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Real Market Concentration Through Partial Acquisitions

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Abstract

We study horizontal partial acquisitions in an oligopolistic industry in the absence of synergies. Contrary to existing results, we …nd that a dominant shareholder may choose to acquire shares in a competitor although the aggregate pro…t of the group of …rms under his control, and even the greater group of …rms in which he has a stake, is reduced. This is due to a "favorite" e¤ect: after the acquisition, the dominant shareholder will favor the …rm in which he eventually holds the relatively higher share to the detriment of shareholders of the other …rms. For this reason, a block of shares can be bought at a discount when the value of the …rm of the initiator decreases post acquisition. Moreover, we show that the existence of initial silent toeholds in rivals enhances the incentive for a dominant shareholder to buy shares in other …rms in the industry, whereas controlling ones may discourage them.

Keywords: horizontal partial acquisitions, real market concentration, dominant shareholder, minority shareholders, silent interests.

JEL classi…cation: D23, D43, G32, G34.

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Résumé

Nous étudions les stratégies de prises de participation dans une industrie oligopolistique en l'absence de synergies. Contrairement aux résultats existants, nous trouvons qu'un actionnaire dominant peut avoir intérêt à acquérir des actions dans une entreprise concurrente alors même que le pro…t joint du groupe d'entreprises qu'il contrôle, et même du groupe élargi d'entreprises dans lesquelles il a une part, diminue. Ce résultat s'explique par un e¤et «favori» : après l'acquisition, l'actionnaire dominant favorisera l'entreprise dans laquelle il détient la part relativement la plus élevée au détriment des actionnaires des autres entreprises. Pour cette raison, il est même possible qu'un bloc de titres soit acheté avec une prime négative dans le cas où la valeur de l'entreprise acquéreuse diminue à la suite de l'acquisition. De plus, nous montrons que la propriété initiale d'intérêts silencieux rend plus pro…tables et encourage ainsi les prises de participation ultérieures dans l'industrie, alors que des intérêts contrôlants peuvent être de nature à les décourager.

Mots clés : prises de participations horizontales, concentration de marché, actionnaire dominant, actionnaires minoritaires, intérêts silencieux.

Classi…cation JEL : D23, D43, G32, G34.

1 Introduction

Evidence suggests that in many continental european countries shareholding power is highly concentrated in the hand of large shareholders (Becht and Röell, 1999). Even in the United States, the presence of a large voting block is not uncommon in listed companies (Becht and Mayer, 2002). When the remaining equity is in the hand of small shareholders, their passivity coupled with the voting rules (quorum) in the annual meetings enable the dominant shareholder to control strategic decisions such as the level of production, investments and acquisitions without a strict majority (50%) of equity or voting rights. Even when they do not control …rms, the presence of blockholders may still have an impact on their policy and pro…ts. Although partial acquisitions are common in corporate life, they received relatively little attention from economists. In particular, several questions remain largely unanswered: do they have causes and consequences similar to mergers and acquisitions of entire companies? How is the toehold in the target determined? What is their impact on the individual and the overall pro…t of the …rms involved in the transaction, and their competitors? How is each category of shareholders' wealth a¤ected? How do existing toeholds a¤ect the future acquisition policy? In this article, we address these questions in the particular case of block trades between large shareholders of …rms from the same industry.

Traditionally, the economic literature concentrates on the consequences of restructurings for the industry (the …rms concerned, their competitors and consumers). In the case of horizontal acquisitions, the classical question since Williamson (1968) of the trade-o¤ between the synergy gains and the possible consumers' welfare loss due to higher prices emerges. Horizontal mergers that claim to achieve economies of scale may actually intend to increase market power. However, Salant, Switzer and Reynolds (1983) (hereafter SSR) show that whereas mergers without economies of scale do result in an increase in prices and pro…ts of the overall oligopolistic industry, they are not pro…table for the merged entity. This surprising result stems from the reaction of the rivals which, in a model à la Cournot, increase their production to pro…t from the higher prices subsequent to the restriction of the quantity o¤ered by the merged entity. Eventually, the restructuring is not pro…table for the merging …rms which therefore have no incentive to initiate the transaction ex ante. Reitman (1994) has extended this result to partial ownership arrangements: if the industry's overall pro…t increases following a partial acquisition, the bene…ciaries are the rival companies which bene…t

from a positive externality (increase in prices) whereas the …rms involved in the transaction lose, which removes any incentive. Similarly, Farrell and Shapiro (1990) show that a marginal increase in an initial toehold is pro…table only when a cost reduction compensates the negative e¤ect of a less aggressive behavior of the companies involved (production restriction) and may be socially desirable in that case.

The …nance literature is on the other hand concerned with the …nancial conditions of mergers and acquisitions, in particular the split of the takeover gain between shareholders of companies involved in the restructuring. Parallel to the surprising result of SSR, Grossman and Hart (1980, 1981) showed in pioneer articles that tender o¤ers that increase shareholders's wealth may actually not occur in equilibrium because of the "free-rider problem": expecting an increase in the value of their shares post acquisition, small stockholders do not sell and therefore no o¤er can succeed. The positive role of large shareholders in this context has been emphasized since the seminal paper of Shleifer and Vishny (1986). However the presence of large shareholders, may also create problems, as they may pursue "private interests" at the expense of minority shareholders (what Johnson, La Porta, Lopez-de Silanes and Shleifer (2000) have called "tunnelling", this term describing "the transfer of resources out of a company to its controlling shareholder"). Thus, the con‡ict between large controlling and weak small shareholders has recently been recognized as probably as important as the traditional agency problem between managers and shareholders.

This article establishes a bridge between these two literatures. We study horizontal partial acquisitions in an oligopolistic industry producing a homogeneous good in the absence of synergies. For the reasons put forward by SSR and Reitman (1994), we …nd that the overall pro…t of the companies involved and the consumers' surplus decrease following the transaction. However, unlike these authors, we show the existence of equilibria with e¤ective acquisition of shares. This partial share acquisition may be controlling or silent. This result comes from the fact that the strategic decision to acquire equity in a rival is made by the dominant shareholder of the bidding …rm. Maximizing his own wealth, this shareholder may engage in privately pro…table transactions at the expense of minority shareholders of his …rm in the case where his toehold is relatively small, or to the detriment of minority shareholders of the target if the reverse is true, or even to the detriment of both when he already has large silent stakes outside the two …rms. The ownership structure of the bidder and the

target turns out to be a key variable: the higher the toehold of the dominant shareholder in the company he initially controls, the better the protection of minority shareholders of this …rm. We also …nd that a shareholder with initial silent toeholds in rivals has more incentive to further make partial acquisitions. This comes from the larger value of these outside interests following the price increase in the whole industry as competition lessens. On the other hand, the impact of initial controlling toeholds is ambiguous. Accounting for the e¤ects in the industry is essential for a good understanding of the acquisition process and the consequences for shareholders' wealth. Integrating productive and …nancial decisions reintroduces an incentive for horizontal equity acquisitions, and initial toeholds in rivals enhance this incentive.

The model is presented in section 2. In section 3, we study the equilibrium in the good market for given toeholds. In section 4, we solve the acquisition game under two di¤erent bidding possibilities, analyze the conditions (quantities and price) of the transactions and their wealth consequences for shareholders.

2 The model

We consider an oligopolistic market with n …rms producing a homogeneous good. The demand is $P(X) = 1 ¡ X$ where X represents the total quantity produced in the industry and P the corresponding price. Each company $i = 1; :::; n$ produces X_i and $X = \sum_{j=1}^{n} X_j$. The marginal cost is supposed to be constant and is normalized to zero for all …rms: Under these assumptions, pro…ts are $¦_i = P(X)X_i$ for all i:

Total equity capital is normalized to 1 for all …rms. Each company is controlled by a dominant shareholder. The dominant shareholder chooses the production of the …rm(s) he controls (possibly with less than 50% of the shares) and maximizes his own wealth. Initially, the dominant shareholder of any company i holds no equity in other companies $j \neq i$ in this market, except for A, the dominant shareholder of …rm 1, who already owns stakes $®_j$ in …rms $j > 2$ in addition to his controlling share $®_1 \in]0; 1]$ in …rm 1. These toeholds $®_j$ held in competitors may be controlling or non controlling ("silent"). In the case of a controlling share $®_k$, A initially chooses the production level X_k of …rm k controlled in addition to X_1: Controlling shareholders compete à la Cournot.

