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Innovate AND Imitate ? : Dynamic Innovation, Patents, and Costly Imitation*

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Innovate AND Imitate?: Dynamic Innovation, Patents, and Costly Imitation*

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Abstract

We build a continuous time dynamic model of sequential innovation with stochastic imitation in which firms can simultaneously undertake original and imitative R&D, to compare the performance of (i) a system with patent protection (with or without licenses) and (ii) a system with costless full spillovers of innovation. We confirm previous findings that if innovation is complementary (i.e. chances of innovating increase with the number of firms, for a given R&D expenditure) and sequential (i.e. innovations arrive one at a time), firms' profits might increase in the second scenario. However, we show that from a social perspective the trade-off between socially costly (but more efficient) imitative R&D and welfare improving reduction of innovators' ex post monopoly rents does not always balance in favour of weak patents, and determine the conditions under which firms' preferences with regards to the intellectual property protection system coincide with social objectives. We also show that the presence of patents in a context with endogenously determined stochastic imitation reduces the socially optimal pace of original innovation, and interpret the observed cross-licensing agreements and weaker patent protection observed in relatively more dynamic industries as the natural outcome of a dynamic R&D game.

Résumé

On présente un modèle dynamique en temps continu d'innovation séquentielle avec de l'imitation stochastique, dans lequel les firmes peuvent entreprendre simultanément des activités de R&D originale et imitative. On compare la performance de (i) un système de protection de l'innovation à travers des brevets (avec ou sans licences) et (ii) un système sans brevets où la diffusion complète des innovations est immédiate et non coûteuse. Le modèle confirme certains résultats dans la littérature indiquant que lorsque l'innovation est complémentaire (c'est à dire la probabilité d'innover s'accroît avec le nombre de firmes, pour des dépenses totales de R&D données) et séquentielle (c'est à dire les innovations arrivent une par une), les profits des firmes peuvent augmenter dans le deuxième cas de figure. Cependant, on montre aussi que, du point de vue du bien-être social, l'arbitrage existant entre le coût social de la recherche imitative (malgré son efficacité accrue) et la dissipation des rentes de monopole ex post ne se résout pas toujours en faveur de brevets faibles, et on détermine les conditions pour que les préférences des firmes par rapport au système de protection intellectuelle soient compatibles avec celles de la société. Finalement, on montre que la présence de brevets (dans un contexte où l'imitation est stochastique et déterminée de façon endogène) réduit le rythme socialement optimal d'innovation originale, et on interprète les accords de licences croisées et la moindre utilisation des brevets observée dans les industries relativement plus dynamiques comme l'issue naturelle d'un jeu dynamique de R&D.

1. Introduction

Patent protection is often presented in the economic literature as the system that solves optimally the trade-off between providing incentives for innovation and allowing diffusion of new ideas. In exchange of legal constraints on potential imitators (that raise the innovator's *ex post* rents, thus providing incentives for incurring the cost of potential innovation), the inventor discloses a very precise[1] description of the idea (that competitors can read and build upon as long as they do not breach the constraints, thus diffusing new knowledge and preventing wasteful duplication of effort).

However, as noticed by Scotchmer (1999) and others, patent protection provides inventors with *ex post* benefits that are independent from the amount of *ex ante* expenditures in research and development (R&D), which is suboptimal from a social perspective.[2] In other words, the standard trade-off in patent protection debates is one between R&D incentives and *ex post* monopoly rents: while increasing firms' rents provides higher incentives for innovation, and thus is socially desirable, *ex post* production at monopoly prices and quantities is socially costly.[3]

Additionally, patents also provide incentives for another, controversial, kind of innovation: imitative R&D, aimed at "inventing around" existing patented innovations. In fact, every time an inventor files a patent, competitors learn not only how to duplicate the innovation without incurring the R&D cost, but also that the market is probably profitable,[4] and which are the R&D directions that have been blocked or ruled out by the patent. Gallini (1992) studies the behaviour of imitators, and shows that they are willing to spend considerable amounts in order to "reverse engineer" some innovations and then modify them to be able to commercialise close substitutes without infringing the original patents. The more a patent proves profitable, the more competitors' R&D efforts will be distorted in that direction to capture part of the inventor's rents. Empirical studies show that even if imitation is less costly than original innovation (by a factor of two thirds, on average), 60% of patented innovations are copied within four years of the initial filing.[5] By conservatively admitting that there is no imitation after that, and that each innovation is only copied once, we find that 30% of total R&D expenditures are socially costly imitation expenditures.[6]

Thus, the existence of imitative R&D plays a mixed, double role in the patents debate: while being *ex post* welfare improving (because it dissipates the original innovator's monopoly rents, thus increasing consumers surplus[7]), it is also costly *ex ante*, so the real

[1] Precise enough to allow duplication by a skilled practitioner, by definition.

[2] In fact, the first best solution would be to reimburse exactly the total R&D costs to the inventor, and then to produce the innovation and sell it at its marginal cost.

[3] In other words, patents solve a problem by creating another. Guell and Fischbaum (1995) and Baker (1996) suggested an interesting mechanism of patent buyouts to avoid this, by which patents would be bought with public funds to be put in the public domain. See Kremer (1998, 1998b), and Ferrando (2000) for a description of the historic experiment provided by the buyout of the patent for the daguerreotype by the French State in the 19th century, and an analysis of a practical implementation of this idea based on auctions.

[4] In practice some firms engage in extensive patenting of unprofitable ideas or "red herrings" in order to confuse rivals.

[5] See for example Levin *et al* (1987), and Mansfield *et al* (1981).

[6] Notice that in some industries, like genomics, it is simply not possible to "invent around": once a gene sequence (or complete group of gene sequences) is found, there is no "close substitute" to it.

[7] For example, Baker (1996) estimates from empirical samples from the pharmaceutical industry a potential 68% reduction in the prices of patented drugs, which would of course bring a huge increase in consumer surplus.

question is whether the total net effect is positive, or negative. It is our purpose to answer this question in this paper.

Although patents are widely considered to be the main explanation of R&D activity, this is only true in a limited number of industries.[8] Bessen and Maskin (2002) go even further by noticing that some industries with "historically weak patent protection" but in which innovation is both sequential and complementary (like semiconductors[9], computers, and software), have been among the most innovative in the last forty years, and argue that stronger patent protection would have inhibited innovation instead of promoting it. Along these lines, some groups that are particularly active in the software industry (for example Copyleft and The Electronic Frontier Foundation, among others) began promoting in the early 90's the free circulation of ideas and knowledge without any legal barriers.[10] Even more recently, the software industry has seen the rise of a new business model based on "open source", that is, freely available computer code that can be modified, altered, or improved, by end users or competitors, which at first became particularly popular in the Unix computer community with the widespread use of GNU licensed software.[11] Since then,[12] defenders of open source argue that patents protect inefficient monopolies and prevent potential innovators from significantly improving patented products without incurring in important R&D expenditures, while freely available code spur innovation through imitation, sequential improvements and customisation of a "single" product in the public domain.

The rationale for this view is that imitation actually fosters innovation, and that firms are willing to forego short term monopoly profits in order to benefit from increased growth. In this context, firms benefit from spillovers from their competitors' R&D efforts while at the same time they save imitation costs, so they should actually welcome competition. All this, weaker patent defenders hold, is good for society. But if imitation is privately desirable because it accelerates technological progress and allows the original inventor to benefit from the imitator's own inventions, then patent holders should be able to appropriate this extra value through the use of licenses. The main problem with this argument is that licenses require an *ex ante* evaluation of the "second generation" innovation's value, which the patent holder is unlikely to have. In any case, the argument shows that licenses are a more than relevant issue in the debate.

The above discussion highlights the importance of the dynamic interactions between firms in the patents debate: if innovators are willing to be imitated, it is in the hope that they, in turn, will benefit from others' R&D efforts. This is only possible if the probability of innovating is increasing in the number of competing firms (because otherwise firms might replicate the effects of increased aggregate R&D spending in-house); or, in other words, if there is complementarity between different firms' R&D, which reflects the fact that different firms follow different approaches to innovate, so overall chances of success are improved. On the other hand, an imitation is only possible after an original innovation, so the different

[8] See for example Levin *et al* (1987), Mansfield (1986), and Schankerman (1998).

[9] See for example Levin (1982) and Hunt (1999) on the issue of patent protection and imitation in the semiconductor industry.

[10] Their motto "information wants to be free" is particularly descriptive.

[11] See Mendys–Kamphorst (2002) for a concise description of the main economic aspects and the history of the open source movement.

[12] The announcement made by Netscape Communications in 1998 to "go open source" with its internet browser *Navigator* was of particular relevance at a time when the market was exploding and had recently seen the aggressive entry of Microsoft's *Internet Explorer*.

systems cannot be assessed unless there is also sequentiality, that is, unless ideas arrive one at a time.

Bessen and Maskin (2002) provide a good review of previous literature studying a single sequential innovation[13] and an infinite sequence of quality improvements for a single technology[14], and build a dynamic model with differentiation that shows that when innovation is sequential and complementary, competition may enhance profits, and patents may interfere with such competition and therefore with hastened innovation. Cadot and Lippman (2002, 1998) use dynamic models close to our specification but with exogenous patent lengths and reverse engineering (imitation) expenditures to show that (i) the relationship between patent length and innovative activity is non-monotone, and (ii) firms may overinvest in R&D to deter entry.

We differ from this literature primarily because in our model firms do not decide between innovating **or** imitating, but rather the levels of simultaneous original **and** imitative R&D, which seems natural if both activities have positive expected returns, and coincides with observed facts: innovating companies have their own original (and confidential) research agendas, but also monitor closely the patenting activities of their rivals, and adapt their R&D efforts to competitors' innovations. It is this interdependence that is our main focus. While literature has often studied settings in which firms are divided in leaders and followers, high tech firms are in practice often both at the same time: ahead in some products, and behind in others. Secondly, the literature has consistently studied imitation of a deterministic nature, neglecting the fact that in reality imitation, like any other creative activity, it is of a fundamentally stochastic nature. This essential feature of our approach also implies that the value of innovations perceived by inventors is endogenously determined: rather than statutory patent life, what matters is effective patent life, and the latter is directly related to the prevailing level of imitative R&D effort. Finally, another important difference with standard hypotheses in the sequential innovation literature is that in our model firms are unable to appropriate the full social value of innovations, which seems natural in a model concerned with *ex post* welfare improvements due to imitation.

