INSTITUT NATIONAL DE LA STATISTIQUE ET DES ETUDES ECONOMIQUES
Série des Documents de Travail du CREST
(Centre de Recherche en Economie et Statistique)

n° 2004-39

Incomplete Regulation, Market Participation and Collusion*

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* We are grateful to Jean-Jacques Laffont for helpful discussions during this work and to Antoine Faure-Grimaud for particularly detailed comments. We also thank Marcel Boyer, Bernard Caillaud, David Ettinger, David Martimort, Patrick Rey and seminar participants in Antwerp, Milano and Paris I, as well as U.C. Berkeley, Toulouse, the World Congress of the Econometric Society in Seattle and the ENTER jamboree at UCL for an older version.

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Incomplete Regulation, Market Competition, and Collusion *

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This version: November 2004

Abstract

Regulators often do not regulate all firms competing in a given sector. Due to product substitutability, unregulated competitors have incentives to bribe regulated firms to have them overstate their costs and produce less. The best collusion-proof contract entails distortions both for inefficient and efficient regulated firms (distortion 'at the top'). But a contract inducing active collusion may do better by allowing the regulator to 'team up' with the regulated firm to indirectly tax its competitor. The best such contract is characterized. It is such that the unregulated firm pays the regulated one to have it truthfully reveals its inefficiency.

JEL Classification: L41, L51.

Keywords: Incomplete Regulation, Collusion, Market Competition, Incentives.

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Abstract

Regulators often cannot regulate all firms competing in a given sector. Due to market competition, unregulated competitors have incentives to bribe regulated firms to have them overstate their costs and produce less. Collusion-proof contracts entail distortions for inefficient but also efficient regulated firm (distortion 'at the top'). Depending on parameter values, a contract inducing active collusion may do better by allowing the regulator to 'team up' with the regulated firm to indirectly tax its competitor. The optimal such contract is characterized. It is such that the unregulated firm pays the regulated one to have it truthfully reveals its efficiency.
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Résumé

Typiquement, les régulateurs sectoriels ne régulent pas l'ensemble des firmes présentes dans le secteur concerné. Du fait de la concurrence sur le marché, les firmes non régulées ont une incitation à corrompre les firmes régulées pour que ces dernières sur-estiment leurs coûts et produisent moins. Les contrats robustes à la collusion génèrent des distorsions à la fois pour les firmes régulées inefficaces mais aussi pour celles efficaces. Un contrat qui laisse la collusion s'instaurer à l'équilibre permet au régulateur de 'taxer' indirectement les firmes non régulées et peut parfois être socialement préférable à un contrat robuste à la collusion. Nous caractérisons ces mécanismes non robustes à la collusion.
JEL: L51
Mots-clés: Régulation incomplète, collusion, concurrence, incitations

1 Introduction

Regulated industries have long been characterized by monopolistic structures. But since the wave of deregulation initiated by the UK and the US in the mid-eighties, and extended to most of the world in the 1990s, entry has been allowed and competition has become a major concern for regulation authorities. Regulating competing firms is complex, and the more so if some market players are not subject to regulation. This paper focuses on the impact of potential collusion in a partially regulated environment, in which the regulator is restricted in its power to control the industry. We will show that fighting collusion is more difficult, and more strikingly, that tolerating some collusion may be beneficial for the regulator in some settings: The 'collusion-proofness principle' does not hold with incomplete regulation and the regulator can take advantage of collusion to some extent.

Asymmetric regulation is a major characteristic of current regulatory structures in many former monopolistic industries, such as telecommunications, electricity or postal services. Industrial regulation can sometimes be too narrow to allow full regulation of the sector.[1] New technologies have also strengthened the asymmetry in regulation: Fixed telephony, for instance, is heavily regulated, particularly the incumbent, whereas regulation of mobile telephony is lighter, and focuses on interconnection charges; Internet telephony is virtually unregulated. Similarly, the main firm providing postal services is more regulated than firms providing imperfect substitutes such as parcel delivery, not to talk about e-mail providers. Last, the bargaining power of the regulator may be too weak to allow for effective regulation of trans-national corporations entering the market, especially in developing economies.

This incompleteness of regulation entails costs, especially when firms can collude. Despite its current practical relevance, the formal literature on this issue is relatively scarce. Caillaud (1990)

[1] One can indeed recall that until 1999, the regulator for electricity in the UK, Offer, could not regulate gas — controlled by Ofgas — although gas constitutes an imperfect substitute. The need to coordinate regulation in those two close industries, and to regulate the whole sector in a coherent way was strongly felt and lead to the merger of the two offices, to create a new regulatory body, Ofgem.

focuses on the informational effects of the existence of a competitive fringe for the regulation of a dominant firm under asymmetric information. Biglaiser and Ma (1995) also analyze the regulation of a dominant firm, faced to competition by an unregulated competitor, both when demands are known and when they are unknown. But again, collusion is not considered. In the Mechanism Design literature, Laffont and Martimort (1997, 2000) offer a new methodology for analyzing collusion under asymmetric information, but with complete regulation. Aubert and Pouyet (2002) use this methodology in a context of incomplete regulation in which the costs of the two firms are correlated. The production of the unregulated competitor conveys information on its costs, and therefore on the one of the regulated firm, hence a collusion stake that is independent from market competition.[2]

This paper analyzes the effects of potential collusion between a regulated firm and an unregulated competitor producing imperfect substitutes. More precisely, collusion is attractive to firms due to product substitutability: the unregulated firm would like to induce a lower production by the regulated one, so as to benefit from an increased residual demand — the standard motive for coordination in cartels. Collusion directly affects the cost of obtaining truthful revelation by the regulated firm, since the latter may be bribed to pretend being inefficient. We characterize the contracts offered by the regulator when faced to collusion. The best collusion-proof contract entails distortions both for an efficient and an inefficient firm: The usual 'no distortion at the top' result does not hold. An interesting result is that, with incomplete regulation, the regulator cannot take advantage of asymmetric information within the coalition: The relevant collusion-proof constraint is the same as if the unregulated firm had been perfectly informed on the type of the regulated one. But in our context, the Collusion-Proofness Principle may not hold: The regulator is unable

[2]In the absence of collusion, the information rent of the regulated firm disappears when the regulator uses a yardstick-like mechanism based on the output of the unregulated competitor. If firms are able to collude, on the other hand, information rents may subside and the regulator may not be able to fully use the information contained in unregulated outputs.

to contract with the unregulated firm, and has therefore limited contracting capacities compared to what can be achieved within the coalition of the two firms. Since the regulator is not able to replicate, with a collusion-proof contract, all the allocations that are feasible within the coalition, it would be restrictive to only consider collusion-proof mechanisms. We characterize the best contract that induces active collusion. In order to do so, we show that an extended version of the Revelation Principle applies with respect to the coalition of the two firms (and not to the regulated firm taken separately), if one takes into account the sub-contract it offers to its members, and the incentive and participation constraints it faces when dealing with them. We define a notion of feasibility given the constraints within the coalition, 'C-incentive feasibility'. The best contract with active collusion induces truthful revelation by the coalition, but would not induce a truthful report by the regulated firm in the absence of collusion.

The best contract with active collusion is as follows: If there was no collusion, the regulated firm would always pretend being efficient. The unregulated firm then pays it a bribe to obtain the revelation of its inefficiency. And this bribe is taxed away by the regulator, through the transfer it imposes on the regulated firm. To summarize, in a collusion-proof contract, the regulator has to pay larger amounts to the regulated firm to ensure that it reveals its *efficiency* despite collusion; by contrast, the best non collusion-proof contract uses collusion by making the unregulated firm pay to ensure that the regulated firm reveals its *inefficiency*. The best regulatory contract inducing active collusion dominates the best collusion-proof contract for sizable sets of parameter values.

The paper is organized as follows: Section 2 describes the model, and Section 3 derives some benchmarks. The contracts with collusion-proof regulation are characterized in Section 4. Section 5 then analyzes the optimal contract inducing active collusion. Section 6 concludes. Proofs are gathered in an appendix, that details in particular how the Revelation Principle can be extended for active collusion.

2 The model

The consumers Consumers can buy two differentiated goods, denoted by a and b, produced respectively by firm F^a and firm F^b. Gross consumers surplus when a quantity q^a of good a and a quantity q^b of good b are sold on the market is given by

$$GS(q^a, q^b) = d^a q^a + d^b q^b - \frac{1}{2}(q^a)^2 - \frac{1}{2}(q^b)^2 - s q^a q^b.$$

Parameter $s \in (0, 1)$ measures the degree of substitutability between the two goods whereas d^i is the size of market for good i. The inverse demand functions for goods a and b are therefore given respectively by

$$\begin{cases} P^a(q^a, q^b) = d^a - q^a - s q^b, \\ P^b(q^a, q^b) = d^b - q^b - s q^a. \end{cases}$$

The firms Firms compete in quantities[3]. Firm F^a has a constant marginal cost θ^a that can take two values, $\underline{\theta}$ or $\overline{\theta}\}$, with $\overline{\theta} - \underline{\theta} \equiv \Delta\theta > 0$. The value of this marginal cost is the private information of F^a. It is common knowledge that firm F^a is efficient ($\theta^a = \underline{\theta}$) with probability $\underline{p}$, and is inefficient ($\theta^a = \overline{\theta}$) with probability $\overline{p} = 1 - \underline{p}$.

The marginal cost of firm F^b, θ^b is also constant and is known by all players in the economy.[4] To simplify notations, we denote by m^b the profit margin of the unregulated firm: $d^b - \theta^b \equiv m^b$.

The regulator Firm F^a —the 'incumbent'— is regulated by regulator R^a, while firm F^b is left unregulated. The regulatory contract is composed of a couple quantity-transfer $\{q^a, t^a\}$, that depends on the message sent by the regulated firm to its regulator, as well as on any other available

[3] One may argue that regulators generally set prices rather than quantities. Considering price competition would give us similar qualitative results, especially regarding the collusion stake. This will be clarified in subsection 4.2.

[4] If the two marginal costs were correlated, an information effect would arise, since the production of the unregulated firm would constitute a correlated signal on the cost of the regulated one. This case is studied in Aubert and Pouyet (2002).

relevant information. For a given contract $\{q^a, t^a\}$, total ex post profits for F^a and F^b respectively are equal to

$$\begin{cases} \pi^a = [P^a(q^a, q^b) - \theta^a]q^a - t^a, \\ \pi^b = [P^b(q^a, q^b) - \theta^b]q^b. \end{cases}$$

Any type of relationship between the regulator and the entrant F^b is ruled out in this paper. The analysis could be extended to a fixed lump-sum tax (shut-down of F^b would occur more often) or to taxes proportional either to profits or to output (this latter type of tax could be assimilated to a decrease in d^b). On the other hand, non linear taxation on firm F^b would constitute some indirect regulation, which is ruled out in this particular setting.[5]

For simplicity, we assume that the firms' profits do not enter the social welfare function of the regulator[6], so that this objective is reduced to net consumers surplus plus the transfer paid by the regulated firm. Hence, the rents left to the regulated firm are socially costly for the regulator. Social welfare can be written as

$$SW^a = GS(q^a, q^b) - \theta^a q^a - P^b(q^a, q^b)q^b - \pi^a.$$

The timing Finally, the chronology of events is the following (collusion will be added in subsection 4.2).

1. Nature draws the private information parameter θ^a. Firm F^a privately learns it.

2. Regulator R^a proposes a contract $\{q^a(.), t^a(.)\}$ to the regulated firm F^a.

3. The regulated firm F^a decides whether to accept or reject this contract. In case of refusal, it gets a reservation gain, exogenously normalized to 0, and the unregulated firm produces the

[5] We thank an anonymous referee for pointing this out.

[6] The results can be extended to positive weights on the sum of firms' profits provided that they be strictly smaller than 1 (this is needed for rent extraction to be desirable for the regulator).

monopolistic quantity. If F^a accepts the contract, the game continues as follows.

4. F^a then sends a message to the regulator and produces the corresponding quantity.

5. The unregulated firm chooses its production after having observed the regulatory contract offered to F^a and the quantity it has produced. A transfer is then paid by the regulated firm to the regulator as specified in the contract.

