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What about Labour Demand ?
A Reinterpretation of the Elasticity of Hours to Wages

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What about labor Demand ? A Reinterpretation of the Elasticity of Hours to Wages

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Abstract : The estimation of the elasticity of hours to wages relies mostly on labor supply models although workers choices of hours may be constrained. This article reexamines the theoretical meaning of this elasticity accounting for labor demand. To do so, we introduce part-time work characteristics within an on-the-job equilibrium search model framework. Workers receive two types of labor contracts : one corresponding to full-time jobs, and the other to part-time jobs. Utility as a function of wages and the number of weekly worked hours is the criterion workers consider when accepting or rejecting job offers. A firm posts part and full-time job offers as a result of the trade-off between its production level, its wage costs and the costs of hiring part- and full-time employees. We propose a representation of the production function that generates a specific demand for part-time jobs. We prove the existence of a Nash equilibrium, in which all firms hire both part- and full-time workers. Moreover, after calibrating the structural search model using French individuals and firms data, we show that the relationship between hours and wages depends dramatically on the firm's demand function parameter in this model.

Résumé :L'estimation de l'élasticité des heures travaillées aux salaires repose surtout sur des modèles d'offre de travail alors que le choix des heures par les travailleurs peut être contraint. Cet article revient sur l'interprétation théorique de cette élasticité en tenant de la demande de travail. Pour cela, nous introduisons le travail à temps partiel dans le cadre d'un modèle de recherche d'équilibre en emploi. Les salariés peuvent se voir offrir deux types de contrat : l'un correspondant à un emploi à temps complet, l'autre à temps partiel. L'utilité, fonction du salaire et du nombre d'heures proposé, est le critère que les travailleurs considèrent lorsqu'ils acceptent ou rejettent les offres d'emploi. Une entreprise propose des offres d'emploi à temps partiel et à temps complet suite à l'arbitrage qu'elle fait entre son niveau de production, ses coûts salariaux et les coûts à embaucher des employés à temps partiel ou à temps complet. Nous proposons une représentation de la fonction de production qui génère une demande spécifique de l'entreprise en emplois à temps partiel. Nous démontrons l'existence d'une situation d'équilibre de Nash, dans laquelle toutes les entreprises emploient à la fois des salariés à temps partiel et à temps complet. De plus, ayant calibré le modèle structurel de recherche sur des données françaises d'individus et d'entreprises, nous montrons que la relation entre heures et salaires dépend beaucoup du paramètre reflétant la demande de travail de l'entreprise dans ce modèle.

Introduction

Wage elasticity of labor supply is an important concept in economics. By measuring labor supply responses to a change in the wage rate, it explains the behavior of workers. Extensive, empirical literature has been devoted to estimate this parameter. Quite naturally, it relies mostly on labor supply models (see, for example, the Blundell-MaCurdy survey [4]). However, a few papers underline that the worker's choices of hours may be constrained by labor demand (see among others, Ilmakunnas and Pudney [11], Lacroix and Fréchette [12], Stratton [23]). In this case, it is important to consider firms behavior when estimating some labor supply parameters. This article reexamines the theoretical meaning of the elasticity of hours to wages when accounting for labor demand.

Some papers in the litterature account for demand-side effects on labor supply behaviors mainly in that they restrict the choice set. In neither of these papers are the demand-side effects jointly determined. When the choice of hours is considered, to our best knowledge, the demand effect is always treated as exogenous[1]. Through staffing and scheduling, however, the number of hours worked by employees is an important aspect of firms' strategies (see, for example, Thompson [24] or Partouche [19]) and thus affects labor demand.

In order to study simultaneously labor supply and labor demand, we use the equilibrium search framework, developed by Burdett and Mortensen [8]. In these models, labor supply and labor demand, but also wage determination, are jointly modeled within a context of imperfect information. Moreover, these models allow for endogeneous wage dispersion, workers and firms being identical. The importance of considering job search and firms' behavior when estimating some labor supply

[1] Few studies have focused on part-time labor demand (see Montgomery [15], Montgomery and Cosgrove [16], Friesen [9]). Most are a reduced form analysis of firms' number of part- and full-time workers.

parameters has already been put forward by Hwang, Mortensen and Reed [10].These authors show the importance of considering firms' behavior in the estimation of the wage returns to amenities. They demonstrate that job search generates differences between firm-level and employee-level data that can cause substantial deviations in the estimates of hedonic wage equations.

We have expanded the standard model to integrate a new dimension : the number of hours worked. To begin with, we only allowed a discrete choice of hours, and only two types of contracts were considered : part-time and full-time. The following arguments have led us to this decision. First, part-time work has been an important feature of the labor market for the last decade. In 1996, nearly one-fifth of the French and U.S. labor force, and more than 36 percent of the Dutch labor force was composed of part-time workers. Second, the variation in hours that is induced by working part-time is the main source of identification of the elasticity of hours to wages. Third, still controlling for gender or skill differences, the variability in firms' proportion of part-time workers remains significant, and the proportion of part-time workers for each gender or skill category within the firm is positively correlated [2]. Differences between employers related to part-time use exist that cannot be explained only by workers' characteristics.

To specify the firm-specific demand of part-time workers, we use a modelization of the production function derived from the recent results in operational research in the area of scheduling and optimization of the allocation of workers during the workweek. The main idea is that part-time work allows for some flexibility, because the number of hours worked is lower than for full-time schedules. Part-time work is supposed to be adapted more easily to peaks in activity.

Utility, as a function of wage and hours, is the criterion workers consider when

[2]An example of these stylized facts from the retail sector in France in 1996 is available upon request.

accepting or rejecting job offers (Pucci Roger Valentin [20], Bloemen [3]). We also assume that firms face some difficulties in obtaining their labor force due to imperfect information. For that reason, they need to post and advertise job offers. This has a strictly positive cost and is denoted as the firm's hiring efforts (Robin and Roux [21]). This variable is a component of the firm's strategy. The introduction of hiring efforts accounts for the firm having two kinds of jobs to fill. Each firm can separately choose to look for a part-time or for a full-time worker. Hiring efforts for part- and full-time workers are thus part of the firm's decision variables set.

Within this framework, we prove the existence of a Nash equilibrium. We derive from this equilibrium the firms' part- and full-time hiring efforts and the distributions of utility offered and earned in the economy. We calibrate the structural search model using French individuals and data from French firms. Then, simulating the economy, we study the sensibility of the probability to be a part-time worker to the worker parameter of preference and to the parameters of the production function. We show that the relationship between hours and wages depends dramatically on the demand-side parameters.

The paper is organized in the following way. Section II is devoted to the model setting. Section III presents the calibration. Section IV presents the simulations, and the last section gives our conclusions.

1 Model Setting

The model is a steady-state equilibrium search model. In this environment, workers receive job offers from employers. The number of vacancies posted by an employer depends on the search behavior of the workers and on the hiring strategies of the other firms. A worker can accept or reject an offer. We assume that there exists a measure m of workers and a number N of firms in the economy.

1.1 Firms' Hiring Strategies

To get their labor force, firms need to post and advertise job offers. This has a strictly positive cost and is denoted below as the firm's hiring efforts. This variable is a component of the firm's strategy. Firms use three different tools to control their level of employment (see Robin and Roux [21]) :

- The utility levels u_P and u_F which firms offer to part-time and full-time workers, respectively. These have some effect on the supply behavior of employed or unemployed workers.
- The effort of hiring full-time workers e_F, which firms make to fill full-time vacancies. This can be interpreted as the number of full-time job offered by a firm on the job market or the number of full-time job vacancies posted by the firm.
- The effort of hiring part-time workers e_P, which firms make to fill part-time vacancies.