This simple model intends to shed some light on diverging results of the previous literature about imitation and dynamic innovation while allowing to revisit most classical findings. Among other new results, we find that in some settings firms might prefer absence of patent protection even though it is better for society as a whole to enforce patent protection. Thus, lobby aimed at softening patent protection from innovating firms should not necessarily be seen as good reason for reform. However, when firms can replicate the effects of a system without patents through cross-licensing agreements, then firm pressure for weaker patents should always be seen with favourable eyes.

In Section 2 we present the basic infinite-horizon dynamic model of repeated innovation and imitation and its main assumptions, and solve for the competitive equilibrium and socially optimal R&D expenditures in a context with patent protection. In Section 3 we move to a system without patents and full costless spillovers of innovation, that may be though of as open source. In Section 4 we compare the results, and illustrate the conditions under which firms and society prefer one system to the other. In Section 5 we explore the effects of licenses, showing that they allow firms to implement the no-patent outcome through cross-licensing agreements, and in Section 6 we conclude, with policy implications.

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[13] See Scotchmer (1991, 1996, 1999, 1999b, 1999c), Scotchmer and Green (1990), Green and Scotchmer (1996).

[14] See O'Donoghue, (1998), O'Donoghue *et al* (1998).

2. Dynamic model of stochastic innovation with patents

Our model is similar to repeated patent race games in the line of Reinganum (1985), Cadot and Lippman (1998, 2002) and Hunt (1999), among others. The main difference is that while in these models the incumbent and followers race for the next innovation only, here there are two simultaneous races: one is a standard patent race for the next innovation at pace l, and the other is an imitation race for existing patents at pace m. Another important difference is that in our model there is no 'replacement effect', that is, new original innovations do not cannibalise the market of previous ones, but so do imitations.

We consider an industry consisting of two *ex ante* symmetric firms,[15] each of which can undertake two kinds of R&D activities: one for original discoveries, and one for imitation of existing, but patented, discoveries. As long as a firm is not copied, it can sustain monopoly profits p^m in the innovation's market (with associated consumers' surplus S^m); but as soon as the competitor imitates, then the profits of the original innovator are reduced, and the imitator gets a share of the total profits in that market (and consumers' surplus increases to S^d). In other words, imitative R&D has positive returns for the imitator, but lower than original research (because its profits are shared), and produces a negative externality on the original innovator.[16] We suppose that after imitation both firms get the same duopoly profits $0 < p^d \leq \frac{1}{2} p^m$, thus abstracting from *ex post* first mover advantages and other similar considerations.[17] We stress that we are using the terms "imitation", "duplication" and "copy" in a very precise sense : imitators are actually "inventing around" the patent, and producing a close substitute (or improved enough version of the product) that can grab one half of the market while not infringing the patent.[18]

We assume that both original and imitative innovations arrive stochastically following Poisson processes of rates l and m, respectively, and that these rates depend on the resources devoted to each of the R&D activities, that is, $l = l(o)$ and $m = m(d)$, where o and d are respectively the total expenditures in original and duplicative R&D. We further suppose that these expenses are constant in time,[19] and that the rates of innovation are increasing in expenses but with decreasing marginal returns, i.e. l and m are continuous twice differentiable functions such that $l' > 0$, $l'' < 0$, $m' > 0$, $m'' < 0$ and $l(0) = m(0) = 0$. For simplicity, we also assume that firms are *ex ante* equally productive in their R&D efforts, which means that both their R&D technologies l and m are the same, but that imitative R&D is relatively more productive than original R&D, that is, $l(x) < m(x)$. We assume a constant intertemporal discount rate of d.

[15] All results extend quite naturally to three or more firms, but we expose the two firm case for simplicity and convenience.
[16] This could be interpreted as an example of Schumpeterian "creative destruction".
[17] These effects would have the same effect than a relative increase in the value of p^m, so one should expect that they should strengthen the case for patents.
[18] See Lichtenberg and Philipson (2002) for an analysis and discussion of "within-patent" and "between-patent" competition.
[19] This is consistent with the observation that R&D expenses are mostly fixed cost flows by nature (infrastructure, salaries, etc), as in Lee and Wilde's (1980) model, and as opposed to Dasgupta and Stiglitz(1980) and Loury's (1979) fixed lump sum cost specification. See also Grossman and Shapiro (1986, 1987) for an analysis focusing on non stationary R&D efforts.

Innovation here is (i) sequential only in the sense that it is necessary to innovate once in order to innovate a second time,[20] and that there must constantly be a base of protected innovations that can be potentially imitated (original innovation precedes imitation); and (ii) complementary in the sense that two firms spending *x* each in R&D stand a better chance at jointly innovating than a single firm spending *2x*.

As stated earlier, we assume that the infinite sequence of potential innovations arriving stochastically have all the same *ex ante* incremental value, modelled as the discounted expected profits of being a monopolist in that new market until imitated, and participating in a duopoly thereafter. Thus, the expected value of a received original innovation by firm *i* is:[21]

$$\underbrace{\Pi^E_{idea,i}}_{\substack{\text{expected value of a}\\ \text{received original idea}}} = \int_0^\infty \left\{ \underbrace{\int_0^t p^m\cdot e^{-dt}\,dt}_{\substack{\text{flow of monopoly profits}\\ \text{between } 0 \text{ and } t}} + \underbrace{\frac{p^d}{d}\cdot e^{-dt}}_{\substack{\text{discounted flow of}\\ \text{duopoly profit thereafter}}} \right\}\cdot \underbrace{m_j(d_j)e^{-m_j(d_j)t}}_{\substack{\text{probability of being}\\ \text{imitated at } t}}\cdot dt = \frac{m(d_j)\frac{p^d}{d}+p^m}{m(d_j)+d}$$

Notice that in this last expression we have assumed that as soon as the new idea is received, all the imitative effort of the competing firm concentrates on copying this innovation and not others, so in a certain sense it corresponds to a conservative, "worst case scenario" evaluation of the value of the received idea. Therefore, the expected value for firm *i* of undertaking original R&D at effort level o_i is:[21]

$$\underbrace{\Pi^E_{O,i}}_{\substack{\text{expected profit of firm } i\\ \text{from engaging in original R\&D}}} = \int_0^\infty \left\{ -\underbrace{\int_0^t o_i\cdot e^{-dt}\,dt}_{\substack{\text{discounted research expenditures}\\ \text{between } 0 \text{ and } t}} + \underbrace{\Pi^E_{idea,i}\cdot e^{-dt}}_{\substack{\text{discounted expected value of}\\ \text{receiving an original idea at } t}} + \underbrace{\Pi^E_{O,i}\cdot e^{-dt}}_{\substack{\text{discounted expected value of}\\ \text{restarting original research at } t}} \right\}\cdot \underbrace{l(o_i)e^{-l(o_i)t}}_{\substack{\text{probability of receving a}\\ \text{"good" idea at instant } t}}\cdot dt$$

$$\Pi^E_{O,i} = \frac{l(o_i)\cdot\Pi^E_{oi,i} + l(o_i)\cdot\Pi^E_{O,i} - o_i}{l(o_i)+d} \Leftrightarrow d\cdot\Pi^E_{O,i} = l(o_i)\cdot\Pi^E_{idea,i} - o_i$$

This last expression tells us that the expected value of undertaking original research at level o_i is equal to the present value of a constant flow of $l(o_i)\cdot\Pi^E_{idea,i}$, which is the instantaneous expected rate of arrival of profits, minus the current expenditures for original research o_i.

On the other hand, the expected value for firm *i* of undertaking imitative R&D at effort level d_i is analogously:[21]

$$\underbrace{\Pi^E_{I,i}}_{\substack{\text{expected profit of firm } i\\ \text{from engaging in imitative R\&D}}} = \int_0^\infty \left\{ -\underbrace{\int_0^t d_i\cdot e^{-dt}\,dt}_{\substack{\text{discounted research expenditures}\\ \text{between } 0 \text{ and } t}} + \underbrace{\frac{p^d}{d}\cdot e^{-dt}}_{\substack{\text{discounted expected value of}\\ \text{succeeding at imitation at } t}} + \underbrace{\Pi^E_{I,i}\cdot e^{-dt}}_{\substack{\text{discounted expected value of}\\ \text{restarting imitative research at } t}} \right\}\cdot \underbrace{m(d_i)e^{-m(d_i)t}}_{\substack{\text{probability of receving an}\\ \text{imitative idea at instant } t}}\cdot dt$$

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[20] This means each firm pursues only one innovation at a time.

[21] See appendix A for a detailed general derivation of the simplified forms.

$$\Pi_{I,i}^{E} = \frac{m(d_i)\cdot\frac{p^d}{d} + m(d_i)\cdot\Pi_{I,i}^{E} - d_i}{m(d_i) + d} \Leftrightarrow d\cdot\Pi_{I,i}^{E} = m(d_i)\cdot\frac{p^d}{d} - d_i$$

As earlier, this last expression tells us that the expected value of undertaking imitative research at level d_i is equal to the present value of a constant flow of $m(d_i)\cdot\frac{p^d}{d}$, which is the instantaneous expected rate of arrival of imitation profits, minus the current expenditures for imitative research d_i.

An implicit assumption of the model is that there is constantly a large base of innovations to be copied, and that in the long run the aggregate original innovation rate is higher than the imitation rate. This stationarity assumption is compatible with observed facts[22] and imposes some conditions on the model parameters. Throughout the analysis we will assume that this condition is verified and we will later check that, at equilibrium, it is generally indeed the case.