Here, regulator R^a can condition the tax paid by the regulated firm on the quantity produced by the unregulated firm, q^b: R^a can wait until q^b is produced to tax the regulated firm. But it cannot condition the quantity produced, q^a, on q^b; Firm F^a has to produce its quantity before the unregulated firm undertakes its production decision.[7]

According to this timing, the unregulated competitor chooses its quantity after the incumbent F^a. The assumption of Stackelberg leadership of firm F^a simplifies our setting by ensuring that the unregulated firm knows the exact value of q^a when it chooses its output (the best response of F^b would otherwise be computed in expectation). It also implies that the regulator can influence q^b *via* q^a, but this Stackelberg effect is mostly orthogonal to the issues we want to study and will play very little role in the absence of collusion. When collusion is concerned, what matters is the timing of the collusion game itself. The timing we use in the following (in which collusion takes place after acceptance by the regulated firm of the regulatory contract, but before a report is made on θ^a) would remain one of the possible timings, though not the only one. The modeling of the collusion game necessarily entails some arbitrariness.

[7] If the regulator had been able to also condition q^a on q^b, he could easily have implemented the socially optimal quantity, under complete information. Consider indeed the following quantity scheme: Produce the socially optimal quantity if firm F^b adopts a perfectly competitive behavior on its market; 'Flood' the market ($q^a \to +\infty$) if firm F^b tries to exploit its market power. If regulator R^a could commit to such a scheme, firm F^b would always prefer to behave competitively and, in equilibrium, the socially optimal allocation would be implemented. Such an equilibrium is not subgame perfect though.

3 Complete information

Let us assume in this section that the firms' efficiency parameters θ^a and θ^b are known to all economic agents.

The unregulated firm's output decision The best response of the unregulated firm F^b to the observation of the quantity $q^a(\theta^a)$ selected by F^a is given by:

$$q^b(q^a(\theta^a)) = \frac{1}{2}[m^b - sq^a(\theta^a)]. \tag{1}$$

Due to product substitutability, the unregulated quantity decreases in the regulated one. As we will see, this best-response function will not change with asymmetric information nor collusion.

The optimal quantity with complete information and complete regulation The socially optimal quantities, q_o^a and q_o^b, are the ones the regulator would ideally choose, but they are not achievable unless there is complete information *and* complete regulation, i.e., regulation of both firms. They are such that price equals marginal cost in each market:

$$\begin{aligned} q_o^a(\theta^a) &= \frac{1}{1-s^2}[(d^a - \theta^a) - sm^b], \\ q_o^b(\theta^a) &= \frac{1}{1-s^2}[m^b - s(d^a - \theta^a)]. \end{aligned}$$

The optimal quantity with complete information and incomplete regulation Let us now come back to our setting, in which the regulator cannot regulate firm F^b. The rent of F^a being socially costly, regulator R^a leaves no profit in equilibrium to the regulated firm F^a. Maximization of his objective function yields the following schedule of output:

$$q^{a*}(\theta^a) = \frac{1}{4-s^2}[4(d^a - \theta^a) - sm^b].$$

This quantity equals the socially optimal one for a regulated monopoly ($q^a(\theta^a) = d^a - \theta^a$) when products are independent ($s = 0$). One can note that the first-order condition of the program of the regulator can be rewritten as $P^a(q^a, q^b) - \theta^a = -\frac{s}{2}q^b(q^a)$. With independent goods, there is no need to distort pricing from marginal cost. But with substitutes, the two firms actually compete; and since the regulator does not benefit from the profits of the unregulated competitor, it requires the regulated firm to price below marginal cost, so as to ensure that it captures a large market share. A larger quantity q^a than under complete regulation is thus required: $q^{a*}(\theta^a, \theta^b) \geq q^a_o(\theta^a, \theta^b)$.

The quantity produced by the unregulated firm[8] F^b can then be obtained using (1):

$$q^b(q^{a*}(\theta^a)) = \frac{2}{4-s^2}[m^b - s(d^a - \theta^a)].$$

4 Market competition and collusion-proof contracts

We assume from now on that the regulator R^a is uninformed on the marginal cost of the firm he regulates.

4.1 No collusion

The unregulated firm F^b, given the timing, knows q^a when it chooses q^b. Its production decision is therefore still given by (1).

Let us now focus on the regulator's problem. From the Revelation Principle[9] we know that R^a

[8] We assume that firm F^b prefers indeed to produce rather than exit the market ($m^b - s(d^a - \theta^a) \geq 0$).

[9] See Green and Laffont (1977) or Myerson (1979) among others. The existence of the unregulated firm does not affect the validity of the Principle, since its actions can be perfectly anticipated, and only depend on q^a. It would

can restrict attention to direct and truthful contracts. The rent of firm F^a with type θ^a when it announces $\tilde{\theta}^a$ to its regulator is $\pi^a(\theta^a, \tilde{\theta}^a) = [P^a(q^a(\tilde{\theta}^a), q^b(q^a(\tilde{\theta}^a))) - \theta^a]q^a(\tilde{\theta}^a) - t^a(\tilde{\theta}^a)$ (we omit parameter θ^b to ease notations).

We denote by $\pi^a(\underline{\theta})$ and $\pi^a(\overline{\theta})$ the rents of an efficient and of an inefficient firm respectively at a truthful equilibrium. Regulator R^a maximizes expected social welfare under incentive compatibility and participation constraints. Incentive compatibility constraints can be written as follows

$$\begin{cases} \pi^a(\underline{\theta}) \geq \pi^a(\overline{\theta}) + \Delta\theta q^a(\overline{\theta}) \quad IC^a(\underline{\theta}), \\ \pi^a(\overline{\theta}) \geq \pi^a(\underline{\theta}) - \Delta\theta q^a(\underline{\theta}) \quad IC^a(\overline{\theta}). \end{cases}$$

Individual rationality constraints, which ensure that in a truthful equilibrium firm F^a is willing to participate to the contract, are defined by

$$\begin{cases} \pi^a(\underline{\theta}) \geq 0 \quad IR^a(\underline{\theta}), \\ \pi^a(\overline{\theta}) \geq 0 \quad IR^a(\overline{\theta}). \end{cases}$$

As usual, the binding constraints are the incentive compatibility constraint of an efficient firm, $IC^a(\underline{\theta})$, and the participation constraint of an inefficient one, $IR^a(\overline{\theta})$. The remaining constraints are easily checked ex post.

Lemma 1 *When firms cannot enter collusive agreements, the optimal quantities for the regulated firm are given by*

$$\begin{aligned} q^a(\underline{\theta}) &= \frac{1}{4-s^2}[4(d^a - \underline{\theta}) - sm^b] \ = q^{a*}(\underline{\theta}), \\ q^a(\overline{\theta}) &= \frac{1}{4-s^2}[4(d^a - \overline{\theta} - \frac{\underline{\underline{p}}}{\overline{p}}\Delta\theta) - sm^b] \ = q^{a*}(\overline{\theta}) - \frac{4}{4-s^2}\frac{\underline{\underline{p}}}{\overline{p}}\Delta\theta. \end{aligned}$$

therefore be useless to limit information revelation on θ^a since that would have no impact on the behavior of F^b.

Moreover the regulated firm obtains a strictly positive rent when efficient.

The standard result of 'no distortion at the top' holds: An efficient firm F^a produces the complete information quantity. But the regulator has to leave an information rent, $\Delta\theta q^a(\overline{\theta})$, to this firm in order to induce revelation of its efficiency. Since the rent increases with the quantity produced by the inefficient firm F^a, this quantity is distorted downward (with respect to the complete information situation). The distortion is larger if it becomes more likely that the firm be efficient (in which case the information rent is paid more often, and the quantity $q^a(\overline{\theta})$ is produced less often).

Using (1), one can determine the unregulated firm's quantity. To focus on the more interesting cases, we will assume throughout the paper that parameters are such that production by both firms is desirable, and that the unregulated firm indeed prefers to stay on the market.[10]

4.2 Bribery from the unregulated competitor

The stake of collusion Due to the substitutability of products, the unregulated firm always prefers a lower production by the regulated firm. The lower q^a is, the larger q^b will be, and unregulated profits can be rewritten as $\pi^b(q^a) = [q^b(q^a)]^2$. Stakes of collusion therefore exist in this context: The unregulated firm would like the regulated one to pretend being inefficient[11], whatever its true cost, since $q^a(\overline{\theta}) < q^a(\underline{\theta})$.

Let us denote by $\Delta\pi^b$ the (positive) variation in the unregulated firm's profit when the regulated

[10] If the regulated quantity was such that firm F^b made negative profits by producing, it would exit the market. The regulated quantities when the regulator anticipates exit are $q_e^a(\underline{\theta}) = d^a - \underline{\theta}$ and $q_e^a(\overline{\theta}) = d^a - \overline{\theta} - \frac{\underline{p}}{\overline{p}}\Delta\theta$. If on the other hand the regulated firm, F^a, was not to produce, the industry would be an unregulated monopoly and would yield a welfare equal to $\frac{1}{8}(m^b)^2$ to the regulator.

[11] Clearly, this result does not depend on the assumption that competition takes place in quantities. Consider instead competition in prices. The best response of the unregulated firm would be given by $p^b(p^a) = \frac{1}{2}(m^b - s(d^a - p^a))$. The corresponding profits would be $\pi^b(p^a) = (p^b(p^a) - \theta^b)^2$ and would increase in p^a. And from incentive compatibility, the output of the regulated firm must be inversely related to marginal cost, so that the regulated price p^a increases in θ^a. The unregulated firm therefore still benefits from having the regulated firm always pretend being inefficient, as with competition in quantities.

firm produces $q^a(\overline{\theta})$ rather than $q^a(\underline{\theta})$.[12]

Modeling collusion We assume that collusion can take place between the two firms after the offer of the regulatory contract and before the report of the regulated firm, between steps 3 and 4 of the previous timing:

1. Nature draws the private information parameters θ^i $(i = a, b)$. θ^b is observed by all agents in the economy.

2. Firm F^a privately learns its cost θ^a.

3. Regulator R^a proposes a contract $\{q^a, t^a\}$ to the regulated firm F^a. If the regulated firm refuses to participate, it gets a reservation gain, normalized to 0. If it accepts, the game goes on as follows.

3′. A coalition may form between the two firms. The collusive game is played as explained below.

4. The regulated firm makes a report on its type to the regulator, which corresponds to a quantity q^a. It potentially receives simultaneously a bribe from F^b.

5. The unregulated firm chooses its production after having observed the quantity produced by the regulated firm. The regulated firm then receives the transfer specified in the regulatory contract.

Since θ^a is private information of firm F^a, bribery takes place under asymmetric information. In order to model the bargaining process within the coalition in such a case, we assume that collusion is organized by a benevolent third party T, that chooses the report by F^a and transfers between the two firms in order to maximize the sum of firms' profits. This parallels the methodology proposed by Laffont and Martimort (1997, 2000).

[12] We have $\Delta\pi^b \equiv \pi^b(q^a(\overline{\theta})) - \pi^b(q^a(\underline{\theta})) = \frac{s}{4}[q^a(\underline{\theta}) - q^a(\overline{\theta})][2m^b - s(q^a(\underline{\theta}) + q^a(\overline{\theta}))]$, which is positive if $q^a(\overline{\theta}) \leq q^a(\underline{\theta})$, a condition imposed by incentive compatibility.

Potential non-validity of the Collusion-Proofness Principle If regulation was complete, and the regulator was able to regulate both firms, then the Collusion-Proofness Principle would apply: There would be no restriction in considering only contracts such that the coalition of the two firms acts exactly as if there was no collusion (collusion is not 'active'). This result relies on the ability of the principal (here the regulator) to perfectly replicate any allocation obtained with collusion, in a contract that does not induce active collusion.

But this may not be the case when regulation is incomplete: Since the regulator is unable to contract with firm F^b, some allocations are not feasible to this regulator, although they can be achieved by the coalition of the two firms. For instance, any allocation in which some money is taken away from F^b and given back to F^a is not available to the regulator — who cannot tax F^b — but is feasible, via a bribe, within the coalition of the two firms[13]. Considering solely collusion-proof contracts constitutes a priori a restriction. This non validity of the Collusion-Proofness Principle is a major departure from the standard framework of full regulation.

The following subsections consider collusion-proof and non collusion-proof contracts to determine which are preferred by the regulator.

4.3 Collusion-proof contracts

A collusion-proof contract is a contract that induces a passive response from the coalition. More precisely, the third party, when faced to such a contract, will ask the regulated firm to truthfully report its type to the regulator, and no bribe will be paid. In our setting, ensuring truthful revelation on θ^a is sufficient to obtain collusion-proofness.

