INSTITUT NATIONAL DE LA STATISTIQUE ET DES ETUDES ECONOMIQUES
Série des Documents de Travail du CREST
(Centre de Recherche en Economie et Statistique)

n° 2005-15

The Optimal Grouping of Commodities for Indirect Taxation

P. BELAN[1]
S. GAUTHIER[2]
G. LAROQUE[3]

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[1] LEN (Université de Nantes) and EUREQua (Université de Paris 1).
[2] CREST and CREM (Université de Caen).
[3] CREST-INSEE and GRECSTA.
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Pascal Belan[1] Stéphane Gauthier[2] Guy Laroque[3]

September 27, 2005

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[2]CREST and CREM (Université de Caen).

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Abstract

Indirect taxes contribute to a sizeable part of government revenues around the world. Typically there are a few different tax rates, and the goods are partitioned into classes associated with each rate. The present paper studies how to group the goods in these few classes. We take as given the number of tax rates and study the optimal aggregation (or classification) of commodities of the fiscal authority in a second best setup. The results are illustrated on data from the United Kingdom.

Les impôts indirects forment une part notable des recettes fiscales. D'ordinaire, on observe un petit nombre de taux différents, et les biens sont répartis en classes associées à chacun de ces taux. On étudie ici comment grouper les biens au mieux. Le nombre de taux est supposé fixé de manière exogène, et on résoud le problème d'agrégation (ou de classement) optimal des biens dans un cadre de second rang. Les résultats sont illustrés sur des données britanniques.

Keywords: indirect tax, Ramsey, aggregation.
JEL classification numbers: H21, H23

1 Introduction

Indirect taxes contribute to a sizeable part of government revenues around the world. Typically there is a small number of different tax rates, possibly including no tax, exemption, and the goods are partitioned into classes associated with each rate. The present paper studies how to allocate the goods in these few classes. We take as exogenously given the number of tax rates and study the optimal aggregation (or classification) of commodities of the fiscal authority in a second best setup[1] under a redistribution motive.

When there are no constraints on the choice of tax rates, the popular Ramsey rule applies. It is most easily described when there are no cross price elasticities between goods. Each commodity then can be assigned two numbers: the elasticity of its demand with respect to price and its social weight, which reflects its relative usage by the consumers in the population. At a given social weight, the optimal tax rate is inversely proportional to the price elasticity; given the elasticity, the optimal tax rate decreases with the social weight.

The relevance of the Ramsey rule is examined here when the government can only use an *a priori* given (small) number of tax rates. Such a situation is likely to occur when there are important costs in the administration of the indirect tax system, as discussed by Yitzhaki (1979) or Slemrod (1990). Given a social weight, should the goods with similar price elasticities be lumped together, the less price elastic group supporting the larger tax rate? Given a price elasticity, does a similar statement hold for social weights?

The literature is almost silent on the topic. Gordon (1989) analyzes how goods should be clustered together in a tax reform perspective, starting from a situation of a uniform tax. In the absence of a redistribution motive, Belan and Gauthier (2004a) and Belan and Gauthier (2004b) show that the Ramsey rule applies in a framework with a finite number of goods, for low levels of collected tax.

We consider an economy with a continuum of goods, each of them being negligible with respect to the total. This allows to abstract from the difficulties associated with discrete optimization: it is then possible to remove an elementary commodity from one group and to insert it into another group leaving unchanged the aggregate tax structure, e.g., the social cost of public funds. In this setup, an important tool for the analysis is the contribution to aggregate welfare of an elementary good, seen as a function of its own tax rate, all other rates being kept

[1]Under strong separability assumptions, indirect taxes are useless in the presence of nonlinear income taxation, as in Atkinson and Stiglitz (1976) and Mirrlees (1976). We place ourselves in the more general situation where indirect taxes are not trivial. This is typically the case when consumers have different tastes (Saez (2002)), for certain types of production functions (Stiglitz (1982), Naito (1999) or Saez (2004)), if it is possible to evade tax (Boadway, Marchand, and Pestieau (1994)), in order to correct externalities (Green and Sheshinski (1976)), in presence of uncertainties (Cremer and Gahvari (1995)), or when the authority implementing direct taxes is not perfectly coordinated with the one that designs indirect taxes, possibly because the decisions are taken at different points in time or in space (federal, state or city levels).

fixed. Indeed, it is only when this function is *unimodal*, or *single-peaked*, that the Ramsey rule, derived from first-order conditions, is sufficient for second best optimality and can be of some guidance to the grouping problem. In terms of fundamentals, the single-peakedness assumption is satisfied if both the elasticity and social weight are non decreasing with the tax rate. However, when the single-peakedness assumption does not hold, i.e. the contribution of a good to social welfare displays several modes, the aggregation problem has a global nature. Then the (local) Ramsey rule becomes irrelevant and the monotonicity properties of the optimal tax rate with respect to the price elasticity or the social weight of the commodity are unlikely to hold any longer.

Under *single-peakedness*, the optimal grouping can be characterized with the help of the tax rate t^R (R for Ramsey) which would maximize social welfare for any given elementary good, when all the other rates are fixed. Namely, such a rate would apply to the corresponding good if the social planner could tax it freely, while the tax rates supported by the remaining commodities are unchanged, set at the constrained optimum. Then, a natural property is that the optimal grouping puts this good in one of the groups which are taxed at the rates closest to t^R, either immediately below or above t^R. By the Ramsey formula, t^R is linked to the price elasticity and social weight of the good, evaluated at this putative free optimum. In this sense, the standard rules of thumb seem essentially preserved.

Indeed, in the special case where the price elasticities and social weights do not vary with the tax rates, *single-peakedness* holds and a stronger result can be shown. In the space of commodity characteristics (price elasticity, social weight), the optimal grouping is associated with nice connected regions. The Ramsey monotonicity properties are satisfied: given the price elasticity, the tax rate is non increasing with the social weight; given the social weight, the tax rate is typically non increasing with the price elasticity.

These results may suggest that *single-peakedness* is the panacea. However, even under *single-peakedness*, there is an important caveat to a blind application of the Ramsey rule. This rule applies to the local properties of the good at the putative free optimum t^R, and *not* at the actual observed (constrained) tax rate. In fact, there are examples in which tax rates are increasing with the (observed) elasticities at the constrained optimum. More strikingly, it is also possible that the ranking of tax rates at the constrained optimum be the reverse of that of unconstrained Ramsey rates.

This analysis is applied to data from the United Kingdom. To this end, the arguments developed in separable economies are adapted to the case where separability does not hold. Assuming that the observed taxes are optimal, we recover the implicit redistributive aims of the government. We find that the social weights that best fit put most of the weight on the fourth decile of consumption. For these social weights, the actual commodity groupings do not look far from optimality. The main departures are Adult clothing and Leisure goods which could be taxed more heavily than at the standard rate. Also Petrol and diesel and Beer should be much less taxed from an equity view point: but it is likely that other considerations (environment, public health) matter in such cases.

The paper is organized as follows. The bulk of the analysis assumes separability between goods. The general framework is laid out in the next section. Then the standard first-order conditions for optimality and the single-peakedness assumption are presented when there are no constraints on the number of tax rates. Sections 4 and 5 derive necessary conditions for optimality for a (small) given number of tax rates. The following section studies in some detail the case of constant elasticities. Caveats in the practical application of the optimality criterion are discussed in section 7. Section 8 indicates how the results can be applied to the real life non separable economies. The analysis is illustrated on data from the United Kingdom in the final section.

2 Consumers

The typical consumer in the economy is designated with an index c in C, and her tastes are represented by an additively separable utility function. Consumer c maximizes her utility function

$$\int_G u(x_g, g, c)\mu(g)dg + m$$

under her budget constraint

$$\int_G (1 + t_g)x_g\mu(g)dg + m \leq w_c.$$

The utility function u, defined over $\mathbb{R}_+ \times G \times C$, is assumed to be concave and twice continuously differentiable with respect to consumption x_g, x_g in $\mathbb{R}_+$, and continuous with respect to the good g, g in G, and consumer c characteristics. The sets C and G are subsets of some Euclidean space. The consumption of numéraire is denoted by m.

The relative importance of the various commodities g is partially captured by their density $\mu(g)$ with respect to the Lebesgue measure. It should be emphasized that all commodities are small, their measure is absolutely continuous with respect to Lebesgue, excluding mass points (see Belan and Gauthier (2004b) for an analysis when there are a finite number of goods). The units of commodities are chosen so that all producer prices equal 1. Commodities are taxed linearly and the tax rate supported by commodity g is denoted t_g (t_g is a number larger than -1); when t_g is negative, the good in fact is subsidized. Finally, w_c is the exogenous income of consumer c.

The strong separability assumptions imply that the overall maximization is equivalent to separate maximizations on each quantity of good x_g

$$u(x_g, g, c) - (1 + t_g)x_g,$$

with m defined residually through the budget constraint. Under the usual Inada conditions, the demand $(\xi_g(t_g, c)))$ of commodity g by consumer c is the unique solution of the first-order condition:

$$u'_x(x, g, c) = 1 + t_g.$$

It is decreasing and twice continuously differentiable with respect to the tax rate. The indirect utility of consumer c from consuming good g, when she is confronted to the tax rate t_g, is:

$$v_g(t_g, c) = u[\xi_g(t_g, c), g, c] - (1 + t_g)\xi_g(t_g, c).$$

The function $v_g(t, c)$ is a convex decreasing differentiable function, and satisfies Roy's identity

$$\frac{\partial v}{\partial t} = -\xi.$$

The overall indirect utility of consumer c is

$$\int_G v_g(t_g, c)\mu(g)dg + w_c.$$

3 Optimal tax schedules

An economy is defined as a probability measure ν on the set C of consumer characteristics. The aggregate quantities, summed over the set of consumers, are denoted with capital letters. The aggregate demand for good g is

$$X_g(t_g) = \int_C \xi_g(t_g, c)d\nu(c).$$

We are interested in the design of indirect taxes. The government takes as given the market behavior of the consumers. It seeks to maximize the sum of the utilities of the consumers in the economy, weighted by some *a priori* weights $\alpha(c)$, $\alpha(c) \geq 0$ for all c, normalized so that

$$\int_C \alpha(c)d\nu(c) = 1.$$

Using the separability of the individual utility functions, the objective of the government can be written as the sum over the goods of the aggregate indirect utility functions V_g, i.e.,

$$\int_G V_g(t_g)\mu(g)dg,$$

where

$$V_g(t_g) = \int_C \alpha(c)v_g(t_g, c)d\nu(c).$$

We shall often use the derivative of V_g with respect to the tax rate, which can be written as

$$\frac{dV_g}{dt_g}(t) = -a_g(t)X_g(t),$$

where

$$a_g(t) = \int_C \alpha(c)\frac{\xi_g(t,c)}{X_g(t)}d\nu(c)$$

is a positive number which measures the social weight of good g. Namely, it is large when the agents c with the largest weights α consume relatively more of the good.

