	0.078233	0.087448	0.000789	0.431261	0.255377	0.008444	0.020237	3.82E-07	0.114885	0.041165
LAFARGE	0.083574	0.099491	0.000576	0.46409	0.347965	0.010337	0.024396	2.11E-07	0.132285	0.079173
LAGARDERE S.C.A.	0.100623	0.085864	0.00189	0.393571	0.316833	0.007572	0.013574	1.49E-06	0.06343	0.045744
LVMH	0.087971	0.095328	0.000482	0.510107	0.281609	0.009605	0.023348	1.36E-07	0.149097	0.045984
MICHELIN	0.141631	0.182732	0.002296	0.997271	0.423821	0.017847	0.05263	1.82E-06	0.330751	0.059811
PERNOD-RICARD	0.074763	0.073995	0.001046	0.342696	0.194887	0.00934	0.018008	8.97E-07	0.095102	0.034075

Table 20: (Continued) Summary of out-sample estimated costs of execution VWAP order for period from September 2, 2003 to December 16, 2003. The dynamics of specific component was described by ARMA model.

	MAPE					MSPE				
Company	Mean	Std	Min	Max	Q95	Mean	Std	Min	Max	Q95
PEUGEOT	0.065013	0.076633	0.00058	0.363543	0.246957	0.003806	0.009907	1.24E-07	0.049376	0.022488
PIN.-PRINT.REDOUTE	0.093109	0.093481	0.000849	0.3926	0.241447	0.014251	0.024458	6.17E-07	0.135122	0.048965
RENAULT	0.109862	0.096077	0.003059	0.45923	0.284489	0.011809	0.019894	5.01E-06	0.119953	0.043966
SAINT GOBAIN	0.09635	0.073971	0.00505	0.311034	0.246172	0.005266	0.007628	9.65E-06	0.034976	0.021783
SANOFI-AVENTIS	0.080981	0.085475	0.000925	0.470069	0.234014	0.007541	0.018552	4.61E-07	0.124607	0.029378
SCHNEIDER ELECTRIC	0.089963	0.137223	0.000771	0.919957	0.276185	0.013613	0.061775	2.83E-07	0.43688	0.038981
SOCIETE GENERALE	0.05898	0.059823	0.000929	0.329148	0.191882	0.004573	0.011581	5.4E-07	0.069661	0.024269
SODEXHO ALLIANCE	0.090263	0.085281	0.006608	0.386401	0.333731	0.003547	0.007206	1E-05	0.03453	0.026587
STMICROELECTRONICS	0.077424	0.082216	0.0011	0.354715	0.258909	0.002905	0.006086	2.83E-07	0.029251	0.015468
SUEZ	0.081059	0.066647	0.0007	0.265047	0.225211	0.001557	0.002317	6.74E-08	0.009764	0.00749
TF1	0.100125	0.08724	0.000237	0.411404	0.261296	0.00469	0.008132	1.45E-08	0.045463	0.018064
THALES	0.090614	0.119713	0.001196	0.544583	0.373197	0.00555	0.015036	3.57E-07	0.073285	0.035508
THOMSON (EX:TMM)	0.088386	0.098192	0.001182	0.577237	0.260797	0.002953	0.008321	2.38E-07	0.055772	0.01154
TOTAL	0.046162	0.048694	0.001695	0.216688	0.145632	0.006017	0.011929	3.84E-06	0.06339	0.02866
VEOLIA ENVIRON.	0.107108	0.097232	0.000563	0.437048	0.306249	0.004014	0.007298	6.09E-08	0.035858	0.019171
VINCI (EX.SGE)	0.069448	0.082488	0.000468	0.413568	0.195252	0.007453	0.019448	1.48E-07	0.11066	0.023845
VIVENDI UNIVERSAL	0.082903	0.064668	6.57E-05	0.235136	0.209637	0.002027	0.002635	8.05E-10	0.010507	0.007376

Table 21: Summary of out-sample estimated costs of execution VWAP order for period from September 2, 2003 to December 16, 2003. The dynamics of specific component was described by SETAR model.

