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A Note on The Pareto Efficiency of General Oligopolistic Equilibria*

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Résumé

Dans cette note nous montrons qu'un équilibre général oligopolistique, une notion d'équilibre introduite récemment par Neary dans la littérature, peut être un optimum de Pareto. Par conséquent, l'allocation des ressources en un tel équilibre peut être identique à celle d'un équilibre général concurrentiel. Nous proposons également une caractérisation des équilibres généraux oligopolistiques qui sont Pareto-efficaces.

Mots clés: Equilibre général oligopolistique, équilibre concurrentiel, Efficacité parétienne.

Classifications JEL : D50, L13, L40.

Abstract

This note shows that a general oligopolistic equilibrium (GOLE), a notion recently introduced in the literature by Neary, may be Pareto-efficient. Consequently, at a GOLE, the allocation of resources can be identical to that of a competitive equilibrium. We also propose a characterization of Pareto-efficient general oligopolistic equilibria.

Key words: General Oligopolistic Equilibrium (GOLE), Competitive Equilibrium, Pareto Efficiency.

JEL Classification: D50, L13, L40.

1 Introduction

Recently, P. Neary introduced a notion of general oligopolistic equilibrium (GOLE), see Neary 2002 (a)-(b), 2003 (a)-(c). He provides us with a new way of modelling oligopoly in general equilibrium. This view takes advantage of the fact that generally firms are large in their own market but small in the economy as a whole.

Using a convenient specification of preferences (i.e. quadratic subutility), Neary illustrates how his notion of GOLE can be fruitful in a wide range of areas (industrial organization, international trade etc...). One of the aims of Neary is to study competition policy in a tractable macro model. He obtains an interesting and surprising result: in a featureless economy (i.e without heterogeneity across production sectors), increasing the number of firms and hence competition in each sector has no effect on welfare (see Neary (2003 (b))). Neary offers the following explanation of this result (Neary, 2003 (b), page 11): "Inducing entry by firms in all sectors raises the demand for labor. Since the aggregate labor supply constraint is binding, this merely redistributes income from profits to wages without any gain in efficiency".

This note complements the explanation of this finding. We show that in fact, under the specific assumptions used by Neary, a GOLE is Pareto efficient. It should then come as no surprise that a competition policy has no effect on welfare (in an economy with a representative agent). Moreover, since in the model used by Neary there is a single competitive equilibrium, this proves that the allocation of resources in a GOLE may be identical to that of a competitive equilibrium. In order to establish these results we shall present a general analysis and establish a characterization of efficient GOLE (with interior allocations).

Our results are related to a paper by Kaas (2001). Kaas proposes a definition of a Cournot-Walras equilibrium without profit feedback. He was able to show that, when firms have identical product technologies, the equilibrium allocation coincides with a perfectly competitive equilibrium solution (and, hence, as it appears, with a Pareto optimum allocation). We also assume that there is no profit feedback. However, we use a subjective demand approach (moreover, firms are price takers on the input markets) and our results obtain even if firms have different technologies.

There is also a link with some results obtained by Dixit and Stiglitz (1977) for second-best optima (but under the assumption of monopolistic competition and different preferences and or technologies).

The plan of this note is as follows. In the next section, we briefly present the notion of GOLE. In section 3, we study the Pareto optimum of this economy. In section 4, we present a characterization of Pareto-optimal GOLE and comment the results obtained by Neary as to the effect of competition policy on welfare.

2 The model

Following Neary (2003 (b)), we consider a simple closed economy inhabited by a representative agent. There is a continuum of production sectors, each of which producing a consumption good.

The preferences of the representative agent are described by the following functional:

$$U(x(.)) = \int_I u(x(z), z)dz, \tag{1}$$

where z is an index of a production sector, $I = [a, b]$ is a interval of the real line. The agent supplies a fixed amount of labor L and considers the price system $p : I \to \mathbb{R}_{++}$ as given. We shall assume that $u(., .)$ is in $C^2(\mathbb{R}_+ \times I, \mathbb{R})$, increasing and strictly concave with respect to its first argument. Moreover, $p(.)$ is in $C^1(I, \mathbb{R}_{++})$.