Let us denote by $\tilde{\theta}(\theta^a)$ the report recommended by the third party when F^a has cost θ^a, and

[13]The regulator can compensate F^a for the bribe it receives from F^b, and therefore induce the same behavior from F^a as with collusion. But it is not possible to tax F^b correspondingly. The extra payment to F^a is borne by R^a under collusion-proof contracts, and by F^b under collusion. The outcomes obtained with collusion-proof contracts are thus a strict subset of all possible allocations.

by $\theta^{nc}(\theta^a)$ the best non cooperative response of this firm (if the coalition does not form). Then if F^a colludes with F^b, it will produce $q^a(\tilde{\theta}(\theta^a))$ (where $q^a(.)$ is specified in the regulatory contract). To simplify mathematical expressions, let us use $\tilde{q}^a(\theta^a) \equiv q^a(\tilde{\theta}(\theta^a))$. We denote by $CIC^k(\theta)$ the incentive compatibility constraint within the coalition for firm F^k, $k = a, b$, when firm F^a has a cost θ, and by $CIR^k(\theta)$ the (coalition) participation constraint. The program of the third party can then be written as:

$$\begin{array}{lll}
\max\limits_{\{\tilde{\theta}(\theta^a), b(\theta^a)\}} & \mathbf{E}_{\theta^a}\left[\pi^a(\tilde{\theta}(\theta^a), \theta^a) + \pi^b(\tilde{q}^a(\theta^a))\right] & \\
\text{s.t.} & \pi^a(\tilde{q}^a(\underline{\theta}), \underline{\theta}) + b(\underline{\theta}) \geq \pi^a(q^a(\theta^{nc}(\underline{\theta})), \underline{\theta}) & CIR^a(\underline{\theta}) \\
& \pi^a(\tilde{q}^a(\overline{\theta}), \overline{\theta}) + b(\overline{\theta}) \geq \pi^a(q^a(\theta^{nc}(\overline{\theta})), \overline{\theta}) & CIR^a(\overline{\theta}) \\
& \pi^a(\tilde{q}^a(\underline{\theta}), \underline{\theta}) + b(\underline{\theta}) \geq \pi^a(\tilde{q}^a(\overline{\theta}), \underline{\theta}) + b(\overline{\theta}) & CIC^a(\underline{\theta}) \\
& \pi^a(\tilde{q}^a(\overline{\theta}), \overline{\theta}) + b(\overline{\theta}) \geq \pi^a(\tilde{q}^a(\underline{\theta}), \overline{\theta}) + b(\underline{\theta}) & CIC^a(\overline{\theta}) \\
& \pi^b(\tilde{q}^a(\underline{\theta})) - b(\underline{\theta}) \geq \pi^b(q^a(\theta^{nc}(\underline{\theta}))) & CIR^b(\underline{\theta}) \\
& \pi^b(\tilde{q}^a(\overline{\theta})) - b(\overline{\theta}) \geq \pi^b(q^a(\theta^{nc}(\overline{\theta}))) & CIR^b(\overline{\theta}).
\end{array}$$

4.3.1 The collusion-proof constraints

Using the methodology of Laffont and Martimort (2000), we obtain the objective maximized by the third party, for a given regulatory contract, as a function of the multipliers associated with incentive and participation constraints within the coalition (see appendix A.2.2.). The multipliers associated with the coalition incentive constraints play a crucial role. These constraints ensure that the regulated firm prefers to truthfully reveal its type to the third party. The weight associated to them therefore measures the internal inefficiencies suffered by the coalition due to its lack of information on θ^a. We show that, if the regulator ensures truthful revelation of its type by the regulated firm, then the coalition incentive constraint for an inefficient ($\overline{\theta}$) regulated firm is slack.

A consequence is that the third party does not have to distort its recommendations to an efficient ($\underline{\theta}$) regulated firm. When inducing revelation of its efficiency from F^a, the regulator faces the same problem as if there was perfect information within the coalition.[14].

Lemma 2 *With incomplete regulation, the regulator cannot take advantage of the fact that the unregulated competitor F^b does not know the type of the regulated firm. The collusion-proofness constraint when the firm is efficient is the same as under complete information.*

The proof is given in Appendix A.2.2. This result strongly differs from the case of complete regulation: Laffont and Martimort (2000) obtain that asymmetric information generates inefficiencies in the functioning of the coalition. Conversely, in our setting, the relevant collusion-proofness constraint is the same as if the unregulated firm had been perfectly informed on θ^a.

It is therefore as if the third party was maximizing the sum of firms' profits, without any distortion arising from asymmetric information. The third party will thus recommend truthfully reporting θ^a if the sum of firms profits with a truthful report is larger than the sum of these profits with a false report, hence the following 'collusion-proofness constraint' that regulator R^a must satisfy to fight collusion:

$$\pi^a(\underline{\theta}) \geq \pi^a(\overline{\theta}) + \Delta\theta q^a(\overline{\theta}) + \Delta\pi^b \quad CPC(\underline{\theta}).$$

This condition can be understood in an intuitive way: An efficient firm F^a accepts to misreport as long as $\pi^a(\underline{\theta})$ is smaller than the sum of the bribe and the rent it can obtain by mimicking an inefficient type (the information rent). Moreover, since collusion takes place after the regulatory contract has been offered, firm F^b can counter the efforts of the regulator to fight collusion, as long as the total rent promised by R^a to an efficient firm is smaller than the information rent plus

[14] Aubert and Pouyet (2002) obtain the same result when there is asymmetric information on the type of the regulated firm, θ^b.

$\Delta\pi^b$. If the total rent was smaller, F^b would be able, and willing, to increase its bribe so as to induce a false report by F^a (the bribe would still be lower than the additional profits generated by collusion). Therefore, whatever the assumptions made regarding the relative bargaining power of firms within the coalition, the regulator needs to give up the maximal amount F^b is willing to pay, $\Delta\pi^b$, to an efficient firm to avoid collusion[15].

Note that the above analysis relies on the assumption that the unregulated firm does not modify its production decision (the reaction function used to compute $\Delta\pi^b$ is the same as without collusion). The underlying reasoning is that any deviation from the output level given by (1) would signal collusion and could be 'punished' by the regulator.[16]

4.3.2 The best collusion-proof contract

Assume that the regulator wants to fight collusion. He has to satisfy the additional collusion-proofness constraint derived above, and this constraint is more stringent than the incentive compatibility constraint of an efficient firm. It will therefore be binding in equilibrium for a collusion-proof contract.

Proposition 1 *The best separating collusion-proof contract is characterized by the following rents*

[15] The appendix derives this result formally.

[16] We could have assumed that the unregulated quantity q^b is modified to maximize collusive profits. Yet since q^b is observable, regulator R^a can infer ex post whether firm F^b has manipulated its quantity or not. Since we allow for regulatory transfers contingent on the competitor's decision, the regulator can design a set of conditional transfers so as to 'punish' the regulated firm in case of obvious collusion on q^b. Note that firm F^b might then try to 'black-mail' firm F^a, threatening to modify its production, thereby inducing a penalty on F^a, unless firm F^a agreed to always announce being inefficient. But we have assumed that the unregulated firm cannot commit and plays last. A threat to adopt a behavior that does not maximize its profits is therefore not credible. It is interesting to note that even when θ^b is known, the regulator can still get information from the observation of the unregulated quantity q^b.

and quantities:

$$
\begin{aligned}
\pi^a_{cp}(\underline{\theta}) &= \Delta\theta q^a(\overline{\theta}) + \Delta\pi^b \\
\pi^a_{cp}(\overline{\theta}) &= 0 \\
q^a_{cp}(\underline{\theta}) &= \frac{1}{4-3s^2}\Big[4(d^a-\underline{\theta}) - 3sm^b\Big], \\
q^a_{cp}(\overline{\theta}) &= \frac{1}{4-s^2(1-2\frac{\underline{p}}{\overline{p}})}\Big[4\Big(d^a-\overline{\theta}-\frac{\underline{p}}{\overline{p}}\Delta\theta\Big) - s\Big(1+2\frac{\underline{p}}{\overline{p}}\Big)(m^b)\Big],
\end{aligned}
$$

Both quantities are distorted with respect to complete information. The best collusion-proof contract is separating provided that $q^a_{cp}(\underline{\theta}) \geq q^a_{cp}(\overline{\theta})$*, i.e., provided that condition* $(\mathcal{C}_{cp})$ *holds:* $2s^2(d^a-\underline{\theta}) + (4-3s^2)\Delta\theta \geq sm^b[2-\underline{p}(4-3s^2)] \qquad (\mathcal{C}_{cp})$.

It otherwise entails pooling, with the same quantity, $q^{a*}(\overline{\theta})$*, produced by both types of regulated firm.*

The computations are given in appendix A.2.2. With collusion-proof contracts, regulator R^a must leave an extra rent to an efficient firm F^a to compensate for the bribe it could receive. Since this bribe is affected by the quantity produced both by an efficient and by an inefficient firm F^a, *both* regulated quantities are downward distorted to minimize the cost of inducing truthful revelation, contrary to the standard result of 'no distortion at the top'.

The larger the profitability of the unregulated competitor (i.e., the larger m^b), and the more substitutable the two goods are (the larger s), the more likely it becomes that the best collusion-proof contract be a pooling one. Fighting collusion under incomplete regulation can thus require large distortions of the quantity profile of the regulated firm. Pooling is an extreme case in which this quantity becomes fully insensitive to the report made by the firm.

4.4 Non collusion-proof contracts

Is collusion-proofness optimal? We have seen that the Collusion Proofness Principle may not apply with incomplete regulation, so non collusion-proof contracts cannot be ruled out.

A potential benefit of contracts inducing active collusion is to allow to extract revenues from the unregulated firm: If F^a receives a bribe from F^b when accepting the regulatory contract, its utility will be higher than without collusion, for given transfers to the regulator. The expected bribe may thus allow to induce participation of F^a in the regulatory contract for higher regulated taxes.

We show in appendix A.2.3. that a version of the Revelation Principle applies, where the 'agent' is the coalition as a whole. The regulator can restrict attention to direct mechanisms that are such that the coalition can be active ('C-incentive feasibility'), and that it reports the truth. The internal incentive compatibility and participation constraints determine the total expected utility of the coalition. The regulator must take them into account, in addition to the constraints related to revelation and participation by the coalition itself to the regulatory contract. The definition of C-incentive feasibility reflects with (the reader is referred to the appendix for a more precise definition):

Definition 1 *An allocation* $\{(\underline{q}^a, \underline{t}^a), (\overline{q}^a, \overline{t}^a)\}$ *is said to be C-incentive feasible if and only if there exists a couple of bribes paid by* F^b *to* F^a, $\{b(\underline{\theta}), b(\overline{\theta})\} \in \mathbb{R}^2$, *such that the collusive constraints* $CIR^a(\underline{\theta})$, $CIR^a(\overline{\theta})$, $CIC^a(\underline{\theta})$, $CIC^a(\overline{\theta})$, $CIR^b(\underline{\theta})$ *and* $CIR^b(\overline{\theta})$ *are simultaneously satisfied.*

We obtain the following result:

Lemma 3 *Assume that the regulator offers the coalition a contract inducing active collusion. There is no loss of generality in considering only direct and truthful C-incentive feasible contracts.*

This lemma does not imply collusion-proofness, since the truthfulness requirement has to be understood with respect to the *coalition*'s incentives, not with respect to the regulated firm's incentives — the coalition, when it is active, will find it preferable to report the true type θ^a, but the regulated firm may not find it so in the absence of collusion with F^b.

A crucial assumption necessary for this 'revelation' result to hold is that the quantity produced by the unregulated firm is a known function of q^a, here the function given by (1). By assuming that firm F^b cannot commit, we consider ex-post participation constraints within the coalition. Moreover, firm F^a has already accepted the regulatory contract at the time of the coalition game, so that the coalition cannot ask this firm to exit the market (this collusive measure would ensure monopoly profits to the unregulated firm).

Rearranging the constraints, we obtain only two relevant constraints (binding in equilibrium), plus a monotonicity constraint (the quantity for an efficient regulated firm, $\underline{q}^a$, must be larger than the one for an inefficient firm, $\overline{q}^a$).

Proposition 2 *The best contract with active collusion is characterized by the following rents and quantities:*

$$\begin{aligned}
\pi^a_{ac}(\underline{\theta}) &= -[q^b(q^a_{ac}(\underline{\theta}))]^2 + \Delta\theta q^a_{ac}(\overline{\theta}) \\
\pi^a_{ac}(\overline{\theta}) &= -[q^b(q^a_{ac}(\overline{\theta}))]^2 \\
q^a_{ac}(\underline{\theta}) &= \frac{1}{4+s^2}\Big[4(d^a - \underline{\theta}) - 3sm^b\Big] \\
q^a_{ac}(\overline{\theta}) &= \frac{1}{4+s^2}\Big[4\Big(d^a - \overline{\theta} - \frac{\underline{p}}{\overline{p}}\Delta\theta\Big) - 3sm^b\Big],
\end{aligned}$$

if $m^b \geq \frac{2}{(4+s^2)s\overline{p}}(4\overline{p} + s^2(1+\overline{p}))\Delta\theta \qquad (\mathcal{C}_{ac})$.