The solution of the symmetric game consists in a couple of R&D expenditures for each firm, coming from the optimality conditions for imitative and original R&D, respectively:

(1) $d_i^* = ArgMax\ \Pi_{I,i}^{E}(d_i) = ArgMax\ d\cdot\Pi_{I,i}^{E}(d_i) = ArgMax\ m(d_i)\cdot\frac{p^d}{d} - d_i \Leftrightarrow m'(d_i^*) = \frac{d}{p^d}$

(2.a) $o_i^* = \underset{o_i}{ArgMax}\ \Pi_{O,i}^{E}(o_i, d_j) = \underset{o_i}{ArgMax}\ d\cdot\Pi_{O,i}^{E}(o_i, d_j) = \underset{o_i}{ArgMax}\ l(o_i)\cdot\Pi_{idea,i}^{E}(d_j) - o_i$

$$\Leftrightarrow l'(o_i^*) = \frac{1}{\Pi_{idea,i}^{E}(d_j)}$$

Condition (1) tells us that the optimal expenditure in imitative R&D is achieved when the marginal benefit of increasing the expenditure (namely an increase in the rate of arrival of imitation profits) equals its direct marginal cost. Likewise, condition (2.a) says that the optimal expenditure in original R&D is achieved when the marginal benefit of increasing the expenditure (that is an increase in the rate of arrival of original innovation profits) equals its direct marginal cost.

Moreover, in equilibrium $d_i^* = d_j^* = d_P^*$ as defined in (1), and $o_i^* = o_j^* = o_P^*$ such that, by replacing d^* in (2.a):

(2) $$l'(o_P^*) = \frac{m(d_P^*) + d}{m(d_P^*)\frac{p^d}{d} + p^m}.$$

From the social welfare standpoint, every time an original innovation arrives, there is value creation, and every time an innovation is imitated, there is a welfare improvement due

[22] Firms engaged in patent competition generally compete in various fronts: they all tend to have large stocks of patents to preclude competitors to enter some markets, while at the same time providing them with specifications and ideas to "invent around".

to the dissipation of monopoly rents. We model this phenomenon by assuming that, in equilibrium, the aggregate rates of innovation and imitation will be respectively $2l(o)$ and $2m(d)$. As earlier, we can then compute the expected present value of the discounted equilibrium social welfare by doing: [21]

$$\underbrace{W_P^E}_{\substack{\text{expected present value}\\ \text{of global R\&D activity}}} = \int_0^\infty \left\{ -\underbrace{\int_0^t (2o+2d)\cdot e^{-dt}\,dt}_{\substack{\text{discounted total research}\\ \text{expenditure flows between 0 and } t}} + \left(W_{event}^E + W_P^E\right)\cdot e^{-dt} \right\} \cdot \underbrace{(2l+2m)\cdot e^{-(2l+2m)t}}_{\substack{\text{probability of an event}\\ \text{happening at instant } t}} \cdot dt \text{, where}$$

$$\underbrace{W_{event}^E}_{\substack{\text{expected present value}\\ \text{of a received idea}}} = \underbrace{\frac{2l}{2l+2m}}_{\substack{\text{probability of event}\\ \text{being an original innovation}}} \left(\underbrace{\frac{p^m+S^m}{d}}_{\substack{\text{discounted "present" value of}\\ \text{new idea incremental welfare}}} \right) + \underbrace{\frac{2m}{2l+2m}}_{\substack{\text{probability of event}\\ \text{being an imitative innovation}}} \left(\underbrace{\frac{2p^d+S^d}{d} - \frac{p^m+S^m}{d}}_{\substack{\text{discounted "present" value of}\\ \text{imitation incremental welfare}}} \right)$$

This simplifies into the following expression:

$$(3) \qquad d \cdot W_P^E = -2o - 2d + 2l\frac{p^m+S^m}{d} + 2m\frac{2p^d+S^d-\left(p^m+S^m\right)}{d}$$

This last expression has the familiar structure previously seen for profits, and it tells us that the expected social value of all research activity at equilibrium levels o and d by each firm is equal to the present value of a constant flow of $2l\frac{p^m+S^m}{d}$, which is the instantaneous expected rate of arrival of original innovation welfare, plus a constant flow of $2m\frac{2p^d+S^d-\left(p^m+S^m\right)}{d}$, which is the instantaneous expected rate of arrival of imitation incremental welfare, minus the current total expenditures for all research $2(o+d)$.

By differentiating expression (3) with respect to o and d we obtain the socially optimal values of symmetric R&D expenditures in this case:[23]

$$(4) \qquad \frac{\partial W_P^E}{\partial o} = 0 \Leftrightarrow l'\left(o_P^{**}\right) = \frac{d}{p^m+S^m}$$

$$(5) \qquad \frac{\partial W_P^E}{\partial d} = 0 \Leftrightarrow m'\left(d_P^{**}\right) = \frac{d}{2p^d+S^d-\left(p^m+S^m\right)}$$

Standard static models of patent races[24] support the idea that patents promote efficient or over-investment in R&D. However, if firms cannot fully appropriate the full social value of their innovations, they will reduce their efforts, so the socially optimal level of R&D

[23] Observe that these two values are completely independent, whereas in the firms optimisation problem the optimal level of original innovation depends on the equilibrium imitation expenditures (see expression (2)).
[24] See Loury (1979), Dasgupta and Stiglitz (1980), and the simplified static model in Bessen and Maskin (2002).

expenditures cannot be correctly decentralized. When taking into account the *ex post* inefficiencies of patents, the dynamic nature of innovation and the complementarities of decentralized R&D, the result of patent-race models is reversed.

Standard Result 1. *In a context with patents, there is always underprovision of original R&D in the competitive equilibrium (with respect to the socially optimal level with two firms).*

Proof. See Appendix B. *Q.E.D.*

Expression (5) implies that there is a positive optimal level of imitation in this context. A fortiori, by comparing expressions (5) and (1), we derive the following:

New Result 1. *If the negative externality between firms coming from imitation is smaller (larger) than the positive externality on consumer surplus, then there will be underprovision (overprovision) of imitative R&D in the competitive equilibrium.*

Proof. See Appendix B. *Q.E.D.*

While imitation R&D expenditures are frequently considered socially wasteful, the preceding result shows that the benefits of imitation (welfare improving dissipation of *ex post* monopoly rents) might actually outweigh its costs. This result is in sharp contrast with those in Gallini (1992), where the socially optimal patent life is shown to be short enough so as to discourage all imitation, and more in line with those suggesting that socially optimal patents should be thin and infinitely lived,[25] and with observed facts. The difference is explained by the way we model the dynamics of imitation: while in her model it is deterministic and with "lump sum" costs, in ours it is stochastic and with "flow" costs, thus enhancing the "active", welfare improving, role of costly but more efficient imitation.

3. Dynamic model of stochastic innovation in the absence of patents

In a context without patent protection, a firm is free to imitate any innovation without incurring any significant imitative R&D cost. In our model, this means that there are full spillovers of innovations between firms, and that monopoly profits are nonexistent, because every time one of the firms innovate, its competitor immediately copies the product and obtains one half of the market.[26]

On the one hand, here firms have only one "line" of R&D, corresponding to the "original R&D" of the model presented in section two, thus avoiding socially inefficient and unnecessary "duplicative" R&D expenses. On the other hand, returns to original R&D are lower, because profits are immediately shared once an innovation occurs, which provides firms with incentives to free-ride on other firms' R&D efforts.

[25] See for example Gilbert and Shapiro (1980) and Klemperer (1990).

[26] This is equivalent to keeping the disclosure requirement of patents but not giving a *de juré*, *ex post*, monopoly right. Although this may seem a strong assumption, it is less so when interpreted as immediate costless reverse engineering by close competitors, *ex ante* cross licensing agreements (see Section 5), or *ex ante* commitment to disclose (as in the GPL license and some other open source licenses, for instance).

In this context, firms care only about the aggregate rate of arrival of innovations, because as soon as one of the firms innovate, both get a constant flow of duopoly profits. We call $\Gamma(r_i, r_j) = l(r_i) + l(r_j)$ this aggregate rate of innovation.

The expected value for firm i of undertaking original R&D at effort level r_i is:[21]

$$\underbrace{\Pi^E_{NP,i}}_{\substack{\text{expected profit of firm } i \text{ from}\\ \text{engaging in (original) R\&D}}} = \int_0^\infty \left\{ \underbrace{-\int_0^t r_i \cdot e^{-dt}\,dt}_{\substack{\text{discounted research expenditures}\\ \text{between } 0 \text{ and } t}} + \underbrace{\frac{p^d}{d} \cdot e^{-dt}}_{\substack{\text{discounted expected profit of}\\ \text{receiving an original idea at } t}} + \underbrace{\Pi^E_{NP,i} \cdot e^{-dt}}_{\substack{\text{discounted expected value of}\\ \text{restarting original research at } t}} \right\} \cdot \underbrace{\Gamma(r_i, r_j) e^{-\Gamma(r_i, r_j)t}}_{\substack{\text{probability of one of the firms}\\ \text{receving a "good" idea at instant } t}} \cdot dt$$

$$\Pi^E_{NP,i} = \frac{\Gamma(r_i, r_j) \cdot \frac{p^d}{d} + \Gamma(r_i, r_j) \cdot \Pi^E_{NP,i} - r_i}{\Gamma(r_i, r_j) + d} \Leftrightarrow d \cdot \Pi^E_{NP,i} = \Gamma(r_i, r_j) \cdot \frac{p^d}{d} - r_i$$

This expression tells us that the expected value of undertaking original research at level r_i is equal to the present value of a constant flow of $\Gamma(r_i, r_j) \cdot \frac{p^d}{d}$, which is the instantaneous expected rate of arrival of profits, minus the current expenditures for original research r_i.

The optimality condition for R&D expenditure translates into:

$$(6) \qquad r_i^* = \underset{r_i}{ArgMax}\, \Pi^E_{NP,i}(r_i, r_j) = \underset{r_i}{ArgMax}\, d \cdot \Pi^E_{NP,i}(r_i, r_j) = \underset{r_i}{ArgMax}\, \Gamma(r_i, r_j) \cdot \frac{p^d}{d} - r_i \Leftrightarrow l'(r_i^*) = \frac{d}{p^d}$$

Like earlier, condition (6) says that the optimal expenditure in original R&D is achieved when the marginal benefit of increasing the expenditure (namely an increase in the rate of arrival of duopoly profits) equals its direct marginal cost. In equilibrium, $r_i^* = r_j^* = r_{NP}^*$ as defined in (6).