If there is no constraint on rates setting, when fiscal income to be collected is R, welfare maximization can be written as

$$\max_{\mathbf{t}} \int_G V_g(t_g)\mu(g)dg$$

under the budget constraint[2]

$$\int_G t_g X_g(t_g)\ \mu(g)dg = R.$$

The Lagrangian function associated with this problem is

$$\int_G \mathcal{L}_g(t_g)\mu(g)dg,$$

where the contribution of good g, after division by $\mu(g)$, to the welfare objective is equal to

$$\mathcal{L}_g(t_g) = V_g(t_g) + \lambda t_g X_g(t_g).$$

Under regularity conditions, at the optimum, one can interpret the multiplier associated with the budget constraint λ as the marginal cost of public funds. If the authority freely chooses the tax rate bearing on good g, the necessary first-order condition for an interior optimum is

$$-a_g(t)X_g(t) + \lambda\left(X_g(t) + tX_g'(t)\right) = 0,$$

or, dropping the index g to simplify notations,

$$\frac{t}{1+t} = \frac{\lambda - a}{\lambda}\frac{X}{-(1+t)X'}. \tag{1}$$

[2]As often in the literature, we assume a linear technology, such that one unit of numeraire can be transformed into any bundle of commodities (X_g) with $\int_G X_g\mu(g)dg = 1$.

This corresponds to the celebrated Ramsey rule, in which the tax rate applying to a consumption good is inversely related to the price elasticity $-(1+t)X'/X$ of the demand for this good[3].

Note that *a priori* the program is *not* well behaved, since the maximand is convex in t_g. The following assumption, which we shall use repeatedly in the paper, helps to put in perspective the use of the first-order condition:

Assumption 1 *A good g satisfies the* single peaked assumption*, given the marginal cost of public funds λ, when the function $\mathcal{L}_g$, defined on* $(-1,+\infty)$*, satisfies one of the following three properties:*

1. *It is increasing;*
2. *It is increasing from -1 to some $\tau_g(\lambda)$ and decreasing from then on;*
3. *It is decreasing.*

Note that when λ is very large, $\mathcal{L}_g/\lambda$ is approximately equal to the tax receipts, so that Assumption 1 implies that the Laffer curve is single peaked. In the normal situation of Assumption 1.2, the Ramsey first-order condition has a unique solution which characterizes the optimum. The analysis is easily extended when the solution goes to the boundaries of the tax domain: Under Assumption 1.1 (resp. Assumption 1.3), the optimal tax rate is equal to $+\infty$: the good is made infinitely expensive (resp. to -1: the good is made free).

One can identify a number of circumstances where the single peaked assumption holds. Since both λ and X are positive, the derivative

$$\mathcal{L}' = -aX + \lambda(X + tX')$$

has the same sign as $(\lambda - a)/\lambda + tX'/X$. In the absence of redistributive motive, a is equal to 1. Whenever the elasticity of aggregate demand with respect to the tax rate, tX'/X, is non increasing in t, $\mathcal{L}'$ at most has one change of sign, and $\mathcal{L}$ is single peaked. This is the case when the price elasticity of demand is constant, $X = A(1+t)^{-\varepsilon}$, since $tX'/X = -\varepsilon t/(1+t)$ is decreasing.

[3] This equation would also hold in a model with endogenous labor supply and non linear direct taxation, provided that separability extends to leisure as in the following utility function

$$\int_G u(x_g, g, c)\mu(g)dg - d(L,c) + m.$$

As is well known since Atkinson and Stiglitz (1976), indirect taxation may be useless when (unconstrained) non linear direct taxes are allowed. This is the case when $\lambda = a_g$ for all consumption goods g, a condition unlikely to be satisfied when the agents do not have the same tastes, as emphasized in the recent literature, e.g. in Saez (2002).

In the case of redistribution, a typically varies with t. Single peakedness still holds if in addition of a decreasing tax elasticity of demand, one assumes that a increases with t: the larger the tax rate, the larger the social weight of the good, i.e. the demand for the good of the socially unfavored (rich) agents relatively decreases in comparison to that of the socially favored (poor) agents.

Still, there are demand functions which do not satisfy Assumption 1. Indeed, the assumption is not preserved under aggregation: the union of goods for which Assumption 1 holds individually does not necessarily satisfy the assumption[4]. For instance, the union of two goods with constant price elasticity of demand may lead to a two peaks Lagrangian: with $X = A_1(1+t)^{-\varepsilon_1} + A_2(1+t)^{-\varepsilon_2}$, when the value of λ is away from 1 and the elasticities are enough apart so that the maximum of each curve occurs in the flat part of the other, the function $\mathcal{L}$ has two different local maxima[5] (see Figure 2 below).

4 Optimization with a fixed number of tax rates

The implementation of the tax system is likely to entail fixed costs for each different tax rate: the associated list of goods has to be defined, the tax rate has to be enforced. In such a circumstance, the Ramsey rule of the preceding section would be too costly to implement, and at the optimum there will only be a finite number of tax rates. In the next three sections, we study the properties of the optimum when the only implementation restriction is that there is an *a priori* given finite number K of different tax rates. We note these tax rates t_k, $k = 1, \ldots, K$, and, without loss of generality, we assume that they are ranked in increasing order, $t_k \leq t_{k+1}$ for all k. In some cases, we add the additional constraint that one of the tax rates is equal to zero, but this feature is inessential for most of the analysis. Let G_k be the subset of goods which are taxed at rate t_k and $\mathbf{G}$ the collection of G_k. The government program becomes:

$$\begin{cases} \max\limits_{\mathbf{t},\mathbf{G}} \sum\limits_{k\in K} \int_{G_k} V_g(t_k)\mu(g)dg \\ \sum\limits_{k\in K} \int_{G_k} t_k X_g(t_k)\ \mu(g)dg = R \\ \bigcup\limits_{k\in K} G_k = G. \end{cases} \tag{2}$$

The government now has to choose the K tax rates (or possibly $K-1$, if one of them is constrained to be equal to zero) and the partition of the set of commodities associated with the various tax rates. Formally, this is a more complicated

[4]The demand function of the aggregate is the sum of the individual demand functions. The sum of single peaked functions is not single peaked in general (while concavity is preserved by summation).

[5]The figure is drawn with $\lambda = 1.25$, $\varepsilon_1 = 0.25$, $A_1 = 1.8$, $\varepsilon_2 = 4$ and $A_2 = 1$.

problem than the Ramsey problem, since it involves the variables $\mathbf{G}$, to which the standard Lagrangian methods do not immediately apply[6].

However, given the partition $\mathbf{G}$, the problem is standard. Under usual regularity conditions, one can write the Lagrangian and the multiplier λ associated with the government budget constraint is equal to the derivative of the objective function with respect to R. When differentiating with respect to the tax rates, to get the analogue of the Ramsey rule, it is natural to consider the *aggregate* commodity G_k, the demand of which is defined as

$$X_{G_k}(t) = \int_{G_k} X_g(t)\mu(g)dg.$$

The necessary first-order condition associated with t_k, first derived in Diamond (1973), is:

$$\int_{G_k} [-a_g(t_k)X_g(t_k) + \lambda(X_g(t_k) + t_k X_g'(t_k))]\mu(g)dg = 0,$$

or

$$\frac{t_k}{1+t_k} = \frac{\lambda - a_{G_k}}{\lambda} \frac{X_{G_k}}{-(1+t_k)X'_{G_k}}. \tag{3}$$

The social weight of the aggregate good is the average of the social weights of the individual commodities:

$$a_{G_k} = \int_{G_k} \frac{X_g(t_k)\mu(g)}{\int_{G_k} X_g \mu(g)dg} a_g(t_k)dg.$$

[6] There is no vector space structure on the variables G, and therefore no way to differentiate with respect to G. A possible device to put a differentiability structure on the set of variables is to abstract from the economic context and to do as if it were possible to tax parts of good g at the various available rates. Let $\pi_k(g)$ be the fraction of good g subject to rate t_k, where $\boldsymbol{\pi}= [\pi_k(g), k = 1, \dots, K]$, is a vector of positive measurable functions, defined on G, of square integrable with respect to the measure $\mu(g)dg$. The program then becomes

$$\begin{cases} \max\limits_{\mathbf{t},\boldsymbol{\pi}} \sum\limits_{k\in K} \int \pi_k(g)V_g(t_k)\mu(g)dg \\ \sum\limits_{k\in K} \int \pi_k(g)t_k X_g(t_k)\ \mu(g)dg = R \\ \pi_k(g) \geq 0, \qquad \sum\limits_k \pi_k(g) = 1. \end{cases} \tag{2'}$$

where the variables maximized upon are $(\mathbf{t}, \boldsymbol{\pi})$ in $\mathbb{R}^K \times L_2^K(G)$ instead of $(\mathbf{t}, \mathbf{G})$. The only solutions of economic relevance are such that the functions π take only two values, either 0 or 1. It is easy to check that both the function to be maximized and the government revenue are Fréchet differentiable with respect to the variables $(\mathbf{t}, \boldsymbol{\pi})$. The standard Lagrangian approach applies (see D.G Luenberger, *Optimization by vector space methods*, Wiley 1969, and L.A. Fernandez, On the Limits of the Lagrange Multiplier Rule, *SIAM Review*, 39(2), 1997, 292-297). Here we proceed by hand, without a differentiability structure, by directly examining small changes in $\mathbf{G}$.

In the first-order condition, the price elasticity of the demand for the aggregate good G_k, say ε_{G_k}, appears in the last term. Note that ε_{G_k} is a weighted sum of the elementary price elasticities of the goods g in the group:

$$\varepsilon_{G_k} = \int \frac{X_g \mu(g)}{\int_{G_k} X_g \mu(g) dg} \frac{-(1+t_k)X'_g}{X_g} dg,$$

the weights being proportional to the quantity consumed X_g of good g multiplied by the density $\mu(g)$ of this good.

