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Optimal Commodity Grouping in a Partial Equilibrium Framework

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Pascal Belan and Stéphane Gauthier

Abstract. The literature on indirect taxation is usually concerned with the case where the number of possible di¤erent tax rates equals the number of commodities. The purpose of this paper is to characterize, in a partial equilibrium framework, which commodities should be taxed at the same rate whenever there is only one possible tax rate. It is shown that if all the consumers are endowed with the same weight by the social planner, i.e., in the representative agent con…guration, then necessities should be taxed while luxuries should be exempted from taxation. In the many-person con…guration, where di¤erent individuals have di¤erent social weights, only one commodity should be taxed; it is the one that is mainly consumed by households with low social weights.

Résumé. Dans la théorie de la taxation indirecte, on suppose qu'il existe autant de taux d'imposition possibles que de biens. Tous les biens peuvent ainsi être taxés à des taux di¤érents ; c'est en général ce qui se produit à l'optimum de Ramsey. Pourtant, la plupart des systèmes de TVA utilisent un ou deux taux d'imposition seulement, le second ne permettant de recueillir qu'une faible part de l'impôt collecté. Cet article caractérise, dans un modèle d'équilibre partiel, le groupe des biens qui devraient être taxés lorsque le plani…cateur n'a qu'un seul taux de taxe à sa disposition. Nous montrons que, si tous les individus sont traités de la même façon, les biens qui devraient être taxés sont ceux dont l'élasticité-prix est la plus faible, en conformité avec la règle de l'élasticité inverse (selon laquelle les taux d'imposition sont d'autant plus élevés que l'élasticité-prix est faible). En outre, peu de biens devraient être taxés si les élasticités-prix sont très di¤érentes les unes des autres, ou si les biens dont la demande est inélastique sont massivement consommés. Dans ce cas, les taux d'imposition qui seraient choisis à l'optimum de Ramsey sont inférieurs au taux unique que doit …xer le plani…cateur. Il est évident que, si les individus que l'Etat favorise consacrent une large part de leur revenus aux biens de première nécessité, imposer une contrainte sur le nombre de taux d'imposition conduit à accentuer le dilemme entre l'équité et l'e¢cacité auquel la société doit faire face. Lorsque des considérations d'équité sont prises en compte (le plani…cateur traitant alors les individus di¤éremment), le groupe optimal revêt en fait une forme particulièrement simple : un seul bien devrait être taxé ; il s'agit du bien le plus massivement consommé par les individus dont les poids sociaux sont les plus faibles.

1 Introduction

The literature on indirect taxation focuses on Ramsey's (1927) configuration in which the number of possible different tax rates equals the number of commodities. In this event the structure of optimal taxation is characterized by the inverse elasticity rule discussed in Baumol and Bradford (1970), as it involves tax rates being inversely proportional to the compensated price elasticity of demand for each commodity. Most countries, however, implement tax policies where the number of available tax rates is less than the number of goods. In a second best tradition this could result from administrative costs of tax collection (see, e.g., Yitzhaki, 1979).

Diamond (1973) takes into account these restrictions and derives optimal partial tax rates under the assumption that the groups of commodities taxed at the same rate are arbitrarily given. It remains, therefore, to determine the optimal grouping structure, i.e., the optimal allocation of commodities to groups. This is the aim of the present paper. Gordon (1989) provides early insights on this issue by using a tax reform methodology. He shows that it may be welfare improving to tax at the same rate consumption goods with possibly very different compensated price elasticities.[1]

This paper characterizes the optimal grouping structure in a partial equilibrium framework[2] where there is a single consumer, an arbitrary number of commodities and only one available tax rate. It highlights that both compensated price elasticities and budget shares of commodities matter for the determination of the optimal tax rule; for efficiency reasons the social planner prefers taxing commodities with large budget shares and low compensated price elasticities. Still, the optimal group is shown to be connected, in the sense that if some commodity is exempted from taxation, then any commodity with a higher compensated price elasticity is also exempted. In other

[1] See Proposition 2 in Gordon (1991), which is concerned with the class of tax reforms starting from a uniform tax rate. Note that if, however, the set of feasible reforms is restricted in a suitable way, then it is socially optimal to group commodities with similar compensated price elasticities.

[2] That is, individual preferences will be represented by a class of utility functions such that there is no income effect and the demand schedules are independent. See Atkinson and Stiglitz, 1980, Section 12.5.

words, the effect of budget shares is dominated, and all the commodities with low compensated price elasticities should be taxed while the others should be exempted at social optimum.

A consequence of this property is to concentrate taxation on necessities and to release luxuries from any tax. As a result, in order to collect a given fiscal revenue, the social planner may tax more heavily commodities with low price elasticity than recommended by the Ramsey rule; such properties should clearly strengthen the traditional conflict between efficiency and equity purposes. Actually, in a many-person framework, the optimal group is shown to consist of only one commodity. It is the one associated with the least distributional characteristic, i.e. it is such that individuals with low social marginal valuations of income do not consume this good in a large proportion. Thus one may say that the optimal group remains connected; it is connected according to distributional characteristics of commodities, and no longer according to compensated price elasticities.

The paper is organized in the following way. First it describes the optimal grouping problem. Then it characterizes the optimal group of taxed commodities in the case of a representative agent. It establishes that this group is connected according to compensated price elasticities, so that the inverse elasticity rule applies. The optimal tax structure in the many-person economy is discussed in the last part of the paper.