Let $dH\left(u_P, u_F, e_P, e_F\right)$ be the measure of firms that adopt the strategy $\left(u_P, u_F, e_P, e_F\right)$. The overall hiring effort for full- and part-time jobs will thus be :

$$\begin{aligned} E_F &= \int_{\underline{u}_P}^{\overline{u}_P} \int_{\underline{u}_F}^{\overline{u}_F} \int_{\underline{e}_F}^{\overline{e}_F} \int_{\underline{e}_P}^{\overline{e}_P} e_F dH\left(u_P, u_F, e_P, e_F\right) \\ E_P &= \int_{\underline{u}_P}^{\overline{u}_P} \int_{\underline{u}_F}^{\overline{u}_F} \int_{\underline{e}_F}^{\overline{e}_F} \int_{\underline{e}_P}^{\overline{e}_P} e_P dH\left(u_P, u_F, e_P, e_F\right) \end{aligned}$$

These aggregate quantities will have some impact on the workers' behavior, by influencing the job-offer arrival rates. E_F and E_P can be interpreted as the average number of full- and part-time job vacancies offered by the firms. If λ is the arrival rate of one job offer, the arrival rate of full- and part-time job offers will be $E_F\lambda$ and $E_P\lambda$, respectively.

1.2 Workers' Behavior

Workers are offered part- and full-time jobs. Their behavior depends on two job characteristics : the wage and the number of hours worked. Utility is the criteria they will consider when accepting or rejecting job offers (Bloemen [3]). The employed workers' utility level depends on the earned wage w and on the number of hours worked h :

$$u(w,h) = w * h + \Gamma(H-h) \tag{1}$$

where H is the total number of hours available, so that $H-h$ can be interpreted as the leisure time. Γ is an increasing concave function (see McCall [14] and Pucci, Roger and Valentin [20]). When unemployed, individuals receive the opportunity cost b. Each worker receives job offers according to a Poisson process with an exogenous parameter λ_U when unemployed and λ_E when employed. Thus, unemployed workers (employed, resp.) receive full-time job offers at the rate $E_F\lambda_U$ ($E_F\lambda_E$, resp.) and part-time job offers at the rate $E_P\lambda_U$ ($E_P\lambda_E$, resp.). Each utility offer is drawn from a known (by the worker) c.d.f. F. Other assumptions are usual. The separation rate is supposed to be exogenous, constant and equal to δ. The individual discount rate is denoted as ρ.

The worker objective is to maximize his/her expected, indirect lifetime utility. The optimal strategy of a worker, either unemployed or unemployed, is to accept any job offer that provides a higher utility level than a threshold value. The workers' strategies are given by proposition 1.1[3].

Proposition 1.1. *The optimal strategy, when unemployed, is to accept any utility offer greater than the reservation utility ϕ, which is implicitly defined as :*

[3] The existence of the threshold values can be proved as in Mortensen and Neuman [18].

$$\phi = b + E_T \left(\lambda_U - \lambda_E \right) \int_{\phi}^{\overline{u}} \frac{\overline{F} \left(x^- \right) dx}{\delta + \rho + E_T \lambda_E \overline{F} \left(x^- \right)}$$

The optimal strategy when employed at a utility level u is to accept any utility offer strictly greater than u.

1.3 Global Flow Equations

The workers determine their strategies taking the utility offer c.d.f. $F(.)$ as given. They may accept or refuse job offers. Thus, their behavior has an impact on the c.d.f. of the earned utilities, denoted as $G(.)$. Under the steady state assumption, the relation between $F(.)$ and $G(.)$ is determined using the workers' transitions within the labor market. Let ν denote the steady-state unemployment rate. Flows into and out of the stock $(1 - \nu) mG(u)$ of workers earning a utility level lower than or equal to u are the following.

Between t and $t + dt$, the measure of workers leaving this category corresponds to :

– workers being laid off :

$$\delta \left(1 - \nu \right) mG \left(u \right) dt$$

– workers receiving an offer higher than u :

$$E_T \lambda_E \overline{F} \left(u \right) \left(1 - \nu \right) mG \left(u \right) dt$$

Between t and $t + dt$, the measure of workers entering this category corresponds to the workers previously unemployed :

$$E_T \lambda_U \left[F \left(u \right) - F \left(\phi \right) \right] \nu m dt$$

At the equilibrium, the inflow and outflow are equal, hence :

$$G(u) = \frac{E_T \lambda_U [F(u) - F(\phi)]}{\delta + E_T \lambda_E \overline{F}(u)} \frac{\nu}{(1-\nu)} \tag{2}$$

For u equals to the higher utility level $\overline{u}$:

$$\nu = \frac{\delta}{\delta + E_T \lambda_U \overline{F}(\phi)}$$

1.4 Local Flow Equations

Under the steady state assumption, we can establish the relationship between the strategy of a firm, summarized by (u_P, u_F, e_P, e_F), and the labor force it obtains. $NdH(u_P, u_F, e_P, e_F)$ represents the measure of firms adopting the strategy (u_P, u_F, e_P, e_F). The labor force $L(u_P, u_F, e_P, e_F)$ of a firm is composed of part- and full-time workers. The firms labor force structure depends on the part- and full-time utility levels (related to the firm-specific hourly wage w) that they offer and on the part- and full-time hiring efforts.

$$L(u_P, u_F, e_P, e_F) = l(u_P, e_P) + l(u_F, e_F)$$

with $l(u_P, e_P)$ ($l(u_F, e_F)$, respectively) the part-time (full-time, respectively) employment level associated to the utility level u_P (u_F, respectively) and the hiring effort e_P (e_F, respectively).

Within a firm, for a given offered utility level u_i and a hiring effort e_i, $i = P,F$, the inflow and outflow of part- or full-time workers are the following.

Between t and $t + dt$, the measure of type-i workers entering the firm is equal to the sum of :

- the workers being hired from unemployment

  The probability of an unemployed worker receiving a type i job offer from a firm with a (u_P, u_F, e_P, e_F) strategy is equal to $e_i \lambda_U dtdH(u_P, u_F, e_P, e_F)$. The inflow depends on the unemployment stock, and on the probability that the workers will accept the job offer. If the offered utility is greater than the reservation utility ϕ, it equals :

$$\nu m e_i \lambda_U dtNdH(u_P, u_F, e_P, e_F)$$

– the workers being hired from a firm

As in the previous case, the probability of an employee receiving a i type job offer from a firm with a (u_P, u_F, e_P, e_F) strategy between t and $t + dt$ depends both on the type i specific hiring efforts of the firm and on the exogenous employee arrival rate of one job offer λ_E. This equals $e_i \lambda_E dtdH(u_P, u_F, e_P, e_F)$. An employee will only accept a job offer if the utility proposed is strictly greater than the one he/she currently earns. Thus, the share of the employed work force that accepts such a job offer is equal to $G\left(u_i^-\right)$. The inflow is equal to :

$$(1 - \nu) mG\left(u_i^-\right) e_i \lambda_E dtdNH(u_P, u_F, e_P, e_F)$$

Between t and $t + dt$, the measure of type-i workers leaving the firm is equal to the sum of :

– the workers being laid off :

$$\delta l\left(u_i, e_i\right) dtNdH(u_P, u_F, e_P, e_F)$$

– the workers receiving a better job proposal

All employees in the economy may be contacted by another firm between t and $t + dt$ with a probability $E_T \lambda_E dt$. Type i employees receiving such an offer only accept if it is strictly greater than their current utility u_i. The probability of this happening equals $\overline{F}(u_i)$. The outflow is given by :

$$E_T\lambda_E\overline{F}(u_i)l\left(u_i,e_i\right)NdH(u_P,u_F,e_P,e_F)$$

At the steady-state equilibrium, the inflow and outflow are equal, hence :

$$\begin{aligned} l\left(u_i,e_i\right) &= \frac{\left[\nu\lambda_U+(1-\nu)\,G\left(u_i^-\right)\lambda_E\right]e_im}{\delta+E_T\lambda_E\overline{F}(u_i)} \\ &= e_i\frac{m\delta\lambda_u}{\left[\delta+E_T\lambda_E\overline{F}(u_i)\right]\left[\delta+E_T\lambda_E\overline{F}(u_i^-)\right]}\times\frac{\left[\delta+E_T\lambda_E\overline{F}(\phi)\right]}{\left[\delta+E_T\lambda_U\overline{F}(\phi)\right]} \\ &= e_i\widetilde{l}(u_i) \end{aligned}$$

This equation shows that a firm's type i labor force is the product of its hiring efforts e_i and of a function $\widetilde{l}(.)$, which depends only on the offered utility u_i. Solving the firm's program under general assumptions is strictly equivalent to solving equations that only imply $\widetilde{l}(u_i)$ and the firm's technology parameters (see Robin and Roux [21]).