From the social welfare standpoint, every time an original innovation arrives, there is value creation and efficient *ex post* duopoly production. We model this phenomenon by assuming that, for any symmetric R&D expenditures, the aggregate rate of innovation will be $2l(r)$. As earlier, we can then compute the expected present value of the discounted equilibrium social welfare by doing:[21]

$$\underbrace{W^E_{NP}}_{\substack{\text{expected present value}\\ \text{of global R\&D activity}}} = \int_0^\infty \left\{ \underbrace{-\int_0^t 2r \cdot e^{-dt}\,dt}_{\substack{\text{discounted total research}\\ \text{expenditure flows between } 0 \text{ and } t}} + \left(\underbrace{\frac{2p^d + S^d}{d}}_{\substack{\text{discounted "present" value of}\\ \text{new idea incremental welfare}}} + W^E_{NP} \right) e^{-dt} \right\} \cdot \underbrace{2l \cdot e^{-2lt}}_{\substack{\text{probability of an event}\\ \text{happening at instant } t}} \cdot dt$$

This simplifies into the following expression:

$$(7) \qquad d \cdot W^E_{NP} = -2r + 2l\,\frac{2p^d + S^d}{d}$$

This last expression has the familiar structure previously seen for profits, by telling us that the expected social value of all research activity at research level r by each firm is equal to the present value of a constant flow of $2l\,\frac{2p^{d}+S^{d}}{d}$, which is the instantaneous expected rate of arrival of incremental innovation welfare flows, minus the current total expenditures for all research $2r$.

By differentiating expression (7) we derive the social optimality condition:

$$\frac{\partial W_{NP}^{E}}{\partial r}=0 \Leftrightarrow l'\left(r_{NP}^{**}\right)=\frac{d}{2p^{d}+S^{d}} \tag{8}$$

Because firms do not internalise neither competitor's profits nor consumer surplus increases when optimising r_{NP}^{*}, we can assert the following:

Standard Result 2. *In a context without patents, there is always underprovision of R&D (with respect to the socially optimal level with two firms) in the competitive equilibrium.*

Proof. See Appendix B. *Q.E.D.*

4. Comparisons

The standard trade-off in patent protection debates is one between R&D incentives and *ex post* monopoly rents. In other words, while increasing firms' rents provides higher incentives for innovation, and thus is socially desirable, *ex post* production at monopoly prices and quantities is socially costly. The existence of imitative R&D plays a mixed, double role in this debate: while being *ex post* welfare improving (because it dissipates the original innovator's monopoly rent thus increasing consumers surplus), it is also costly, so the real question is if the net effect is positive or negative. Additionally, imitation reduces patent protection effectiveness, and thus indirectly alters the pace of technological progress. This section explores the outcome when these effects are simultaneously taken into account.

To begin with we recall the conditions defining the optimal expenditures in R&D with and without patents:

Context		Competitive Equilibrium		Social Optima	
		variable name	condition	variable name	condition
Patents	Original R&D	o_P^{*}	$l'\left(o_P^{*}\right)=\frac{m\left(d_P^{*}\right)+d}{m\left(d_P^{*}\right)\frac{p^{d}}{d}+p^{m}}$	o_P^{**}	$l'\left(o_P^{**}\right)=\frac{d}{p^{m}+S^{m}}$
	Imitative R&D	d_P^{*}	$m'\left(d_P^{*}\right)=\frac{d}{p^{d}}$	d_P^{**}	$m'\left(d_P^{**}\right)=\frac{d}{2p^{d}+S^{d}-\left(p^{m}+S^{m}\right)}$
No Patents	Original R&D	r_{NP}^{*}	$l'\left(r_{NP}^{*}\right)=\frac{d}{p^{d}}$	r_{NP}^{**}	$l'\left(r_{NP}^{**}\right)=\frac{d}{2p^{d}+S^{d}}$

Table 1. Optimality conditions for R&D expenditures

Standard Result 3. *Firms engage in more original R&D with patent protection than without.*

Proof. See Appendix B. *Q.E.D.*

This is a standard result in the literature, and confirms the straightforward intuition that incentives for original R&D are higher when firms can appropriate a larger part of the value of their innovations.

However, patents also provide incentives to engage in socially wasteful duplicative R&D, so from the social welfare standpoint, what matters is if the increased incentives to innovate outweigh the cost of imitation. In fact, it is useful to begin by bearing in mind that the existence of costly imitation reduces the desirability of original innovation. The intuition behind this is that patents allow firms to sustain monopoly profits (with the accompanying reduction in consumers surplus) for a certain period, during which competitors try to (costly) imitate them, and then attain the final duopoly equilibrium, whereas the exact same final result could be achieved costlessly in the absence of patents (with immediate, costless imitation). We summarize this result in the following proposition:

Proposition 1. *The socially optimal pace of original innovation is higher in a world without patent protection than in a world with patent protection and imitation.*

Proof. See Appendix B. *Q.E.D.*

Defenders of weak patents and open source often support their claims on the increased speed of technological change in a context lacking strong patent protection. However, even though this last proposition supports this view, it does not mean that a system without patents should always be preferred to a system with patents, because global welfare might actually be higher in this last scenario than in the former. In other words, *a higher pace of innovation is not necessarily good for society as a whole*.

In fact, the condition for a system without patents to be socially preferred, for any symmetric R&D expenditures by firms, is:

$$(9) \qquad W_{NP}(r) > W_P(o,d) \Leftrightarrow [l(r) - m(d)]\frac{W^d}{d} > [l(o) - m(d)]\frac{W^m}{d} + r - (o+d)$$

Analogously, the condition for a system without patents to be preferred by firms, for any symmetric R&D expenditures, is:[27]

$$(10) \qquad \Pi^E_{NP,i}(r) > \Pi^E_{O,i}(o) + \Pi^E_{I,i}(d) \Leftrightarrow [2l(r) - l(o) - m(d)]\frac{p^d}{d} > l(o)\frac{p^m - p^d}{m(d)+d} + r - (o+d)$$

Both these conditions are not easy to sign or order for general forms of the R&D technologies. In a context in which there is imperfect appropriability of the social value of innovations, it should come as no surprise that the preferences of firms and society with

[27] The condition on profits is straightforward, and can be easily interpreted: it says that firms will prefer a system without patents when the expected increase in the arrival rate of permanent duopoly profits is greater than the difference in current R&D expenses plus the expected arrival rate of temporary incremental monopoly profits.

regards to intellectual property protection systems do not always coincide. What is perhaps more surprising is what follows:

Proposition 2. *Even though stronger patents always increase the competitive equilibrium pace of original innovation, (i) they do not necessarily improve welfare, and (ii) firms do not always favour stronger patent protection more than society does.*

Proof. See Standard Result 3 and the following example. *Q.E.D.*

Most important points might be illustrated by means of the following example,[28] that was built for a particular specification of the R&D technology and *ex post* market competition.

Example 1: Firms want weak patents but society resists. *Square root R&D technologies.*

The specification shown in Figure 1 and described in detail in Appendix C is ideal for describing a "high productivity" R&D technology that allows firms to innovate relatively faster, especially for small R&D expenditures. The advantage of imitation over original innovation was supposed to be of a "fixed" nature, in the sense that it is equivalent to having already spent part of the R&D expenses.[29] This specification ensures that stationarity constrains are verified at the competitive equilibrium and has the appealing feature that both R&D technologies converge for large expenditures, which means that the advantage provided by the available information in the patent gets "diluted" for large R&D efforts.

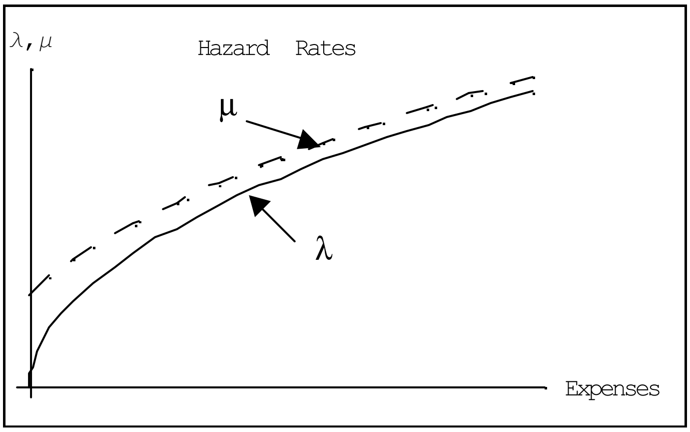

Figure 1. Square root R&D technologies with fixed advantage for imitators

In some circumstances, the information contained in patent filings may be of great direct help to potential imitators. In this case, patent specifications provide the imitator with a considerable advantage when trying to invent around an existing patent, with respect to pursuing a completely original innovation. If product (or process) invention descriptions are precise enough to allow duplication, they also facilitate the discovery of alternative, non infringing, close substitutes. Industries like software and informatics are likely to fall within this category, because on the one hand the algorithmic descriptions contained in patent filings

[28] See Appendix C for details.