We look at the case where shareholder A (and only him by assumption) may buy, directly

or indirectly, all or part of the block of shares $\bar{\alpha}_2$ ($\bar{\alpha}_2 \in]0;1]$) initially held by B, the dominant shareholder of firm 2. The remaining equity of 2 is supposed to be widely held. In the case of an indirect acquisition, firm 1 rather than A buys the shares, still under A's initiative. After this acquisition, he may therefore control company B if he (or firm 1) becomes the largest shareholder. In this case, he decides also on the production level X_2. If after the acquisition of stock by A (or firm 1), B remains the largest shareholder in firm 2, B keeps controlling X_2: Let $\alpha_2 \in]0;1]$ be the share of equity acquired in firm 2 by A directly or indirectly from B; in the case of an indirect acquisition, $\beta\alpha_2 \in]0;1]$ represents the amount acquired by firm 1; $\alpha_2 p$ (resp. $\beta\alpha_2 p$) is the total amount paid for the transaction by A (resp. by firm 1). After the acquisition, the wealth of shareholder A is therefore $\alpha_1 \pi_1 + \alpha_2(\pi_2 - p) + \sum_{j>2} \alpha_j \pi_j$.[1] The wealth of B becomes $(\bar{\alpha}_2 - \alpha_2)\pi_2 + \alpha_2 p$ after a direct acquisition, or $(\bar{\alpha}_2 - \beta\alpha_2)\pi_2 + \beta\alpha_2 p$ after an indirect one. The other shareholders' wealth solely depends on the profits of their company. The objective of shareholders is to maximize the value of their financial wealth.[2]

The timing of the global game is therefore the following: A (or firm 1) acquires equity in 2 in the first stage of the game, determining α_2 and p; in the second stage, dominant shareholders choose simultaneously the production level of the firm(s) they control given α_2: Next section examines the Nash equilibrium of this production game. The first stage of the game, the acquisition of equity by A in firm 2, is presented in section 4 where we analyze the existence of bidding Nash equilibria.

3 The equilibrium in the real sector

In this section, we analyze the equilibrium in the real sector after the bidding game has taken place. In the case of a successful acquisition, A has acquired (directly or indirectly) a toehold α_2 in firm 2 (in addition to the shares α_j he already owns in firms $j > 2$) that may or may not give him the control of this firm. At this stage the amount $\alpha_2 p$ paid by A to shareholder B is a sunk cost and therefore does not influence the production decisions of any firm. The toeholds α_2 and α_j affect the production decisions of A in two ways. When setting

[1] In the case of an indirect acquisition, A's final wealth is: $\alpha_1[\pi_1 + \beta\alpha_2(\pi_2 - p)] + \sum_{j>2} \alpha_j \pi_j = \alpha_1 \pi_1 + \alpha_2(\pi_2 - p) + \sum_{j>2} \alpha_j \pi_j$ where share α_2 of A in 2 is $\alpha_1 \beta\alpha_2$:

[2] This important hypothesis differs from most existing economic literature where managers maximize profits in the interest of shareholders in general and do not consider the dominant shareholder's interest.

the output of the …rm(s) he controls (at least …rm 1), A takes into account the direct e¤ect of the output level of the controlled …rm(s) on its (their) own pro…ts, but also the indirect e¤ect on the pro…ts of the companies in which he holds an interest, silent or controlling. Let C represent the set of …rms (the "core" group) controlled by A, and G the set of …rms in which A holds a stake, controlling or not (the "greater" group). The number of …rms in which A has a silent stake is denoted n_s and n_c is the number of …rms controlled by A (i.e. the cardinal of C).

Thus we have a Cournot game with $(n - n_c)$ dominant shareholders choosing the output of the …rm they control in order to maximize its pro…ts, and A choosing the production of the n_c …rms controlled in order to maximize his wealth $\sum_{i=1}^{n} \alpha_i \pi_i$.

Obviously, the quantities produced in equilibrium depend on n_c: It turns out that they also depend on the ratio of the sum of the silent interests held by A in his rivals denoted α_s to his highest controlling stake denoted α_c. Let ρ represent this ratio:[3]

$$\rho = \frac{\alpha_s}{\alpha_c} \text{ with } \alpha_s = \sum_{i \notin C} \alpha_i \text{ and } \alpha_c = Max\{\alpha_i; i \in C\}.$$

The following proposition gives the quantities and the pro…ts of the …rms in equilibrium.

Proposition 1 Equilibrium quantities and pro…ts are given by the following equations (1)-(4). Among the n_c …rms controlled by A, only the …rm with the highest weight may have positive production and pro…ts ($X_i^* = \pi_i^* = 0$ for $i \in C$ and $\alpha_i < \alpha_c$).

$$X_j^*(\alpha_1; \alpha_2; ...; \alpha_n) = \frac{1}{n - n_c + 1 + \max(1 - \rho; 0)} \text{ for } j \notin C \quad (1)$$

$$\sum_{i \in C} X_i^*(\alpha_1; \alpha_2; ...; \alpha_n) = \frac{\max(1 - \rho; 0)}{n - n_c + 1 + \max(1 - \rho; 0)} \quad (2)$$

$$\pi_j^*(\alpha_1; \alpha_2; ...; \alpha_n) = \frac{1}{[n - n_c + 1 + \max(1 - \rho; 0)]^2} \text{ for } j \notin C \quad (3)$$

$$\sum_{i \in C} \pi_i^*(\alpha_1; \alpha_2; ...; \alpha_n) = \frac{\max(1 - \rho; 0)}{[n - n_c + 1 + \max(1 - \rho; 0)]^2}. \quad (4)$$

All the proofs are given in the appendix.

[3] For instance, when A holds shares in only three …rms, say 1, 2 and 3, and A controls …rm 1 only; $C = \{1\}$, $\alpha_s = \alpha_2 + \alpha_3$, $\alpha_c = \alpha_1$ and $\rho = \frac{\alpha_2 + \alpha_3}{\alpha_1}$; when A controls two …rms, say 1 and 2, $C = \{1; 2\}$, $\alpha_s = \alpha_3$, $\alpha_c = Max(\alpha_1; \alpha_2)$ and $\rho = \frac{\alpha_3}{Max(\alpha_1; \alpha_2)}$; when A controls all three …rms, $C = \{1; 2; 3\}$, $\alpha_s = 0$ and $\rho = 0$.

3.1 The di¤erent e¤ects at play

Proposition 1 re‡ects the restructuring that A operates within the group C of the n_c …rms he controls given the reaction of his rivals, including the …rms in which he has silent interests (belonging to G but not in C), in equilibrium. Di¤erent mechanisms or "e¤ects" are actually at play.

The "favorite" e¤ect

Proposition 1 implies $X_i^{¤} = ¦_i^{¤} = 0$ for $i \in C$ and $®_i < ®_c$: This means that among the n_c …rms controlled by A, only the …rm which has the highest weight ($®_c$) in his portfolio (say …rm 1) possibly remains active. A chooses to stop the production of all other controlled companies. In other words, A favors the controlled …rm in which he has the highest stake. This "favorite" e¤ect obviously harms shareholders of the other controlled …rms. Controlling acquisitions are in this model formally equivalent to a complete merger of the n_c …rms into at most one company. Indeed, only in the case where the highest controlling stake (say $®_1$) is greater than the sum of A's silent interests $®_s$ does …rm 1 remains active.

The "Hara Kiri" e¤ect

When the highest controlling stake $®_c$ (say $®_1$) is lower than the sum of the silent interests in competitors $®_s$, i.e. $½ > 1$, A also shuts down the corresponding controlled …rm (say 1) in order to concentrate the production and pro…t where his stake is the highest: all the controlled production units are closed. This "Hara Kiri" e¤ect bene…ts shareholders of all rivals to the detriment of those of all controlled companies, in particular …rm 1.