The regulator extracts the unregulated firm's profits, $[q^b(q^a_{ac}(\theta^a))]^2$*, but gives up an information rent when firm* F^a *is efficient.*

The best quantities are smaller than if there was no possibility of collusion between the two *firms.*

This non collusion-proof contract is such that the regulated firm would always pretend being efficient if there was no collusion. This is guaranteed by condition ($\mathcal{C}_{ac}$). From this condition, we can note that it is necessary that goods be sufficiently substitutes (s cannot be too close to zero) and that the unregulated firm be sufficiently profitable (a condition also needed for firm F^b to produce in all states of nature). It is quite intuitive that if goods are nearly independent from each other, the stake of collusion disappears, since a decrease in q^a no longer generates a sizable increase in firm F^b's profits.

The fact that $q^a_{ac}(\overline{\theta})$ and $q^a_{ac}(\underline{\theta})$ are smaller than $q^a_{cp}(\overline{\theta})$ and $q^a_{cp}(\underline{\theta})$, respectively, can be understood easily: With collusion-proof contracts, the regulator has an incentive to have the regulated firm produce more, so as to appropriate a larger market share. But since the regulator extracts unregulated profits with contracts entailing active collusion, he now internalizes not only the benefit that consumers derive from consumption of good b, but also the profits that this consumption generates. A relative restriction in quantities ensues.

One can interpret the optimal contract with active collusion as follows. The regulator uses his Stackelberg position to give the regulated firm some 'bargaining power' in the collusion stage: if there is no collusion, the regulated quantity will be high, which is bad for the unregulated firm. The regulator maximizes the bribe this firm will obtain, since he ultimately extracts it.

As a final remark on this contract, note that the equilibrium concept we use matters. The above contract is feasible because firm F^a perfectly anticipates the coalition game, and knows that it will be compensated, within the coalition, for the negative profits it makes when accepting the contract. It would otherwise refuse it, since this contract does not satisfy its individual participation constraint without collusion. Firm F^b would like to commit not to participate in the coalition, so as to induce refusal of the regulatory contract by F^a. But it cannot commit, and a simple threat is not credible (since firm F^b is better off participating). The credibility requirements imposed by sub-game perfect Nash equilibria are important by eliminating situations in which firm F^a would not participate in the regulatory contract.

4.5 When are contracts inducing active collusion preferable?

We have seen that the regulated quantities are lower when active collusion is induced than when it is prevented. Yet the transfer paid to the regulated firm by the regulator is also lower, since the bribe is substracted to the information rent that firm F^a obtains. Contracts inducing active collusion are optimal from the point of view of the regulator when the difference between its welfare with the best available non collusion-proof contracts, and its welfare with the best available collusion-proof contracts is positive. This difference, that we will denote $\Delta^{(ac)-(cp)}$ is given by:

$$\begin{aligned}
&\Delta^{(ac)-(cp)} = \mathbf{E}_{\theta^a}\{SW^{ac}(q^a_{ac}(\underline{\theta}), q^a_{ac}(\overline{\theta}))\} - \mathbf{E}_{\theta^a}\{SW^{cp}(q^a_{cp}(\underline{\theta}), q^a_{cp}(\overline{\theta}))\} \\
&= \underline{p}\Big[d^a - \underline{\theta} + \frac{1}{2}(q^a_{ac}(\underline{\theta}) + q^a_{cp}(\underline{\theta}))\Big](q^a_{ac}(\underline{\theta}) - q^a_{cp}(\underline{\theta})) \\
&+\overline{p}\Big[d^a - \overline{\theta} - \frac{\underline{p}}{\overline{p}}\Delta\theta + \frac{1}{2}(q^a_{ac}(\overline{\theta}) + q^a_{cp}(\overline{\theta}))\Big](q^a_{ac}(\overline{\theta}) - q^a_{cp}(\overline{\theta})) \\
&+\frac{\underline{p}}{2}\Big[3(q^b_{ac}(\underline{\theta}))^2 - (1+\underline{p})(q^b_{cp}(\underline{\theta}))^2\Big] + \frac{\overline{p}}{2}\Big[3(q^b_{ac}(\overline{\theta}))^2 - \Big(1 - \frac{(\underline{p})^2}{\overline{p}}\Big)(q^b_{cp}(\overline{\theta}))^2\Big].
\end{aligned}$$

We cannot derive general rules as to when the difference is positive, but particular calibrations show that contracts inducing active collusion dominate collusion-proof ones, for sizable sets of parameter values (hence the following proposition).

Proposition 3 *The regulator may prefer to offer the best contract inducing active collusion rather than the best collusion-proof contract, depending on the value of the parameters.*

In a number of cases, the regulator benefits from tolerating active collusion rather than preventing it. Allowing collusion is a way to use the regulated firm as an intermediary so as to tax in an indirect way the unregulated profits. This remains quite incomplete, since a regulator contracting with both firms would be able to choose the output for both, but it may be optimal given the strong distortions needed to prevent collusion.

The following figures illustrate, for $d^a - \underline{\theta} = 2$, $d^a - \overline{\theta} = 1$ and $m^b = 2$, situations in which the regulator may prefer active collusion or collusion-proofness, depending on the value of a parameter (s or $\underline{p}$). Figure 1 shows how active collusion becomes preferred when products become less substitutable[17]. Figure 2 shows the same, but for increases in the probability that the regulated firm be efficient, when $s = 0.6$.

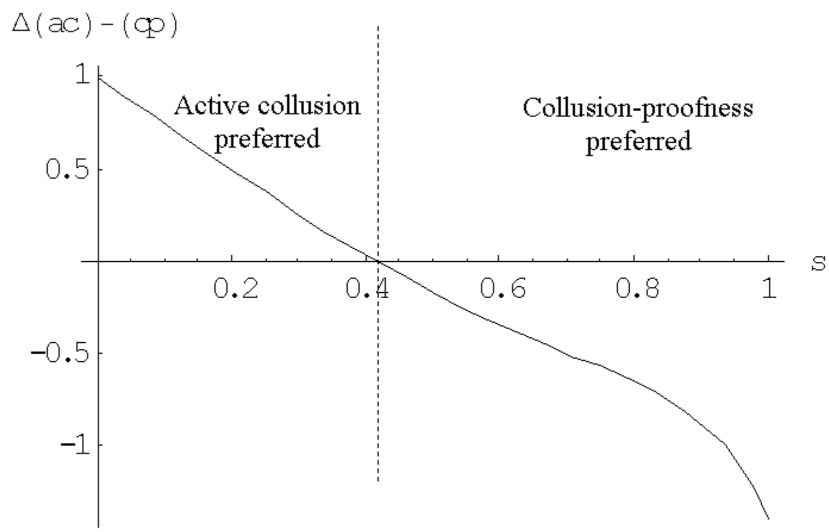

Figure 1: Optimal regulatory contracts depending on product substitutability s

Note that when products are nearly independent (s goes to zero), the regulated quantities, both with collusion-proof contracts and with contracts inducing active collusion, tend towards the socially optimal quantities with a single firm ($q^a(\theta) = d^a - \theta$). The collusion stake, $\Delta\pi^b$ also go to zero (the profits of the unregulated firm become less and less sensitive to the regulated quantity),

[17] With these parameters, active collusion is preferred whenever s is smaller than some threshold (approximately 0.41309) and collusion-proofness is preferred for larger degrees of substitutatibility.

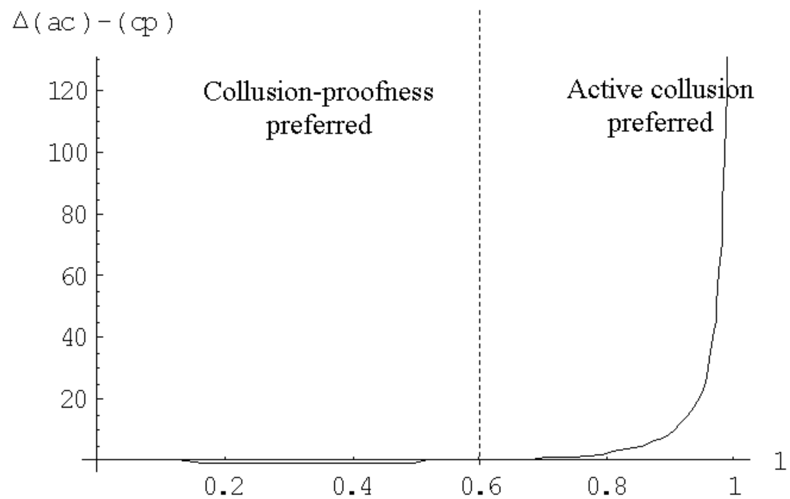

Figure 2: Optimal regulatory contracts depending on probability that F^a be efficient, $\underline{p}$

and therefore so do i) the bribe paid by F^b in case of active collusion, and ii) the cost of ensuring collusion-proofness in collusion-proof contracts. Both types of contracts therefore yield a regulatory surplus that tends towards the one for a regulated monopoly.

As a last comment, let us consider what would happen if the type of the unregulated firm was also private information. Then the regulator would have to obtain the revelation of two information parameters, although he has only one tool available. Bunching would therefore be likely to occur in a non collusion-proof contract (see e.g., Armstrong and Rochet, 1999, for a simple exposition on screening with multi-dimensional types). This would obviously lessen the gain that the regulator can obtain with non collusion-proof contracts.

5 Conclusion

We have shown that the incompleteness of regulation makes collusion-proofness very costly to obtain. More importantly, the Collusion-Proofness Principle does not apply, and non collusion-proof contracts may yield a better outcome from the point of view of the regulator. The best contract inducing active collusion can be computed using an extension of the Revelation Principle between the regulator and the coalition, with some additional constraints coming from the internal functioning of the coalition. This contract is such that the unregulated firm has to bribe the regulated one to make it reveal its inefficiency to the regulator. This allows the latter to indirectly tax unregulated profits. The timing chosen to describe the working of the coalition is somewhat arbitrary. Yet the possibility that the regulator 'teams up' with the regulated firm to partially extract the unregulated firm's profits is an interesting insight, and corresponds to some of the fears stated by entrants on a regulated market.

The potential costs of incomplete regulation leads us to insist on the need to adapt the regulatory structure before allowing entry, whenever possible — as recommended, e.g., by Laffont (1998). Yet having a single body regulate all the firms that produce imperfect substitutes may be difficult in practice. First, regulating a heterogenous sector composed of very diverse firms may require more effort in information acquisition than regulating a well defined, homogeneous, industry. This is particularly true when firms can have very diverse technologies (as with letter delivery, parcel delivery and e-mail messages). Second, restricting the discretion of the regulator may be desirable, especially if this regulator can have private motivations. If it is necessary to limit the scope of the regulator, a justification arises for having a competition authority monitor the regulated sector, in order to lessen the costs of potential collusion.[18] But as we have seen, if antitrust oversight does

[18]A different justification of intervention by a competition authority in regulated industries is provided by Sappington and Sidak (2002), when the firm is not only regulated, but state-owned. The authors emphasize the fact that State-Owned Enterprises benefit from more power, and have more incentives to engage in anti-competitive activities than private firms.

not suffice to fully deter collusion, then the regulator may be better off tolerating it: A conflict may thus emerge, in situations of incomplete regulation, between the mission of the competition authority and the regulatory objective.

Appendix

A.1. The benchmark of complete information

The reaction function of the unregulated firm The unregulated firm is under complete information on q^a when it chooses its own quantity. It simply solves the following program:

$$\max_{q^b} \pi^b = (d^b - sq^a - \theta^b)q^b - (q^b)^2,$$

which gives $q^b(q^a) = \frac{1}{2}(d^b - sq^a - \theta^b)$.

The quantities under complete information and complete regulation When regulation is complete, the regulator is able to choose the quantities produced by both firms. It maximizes the following objective over q^a and q^b:

$$SW = GS(q^a, q^b) - \theta^a q^a - \theta^b q^b = (d^a - \theta^a)q^a + m^b q^b - \frac{1}{2}((q^a)^2 + (q^b)^2) - sq^a q^b.$$

The first-order conditions yield a system of 2 equations for 2 unknowns, the solution of which is stated in the text.