1.5 Firms' Behavior

We provide a framework where firms offer both part- and full-time labor contracts. In the last thirty years, practicing managers and academic researchers interest in staffing and scheduling problems has increased. Attention has been devoted to scheduling of a composite workforce constisting of part-time and full-time workers[4]. A comprehensive survey of this litterature can be found in Partouche [19]. We present below a simplified scheduling model.

The feasible set of alternative labor schedules is restricted by the following assumptions. Full-time (part-time, respectively) schedules are assumed to cover h_F (h_P, respectively) hours per week. The members of the same shift have the same

[4]See, for example, Showalter and Maber [22], Maber and Showalter [13]

work schedule. To begin with, we allow only one full-time shift and three part-time shifts.

The demand pattern of the firms is defined as follows. All firms produce the same good, perishable[5], during a period T such that $T < h_P + h_F$. The good price is normalized at 1. Between t and $t+dt$, a firm i faces a known exogenous demand for this good : $f_i(t)\,dt$. The firms' technology $p_i(.)$ is an increasing function of the number $[L(t)]$ of employees in the firm at date t. Thus, production is equal to $\min(f_i(t), p_i[L(t)])\,dt$. If every kind of hourly contract were allowed, a firm could produce exactly $f_i(t)\,dt$. As only two kinds of contracts are allowed, a firm cannot adjust its labor force to the demand for goods at time t. Introducing part- and full-time contracts into the firm's labor force gives the firm more flexibility.

To simplify, we assume that the time worked by employees can be indefinitely divided. Under this assumption, the firm can allocate its labor force as if the demand function were decreasing. This new demand function is built in the following way. Let $t_i(q)$ be the time that the demand function is greater than q

$$t_i(q) = \mu(\{x/f_i(x) \geq q\})$$

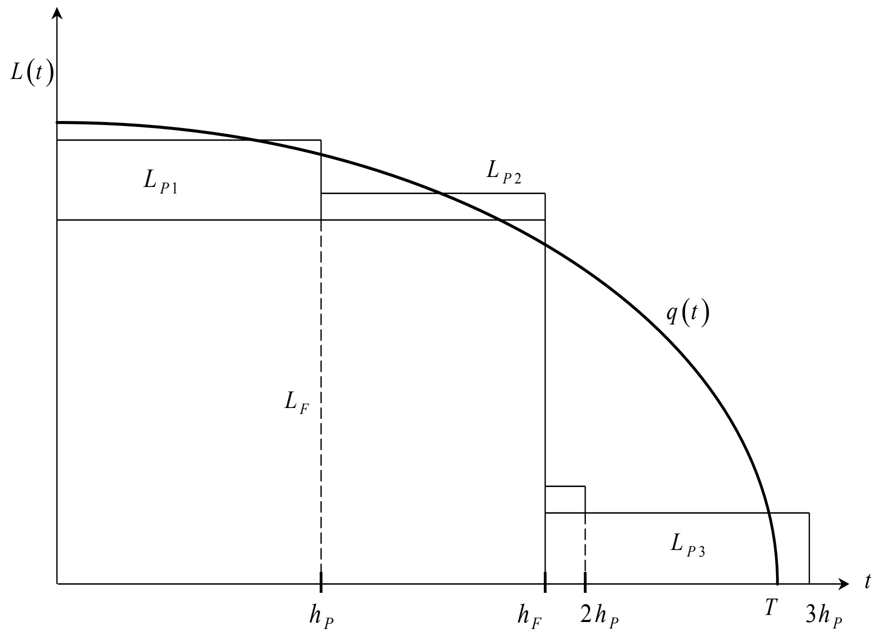
where μ is the Lebesgue measure. The inverse of this function, denoted as $q_i(t)$, is decreasing and corresponds to the new demand for goods. Thanks to the previous assumption, the firms confronted with the demand functions f_i and q_i, all face exactly the same optimization problem - the optimal allocation of their labor force during the opening period T.

As the demand function is strictly decreasing with time, it might be more natural to decide on the maximum number of employees at the beginning of the period (See Figure 1). Thus, full-time employees are working at the beginning of the period and

[5] This means that firms can not stock their production, see services for instance

continue working until $t = h_F$. The total number of these workers is denoted as L_F. Let L_P be the number of part-time employees in the firm. Some of them, share p_1, are working at the beginning of the period and continue working until $t = h_P$. The firms hire other part-time workers (share p_2) between $t = h_P$ and $t = 2h_P$, and the remaining part-time workers between $t = 2h_p$ and $t = T$.

FIG. 1 – Production plan of the firm



The number of employees in a firm is given by[6] :

$$L(t) = \begin{cases} L_F + p_1 L_P, \ 0 \leq t < h_P \\ L_F + p_2 L_P, \ h_P \leq t < h_C \\ (p_2 + p_3) L_P, \ h_F \leq t < 2h_P \\ p_3 L_P, \ 2h_P \leq t < T \end{cases}$$

where $p_3 = 1 - p_1 - p_2$.

[6]Other organizations and number of shifts may be possible. We restrict our study to this type of organization strategy.

The production function is thus :

$$\begin{aligned} Q_i^*\left(L_P, L_F, p_1, p_2\right) &= \int_0^T \min\left\{q_i\left(t\right), p_i\left[L\left(t\right)\right]\right\} dt \\ &= \int_0^{h_P} \min\left\{q_i\left(t\right), p_i\left(L_F + p_1 L_P\right)\right\} dt \\ &\quad + \int_{h_P}^{h_C} \min\left\{q_i\left(t\right), p_i\left(L_F + p_2 L_P\right)\right\} dt \\ &\quad + \int_{h_C}^{2h_P} \min\left\{q_i\left(t\right), p_i\left[\left(p_2 + p_3\right) L_P\right]\right\} dt \\ &\quad + \int_{2h_P}^{T} \min\left\{q_i\left(t\right), p_i\left(p_3 L_P\right)\right\} dt \end{aligned}$$

A firm chooses the optimal allocation (p_1^*, p_2^*) so that the production will be maximized. The couple (p_1, p_2) belongs to the compact $[0,1] \times [0,1]$. Thus, there exists (p_1^*, p_2^*) , such that

$$\begin{aligned} Q_i\left(L_P, L_F\right) &= Q_i^*\left(L_P, L_F, p_1^*, p_2^*\right) \\ &= \sup_{(p_1, p_2)} Q_i\left(L_P, L_F, p_1, p_2\right) \end{aligned}$$

To obtain its labor force, a firm needs to produce hiring efforts e_P and e_F, which cost $c_i\left(e_P, e_F\right)$ where $c_i\left(.,.\right)$ is strictly positive and increases with e_P and e_F. The firm's objective is to maximize the steady-state profit flow, which is given by the following expression :

$$\Pi_i\left(u_P, u_F, e_P, e_F\right) = Q_i\left(L_P, L_F\right) - \left(w_P h_P L_P + w_F h_F L_F\right) - c_i\left(e_P, e_F\right)$$

where w_P (w_F, respectively) is the hourly wage offered to part-time workers (full-time, respectively). From the specification of the utility function, we can deduce that the wage paid by a firm offering u_P to part-time workers equals $w_P h_P = u_P - \Gamma\left(H - h_P\right)$, and the wage paid to full-time workers equals $w_F h_F = u_F - \Gamma\left(H - h_F\right)$.

The firm-specific part-time (full-time, respectively) labor force L_P (L_F, respectively) is a function of the offered utility u_P (u_F, respectively) and of the hiring effort e_P (e_F, respectively). Thus, the firm chooses (u_P, u_F, e_P, e_F), the utility and the hiring effort in part-time or full-time workers so that profits will be maximized. Regularity conditions on the cost $c_i(.,.)$ and production $p_i(.)$ functions must be verified so that the program $\sup_{e_P \geq 0,\, e_F \geq 0} \Pi_i(u_P, u_F, e_P, e_F)$ has a unique finite solution[7]. Let $(e_P^*(u_P, u_F), e_F^*(u_P, u_F))$ be this solution. The firm's profits are defined as $\Pi_i^*(u_P, u_F) = \Pi_i(u_P, u_F, e_P^*, e_F^*)$.