5 How to tax the goods

The aim of the paper is to study the optimal partition $\mathbf{G}$ of the goods. At the optimum, the goods are taxed at one of the K optimal tax rates, and there is an associated marginal cost of public funds λ. This cost obviously depends on institutional constraints: it is different from the marginal cost of public funds that would prevail if all goods were taxed freely. Consider an individual commodity g, small with respect to the whole economy. Under the continuum hypothesis, a change in its tax rate leaves λ unchanged, and a necessary condition for optimality[7] is that good g be attached to the group k such that

$$\mathcal{L}_g(t_k) = \max_{h=1,\dots,K} \mathcal{L}_g(t_h). \tag{4}$$

[7] A formal proof proceeds as follows. Take the optimal partition of goods $\mathbf{G}^*$, given the level R^* of public receipts. Consider the program associated with the fixed $\mathbf{G}^*$, when R varies:

$$\begin{array}{l} \max_{\mathbf{t}} \sum_k \int_{G_k^*} V_g(t_k)\mu(g)dg \\ \sum_k \int_{G_k^*} t_k X_g(t_k)\mu(g)dg = R. \end{array}$$

Let $W(R)$ be the value of the program at a regular point, $t^*(R)$ the optimal tax rates, and $\mathcal{L}_g$ the corresponding Lagrangian. Suppose that, by contradiction, at the point R^*, there is a non-negligible set of goods which belong to G_h^* while $\mathcal{L}_g(t_k^*(R^*)) > \mathcal{L}_g(t_h^*(R^*))$. By continuity, in an open neighborhood of R^*, there also is a non-negligible set of goods, say Γ, fixed independently of R in the neighborhood, such that the inequality holds.

Let $\Delta(R) = \int_\Gamma [t_k X_g(t_k) - t_h X_g(t_h)]\mu(g)dg$, where the expression depends on R through the tax rates, which are the solutions to the program $\mathbf{G}^*$. If $\Delta(R^*)$ is equal to zero, the desired contradiction is obtained by switching Γ from G_h^* into G_k^*. Otherwise, suppose for instance that $\Delta(R^*) > 0$, and therefore stays positive in the appropriate neighborhood of R^* (the negative case can be treated similarly). Take (any) positive small dR with $R' = R^* - dR$ in the neighborhood. By the continuum assumption on the set of goods, from Lyapunov theorem (Hildenbrand, p.45), there exists a subset $d\Gamma$ of Γ such that

$$dR = \int_{d\Gamma} [t'_k X_g(t'_k) - t'_h X_g(t'_h)]\mu(g)dg,$$

where the tax rates $\mathbf{t}'$ stand for $\mathbf{t}^*(R')$. The desired contradiction obtains starting from $\mathbf{G}^*$ at R' by switching $d\Gamma$ from G_h^* into G_k^*, while keeping the tax rates at their $\mathbf{t}'$ values. By construction, government revenue is equal to $R' + dR = R^*$ after the transformation. The value of the objective is $W(R') + \int_{d\Gamma} [V_g(t'_k) - V_g(t'_h)]\mu(g)dg$, i.e. at the first-order for small dR (the

Let t_g^R be the tax rate that this good would support in the hypothetical situation where it would be taxed individually. For an interior solution, t_g^R satisfies the Ramsey rule (1). A direct consequence of the shape of the function $\mathcal{L}$ described in Assumption 1 is

Lemma 1 *Under Assumption 1, at the optimum,*

1. *If $\mathcal{L}_g$ is increasing, good g belongs to the more heavily taxed group K; if it is decreasing, it belongs to the less taxed group.*
2. *Otherwise, with t_g^R the tax rate that maximizes $\mathcal{L}_g$,*
   (a) *if t_g^R is larger than t_K, commodity g supports the maximal rate;*
   (b) *if there exists k, $k < K$, such that $t_k \leq t_g^R \leq t_{k+1}$, then g is taxed either at rate t_k or at rate t_{k+1};*
   (c) *if $t_g^R \leq t_1$, g is taxed at rate t_1.*

This lemma can be used to describe interesting features of the optimal groups of commodities in more economic terms. Let

$$\varepsilon_g(t) = -\frac{(1+t)X_g'(t)}{X_g(t)},$$

$$a_g(t) = \frac{\int_C \alpha(c)\xi_g(t,c)d\nu(c)}{X_g(t)}.$$

Define the demand elasticity and the social weight at the peak: $\varepsilon_g^R = \varepsilon_g(t_g^R)$ and $a_g^R = a_g(t_g^R)$. When commodity g varies in G, the couple (ε_g^R, a_g^R) describes a subset of the positive orthant. It turns out that, under Assumption 1, the position of (ε_g^R, a_g^R) in the plan yields some information on the rate at which it should be taxed.

First consider the efficiency criterion, where a_g is identically equal to one for all g. The monotonicity of the Ramsey formula (1) in elasticities allows to translate Lemma 1 into:

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derivative of W with respect to R at the point R' is denoted $-\lambda'$):

$$W(R^*) + \lambda' dR + \int_{d\Gamma} [V_g(t_k') - V_g(t_h')]\mu(g)dg,$$

or

$$W(R^*) + \int_{d\Gamma} [\mathcal{L}_g(t_k') - \mathcal{L}_g(t_h')]\mu(g)dg + (\lambda' - \lambda)dR.$$

Since the last term is of second order, we have a quantity strictly larger than $W(R^*)$, as desired.

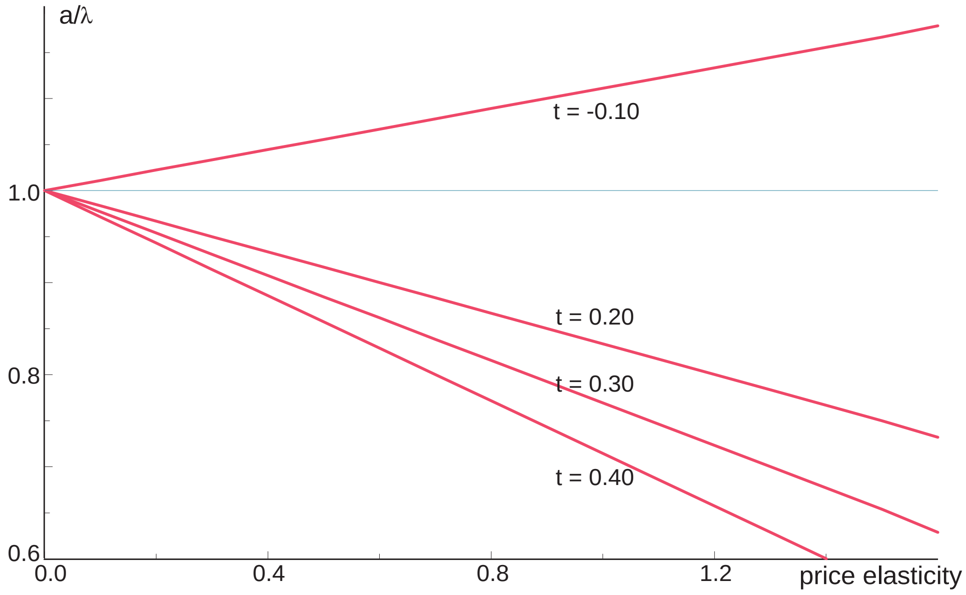

Figure 1: Plan division with four tax rates

Theorem 1 *At an optimum, in the absence of redistribution motive, if the Ramsey price elasticity of good g, ε_g^R, is smaller than $\varepsilon(G_K)$, good g is taxed at the maximal rate t_K. If ε_g^R is larger than ε_{G_1}, good g is untaxed. Otherwise, g is taxed at one of the k or $k+1$ rates such that*

$$\varepsilon_{G_k} \geq \varepsilon_g^R \geq \varepsilon_{G_{k+1}}.$$

When the government has a redistributive objective, the social weights of the commodities typically differ from one. In the plan $(\varepsilon, a/\lambda)$, the goods that satisfy the first-order Ramsey condition for a given tax rate t belong to a half line, with intercept equal to 1 and slope $-t/(1+t)$

$$\frac{a}{\lambda} = 1 - \frac{t}{1+t}\varepsilon.$$

Given the optimal tax rates t_k, $k = 1, \ldots, K$ (in Figure 1, we took four rates, $t = -0.1, 0.2, 0.3$ and 0.4), the plan looks like a fan and is divided into $K+1$ regions. From Lemma 1 and the first-order condition (1), when a good belongs to some interior region, at the optimum it is taxed at one of the two rates that correspond to the boundaries of this region (in the top left region, it is subsidized at the most favorable rate; in the bottom left, it is taxed at the maximal rate). Figure 1 also shows the relative roles of the social weight and price elasticity of the commodity. When the zero rate is observed at the optimum, a good is subsidized (or taxed) when a_g is larger (or smaller) than λ. For a_g larger than λ, smaller price elasticities tend to be associated with larger subsidies. For a_g

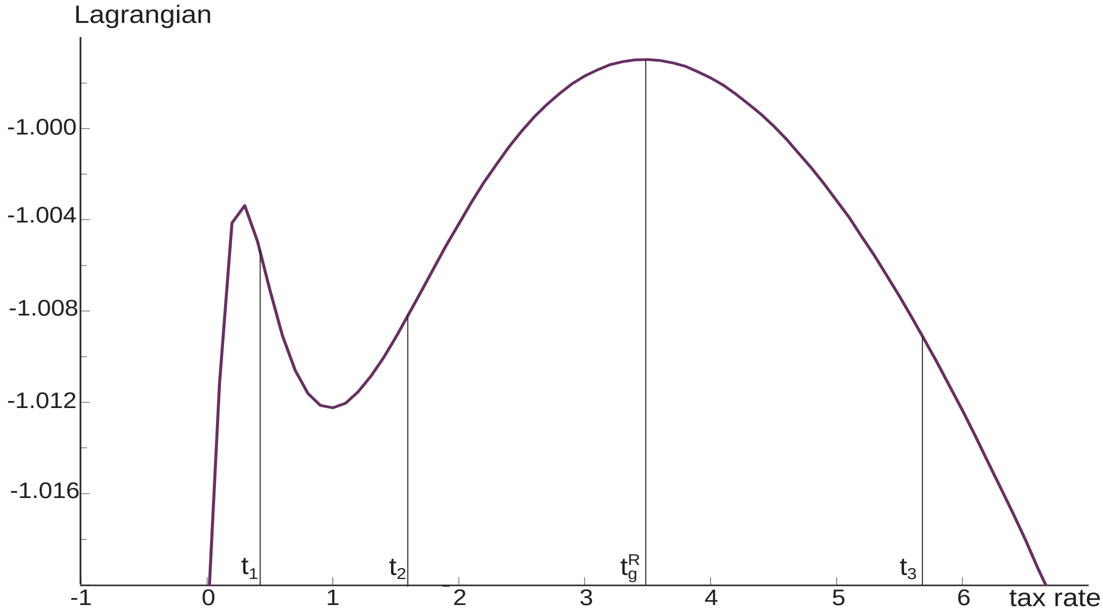

Figure 2: A Lagrangian function with two peaks

smaller than λ, smaller price elasticities tend to be associated with larger tax rates. Without further assumptions, we cannot say more on the shape of the regions in the $(\varepsilon, a/\lambda)$ space corresponding to a given tax rate. For instance, given a smaller than λ, the tax rate is not always decreasing with the price elasticity at the optimum.