The quantities under complete information and incomplete regulation The problem of the regulator when he cannot tax firm F^b is very similar to the program for complete regulation, except that maximization is done on q^a only (but using $\frac{dq^b}{dq^a} = -\frac{s}{2}$) and that the profits of firm F^b are no longer taken into account. The first-order condition gives $q^a = d^a - \theta^a - \frac{s}{2}q^b(q^a)$. This can be rewritten as $P^a(q^a, q^b) - \theta^a + \frac{s}{2}q^b(q^a) = 0$, since $P^a(q^a, q^b) = d^a - q^a - sq^b$. Replacing q^b by its expression yields the solution, $q^{a*}(\theta^a)$.

A.2. Asymmetric information

The expected welfare of the regulator is

$$\begin{aligned}\mathbf{E}_{\theta^a}\{SW\} = \quad & \underline{p} \quad \Big\{GS(q^a(\underline{\theta}), q^b(q^a(\underline{\theta}))) - \underline{\theta} q^a(\underline{\theta}) - P^b(q^a(\underline{\theta}), q^b(q^a(\underline{\theta})))q^b(q^a(\underline{\theta})) - \pi^a(\underline{\theta})\Big\} \\ & +\overline{p} \quad \Big\{GS(q^a(\overline{\theta}), q^b(q^a(\overline{\theta}))) - \overline{\theta} q^a(\overline{\theta}) - P^b(q^a(\overline{\theta}), q^b(q^a(\overline{\theta})))q^b(q^a(\overline{\theta})) - \pi^a(\overline{\theta})\Big\}.\end{aligned}$$

The constraints the regulator has to satisfy are:

- the participation constraints, given by

$$\begin{aligned}\pi^a(\underline{\theta}) & \geq 0 \quad IR^a(\underline{\theta}), \\ \pi^a(\overline{\theta}) & \geq 0 \quad IR^a(\overline{\theta});\end{aligned}$$

- and the incentive compatibility constraints:

$$\begin{aligned}\pi^a(\underline{\theta}) & \geq \pi^a(\overline{\theta}) + \Delta\theta q^a(\overline{\theta}) \quad IC^a(\underline{\theta}), \\ \pi^a(\overline{\theta}) & \geq \pi^a(\underline{\theta}) - \Delta\theta q^a(\underline{\theta}) \quad IC^a(\overline{\theta}).\end{aligned}$$

A.2.1. Asymmetric information and no collusion

When collusion is not possible, the constraints above are the only ones to be satisfied. When $IC^a(\underline{\theta})$ and $IR^a(\overline{\theta})$ are binding, we obtain $\pi^a(\overline{\theta}) = 0$ and $\pi^a(\underline{\theta}) = \Delta\theta q^a(\overline{\theta})$. The expected welfare of the regulator can indeed be rewritten as:

$$\begin{aligned}\mathbf{E}_{\theta^a}\{SW\} = \quad & \underline{p} \quad \Big[(d^a - \underline{\theta})q^a(\underline{\theta}) - \frac{1}{2}(q^a(\underline{\theta}))^2 + \frac{1}{2}(q^b(q^a(\underline{\theta})))^2 - \Delta\theta q^a(\overline{\theta})\Big] \\ & +\overline{p} \quad \Big[(d^a - \overline{\theta})q^a(\overline{\theta}) - \frac{1}{2}(q^a(\overline{\theta}))^2 + \frac{1}{2}(q^b(q^a(\overline{\theta})))^2\Big],\end{aligned}$$

with $q^b(q^a) = \frac{1}{2}(m^b - sq^a)$.

Optimizing with respect to quantities yields the following first-order conditions:

$$\begin{aligned} \underline{p}[q^a - \underline{\theta} - \frac{s}{4}(m^b - sq^a(\underline{\theta})) - q^a(\underline{\theta})] &= 0 \\ \overline{p}[q^a - \overline{\theta} - \frac{s}{4}(m^b - sq^a(\overline{\theta})) - q^a(\overline{\theta})] - \underline{p}\Delta\theta &= 0 \end{aligned}$$

The second-order condition are satisfied (the problems we consider being concave, we will generally omit them in the following). Rearranging terms, we obtain the quantity profile stated in the proposition.

Finally it is immediate to check that the remaining individual constraints are satisfied in equilibrium. We have $q^a(\underline{\theta}) - q^a(\overline{\theta}) = \frac{1}{4+s^2}\Delta\theta(1 + \frac{\underline{p}}{\overline{p}}) > 0$.

A.2.2. Collusion-proof contracts

Collusion-proofness constraints We denote by $\tilde{\theta}(\theta^a)$ the report recommended by the third party for a cost θ^a, and by $\theta^{nc}(\theta^a)$ the best non cooperative response of firm F^a. With collusion, F^a finally produces $q^a(\tilde{\theta}(\theta^a))$, where $q^a(.)$ is specified in the regulatory contract offered by R^a. We denote $\tilde{q}^a(\theta^a) \equiv q^a(\tilde{\theta}(\theta^a))$ to simplify notations. $CIC^k(\theta)$ denotes the incentive compatibility constraint, within the coalition, of firm F^k, $k = a, b$, when firm F^a has a cost θ; $CIR^k(\theta)$ is the constraint ensuring participation of this firm in the coalition. The program of the third party can then be written as:

$$
\begin{array}{lll}
\max\limits_{\{\tilde{\theta}(\theta^a), b(\theta^a)\}} & \mathbf{E}_{\theta^a}\left[\pi^a(\tilde{\theta}(\theta^a), \theta^a) + \pi^b(\tilde{q}^a(\theta^a))\right] & \\
\text{s.t.} & \pi^a(\tilde{q}^a(\underline{\theta}), \underline{\theta}) + b(\underline{\theta}) \geq \pi^a(q^a(\theta^{nc}(\underline{\theta})), \underline{\theta}) & CIR^a(\underline{\theta}) \\
& \pi^a(\tilde{q}^a(\overline{\theta}), \overline{\theta}) + b(\overline{\theta}) \geq \pi^a(q^a(\theta^{nc}(\overline{\theta})), \overline{\theta}) & CIR^a(\overline{\theta}) \\
& \pi^a(\tilde{q}^a(\underline{\theta}), \underline{\theta}) + b(\underline{\theta}) \geq \pi^a(\tilde{q}^a(\overline{\theta}), \underline{\theta}) + b(\overline{\theta}) & CIC^a(\underline{\theta}) \\
& \pi^a(\tilde{q}^a(\overline{\theta}), \overline{\theta}) + b(\overline{\theta}) \geq \pi^a(\tilde{q}^a(\underline{\theta}), \overline{\theta}) + b(\underline{\theta}) & CIC^a(\overline{\theta}) \\
& \pi^b(\tilde{q}^a(\underline{\theta})) - b(\underline{\theta}) \geq \pi^b(q^a(\theta^{nc}(\underline{\theta}))) & CIR^b(\underline{\theta}) \\
& \pi^b(\tilde{q}^a(\overline{\theta})) - b(\overline{\theta}) \geq \pi^b(q^a(\theta^{nc}(\overline{\theta}))) & CIR^b(\overline{\theta}).
\end{array}
$$

Let us denote by $\nu^k(\theta^a)$ the multiplier of the coalition participation constraint $CIR^k(\theta^a)$, $k = a, b$, and $\delta^a(\theta^a)$ the multiplier of the coalition incentive constraint $CIC^a(\theta^a)$. Optimizing the program of the third party with respect to the bribes $b(\underline{\theta})$ and $b(\overline{\theta})$ yields two first-order conditions that can be combined to obtain the following relationship between the multipliers: $\nu^a(\underline{\theta}) - \nu^b(\underline{\theta}) = -(\nu^a(\overline{\theta}) - \nu^b(\overline{\theta})) = \delta^a(\overline{\theta}) - \delta^a(\underline{\theta})$.

Using the above relationship between multipliers, one can separate the program of the third party with respect to reports into two parts:

$$
\max_{\tilde{q}^a(\underline{\theta})} \left\{ \pi^a(\tilde{q}^a(\underline{\theta}), \underline{\theta}) + \pi^b(\tilde{q}^a(\underline{\theta})) + \frac{\delta^a(\overline{\theta})}{\underline{p} + \nu^b(\underline{\theta})} \Big(\pi^a(\tilde{q}^a(\underline{\theta}), \underline{\theta}) - \pi^a(\tilde{q}^a(\underline{\theta}), \overline{\theta}) \Big) \right\}
$$
$$
\max_{\tilde{q}^a(\overline{\theta})} \left\{ \pi^a(\tilde{q}^a(\overline{\theta}), \overline{\theta}) + \pi^b(\tilde{q}^a(\overline{\theta})) + \frac{\delta^a(\underline{\theta})}{\overline{p} + \nu^b(\overline{\theta})} \Big(\pi^a(\tilde{q}^a(\overline{\theta}), \overline{\theta}) - \pi^a(\tilde{q}^a(\overline{\theta}), \underline{\theta}) \Big) \right\}
$$

To ensure collusion-proofness, the regulator must offer a contract such that $\tilde{\theta}(\underline{\theta}) = \underline{\theta}$ and $\tilde{\theta}(\overline{\theta}) = \overline{\theta}$ give a higher value for the two objective functions above than other reports. Hence the two collusion-proofness constraints:

$$\pi^a(q^a(\underline{\theta}),\underline{\theta}) + \pi^b(q^a(\underline{\theta})) + \frac{\delta^a(\overline{\theta})}{\underline{p}+\nu^b(\underline{\theta})}\Big(\pi^a(q^a(\underline{\theta}),\underline{\theta}) - \pi^a(q^a(\underline{\theta}),\overline{\theta})\Big) \geq$$
$$\pi^a(q^a(\overline{\theta}),\underline{\theta}) + \pi^b(q^a(\overline{\theta})) + \frac{\delta^a(\overline{\theta})}{\underline{p}+\nu^b(\underline{\theta})}\Big(\pi^a(q^a(\overline{\theta}),\underline{\theta}) - \pi^a(q^a(\overline{\theta}),\overline{\theta})\Big),$$
$$\pi^a(q^a(\overline{\theta}),\overline{\theta}) + \pi^b(q^a(\overline{\theta})) + \frac{\delta^a(\underline{\theta})}{\overline{p}+\nu^b(\overline{\theta})}\Big(\pi^a(q^a(\overline{\theta}),\overline{\theta}) - \pi^a(q^a(\overline{\theta}),\underline{\theta})\Big) \geq$$
$$\pi^a(q^a(\underline{\theta}),\overline{\theta}) + \pi^b(q^a(\underline{\theta})) + \frac{\delta^a(\underline{\theta})}{\overline{p}+\nu^b(\overline{\theta})}\Big(\pi^a(q^a(\underline{\theta}),\overline{\theta}) - \pi^a(q^a(\underline{\theta}),\underline{\theta})\Big).$$

They can be rewritten as:

$$\pi^a(q^a(\underline{\theta}),\underline{\theta}) \geq \pi^a(q^a(\overline{\theta}),\overline{\theta}) + \Delta\theta q^a(\overline{\theta}) + \Delta\pi^b - \tfrac{\delta^a(\overline{\theta})}{\underline{p}+\nu^b(\underline{\theta})}[q^a(\underline{\theta}) - q^a(\overline{\theta})]\Delta\theta \qquad CPC(\underline{\theta})$$
$$\pi^a(q^a(\overline{\theta}),\overline{\theta}) \geq \pi^a(q^a(\underline{\theta}),\underline{\theta}) - \Delta\theta q^a(\underline{\theta}) - \Delta\pi^b - \tfrac{\delta^a(\underline{\theta})}{\overline{p}+\nu^b(\overline{\theta})}[q^a(\underline{\theta}) - q^a(\overline{\theta})]\Delta\theta \qquad CPC(\overline{\theta})$$

One can take into consideration $CPC(\underline{\theta})$ only in a first step, and check that $CPC(\overline{\theta})$ is also satisfied afterwards.

But one should note that if the regulator induces a truthful report on θ^a, firm F^b is not willing to pay any bribe ($b(\overline{\theta}) = b(\underline{\theta}) = 0$), and $CIC^a(\overline{\theta})$ can be rewritten as $\pi^a(q^a(\overline{\theta}),\overline{\theta}) \geq \pi^a(q^a(\underline{\theta}),\underline{\theta}) - \Delta\theta q^a(\underline{\theta})$: It is exactly the same constraint as the incentive compatibility constraint for an inefficient firm in the program of the regulator, $IC^a(\overline{\theta})$. Therefore, if this incentive constraint is not binding in the program of the regulator for the best collusion-proof contract, the multiplier $\delta^a(\overline{\theta})$ equals zero, and the collusion-proof constraint $CPC(\underline{\theta})$ is the same as with perfect information within the coalition: $\pi^a(q^a(\underline{\theta}),\underline{\theta}) \geq \pi^a(q^a(\overline{\theta}),\overline{\theta}) + \Delta\theta q^a(\overline{\theta}) + \Delta\pi^b$.