[29] Alternative relationships between original and imitative R&D technologies, like "fixed rate increase" ($m(x)=l(x)+ m_0$) or "cost division effect" ($m(x)=l(k\,x)+ m_0$), as well as combinations of these three were also explored but are not presented here, because they add to the model in complexity without significantly enriching the results.

are likely to provide imitators not only with ideas of profitable markets, but also with "how-to" guides allowing easy, immediate non-infringing replication. Besides, higher R&D spending is not likely to further increase the higher productivity of imitative R&D (relative to original R&D), because patents contain much of what can be "extracted" from the existing innovation, and increased spending resembles original R&D expenditures. Figure 1 above illustrates this scenario, corresponding to patents containing "soft" information (in the sense that descriptions have to be vague in order to block close imitations and constitute "prior art"), and/or to industries with higher R&D productivity (in the sense of the increase of the innovation rate per unit of R&D spending).[30]

A reduced form of an *ex post* duopoly competition in a product market with linear demand was considered for simplicity. Figure 2 shows the preferences of the firms and society as a whole regarding the strength of patent protection in the competitive equilibrium. In general, for high values of the discount rate d (i.e. slow innovation), both firms and the social planner prefer strong patent protection to weak patents; and for low values of d, both firms and the social planner prefer weak patents. However, for values of d in the intermediate, shaded zone, firms will prefer no patents while society would be better off with strong patent protection.[31]

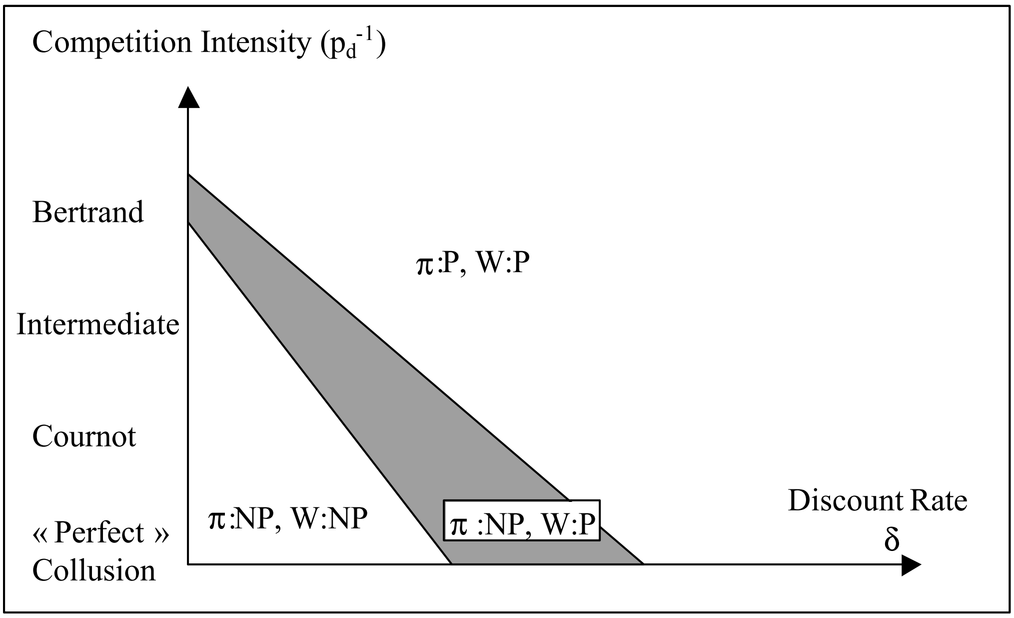

Figure 2. Preferences at the competitive equilibrium with square root R&D
(π: firms, W: society)

This simple example shows that, while in most cases what is good for firms is also good for society, in some others both preferences are incompatible. Moreover, and contrary to previous findings in the literature, the endogenous determination of imitation efforts of a stochastic nature gives rise to situations in which the absence of patents is good for firms but detrimental to society as a whole.[32] This implies the following corollary to Proposition 2:

[30] Notice that high productivity R&D can also be interpreted as the industry's "natural" dynamism, because the hazard rates l and m should be compared to the discount rate d.
[31] See Appendix C for the formal proof of this ordering of preferences.
[32] Notice however that the proof depends on the particular R&D technology chosen, see Appendix C for a counterexample.

Corollary 2. *Firms' lobby for weaker or stronger patent protection should be considered with great care: what is good for firms is not necessarily good for society, and vice-versa.*

Proof. See Example 1, Proposition 2 and Example 2 in Appendix C. *Q.E.D.*

Nevertheless, and despite the fact that the threshold values are different for society and for the firms, some general recommendations can be drawn from the analysis and the example above. In particular, for extreme values of the discount rate d and the intensity of *ex post* competition, and independent of the R&D technology specification, what is good for firms is also good for society and vice versa, so it is wise to listen to firms when they call for stronger or weaker patent protection. In general, when d is low and *ex post* competition soft, a system without patents and full information spillovers performs best for firms and society, and when d is high and the competition fierce, strong patents are to be preferred. Notice that this seems broadly in line with observed facts : in industries where hazard rates are relatively larger (low d, like semiconductors), firms tend to prefer weak patents and implement cross-licensing agreements; on the contrary, in industries where innovation takes longer (high d, like the automotive industry), patents are strong and widely used by firms. As regarding competition intensity, the intuition is straightforward: the stronger the *ex post* competition, the higher the externality of imitation, so the more firms will prefer patents.

The incompatibility between social and firms' private objectives shown in these results is often seen as a direct consequence of the firms not being able to fully internalise the total social value of their innovations, which is of particular relevance in the case of sequential innovation. In this scenario, the incapacity of the "first generation" innovator to (at least partially) capture the value of the "second generation" innovation creates distortions that reduce first generation R&D investment. The standard way to address this problem within the patent system is by way of licenses, that allow transfers between the second and first generation innovators, thus restoring the right incentives. In the following section we reinterpret our model to address this important issue and study the impact of licenses in the dynamic process of sequential innovation.

5. Licenses

We have seen in the previous section that for extreme values of the discount rate d and the intensity of *ex post* competition, the preferences of society regarding the strength of patent protection coincide with those of the society. In other words, for these extreme scenarios, what is good for firms is also good for society, so it is wise to listen to firms when they call for stronger or weaker patent protection. However, for intermediate values of these two parameters, firms' preferences may be detrimental for society as a whole, and in particular in some cases firms might prefer no patents even though patents improve social welfare. In this section we will focus in this particular scenario in which firms would prefer the no-patents outcome (the shaded zone in Figure 2), but assume patent protection exists.

It is of course always possible in a system with patents to replicate the market outcome of a system without patents simply by committing not to patent innovations. However, if patents exist, invariably choosing not to patent innovations is not a Nash equilibrium of the R&D game. In fact, every time an innovation arrives, the patenting game is a standard prisoner's dilemma: patenting is always a dominant strategy, regardless of the action chosen by the rival. Thus, not patenting is not a sub-game perfect equilibrium and the commitment not to patent innovations is not credible.

Another way by which firms might partially achieve the no-patents outcome is through automatic licensing or cross-licensing agreements.[33] Practice shows that some industries have developed cross-licensing agreements, and that others use standard licenses to spur enhancements to existing products. One should expect that these arrangements appear in those cases in which the no-patent regime is preferred by firms.

Basically, when deciding on whether to sell a license, a firm compares its price[34] to the expected value of temporary monopoly profits until imitation; when deciding on whether to buy a license, it compares its price to the expected cost of temporary imitative R&D expenses plus the expected opportunity cost of not being on the market during the research phase.[35] So the market for licenses exists if and only if the expected value of temporary monopoly profits is smaller than the expected (total) cost of the temporary imitative R&D phase. Of course, licenses should increase the returns to original R&D, so the equilibrium expenses should change, and so should the pace of innovation.

The expected value of the temporary monopoly profits that the potential licensor foregoes when selling a license is given by:[21]

$$\underbrace{\Delta\Pi^{E}_{Seller}}_{\substack{\text{expected incremental profit of}\\ \text{firm } i \text{ from not selling a license}}} = \int_0^\infty \left\{ \underbrace{\int_0^t \left(p^m - p^d\right) e^{-dt}\, dt}_{\substack{\text{discounted incremental profits}\\ \text{between } 0 \text{ and } t}} \right\} \cdot \underbrace{m\left(d^*\right) e^{-m\left(d^*\right)t}}_{\substack{\text{probability of being}\\ \text{imitated at instant } t}} \cdot dt = \frac{p^m - p^d}{m\left(d^*\right) + d}$$

And the expected total cost of the imitative R&D period that the potential licensee avoids when buying a license is given by:[21]

$$\underbrace{\Delta\Pi^{E}_{Buyer}}_{\substack{\text{expected avoided cost of}\\ \text{firm } j \text{ from buying a license}}} = \int_0^\infty \left\{ \underbrace{\int_0^t \left(d^* + p^d\right) e^{-dt}\, dt}_{\substack{\text{discounted flow of costs}\\ \text{between } 0 \text{ and } t}} \right\} \cdot \underbrace{m\left(d^*\right) e^{-m\left(d^*\right)t}}_{\substack{\text{probability of}\\ \text{imitating at instant } t}} \cdot dt = \frac{d^* + p^d}{m\left(d^*\right) + d}$$

So the market for licenses exists if and only if there exists a license price p_{lic} such that:

$$\text{(11)} \qquad \overbrace{\frac{p^m - p^d}{m + d} \le p_{lic}}^{\text{licensor sells iff}} \underbrace{\le \frac{d^* + p^d}{m + d}}_{\text{licensee buys iff}}$$

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[33] See Fershtman and Kamien (1991) for an analysis of the cross-licensing of complementary innovations and the pace of the innovation race.

[34] We restrict our analysis to simple licenses sold with a lump sum payment or price. Of course, licensing contracts can take more complex forms. See Shapiro (1985) and Erutku and Richelle (2000) for discussions on the licensor's ability to extract monopoly profits out of licensees.

[35] This is an *ex post* licensing decision in the sense of Gallini and Winter (1985) as opposed to an *ex ante* licensing agreement struck before R&D expenses are engaged.

This interval will be nonempty iff $p^m \leq 2p^d + d^*$, so licenses will be sold when the equilibrium imitative expenses are higher than the incremental industry profits in the monopoly situation or, in other words, when imitative expenses are high and *ex post* competition is soft.

If licensing occurs, profits are brought back to the no patents case, because firms will expect to sell and buy licenses with probability ½, as can be seen in the following expression:

$$\Pi_{lic}^{E} = \int_0^\infty \left\{ -\int_0^t r_i \cdot e^{-dt} dt + \underbrace{\frac{l(r_i)}{\Gamma(r_i,r_j)}\left[\frac{p^d}{d}\cdot e^{-dt} + p_{lic} + \Pi_{lic}^{E}\right]}_{\text{firm } i \text{ innovates and sells a license}} + \underbrace{\frac{l(r_j)}{\Gamma(r_i,r_j)}\left[\frac{p^d}{d}\cdot e^{-dt} - p_{lic} + \Pi_{lic}^{E}\right]}_{\text{firm } j \text{ innovates and firm } i \text{ buys a license}} \right\} \cdot \underbrace{\Gamma(r_i,r_j) e^{-\Gamma(r_i,r_j)t}}_{\text{probability of one of the firms receving a "good" idea at instant } t} \cdot dt$$

$$\Pi_{lic}^{E} = \int_0^\infty \left\{ -\int_0^t r \cdot e^{-dt} dt + \frac{p^d}{d}\cdot e^{-dt} + \Pi_{lic}^{E} \right\} \cdot 2\,l(r) e^{-2l(r)t} \cdot dt$$

We recognize in this last expression the one for $\Pi_{NP,i}^{E}$ in Section 3, which implies:

Lemma. *If $p^m \leq 2p^d + d^*$, then firms can and will replicate the outcome of a system without patents under a system with patents and licenses.*

This last lemma implies that the social preference for a system with strong patents and imitation cannot be implemented when firms prefer weak patents,[36] because firms would prefer to implement the no-patent outcome by cross-licensing all their innovations. On the contrary, when firms prefer strong patents but society is better off without patent protection,[37] firms cannot implement the dominated outcome. Thus, we conclude that:

Proposition 3. *If firms prefer no patent protection, society should follow suit.*

6. Conclusions

By means of a simple stochastic model of Poisson innovation and imitation, we have described the behaviour of a duopoly of innovating firms that can simultaneously invest in original and imitative R&D. Instead of focusing in the more traditional decision on *whether* to innovate or imitate, here we explore the question of *how much* to innovate and *how much* to imitate, and study the resulting industry dynamics when both processes coexist.