In this case, partial acquisitions are formally equivalent to a complete merger of $n_c + 1$ production units, and the level of the pro…ts of the $(n - n_c)$ …rms which remain active is equivalent to the pro…ts of the merged entity in SSR's model.[4]

The "silent" e¤ect

Conversely, when his highest controlling toehold in a …rm (say $®_1$) is greater than the total of silent stakes in rivals $®_s$ (i.e. $½ < 1$), A would like to concentrate the production in the controlled …rm and shut down the …rms in which he has a silent stake but, being a

[4] This extreme result is due to the linearity of the model; similar but less radical e¤ects would be obtained in other frameworks. With quadratic costs, production restructuring would still imply reallocations of production between …rms, but far less cases of shutting down a production unit.

minority shareholder, does not have the power to do so. On the contrary, A internalizes the negative consequences of the production of the active controlled company on the value of his silent interests, and therefore restricts its output to limit the negative externalities on the value of his outside interests ("silent" e¤ect).

Rivals (including competitors in which A has a stake) react to this restriction of the controlled …rm (say 1) output by increasing their production to take advantage of the price increase. The higher the value of ½, the higher the restriction of 1's output, the stronger its competitors' reaction and the higher their pro…ts.

This discussion sheds some light on the crucial role played by ½ (the ratio of the global silent interests held by A in his rivals to his highest controlling stake) in equilibrium. Actually, keeping the production of non controlled …rms constant (out of equilibrium), toeholds always make shareholder A (i.e. the group C of …rms under his control) less "aggressive": he restricts the global production of the controlled group C to increase its pro…t. For given production levels of non controlled …rms, the global pro…t of group C would increase. But as mentioned above, controlling shareholders of rivals outside of C do not remain passive and react to the restriction of the output by C and the higher resulting price. This is the strategic e¤ect underlined by Fudenberg and Tirole (1984). We are in their classical case of strategic substitutes. In the Cournot model, when a …rm is less aggressive, other …rms respond by more aggressiveness.

Figures[5][6] 1 (resp. 2) illustrate the aggregate best-response curve of …rms in group C (resp. in the "greater" group G), and the aggregate best response curve of their competitors. The Cournot equilibrium lies at the intersection of the best-response curves. In either case, when ½ increases, the best-response curve of the group (C or G) moves to the left (from the Cournot best response curve for ½ = 0 at one extreme to that of the merged entity of the SSR model

[5]Figure 1 is built as follows. Suppose the production of group C, X is given. The best response of a …rm i veri…es $X_i = 1 - X - \sum_{j \neq C} X_j$: For given X, one can calculate the game equilibrium among the other …rms, i.e. the response of all the other …rms to the choice of X by shareholder A. This response is given by: $Y = \sum_{j \neq C} X_j = \frac{n - n_c}{n - n_c + 1}(1 - X)$: The best response of group C controlled by A is then: $2X = Max(0; 1 - \frac{n - n_c + ½}{n - n_c} Y)$:

[6]Using the same method, we can determine the response of the …rms outside the enlarged group, Y to a quantity X^G. We have $Y = \frac{n - n_c - n_s}{n - n_c - n_s + 1}\left[1 - X^G\right]$ and $X^G = Max\{\frac{(2 - \tau)n_s + 1 - [(2 - \tau)n_s + \tau]Y}{(2 - \tau)n_s + 2}; \frac{n_s(1 - Y)}{n_s + 1}\}$ where $\tau = \frac{n - n_c + ½}{n - n_c}$.

when $\theta > 1$ at the other extreme). Thus both groups C and G become less aggressive as θ increases.

Insert Figure 1 and Figure 2.

3.2 Welfare effects of partial acquisitions

Starting from an initial situation characterized by $\alpha_s; \alpha_c$ and n_c; we now examine the consequences of an increase in one of A's stakes (say α_2 in firm 2) on the equilibrium profits of the whole industry (and welfare), of the "greater" group G of firms in which A has a positive stake, and of group C of firms controlled by A.

Are partial acquisitions anticompetitive?

From Proposition 1, it is clear[7] that an increase in a controlling share (say α_2) does not change the equilibrium as long as it remains below the highest controlling share ($\alpha_2 \cdot \alpha_c$); and simply results in a continuous decrease in θ when it becomes the highest controlling share. Thus, any partial acquisition that increases a toehold in an already controlled company benefits (or at least does not harm) consumers: the larger the controlling shareholder, the higher the welfare; this is due to a reverse "silent" effect: his controlling stake becoming relatively higher than his outside interests, A becomes more aggressive since the negative impact on silent stakes counts less relatively.

On the contrary, as a silent share (say α_2) goes up while remaining silent, θ increases continuously. Therefore any silent partial acquisition is harmful for consumers: as seen before, a higher silent stake (say α_2) encourages the acquirer (say shareholder A) to restrict the production in the companies he controls to protect the value of his higher outside interests (the "silent" effect).

Finally, the overall effect of a partial acquisition which turns a silent toehold into a controlling stake is a priori ambiguous. Indeed, as α_2 keeps increasing, it eventually reaches the control threshold $\frac{\bar{}}{2}$; at this level, θ drops from $\frac{\alpha_s + \frac{\bar{}}{2}}{\alpha_c}$ to $\frac{\alpha_s}{Max(\alpha_c; \frac{\bar{}}{2})}$ and the number of controlled firms goes up from n_c to $n_c + 1$: The simultaneous decrease in θ and increase

[7] The quantity produced by the entire industry (X^{ind}), and therefore welfare, is a decreasing function of n_c and θ. In our Cournot model with a constant unit cost, a restriction of the total supply results in an increase in the profit of the whole industry and a decrease in total welfare.

in the number of …rms under A's control a¤ect total production (and welfare) in opposite directions. However, calculation shows that it actually drops. Acquiring control of a new company leads A to shut an additional …rm; this direct e¤ect outweighs the simultaneous indirect e¤ect (higher production of the active controlled …rm following the decrease in outside interests). Overall, increasing the number of …rms under the control of A is always anticompetitive. Moreover, although production (and welfare) increase as the share in the newly controlled company gets larger, it never reaches the level prevailing before the acquisition of control: the highest possible silent stake in …rm 2 (close to $\frac{\bar{}_2}{2}$) is always less harmful than the highest controlling share ($®_2 = 1$):

Figure 3 illustrates how industry output (or welfare) varies with the toehold $®_2$ held by A in …rm 2, keeping other stakes constant.

Insert Figure 3.

Who gains, who loses?

Obviously, silent partial acquisitions always bene…t non controlled companies whereas group C loses (as well as the greater group G if the number n_s of companies in which A has a silent interest is small enough). The reverse is true for operations which increase already controlling shares. As a consequence, partial acquisitions which do not change n_c however lead to a production restructuring and a reallocation of production and pro…ts within the greater group.

Controlling acquisitions also clearly bene…t companies that remain out of the control of A. The production and pro…t of the group of $(n_c + 1)$ …rms under A's control is however always lower than the sum of the output of the n_c …rms initially controlled by A and the output of the newly controlled …rm before the acquisition (the price increase is outweighed by the quantity decrease). Therefore, the pro…ts of the group of …rms involved in partial acquisitions generally decrease.

However, A may gain from these operations for two reasons. First, the pro…tability of the acquisition depends on the price paid for the toehold. Second, A's wealth ($\sum_i ®_i ¦_i$) is not proportional to the pro…t of the group ($\sum_i ¦_i$): When the toeholds are asymmetric, shareholder A may well become wealthier whereas the total pro…t of the …rms in which he has a stake decreases. For example, if group C loses while rivals (including …rms in which A

has a silent interest) gain, the loss incurred by A in C may be more than compensated by the increase in the value of his silent stakes (for $\beta_c < \beta_s$). The next section examines these questions.

4 The acquisition game

In this section, we study the …rst stage of the global game in which shareholder A may acquire a share of the equity of …rm 2. We showed in the previous section that the strategic choices of output in the second stage depend on the level of the toehold acquired and its nature, controlling or not.

In practice, there exist multiple ways to acquire a share or the entire equity of a …rm: private negotiation, snapping up shares on the stock market, block trades, tender o¤ers, etc...

We explore two hypotheses. In the …rst sub-section, shareholder A makes a "take it or leave it" block o¤er to B. In this case, A acquires a stake in 2 directly. Since he controls …rm 1's productive and …nancial decisions, its M&A policy in particular, A may have …rm 1 acquire a share of equity in …rm 2. This case of an indirect acquisition is studied in the second sub-section in which …rm 1 (rather than shareholder A) makes a "take it or leave it" block o¤er to B. At this stage of the analysis, there is no reason why these two modes of acquisition should be equivalent. A priori neither dominates from A's point of view.