The agent solves the following problem:

$$\max U(x(.)) = \int_I u(x(z), z)dz, \tag{2}$$

s.t.

$$\int_I p(z)x(z) \; dz = R, \tag{3}$$

where R is the agent's income.

Let us assume that the solution of the agent's problem is "interior" (i.e., for all z in I, $x(z) > 0$). Then, a standard argument shows that there exists $\lambda > 0$, such that the agent's optimal choice $x(.)$ solves (3) and the following equation[1]:

$$u'(x(z), z) \;\; = \;\; \lambda p(z). \tag{4}$$

[1]In this paper, for simplicity, the notation $f'(x, z)$ denotes the partial derivative of f with respect to its first argument x.

Let us now turn to the production side of the economy. Technology is ricardian (labor is the sole input). The unit cost in sector z writes $\alpha(z)w$, where w is the wage rate. The function $\alpha(.)$ is assumed to be in $C^1(I, \mathbb{R}_{++})$.
Firms are symmetric within each sector and $n(z)$ denotes the number of firms in sector z. Firms maximize profits taking both a subjective inverse demand function $P(x, z)$ and the wage rate as given. The function $P(.,.)$ is in $C^1(\mathbb{R}_+ \times I, \mathbb{R}_+)$ and is decreasing with respect to its first argument. Firms compete à la Cournot in their market.

Denoting by x_i firm i's production in sector z, we define its profit as:

$$(P(\sum_{j=1}^{n(z)} x_j, z) - \alpha(z)w)x_i.$$

In a Cournot equilibrium each firm is active and its production must satisfy:

$$P(\sum_{j=1}^{n(z)} x_j, z) - \alpha(z)w + P'(\sum_{j=1}^{n(z)} x_j, z)x_i = 0, \ i = 1, .., n(z).$$

Firms being symmetric, they produce the same quantity in equilibrium, and the total equilibrium quantity produced in sector z, say $x(z)$, solves

$$P(x(z), z) - \alpha(z)w + P'(x(z), z)\frac{x(z)}{n(z)} = 0. \tag{5}$$

We shall denote $\pi(x(z), z)$ the aggregate profit of sector z when the production of this sector is $x(z)$.

We now introduce two definitions of a general equilibrium with imperfect competition for our model.

Definition 1 *A general equilibrium with imperfect competition given subjective demand functions $P(., z)$, $z \in I$, is a price function p(.), a wage rate w, and an allocation function x(.) such that:*

1) The allocation function $x(.)$ solves the consumer's problem for the price $p(.)$ and the income $R = \int_I \pi(x(z), z)dz + wL$, that is (3) and (4) are satisfied.

2) For all z in I, the quantity $x(z)$ is the Cournot equilibrium production of this sector, that is equation (5) is satisfied for all z.

3) The labor market clears

$$\int_I \alpha(z)x(z)dz = L. \tag{6}$$

4) Price expectations of firms and consumer are compatible, i.e.

$$p(z) = P(x(z), z), \quad \forall z \in I. \tag{7}$$

Remark 1. Note that in this definition, price expectations of firms and consumer are compatible only for equilibrium quantities. That is, we do not require that the inverse function is equal to its "true" value off equilibrium. Hence, we use a definition in the spirit of Negishi's approach to general equilibrium with imperfect competition. Note also that if $P'(x, z) = 0$, firms in sector z behave as price takers so our definition boils down to that of a competitive equilibrium. Finally, inspecting the above equations shows that the equilibrium allocation does not change when all the nominal variable - $P(.,.)$, $p(.)$, w - are multiplied by a same positive real number. As a consequence, we are free to choose any convenient normalization device.

We can now define the notion of GOLE introduced by Neary which is a general equilibrium with imperfect competition with a peculiar price expectations function.