In order to highlight how Theorem 1 relies on Assumption 1, one can for instance consider the demand function $X = A_1(1+t)^{-\varepsilon_1} + A_2(1+t)^{-\varepsilon_2}$ of the end of Section 3, with a marginal cost of public funds λ equal to 1.25. The corresponding Lagrangian $\mathcal{L}_g$ has two different peaks, as depicted in Figure 2. The tax rate t_g^R which maximizes $\mathcal{L}_g$ is between t_2 and t_3. Still, in the circumstances of the Figure, the good should be taxed at rate t_1, in clear violation of Theorem 1.

6 The case of constant elasticities

It is possible to derive a more precise characterization of the optimal classification when, for each good, all the consumers' demands have the same constant price elasticities.

6.1 The model and classification criterion

The utility functions which yield demand functions whose price elasticities are constant are of the form[8]

$$u(x,g,c) = \begin{cases} [A_g(c)]^{1/\varepsilon_g(c)} \dfrac{x^{1-1/\varepsilon_g(c)}}{1-1/\varepsilon_g(c)} & \text{for } \varepsilon_g(c) > 0,\ \varepsilon_g(c) \neq 1 \\ A_g(c) \ln \dfrac{x}{A_g(c)} & \text{for } \varepsilon_g(c) = 1. \end{cases}$$

The associated demand and indirect utility functions are:

$$\xi_g(t,c) = \frac{A_g(c)}{(1+t)^{\varepsilon_g(c)}}$$

and

$$v_g(t,c) = \begin{cases} \dfrac{A_g(c)}{\varepsilon_g(c)-1}(1+t)^{1-\varepsilon_g(c)} & \text{for } \varepsilon_g(c) > 0,\ \varepsilon_g(c) \neq 1 \\ -A_g(c) - A_g(c)\ln(1+t) & \text{for } \varepsilon_g(c) = 1. \end{cases}$$

We have seen that, for any individual c and good g, the function $v_g(t,c)+\lambda t \xi_g(t,c)$ is single peaked.

We are able to go further in the description of the sales tax under the additional assumption that, for any given commodity, the price elasticities $\varepsilon_g(c)$ are identical across consumers: $\varepsilon_g(c) = \varepsilon_g$. This assumption is a sufficient condition for the aggregate

$$\mathcal{L}_g(t) = \int_C (\alpha(c) v_g(t,c) + \lambda t \xi_g(t,c))\, d\nu(c)$$

to be also single peaked, so that this function satisfies Assumption 1.

Straightforward computations give the contribution of commodity g to social welfare

$$\mathcal{L}_g(t) = \begin{cases} A_g \dfrac{(1+t)^{-\varepsilon_g}}{\varepsilon_g - 1} \left[a_g(1+t) + \lambda t\,(\varepsilon_g - 1)\right] & \text{for } \varepsilon_g > 0,\ \varepsilon_g \neq 1 \\ A_g \left[a_g \left[-\ln(1+t) - 1\right] + \dfrac{\lambda t}{1+t}\right] & \text{for } \varepsilon_g = 1. \end{cases} \tag{5}$$

where $A_g = \int_C A_g(c) d\nu(c)$, and a_g is the distributional characteristic of commodity g, which here is independent of t

$$a_g = \int_C \alpha(c) \frac{A_g(c)}{A_g} d\nu(c).$$

[8] Gordon (1989) uses such a setup to numerically compute the optimal tax structure in an economy with six goods and a single consumer.

For this specification, one can go further than Theorem 1 and obtain a precise description of the optimal classification of goods in the different tax groups. Good g is taxed at rate t' rather than rate t when

$$\mathcal{L}_g(t') > \mathcal{L}_g(t).$$

This inequality is equivalent to

$$\frac{a_g}{\lambda} < \phi(\varepsilon_g, t, t'),$$

where we show in the Appendix that the function ϕ has the following properties

Lemma 2 *1. The function $\phi(., t, t')$ is convex on $(0, +\infty)$;*

2. When ε goes to infinity, for $t' > t$, $\phi(., t, t')$ is asymptote to $(1-\varepsilon)t/(1+t)$.

From Theorem 1, the graph of the function ϕ in the $(\varepsilon, a/\lambda)$ plan is contained in the cone delimited by the two lines of equations

$$1 - \frac{t}{1+t}\varepsilon \quad \text{and} \quad 1 - \frac{t'}{1+t'}\varepsilon.$$

Note that the asymptote to ϕ has the same slope as the upper boundary of the cone, which makes ϕ always decreasing in the cones situated under the horizontal line of intercept equal to λ, and always increasing in the cones above this line.

Typical pictures of the fan and of the tax groups in the (ε, a) plane appear in Figures 3 to 5. In Figures 3 and 5, there are three tax rates $t_1 = 0$, $t_2 = 0.2$ and $t_3 = 0.4$, and λ is equal to 1.5. In Figure 4, the three tax rates are equal to -0.10, 0.30 and 0.40.

6.2 The tax structure

In this simple framework, a good g is characterized with three numbers $(A_g, a_g, \varepsilon_g)$, respectively describing the level of its demand, its social weight and its price elasticity. The previous computations allow us to find how to optimally tax the goods, given a finite set of tax rates $\mathbf{t}$ and a value of the marginal cost of public funds λ.

A first notable fact is that the levels of demand play no role whatsoever in the classification, so that for taxation one can work in the two dimensional plane (ε, a). In this plan, it is important to draw the curves associated with the first-order condition (3)

$$\frac{t_k}{1+t_k} = \frac{\lambda - a_{G_k}}{\lambda} \frac{1}{\varepsilon\,(G_k)}.$$

These are straight (broken on the Figures) half lines originating at the point of ordinate λ on the a axis, of slope equal to $-\lambda t_k/(1+t_k)$. Theorem 1 tells us