4.1 Shareholder A makes a direct block o¤er to B

Let us suppose that shareholder A makes a block o¤er (quantity, price) to the dominant shareholder of 2, namely B, who accepts or rejects it.[8] Proposition 2 describes the equilibrium.

Proposition 2 The optimal share β_2^{α} maximizes the joint wealth of A and B. Depending on the value of $\bar{\theta}_2$, acquisitions are hostile, friendly or do not occur. In:

² zone I ($\bar{\theta}_2 \cdot \bar{\theta}_1$): A acquires a controlling share in …rm 2 with any $\beta_2^{\alpha} \in]\frac{\bar{\theta}_2}{2}; \bar{\theta}_2]$ (hostile acquisitions);

[8] The results obtained under this hypothesis can be generalized to the case where shareholders A and B negotiate at the …rst stage a block sale. See Charléty, Fagart and Souam (2003).

- zone II ($\bar{\alpha}_I < \bar{\alpha}_2 \cdot \bar{\alpha}_{II}$): A acquires no share in firm 2 ($\beta_2^* = 0$) when his silent interest β_s is small, and is indifferent between any share $[0; \frac{\bar{2}}{2}]$ when it is high (no acquisition or friendly acquisitions);

- zone III ($\bar{\alpha}_{II} < \bar{\alpha}_2 \cdot \bar{\alpha}_{III}$): A acquires a silent interest in firm 2, $\beta_2^* = 2\bar{\alpha}_2 + \beta_s - \beta_c(n - n_c)$ (friendly acquisitions);

- zone IV ($\bar{\alpha}_{III} < \bar{\alpha}_2$): any silent interest $\beta_2^* \in [\beta_c - \beta_s; \frac{\bar{2}}{2}]$ in firm 2 is an equilibrium when $\beta_s > 0$. A is indifferent between acquiring a controlling or a silent interest in firm 2 when $\beta_s = 0$ and $\beta_2^* \in [\beta_c; \bar{\alpha}_2]$ (friendly acquisitions).

The values taken by the three thresholds $\bar{\alpha}_I$, $\bar{\alpha}_{II}$ and $\bar{\alpha}_{III}$ are given by the following equations:

$$\bar{\alpha}_I = Max(\beta_c; \beta_s)\frac{2(n - n_c + 1 - \frac{\beta_s}{Max(\beta_c; \beta_s)}) + 1}{[n - n_c + 1 - \frac{\beta_s}{Max(\beta_c; \beta_s)}]^2} \quad (5)$$

$$\bar{\alpha}_{II} = \beta_c\frac{[n - n_c - \frac{\beta_s}{\beta_c}]}{2} \text{ when } \beta_s \cdot \beta_c \text{ and } \bar{\alpha}_{II} = 1 \text{ otherwise.} \quad (6)$$

$$\bar{\alpha}_{III} = \beta_c\frac{[n - n_c + 1 - \frac{2\beta_s}{\beta_c}]}{2} \text{ when } \beta_s \cdot \beta_c \text{ and } \bar{\alpha}_{III} = 1 \text{ otherwise.} \quad (7)$$

Figure 4 presents the different zones as a function of $\bar{\alpha}_2$ and β_s for a given value of β_c.

Insert Figure 4.

As already suggested at the end of the third section, the decision criterion for an acquisition is not the sum of the profits of firms 1 and 2 involved in the operation, but the joint profit of shareholders A and B. Thus, even though the joint profit of 1 and 2 combined always decreases, A and B joint wealth may increase when their respective controlling shares are "not too close". Only when β_c (= β_1 by assumption) and $\bar{\alpha}_2$ are equal and $\beta_s = 0$ is the joint shareholders's wealth proportional to the joint firms' profit, in which case we go back to SSR's results. As long as the joint wealth of A and B increases following the acquisition (possibly at the expense of other shareholders), A and B are able to share this gain in this game where other shareholders play no role.

Remembering that among the firms in which A holds a controlling stake, only the firm in which A has the highest interest may remain active in equilibrium (due to the "favorite" and

the "Hara Kiri" e¤ects), we look more closely at the equilibrium depending on the relative size of α_s:

Small silent interest (α_c , α_s , 0)

Let us begin with the case where A has a small initial silent interest in …rms outside of 1 and 2.

² In zone I, as $\bar{}_2$ is small relatively to α_c, in order to maximize the joint wealth of A and B, …rm 2 should be closed, the increase in the value of the stake in 1 far outweighing the decrease in the value of the stake in 2. Therefore, A buys enough equity in 2 to get its control[9] and favors …rm 1 afterwards. This also bene…ts his silent stakes. The acquisition can be considered as hostile, the "favorite" e¤ect always playing in favor of the previously controlled …rm with the highest interest (…rm 1).

² In zone II, no acquisition takes place: the weights of A and B in their original …rms are too close, and α_s too small, to make any acquisition pro…table. Indeed, the joint wealth of A and B is closely related to the sum of the pro…ts of …rms 1 and 2, which would decrease following the acquisition. This generalizes SSR's results.

² Zones III and IV correspond to friendly acquisitions, in the sense that …rm 2's pro…t increases at the expense of the minority shareholders of the …rms initially controlled by A.

In zone III, A acquires a silent interest in …rm 2, $\bar{}_2$ is greater than α_c; but not enough to make it pro…table for A to close …rm 1 (both …rms keep a distinct control and positive productions, the "silent" e¤ect is at play).

In zone IV where $\bar{}_2$ is high enough, maximizing A and B joint wealth implies closing …rm 1. Thus A acquires a su¢ciently high interest in …rm 2 and stops production in all the previously controlled …rms; the "Hara Kiri" e¤ect is at play. When $\alpha_s = 0$; he is actually indi¤erent between controlling …rm 2 or not. In fact, when A controls …rm 2, he maximizes its value exactly as B would if he remained the controlling shareholder. Their interests are completely aligned. When $\alpha_s > 0$, A is no longer indi¤erent between acquiring a controlling or a silent stake in …rm 2. Indeed, when A controls …rm 2, he takes into account the negative impact of …rm 2's output on the value of his silent interests, and therefore chooses a level of production for …rm 2 that is lower than what would prevail under B's control. As a consequence, when $\bar{}_2$ is high, the overall wealth of A and B would actually be smaller under

[9] In the production game, the number of …rms controlled by A increases by one (n_c is replaced by $(n_c + 1)$).

A's control rather than B's. Acquiring a silent share is a commitment not to decrease 2's value afterwards, which explains the rather counter-intuitive result that silent acquisitions dominate.

High silent interest ($\alpha_s > \alpha_c$)

In this case, since A's silent interest is high compared with his highest initial controlling share ($\alpha_1 = \alpha_c$), the output of 1 is initially null in order to favor competitors in which he has a high stake. A silent acquisition in …rm 2 would of course not a¤ect this equilibrium in the real sector (α_s would become even higher). Therefore, no silent partial acquisition possibly increases the joint wealth of A and B. Since A acquires shares at a price which re‡ects the initial value of …rm 2 in our take-it or leave-it game, and nothing really changes after a silent acquisition, A is obviously indi¤erent between acquiring a silent interest or nothing.

On the other hand, when A takes the control of …rm 2, he considers the impact of 2's production level on competitors in which he holds silent interests and reduces the production of 2. A controlling acquisition in …rm 2 thus reduces the value of …rm 2 and increases the value of competitors. This dilution of 2 is pro…table, from the point of view of A and B together, only when $\bar{\alpha}_2$ is low enough (zone I). This explains why we only have two zones depending on the value of $\bar{\alpha}_2$ as illustrated by Figure 4:[10]

Prices, block premia and minority shareholders' wealth

In this take-it or leave-it framework, the level of the o¤er (when it exists) made by A is such that it leaves B's wealth unchanged.[11] Therefore, when the acquisition is hostile (leading to a decrease in 2's pro…ts), the price paid for each share acquired by A must include a premium compared with the initial price in order to compensate B for the lower value of his remaining holdings post acquisition. On the other hand, when the acquisition is friendly (leading to an increase in 2's value), the price actually displays a discount compared with the previously prevailing price, since B bene…ts from the increased value of his remaining holdings post acquisition. Of course, in the case where A acquires the entire block $\bar{\alpha}_2$; there

[10] The fact that …rm 1 which initially produces nothing is active on the market for corporate control may seem strange; however the extreme result concerning the output is once again due to the linearity of the model; another interpretation is that …rm 1 is as an "empty shell" serving shareholder A's interests.