Definition 2 *A GOLE is a general equilibrium with imperfect competition where* $P(x, z) = u'(x, z)$, $\forall x \geq 0, \forall z \in I$.

Remark 2. Several comments are in order. First of all, the choice of the inverse demand function above generates a particular normalization of nominal variables. Indeed, inspecting equation (4) reveals that in equilibrium the marginal utility of income - λ - is equal to one. Second, since the inverse subjective demand function is given by (4) (with a constant λ), this implies that firms do not take into account effects of their choices on the consumer's income (Ford effects are disregarded). This is in line with Neary's idea that firms are large in their own market but small in the economy as a whole. Finally, in the sequel, we shall use the following equation in order to use the notion of GOLE (it is easily deduced from equation (5)):

$$\frac{u'(x(z), z)}{\alpha(z)} + \frac{u''(x(z), z)x(z)}{\alpha(z)n(z)} = \frac{u'(x(\widehat{z}), \widehat{z})}{\alpha(\widehat{z})} + \frac{u''(x(\widehat{z}), \widehat{z})x(\widehat{z})}{\alpha(\widehat{z})n(\widehat{z})} \text{ for all } z, \widehat{z} \text{ in } I. \tag{8}$$

Example 1. In Neary (2003 (b)), the author assumes that the subutility functions are quadratic and do not depend upon z. Moreover, he supposes that there is diversity across production sectors (i.e. $\alpha(z)$ is not a constant function)[2]. In this paper, we shall present our results assuming that the subutility functions and the number of firms may depend upon z. In the quadratic case, this would amount to assume:

$$u(x,z) \equiv a(z)x - \frac{1}{2}b(z)x^2. \tag{9}$$

To obtain an interior solution, assume that for all $z \in I$, $a(z) > 0$, $b(z) > 0$, $\frac{a(z)}{b(z)} < +\infty$, $x(z) \in [0, \frac{a(z)}{b(z)}]$. Such a specification of utility implies a linear demand function for good z, that is:

$$P(x,z) = a(z) - b(z)x. \tag{10}$$

A direct computation yields the equilibrium values of consumptions and the wage rate:

$$x(z) = \frac{n(z)}{b(1+n(z))}(a(z) - \alpha(z)w), \tag{11}$$

where:

$$w = \frac{\int_I \frac{\alpha(z)n(z)a(z)}{b(1+n(z))}dz - L}{\int_I \frac{\alpha(z)^2 n(z)}{b(1+n(z))}dz} \tag{12}$$

Example 2. Consider the case where no heterogeneity exists[3], i.e., $u(x,z) = v(x)$, $\alpha(z) = \alpha$ and $n(z) = n$. Denote by $\mu(I)$ the measure of I, the equilibrium value of $x(z)$ is then:

$$x(z) = \frac{L}{\alpha\mu(I)} \tag{13}$$

[2]The number of firms is the same across sectors.

[3]Neary (2003 (b)) calls this very simple case a *featureless economy* (in the setting of example 1, he assumes that $\alpha(z) = \alpha$, $a(z) = a$, $b(z) = b$ and $n(z) = n$, $\forall z \in I$).

3 Pareto Optimum

We now study the Pareto optimum of the economy, denoted by $x^*(.)$, which is the solution of the following problem:

$$\max_{x(.)} U(x(.)) = \int_I u(x(z), z)dz, \tag{14}$$

s.t.

$$\int_I \alpha(z)x(z)\ dz = L. \tag{15}$$

We assume that the optimum is "interior". The optimum is unique (since the subutilities are strictly concave) and solves (15) and:

$$\frac{u'(x^*(z), z)}{\alpha(z)} = \frac{u'(x^*(\widehat{z}), \widehat{z})}{\alpha(\widehat{z})} \text{ for all } z, \widehat{z} \text{ in } I. \tag{16}$$

In example 1, the interior Pareto optimum is given by[4]

$$x^*(z) = \frac{a(z)}{b(z)} - \frac{\alpha(z)}{b(z)}(\frac{\int_I \frac{\alpha(z)a(z)}{b(z)}\ dz - L}{\int_I \frac{\alpha(z)^2}{b(z)}\ dz}). \tag{17}$$

In example 2, the common value of consumption reduces to:

$$x^* = \frac{L}{\alpha\mu(I)}. \tag{18}$$

It is immediate to see in this example that the efficient interior allocation of resources is identical to that of a GOLE. Hence, a GOLE may be Pareto efficient. In the next section, we address the issue of the robustness of this result.