We show below that $IC^a(\overline{\theta})$ is not binding in the program of the regulator for the best collusion-proof contract.

The best separating collusion-proof contract Let us here assume that $IC^a(\overline{\theta})$ is not binding in the program of the regulator. Then $CPC(\underline{\theta})$ is always more stringent than $IC^a(\underline{\theta})$. The binding constraints are therefore $IR^a(\overline{\theta})$, as usual, and $CPC(\underline{\theta})$. We will have to check that the solutions obtained indeed satisfies $IC^a(\overline{\theta})$, i.e., that $q^a(\underline{\theta}) \geq q^a(\overline{\theta})$.

Since $\pi^b(q^a) = [q^b(q^a)]^2$, $\Delta\pi^b = \pi^b(\overline{\theta}) - \pi^b(\underline{\theta}) = [q^b(q^a(\overline{\theta}))]^2 - [q^b(q^a(\underline{\theta}))]^2 = \frac{s}{4}[q^a(\underline{\theta}) - q^a(\overline{\theta})][2(m^b - s(q^a(\underline{\theta}) + q^a(\overline{\theta}))] \geq 0$. From the binding constraints, we have:

$$\begin{aligned}
\pi^a(\overline{\theta}) &= 0 \\
\pi^a(\underline{\theta}) &= \Delta\theta q^a(\overline{\theta}) + \frac{s}{4}[q^a(\underline{\theta}) - q^a(\overline{\theta})][2m^b - s(q^a(\underline{\theta}) + q^a(\overline{\theta}))] \\
&\quad - \frac{\delta^a(\overline{\theta})}{\underline{p} + \nu^b(\underline{\theta})}[q^a(\underline{\theta}) - q^a(\overline{\theta})]\Delta\theta.
\end{aligned}$$

Rearranging the first-order conditions, one obtains the best collusion-proof regulated quantities:

$$\begin{aligned}
q^a(\underline{\theta}) &= \frac{1}{4 - 3s^2}\Big[4\Big(d^a - \underline{\theta} + \frac{\delta^a(\overline{\theta})}{\underline{p} + \nu^b(\underline{\theta})}\Delta\theta\Big) - 3sm^b\Big] \\
q^a(\overline{\theta}) &= \frac{1}{4 - s^2\Big(1 - 2\frac{\underline{\underline{p}}}{\overline{p}}\Big)}\Big[4\Big(d^a - \overline{\theta} - \frac{\underline{\underline{p}}}{\overline{p}}\Delta\theta\Big(1 + \frac{\delta^a(\overline{\theta})}{\underline{p} + \nu^b(\underline{\theta})}\Big)\Big) - s\Big(1 + 2\frac{\underline{\underline{p}}}{\overline{p}}\Big)m^b\Big].
\end{aligned}$$

Simple computations yield the quantities given in the text.

Let us now check that $IC^a(\overline{\theta})$ (and therefore $CIC^a(\overline{\theta})$) is satisfied and not binding, for the quantities obtained: One must check whether condition $q^a(\underline{\theta}) \geq q^a(\overline{\theta})$ holds. Computations yield:

$$
\begin{aligned}
& q^a(\underline{\theta}) - q^a(\overline{\theta}) \\
= \; & \frac{1}{4-3s^2}\Big[4(d^a - \underline{\theta}) - 3sm^b\Big] - \frac{1}{4 - s^2(1 - 2\frac{\underline{p}}{\overline{p}})}\Big[4\Big(d^a - \overline{\theta} - \frac{\underline{p}}{\overline{p}}\Delta\theta\Big) - s\Big(1 + 2\frac{\underline{p}}{\overline{p}}\Big)m^b\Big] \\
= \; & \frac{4}{\overline{p}(4 - s^2(1 - 2\frac{\underline{p}}{\overline{p}}))(4-3s^2)}\Big[2s^2(d^a - \underline{\theta}) + (4-3s^2)\Delta\theta - sm^b[2 - \underline{p}(4-3s^2)]\Big] \\
= \; & \frac{4}{\overline{p}(4 - s^2(1 - 2\frac{\underline{p}}{\overline{p}}))(4-3s^2)}\Big[2[s^2(d^a - \underline{\theta}) - sm^b] + (4-3s^2)[\Delta\theta + \underline{p}sm^b]\Big] \\
= \; & \frac{4}{\overline{p}(4 - s^2(1 - 2\frac{\underline{p}}{\overline{p}}))}\Big[-2q^b(q^{a*}(\underline{\theta}))\frac{4-s^2}{4-3s^2} + [\Delta\theta + \underline{p}sm^b]\Big].
\end{aligned}
$$

The best separating collusion-proof contract is thus feasible if the following condition is satisfied:

$$(4-3s^2)[\Delta\theta + \underline{p}sm^b] \geq 2(4-s^2)q^b(q^{a*}(\underline{\theta})),$$

or equivalently

$$2s^2(d^a - \underline{\theta}) + (4-3s^2)\Delta\theta \geq sm^b[2 - \underline{p}(4-3s^2)] \qquad (\mathcal{C}_{cp}).$$

It is always satisfied when s tends to zero.

If this condition is satisfied, then $CIC^a(\overline{\theta})$ is not binding for the best collusion-proof contract, and the relevant collusion-proofness constraint is indeed the same as if the coalition was under complete information on θ^a. If it is not satisfied, on the other hand, then the best collusion-proof contract entails pooling, and the collusion-proofness constraints are no longer relevant. This proves our claim that $\delta^a(\overline{\theta}) = 0$ in equilibrium.

The best pooling contract If the previous condition is not satisfied, the collusion-proof contract will be a pooling one: $q^a(\underline{\theta}) = q^a(\overline{\theta}) = \hat{q}^a$. The incentive compatibility constraint of an inefficient firm, $IC^a(\overline{\theta})$ is then always satisfied, and the stake of collusion obviously disappears.

In a pooling contract, it is no longer possible to separate between a high-cost and a low-cost regulated firm. It is as if the cost of production was $\overline{\theta}$ for both types of firms. Collusion is in addition not an issue any longer. As a consequence, the best pooling quantity is the full information one for a firm with type $\overline{\theta}$: $\hat{q}^a = q^{a*}(\overline{\theta})$.

A.2.3. Non collusion-proof contracts

Let us consider mechanisms that trigger an active response from the coalition.

Outline of the proof: In steps 1 and 2, we show that some version of the Revelation Principle may apply when the agent is taken to be the coalition, and not the regulated firm. The utility of this particular agent is the maximal total profit it can obtain given its asymmetric information on firm F^a. We show the following: First, restricting an initial set of messages to the couple of incentive feasible messages that are optimal for the coalition does not restrict the set of outcomes available for the principal; Second, there is no loss of generality in considering only direct and truthful contracts, provided they are also incentive feasible at the level of the coalition. In step 3, we turn to the choice of contract by the regulator.

The regulator offers a mechanism $g : \mathcal{M} \rightarrow I\!R^+ \times I\!R$ that associates to a message m, in some message space $\mathcal{M}$, a quantity $q^a(m)$ and a transfer $t^a(m)$.

This corresponds, for a firm F^a with type θ^a, to a profit of $\pi^a(m, \theta^a)$ (not including collusive transfers). Remember now that the profits of the unregulated firm are $\pi^b(m) = [q^b(q^a(m))]^2$, where $q^b(.)$ is the best response function characterized in equation (1). Hence, the mechanism yields total profits $\pi^a(m, \theta^a) + [q^b(q^a(m))]^2$ for the coalition (bribes are pure transfers and do not appear in this sum).

A benevolent third party maximizes the sum of the profits of the two firms, under asymmetric information on θ^a. It determines the message sent to the regulator and the bribe b paid to F^a by F^b for a message $\hat{m}$ to the regulator. We assume that the coalition budget must be balanced (the third party cannot provide or keep any share of the collusive transfers) and that the firms can exit the coalition at all times.[19]

* *Step1: Can we restrict attention to direct mechanisms?*

Let us first consider the problem of the third party. The Revelation Principle applies at its level. There is therefore no restriction in considering only direct truthful mechanisms $\hat{m} : \{\underline{\theta}, \overline{\theta}\} \to \mathcal{M} \times I\!R$, associating to a truthful report θ^a by F^a an allocation $\{\hat{m}(\theta^a), b(\theta^a)\}$. Let us denote for simplicity $\hat{m}(\overline{\theta}) \equiv \overline{m}$, $b(\overline{\theta}) \equiv b(\overline{\theta})$, etc., and, with a slight abuse of notations, $\pi^b(m) = \pi^b(q^b(q^a(m)))$. We denote by $CIC^k(\theta)$ the incentive compatibility constraint, within the coalition, of firm F^k, $k = a, b$, when firm F^a has a cost θ, and by $CIR^k(\theta)$ the constraint ensuring participation of this firm in the coalition. The program of the third party can then be written as:

$$
\begin{array}{lll}
\max\limits_{\{\hat{m}(\theta^a), b(\theta^a)\}} & \mathbf{E}_{\theta^a}[\pi^a(\hat{m}(\theta^a), \theta^a) + \pi^b(\hat{m}(\theta^a))] & \\
\text{s.t.} & \pi^a(\underline{m}, \underline{\theta}) + b(\underline{\theta}) \geq \pi^a(m^{nc}(\underline{\theta}), \underline{\theta}) & CIR^a(\underline{\theta}) \\
& \pi^a(\overline{m}, \overline{\theta}) + b(\overline{\theta}) \geq \pi^a(m^{nc}(\overline{\theta}), \overline{\theta}) & CIR^a(\overline{\theta}) \\
& \pi^a(\underline{m}, \underline{\theta}) + b(\underline{\theta}) \geq \pi^a(\overline{m}, \underline{\theta}) + b(\overline{\theta}) & CIC^a(\underline{\theta}) \\
& \pi^a(\overline{m}, \overline{\theta}) + b(\overline{\theta}) \geq \pi^a(\underline{m}, \overline{\theta}) + b(\underline{\theta}) & CIC^a(\overline{\theta}) \\
& \pi^b(\underline{m}) - b(\underline{\theta}) \geq \pi^b(m^{nc}(\underline{\theta})) & CIR^b(\underline{\theta}) \\
& \pi^b(\overline{m}) - b(\overline{\theta}) \geq \pi^b(m^{nc}(\overline{\theta})) & CIR^b(\overline{\theta}).
\end{array}
$$

[19]This assumption implies that the collusive participation constraint of firm F^b is an ex post one, as if F^b was informed on θ^a. The Bayesian (interim) constraint also exists but will always be less stringent than the participation constraint in one state, so we will ignore it in the remaining of the analysis.

We define below a concept of incentive feasibility at the level of the coalition. This concept is not fundamentally different from standard incentive feasibility for one agent only, but we want to distinguish clearly the incentive constraints that come from the program of the coalition from the ones that come from the program of the regulator.

Definition 2 *A message response $m(.)$ that associates some messages $\underline{m}$ and $\overline{m}$ in $\mathcal{M}$ to respectively $\underline{\theta}$ and $\overline{\theta}$ is said to be C-incentive feasible if and only if there exists a couple $\{b(\underline{\theta}), b(\overline{\theta})\} \in I\!R^2$ such that the constraints $CIR^a(\underline{\theta})$, $CIR^a(\overline{\theta})$, $CIC^a(\underline{\theta})$, $CIC^a(\overline{\theta})$, $CIR^b(\underline{\theta})$ and $CIR^b(\overline{\theta})$ are simultaneously satisfied, for $\{\underline{m}, \overline{m}, b(\underline{\theta}), b(\overline{\theta})\}$.*

We will denote by $\mathcal{A}$, a subset of $\mathcal{M} \times \mathcal{M}$, the set of C-incentive feasible message responses.