[11] In equilibrium, $\alpha_2^{\star} p(\alpha_2^{\star}) + (1 - \alpha_2^{\star}) \pi_2^{\text{post acquisition}} = \bar{\alpha}_2 \pi_2^{\text{before acquisition}}$: In a negotiation game, the same acquisitions would occur but the increase in A and B joint wealth would be shared.

is no premium or discount.[12]

Whereas partial acquisitions always bene…t A and B, they always harm minority shareholders of either …rm 1 (friendly acquisitions) or …rm 2 (hostile acquisitions). As already noticed, they bene…t shareholders from other companies. The asymmetry of the weights of A and B in their original …rms is crucial for partial acquisitions to be (privately) pro…table.

The incentive to acquire partial interests and the role of initial toeholds

The incentive to acquire a toehold obviously depend of the total number of …rms in the industry: the …ercer is competition in this model à la Cournot, the less frequent are acquisitions (the size of zone II increases with the number of …rms in the industry). As increasing the number n_c of …rms controlled by A (leaving $®_c$ unchanged) is equivalent to closing one …rm, more control in this sense not only reduces competition and welfare, but also enhances the incentives to make further acquisitions.

More control can as well be interpreted as an increase in the controlling toehold. For small values of $®_c$ ($®_c < ®_s$; which corresponds to a high silent interest), no …rm belonging to group C produces in equilibrium. Thus a small increase in $®_c$; as long as the silent interest remains high, plays no role in either the production or the acquisition policy ($¯_I$ is independent of $®_c$ and there are only two zones). For higher values of $®_c$ ($®_c \geq ®_s$; which corresponds to a small silent interest), $¯_I$, $¯_{II}$ and $¯_{III}$ increase with $®_c$. As the controlling interest gets larger, zone I (hostile acquisitions) increases and zones III and IV (friendly acquisitions) shrink. In zone I, shareholder A acquires the control of …rm 2 and favors …rm 1 afterwards. Therefore, the larger shareholder A, the more pro…table the acquisition for him, and the higher the incentive to acquire. On the other hand, in zones III and IV where $®_c$ is relatively small w.r.t $¯_2$; the acquisition favors …rm 2 at the expense of 1. Thus, friendly acquisitions are less pro…table for A and occur less frequently when $®_c$ increases.

Whereas controlling toeholds have an ambiguous e¤ect on the incentive to make additional partial acquisitions, silent toeholds always encourage them. It can easily be shown that zone I and zones III and IV together (where shareholder A acquires shares in …rm 2) are larger when $®_s$ increases. Acquisitions are more frequent for …rms which already hold silent interests in rivals; this is obviously due to the fact that being anti-competitive, they

[12] Premia would of course be more frequent in a negotiation game and would in particular be present when the whole block $¯_2$ is acquired.

always bene…t …rms outside of these operations. Thus, the higher A's silent interest in rivals, the higher the incentive to acquire new toeholds.[13]

4.2 Shareholder A makes an indirect block o¤er to B

In this sub-section, we study the case of an indirect acquisition[14] in which …rm 1 (rather than shareholder A) makes, under the control of A, a "take it or leave it" block o¤er to B. In this framework, θ_2 represents the toehold acquired by …rm 1, and α_2 the share acquired by shareholder A indirectly through his holdings in 1: Proposition 3 states that only zone I (hostile acquisitions) and zone II (no partial acquisition) remain in this context.

Proposition 3 In the indirect acquisition game of shares from the dominant shareholder of …rm 2 by …rm 1 controlled by A :

- if $\bar{\theta}_2 \leq \bar{\theta}_I^{ind}$ (zone I), any $\theta_2^* \in [\frac{\bar{\theta}_2}{2}; \bar{\theta}_2]$ is an equilibrium, …rm 1 acquires the control of …rm 2;
- if $\bar{\theta}_2 > \bar{\theta}_I^{ind}$ (zone II), $\theta_2^* = 0$ (…rm 1 acquires no share in …rm 2). The threshold $\bar{\theta}_I^{ind}$ is given by:

$$\bar{\theta}_I^{ind} = Max(1; \frac{\alpha_s}{\alpha_c}) \frac{2(n - n_c + 1 - \frac{\alpha_s}{Max(\alpha_c; \alpha_s)}) + 1}{[n - n_c + 1 - \frac{\alpha_s}{Max(\alpha_c; \alpha_s)}]^2}. \quad (8)$$

Let as before $\alpha_c = \alpha_1$: A's …nal wealth after an indirect acquisition is: $\alpha_1[\pi_1 + \theta_2(\pi_2 - p)] + \sum_{j>2} \alpha_j \pi_j = \alpha_1 \pi_1 + \alpha_2(\pi_2 - p) + \sum_{j>2} \alpha_j \pi_j$ where share α_2 of A in 2 is $\alpha_1\theta_2$: Therefore, in the production game (p is a sunk cost), if A controls 2 indirectly, he always shuts it down since his stake in 2 is by construction lower than his holding in 1 ($\alpha_2 = \alpha_1\theta_2 \leq \alpha_1\bar{\theta}_2 < \alpha_1$): The "favorite" e¤ect is at play. On the other hand, if A holds (indirectly) α_2 silent, although $\alpha_1 > \alpha_2$, A restricts the output of …rm 1 due to the "silent" e¤ect.[15]

For these reasons, ex ante, it is never pro…table for A to acquire a silent interest in 2 indirectly: the loss incurred on his stake in 1 being by construction relatively large compared

[13] It can also be shown that the size of the toehold acquired in 2 is non-decreasing in α_s in the sense that when α_s increases, there always exist a higher new equilibrium α_2^*:

[14] By assumption, the initial toeholds of A in other …rms are owned directly.

[15] This comes directly from Proposition 1.

to the gain made on his indirect share in 2. Only controlling indirect acquisitions which increase …rm 1's operating pro…ts possibly occur.[16]

Whereas such indirect partial acquisitions always harm minority shareholders of 2 (shareholder B being indi¤erent in this take-it-or-leave game), they do not necessarily bene…t minority shareholders of …rm 1. First, although it is true that …rm 1's equilibrium operating pro…ts Π_1^{α} always increase, …rm 1's equilibrium value $\Pi_1^{\alpha} - p\theta_2^{\alpha}$ may decrease after the acquisition: …rm 1 pays the pre-acquisition value of B's stake which is worth nothing afterwards. If A has no silent interest, A's …nal wealth is proportional to …rm 1's value; acquisitions that bene…t A also create value for minority shareholders of …rm 1. But when A has silent stakes in rivals, the increase in their value may compensate a loss in the value of A's interest in 1. This is likely to be true when α_s is high and $\alpha_c = \alpha_1$ low. The pro…tability threshold[17] β_I^{ind} for the minority shareholders of …rm 1 is lower than shareholder A's threshold γ_I^{ind}. Therefore, for $\gamma_2 \in [\beta_I^{ind}; \gamma_I^{ind}]$, A initiates controlling partial acquisitions that also expropriate the minority shareholders of …rm 1.

Moreover, it can be checked that $\gamma_I^{ind} > \gamma_I$: controlling indirect partial acquisitions are more likely than controlling direct acquisitions by A. This is of course due to the fact that A pays only a fraction α_1 of the amount o¤ered to B (and also gets α_1 of …rm 1's operating pro…ts) in indirect operations but receives the total increase in value of his silent stakes as in direct acquisitions. Therefore, indirect acquisitions may dominate for low values of γ_2: On the other hand, for high values of γ_2, indirect acquisitions are never pro…table for A. Direct acquisitions will be preferred by A in that case.

Finally, the e¤ect of higher controlling initial interests is no more ambiguous, contrary to the case of direct acquisitions: they always discourage further acquisitions. As silent initial toeholds encourage them, they play in opposite directions. This is due to the fact that acquisitions are always hostile.