[4]It is assumed that $\inf_{z\in I} \frac{a(z)}{\alpha(z)} > (\frac{1}{\int_I \frac{\alpha(z)^2}{b(z)}\ dz})(\int_I \frac{\alpha(z)a(z)}{b(z)}\ dz - L)$ and $\int_I \frac{\alpha(z)a(z)}{b(z)} dz > L$.

4 Characterization of Pareto Efficient GOLE

Let us now determine under what conditions a GOLE with an interior allocation is Pareto efficient.

Proposition

• *If x is a GOLE allocation, it is Pareto efficient if and only if*

$$\frac{x(z)u''(x(z),z)}{n(z)u'(x(z),z)} = \frac{x(\widehat{z})u''(x(\widehat{z}),\widehat{z})}{n(\widehat{z})u'(x(\widehat{z}),\widehat{z})} \text{ for all } z,\widehat{z} \in I. \tag{19}$$

• *Moreover, assume that $u(.,.)$ is in $C^3(\mathbb{R}_+ \times I, \mathbb{R})$ and $2n(z)u''(y+\frac{n(z)-1}{n(z)}x(z),z)+ yu'''(y+\frac{n(z)-1}{n(z)}x(z),z) \leq 0$ for all positive y and for all z. Then, if x is efficient, it is a GOLE allocation if and only (19) is satisfied.*

Proof. Let us prove the first assertion. Assume that $x(.)$ is a GOLE. As for the only if part, notice that $x(z)/n(z)$ realizes the maximum of the profit function $(P(y+x(z)\left((n(z)-1)/n(z)\right),z)-\alpha(z)w)y$ with respect to y. Recall that the necessary condition for optimality writes (equation (5)):

$$P(x(z),z) - \alpha(z)w + \frac{x(z)}{n(z)}P'(x(z),z) = 0 \ \forall z.$$

As $P(x(z),z) \equiv u'(x(z),z)$, this equation writes as already noticed:

$$u'(x(z),z) - \alpha(z)w + \frac{x(z)}{n(z)}u''(x(z),z) = 0, \ \forall z. \tag{20}$$

So, for all z, $\widehat{z}$, one has:

$$\frac{1}{\alpha(z)}(u'(x(z),z) + \frac{x(z)}{n(z)}u''(x(z),z)) = \frac{1}{\alpha(\widehat{z})}(u'(x(\widehat{z}),\widehat{z}) + \frac{x(\widehat{z})}{n(\widehat{z})}u''(x(\widehat{z}),\widehat{z})). \tag{21}$$

For a GOLE to be an optimum, equation (16) must be satisfied. This clearly implies condition (19).

As for the if part, assume that equation (19) holds true and that $x(.)$ is a GOLE allocation. It is immediate to see that the necessary condition for Pareto optimality (16) is satisfied. Since this condition is sufficient, the GOLE allocation is indeed a Pareto-optimum.

Now, consider the second assertion. We have already proved that if $x(.)$ is a GOLE allocation and if $x(.)$ is efficient, then equation (16) must be true. Assume now that $x(.)$ is a Pareto optimum (and $x(z) > 0$, $\forall z$). Let us prove that under our additional assumption, if condition (19) holds true, $x(.)$ is a GOLE allocation. Since equation (16) is satisfied, there exists $\rho > 0$ such that for all z: $u'(x(z), z)/\alpha(z) = 1/\rho$. Multiplying both sides of equation (19) by $1/\rho$ and adding each side to its corresponding part in equation (16), one sees that there exists a positive h such that for all z:

$$\frac{u'(x(z), z)}{\alpha(z)} + \frac{x(z)}{\rho n(z)} \frac{u''(x(z), z)}{u'(x(z), z)} = h. \tag{22}$$