	MAPE					MSPE				
Company	Mean	Std	Min	Max	Q95	Mean	Std	Min	Max	Q95
ACCOR	0.090626	0.137886	0.002348	0.649917	0.536377	0.009189	0.029316	1.93E-06	0.144187	0.096162
AGF-ASS.GEN.FRANCE	0.102291	0.133633	0.000582	0.779412	0.286914	0.012129	0.039268	1.41E-07	0.257362	0.03677
AIR LIQUIDE	0.072565	0.066163	0.000213	0.300349	0.176441	0.012488	0.021743	6.07E-08	0.112945	0.041765
ALCATEL	0.084513	0.090354	0.000423	0.457669	0.278512	0.00167	0.003795	2.06E-08	0.022925	0.008392
ARCELOR	0.066488	0.062096	0.005302	0.275934	0.179569	0.001007	0.001882	3.76E-06	0.008858	0.003864
AXA	0.072036	0.105963	0.001963	0.634953	0.220673	0.002605	0.009807	5.93E-07	0.065383	0.007983
BNP PARIBAS	0.07099	0.058753	0.001241	0.324374	0.184274	0.003878	0.007371	6.93E-07	0.04794	0.015302
BOUYGUES	0.162325	0.083088	0.015894	0.362491	0.310375	0.008194	0.007476	5.86E-05	0.031943	0.023393
CAP GEMINI	0.144827	0.195543	0.001048	1.066173	0.573595	0.023025	0.068357	4.88E-07	0.433125	0.121685
CARREFOUR	0.053692	0.04869	0.000588	0.173051	0.162094	0.00233	0.003645	1.57E-07	0.013611	0.01148
CASINO GUICHARD	0.105362	0.187273	0.001999	1.319837	0.205416	0.035152	0.189768	3.02E-06	1.346507	0.032469
CREDIT AGRICOLE	0.090236	0.132885	2.55E-05	0.838342	0.186043	0.004565	0.01835	1.18E-10	0.125109	0.005974
DANONE	0.045923	0.040887	0.003067	0.200447	0.122816	0.00486	0.009038	1.21E-05	0.052298	0.019593
DEXIA	0.084817	0.184903	0.001923	1.320052	0.207549	0.005415	0.032714	4.86E-07	0.231879	0.00577
EADS	0.143373	0.197349	0.003053	1.31882	0.353095	0.010522	0.044449	1.52E-06	0.315216	0.024352
FRANCE TELECOM	0.100619	0.190161	0.001971	1.098809	0.2805	0.009736	0.041666	8.27E-07	0.25859	0.01709
L'OREAL	0.0698	0.084434	0.002911	0.428982	0.208267	0.007348	0.020075	5.27E-06	0.113674	0.027378
LAFARGE	0.096378	0.13112	7.31E-05	0.637217	0.473039	0.016177	0.044913	3.48E-09	0.249392	0.131625
LAGARDERE S.C.A.	0.081552	0.070788	0.001943	0.307987	0.254365	0.005055	0.008835	1.57E-06	0.038843	0.028124
LVMH	0.091313	0.12251	0.000786	0.766689	0.285854	0.013293	0.048341	3.72E-07	0.336811	0.048951
MICHELIN	0.13799	0.174418	0.002542	0.914142	0.573479	0.016481	0.047011	2.17E-06	0.277909	0.109736
PERNOD-RICARD	0.053228	0.055811	0.000956	0.281365	0.182382	0.004956	0.010665	7.61E-07	0.064108	0.026978

Table 22: (Continued)Summary of out-sample estimated costs of execution VWAP order for period from September 2, 2003 to December 2, 2003. The dynamics of specific component was described by SETAR model.

	MAPE					MSPE				
Company	Mean	Std	Min	Max	Q95	Mean	Std	Min	Max	Q95
PEUGEOT	0.059045	0.06552	0.000591	0.31294	0.225685	0.002938	0.007139	1.35E-07	0.036587	0.019138
PIN.-PRINT.REDOUTE	0.077759	0.077514	0.000343	0.237852	0.223485	0.009731	0.014472	9.18E-08	0.045605	0.04205
RENAULT	0.107648	0.093682	0.001188	0.41558	0.266571	0.011348	0.017524	7.85E-07	0.098234	0.040327
SAINT GOBAIN	0.089499	0.064233	0.001668	0.305529	0.225959	0.004355	0.006521	9.98E-07	0.034689	0.018459
SANOFI-AVENTIS	0.070683	0.081308	0.000302	0.468029	0.21672	0.006318	0.018496	4.91E-08	0.123527	0.025925
SCHNEIDER ELECTRIC	0.078796	0.128382	0.003367	0.863407	0.219359	0.011501	0.05438	5.79E-06	0.38482	0.024058
SOCIETE GENERALE	0.065305	0.068449	0.009275	0.37611	0.199134	0.005834	0.014646	5.41E-05	0.090957	0.027321
SODEXHO ALLIANCE	0.080608	0.085728	0.001775	0.380575	0.323014	0.003182	0.007198	7.18E-07	0.033497	0.02476
STMICROELECTRONICS	0.080173	0.099279	0.002199	0.585146	0.246765	0.003717	0.011863	1.11E-06	0.079598	0.014051
SUEZ	0.072454	0.066261	0.00277	0.325311	0.206004	0.001351	0.002569	1.06E-06	0.014708	0.005891
TF1	0.089901	0.080429	0.001446	0.370885	0.260495	0.003856	0.006832	5.38E-07	0.036949	0.018702
THALES	0.078236	0.086654	0.000947	0.364021	0.333167	0.003388	0.007474	2.24E-07	0.032745	0.027787
THOMSON (EX:TMM)	0.078428	0.062133	0.003129	0.26672	0.218878	0.001717	0.002593	1.54E-06	0.01207	0.008421
TOTAL	0.049591	0.05375	0.000242	0.222192	0.178904	0.007152	0.01504	7.68E-08	0.066651	0.04288
VEOLIA ENVIRON.	0.089907	0.092122	0.000975	0.416781	0.309455	0.003187	0.006369	1.76E-07	0.033918	0.019574
VINCI (EX.SGE)	0.055948	0.0706	0.000422	0.376909	0.147256	0.005221	0.016464	1.21E-07	0.091912	0.014218
VIVENDI UNIVERSAL	0.074614	0.06696	0.002236	0.270463	0.22277	0.001851	0.002939	9.75E-07	0.013901	0.009113