[16] Proposition 3 derives from Proposition 2 where γ_2 is simply replaced by $\alpha_1\gamma_2$; it is then clear that zones III and IV can never emerge (zones III and IV correspond to values of $\gamma_2 > \alpha_1$; which can never hold when γ_2 is replaced by $\alpha_1\gamma_2$):

[17] Simple calculus shows that $\beta_I^{ind} = \left(1 - \frac{\alpha_s}{Max(\alpha_s;\alpha_c)}\right)\frac{2(n - n_c) + 3 - \frac{2\alpha_s}{Max(\alpha_c;\alpha_s)}}{\left[n - n_c + 1 - \frac{\alpha_s}{Max(\alpha_c;\alpha_s)}\right]^2} = Max[\alpha_c(\alpha_c - \alpha_s); 0] - \gamma_I^{ind}$: This threshold decreases with $\frac{\alpha_s}{\alpha_c}$.

5 Conclusion

In a standard framework à la Cournot, we show that partial acquisitions, both controlling and silent, always lead to a decrease in the overall pro…t of the companies involved in the transaction, an increase in their competitors' pro…t and have a negative impact on the consumer surplus. In this sense, our results generalize to the case of partial acquisitions the …ndings of SSR that, in the absence of cost savings, mergers are not pro…table operations.

Still, we do obtain equilibria where a large shareholder gains from acquiring a block of shares, silent or controlling, from a shareholder in a rival …rm. This result is partly due to the assumption that a blockholder may actually control a …rm when the remaining shares are dispersed among atomistic shareholders. In our model, the relative separation of ownership and control gives rise to "favoritism" within the group of …rms in which the dominant shareholder has a stake. When choosing the production level of the …rms he controls, he "favors" the …rm in which he holds the relatively highest share. If, for example, the controlling block in the target is relatively small compared with his initial toeholds in the other companies, he closes the new …rm under his control after the acquisition. Such a hostile acquisition implies the payment of a premium to the selling shareholder, and a decrease in the value of the target following the transaction. On the contrary, the acquisition of a relatively high share in a rival will lead the acquiring shareholder to close his own company at the expense of its minority shareholders and to the bene…t of the target. For such a friendly operation, the block may be bought at a discount since, if he keeps a stake in the company, the seller will pro…t from its increase in value. When the acquirer initially holds high silent toeholds in rivals, he may even sacri…ce the controlled companies to the bene…t of competitors only. Thus the presence of large shareholders generally tends to protect minority shareholders of their company, as already pointed out in other articles.[18] Another interesting result concerns the choice between a direct acquisition of shares and an indirect acquisition through the controlled company (a stock "pyramid", see Faccio and Lang, 2000). In particular, even in the absence of …nancial constraints, we show that an indirect controlling acquisition may be preferred when the acquirer already owns silent toeholds in rivals and the targeted block is relatively small. On the other hand, relatively large blocks should be acquired directly according to our …ndings. The initial ownership structure of …rms and the

[18] See for example Shleifer and Vishny (1986).

presence of initial stakes in rivals[19] therefore play important roles in determining the share of the target acquired, the nature of the toehod (controlling or silent), the type of acquisition (direct or indirect) and its feasibility for the dominant shareholder.

An important feature of our model is the linkage between productive and …nancial decisions. As for Brander and Lewis (1986) in another context (the choice of debt), the functioning of …nancial markets has an important impact on the economic performance of the real sector.

Another implication of our analysis is that in particular, when the share acquired in the target by the dominant shareholder is higher than his toehold in his original …rm, the minority shareholders of the latter are expropriated. In most countries, …nancial markets regulation aims at protecting the interest of target …rms' shareholders and overlooks the interest of bidding …rms' shareholders. This …nding may explain, at least partly, the well documented poor …nancial performance of acquiring …rms in takeovers.

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6 Appendix

6.1 Proof of proposition 1

We look for the Cournot-Nash equilibrium of the production game between …rms outside of A's control and group C of …rms under A's control.

(i) Any independent …rm k 2 C simply maximizes its pro…t given by:

$$(1 - \sum_{i=1}^{n} X_i)X_k: \tag{9}$$

Thus its best response quantity is:

$$X_k = \frac{Max(1 - \sum_{i=1; i \neq k}^{n} X_i; 0)}{2}: \tag{10}$$

Let $X = \sum_{i \in C} X_i$; represent the total quantity produced by the …rms controlled by A and de…ne $\overline{X} = \sum_{i \notin C} X_i$ the total quantity produced by the …rms outside group C of …rms controlled by A. Equation (10) can be rewritten as follows:

$$2X_k = Max(1 - X - \overline{X} + X_k; 0); k \in C. \tag{11}$$

For X given; since all independent …rms respond in the same way, by symmetry we can write:

$$X_k = \frac{1 - X - \overline{X}}{n - n_c} = \frac{1 - X}{n + 1 - n_c}: \tag{12}$$

(ii) We now determine the behavior of group C. A's wealth can be written as:

$$(1 - \sum_{i=1}^{n} X_i)(\sum_{j=1}^{n} \alpha_j X_j): \tag{13}$$

For $j \in C$, A chooses the production X_j of …rm j and $X = \sum_{j \in C} X_j$ in order to maximize his wealth:

$$(1 - X - \overline{X})(\sum_{j \in C} \alpha_j X_j + \sum_{i \notin C} \alpha_i X_i): \tag{14}$$

Given X, A's wealth is maximized for $X_j = 0$ when $\alpha_j < \max(\alpha_i; i \in C) \times \alpha_c$ and $\sum_{j \in C} X_j = X$. It can then be rewritten as $(1 - X - \sum X)(\alpha_c X + \frac{\sum_{i \in C} \alpha_i}{n - n_c} \sum X) = \alpha_c(1 - X - \sum X)(X + \frac{\gamma}{n - n_c} \sum X)$ where $\gamma = \frac{\sum_{i \in C} \alpha_i}{\alpha_c}$:

The best response of shareholder A is thus $2X = \max(1 - \frac{n - n_c + \gamma}{n - n_c} \sum X; 0)$:

(iii) Finally, equilibrium quantities are given by:

$$X = \frac{1 - \gamma}{n - n_c + 2 - \gamma} \text{ and } \sum X = \frac{n - n_c}{n - n_c + 2 - \gamma} \text{ when } \gamma \leq 1$$

$$X = 0 \text{ and } \sum X = \frac{n - n_c}{n - n_c + 1} \text{ when } \gamma > 1:$$

The equilibrium price is then given by $p^e = \frac{1}{n - n_c + 1 + \max(1 - \gamma; 0)}$ and A's wealth is $\alpha_c \frac{\max(1 - \gamma; 0) + \gamma}{[n - n_c + 1 + \max(1 - \gamma; 0)]^2}$:

6.2 Proof of proposition 2

Assume that A makes a block offer $(p; \alpha_2)$ to B: If B rejects this offer, his wealth is $\bar{\omega}_2 - \pi_2(\alpha_1; 0; ...; \alpha_n)$. If he accepts, he gets $(\bar{\omega}_2 - \alpha_2) - \pi_2(\alpha_1; \alpha_2; ...; \alpha_n) + \alpha_2 p$: He will therefore accept provided that the proposed price is such that $(\bar{\omega}_2 - \alpha_2) - \pi_2(\alpha_1; \alpha_2; ...; \alpha_n) + \alpha_2 p \geq \bar{\omega}_2 - \pi_2(\alpha_1; 0; ...; \alpha_n)$. Anticipating this behavior, A offers the smallest price compatible with this condition, such that his participation constraint is binding. Shareholder A's wealth is then given by:

$$\begin{aligned} & \sum_{i \in 2} \alpha_i - \pi_i(\alpha_1; \alpha_2; ...; \alpha_n) + \alpha_2 - \pi_2(\alpha_1; \alpha_2; ...; \alpha_n) - \alpha_2 p \\ = & \sum_{i \in 2} \alpha_i - \pi_i(\alpha_1; \alpha_2; ...; \alpha_n) + \bar{\omega}_2 - \pi_2(\alpha_1; \alpha_2; ...; \alpha_n) - \bar{\omega}_2 - \pi_2(\alpha_1; 0; ...; \alpha_n); \end{aligned}$$

and he will propose $\alpha_2 \in [0; \bar{\omega}_2]$ which maximizes the above expression. Define $L(\alpha_2) = \sum_{i \in 2} \alpha_i - \pi_i(\alpha_1; \alpha_2; ...; \alpha_n) + \bar{\omega}_2 - \pi_2(\alpha_1; \alpha_2; ...; \alpha_n)$. In equilibrium the optimal share α_2 maximizes $L(\alpha_2)$ in $[0; \bar{\omega}_2]$ since $\bar{\omega}_2 - \pi_2(\alpha_1; 0; ...; \alpha_n)$ does not depend on α_2:

The optimum among the controlling interests

Consider the case where A controls firm 2. As n_c denotes the number of firms controlled by A before the partial acquisition in firm 2, the number of controlled firms becomes $n_c + 1$.