Taking into account the definition of ρ, the above equation can be re-arranged, and one gets for all z:

$$u'(x(z), z) + \frac{x(z)}{n(z)} u''(x(z), z) = \alpha(z)h. \tag{23}$$

Let us set $w \equiv h$. Under our additional condition, the profit function of each firm in each sector is concave. So, the preceding equation shows that $x(z)/n(z)$ satisfies a necessary and sufficient condition for optimality. From now on, it easy to see that all the remaining equations defining a GOLE are satisfied. Q.E.D.

Remark 3. Equation (19) is clearly satisfied in example 2. It is satisfied in example 1, if there exists a constant c such that for all z:

$$\frac{\frac{a(z)}{\alpha(z)} - w}{\frac{a(z)}{\alpha(z)} + n(z)w} = c. \tag{24}$$

This equality is true whenever $a(z)/\alpha(z)$ and $n(z)$ are constant across sectors.

Remark 4. The additional condition given in the second part of the Proposition is always satisfied with $u(x, z) = \theta(z)v(x)$ where $v(x) = (x^{1-\sigma} - 1) \Big/ (1 - \sigma)$, $0 < \sigma \leq 1$ and $\theta(z) > 0$. It is also satisfied with $u(x, z) = \alpha(z)x - (1/2)b(z)x^2$.

What is the intuition that drives the result of the above Proposition? It is a rather common view that imperfect competition is inefficient because it induces underproduction compared to perfect competition. However, such a conclusion only holds in a partial equilibrium analysis, and is no longer robust when general equilibrium is considered. Indeed, the labor market equilibrium condition (6) ensures that increasing production in each sector is infeasible in equilibrium. So when a GOLE is inefficient, imperfect competition induces overproduction in some sectors and underproduction in the others (compared to perfect competition).

It is interesting to reformulate condition (19) in terms of the inverse demand elasticity. Denoting $\sigma(x, z)$ the demand elasticity, we have in equilibrium

$$\frac{P(x(z), z) - \alpha(z)w}{P(x(z), z)} = -\frac{P'(x(z), z)x(z)}{P(x(z), z)n(z)} = \frac{\sigma(x(z), z)}{n(z)}.$$

In plain english, condition (19) states that the value of the inverse demand elasticity divided by the number of firms is the same across sectors. To put it another way, the markups are the same for all goods. Note that the above condition obviously holds true under perfect competition. The interesting point is that what matters is not the fact that prices equate marginal costs in equilibrium, but that they are proportional to marginal costs.

Such a property is obviously satisfied in a featureless economy. As a consequence, when there is no heterogeneity across sectors, (that is $\alpha(z) = \alpha, n(z) = n$ and $u(x, z) = u(x)$), a GOLE with an interior allocation is *always* efficient. This is what happens in Neary's analysis and explains his result with regard to competition policy.

Let us explain in a more intuitive way why a general oligopolistic equilibrium is always Pareto efficient in a featureless economy. In this kind of economy the equilibrium allocation is symmetrical. Indeed, since the technology is ricardian, it is immediate that the production x in each sector is such that $\mu(I)\alpha x = L$, and does not depend on the number of firms. In fact, this reasoning does not depend on the equilibrium concept at hand.

Moreover, note that without heterogeneity, all prices $p(z)$ are equal, which implies that they are identical in a GOLE and a competitive equilibrium[5]. The only price which takes different values under imperfect competition and

[5]This fact also ensures that the income of the representative agent, which is equal to the equilibrium value of the production, is the same in a competitive equilibrium and in a GOLE.

perfect competition is that of labor (and indeed, Neary notes that an increase in competition yields an increase in the wage rate). However, since labor is exogenously supplied by the representative agent, the difference in the equilibrium values of wages has no supply-side effect.

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