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**Indirect Taxation is Superfluous  
under Separability and Taste  
Homogeneity : A Simple Proof**

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I have benefited from discussions with Stéphane Gauthier and Bernard Salanié, and from the comments of Anthony Atkinson. After I had completed this note, Robin Boadway and Emmanuel Saez draw my attention to a recent paper of Kaplow (April 2004) which is based on similar ideas.

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# Indirect taxation is superfluous under separability and taste homogeneity: a simple proof

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## **Résumé**

Quand tous les consommateurs ont les mêmes préférences pour les biens, indépendamment de leur offre de travail, et quand l'impôt sur le revenu peut être non linéaire, la taxation indirecte est inutile. Cette note fournit une démonstration élémentaire de ce résultat due à Atkinson et Stiglitz (1976).

## **Abstract**

Indirect taxation is of no use when nonlinear income taxation is available in an economy where everyone has the same taste for goods: an elementary proof of this result, due to Atkinson and Stiglitz (1976), is provided.

JEL: H21, H31

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In a celebrated paper, Atkinson and Stiglitz (1976) present a situation where, when the government has access both to direct and indirect taxation, indirect taxation is of no use. This result holds in an economy with constant returns to scale, so that the production prices  $p$ , measured in efficient labor units, are fixed exogeneously. The typical consumer  $h$  supplies labor  $L^h$  and consumes goods  $x_i^h$ ,  $i = 1, \dots, n$ . Her tastes are represented with a utility function  $U(V(x_1, \dots, x_n), L, h)$ , separable between goods and labor, and such that  $V$  is identical across consumers. The consumers differ both through the shapes of  $U$  and their productivities  $w^h$ , both characteristics that the government does not observe at the individual level, while it sees total before tax income  $Y^h = w^h L^h$ .

The government can tax income, using a non linear schedule:  $R(wL)$  denotes after tax income. It can also impose a linear tax  $t$  on consumption, so that the vector of consumption prices is  $q = p + t$ . Total government income after tax and transfers (income tax may be negative, some goods can be subsidized) is<sup>1</sup>

$$G = \int_h [w^h L^h - R(w^h L^h) + (q - p) \cdot x^h].$$

In this setup, indirect taxes are of no use. While the result is intuitive, the formal proofs available in the literature are rather involved and mostly consider a situation where the government has *a priori* chosen the optimal nonlinear income tax. Mirrlees (1976) gives the first complete proof under regularity assumptions for the smoothness of the optimal nonlinear optimal tax schedule, when the agents differ along a single dimension of heterogeneity. Christiansen (1984) works in the tax reform tradition. Starting with an optimal income tax schedule in the absence of indirect taxes, under strong regularity assumptions, he shows that there is no gain at the first order in introducing indirect taxes. Konishi (1995) is the first to note that optimality of the income tax is not necessary for the result.

Here, under separability and taste homogeneity, the inutility of indirect taxes is derived from a simple global argument that does not rely on differentiability and applies whether the labor choices are continuous or discrete, without any *a priori* optimality property of the income tax schedule which, for instance, may involve bunching, and with no restriction on the shape of the set of agents' characteristics<sup>2</sup>.

**Assumption 1.** *The utility function  $V(x)$  is continuous and exhibits non satiation.*

**Theorem 1.** *Let  $(t, R)$  be any government policy such that, for all  $h$ ,  $U^h$  the utility level attained by agent  $h$*

$$U^h = \max_{x,L} \{U(V(x_1, \dots, x_n), L, h) \mid (p + t) \cdot x = R(w^h L)\}$$

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<sup>1</sup>Under constant returns to scale, this equality serves also as the feasibility condition.

<sup>2</sup>Proposition 1, p.8, of Kaplow (2004) is very close to my Theorem 1.

is well defined.

Then there exists another government policy  $(0, \bar{R})$ , with no indirect taxes, with the following properties

1. All the agents in the economy have the same utility under  $(0, \bar{R})$  as under  $(t, R)$ ;
2. All the agents supply the same amount of labor in the two allocations;
3. Government revenue is higher under  $(0, \bar{R})$ :

$$\bar{G} = \int_h w^h L^h - \bar{R}(w^h L^h) \geq G,$$

and the inequality is strict provided that  $t$  is not proportional to  $p$  and there is a non negligible set of income levels at which  $V$  is continuously differentiable.