Table 23: Comparison of VWAP predictions, based on mean absolute percentage error (MAPE).

Models	*Mean*	*STD*	*Min*	*Max*	*Q95*
Result of in-sample estimation					
SETAR	0.0706	0.0825	0.0017	0.4526	0.2030
ARMA	0.0772	0.0877	0.0019	0.4813	0.2173
Classical approach	0.1140	0.1358	0.0017	0.7054	0.3702
Result of out-sample estimation					
SETAR	0.0770	0.0942	0.0020	0.5070	0.2432
ARMA	0.0833	0.0956	0.0017	0.5009	0.2498
Classical approach	0.1006	0.1171	0.0025	0.5787	0.3427

Note:The cost is expressed in as percentage of the end of day volume weighted price. The classical approach based on calculation averages from historical volume data.

Table 24: Comparison of VWAP predictions, based on mean square percentage error (MSPE).

Models	*Mean*	*STD*	*Min*	*Max*	*Q95*
Result of in-sample estimation					
SETAR	0.0052	0.0156	2.28E-06	0.0988	0.0187
ARMA	0.0060	0.0173	2.37E-06	0.1119	0.0211
Classical approach	0.0138	0.0366	2.52E-06	0.2124	0.0644
Result of out-sample estimation					
SETAR	0.0077	0.0240	4.48E-6	0.1484	0.0335
ARMA	0.0083	0.0231	3.84E-06	0.1449	0.0318
Classical approach	0.0120	0.0318	4.50E-06	0.1810	0.0645

Note:The cost is expressed in as percentage of the end of day volume weighted price. The classical approach based on calculation averages from historical volume data.

Figure 4: The graphs present the in sample absolute percentage errors of VWAP's prediction against time. The forecast for classical approach is denoted by solid line. The forecast for models based on PCA volume decomposition is denoted by dash line. The left panel presents outcome for application ARMA model for specific part, and right panel for SETAR model respectively.