α_c is the highest initial controlling share ($\alpha_c = Max(\alpha_i; i \in C \setminus \{2\})$) and α_s the sum of A's silent interests. Thus $\gamma = \frac{\alpha_s}{Max(\alpha_c; \alpha_2)}$. L can be nicely expressed:

$$L(\alpha_2) = \frac{Max(\alpha_c; \alpha_2; \alpha_s) + (\bar{\alpha}_2 - \alpha_2)\delta Max(1 - \gamma; 0)}{[n - n_c + Max(1 - \gamma; 0)]^2}$$
$$\text{with } \delta = 1 \text{ if } \alpha_2 > \alpha_c \text{ and } \delta = 0 \text{ otherwise.}$$

We want to show that the optimal value of L is:

$$L_c^* = \frac{Max(\bar{\alpha}_2; \alpha_c; \alpha_s)}{[n - n_c + 1 - \frac{\alpha_s}{Max(\bar{\alpha}_2; \alpha_c; \alpha_s)}]^2}. \quad (15)$$

(15) obviously holds when $Max(\bar{\alpha}_2; \alpha_c; \alpha_s) = \alpha_c$ or $Max(\bar{\alpha}_2; \alpha_c; \alpha_s) = \alpha_s$. Indeed, in these two cases, L(:) does not depend on α_2, either because $\delta = 0$ or because $\gamma \geq 1$. As a consequence, when $\bar{\alpha}_2 \leq Max(\alpha_c; \alpha_s)$, the optimal solution α_2^* is any share in $]\frac{\bar{\alpha}_2}{2}; \bar{\alpha}_2]$. We thus assume in the following that $\bar{\alpha}_2 > Max(\alpha_c; \alpha_s)$.

When $\alpha_2 \leq Max(\alpha_c; \alpha_s)$, L(:) does not depend on α_2. Conversely, when $\alpha_2 > Max(\alpha_c; \alpha_s)$, we have $\delta = 1$; and $\gamma < 1$ so L can be written $\frac{\alpha_s + \bar{\alpha}_2(1 - \frac{\alpha_s}{\alpha_2})}{[n - n_c + 1 - \frac{\alpha_s}{\alpha_2}]^2}$. L(:) does not depend on α_2 when $\alpha_s = 0$. When $\alpha_s > 0$; the derivative of L with respect to α_2 has the sign of:

$$\bar{\alpha}_2[n - n_c + 1 - \frac{\alpha_s}{\alpha_2}] - 2[\alpha_s + \bar{\alpha}_2(1 - \frac{\alpha_s}{\alpha_2})] = \bar{\alpha}_2[n - n_c - 1 + \frac{\alpha_s}{\alpha_2}] - 2\alpha_s$$
$$\propto \bar{\alpha}_2[n - n_c - 1] - \alpha_s \text{ as } \alpha_2 \leq \bar{\alpha}_2:$$

This latter expression is positive since $\bar{\alpha}_2 > \alpha_s$ and $n - n_c > 2$. Hence L(:) increases with respect to α_2, and the optimal share, constrained to be higher than $Max(\alpha_c; \alpha_s)$, is $\bar{\alpha}_2$. We have to prove now that the optimal share is actually not smaller than $Max(\alpha_c; \alpha_s)$ when feasible (that is when $\frac{\bar{\alpha}_2}{2} < Max(\alpha_c; \alpha_s) < \bar{\alpha}_2$). When $\alpha_s \geq \alpha_c$, L(:) is continuous w.r.t. α_2, so the optimal share is $\bar{\alpha}_2$. When conversely $\alpha_c > \alpha_s$, L(:) is discontinuous at $\alpha_2 = \alpha_c$, equates $L(\alpha_c) = \frac{\alpha_c}{[n - n_c + 1 - \frac{\alpha_s}{\alpha_c}]^2}$ for any $\alpha_2 \leq \alpha_c$. Moreover $L(\alpha_c^+) < L(\alpha_c^j)$. However, it is easy to show that $L(\alpha_c)$ increases with respect to α_c whenever $n - n_c > 2$, so finally $L(\alpha_c) \leq L(\bar{\alpha}_2)$ as $\bar{\alpha}_2 > \alpha_c$.

Comparing controlling and silent acquisitions

Assume now that A does not control firm 2. Define $\alpha_s = \sum_{i \neq 2;\, i \in C} \alpha_i$ the sum of the initial silent interests held by shareholder A in other firms. Thus $\gamma = \frac{\alpha_2 + \alpha_s}{\alpha_c}$ and $\alpha_2 \in [0; \frac{\bar{\alpha}_2}{2}]$ so

$L(\alpha_2)$ can be written as:

$$L_s(\alpha_2) = \frac{\alpha_c Max(1 - \frac{\alpha_2 + \alpha_s}{\alpha_c}; 0) + \bar{}_2 + \alpha_s}{[n - n_c + 1 + Max(1 - \frac{\alpha_2 + \alpha_s}{\alpha_c}; 0)]^2}: \quad (16)$$

² As a …rst step, note that $L_s(\alpha_2)$ does not depend on α_2 in case of high silent interests that is when $\alpha_s > \alpha_c$. Comparing $L_s(0)$ and $L_c^¤$ in that case gives

$$L_s(0) - L_c^¤ = \frac{\bar{}_2 + \alpha_s}{[n - n_c + 1]^2} - \frac{Max(\bar{}_2; \alpha_s)}{[n - n_c + 1 - \frac{\alpha_s}{Max(\bar{}_2; \alpha_s)}]^2}: \quad (17)$$

When $\bar{}_2 \cdot \alpha_s$, $L_s(0) - L_c^¤ > 0$ if and only if $\bar{}_2 > \bar{}_I$. When $\bar{}_2 > \alpha_s$; $L_s(0) - L_c^¤ > 0$:

So when $\bar{}_2 > \bar{}_I$ (i.e. in zone II for high silent interest), A prefers acquiring any silent interest to a controlling one.

² In case of small silent interests (i.e. when $\alpha_c > \alpha_s$), $L_s(:)$ is a continuous function of α_2. Taking the derivative of $L_s(\alpha_2)$ w.r.t. α_2 in $[0; \alpha_c - \alpha_s]$ shows that L_s increases with α_2 if and only if $\alpha_2 \cdot \alpha_2^¤ = 2\bar{}_2 + \alpha_s - \alpha_c[n - n_c]$. When $\alpha_2^¤ \cdot 0$ ($\bar{}_2 \cdot \bar{}_{II}$), the optimal silent interest is thus zero (zones I and II). Conversely, when $0 < \alpha_2^¤ \cdot \alpha_c - \alpha_s$ ($\bar{}_{II} < \bar{}_2 \cdot \bar{}_{III}$) the optimal silent interest is $\alpha_2^¤$ (zone III): Finally, if $\alpha_2^¤ > \alpha_c - \alpha_s$, ($\bar{}_2 > \bar{}_{III}$) $L_s(\alpha_2)$ does not depend on α_2. A is thus indi¤erent between any silent share higher than $\alpha_c - \alpha_s$ (zone IV).

² Let us now show that A always prefers controlling in zone I under small silent interests. As a technical point, it is easy to state that the function $f(x) = \frac{x}{[n - n_c + 1 - \frac{\alpha_s}{x}]^2}$ increases with respect to $x > \alpha_s$ whenever $n - n_c \geq 2$. Thus the di¤erence $L_s(0) - L_c^¤$ is such that:

$$L_s(0) - L_c^¤ \cdot \frac{\alpha_c + \bar{}_2}{[n - n_c + 2 - \frac{\alpha_s}{\alpha_c}]^2} - \frac{\alpha_c}{[n - n_c + 1 - \frac{\alpha_s}{\alpha_c}]^2}. \quad (18)$$

This last expression is negative if $\bar{}_2 \cdot \bar{}_I$, that is in zone I.