**Proof:** From the point of view of the agents in the economy, a government policy is equivalent to a set  $\mathcal{V}$  of  $(\tilde{v}(Y), Y)$ , with  $Y$  in  $\mathbb{R}_+$ , where  $\tilde{v}(Y)$  is the utility derived from consumption when before tax income is  $Y$ :

$$\tilde{v}(Y) = \max_x \{V(x_1, \dots, x_n) \mid (p+t) \cdot x = R(Y)\}.$$

Indeed, consumer  $h$  chooses her labor supply by maximizing  $U(v, L, h)$  for  $(v, w^h L)$  in  $\mathcal{V}$ .

The new allocation, designated with an upper bar, is obtained by keeping  $\mathcal{V}$  unchanged. Since the agents have access to exactly the same menu  $(\tilde{v}(Y), Y)$  as before, they choose the same labor supply. The menu can be supported with a more efficient choice of prices and incomes than initially. Define

$$\bar{x} = \arg \min_x \{p \cdot x \mid V(x) \geq \tilde{v}(Y)\},$$

and  $\bar{R}(Y) = p \cdot \bar{x}$ . The quantity  $\bar{R}(Y)$  is equal to the value of the expenditure function  $e(p, v(Y))$  (see e.g. Mas-Colell, Whinston, and Green (1995), p.59): under Assumption 1, the maximum of  $V(x)$  on the budget set  $p \cdot x \leq \bar{R}(Y)$  is attained at  $\bar{x}$ . By definition  $\bar{R}(Y)$  is smaller than  $p \cdot x$ , where  $x$  is the consumption under the reference allocation, provided  $p+t$  is not proportional to  $p$  and  $V$  is differentiable at  $x$ . The result then follows since:

$$G = \int_h w^h L^h - p \cdot x^h < \int_h w^h L^h - \bar{R}(w^h L^h) = \bar{G}.$$

■

*Remark 1.* It is a minimal requirement to ask that the consumers' programs have a solution. This is the case whenever after tax income is a continuous function of before tax income, and labor supply is bounded. Under an additional mild regularity requirement, removing indirect taxes allows a strict Pareto improvement for every agent when  $\bar{G}$  is larger than  $G$ . Assume, for instance, that at the bar allocation, aggregate labor supply varies continuously with  $dr$  where  $dr$  is a uniform increase in after tax income ( $\bar{R}(Y)$  is changed into  $\bar{R}(Y) + dr$  for all  $Y$ ): then reducing  $\bar{G}$  for a positive  $dr$  at the margin makes every one better off. Then, *under the homogeneity and separability assumption, any incentive compatible allocation is Pareto dominated by another incentive compatible allocation without indirect taxes.* This property generalizes the result of Mirrlees (1976), p.337, who showed that Pareto efficiency required no (local) indirect taxes.

*Remark 2.* The homogeneity of tastes of course is a crucial assumption: take an economy where the agents have the same productivities and the same inelastic labor supplies and therefore the same before tax incomes, but differ only by their tastes of commodities. In such a case, given the instruments available to the government, transfers can only occur through indirect taxes.

*Remark 3.* The argument is easily extended to a a situation where utility is separable into groups of goods, in the spirit of Mirrlees (1976) p.338. Suppose that the typical utility function is of the form  $U(V(x), W(y), L, h)$ , where the subutilities  $V$  and  $W$  are identical across agents. Consider a government policy  $(t_x, t_y, R)$ . Then if  $t_x$  is not proportional to  $p_x$  or  $t_y$  is not proportional to  $p_y$ , there is a Pareto improving policy where the relative consumption and production prices within groups coincide (but typically not between groups:  $t_x/p_x \neq t_y/p_y$ ).

*Remark 4.* The result holds, provided that the preferences for commodities are homogeneous, *conditional on the information  $I$  on which the income tax is based*, as a close look at the proof indicates (replace  $Y$  with  $I$  in the argument). This may allow to relax somewhat the separability assumption. For example, consider an economy where the decision to work is either 0 (non employment) or 1 (full time work), productivity differs across agents, but the preferences for goods are homogeneous respectively for the non employed and for the employed (the difference stemming from incidental expenses associated with work, such as clothing, transportation, food, etc.). Then no indirect taxes are needed, provided income tax (or subsidy) can be based both on the employment status and on income when employed.

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