1.6 Equilibrium

The workers choose their reservation utility on the basis of the utility c.d.f. generated by the firms' utility posting behavior and total hiring effort. The firms' optimal strategies are determined by those of the other firms and by workers' behavior.

All firms are ex-ante identical. They have the same technology, face the same demand function and have the same hiring cost function. Thus, they get the same profit flow. Under this assumption, we can establish the relationship between the utility offer distribution $F(.)$ and the firm's utility distribution $H(.)$ using the relationship between the firm-specific part- and full-time hiring efforts and the offered utility. H and F obviously have the same support Λ.

The number of part-time (full-time, respectively) job offers posted by the firm offering a utility u is the same as the number of part-time (full-time, respectively) job offers received by the workers with a utility u. We get :

$$\begin{aligned} E_P dF_P(u) &= N \int_{\underline{u}_F}^{\overline{u}_F} e_P^*(u, u_F)\, dH(u, u_F) \\ E_F dF_F(u) &= N \int_{\underline{u}_P}^{\overline{u}_P} e_F^*(u_P, u)\, dH(u_P, u) \end{aligned}$$

[7]If, for example, $p_i(.)$ is concave (which implies the quasi-concavity of $Q_i^*(.,.)$) and $c_i(.,.)$ convex, Π_i is sure to be quasi-concave.

where N is the number of firms, which gives :

$$\begin{aligned} E_T F(u) &= N \int_{\underline{u}_P}^{u} \int_{\underline{u}_F}^{\overline{u}_F} e_P^*(u_P, u_F)\, dH(u_P, u_F) \\ &\quad + N \int_{\underline{u}_F}^{u} \int_{\underline{u}_P}^{\overline{u}_P} e_F^*(u_P, u_F)\, dH(u_P, u_F) \end{aligned}$$

The equilibrium concept is a Nash equilibrium. The definition below is inspired by the one used by Robin and Roux [21].

Definition 1.2. *An equilibrium consists of a measure of N active firms, a distribution H of the firm's strategies with support Λ, a reservation utility ϕ for unemployed workers and a global hiring effort E_T such that :*

1. *(Optimal Stopping Rule for Unemployed) Unemployed workers accept any utility offer greater than their reservation utility ϕ, knowing H and E_T.*
2. *(Quitting Rule for Employees) Employed workers at utility u accept any alternative utility offer where $u' > u$.*
3. *(Optimal Wage Offers)* $\forall (u_P, u_F) \in \Lambda,\ \forall (u'_P, u'_F),\ \Pi^*(u_P, u_F) \geq \Pi^*(u'_P, u'_F)$
4. *(Entry Condition)* $\forall (u_P, u_F) \in \Lambda,\ \Pi^*(u_P, u_F) \geq \pi_0$.

The equilibrium is sequential. First, firms decide whether or not to enter the market. Second, firms choose a wage offer that maximizes their steady-state profit flow. Third, firms choose their part- and full-time job hiring efforts and organization. Workers determine their search behavior and reservation utility according to the distribution of utility offers.

Proposition 1.3. *If on-the-job transitions are allowed* $(\lambda_E > 0)$, *then the utility offer distribution is non-degenerate.*

Démonstration. See appendix. □

The equilibrium illustrates that firms must face a trade-off between the wage and hiring costs and the level of employment they need in order to produce. Knowing the wages they post (which means the utility level they offer), firms choose part- and full-time job-hiring efforts that maximize their steady-state profit flow. The function $\widetilde{l}\,(.)$ establishes the link between the decision to offer the utility u and the labor force the firm will hire. If a discontinuity in $\widetilde{l}\,(.)$ occurred, a significant measure of firms would offer the same utility u. In that case, since all firms have the same information, they are all aware that a significant measure of the firms offers the same utility u. Thus, a firm offering a slightly greater utility at the equilibrium instead of u increases its profits if no other firm changes its strategy. Since the utility u is not profit-maximizing, no firm will post it. This is contrary to the assumption that a measurable share of firms post this utility. The argument used here is the same as that used in Burdett and Mortensen [8].

The lowest offered utility is greater than $\max(\phi, \underline{u})$ where $\underline{u}$ is the lowest utility associated with the legal minimum hourly wage $\underline{w}$. This is because no firm offering a lower utility could exist for legal reasons. Moreover, the firm could not attract any workers if the offered utility were lower than the reservation utility[8]. As a consequence, the equilibrium unemployment rate is

$$\nu = \frac{\delta}{\delta + E_T \lambda_U}$$

This rate depends on both the exogenous structure (δ, λ_U) and the endogenous structure (firms behavior) E_T.

Proposition 1.4. *If Q is quasi-concave, $\widetilde{l}$ is continuous and differentiable with $2\widetilde{l}'^2 -$*

[8]The introduction of a legal minimum hourly wage can create some computational complications, because some firms might only hire one kind of worker. To avoid this problem, we do not consider the effects of a legal minimum wage.

$\widetilde{l}''\widetilde{l} > 0$ [9], *and the firm produces the optimal hiring efforts* (e_P^*, e_F^*) *with*

$$\frac{\partial c}{\partial e_P}\left(e_P^*, e_F^*\right) = \frac{\partial c}{\partial e_F}\left(e_P^*, e_F^*\right) \tag{3}$$

then it is optimal for the firm to offer the same utility to part-time and full-time workers.

Démonstration. See appendix. □

If (3) applies, the firms' strategies are completely determined by the choice of one utility level $u = u_P = u_F$. Hence, $e_P^*(u_P, u_F) = e_P^*(u)$, $e_F^*(u_P, u_F) = e_F^*(u)$, and the firm's utility distribution H is related to the c.d.f F by equation :

$$(E_P + E_F)dF\left(u\right) = N(e_P^*\left(u\right) + e_F^*\left(u\right))dH\left(u\right)$$

derived from (3).

As stated by the definition of the equilibrium, the firm's decisions are sequential. The firm maximizes its profits at a given level of utility u, knowing $\widetilde{l}\left(u\right) = l$:

$$\widehat{\pi}\left(u, l\right) = \sup_{e_P, e_F \geq 0} Q\left[e_P l, e_F l\right] - l\left\{\left[u - \Gamma\left(H - h_P\right)\right] e_P + \left[u - \Gamma\left(H - h_F\right)\right] e_F\right\} - c\left(e_P + e_F\right)$$

The resolution of this program gives the firm's production plan and establishes its hiring effort of part- and full-time workers. The condition of a constant profit level when all firms are ex-ante identical (free Entry) is given by :

$$\widehat{\pi}\left[u, \widetilde{l}\left(u\right)\right] = \pi_0$$

This condition gives the relation between u and $\widetilde{l}\left(u\right)$. This condition can be seen as an entry condition : firms which do not have enough profits cannot survive. Using

[9] The condition $2\widetilde{l}'^2 - \widetilde{l}''\widetilde{l} > 0$ is verified if F is not too convex.

this relation and the firm's maximization program, we know the value of $\widetilde{l}(u)$ and the hiring efforts for part- and full-time workers for each firm that offers a utility level u. The minimum utility $\underline{u}$ is given by the minimum of the reservation utility and the legal minimum utility (if it exists). Using the relations presented above, $\widetilde{l}(\underline{u})$ can be computed. Flow equations give a relation between the friction parameters and $\widetilde{l}$:

$$E_T\widetilde{l}(\underline{u}) = \frac{m\delta E_T\lambda_u}{[\delta + E_T\lambda_U][\delta + E_T\lambda_E]}$$

where $E_T\lambda_U$, $E_T\lambda_E$ and δ are transition intensities, which can be estimated from the observed transitions on the labor market. From these relations, we can obtain the global hiring effort E_T and compute the exogenous parts of the transition parameters λ_U and λ_E. Thus, the maximum value of $\widetilde{l}$ is obtained by the following relation :

$$\widetilde{l}(\overline{u}) = \frac{m\lambda_u[\delta + E_T\lambda_E]}{[\delta + E_T\lambda_U]\delta}$$

The inversion of $\widetilde{l}$ gives the utility support's upper bound $\overline{u}$. Thus, the support of the offered utilities is fully determined. The c.d.f. F of the offered utilities to the workers is a function of $\widetilde{l}$. Using the derivative of this function, the following relation can be established for the utility offer distribution among workers f and the utility distribution among firms h :

$$E_T f(u) = N[e_P^*(u) + e_F^*(u)]h(u)$$

From this relation, h normalized to 1 , the value of the number of firms can be obtained :

$$N = \int_{\underline{u}}^{\overline{u}} \frac{E_T dF(u)}{e_P^*(u) + e_F^*(u)}$$

The entry condition limits the number of active firms on the market[10].