Let us now denote by $\{\underline{m}^T, \overline{m}^T\}$ the solution to the maximization of the sum of firms profits:

$$\{\underline{m}^T, \overline{m}^T\} \in \underset{\{\underline{m},\overline{m}\}\in\mathcal{A}}{\arg\max}\ \underline{p}\Big[\pi^a(\underline{m}, \underline{\theta}) + \pi^b(\underline{m})\Big] + \overline{p}\Big[\pi^a(\overline{m}, \overline{\theta}) + \pi^b(\overline{m})\Big].$$

Result 1 *If $\{\underline{m}, \overline{m}\}$ is C-incentive feasible given an initial message space $\mathcal{M}$, then it remains so when the message space is reduced to $\{\underline{m}, \overline{m}\}$.*

Proof: All the 6 constraints can be satisfied for the restricted message space. Let us begin with the participation constraint of, say, an efficient regulated firm. If $\{\underline{m}, \overline{m}\}$ is feasible for $\mathcal{M}$, then constraint $CIR^a(\underline{\theta})$ is satisfied, i.e., there exists $b(\underline{\theta})$ such that $\pi^a(\underline{m}, \underline{\theta}) + b(\underline{\theta}) \geq \pi^a(m^{nc}(\underline{\theta}), \underline{\theta})$. And by definition of the non collusive best response, $\pi^a(m^{nc}(\underline{\theta}), \underline{\theta}) \geq \max\{\pi^a(\underline{m}, \underline{\theta}), \pi^a(\overline{m}, \underline{\theta})\}$. Hence, $CIR^a(\underline{\theta})$ is satisfied as well when the message space is restricted: There exists $b(\underline{\theta})$ such that $\pi^a(\underline{m}, \underline{\theta}) + b(\underline{\theta}) \geq \max\{\pi^a(\underline{m}, \underline{\theta}), \pi^a(\overline{m}, \underline{\theta})\}$. The same reasoning applies for an inefficient firm F^a, and for firm F^b in both states of nature. The collusive incentive compatibility constraints are unaffected. All constraints are therefore satisfied, which proves the result.

Now, from the point of view of the regulator, there is no restriction in offering message space

$\{\underline{m}^T, \overline{m}^T\}$ instead of $\mathcal{M}$, since the couple is incentive feasible, and that other messages are never played. There is therefore no restriction as well in considering *direct* mechanisms, i.e., $\mathcal{M} = \{\underline{\theta}, \overline{\theta}\}$, provided that they are C-incentive feasible.

* *Step2: Can we restrict attention to truthful mechanisms?*

There are only two possible messages. Therefore,

- either the coalition always announces the same type, whatever the true type θ, and this is also implementable with a truthful contract by offering only one message,
- or the coalition makes a different announcement given its type, which corresponds to a truthful mechanism (possibly re-labeling the messages),
- or the coalition randomizes over the two messages in at least one state of nature.

Randomization only occurs if the coalition's total profits are identical for both messages in this state of nature, i.e., $\pi^a(\overline{\theta}, \theta^a) + [q^b(q^a(\overline{\theta}))]^2 = \pi^a(\underline{\theta}, \theta^a) + [q^b(q^a(\underline{\theta}))]^2$, or $\pi^a(\underline{\theta}, \theta^a) - \pi^a(\overline{\theta}, \theta^a) = [q^b(q^a(\overline{\theta}))]^2 - [q^b(q^a(\underline{\theta}))]^2$. The welfare of the regulator only depends on the quantity q^a produced by the regulated firm (q^b being a given function of this quantity) and on the transfers t^a. One of the two possible couples $\{q^a(\underline{\theta}), t^a(\underline{\theta})\}$ and $\{q^a(\overline{\theta}), t^a(\overline{\theta})\}$ is thus preferred by the regulator. Assuming that the coalition is indifferent between the two couples, the regulator can break this indifference by offering some additional profits of ϵ, very close to zero, for the message for which his welfare is higher. Considering only truthful mechanisms is therefore no restriction.

To summarize, we obtain Lemma 3, that we restate here:

Result 2 *There is no loss of generality in considering that only direct and truthful (C-incentive feasible) mechanisms are offered to the coalition.*

Note once again that this does not imply collusion-proofness.

* *Step 3: What is the optimal non collusion-proof direct and truthful mechanism?*

The set of regulatory contracts has now been characterized as the subset of direct truthful mechanisms that are C-incentive feasible. The constraints written below correspond to the characterization of this set: The first four constraints are incentive and participation constraints at the level of the coalition (for the hypothetical third party, hence the superscript T), and the following six constraints guarantee that the contract is C-incentive feasible. The program of the regulator can be written as follows:

$$
\begin{aligned}
\max_{\{q^a(.),\pi^a(.),b(.)\}} \quad & \underline{p}\Big[(d^a-\underline{\theta})q^a(\underline{\theta})-\tfrac{1}{2}(q^a(\underline{\theta}))^2+\tfrac{1}{2}(q^b(q^a(\underline{\theta})))^2-\pi^a(\underline{\theta})\Big] \\
& +\overline{p}\Big[(d^a-\overline{\theta})q^a(\overline{\theta})-\tfrac{1}{2}(q^a(\overline{\theta}))^2+\tfrac{1}{2}(q^b(q^a(\overline{\theta})))^2-\pi^a(\overline{\theta})\Big] \\
\text{subject to} \quad & \pi^a(\underline{\theta})+\pi^b(q^a(\underline{\theta}))\geq \pi^a(\overline{\theta})+\Delta\theta q^a(\overline{\theta})+\pi^b(q^a(\overline{\theta})) \qquad IC^T(\underline{\theta}) \\
& \pi^a(\overline{\theta})+\pi^b(q^a(\overline{\theta}))\geq \pi^a(\underline{\theta})-\Delta\theta q^a(\underline{\theta})+\pi^b(q^a(\underline{\theta})) \qquad IC^T(\overline{\theta}) \\
& \pi^a(\underline{\theta})+\pi^b(q^a(\underline{\theta}))\geq 0 \qquad IR^T(\underline{\theta}) \\
& \pi^a(\overline{\theta})+\pi^b(q^a(\overline{\theta}))\geq 0 \qquad IR^T(\overline{\theta}) \\
& \pi^a(\underline{\theta})+b(\underline{\theta})\geq \pi^a(m^{nc}(\underline{\theta})) \qquad CIR^a(\underline{\theta}) \\
& \pi^a(\overline{\theta})+b(\overline{\theta})\geq \pi^a(m^{nc}(\overline{\theta})) \qquad CIR^a(\overline{\theta}) \\
& \pi^a(\underline{\theta})+b(\underline{\theta})\geq \pi^a(\overline{\theta})+\Delta\theta q^a(\overline{\theta})+b(\overline{\theta}) \qquad CIC^a(\underline{\theta}) \\
& \pi^a(\overline{\theta})+b(\overline{\theta})\geq \pi^a(\underline{\theta})-\Delta\theta q^a(\underline{\theta})+b(\underline{\theta}) \qquad CIC^a(\overline{\theta}) \\
& \pi^b(\underline{\theta})-b(\underline{\theta})\geq \pi^b(m^{nc}(\underline{\theta})) \qquad CIR^b(\underline{\theta}) \\
& \pi^b(\overline{\theta})-b(\overline{\theta})\geq \pi^b(m^{nc}(\overline{\theta})) \qquad CIR^b(\overline{\theta}).
\end{aligned}
$$

One can immediately note that combining the two incentive compatibility constraints for the coalition yields $q^a(\underline{\theta}) \geq q^a(\overline{\theta})$. Hence, $\pi^b(\overline{\theta}) \geq \pi^b(\underline{\theta})$.

Still denoting by $m^{nc}(\theta^a)$ the non collusive best response by firm F^a when it has type θ^a, the coalition's best response in a truthful contract is to report θ^a. Hence the coalition is not active if $m^{nc}(\theta^a) = \theta^a$ in both states of nature. If the coalition always makes the same announcement as the non cooperative choice of F^a, then no side-transfers occur and the outcome can be replicated by a collusion-proof contract. We can therefore focus on contracts such that $m^{nc}(\theta^a) \neq \theta^a$ in at least one state of nature.

1- Consider the case in which firm F^a always announces being inefficient when there is no collusion ($m^{nc}(\theta^a) = \overline{\theta}$). Since this leads to a lower quantity (due to constraint $q^a(\underline{\theta}) \geq q^a(\overline{\theta})$), the unregulated firm F^b makes higher profits than in any other configuration. It is therefore not willing to pay a bribe to have F^a alter its report.

2 - Consider now the opposite case in which firm F^a always announces being efficient when there is no collusion ($m^{nc}(\theta^a) = \underline{\theta}$). A stake of collusion now exists, for $\theta^a = \overline{\theta}$. We necessarily have $b(\underline{\theta}) = 0$, from the ex post participation constraint of F^b to the coalition ($CIR^b(\theta^a)$). The set

of constraints that the regulator faces can be rewritten as follows:

$$\begin{array}{lll}
\max\limits_{\{q^a(.),\pi^a(.),b(\overline{\theta})\}} & \underline{p}\Big[(d^a-\underline{\theta})q^a(\underline{\theta})-\frac{1}{2}(q^a(\underline{\theta}))^2+\frac{1}{2}(q^b(q^a(\underline{\theta})))^2-\pi^a(\underline{\theta})\Big] & \\
& +\overline{p}\Big[(d^a-\overline{\theta})q^a(\overline{\theta})-\frac{1}{2}(q^a(\overline{\theta}))^2+\frac{1}{2}(q^b(q^a(\overline{\theta})))^2-\pi^a(\overline{\theta})\Big] & \\
\text{subject to} & \pi^a(\underline{\theta})+\pi^b(q^a(\underline{\theta}))\geq\pi^a(\overline{\theta})+\Delta\theta q^a(\overline{\theta})+\pi^b(q^a(\overline{\theta})) & IC^T(\underline{\theta}) \\
& \pi^a(\overline{\theta})+\pi^b(q^a(\overline{\theta}))\geq\pi^a(\underline{\theta})-\Delta\theta q^a(\underline{\theta})+\pi^b(q^a(\underline{\theta})) & IC^T(\overline{\theta}) \\
& \pi^a(\underline{\theta})+\pi^b(q^a(\underline{\theta}))\geq 0 & IR^T(\underline{\theta}) \\
& \pi^a(\overline{\theta})+\pi^b(q^a(\overline{\theta}))\geq 0 & IR^T(\overline{\theta}) \\
& \pi^a(\underline{\theta})\geq\pi^a(\overline{\theta})+\Delta\theta q^a(\overline{\theta}) & CIR^a(\underline{\theta}) \\
& \pi^a(\overline{\theta})+b(\overline{\theta})\geq\pi^a(\underline{\theta})-\Delta\theta q^a(\underline{\theta}) & CIR^a(\overline{\theta}) \\
& \pi^a(\underline{\theta})\geq\pi^a(\overline{\theta})+\Delta\theta q^a(\overline{\theta})+b(\overline{\theta}) & CIC^a(\underline{\theta}) \\
& \pi^a(\overline{\theta})+b(\overline{\theta})\geq\pi^a(\underline{\theta})-\Delta\theta q^a(\underline{\theta}) & CIC^a(\overline{\theta}) \\
& \pi^b(q^a(\underline{\theta}))\geq\pi^b((q^a(\underline{\theta})) & CIR^b(\underline{\theta}) \\
& \pi^b(q^a(\overline{\theta}))-b(\overline{\theta})\geq\pi^b(q^a(\underline{\theta})) & CIR^b(\overline{\theta}),
\end{array}$$

and in addition, for firm F^a to always prefer announcing being efficient, we have the following individual constraints:

$$\begin{array}{ll}
\pi^a(\underline{\theta})\geq\overline{\pi^a}+\Delta\theta q^a(\overline{\theta}) & I_1 \\
\pi^a(\underline{\theta})-\Delta\theta q^a(\underline{\theta})>\pi^a(\overline{\theta}) & I_2.
\end{array}$$

Constraints $CIR^a(\overline{\theta})$ and $CIC^a(\overline{\theta})$ are identical, and $CIR^b(\underline{\theta})$ is trivially satisfied. $CIC^a(\underline{\theta})$ implies I_1, and $IR^T(\underline{\theta})$ is implied by $IR^T(\overline{\theta})$ and $IC^T(\underline{\theta})$. Last, I_2 and $q^a(\overline{\theta})\leq q^a(\underline{\theta})$ imply $CIR^a(\underline{\theta})$.