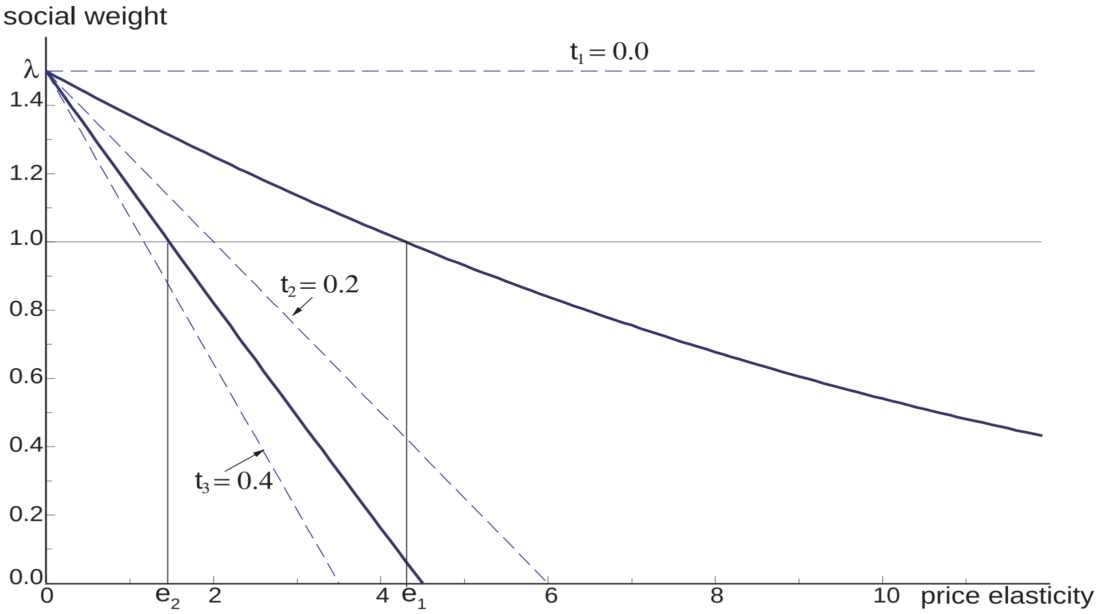

Figure 3: The efficient tax structure

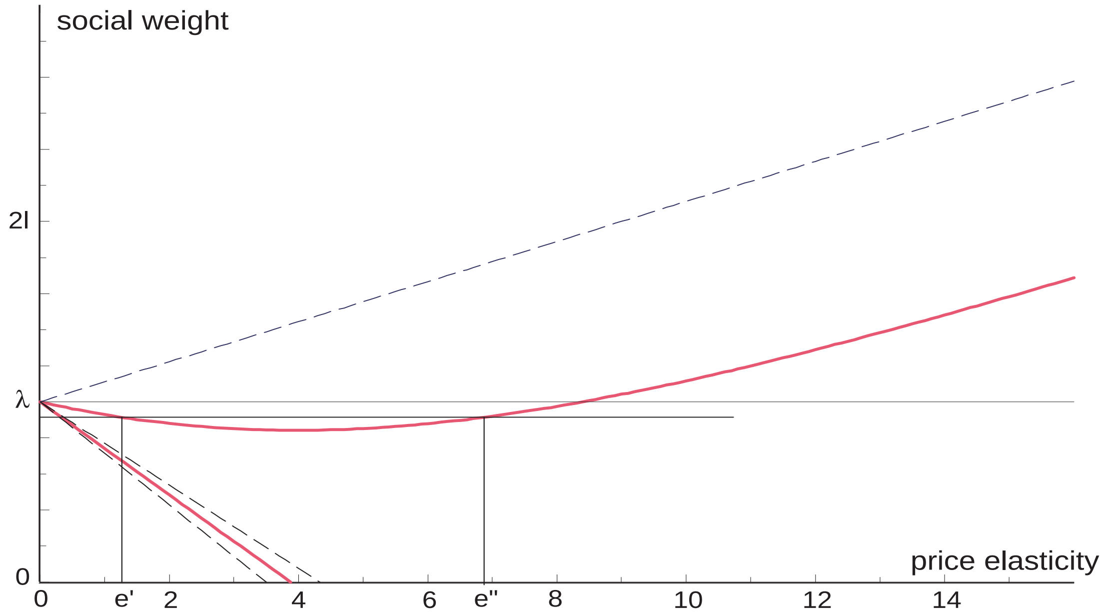

Figure 4: Taxes may not be decreasing with elasticity

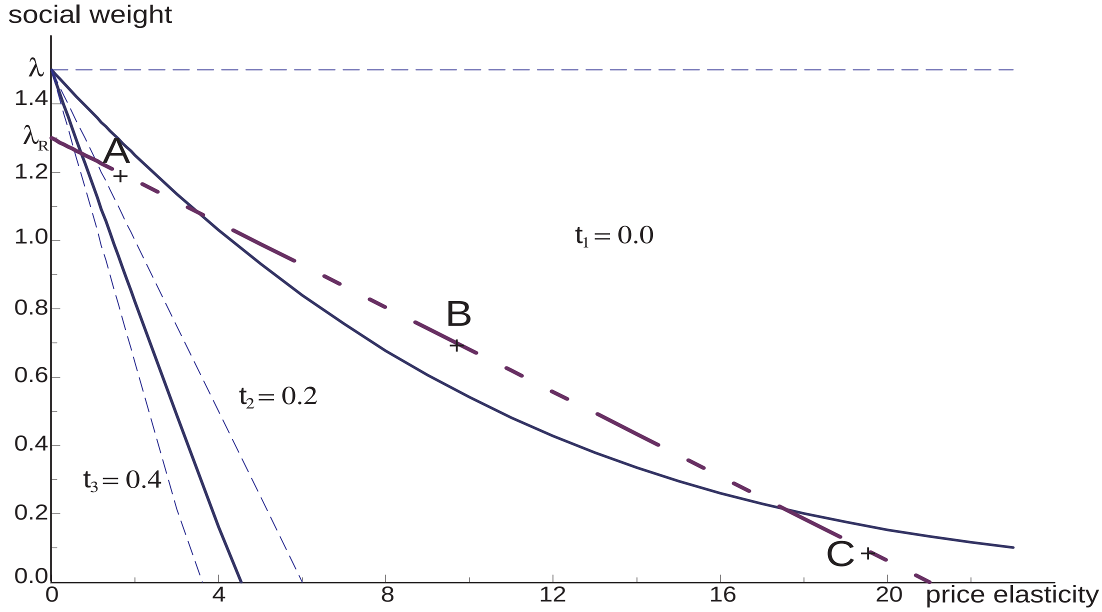

Figure 5: Social weights and classification

that, when commodity g, i.e. the point (ε_g, a_g), belongs to the cone between lines k and $k+1$, good g is taxed either at rate t_k or at rate t_{k+1}. From Lemma 2, inside each cone $(k, k+1)$, there is a curve of equation $a/\lambda = \phi(\varepsilon, t_k, t_{k+1})$ (thick solid lines on the figures) which partitions the cone into two regions: the goods above the curve are taxed at the lower rate, those below the curve are taxed at the upper rate. We therefore have a full characterization of the optimal goods classification.

For instance, consider the case where all the goods have the same social weight, equal to 1. There is a unique threshold e_k solution to the equation $1/\lambda = \phi(e_k, t_k, t_{k+1})$: all the goods of price elasticity ε between e_k and e_{k+1} are taxed at rate t_k (see Figure 3).

More generally, when a_g is not constrained to be equal to 1, we have a partition of the plane into connected regions. It is easy to see that for fixed elasticity ε, along vertical lines, the tax rates are decreasing with the social weight, which plays a leading role in the classification. For a fixed social weight, along a horizontal line, the situation differs depending on whether a is larger or smaller than λ. When it is larger (resp. smaller), the goods typically are subsidized (resp. taxed). When some goods are exempted (0 is among the tax rates), the subsidy (resp. tax) rates decrease with the price elasticity. When 0 is not among the tax rates, there is a theoretical case, exemplified on Figure 4, where the tax rate is not monotone in elasticity: for a slightly smaller than λ, there is a finite interval of elasticities, say (e', e''), for which the goods are subsidized, while the goods in $(0, e')$ and in $(e'', +\infty)$ are taxed at the smallest positive rate.

Remark 1: Good g may be taxed at a higher rate than good g' at the unconstrained Ramsey optimum but at a lower rate at the constrained optimum. This is shown on Figure 5. At the unconstrained optimum, the marginal cost of public funds λ_R is usually smaller than in the constrained situation λ. The points ABC, being aligned on a half line with intercept λ_R are subject to the same tax rate at the unconstrained optimum. But A and C are taxed at t_2 in the constrained situation, while B is exempted.

Remark 2: A solution to the full optimum might be obtained through an algorithm which could run as follows. At even dates, given the good classification, the program solves for the K optimal tax rates and the marginal cost of public fund using the Diamond conditions (3) and the government budget constraint. At odd dates, given λ and the tax rates, the classification of goods is derived as above. The optimum is a fixed point of this algorithm, which, unfortunately, since the program is not concave, may have many fixed points, including local minima.

7 Beware of the Ramsey formula

In the absence of redistribution motive ($a_g = 1$), when there are no constraints on the number of tax rates, the lower the actual price elasticity of the demand for a good, the larger the tax rate which it bears, according to the Ramsey formula. This monotonicity property no longer holds true when the number of available tax rates is fixed, even when the single peaked assumption is satisfied.

To illustrate, consider a case with two tax rates, say t_1 and t_2. Then, it is easy to find examples where there are two goods, g_1 and g_2, respectively taxed at t_1 and t_2, $t_1 < t_2$, at the optimum, such that the actual elasticities computed at the optimal allocation verify

$$\varepsilon_{g_1}(t_1) < \varepsilon_{g_2}(t_2).$$

Note that the elasticities are computed at the current allocation (and not at the putative optimal point of Lemma 1).

Here is an example where all the agents have the same preferences. There is a continuum of goods, indexed with a positive scalar γ. The demand for good γ is equal to

$$\exp\left[-(1+t)\gamma\right],$$

so that the price elasticity of demand, when the good is taxed at rate t is

$$\varepsilon_g = \gamma(1+t).$$

Assumption 1 is satisfied for this good. Simple calculations show that there exists a threshold γ^* such that goods with γ smaller than γ^* are taxed at t_2, the remainder at t_1. The threshold good γ^* itself, which can be allocated indifferently to any of the two groups, provides the example we are looking for: its elasticity is smaller when taxed at t_1 than when taxed at t_2. Examples are probably harder to get with types of goods whose price elasticities of demand decrease with the price. In general, the elasticity, being a local characteristics of demand, is unlikely to be a reliable guide to the global problem of goods assignments.

8 Non separability

For simplicity, we have assumed until now that the utility functions were separable. To bring theory closer to the data, we both have to relax this assumption and to introduce labor supply together with direct taxes. We sketch the broad lines of the largely standard argument, without entering into technical issues.

Let the tastes of agent c be represented with the utility function $U(\mathbf{x}, -L, c)$, where $\mathbf{x}$ is the consumption of goods, a measurable mapping from the set of commodities G into $\mathbb{R}_+$, and L is labor supply, a positive number. The typical consumer maximizes her utility under her budget constraint:

$$\int_G (1+t_g)x(g)\mu(g)dg = w_c L - T(w_c L),$$

where w_c is before tax wage and T is the possibly non linear tax on income. *We assume that the program has a regular interior solution, with positive consumptions of all goods, and that we can use differential calculus*[9].

Let $\mathbf{t}$ be the collection of tax rates (t_g) for g in G. The indirect utility function $V(\mathbf{t}, T, c)$ is obtained as usual by plugging the solution into U and depends on the tax rates $\mathbf{t}$ and the income tax schedule T. Under regularity conditions, if ρ_c is the marginal utility of income of consumer c (the Lagrange multiplier associated with the budget constraint), an application of the enveloppe theorem yields:

$$\frac{\partial V}{\partial t_g} = -\rho_c \xi_g(\mathbf{t}, T, c),$$

where $\xi_g(\mathbf{t}, T, c)$ is the demand for good g of consumer c.

The government program can then be written as

$$\begin{cases} \max\limits_{\mathbf{t},T} \displaystyle\int_C \alpha_c V(\mathbf{t}, T, c) d\nu(c) \\ \displaystyle\int_C \left[\int_G t_g \xi_g(\mathbf{t}, T, c)\mu(g)dg + T(Y(\mathbf{t}, T, c)) \right] d\nu(c) = R, \end{cases}$$

[9]This assumption holds in the separable case. In general, it requires not too much substitutability between commodities.

where $Y(\mathbf{t}, T, c)$ denotes before tax income of consumer c, when she faces taxes $(\mathbf{t}, T)$. The Lagrangian of this problem is

$$\mathcal{L}(\mathbf{t}, T) = \int_C \alpha_c V(\mathbf{t}, T, c) d\nu(c) + \lambda \left\{ \int_C \left[\int_G t_g \xi_g(\mathbf{t}, T, c)\mu(g)dg + T(Y(\mathbf{t}, T, c)) \right] d\nu(c) - R \right\}.$$

Now for a (small) group of goods $\{g\} + dG$ around g, define $\boldsymbol{\tau}_{\{g\}+dG}(\mathbf{t}, s)$ to be the set of tax rates $\mathbf{t}'$ such that $t'_\gamma = t_\gamma$ for γ not in $\{g\} + dG$ and $t'_\gamma = s$ for γ in $\{g\} + dG$. Adapting the argument of footnote 7, a necessary condition for the optimality of a partition $\mathbf{G}$ associated with tax rates $\mathbf{t}$ is that, for all h, for all g and all small enough dG such that $\{g\} + dG$ is in G_h, and for all k

$$\mathcal{L}[\boldsymbol{\tau}_{\{g\}+dG}(\mathbf{t}, t_h), T] \geq \mathcal{L}[\boldsymbol{\tau}_{\{g\}+dG}(\mathbf{t}, t_k), T].$$

Continuing to reproduce the line of argument of the paper, we are interested in the tax rate t_g^R which maximizes $\mathcal{L}[\boldsymbol{\tau}_{\{g\}+dG}(\mathbf{t}, s), T]$ over s, for small dG. The first-order condition for an interior maximum is

$$\begin{aligned}
0 = &- \int_C \alpha_c \rho_c \int_{\{g\}+dG} \xi_\ell[\boldsymbol{\tau}_{\{g\}+dG}(\mathbf{t}, s), T, c]\mu(\ell)d\ell \; d\nu(c) \\
&+ \lambda \int_C \int_{\{g\}+dG} \xi_\ell[\boldsymbol{\tau}_{g+dG}(\mathbf{t}, s), T, c]\mu(\ell)d\ell \; d\nu(c) \\
&+ \lambda \int_C \left[\int_G t_\ell \frac{\partial \xi_\ell[\boldsymbol{\tau}_{\{g\}+dG}(\mathbf{t}, s), T, c]}{\partial s} \mu(\ell)d\ell \right] d\nu(c) \\
&+ \lambda \int_C \left[T'\{Y[\boldsymbol{\tau}_{\{g\}+dG}(\mathbf{t}, s), T, c]\} \frac{\partial Y[\boldsymbol{\tau}_{\{g\}+dG}(\mathbf{t}, s), T, c]}{\partial s} \right] d\nu(c)
\end{aligned}$$