1.7 Mean hours and wages

The model allows for the joint determination of the probability to be a part-time worker. The measure of part-time workers in a firm offering a utility u is :

$$e_P \widetilde{l}(u) dH(u)$$

Thus, the measure of part-time workers owning a wage w is equal to

$$\begin{aligned} e_P \widetilde{l}(u_P) dH(u_P) &= e_P \widetilde{l}(u_P) h_P h(u_P) dw \\ &= E_P h_P \widetilde{l}(u_P) f(u_P) dw \\ &= E_P h_P \frac{\widetilde{l}'(u_P)}{\sqrt{\widetilde{l}(u_P)}} C dw \end{aligned}$$

The measure of full-time workers owning a wage w is given by

$$\begin{aligned} e_F \widetilde{l}(u_F) dH(u_F) &= e_F \widetilde{l}(u_F) h_F h(u_F) dw \\ &= E_F h_F \widetilde{l}(u_F) f(u_F) dw \\ &= E_F h_F \frac{\widetilde{l}'(u_F)}{\sqrt{\widetilde{l}(u_F)}} C dw \end{aligned}$$

[10] As π_0 tends towards 0, the number of active firms increases, as well as the global hiring effort. In this model, the existence of friction on the labor market depends on the existence of barriers to the entry of the active firms, materialized here by a condition on the minimum level of profit that is required.

Thus the conditional probability to be a part-time worker equals :

$$\begin{aligned} P(h=h_P/w) &= P(h=h_P/u=w_P*h_P+\Gamma(H-h_P)) \\ &= \frac{E_P h_P \frac{\widetilde{l}'(u_P)}{\sqrt{\widetilde{l}(u_P)}}}{E_P h_P \frac{\widetilde{l}'(u_P)}{\sqrt{\widetilde{l}(u_P)}} + E_F h_F \frac{\widetilde{l}'(u_F)}{\sqrt{\widetilde{l}(u_F)}}} \\ &= \frac{1}{1+\frac{E_F h_F \widetilde{l}'(u_F)}{E_P h_P \widetilde{l}'(u_P)}\sqrt{\frac{\widetilde{l}(u_P)}{\widetilde{l}(u_F)}}} \end{aligned}$$

with

$$\begin{aligned} u_P &= w*h_P+\Gamma\left(H-h_P\right) \\ u_F &= w*h_F+\Gamma\left(H-h_F\right) \end{aligned}$$

The mean number of weekly hours worked by people with an hourly wage w is :

$$\begin{aligned} \bar{h} &= h_P P(h=h_P/w)+h_F\left(1-P\left(h=h_P/w\right)\right) \\ &= h_F-\left(h_F-h_P\right)P\left(h=h_P/w\right) \end{aligned}$$

From this relation, we get the observed elasticity of hours to wages as a function of the hourly wage w

$$\hat{\alpha}=\frac{\partial\bar{h}}{\partial w}=-\left(h_F-h_P\right)\frac{\partial P\left(h=h_P/w\right)}{\partial w} \tag{4}$$

This relationship shows how the elasticity of hours to wages depend on the structural parameters of the model. We can notice that $\hat{\alpha}$ is a function of the firms' parameters, through the function $\tilde{l}$. We can not exhibit a closed form for this relation. For this reason, we conduct some simulations to look at the way this relation is affected by changes in demand or supply parameters. To do this, we need some plausible values of the structural parameters. This is the aim of the following section which is devoted to the calibration of the model.

2 Calibration of the model

Values of most structural parameters of the model could be easily found in the litterature. For instance, mean hourly productivity p_0 can be found in national accounts. This holds for usual economic concepts. However, the specificity of the production function makes it impossible to find relevant values of its structural parameters in the literature. For this reason we have conducted some reduced form estimations of the model using firm and individual data. They give us values for all the parameters of the structural model.

As in the theoretical section, we do not account for firm and individual heterogeneity. This may affect the values of the estimated parameters. Among others, the endogeneous utility distribution does not fit the data : the distribution is increasing which tends to overweight the biggest firms[11]. However, even being possibly biased, the estimated values will allow us to show the impact of labor demand on the relationship between hours and wages.

The model is calibrated on the Retail sector. In this sector, the number of part-time workers is particularly high and the demand verifies the "no stock" assumption (the answer to demand cannot be delayed). For these reasons, we expect the model to fit best with this sector.

The data have been taken from the French survey "Declarations Annuelles de Salaires" (DADS) collected by INSEE. The DADS are based on employers' reports of their employees' status and earnings. Two distinct samples from the DADS data have been used : one to estimate the parameters linked to the worker's side and one

[11] This is a usual problem in on-the-job equilibrium search models without exogenous heterogeneity. See for example Burdett and Mortensen [8]

to estimate the parameters linked to the firm's side. Some parameters have been held constant : the period T is set at 50 hours; the number of part-time hours worked h_P is equal to 20 hours; the number of full-time hours worked h_F is equal to 39 hours. The number of available hours H is set to 80.

2.1 Parameters Linked to the Firm's Side

The Data The sample consists of $7,329$ stores, employing $680,000$ workers, issued from the administrative data sets "Declarations Annuelles de Données Sociales" (DADS) collected by INSEE in 1996 and the "Bénéfices Industriels et Commerciaux" (BIC). We keep in the merged data set all firms with more than 20 employees and containing both part- and full-time workers, with the goal of studying the substitution effects between these categories. Thus, our final sample contains 5853 observations, corresponding to $668,000$ workers. Information on wages and on part- and full-time work are issued from the DADS. Information on production is issued from the BIC.

The Empirical Model The specification of the production function is a major concern here. We chose the following specification

$$Q\left(L_P, L_F\right) = p_0 \int_0^T \min\left(q_0 \left(T-t\right)^{\chi}, L\left(t\right)\right) dt$$

where $L(t)$ is the active labor force at time t, as defined in Section 1.5. With this specification, we assume that the hourly productivity is constant and equal to p_0. χ will be denoted thereafter as the firm's demand shape parameter. For low values of χ, the demand is rather constant through the opening period, which induces the firm to hire part-time workers only for the third team (L_{P3}, Section 1.5), and thus a higher share of full-time workers. For high values of χ, the demand function is fluctuant and the firm needs part-time workers not only in the third team, but also when the demand is high.

We estimate a nonlinear simultaneous equation system using maximum likelihood methods (Amemiya [1]) under the assumption that the value added Q_i, and the part-time and full-time wage w_P and w_F are observed with measurement errors. We assume the error terms to be Gaussian. The cost function is defined by $c(e_p, e_F) = (e_p + e_F)^{\gamma}$. Estimations are made for $\gamma = 2$. The utility function is defined by $u(w, h) = w * h + \Gamma(H - h)$ with w being the earned hourly wage and h the number of worked hours. As in Pucci, Roger and Valentin [20], we have chosen the specification $\Gamma(H - h) = (H - h)^{\epsilon}$. The total wage cost is denoted $w_{tot} = w_P h_P L_P + w_F h_F L_F$.