In most cases, the preferences of firms and of society as a whole regarding the strength of patent protection coincide, particularly for extreme values of the discount rate d and the intensity of *ex post* competition. In other words, for these scenarios, what is good for firms is also good for society and vice versa, so it is wise to listen to firms when they call for stronger or weaker patent protection. Due to the welfare improving properties of costly imitation, we find that in general when d is low and competition is soft a system without patents and full information spillovers performs best for firms and society, and when d is high and competition is intense strong patents are to be preferred. However, for intermediate values of d and competition intensity, the preferences of the firms and society are divergent, and the recommendation depends on the shapes of the original and imitative R&D technologies, which calls for some empiric work on that issue. In particular, we show that in some (likely)

[36] This scenario corresponds to the preferences illustrated in the shaded region of Figure 2.
[37] This scenario corresponds to the preferences illustrated by the counterexample in Appendix C.

circumstances, firms might prefer no patent protection even though society would benefit from enforcing strong patents.

Firms can always to replicate the market outcome of a system without patents in a system with patents simply by committing not to patent innovations. However, if patents exist, invariably choosing not to patent innovations is not a Nash equilibrium of the R&D game. Nevertheless, firms can still implement the no-patent equilibrium through the use of licenses when the *ex post* licensing conditions are satisfied. Thus, cross-licensing agreements appear as a natural equilibrium outcome in situations where patents exist but firms are better-off without patent protection. This also means that firms can always implement the no-patent outcome when they prefer it, making strong patent protection (and the associated, needed, welfare improving imitation) difficult to enforce by law. As a consequence, weakening patent protection when firms call for it can do no harm.

As noticed by Reinganum (1984), the stochastic nature of innovation and industry evolution must be addressed by asymmetric stochastic models. The model presented in this paper allows for the traditional asymmetric leader/follower interpretation, except for the fact that here both firms may be leaders *and* followers at the same time, thus restoring structural symmetry to the industry. An interesting extension would be one in which such asymmetry would play a more active role, by considering, for example, R&D expenses that were contingent to the "lag" between the leader and the follower of each product market.

While in our model weak and strong patents cannot coexist (if firms can choose between weak and strong patents, all of them have incentives to pick strong patents in order to avoid free riding on their own R&D expenses), it would be interesting to add an extra layer to the model in which firms would choose whether to patent or not. Such a model would probably require considering fixed costs of entry and/or heterogeneity of firms to explain, for instance, the coexistence of strong patents and open source observed in the software industry. One could also extend the model allowing firms to use secrecy as an additional instrument to sustain monopoly profits. If they can make imitation costlier by avoiding to disclose the information required in the patent filing process, they might prefer withholding information to patenting. Within the present model, this would probably translate into higher monopoly profits and lower incentives to imitate. Another natural question that arises when considering two simultaneous processes of innovation is that of free entry. In fact, if both activities have positive returns, one should expect that more markets participants would enter the market.[38] Whereas the qualitative results of this paper would not radically change with more (*ex ante* symmetric) firms, lower returns to imitation would probably weaken some of the crossed effects. A possible extension of the model in this sense should accommodate *ex post* inefficiencies as a counterweight for costly imitation R&D expenditures and zero profit entry conditions.

Finally, an alternative interpretation to the present model is one in which two firms compete to obtain the "next generation" enhancement for a single product. If patents are broad enough to protect current generation innovations but not broad enough to protect subsequent innovations,[39] sequential innovation may be though of as a sequence of independent innovations of the same *incremental* value (shared in constant proportions between innovators and consumers). In such a context, firms can also have two parallel lines of R&D, one

[38] See Hunt (2002) for a repeated patent race game with endogenous entry and deterministic imitation.
[39] That is, once the generation t enhancement has been discovered, its patent confers a *de juré* monopoly on the market for the t-enhanced product, but not on any subsequent or previous version of the product.

intending to catch-up with the leading, by inventing around the "current generation" innovation, and a second aimed at leapfrogging the competitor by developing the "next generation" innovation.[40] Notice that in this case we also abstract from the complications arising with obsolescence (also known as the replacement effect) by assuming that every generation has its own market and that the production of new enhancements does not hinder the market for older versions of the product.[41] This interpretation may be of particular relevance for discussing the effects of licenses on the incentives for sequential innovation,[42] which we leave for further research.

References

[1] Baker, D. (1996), "The high cost of protectionsim : the case of intellectual property claims", Economic Policy Institute, Technical Paper No. 230 mimeo, September 1996.

[2] Bessen, J., and Maskin, E. (2000), "Sequential innovation, patents, and imitation", working paper No. 00-01, Department of Economics, Massachusetts Institute of Technology, January 2000.

[3] Cadot, O. and Lippman, S. (1998), "Fighting Imitation with Fast-Paced Innovation", INSEAD working paper 97/73/EPS.

[4] Cadot, O. and Lippman, S. (2002), "Barriers to Imitation and the Incentive to Innovate", INSEAD working paper 95/23/EPS.

[5] Dasgupta, P. and Stiglitz, J. (1980), "Uncertainty, Industrial Structure, and the Speed of R&D", Bell Journal of Economics, Vol. 11, No. 1, 1-28.

[6] Encaoua, D. and Ulph, D. (2000), "Catching-up or leapfrogging? The effects of competition on innovation and growth", Cahier de la MSE, EUREQua n.2000.97.

[7] Erutku, C., and Richelle, Y. (2000), "Optimal licensing contracts and the value of a patent", Cahier 2000-07, Département des sciences économiques, Université de Montréal, février 2000.

[8] Ferrando, J. (2000), "Les formes de protection alternatives du brevet: les aspects stratégiques liés à l'évaluation d'une innovation par mise aux enchères", mémoire de DEA, Université Paris 1 Panthéon Sorbonne, available from the author upon request.

[9] Fershtman, C., and Kamien, M. (1992), "Cross licensing of complementary technologies", International Journal of Industrial Organization, Vol. 10, No. 3, 329-348.

[10] Gallini, N. (1992), "Patent policy and costly imitation", The RAND Journal of Economics, Vol. 23, No. 1, 52-63.

[11] Gallini, N. and Winter, R. (1985), "Licensing in the theory of innovation", The RAND Journal of Economics, Vol. 16, No.2, 237-252.

[40] See Encaoua and Ulph (2000) for a different description of catching-up and leapfrogging.
[41] This assumption is arguably equivalent to lowering the ex ante expected value of the discounted permanent duopoly profits.
[42] See particularly the articles by Suzanne Scotchmer for in-depth discussions of this issue.

[12] Gilbert, R. and Shapiro, C. (1980), "Optimal Patent Length and Breadth", The RAND Journal of Economics, Vol. 21, No.1, 106-112.

[13] Green, J. and Scotchmer, S. (1995), "On the division of profit in sequential innovation", The RAND Journal of Economics, Vol. 26, No. 1, 20-33.

[14] Grossman, G.M., and Shapiro, C. (1986), "Optimal dynamic R&D programs", The RAND Journal of Economics, Vol. 17, No. 4, 581-593.

[15] Grossman, G.M., and Shapiro, C. (1987), "Dynamic R&D competition", The Economic Journal, No. 97, 372-387.

[16] Guell, R., and Fischbaum, M. (1995), "Toward allocative efficiency in the prescription drug industry", The Milbank Quarterly, Vol. 73, No. 2, June 1995, 213-229.

[17] Hunt, R. (1999), "Nonobviousness and the incentive to innovate: An economic analysis of intellectual property reform", working paper No. 99-3, Federal Reserve Bank of Philadelphia, March 1999.

[18] Hunt, R. (2002), "Patentability, industry structure, and innovation", working paper No. 01-13/R, Federal Reserve Bank of Philadelphia, September 2002.

[19] Klemperer, P. (1990), "How broad should the scope of patent protection be ?", The RAND Journal of Economics, Vol. 21, No.1, Spring 1990, 113-130.

[20] Kremer, M. (1998), "Patent buyouts : A mechanism for encouraging innovation", Quarterly Journal of Economics, Vol. 113, November 1998, 1137-1168.

[21] Kremer, M. (1998b), "Patent buyouts : A mechanism for encouraging innovation", *working paper version*, disponible sur demande au MIT, NBER 6304.

[22] Lee, T., and Wilde, L. (1980), "Market structure and innovation : a reformulation", Quarterly Journal of Economics, No. 21, 113-130.

[23] Levin, R.C. (1982), "The Semiconductor Industry", in Richard Nelson (ed.), "Government and Technical Progress: A Cross-Industry Analysis", Pergamon Press, 1982, 9-100.

[24] Levin, R.C., Klevorick, A.K., Nelson, R.R., and Winter, S.G. (1987), "Appropriating the returns from industrial research and development", Brooking Papers on Economic Activity, Vol. 14, 783-820.

[25] Lichtenberg, F. and Philipson, T. (2002), "The dual effects of intellectual property regulations: within- and between- patent competition in the US pharmaceuticals industry", NBER working paper, No. 4161, 1992.

[26] Loury, G. (1979), "Market structure and innovation", Quarterly Journal of Economics, XCIII, 395-401.

[27] Mansfield, E., Schwartz, M., and Wagner, S. (1981), "Imitation costs and patents: An empirical study", Economic Journal, Vol. 91, 907-918.

[28] Mansfield, E. (1986), "Patents and innovation", Journal of Management Science, Vol. 32, 173-81.