Rearranging the constraints that are not redundant, we obtain:

$$\begin{aligned}
\pi^a(\underline{\theta}) - \pi^a(\overline{\theta}) &\geq [q^b(q^a(\overline{\theta}))]^2 - [q^b(q^a(\underline{\theta}))]^2 + \Delta\theta q^a(\overline{\theta}) \\
\pi^a(\underline{\theta}) - \pi^a(\overline{\theta}) &\leq [q^b(q^a(\overline{\theta}))]^2 - [q^b(q^a(\underline{\theta}))]^2 + \Delta\theta q^a(\underline{\theta}) \\
\pi^a(\overline{\theta}) &\geq -[q^b(q^a(\overline{\theta}))]^2.
\end{aligned}$$

Assume now that the third constraint is binding (the regulator minimizes the profits of the regulated firm). Then the first two inequalities become $-[q^b(q^a(\underline{\theta}))]^2 + \Delta\theta q^a(\overline{\theta}) \leq \pi^a(\underline{\theta}) \leq -[q^b(q^a(\underline{\theta}))]^2 + \Delta\theta q^a(\underline{\theta})$, and the regulator is better off minimizing $\pi^a(\underline{\theta})$, i.e., having $\pi^a(\underline{\theta}) = -[q^b(q^a(\overline{\theta}))]^2 + \Delta\theta q^a(\overline{\theta})$. The regulator's program then becomes:

$$\begin{aligned}
\max_{\{q^a(\underline{\theta}), q^a(\overline{\theta})\}} \quad & \underline{p}\Big[(d^a - \underline{\theta})q^a(\underline{\theta}) - \tfrac{1}{2}(q^a(\underline{\theta}))^2 + \tfrac{3}{2}(q^b(q^a(\underline{\theta})))^2 - \Delta\theta q^a(\overline{\theta})\Big] \\
& + \overline{p}\Big[(d^a - \overline{\theta})q^a(\overline{\theta}) - \tfrac{1}{2}(q^a(\overline{\theta}))^2 + \tfrac{3}{2}[q^b(q^a(\overline{\theta}))]^2\Big].
\end{aligned}$$

The first-order conditions yield the optimal non collusion-proof quantities:

$$\begin{aligned}
q^a_{ac}(\underline{\theta}) &= \frac{1}{4+s^2}\Big[4(d^a - \underline{\theta}) - 3sm^b\Big] \\
q^a_{ac}(\overline{\theta}) &= \frac{1}{4+s^2}\Big[4\Big(d^a - \overline{\theta} - \frac{\underline{p}}{\overline{p}}\Big) - 3sm^b\Big].
\end{aligned}$$

The condition necessary for this regulatory contract to indeed induce a report of efficiency by all types of firm F^a when there is no collusion is $m^b - \frac{2}{s}\Delta\theta \geq \frac{s}{2}(q^a(\underline{\theta})_{ac} - q^a(\overline{\theta})_{ac})$, that is $m^b \geq \frac{2}{(4+s^2)s\overline{p}}(4\overline{p} + s^2(1+\overline{p}))\Delta\theta$, condition $(\mathcal{C}_{ac})$. There is otherwise bunching.

Finally, the ranking between the regulated quantities without the possibility of collusion and with active collusion contracts is easily obtained by substracting the expressions of the two quantities. For an efficient firm, we obtain:

$$q^a(\underline{\theta}) - q^a(\underline{\theta})_{ac} = \frac{8s}{(4-3c^2)(4+s^2)}[m^b - s(d^a - \underline{\theta})] < 0.$$

The same computation can be done for an inefficient firm.

A.2.3. When are contracts inducing active collusion optimal?

The difference in welfare for the regulator with active collusion, and collusion-proofness is given by:

$$\begin{aligned}
&\Delta^{(ac)-(cp)} = \mathbf{E}_{\theta^a}\{SW^{ac}(q^a_{ac}(\underline{\theta}), q^a_{ac}(\overline{\theta}))\} - \mathbf{E}_{\theta^a}\{SW^{cp}(q^a_{cp}(\underline{\theta}), q^a_{cp}(\overline{\theta}))\} \\
&= \Big\{\mathbf{E}_{\theta^a}\Big[(d^a - \theta^a)q^a_{ac}(\theta^a) - \frac{1}{2}(q^a_{ac}(\theta^a))^2 + \frac{1}{2}(q^b(q^a_{ac}(\theta^a)))^2\Big] \\
&+\underline{p}\Big[\Delta\theta q^a_{ac}(\overline{\theta}) + (q^b(q^a_{ac}(\underline{\theta})))^2\Big] + \overline{p}(q^b(q^a_{ac}(\overline{\theta})))^2\Big\} \\
&-\Big\{\mathbf{E}_{\theta^a}\Big[(d^a - \theta^a)q^a_{cp}(\theta^a) - \frac{1}{2}(q^a_{cp}(\theta^a))^2 + \frac{1}{2}(q^b(q^a_{cp}(\theta^a)))^2\Big] \\
&-\underline{p}\Big[\Delta\theta q^a_{cp}(\overline{\theta}) + \Delta\pi^b\Big]\Big\}.
\end{aligned}$$

Using the fact that $\Delta\pi^b = (q^b(q^a_{ac}(\overline{\theta})))^2 - (q^b(q^a_{ac}(\underline{\theta})))^2 = \frac{s}{4}[q^a_{cp}(\underline{\theta}) - q^a_{cp}(\overline{\theta})][2m^b - s(q^a_{cp}(\underline{\theta}) + q^a_{cp}(\overline{\theta}))]$ together with the identity "$A^2 + B^2 = (A-B)(A+B)$", one obtains the following expression:

$$\begin{aligned}
\Delta^{(ac)-(cp)} = \quad & \underline{p}\Big[d^a - \overline{\theta} - \frac{1}{2}(q^a_{ac}(\underline{\theta}) + q^a_{cp}(\underline{\theta}))\Big]\Big(q^a_{ac}(\underline{\theta}) - q^a_{cp}(\underline{\theta})\Big) \\
&+\underline{p}\Big[\frac{3}{2}\Big(q^b(q^a_{ac}(\underline{\theta}))\Big)^2 - \frac{1}{2}\Big(q^b(q^a_{cp}(\underline{\theta}))\Big)^2\Big] \\
&+\overline{p}\Big[d^a - \overline{\theta} - \frac{1}{2}(q^a_{ac}(\overline{\theta}) + q^a_{cp}(\overline{\theta}))\Big]\Big(q^a_{ac}(\overline{\theta}) - q^a_{cp}(\overline{\theta})\Big) \\
&+\overline{p}\Big[\frac{3}{2}\Big(q^b(q^a_{ac}(\overline{\theta}))\Big)^2 - \frac{1}{2}\Big(q^b(q^a_{cp}(\overline{\theta}))\Big)^2\Big] \\
&+\underline{p}\Big[\Big(q^b(q^a_{cp}(\overline{\theta}))\Big)^2 - \Big(q^b(q^a_{cp}(\underline{\theta}))\Big)^2\Big],
\end{aligned}$$

or equivalently,

$$
\begin{aligned}
\Delta^{(ac)-(cp)} = \quad & \underline{p}\Big[d^a - \overline{\theta} - \frac{1}{2}(q^a_{ac}(\underline{\theta}) + q^a_{cp}(\underline{\theta}))\Big]\Big(q^a_{ac}(\underline{\theta}) - q^a_{cp}(\underline{\theta})\Big) \\
& + \underline{p}\frac{3}{2}\Big[\Big(q^b(q^a_{ac}(\underline{\theta}))\Big)^2 - \Big(q^b(q^a_{cp}(\underline{\theta}))\Big)^2\Big] \\
& + \overline{p}\Big[d^a - \overline{\theta} - \frac{1}{2}(q^a_{ac}(\overline{\theta}) + q^a_{cp}(\overline{\theta}))\Big]\Big(q^a_{ac}(\overline{\theta}) - q^a_{cp}(\overline{\theta})\Big) \\
& + \overline{p}\frac{1}{2}\Big[3\Big(q^b(q^a_{ac}(\overline{\theta}))\Big)^2 - \Big(1 - 2\frac{\underline{p}}{\overline{p}}\Big)\Big(q^b(q^a_{cp}(\overline{\theta}))\Big)^2\Big].
\end{aligned}
$$

The expression is too complex to allow for deriving general rules as to when it is positive. This is why we have been using graphical representations (under Mathematica) for particular, simple, parameter values. Examples show that the expression is positive for large sets of the parameters. This proves that contracts inducing active collusion can dominate collusion-proof contracts, hence Proposition 3. For instance, for $d^a - \underline{\theta} = 2$, $d^a - \overline{\theta} = 1$, $m^b = 2$, $\underline{p} = 0.4$ and $s = 0.2$, all quantities are positive, and the best contract inducing active collusion is preferred. For information, the approximated values are: $q^a_{cp}(\underline{\theta}) = 1.75258$, $q^a_{cp}(\overline{\theta}) = 1.42857$, $q^a_{ac}(\underline{\theta}) = 1.68317$, $q^a_{cp}(\overline{\theta}) = 1.35314$, $q^b(q^a_{cp}(\underline{\theta})) = 0.824742$, $q^b(q^a_{cp}(\overline{\theta})) = 0.857143$, $q^b(q^a_{ac}(\underline{\theta})) = 0.831683$, $q^b(q^a_{cp}(\overline{\theta})) = 0.864686$, and the value taken by $\Delta^{(ac)-(cp)}$ is approximately 0.494234. With the parameters above, active collusion is preferred whenever $s \leq 0.41309$ (in approximate value), and collusion-proofness is preferred otherwise.

To the contrary, if one changes the value of s to 0.6, the best collusion-proof contract is preferred. The approximated values are then: $q^a_{cp}(\underline{\theta}) = 1.50685$, $q^a_{cp}(\overline{\theta}) = 0.938511$, $q^a_{ac}(\underline{\theta}) = 1.00917$, $q^a_{cp}(\overline{\theta}) = 0.703364$, $q^b(q^a_{cp}(\underline{\theta})) = 0.547945$, $q^b(q^a_{cp}(\overline{\theta})) = 0.718447$, $q^b(q^a_{ac}(\underline{\theta})) = 0.697248$, $q^b(q^a_{cp}(\overline{\theta})) = 0.788991$, and the value taken by $\Delta^{(ac)-(cp)}$ is approximately -0.34522.

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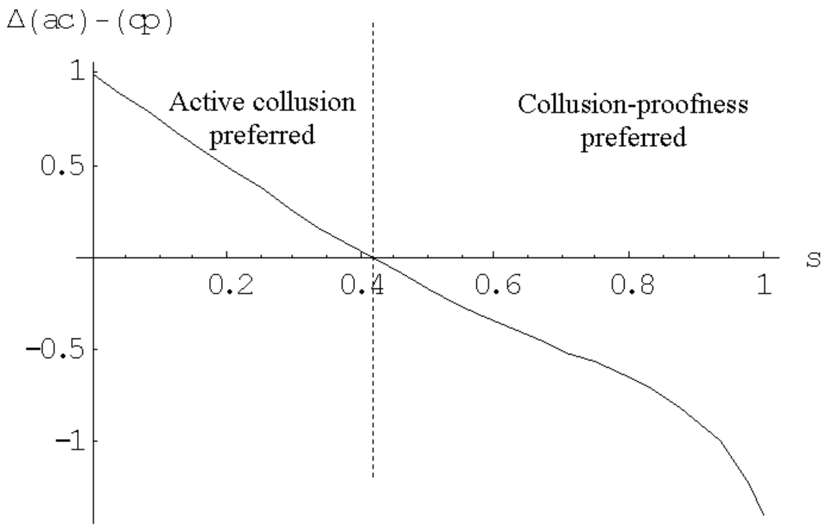

FIGURE 1: Optimal regulatory contracts depending on product substitutability s

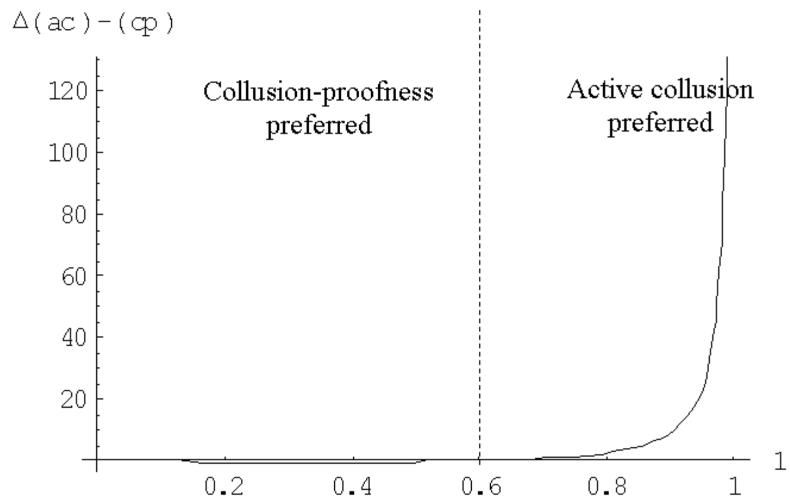

FIGURE 2: Optimal regulatory contracts depending on probability that F^a be efficient, $\underline{p}$