The estimated equations are given below.

$$\begin{cases} Q_i = Q\left(L_{Pi}, L_{Fi}\right) + \varepsilon_{1i} \\ w_{toti} = \frac{1}{1-\gamma}\left[L_{Pi}\left(\frac{\partial Q}{\partial L_P}\right) + L_{Fi}\left(\frac{\partial Q}{\partial L_F}\right) - \gamma Q\left(L_{Pi}, L_{Fi}\right) + \gamma \pi_0\right] + \varepsilon_{2i} \\ w_{Fi} h_F - w_{Pi} h_P = \left[(H - h_F)^{\varepsilon} - (H - h_P)^{\varepsilon}\right] + \varepsilon_{3i} \end{cases}$$

Parameter Estimates The results are gathered in Table 1.

TAB. 1 – Firm-side parameters (the standard errors are in parentheses)

p_0	q_0	χ	π_0	ε
8.14	2.12	0.55	502	0.4
(1.89)	(0.87)	(0.13)	(53.6)	(0.01)

We can compute the total potential weekly demand adressed to the firm, $\int_0^T q_0 (T-t)^{\chi} = 588$. It is happily greater than the expected profit π_0. We also conclude that the markup (defined as the ratio of the profit on the production) is greater than $502/588 = 0.85$. This is a very high value with respect to the observed one in the French retail sector. It can be explained by the absence of any other profit sharing mechanism than the between firm competition on wages induced by on-the-job search.

Under the asumptions of the model, utilities of part-time and full-time workers are equal within a firm. This equality provides a way to estimate the supply-side parameter ϵ from firm data. To our best knowledge, this is the first estimation of this parameter using, as source of identification, between firm variability.

2.2 Parameters Linked to the Worker's Side

The Data The first sample consists of 32473 individuals working part-time or full-time in the Retail Sector in 1996. The observations were extracted from a survey in which workers' career paths were followed for 20 years (1976-1996, DADS panel data set). For these workers, the data provide information each year on their wages, employers and current occupations (full-time work, part-time work or non-work). From the complete data set, we can rebuild individual labor market histories. To be as consistent as possible with the firm's data, we did not use the panel's information on past wages. Thus, we only used information for 1996.

The Empirical Model The relevant variables avalaible in the data set are the following.

- S_a is a variable which takes value 1 if the individual had a part-time job and value 2 if he/she had a full-time job in March 1996 ;
- S_f is a variable which takes value 0 if the individual is unemployed, value 1 if he/she had a job during the spell following the observed period ;
- T_e : the elapsed duration in the current state in March 1996 ;
- T_r : the residual duration in the current state in March 1996 ;
- w : the hourly wage associated with the employment spell.

Knowing S_a and w, the estimated value of ε can be used to calculate the individual earned utility level. For a given specification of the utility function, the model determines a unique reservation utility rule implying that any utility level accepted by workers is greater than ϕ. Theoretical results predict that no firm offering a lower utility level than ϕ could exist because it could not attract any workers. Thus, $F(\phi)$ is equal to 0. We get the following relation between ν, δ and $E_T\lambda_U$:

$$\nu = \frac{\delta}{\delta + E_T\lambda_U}$$

As there is no data on unemployed individuals of the Trade Sector in 1996[12], this relation is introduced into the estimations. The value of ν is equal to the observed value in the French economy in 1996 : $\nu = 0.12$.

The model allows for an on-the-job search. The oberved utility distribution is thus not the same as the offered utility distribution. The relation between these two distributions is given by the theoretical model and is derived from the global flow equation presented above. Like Bontemps, Robin and Van den Berg [5], we have used a kernel estimator and the data on employed-worker earning utility to fit the utility distribution G. Conditional to this estimate, the parameters $E_T\lambda_E$ and δ are estimated using maximum likelihood methods. The contribution to the likelihood function for the individual i $(i = 1, ..., N)$ is the joint conditional probability of the endogeneous variables. Under the steady-state assumption, the elapsed duration T_e and the residual duration T_r have exponential distributions. For a worker employed at the utility level u, the parameters of the exponential distributions are the same and equal to $\delta + E_T\lambda_E\overline{F}(u)$.

Parameter Estimates The results are presented in Table 2. The parameters are estimated under the exponentiel link function to ensure their positivity.

The durations of employment depend on the job offer arrival rate, on the separation rate but also on the utility level earned in a given firm. They are in the range

[12]These kinds of data are difficult to define and collect because unemployed individuals are not attached to a special sector.

from 15 months for the firm offering the lowest utility level to 172 months for the one offering the highest one.

TAB. 2 – Worker-side parameters (the standard errors are in parentheses)

δ	$E_T\lambda_E$
-5.17	-2.79
(0.05)	(0.03)

3 The relationship between hours and wages

The model allows for economies where wages are dispersed and hours differentiated. We can thus examine the relationship between the number of hours worked (part-time or full-time) and the wage. How does this relationship depend on the demand- or supply-side parameters ? This section aims at answering this question.

To do this, we first simulate the economy and then provide a sensitivity analysis of the elasticity of hours to wages.

3.1 Simulations

Knowing all the structural parameters, we can calculate the number of part-time and full-time workers in a firm, the global hiring effort E_T, the number of firms, the cdf of the offered and observed utilities and the minimum utility. The method proposed is given in the Appendix. Table 3 and Figures 2 and 3 present the model's features. In Table 3, it appears that the calibrated share of part-time workers is about the same as the share of part-time workers in the workers' sample (39%).

Figure 2 represents the firm's production plans. The left axis corresponds to the utility level u offered by the firm, the right axis corresponds to the elapsed time t of the opening period and the ordinate corresponds to the minimum of the demand function at time t and the firm's potential production level. A cross section of this figure corresponds to the production plan of one firm : how it chooses to allocate its part-time workers throughout the day. This figure shows that the firm offering the lowest utility has the lowest number of part-time workers, who are all working at the end of the opening period. The firm offering the highest utility allocates its part-time workers during the entire opening period.

TAB. 3 – Calibration ($h_F = 39$)

total hiring effort E	197868
Number of firms N	636014
Share of part-time offers	37%
Job offer* arrival rate λ_U	$2.22.10^{-7}$
On-the-job arrival rate λ_E	$3.03.10^{-7}$
Share of part-time workers	34%
Max earned utility $\overline{u}$	32.6
Min earned utility $\underline{u}$	5.7

FIG. 2 – Simulated production plan of the firms

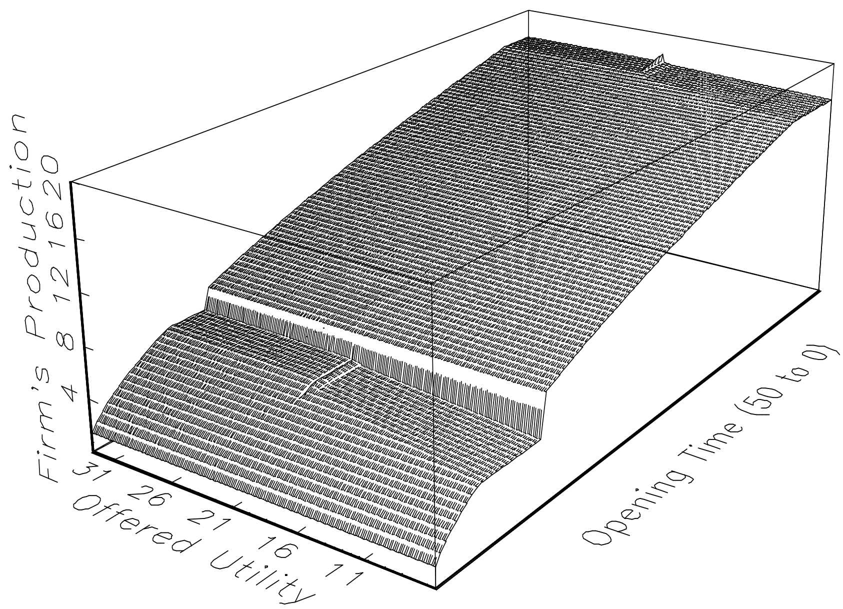
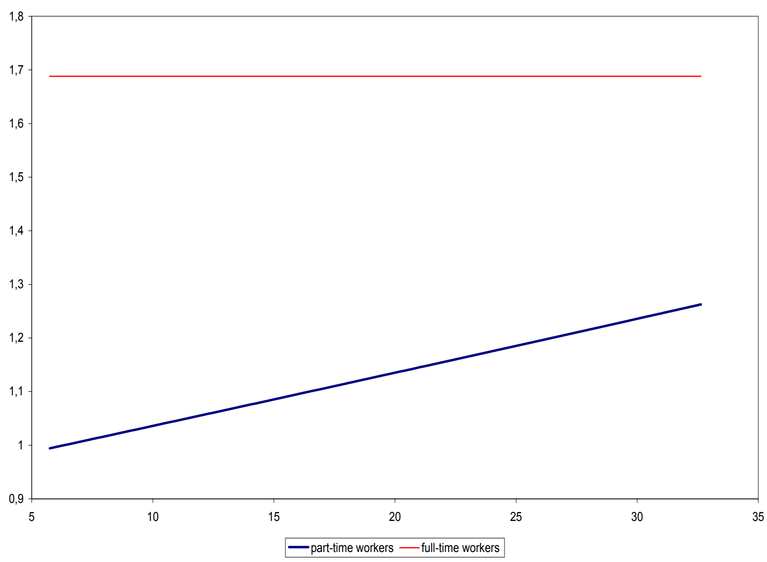