We want to get the limit of the above expression when dG goes to zero, after division by the weight $\mu(dG) = \int_{dG} \mu(\ell)d\ell$. The two first terms, as well as the last one, are easily dealt with. Indeed, define, with some abuse of notation:

$$\xi_g[\boldsymbol{\tau}_g(\mathbf{t}, s), T, c] = \lim_{dG \to 0} \int_{\{g\}+dG} \frac{\xi_\ell[\boldsymbol{\tau}_{\{g\}+dG}(\mathbf{t}, s), T, c]}{\mu(dG)} \mu(\ell)d\ell,$$

$$\frac{\partial Y[\boldsymbol{\tau}_g(\mathbf{t}, s), T, c]}{\partial s} = \lim_{dG \to 0} \frac{1}{\mu(dG)} \frac{\partial Y[\boldsymbol{\tau}_{\{g\}+dG}(\mathbf{t}, s), T, c]}{\partial s}.$$

Also let:

$$X_g[\boldsymbol{\tau}_g(\mathbf{t}, s), T] = \int_C \xi_g[\boldsymbol{\tau}_g(\mathbf{t}, s), T, c] d\nu(c), \tag{6}$$

and

$$a_g[\boldsymbol{\tau}_g(\mathbf{t}, s), T] = \int_C \frac{\xi_g[\boldsymbol{\tau}_g(\mathbf{t}, s), T, c]}{X_g[\boldsymbol{\tau}_g(\mathbf{t}, s), T, c]} \alpha_c \rho_c d\nu(c). \tag{7}$$

Then, the two first terms tend to

$$\{-a_g[\boldsymbol{\tau}_g(\mathbf{t},s),T]+\lambda\}\ X_g[\boldsymbol{\tau}_g(\mathbf{t},s),T],$$

while the last one can be written as

$$\lambda\int_C T'(Y[\boldsymbol{\tau}_g(\mathbf{t},s),T])\frac{\partial Y[\boldsymbol{\tau}_g(\mathbf{t},s),T]}{\partial s}d\nu(c).$$

The third term needs some more care. When taking the limit, one must separate the own price effect from the substitution effect on other goods:

$$\frac{\partial\xi_g[\boldsymbol{\tau}_g(\mathbf{t},s),T,c]}{\partial s}=\lim_{dG\to 0}\int_{\{g\}+dG}\frac{1}{\mu(dG)}\frac{\partial\xi_\ell[\boldsymbol{\tau}_{\{g\}+dG}(\mathbf{t},s),T,c]}{\partial s}\mu(\ell)d\ell,$$

$$\frac{\partial\xi_{G_k\setminus\{g\}}[\boldsymbol{\tau}_g(\mathbf{t},s),T,c]}{\partial s}=\lim_{dG\to 0}\int_{G_k\setminus\{\{g\}+dG\}}\frac{1}{\mu(dG)}\frac{\partial\xi_\ell[\boldsymbol{\tau}_{\{g\}+dG}(\mathbf{t},s),T,c]}{\partial s}\mu(\ell)d\ell.$$

The former limit is the own price elasticity, while the latter is the average substitution effect on the commodities[10] in the set $G_k\setminus\{g\}$, which only exists when substitution between commodities is not too 'large'. Finally, summing up on agents, define

$$\frac{\partial X_g[\boldsymbol{\tau}_g(\mathbf{t},s),T]}{\partial s}=\int_C\frac{\partial\xi_g[\boldsymbol{\tau}_g(\mathbf{t},s),T,c]}{\partial s}d\nu(c),$$

and

$$\frac{\partial X_{G_k\setminus\{g\}}[\boldsymbol{\tau}_g(\mathbf{t},s),T]}{\partial s}=\int_C\frac{\partial\xi_{G_k\setminus\{g\}}[\boldsymbol{\tau}_g(\mathbf{t},s),T,c]}{\partial s}d\nu(c).$$

With all the above definitions, when all commodities in group G_k are taxed at the same rate t_k, the first-order condition associated with the tax rate t_k, following as before Diamond (1973), is

$$\int_{g\in G_k}\left\{(-a_g+\lambda)X_g+\lambda\left[t_g\frac{\partial X_g}{\partial t_g}+\int_{g'\neq g}t_{g'}\frac{\partial X_{g'}}{\partial t_g}+\int_C T'(Y)\frac{\partial Y}{\partial t_g}d\nu(c)\right]\right\}\mu(g)dg=0. \tag{8}$$

The first-order condition verified by $t_g^R=s$ can be rewritten as

$$(-a_g+\lambda)X_g+\lambda\left[s\frac{\partial X_g}{\partial s}+\sum_{k=1}^K t_k\frac{\partial X_{G_k\setminus\{g\}}}{\partial s}+\int_C T'(Y)\frac{\partial Y}{\partial s}d\nu(c)\right]=0, \tag{9}$$

where a_g, X_g and the partial derivatives of X_g and $X_{G_k\setminus\{g\}}$ are evaluated at $[\boldsymbol{\tau}_g(\mathbf{t},s),T]$ while Y and its derivative is evaluated at $[\boldsymbol{\tau}_g(\mathbf{t},s),T,c]$.

[10] We use the notation $G_k\setminus\{\{g\}+dG\}$ as a short hand for $G_k\setminus[G_k\cap\{\{g\}+dG\}]$. Note that since, by construction, $\{g\}+dG$ is contained in a single member of the partition, say G_h, all the $G_k\setminus\{\{g\}+dG\}$'s coincide with G_k, for all k different from h.

The analysis of Section 5 then can be adapted to this more general setup, provided Assumption 1 applies to the (newly defined) function $\mathcal{L}_g$. Rewriting the first-order condition as

$$\frac{a_g}{\lambda} - b_g = 1 - \frac{t}{1+t}\varepsilon_g, \tag{10}$$

where

$$b_g = \frac{1}{X_g}\left[\sum_{k=1}^{K} t_k \frac{\partial X_{G_k\setminus\{g\}}}{\partial s} + \int_C T'(Y)\frac{\partial Y}{\partial s} d\nu(c)\right], \tag{11}$$

one can draw Figure 1 in the plan $(\varepsilon, a/\lambda - b)$ with a similar interpretation, provided that the Lagrangian is single peaked (so that the first-order condition characterizes the optimal tax rate for good g all other rates being kept fixed) and that the quantities in (10), $(a_g, \varepsilon_g, b_g)$, all are evaluated at the optimal tax rate.

9 Illustration with data from the United Kingdom

Professor Ian Crawford, from the Institute for Fiscal Studies, has provided us with uncompensated cross price elasticities for consumption in the UK, grouped into twenty categories[11], homogenous by tax rates, computed along the lines initiated by Blundell and Robin (1999) (see Appendix). We also have the budget shares by deciles of consumption expenditures in the population.

A large part of consumption, 49%, is subject to the 'standard' (17.5%) tax rate, and a substantial part, 27%, necessities including basic food, is exempted. Domestic fuel, 10% of consumption, is taxed at the 'reduced' (5%) rate. Tobacco, alcohol and petrol bear large excise tax rates.

We would like to recover from this data the implicit redistributive weights of the UK government on the ten population deciles, and to see whether the actual grouping of commodities fits with the theory developed above. Our strategy bears on the Diamond first-order conditions (8) for the basic three groups, exempted, reduced rate and standard rate. The tax rates on alcohol and tobacco on one side, petrol on the other, are likely to depend on other considerations than mere redistribution (public health, environmental issues,..), and we do not consider them at this stage.

Given the observed tax rates, budget shares and price elasticities[12], the unknowns in (8) are the non negative social weights $\alpha_c\rho_c$ and marginal cost of public funds λ, where a_g is linked to the social weights by (7). The equations

[11] In the analysis, we dropped children clothing, which represent less than 1% of aggregate consumption expenditure, because the estimated price elasticities are somewhat out of the ball park.

[12] In practice, we have to rewrite (8), given the available statistics. We do not know the income elasticities with respect to indirect tax rates: we take $\partial Y_c/\partial t_g = 0$. There is a finite

are homogenous of degree one in $(\alpha_c\rho_c, \lambda)$, so that we only can recover the ratios $\alpha_c\rho_c/\lambda$. We minimize the sum of the squares of the three left-hand sides of the Diamond conditions (8), which yields values for the ratios $(\alpha_c\rho_c/\lambda)$. For the sake of the presentation, we normalize the sum of $\alpha_c\rho_c$ over the deciles to unity, and compute λ accordingly[13]. The normalization yields an implicit choice of units: an increase of aggregate consumption of dC, uniformly distributed, gives $dC/10$ to each decile and therefore, for this choice of normalization, increases social welfare by $dC/10$: Social welfare is measured in tenths of aggregate consumption. This procedure gives

$$\hat{\lambda} = 1.16,$$

and puts $\alpha_c\rho_c$ at zero for the second and third decile, as well as for all deciles from the fifth and above, while

$$\alpha_1\rho_1 = 0.04 \qquad \alpha_4\rho_4 = 0.96.$$

The bulk of the weight is concentrated on the fourth decile. The left-hand sides of the Diamond conditions are respectively equal to 0.0026 for the exempted goods, -0.0060 for domestic fuel (the only good taxed at the reduced rate), and to -0.0003 for goods taxed at the standard rate. These numbers are proportional, up to a positive factor, to the derivatives of the social objective $\mathcal{L}(\mathbf{t}, T)$ with respect to the corresponding tax rates. If one is willing to retain 1.16 as the marginal cost of public funds, they are equal to the social values of marginal changes of the

---

number of commodities, so that the equality becomes

$$\sum_{g\in G_k} \left\{ (-a_g + \lambda)X_g + \lambda \sum_{g'\in G} t_{g'} \frac{\partial X_{g'}}{\partial t_g} \right\} = 0.$$

The consumption, rather than production, price is the numeraire. Using tildas for the variables measured with the new numeraire:

$$\tilde{X}_g = (1 + t_g)X_g,$$

and

$$\frac{1}{1+t_g} = 1 - \tilde{t}_g.$$

After some manipulations, with appropriate definition of the aggregates, (8) becomes

$$(-a_{G_k} + \lambda)(1 - \tilde{t}_k)\tilde{X}_{G_k} + \lambda \sum_{k'} \tilde{t}_{k'} \tilde{X}_{G_{k'}} \tilde{\varepsilon}_{G_{k'}G_k} = 0.$$

Finally we work in shares of total consumption, dividing the equalities by total consumption.

[13]If the allocation were optimal with respect to uniform transfers from the public to the government, the equality

$$\sum_c \alpha_c\rho_c = \lambda$$

would hold. In practice however this constraint appears not to be satisfied by the data.