² Finally we have to show that within the zones II, III and IV, A prefers acquiring silent shares rather than controlling ones if $\alpha_s > 0$.

In zone IV, $\bar{}_2 > \bar{}_{III} > \alpha_c$, a straighforward calculus shows that $L_s^¤ - L_c^¤$ increases with α_s and equates 0 for $\alpha_s = 0$ (in this case, A is indi¤erent between controlling …rm 2 and acquiring a silent interest in it).

Note that whenever $n - n_c \geq 3$, $L_s(0) - L_c^¤$ is strictly positive at $\bar{}_2 = \alpha_c$. Moreover, $L_s(0) - L_c^¤$ increases with respect to α_c when $\alpha_c \cdot \bar{}_2 < \bar{}_{III}$ so is strictly positive in zones

II and III. As a consequence, whenever $\bar{\beta}_2 \geq \beta_c$, $L_s(\beta_2^{\natural}) \geq L_s(0) > L_c^{\natural}$ and the optimal silent interest dominates the optimal controlling one.

Lastly, in zone II when $\bar{\beta}_2 < \beta_c$, we have $L_s(0) - L_c^{\natural} > 0$: Indeed, it is easy to see that $L_s(0) - L_c^{\natural}$ increases w.r.t. $\bar{\beta}_2$ and equals zero when $\bar{\beta}_2 = \bar{\beta}_1$. So in this case, acquiring a silent interest is better than controlling firm 2.

6.3 Proof of proposition 3

Assume now that shareholder A buys shares through firm 1 in which he has the highest controlling interest. When firm 1 buys $\delta_2 \in [0; \bar{\beta}_2]$ shares from shareholder B, control is obtained when $\delta_2 > \frac{\bar{\beta}_2}{2}$. Moreover, for given quantities $X_1; X_2; :::; X_n$, the wealths of shareholders A and B are given by:

$$W_A = \sum_{i \neq 2} \alpha_i \pi_i(X_1; X_2; :::; X_n) + \alpha_c \delta_2 f \pi_2(X_1; X_2; :::; X_n) - p g$$

$$W_B = (\bar{\beta}_2 - \delta_2) \pi_2(X_1; X_2; :::; X_n) + \delta_2 p:$$

Recall that A owns α_i shares of firm i, with $\alpha_2 = \alpha_c \delta_2$ shares of firm 2. Taking into account the equilibrium in the real sector, we obtain:

$$W_A = \sum_i \alpha_i \pi_i^{\natural}(\alpha_1; \alpha_2; \alpha_3; :::; \alpha_n) - \alpha_2 p \text{ with } \alpha_2 \leq \alpha_c \bar{\beta}_2$$

$$\text{and } W_B = (\bar{\beta}_2 - \frac{\alpha_2}{\alpha_c}) \pi_2^{\natural}(\alpha_1; \alpha_2; \alpha_3; :::; \alpha_n) + \frac{\alpha_2}{\alpha_c} p:$$

As in the preceding proof, assume that firm 1 offers to buy $\delta_2 = \frac{\alpha_2}{\alpha_c}$ shares at a price p. If B rejects the offer, he gets $\bar{\beta}_2 \pi_2(\alpha_1; 0; \alpha_3; :::; \alpha_n)$. If he accepts it, his wealth becomes W_B. He therefore accepts the offer if:

$$(\bar{\beta}_2 - \frac{\alpha_2}{\alpha_c}) \pi_2^{\natural}(\alpha_1; \alpha_2; \alpha_3; :::; \alpha_n) + \frac{\alpha_2}{\alpha_c} p \geq \bar{\beta}_2 \pi_2^{\natural}(\alpha_1; 0; \alpha_3; :::; \alpha_n):$$

Anticipating this behavior, A offers B a pair quantity-price such that his participation constraint is binding, involving a wealth:

$$W_A = \sum_{i \neq 2} \alpha_i \pi_i^{\natural}(\alpha_1; \alpha_2; \alpha_3; :::; \alpha_n) + \bar{\beta}_2 \alpha_c \pi_2^{\natural}(\alpha_1; \alpha_2; \alpha_3; :::; \alpha_n)$$
$$- \bar{\beta}_2 \alpha_c \pi_2^{\natural}(\alpha_1; 0; \alpha_3; :::; \alpha_n) g:$$

The optimal share $\alpha_2 \in [0; \alpha_c \bar{\beta}_2]$ maximizes the above expression. Finally, we obtain the same problem as in proposition 2, where $\bar{\beta}_2$ is replaced by $\bar{\beta}_2 \alpha_c$.

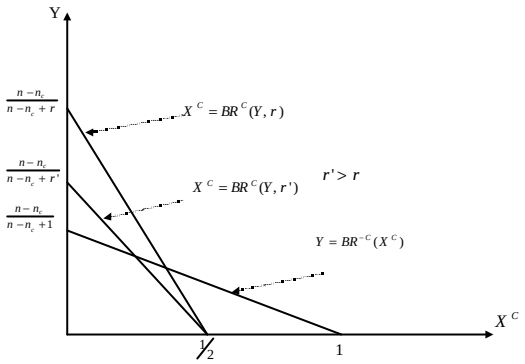

Figure 1: Aggregate best-response curve of group C.

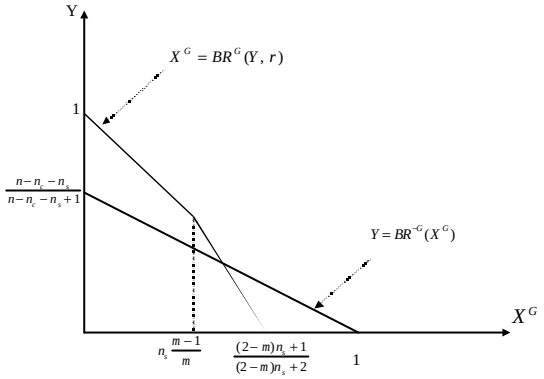

Figure 2: Aggregate best-response curve of group G.

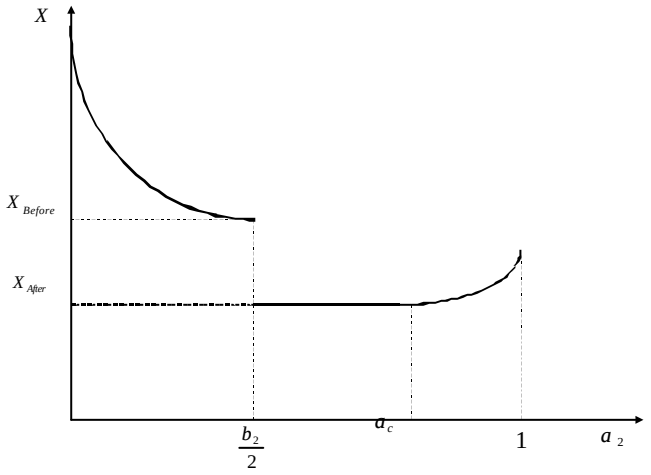

Figure 3: Variation of the total output with the toehold ®$_2$:

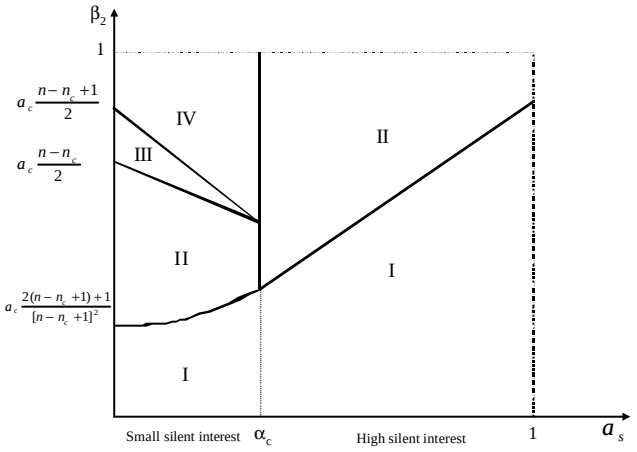

Figure 4: Zones with or without acquisitions.