Figure 3 illustrates more precisely the relationship between the firm-specific part- or full-time labor force and the utility level offered by the firm. As previously stated, the share of part-time workers increases with the offered utility. Moreover, the number of full-time employees often remains constant among firms. This result illustrates a particular feature of the model : firms choose a full-time worker hiring effort, which totally compensates for the increase in $\widetilde{l}$. Should ex-ante heterogeneity be allowed between firms, their full-time workforce would be more differentiated.

FIG. 3 – Relationship between part-time and full-time workforce and firm's offered utility



The figures above illustrate the between-firm differentiation on their production plans and on their use of their workforce. Although the workers are all ex-ante identical, they will be ex-post different in their utility levels. Indeed, they are confronted with different firms offering different levels of utilities depending on the wages and hours worked proposed by each firm. The next section will examine the relationship between hours and wages at the individual level, which is usually done in the labor supply literature (see Blundell and Mc Curdy [4]).

3.2 Part-time probability and hourly wage

We will now focus on the probability of being a part-time worker, conditional on a given level w of the hourly wage. This relation is very important since it bounds the hours to the wages. Let us consider what we would do to estimate the elasticity of hours to wages using the variability generated by part-time or full-time status. The first idea would be to exploit the observed variability in workers' wages and hours. Thus, regressing the number of hours on the wages would bring the elasticity of hours to wages. For this regression to give the "true" value of the elasticity, the exogeneity of wages to hours is required. It cannot be the case here since wages and hours are simultaneously proposed by firms to workers[13].

To underline this point, we will consider the derivative of the part-time probability to wages. We showed in section 1.7 that our model predicts a simple relation (see equation 4), between this derivative and the elasticity of hours to wages.

Figure 4, simulating the model holding all parameters constant except the firm's demand shape parameter χ, shows that the estimated elasticity depends on labor demand. The relation between part-time probability and wages is estimated for values of the wages for which part-time and full-time jobs coexist. For wages that are too high or too low, the model predicts only part-time workers[14]. On the common support of the wage distributions of part- and full-time workers, figure 4 shows that the slope of the part-time probability relative to wages depends heavily on the firm's demand

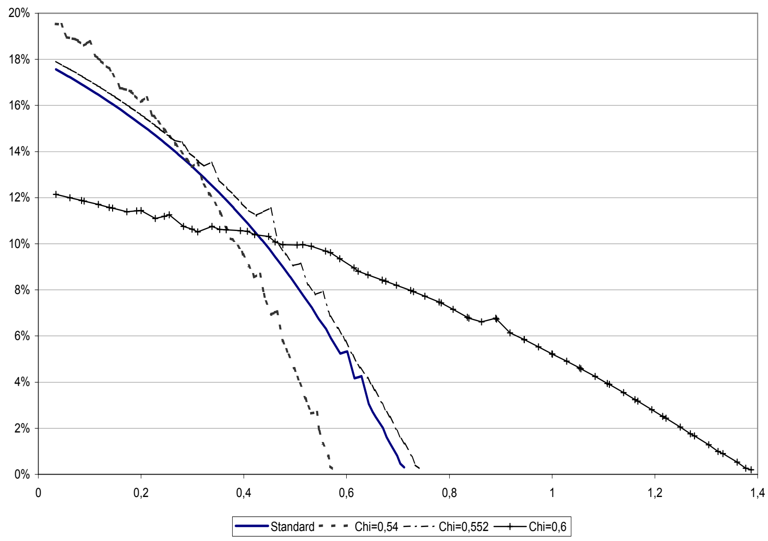
[13] A way to solve this problem could be to find some instrumental variable that would theoretically affect wages but not hours. This does not seem possible, however, if both wages and hours are part of a firm's consistent hiring strategy, unless this instrument is a well-chosen firm-specific variable.

[14] In firms which offer the greatest utilities, full-time hourly wages are lower than part-time hourly wages. As underlined previously, the firm's equilibrium utility distribution is increasing. These two facts induce a large number of the part-time workers to obtain a wage that is greater than the maximum full-time wage due to the bigger size and the high number of firms offering a high level utility.

shape parameter χ.

Simulating the economy with different values of the supply-side parameter ϵ, that varies in the same magnitude as χ, does not exhibit such results. We can see in Figure 5 that the slopes of the part-time probability curves are very close.

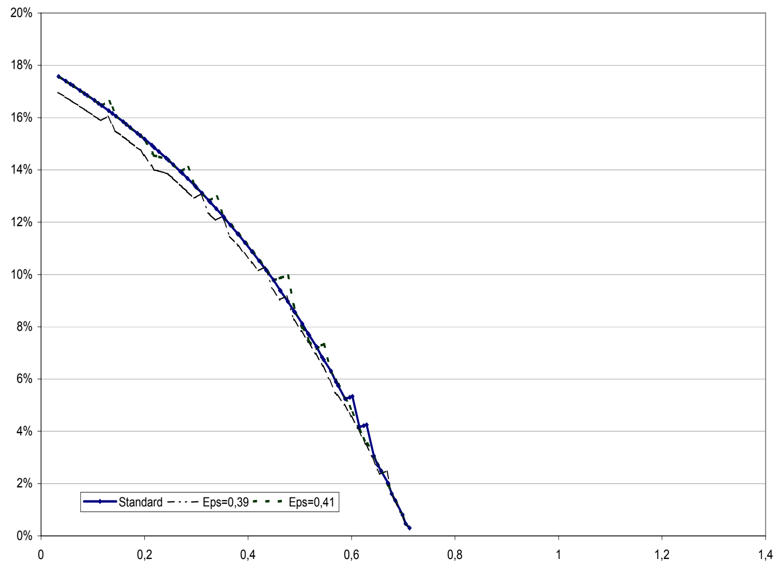
FIG. 4 – Variation of the relationship between part-time probability and wage with labor demand parameters



4 Conclusion

In this paper, we prove the existence of a Nash equilibrium, in which all firms hire both part- and full-time workers in a search framework. The main contribution of this paper is to show how the relation between hours and wages can be affected by firms' behavior. It gives a reinterpretation of the elasticity of hours to wages, which is usually implicitly assumed to be only supply driven and interpreted as reflecting individual preference for leisure. In the theoretical model presented in this paper, we

FIG. 5 – Variation of the relationship between part-time probability and wage with labor supply parameters



show that the relation between part-time probability and wages does not only reflect individual preference for leisure, but overall, the firm's demand shape parameter.

Presenting a model in which all agents are ex-ante identical makes it possible to underline the phenomena caused by agents' behavior. To go further, two directions are possible.

The first direction would be to rely on the growing on-the-job search equilibrium literature to account for firms and individual heterogeneity (see Bontemps Robin Van den Berg [5]) or participation (see Bowlus [6]). With respect to this literature, our contribution is that we allow firms to have two different types of jobs on the same market.