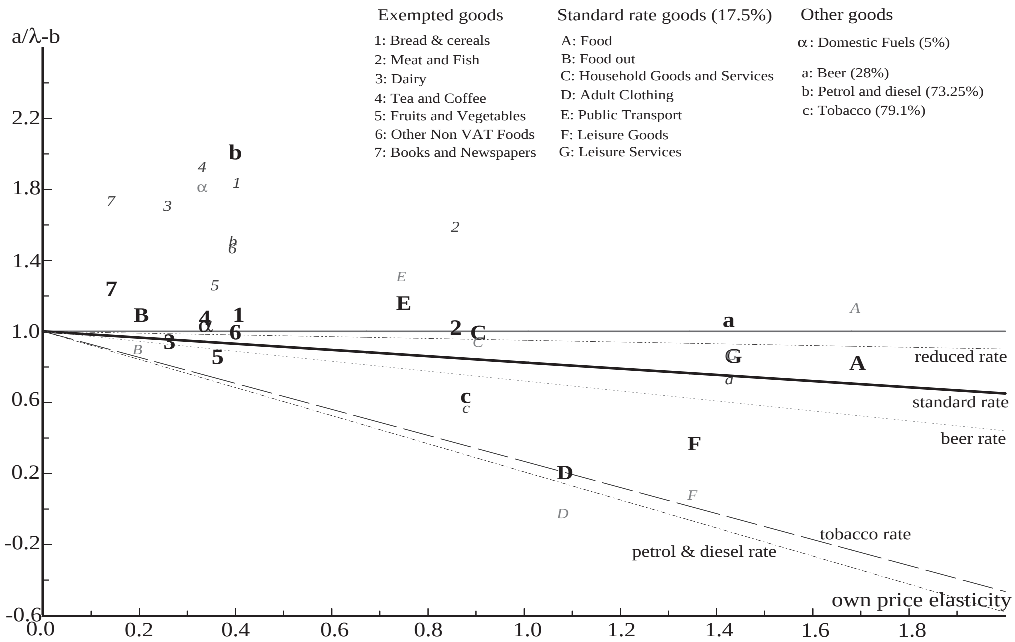

Figure 6: The fan for the UK

tax rates, measured as tenths of aggregate consumption. For instance, increasing by 1 point the standard rate, from 17.5% to 18.5% would induce a social loss of $0.0003 \times 0.01 \times 10 = 0.003\%$ of aggregate consumption. The Diamond first-order conditions therefore are close to be satisfied.

Is the grouping of commodities optimal? Figure 6 allows to approach this difficult question, building on the previous results in the paper. It plots the discounted adjusted social weight $a/\lambda - b$ as a function of the own price elasticity. From (10), the first-order condition for an individual commodity (supposed to be small enough, so as to have no influence on the marginal cost of the public fund) is

$$\frac{a}{\lambda} - b = 1 - \tilde{t}\varepsilon,$$

where $\tilde{t} = t/(1+t)$ is the unknown optimal tax rate, computed as a fraction of the consumption price. We have set a, b and ε at their current observed values[14], so that $\tilde{t}$ is the solution of a linear equation. The graph in the plan $(\varepsilon, a/\lambda - b)$

[14]We have done some experimentation with more sophisticated computations for b and ε, using (11) but maintaining $T'(Y)$ equal to zero, and deriving a linear approximation of the equation around the observed point. In particular we have looked at cases where all the elasticities are constant, equal to their observed values, where demand functions are linear, and at a couple of other variants, including QAIDS which underlies the empirical estimation. Unsurprisingly, the results are quite sensitive to the specification of the shape of the demand functions: in particular single peakedness is easily lost, and the Lagrangian may be locally convex at the observed point. More work is needed in this area.

shows the half lines corresponding to the current tax rates. The representative points of eighteen[15] commodities are also shown: in fact two points are drawn for each good, one in large bold type corresponds to the implicit social weights computed above, the other one in small italic type represents the good location for a Rawlsian government which would put all the social weight on the first population decile.

If Assumption 1 holds, optimality requires that the large bold representative points of all the exempted goods be above the reduced rate half line, the point associated with 'Domestic Fuels' (the only good supporting the reduced rate) be between the standard rate line and the horizontal, and all the goods bearing the standard rate be below the reduced rate half line. Given the possible measurement errors on elasticities, these necessary conditions fare rather well on the graph. The few violations are not large: rather than being exempted, (some of the) 'Dairy products' should be taxed at the reduced or at the standard rate, and (some of the) 'Fruits and Vegetables' at the standard rate; (some of the) 'Domestic Fuels' should be exempted; (some of) 'Public Transport' and 'Food Out', two goods complementary with work, should be exempted, instead of being taxed at the standard rate[16]. If the government wants to raise more money by creating a larger tax rate, (some of) 'Adult Clothing' and 'Leisure Goods' seem to be good candidates to enter its basis.

There are four specific categories subject to excise taxes. The overall tax rate on 'Wine and spirits', which is not reported on the graph, is optimal purely on redistributive grounds. All the other goods appear to be taxed more than the redistributive social objective would recommend, maybe on public health or environmental protection grounds. The difference is not large for tobacco, but, according to the figure, beer should be exempted and 'Petrol and Diesel' should be strongly subsidized.

It is also of interest to look at the impact of the redistributive stance of the government on the diagram. Going from the fourth decile voter just discussed to a Rawlsian government seems to spread out the figure: goods that are presently exempted or lightly taxed move up into the subsidy region, above the horizontal line, while goods below the standard rate line appear to move down: more redistribution through indirect taxes seems to require more dispersion of the tax

[15]Theory would require partial optimization to compute the first-order conditions at the optimal tax rate for each considered commodity: for lack of better information, we assume that the observed elasticities are good enough approximations to be used to compute the graph coordinates. In the interest of readability, wine and spirits do not appear on the graph: its own price elasticity is (-)3, much larger than that of the other goods. Its excise rate seems optimal from the viewpoint adopted here, solely based on redistributive motives: the bold point in fact lies on the corresponding half line.

[16]In practice, we do not have access to data on a continuum of goods, contrary to the setup used in the theoretical part of the paper. We work on (small) aggregates of 'elementary', probably diverse, commodities. The qualifier 'some of' designates some of the elementary commodities composing the aggregate.

rates.

All things considered, these results look plausible and may be worth independent confirmation and further refinement.

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Appendix
Constant elasticity

Under the assumptions of Section 6, one can derive the following results:

Lemma 3 *For any commodity g, with price elasticity $\varepsilon > 0$ and distributional characteristic a, the inequality $\mathcal{L}_g(t') > \mathcal{L}_g(t)$ is equivalent to*

$$\frac{a}{\lambda} < \phi\left(\varepsilon, t, t'\right)$$

where

$$\phi\left(\varepsilon, t, t'\right) = (1-\varepsilon)\left[1 + \frac{1}{\sqrt{(1+t')\,(1+t)}}\frac{\sinh\left(r\frac{\varepsilon}{2}\right)}{\sinh\left(r\frac{1-\varepsilon}{2}\right)}\right], \text{ and } r = \ln\left(\frac{1+t'}{1+t}\right).$$

Proof: Using $t'(1+t')^{-\varepsilon} = (1+t')^{1-\varepsilon} - (1+t')^{-\varepsilon}$, the inequality $\mathcal{L}_g(t') > \mathcal{L}_g(t)$ rewrites

$$\left[(1+t')^{1-\varepsilon} - (1+t)^{1-\varepsilon}\right]\left(\frac{a}{\varepsilon-1} + \lambda\right)\frac{1}{\lambda} > (1+t')^{-\varepsilon} - (1+t)^{-\varepsilon}.$$

Note that, for any real number σ,

$$(1+t')^{\sigma} - (1+t)^{\sigma} = 2\,(1+t)^{\sigma/2}\,(1+t')^{\sigma/2}\sinh\left(\frac{\sigma r}{2}\right)$$

where r is as defined in the Lemma. Thus, we get that $\mathcal{L}_g(t') > \mathcal{L}_g(t)$ is equivalent to

$$\left[(1+t')\,(1+t)\right]^{\frac{1}{2}}\sinh\left(r\frac{1-\varepsilon}{2}\right)\left(\frac{a}{\varepsilon-1} + \lambda\right)\frac{1}{\lambda} + \sinh\left(r\frac{\varepsilon}{2}\right) > 0.$$

Since $\sinh\left(r\frac{1-\varepsilon}{2}\right)$ has the same sign as $1-\varepsilon$, the last inequality rewrites

$$\frac{a}{\lambda} < \phi\left(\varepsilon, t, t'\right)$$

where

$$\phi\left(\varepsilon,t,t'\right)=(1-\varepsilon)\left[1+\frac{1}{\sqrt{\left(1+t'\right)\left(1+t\right)}}\frac{\sinh\left(r\frac{\varepsilon}{2}\right)}{\sinh\left(r\frac{1-\varepsilon}{2}\right)}\right].$$

In the particular case $\varepsilon=1$, it is sufficient to show that $\frac{a}{\lambda}<\phi\left(1,t,t'\right)$ is equivalent to $\mathcal{L}_g(t')>\mathcal{L}_g(t)$ for $\varepsilon=1$. Since $\sinh x$ is equivalent to x in the neighborhood of $x=0$, we have

$$\phi\left(1,t,t'\right)=\frac{1}{\sqrt{\left(1+t'\right)\left(1+t\right)}}\frac{\sinh\left(\frac{r}{2}\right)}{\frac{r}{2}}=\frac{\left(1+t\right)^{-1}-\left(1+t'\right)^{-1}}{\ln\left(1+t'\right)-\ln\left(1+t\right)}$$

and $\mathcal{L}_g(t')>\mathcal{L}_g(t)$ is equivalent to

$$\frac{\lambda t'}{(1+t')}-a\ln(1+t')>\frac{\lambda t}{(1+t)}-a\ln(1+t)$$

$$\Leftrightarrow\lambda-\frac{\lambda}{(1+t')}-a\ln(1+t')>\lambda-\frac{\lambda}{(1+t)}-a\ln(1+t)$$

$$\Leftrightarrow\lambda\left[\frac{1}{(1+t)}-\frac{1}{(1+t')}\right]>a\left[\ln(1+t')-\ln(1+t)\right]$$

∎

Lemma 4 *For $t'>t$, the function $\phi(\varepsilon,t,t')$ is convex in its first argument.*

1. *Its slope at the origin is*

$$\frac{\partial\phi}{\partial\varepsilon}=\frac{1}{t'-t}\left[\ln\left(\frac{1+t'}{1+t}\right)-(t'-t)\right].$$

2. *When ε goes to ∞, ϕ is equivalent to*

$$(1-\varepsilon)\frac{t}{1+t}.$$

Proof: 1) Using the identity

$$\sinh a\cosh b+\sinh b\cosh a=\sinh(a+b),$$

a direct computation yields

$$\frac{\partial\phi}{\partial\varepsilon}=-1-\frac{1}{\sqrt{(1+t')(1+t)}[\sinh\left(1-\varepsilon\right)r/2]^2}\left[\sinh\frac{(1-\varepsilon)r}{2}\sinh\frac{\varepsilon r}{2}-\frac{(1-\varepsilon)r}{2}\sinh\frac{r}{2}\right].$$

The desired formula follows when $\varepsilon=0$, using the equality $\sqrt{(1+t')(1+t)}\sinh r/2=(t'-t)/2$.