[29] Mendys–Kamphorst, E. (2002), "Open vs. Closed: Some consequences of the open source movement for software markets", CPB discussion paper No 13, September 2002.

[30] O'Donoghue, T. (1998), "A Patentability Requirement for Sequential Innovation", The RAND Journal of Economics, Vol. 29, No. 4, 654-679.

[31] O'Donoghue, T., Scotchmer, S., and Thisse, J-F. (1998), "Patent breadth, patent life, and the pace of technological progress", Journal of Economics and Management Studies, Vol. 7, No.1, Spring 1998, 1-32.

[32] Reinganum, J.F. (1984), "Practical implications of game theoretic models of R&D", American Economic Review, Vol. 74, No. 2, 61-66.

[33] Schankerman, M. (1998), "How valuable is patent protection? Estimates by technology field.", The RAND Journal of Economics, Vol. 29, No. 1, Spring 1998, 77-107.

[34] Scotchmer, S. and Green, J. (1990), "Novelty and disclosure in patent law", The RAND Journal of Economics, Vol. 21, No. 1, 131-146.

[35] Scotchmer, S. (1991), "Standing on the shoulders of giants : Cumulative research and the patent law", Journal of Economic Perspectives, Vol. 5, No. 1, 29-41.

[36] Scotchmer, S. (1996), "Protecting Early Innovators: Should Second-Generation Products be Patentable?", The RAND Journal of Economics, Vol. 27, No. 2, 322-331.

[37] Scotchmer, S. (1999), "On the optimality of the patent renewal system", The RAND Journal of Economics, Vol. 30, No. 2, Spring 1999, 181-196.

[38] Scotchmer, S. (1999b), "Cumulative innovation in theory and practice", GSPP working paper 240, U.C. Berkeley, February 1999.

[39] Scotchmer, S. (1999c), "Delegating investment in a common-value project", Institute of Business and Economic Research, UC Berkeley, Working Paper 99-266.

[40] Segerstrom, P. (1988), "Demons and repentance", Journal of Economic Theory, 45, 32-52.

[41] Shapiro, C. (1985) "Patent Licensing and R&D Rivalry", American Economic Review, 75, 25-30.

Appendix A

The following is a useful simplification of *ex ante* expected net present value of discounted cash flows in the case of a flow changing event happening at an unknown moment *t* in time with an exponential distribution of hazard rate a . It is assumed that before the event there is a constant instantaneous net cash flow of *a*, and that the event puts an end to this flow and produces a fixed flow of *b* when the event happens (or equivalently an expected discounted net present value of *b* at instant *t*). A constant discount rate of d is considered.

By conditioning on the instant *t* at which the event happens we get:

$$\underset{\substack{\text{expected value}\\ \text{of future flows}}}{V} = \int_0^\infty \left\{ \underset{\substack{\text{constant flow before event}\\ \text{between } 0 \text{ and } t}}{\int_0^t a \cdot e^{-dt}\, dt} + \underset{\substack{\text{instantaneous flow}\\ \text{received with event at } t}}{b \cdot e^{-dt}} \right\} \cdot \underset{\substack{\text{probability of event}\\ \text{happening at } t}}{a \cdot e^{-a \cdot t}} \cdot dt$$

$$V = \int_0^\infty \left\{ \frac{a}{-d} \cdot \int_0^{-dt} e^{-dt}\, d(-dt) + b \cdot e^{-dt} \right\} \cdot a \cdot e^{-a \cdot t} \cdot dt = \int_0^\infty \left\{ \frac{a}{-d} \cdot e^x \Big|_{x=0}^{x=-dt} + b \cdot e^{-dt} \right\} \cdot a \cdot e^{-a \cdot t} \cdot dt$$

$$V = \int_0^\infty \left\{ \frac{a}{-d} \cdot \left(e^{-dt} - 1\right) + b \cdot e^{-dt} \right\} \cdot a \cdot e^{-a \cdot t} \cdot dt = \int_0^\infty \left\{ \left(\frac{a}{-d} + b \right) e^{-dt} + \frac{a}{d} \right\} \cdot a \cdot e^{-a \cdot t} \cdot dt$$

If $d > 0$ and $a > 0$ then we can split this converging integral in the following two converging integrals:

$$V = \int_0^\infty \left(\frac{a}{-d} + b \right) \cdot a \cdot e^{-(a+d)t} \cdot dt + \int_0^\infty \frac{a}{d} \cdot a \cdot e^{-a \cdot t} \cdot dt$$

$$V = \left(\frac{a}{d} - b \right) \frac{a}{a+d} \int_0^{-\infty} e^{-(a+d)t} \cdot d\left[-(a+d)t\right] - \frac{a}{d} \int_0^{-\infty} e^{-a \cdot t} \cdot d\left[-at\right]$$

$$V = \left(\frac{a}{d} - b \right) \frac{a}{a+d} \int_0^{-\infty} e^x \cdot dx - \frac{a}{d} \int_0^{-\infty} e^x \cdot dx = \left[\left(\frac{a}{d} - b \right) \frac{a}{a+d} - \frac{a}{d} \right] \cdot e^x \Big|_{x=0}^{x=-\infty}$$

$$V = -\left[\left(\frac{a}{d} - b \right) \frac{a}{a+d} - \frac{a}{d} \right] = -\frac{a - b \cdot d}{d} \frac{a}{a+d} + \frac{a}{d} \frac{a+d}{a+d} = \frac{-a \cdot a + b \cdot d \cdot a + a \cdot a + a \cdot d}{d(a+d)}$$

$$V = \frac{b \cdot d \cdot a + a \cdot d}{d(a+d)} = \frac{d(a + b \cdot a)}{d(a+d)} = \frac{a + b \cdot a}{a+d}$$

$$V = \frac{a + b \cdot a}{a+d}$$

Appendix B: Proofs

Standard Result 1. *In a context with patents, there is always underprovision of original R&D in the competitive equilibrium (with respect to the socially optimal level with two firms).*

Proof.

This is equivalent to showing that $o_P^{**} > o_P^{*}$, or equivalently, because l' is a decreasing function, that $l'\left(o_P^{**}\right) < l'\left(o_P^{*}\right)$. But, from expressions (4) and (2) we have:

$$l'\left(o_P^{**}\right) < l'\left(o_P^{*}\right) \Leftrightarrow \frac{d}{p^m + S^m} < \frac{m\left(d_P^*\right) + d}{m\left(d_P^*\right)\frac{p^d}{d} + p^m} \Leftrightarrow d\left(m_P^* \frac{p^d}{d} + p^m\right) < \left(m_P^* + d\right)\left(p^m + S^m\right)$$

$$\Leftrightarrow m_P^* p^d + dp^m < m_P^* p^m + dp^m + \left(m_P^* + d\right)S^m \Leftrightarrow 0 < m_P^*\left(p^m - p^d\right) + \left(m_P^* + d\right)S^m$$

As these two last terms are positive, this last inequality holds. *Q.E.D.*

New Result 1. *If the negative externality between firms (coming from imitation) is smaller (larger) than the positive externality on consumer surplus, then there will be underprovision (overprovision) of imitative R&D in the competitive equilibrium.*

Proof.

This is equivalent to showing that $d_P^{**} > d_P^{*}$ if and only if $p^m - p^d < S^d - S^m$, or equivalently, because m' is a decreasing function, that $m'\left(d_P^{**}\right) < m'\left(d_P^{*}\right) \Leftrightarrow p^m - p^d < S^d - S^m$. But from expressions (5) and (1) we know that :

$$m'\left(d_P^{**}\right) < m'\left(d_P^{*}\right) \Leftrightarrow \frac{d}{2p^d + S^d - \left(p^m + S^m\right)} < \frac{d}{p^d} \Leftrightarrow p^d < 2p^d + S^d - \left(p^m + S^m\right) = W^d - W^m$$

$$\Leftrightarrow p^m - p^d < S^d - S^m$$ *Q.E.D.*

Standard Result 2. *In a context without patents, there is always underprovision of R&D (with respect to the socially optimal level with two firms) in the competitive equilibrium.*

Proof.

This is equivalent to showing that $r_{NP}^{*} < r_{NP}^{**}$, or equivalently, because l' is a decreasing function, that $l'\left(r_{NP}^{**}\right) < l'\left(r_{NP}^{*}\right)$. But, from expressions (8) and (6) we have:

$$l'\left(r_{NP}^{**}\right) < l'\left(r_{NP}^{*}\right) \Leftrightarrow \frac{d}{2p^d + S^d} < \frac{d}{p^d} \Leftrightarrow p^d < 2p^d + S^d \Leftrightarrow 0 < p^d + S^d$$ *Q.E.D.*

Standard Result 3. *Firms engage in more original R&D with patent protection than without.*

Proof.

This is equivalent to showing that $r_{NP}^{*} < o_{P}^{*}$, or equivalently, because l' is a decreasing function, that $l'\left(r_{NP}^{*}\right) > l'\left(o_{P}^{*}\right)$. But

$$l'\left(r_{NP}^{*}\right) > l'\left(o_{P}^{*}\right) \Leftrightarrow \frac{d}{p^{d}} > \frac{m\left(d_{P}^{*}\right)+d}{m\left(d_{P}^{*}\right)\frac{p^{d}}{d}+p^{m}} = \frac{\left(m\left(d_{P}^{*}\right)+d\right)d}{m\left(d_{P}^{*}\right)p^{d}+d\cdot p^{m}} = \frac{\left(m\left(d_{P}^{*}\right)+d\right)d}{\left(m\left(d_{P}^{*}\right)+d\right)p^{d}+d\left(p^{m}-p^{d}\right)}$$

$$\Leftrightarrow \left(m\left(d_{P}^{*}\right)+d\right)p^{d}+d\left(p^{m}-p^{d}\right) > \left(m\left(d_{P}^{*}\right)+d\right)p^{d} \Leftrightarrow d\left(p^{m}-p^{d}\right) > 0$$

Q.E.D.

Proposition 1. *The socially optimal pace of original innovation is higher in a world without patent protection than in a world with patent protection and imitation.*

Proof.