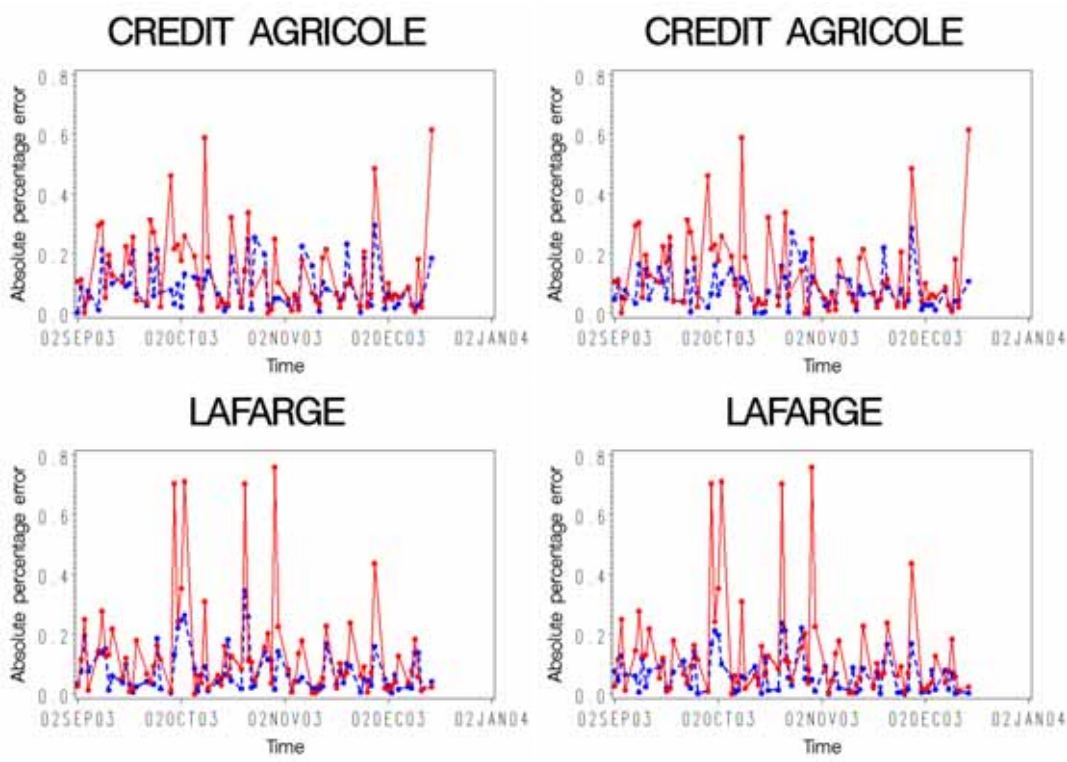

Figure 5: The graphs present the out sample absolute percentage errors of VWAP's prediction against time. The forecast for classical approach is denoted by solid line. The forecast for models based on PCA volume decomposition is denoted by dash line. The left panel presents outcome for application ARMA model for specific part, and right panel for SETAR model respectively.

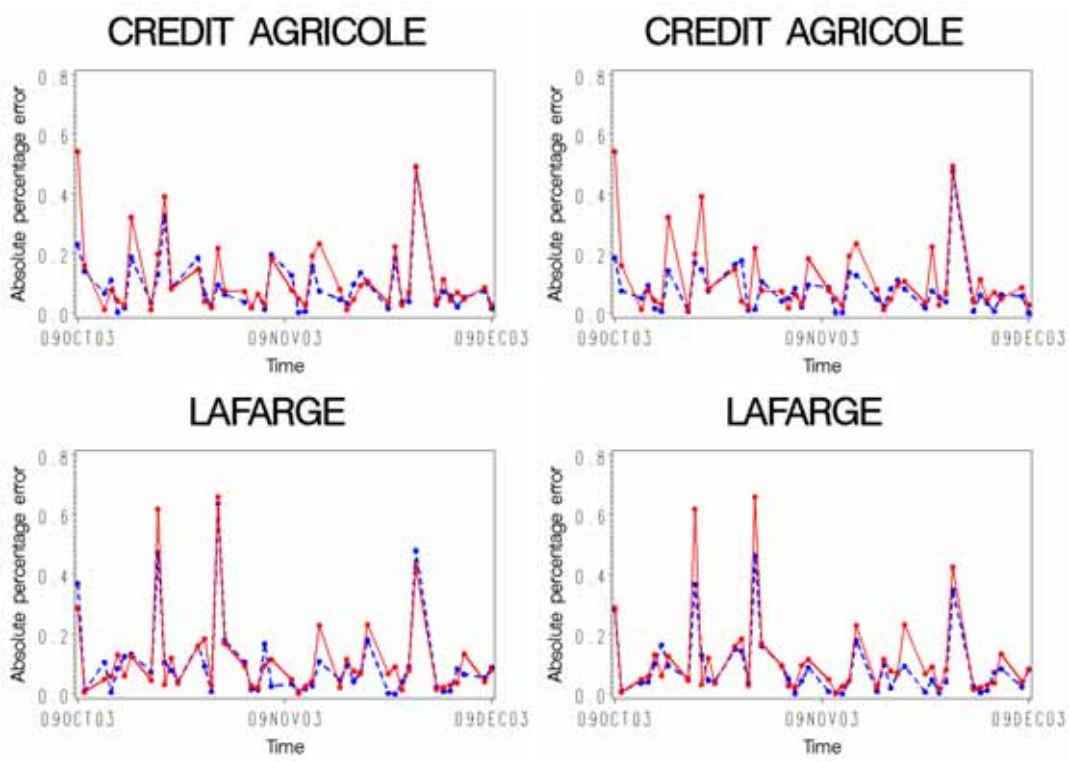

Table 25: Summary of estimated costs of execution VWAP order for different intraday volume models. The panels present summary in cases when estimated volume weighed prices are smaller or higher from observed ones, upper and lower panel respectively.

Models	*Mean*	*Frequency*	*STD*	*Min*	*Max*	*Q95*
SETAR	0.0751	49.2	0.0924	0.0016	0.5681	0.2032
ARMA	0.0824	49.9	0.0915	0.0016	0.5291	0.2300
Classical approach	0.1122	52.0	0.1358	0.00158	0.7661	0.3527
SETAR	0.0795	50.8	0.0881	0.0013	0.5023	0.2340
ARMA	0.0856	50.1	0.0910	0.0020	0.5040	0.2471
Classical approach	0.1147	48.0	0.1310	0.0019	0.7231	0.3390

Note:The cost is expressed in as percentage of the end of day volume weighted price. The classical approach based on calculation averages from historical volume data.