The second direction is to develop a representation of the firm's production function that explicitly accounts for a demand in part-time work. This representation is of interest not only in that it improves the representation of the firm's organization but also because it can improve the analysis of the consequences of economic policies that affect the number of hours worked, like the reduction of the workweek or subsidies to part-time work.

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APPENDIX

Proof of proposition 1.3 Let u^* be a mass point of the utility offer distribution.

The existence of a utility offer distribution mass point implies the existence of a mass point in the firm's strategy distribution, either on the offered utility to part-time workers or to full-time workers. Let's assume the mass point corresponds to part-time workers.

We get $F(u^*) = F(u^{*-}) + \eta$, $\eta > 0$. We can obtain :

$$E_P dF_P(u^*) = N \int_{\underline{u}_F}^{\overline{u}_F} e_P^*(u, u_F)\, dH(u, u_F)$$

H has a mass point in u^* since $e_P(u^*) > 0$ (otherwise no firm offering the utility u^* would have any part-time employees). A mass of firms exist that offer the same utility level to their part-time workers.

Let

$$\begin{aligned}\widehat{\pi}(u, u_L, l_P, l_F) &= \sum_{(e_P, e_F) \geq 0} Q(e_P l, e_F l) - [u - \Gamma(H - h_P)]\, e_P l_P + \\ &\quad [u_F - \Gamma(H - h_F)]\, e_F l_F - c(e_P, e_F)\end{aligned}$$

the firm's profit is $\Pi^*(u, u_F) = \widehat{\pi}\left[u, u_F, \widetilde{l}(u_P), \widetilde{l}(u_F)\right]$.

Using the first order conditions and the envelope theorem, the value of the partial derivative of $\widehat{\pi}(u, u_F, l_P, l_F)$ with respect to l_P is :

$$\begin{aligned}\frac{\partial \widehat{\pi}(u, u_F, l_P, l_F)}{\partial l_P} &= e_P \left(\frac{\partial Q}{\partial L_P} - u \right) \\ &= \frac{e_P}{l_P} \frac{\partial c}{\partial e_P}\end{aligned}$$

If $u > u^*$, there exists $(u^{**}, l^{**}) \in [u^*, u] \times \left[\widetilde{l}(u^*), \widetilde{l}(u)\right]$ such that :

$$\begin{aligned}\Pi(u, u_F) - \Pi(u^*, u_F) &= (u - u^*) \frac{\partial \widehat{\pi}}{\partial u_P} (u^{**}, l^{**}, u_F, l_F) \\ &\quad + \left[\widetilde{l}(u) - \widetilde{l}(u^*)\right] \frac{\partial \widehat{\pi}}{\partial l_P} (u^{**}, l^{**}) \\ &= (u - u^*) \frac{\partial \widehat{\pi}}{\partial u} (u^{**}, l^{**}) \\ &\quad + \left[\widetilde{l}(u) - \widetilde{l}(u^*)\right] \frac{e_P}{l_P^{**}} \frac{\partial c}{\partial e_P}\end{aligned}$$

$\widetilde{l}(u^*) = \frac{C}{\left(\delta + E_T \lambda_E \overline{F}(u^*)\right)\left(\delta + E_T \lambda_E \overline{F}(u^{*-})\right)}$ Thus $\widetilde{l}(.)$ is non continuous in u^*.

There exists A such that $\forall u > u^*$, $\widetilde{l}(u) - \widetilde{l}(u^*) \geq A > 0$. Since $e_P^*(u, u_F)$ is continuous and $\frac{e_P(u^*, u_F)}{\widetilde{l}(u^*)} \frac{\partial c}{\partial e_P} > 0$[15], it is possible to get a utility level greater than u^* such that the associated profit is strictly greater. That means that a firm offering a utility to part-time workers equal to u^* increases its profits by offering a slightly greater utility. Thus, u^* cannot correspond to an equilibrium situation since it is not a profit-maximizing wage. u^* cannot be a utility offer distribution mass point. The argument is exactly the same if full-time workers are considered instead of part-time workers.

[15]because it is assumed that the firm employs part-time workers.

Proof of proposition 1.4 The objective of the firm is to maximise its profit i-e

$$\begin{aligned}\Pi\left(L_P, L_F\right) &= \sup_{u_P, u_F \geq 0} Q\left[L_P, L_F\right] - \left[u_P - \Gamma\left(H - h_P\right)\right] L_P \\ &\quad + \left[u_F - \Gamma\left(H - h_F\right)\right] L_F - c\left(\frac{L_P}{\widetilde{l}\left(u_P\right)}, \frac{L_F}{\widetilde{l}\left(u_F\right)}\right)\end{aligned}$$

If L_P and L_F are given, it is equivalent for the firm to minimize the cost function.

$$\widetilde{C}(u_P, u_F) = u_P L_P + u_F L_F + c\left(\frac{L_P}{\widetilde{l}\left(u_P\right)}, \frac{L_F}{\widetilde{l}\left(u_F\right)}\right)$$

Thus, u_P and u_F are the solutions to the first order equations

$$\begin{aligned}L_P - \frac{\widetilde{l}'}{\widetilde{l}^2}\left(u_P\right) L_P \frac{\partial c}{\partial e_P}\left(\frac{L_P}{\widetilde{l}\left(u_P\right)}, \frac{L_F}{\widetilde{l}\left(u_F\right)}\right) &= 0 \\ L_F - \frac{\widetilde{l}'}{\widetilde{l}^2}\left(u_F\right) L_F \frac{\partial c}{\partial e_F}\left(\frac{L_P}{\widetilde{l}\left(u_P\right)}, \frac{L_F}{\widetilde{l}\left(u_F\right)}\right) &= 0\end{aligned}$$

If the condition 3 is verified, we get the following equality

$$\frac{\widetilde{l}'\left(u_P\right)}{\widetilde{l}^2\left(u_P\right)} = \frac{\widetilde{l}'\left(u_F\right)}{\widetilde{l}^2\left(u_F\right)}. \tag{5}$$

The quasi-convexity of the cost function in both u_P and u_F induces the following necessary conditions to hold.

$$\frac{2\widetilde{l}'^2 - \widetilde{l}''\widetilde{l}}{\widetilde{l}^2} \frac{\partial c}{\partial e_P}\left(\frac{L_P}{\widetilde{l}\left(u_P\right)}, \frac{L_F}{\widetilde{l}\left(u_F\right)}\right) + \frac{\widetilde{l}'^2}{\widetilde{l}^4}\left(u_P\right) L_P^2 \frac{\partial^2 c}{\partial e_P^2}\left(\frac{L_P}{\widetilde{l}\left(u_P\right)}, \frac{L_F}{\widetilde{l}\left(u_F\right)}\right) > 0$$

which is verified if $2\widetilde{l}'^2 - \widetilde{l}''\widetilde{l} > 0$. Using the same condition, the equality 5 implies $u_P = u_F$, since the function $\frac{\widetilde{l}'}{\widetilde{l}^2}$ decreases.

Simulation method Knowing all these parameters, we can calculate the number of part-time and full-time workers in a firm, the global hiring effort E_T, the number of firms, the cdf of the offered and observed utilities and the minimum utility. The method proposed is the following :

1. The first step is to compute the value of $\widetilde{l}$ in the minimum utility $\underline{u}$.
2. From the flow equations, we can establish that $\widetilde{l}\,(\underline{u})$ is a function of the exogenous friction parameters and of the total hiring effort E_T. $\widetilde{l}\,(\underline{u})$ also given in the first step of the algorithm, we obtain the value E_T.
3. $\widetilde{l}\,(\overline{u})$ is also a function of the friction parameters and of the total hiring effort. The value of $\overline{u}$ is given by the inversion of the function $\widetilde{l}$.
4. The last step of the iteration consists of finding the value of e_P, e_F and $\widetilde{l}$ for a number N_{sim} of points between $\underline{u}$ and $\overline{u}$. We can then deduce the parameters' values to those that can be used in our model as the hiring effort in part- and full-time jobs E_p and E_F, or the proportion of part-time or full-time workers.