In our context this is equivalent to showing that $r_{NP}^{**} > o_{P}^{**}$, or equivalently, because l' is a decreasing function, that $l'\left(r_{NP}^{**}\right) < l'\left(o_{P}^{**}\right)$. But

$$l'\left(r_{NP}^{**}\right) < l'\left(o_{P}^{**}\right) \Leftrightarrow \frac{d}{2p^{d}+S^{d}} < \frac{d}{p^{m}+S^{m}} \Leftrightarrow p^{m}+S^{m} < 2p^{d}+S^{d} \Leftrightarrow W^{m} < W^{d}$$

So if the market duopoly is welfare improving with respect to the monopoly, then the socially optimal pace of innovation without patents will be higher. *Q.E.D.*

Appendix C: Examples

Equilibrium conditions under "fixed spending" advantage for imitators.

If the advantage of imitators over innovators is of a "fixed spending" nature, that is, if $m(x) = l(x + x_0)$, then $m'(x) = l'(x + x_0)$, and the equilibrium conditions can be rewritten:

(1) $m'(d_P^*) = \frac{d}{p^d} \Leftrightarrow l'(d_P^* + x_0) = \frac{d}{p^d}$

(2) $l'(o_P^*) = \frac{m(d_P^*) + d}{m(d_P^*)\frac{p^d}{d} + p^m} = \frac{l(d_P^* + x_0) + d}{l(d_P^* + x_0)\frac{p^d}{d} + p^m}$

(6) $l'(r_{NP}^*) = \frac{d}{p^d}$

Expressions (1) and (3) imply $l'(d_P^* + x_0) = l'(r_{NP}^*) \Leftrightarrow d_P^* + x_0 = r_{NP}^* \Leftrightarrow d_P^* = r_{NP}^* - x_0$, and this allows us to evaluate $m(d_P^*) = m(r_{NP}^* - x_0) = l(r_{NP}^* - x_0 + x_0) = l(r_{NP}^*)$ and to rewrite (2) as follows:

(2) $l'(o_P^*) = \frac{l(r_{NP}^*) + d}{l(r_{NP}^*)\frac{p^d}{d} + p^m}$

Stationarity conditions. *If* $0 < x_0 < r_{NP}^*$*, then the stationarity and non negativity constraints on hazard rates are always verified at the competitive equilibrium.*

Proof.

The stationarity condition ensure that in the long run there are always protected innovations that imitators can copy, and can be written $m(d_P^*) < l(o_P^*)$, that is, the rate of imitation must be lower than the rate of original innovation.

But $m(d_P^*) = l(r_{NP}^*) < l(o_P^*)$ (by Standard Result 3).

By observing that $d_P^* < r_{NP}^* < o_P^*$, that $r_{NP}^* > 0$ and that $x_0 < r_{NP}^*$, proofs of the non negativity constraints are straightforward. *Q.E.D.*

Ordering of preferences regarding intellectual protection system.

We know from conditions (9) and (10) in Section 4 that, for symmetric R&D expenditures:

$$W_{NP}(r) > W_P(o,d) \Leftrightarrow [l(r) - m(d)]\frac{W^d}{d} > [l(o) - m(d)]\frac{W^m}{d} + r - (o + d)$$

$$\Pi_{NP,i}^E(r) > \Pi_{O,i}^E(o) + \Pi_{I,i}^E(d) \Leftrightarrow [2l(r) - l(o) - m(d)]\frac{p^d}{d} > l(o)\frac{p^m - p^d}{m(d) + d} + r - (o + d)$$

By replacing the equilibrium values under the "fixed spending" advantage for imitators hypothesis in these equivalences we obtain:

$$W_{NP}\left(r_{NP}^*\right) > W_P\left(o_P^*, d_P^*\right) \Leftrightarrow o_P^* - x_0 > \left[l\left(o_P^*\right) - l\left(r_{NP}^*\right)\right]\frac{W^m}{d}$$

$$\Pi_{NP,i}^E\left(r_{NP}^*\right) > \Pi_{P,i}^E\left(o_P^*, d_P^*\right) \Leftrightarrow o_P^* - x_0 > \left[l\left(o_P^*\right) - l\left(r_{NP}^*\right)\right]\frac{p^d}{d} + \frac{l\left(o_P^*\right)}{l\left(r_{NP}^*\right) + d}\left(p^m - p^d\right)$$

By observing that the left hand side of both inequalities is the same, it is possible to order the preferences of society and the firms by comparing the right hand side terms only. In other words, if the following inequality holds, then $\left\{W_{NP}\left(r_{NP}^*\right) > W_P\left(o_P^*, d_P^*\right)\right\} \Rightarrow \left\{\Pi_{NP,i}^E\left(r_{NP}^*\right) > \Pi_{P,i}^E\left(o_P^*, d_P^*\right)\right\}$:

$$\left[l\left(o_P^*\right) - l\left(r_{NP}^*\right)\right]\frac{W^m}{d} > \left[l\left(o_P^*\right) - l\left(r_{NP}^*\right)\right]\frac{p^d}{d} + \frac{l\left(o_P^*\right)}{l\left(r_{NP}^*\right) + d}\left(p^m - p^d\right) \tag{C.1}$$

Example 1. *Square root R&D technologies:* $l(x) = \sqrt{x}$.

Equilibrium values are:

$$r_{NP}^* = \left(\frac{p^d}{2d}\right)^2 \Rightarrow l\left(r_{NP}^*\right) = \frac{p^d}{2d}$$

$$d_P^* = r_{NP}^* - x_0 \Rightarrow m\left(d_P^*\right) = l\left(r_{NP}^*\right) = \frac{p^d}{2d}$$

$$o_P^* = \left(\frac{p^d}{2d} + \frac{d\left(p^m - p^d\right)}{p^d + 2d^2}\right)^2 \Rightarrow l\left(o_P^*\right) = \frac{p^d}{2d} + \frac{d\left(p^m - p^d\right)}{p^d + 2d^2}$$

The increase in the hazard rate of original R&D when moving to a system with patents from a system without patent protection is in this case given by the following expression, which is as expected increasing in p^m, decreasing in p^d, and increasing in d iff $p^d > 2d^2$:

$$l\left(o_P^*\right) - l\left(r_{NP}^*\right) = \frac{d\left(p^m - p^d\right)}{p^d + 2d^2}$$

By evaluating condition (C.1) we obtain:

$$\frac{d\left(p^m - p^d\right)}{p^d + 2d^2}\frac{W^m}{d} > \frac{d\left(p^m - p^d\right)}{p^d + 2d^2}\frac{p^d}{d} + \frac{\frac{p^d}{2d} + \frac{d\left(p^m - p^d\right)}{p^d + 2d^2}}{\frac{p^d}{2d} + d}\left(p^m - p^d\right) = \frac{\left(p^m - p^d\right)}{p^d + 2d^2}\frac{2d^2\left(p^m - p^d\right)}{p^d + 2d^2}$$

$\Leftrightarrow W^m > \frac{2d^2}{p^d + 2d^2}\left(p^m - p^d\right)$. But $\frac{2d^2}{p^d + 2d^2} < 1$, so $\frac{2d^2}{p^d + 2d^2}\left(p^m - p^d\right) < \left(p^m - p^d\right) < W^m$.

Thus we demonstrate that $\{W_{NP}(r^*_{NP}) > W_P(o^*_P, d^*_P)\} \Rightarrow \{\Pi^E_{NP,i}(r^*_{NP}) > \Pi^E_{P,i}(o^*_P, d^*_P)\}$ for all values of the model parameters.

Example 2. *Logarithmic R&D technologies :* $l(x) = \ln(x+1)$.

Equilibrium values are:

$$r^*_{NP} = \frac{p^d}{d} - 1 \Rightarrow l(r^*_{NP}) = \ln\left(\frac{p^d}{d}\right)$$

$$d^*_P = r^*_{NP} - x_0 \Rightarrow m(d^*_P) = l(r^*_{NP}) = \ln\left(\frac{p^d}{d}\right)$$

$$o^*_P = \frac{p^d}{d}\frac{\ln\left(\frac{p^d}{d}\right) + d\frac{p^m}{p^d}}{\ln\left(\frac{p^d}{d}\right) + d} - 1 \Rightarrow l(o^*_P) = \ln\left[\frac{p^d}{d}\frac{\ln\left(\frac{p^d}{d}\right) + d\frac{p^m}{p^d}}{\ln\left(\frac{p^d}{d}\right) + d}\right] = \ln\left(\frac{p^d}{d}\right) + \ln\left[\frac{\ln\left(\frac{p^d}{d}\right) + d\frac{p^m}{p^d}}{\ln\left(\frac{p^d}{d}\right) + d}\right]$$

Like earlier, it is easy to compute the increase in the hazard rate of original R&D when moving to a system with patents from a system without patent protection. In this case it is also as expected increasing in p^m, decreasing in p^d, and increasing in d if $p^m, p^d > 1$:

$$l(o^*_P) - l(r^*_{NP}) = \ln\left[\frac{\ln\left(\frac{p^d}{d}\right) + d\frac{p^m}{p^d}}{\ln\left(\frac{p^d}{d}\right) + d}\right]$$

Inequality (C.1) translates into the following, which cannot be generally signed :

$$\ln\left[\frac{\ln\left(\frac{p^d}{d}\right) + d\frac{p^m}{p^d}}{\ln\left(\frac{p^d}{d}\right) + d}\right]\left[\left(\ln\left(\frac{p^d}{d}\right) + d\right)S^m + \ln\left(\frac{p^d}{d}\right)\left(p^m - p^d\right)\right] > d\ln\left(\frac{p^d}{d}\right)\left(p^m - p^d\right)$$

By taking for instance $d = 1/2$, $p^m = 450$, $p^d = 125$, and $S^m = 225$ we show that this inequality does not always hold :[43]

$$\ln\left[\frac{\ln(250) + \frac{9}{5}}{\ln(250) + \frac{1}{2}}\right]\left[225 + 1100\ln(250)\right] \overset{?}{>} 325\ln(250) \Leftrightarrow 1231.25 \overset{?}{>} 1794.48$$

[43] These values correspond to an *ex post* product market with linear, unit slope demand, and a reduced form duopoly competition with equilibrium prices between the monopoly and Bertrand values (here $p^d = \frac{1}{3}p^m$). With this specification there is a zone in which society prefers no patents but firms prefer patents.