2) One can rewrite

$$\phi(\varepsilon, t, t') = (1-\varepsilon)\left[1 - \frac{1}{1+t}\ \frac{1-\exp(-\varepsilon r)}{1-\exp[(1-\varepsilon)r]}\right].$$

When ε goes to infinity, the result follows.

We finally show the convexity of ϕ with respect to ε, which is derived from the fact that its second derivative is positive. Indeed differentiating the expression obtained in 1) for the first derivative gives:

$$\frac{\partial^2\phi}{\partial\varepsilon^2} = \frac{r}{\sqrt{(1+t')(1+t)}}\frac{\sinh r/2}{[\sinh r(1-\varepsilon)/2]^2}\left[-1 + \frac{(1-\varepsilon)r}{2}\frac{\cosh\dfrac{(1-\varepsilon)r}{2}}{\sinh\dfrac{(1-\varepsilon)r}{2}}\right].$$

It is positive since $(x/\tanh x)$ is larger than 1 for all x (the tanh curve is below the 45^0 line for positive x, and above for negative x). ∎

	B&C	M&F	D	T&C	F&V	OUF	F	FO	B	W&S	T	DF	HGS	AC	CC	P&D	PT	LG	B&N	LS
Bread & Cereals	**-0.391**	-0.085	-0.154	-0.02	-0.175	0.385	-0.441	0.812	-0.227	-0.551	0.151	0.232	0.203	-0.381	0.093	0.146	-0.005	-0.474	-0.489	0.371
Meat & Fish	-0.026	**-0.843**	0.038	-0.019	-0.107	0.072	-0.082	0.003	-0.029	-0.187	-0.05	-0.088	-0.173	0.084	-0.024	0.171	0.064	-0.066	-0.062	0.325
Dairy	-0.07	0.057	**-0.247**	0.073	-0.175	-0.206	0.012	-0.043	-0.353	0.142	-0.072	0.063	0.148	-0.044	-0.281	0.175	-0.047	-0.395	-0.101	0.367
Tea & Coffee	-0.045	-0.139	0.359	**-0.32**	-0.266	-0.037	-0.089	-0.322	0.312	-0.338	0.086	-0.034	0.487	-0.768	0.191	0.159	-0.149	-0.077	0.017	-0.029
Fruits and Vegetables	-0.088	-0.176	-0.192	-0.059	**-0.345**	-0.173	-0.04	0.275	0.081	-0.005	-0.045	-0.135	-0.159	-0.37	-0.107	0.146	0.088	0.061	-0.1	0.344
Other Untax. Foods	0.291	0.178	-0.34	-0.013	-0.26	**-0.381**	-0.22	0.4	0.756	0.058	-0.356	-0.403	-0.205	-0.52	-0.367	0.037	0.047	0.306	-0.096	0.086
Food	-0.299	-0.184	0.017	-0.027	-0.054	-0.198	**-1.68**	0.486	0.354	-0.338	-0.221	-0.554	0.274	1.615	-0.016	0.052	-0.036	-1.11	0.118	0.8
Food out	0.356	0.004	-0.041	-0.063	0.24	0.232	0.314	**-0.19**	-0.674	0.044	0.336	-0.143	-0.875	-0.517	0.338	-0.181	-0.085	0.307	0.111	-0.514
Beer	-0.137	-0.058	-0.468	0.084	0.098	0.606	0.317	-0.932	**-1.408**	-0.077	0.388	0.883	0.244	0.285	0.489	-0.406	0.111	-0.3	-0.292	-0.427
Wine & Spirits	-0.637	-0.711	0.358	-0.174	-0.011	0.089	-0.576	0.116	-0.146	**-3.024**	0.603	0.063	2.455	1.975	0.877	0.3	-0.31	-0.599	-0.341	-1.307
Tobacco	0.084	-0.092	-0.088	0.021	-0.05	-0.262	-0.181	0.427	0.356	0.29	**-0.864**	0.19	-0.763	-0.089	-0.144	-0.204	0.1	0.203	-0.065	0.132
Domestic Fuels	0.054	-0.067	0.032	-0.003	-0.063	-0.124	-0.19	-0.076	0.339	0.013	0.08	**-0.324**	-0.269	-0.817	-0.07	-0.042	0.098	-0.085	0.179	0.335
Household G & S	0.026	-0.073	0.042	0.028	-0.041	-0.035	0.052	-0.257	0.052	0.274	-0.177	-0.149	**-0.892**	-0.074	-0.037	-0.199	0.007	0.044	0.127	0.281
Adult Clothing	-0.122	0.089	-0.031	-0.11	-0.237	-0.221	0.764	-0.378	0.151	0.548	-0.051	-1.127	-0.184	**-1.07**	0.01	0.111	0.083	1.213	0.141	-0.579
Children's Clothing	0.236	-0.204	-1.558	0.215	-0.539	-1.23	-0.06	1.953	2.045	1.923	-0.656	-0.76	-0.719	0.077	**-0.092**	0.148	0.538	-0.047	-0.678	-1.592
Petrol and Diesel	0.073	0.282	0.191	0.036	0.146	0.025	0.039	-0.207	-0.336	0.13	-0.184	-0.09	-0.771	0.174	0.029	**-0.383**	-0.083	0.124	-0.095	-0.1
Public Transport	-0.006	0.259	-0.125	-0.081	0.215	0.076	-0.065	-0.237	0.223	-0.329	0.219	0.514	0.07	0.318	0.26	-0.202	**-0.736**	-0.254	-0.691	-0.428
Leisure Goods	-0.241	-0.11	-0.441	-0.017	0.062	0.206	-0.834	0.357	-0.252	-0.264	0.186	-0.186	0.174	1.926	-0.009	0.126	-0.106	**-1.341**	-0.148	-0.089
Books & Newspapers	-0.462	-0.193	-0.21	0.007	-0.188	-0.12	0.165	0.239	-0.455	-0.279	-0.11	0.728	0.933	0.414	-0.253	-0.179	-0.534	-0.274	**-0.127**	-0.103
Leisure Services	0.087	0.251	0.188	-0.003	0.161	0.027	0.277	-0.276	-0.166	-0.266	0.056	0.339	0.512	-0.424	-0.148	-0.047	-0.082	-0.041	-0.025	**-1.421**

I. Elasticities of the demand for goods (rows) with respect to prices (columns)

	Decile 1	Decile 2	Decile 3	Decile 4	Decile 5	Decile 6	Decile 7	Decile 8	Decile 9	Decile 10	Overall	Av. Tax rate
Bread & Cereals	0.03877	0.03093	0.0274	0.02515	0.02306	0.02102	0.01902	0.01702	0.01461	0.01037	0.02273	0%
Meat & Fish	0.11791	0.10042	0.09222	0.08495	0.0778	0.07142	0.06378	0.05618	0.04764	0.03509	0.07474	0%
Dairy	0.08713	0.06931	0.06137	0.05619	0.05036	0.04583	0.04071	0.03565	0.03007	0.02107	0.04977	0%
Tea & Coffee	0.01854	0.01459	0.01245	0.01129	0.0101	0.00911	0.00805	0.00708	0.00587	0.0041	0.01012	0%
Fruits and Vegetables	0.06246	0.05681	0.05306	0.04979	0.04704	0.04451	0.04137	0.0382	0.03413	0.02606	0.04534	0%
Other Untax. Foods	0.04277	0.03724	0.03393	0.03238	0.03079	0.02921	0.02742	0.02586	0.02335	0.01817	0.03011	0%
Food	0.04098	0.03877	0.03733	0.03551	0.03464	0.03366	0.03226	0.0308	0.02864	0.02279	0.03354	17.50%
Food out	0.02647	0.03441	0.03994	0.04437	0.05048	0.05489	0.06071	0.06613	0.07147	0.06985	0.05187	17.50%
Beer	0.02131	0.03223	0.03664	0.03962	0.04116	0.0421	0.04287	0.04237	0.04194	0.03509	0.03753	28%
Wine & Spirits	0.0108	0.01427	0.01685	0.01828	0.01994	0.02109	0.02252	0.02337	0.02487	0.02487	0.01969	55.80%
Tobacco	0.04183	0.05082	0.05255	0.05075	0.04797	0.04394	0.03996	0.03516	0.02802	0.01803	0.0409	79.10%
Domestic Fuels	0.16981	0.14192	0.12375	0.11068	0.0996	0.08992	0.07856	0.0681	0.05654	0.03936	0.09782	5%
Household G & S	0.13618	0.15314	0.16222	0.17019	0.17491	0.1814	0.18828	0.1958	0.19979	0.20161	0.17635	17.50%
Adult Clothing	0.03831	0.05315	0.06014	0.06612	0.07077	0.07433	0.07939	0.08317	0.08745	0.09643	0.07093	17.50%
Children's Clothing	0.00202	0.00533	0.00752	0.00898	0.01	0.01058	0.01102	0.01164	0.01177	0.01097	0.00898	0%
Petrol and Diesel	0.01385	0.02871	0.03672	0.04445	0.04988	0.05418	0.05819	0.05899	0.05909	0.04989	0.04539	73.25%
Public Transport	0.02006	0.02115	0.02119	0.02007	0.01939	0.01912	0.0178	0.01697	0.01542	0.01481	0.0186	17.50%
Leisure Goods	0.01523	0.02362	0.03047	0.03485	0.04046	0.04615	0.05027	0.05718	0.06735	0.08105	0.04466	17.50%
Books & Newspapers	0.03357	0.0297	0.02724	0.02514	0.0245	0.02337	0.02161	0.02042	0.01878	0.01633	0.02407	0%
Leisure Services	0.06199	0.06348	0.06704	0.07124	0.07714	0.08419	0.09621	0.10993	0.13318	0.20406	0.09685	17.50%

II. Budget shares by consumption deciles and aggregate; indirect VAT and excise tax rate (fraction of consumption price)