2 The optimal grouping problem

We consider a standard partial equilibrium analysis with n commodities and labor as numeraire. We ignore for the moment the equity viewpoint in the design of tax rules; our attention is only focused on efficiency considerations. The preferences of the representative consumer are described by

$$U(X_1, \ldots, X_n, L) = \sum_{i=1} U_i(X_i) - L, \tag{1}$$

where X_i $(i = 1, \ldots, n)$ is the amount of consumption good i purchased, and L is the amount of labor supplied. The function $U_i(\cdot)$ is increasing and concave. Following the literature on excess burden, it is assumed that producer prices are constant, and they are set at unity without loss of generality.

Hence consumer prices are $(1 + t_i)$, where t_i is the tax rate on commodity i. As usual the consumer is assumed to maximize his utility function (1) subject to the budget constraint

$$\sum_{i=1} (1 + t_i) X_i \leq L. \tag{2}$$

Let $X_i(t_i)$ and $L(t_1; \ldots; t_n)$ be the solution of the consumer program.

The classical Ramsey problem amounts to choose the tax rates maximizing welfare $U(X_1(t_1); \ldots; X_n(t_n); L(t_1; \ldots; t_n))$ under the constraint that the government collects a certain fiscal revenue R, i.e.,

$$\sum_{i=1} t_i X_i(t_i) \geq R. \tag{3}$$

It is well known (see Atkinson and Stiglitz, 1980, Lecture 12) that the solution to this problem involves the tax rate being in inverse proportion to the price elasticity of demand $\varepsilon_i(t_i) = -(1 + t_i) X_i'(t_i) / X_i(t_i)$. Hence Ramsey tax rates are not uniform whenever commodities differ according to price elasticities.

In this paper, we depart from this framework in restricting the number of possible different tax rates. We actually consider that some commodities are taxed at a unique rate, while the others (and labor) are exempted from taxation.[3] Let G denote the group of taxed commodities, i.e., $t_i = t > 0$ if $i \in G$, and $t_i = 0$ otherwise. Let also

$$\varepsilon^G(t) \equiv \sum_{i \in G} \frac{\pi_i(t)}{\pi^G(t)} \varepsilon_i(t) \tag{4}$$

[3] Most of value-added tax systems use two different tax rates, a normal and a reduced one. In general, however, the first rate allows to collect most of the tax, which may justify our assumption of a single tax rate. The main VAT rate is charged at 17.5% in the UK, and at 19,6% in France (it yields about 90% of the french VAT tax). The United States relies on a system of retail sales taxes defined at the state level. Although the US have a low reliance on such taxes, some have recently proposed to introduce a single tax rate on sales on consumption at 15% (National Retail Sales Tax Act of 1997). It has been argued by opponents to this reform that this would raise the tax burden on low and middle income households while sharply cut taxes on rich individuals. From the efficiency viewpoint, our results actually tend to confirm this view if wealth is unevenly distributed, so that there is a large proportion of low income consumers.

be the price elasticity of group G, where

$$\pi_i(t) \equiv \frac{(1+t)X_i(t)}{\sum_{j\in G}(1+t)X_j(t) + \sum_{j\notin G} X_j(0)}, \tag{5}$$

$$\text{and} \quad \pi^G(t) \equiv \sum_{i\in G} \pi_i(t). \tag{6}$$

stand for the budget share of commodity i and group G, respectively. For a given group G, the government constraint (3) at equality implicitly defines the tax rate $t^G(R)$ as a function of fiscal revenue. By definition, the optimal group G^* maximizes

$$V(R;G) \equiv U(X_1(t_1^G(R)); \dots; X_n(t_n^G(R)); L(t_1^G(R); \dots; t_n^G(R)),$$

with the restriction that $t_i^G(R) = t^G(R) > 0$ if $i \in G$, and $t_i^G(R) = 0$ if $i \notin G$.

3 Characterization of the optimal group

Our first result allows us to compare social welfare for two different groups of taxed commodities. It is derived under the assumption that fiscal revenue is low enough, i.e. in the so-called small government case.

Lemma 1 Let R be arbitrarily close to 0. Consider two groups G_1 and G_2 of taxed commodities. Then $V(R;G_1) > V(R;G_2)$ if and only if

$$\frac{\varepsilon(G_1)}{\pi(G_1)} < \frac{\varepsilon(G_2)}{\pi(G_2)},$$

where $\varepsilon(G_i)$ and $\pi(G_i)$ stand for $\varepsilon^{G_i}(0)$ and $\pi^{G_i}(0)$, respectively.

Proof. Observe that if R is close enough to 0, then the shadow price of fiscal revenue is independent of the group of commodities taxed at a positive rate. Indeed $V'(R;G)$, the first derivative of indirect utility function V with respect to fiscal revenue R, is equal to

$$\frac{-\sum_{i\in G} X_i(t^G(R))}{\sum_{i\in G} X_i(t^G(R)) + t^G(R)\sum_{i\in G} X_i'(t^G(R))}.$$

By (3), we have $t^G(0) = 0$ for $R = 0$. Thus $V'(0;G) = -1$ whatever G is. This shadow price represents the social marginal loss of raising one additional unit of fiscal liabilities; when there is an increase in fiscal revenue, the lower the increase in this price, the higher social welfare is. The condition for $V(R;G_1) > V(R;G_2)$ is consequently $V''(R;G_1) > V''(R;G_2)$, where $V''(R;G)$ stands for the second derivative of V with respect to R. It is then straightforward to show that, for $R = 0$, and so $t^G(R) = 0$,

$$V''(0;G) = \left(\sum_{i \in G} X_i(0)\right)^{-2} \left(\sum_{i \in G} X'_i(0)\right).$$

Evaluated at $t^G(R) = 0$, the compensated price elasticity of commodity i equals $-X'_i(0)/X_i(0)$. Thus, after using both (4), (5) and (6), $V''(R;G_1) > V''(R;G_2)$ rewrites as in Lemma 1, which concludes the proof. ■

When a constraint on the number of possible different tax rates is taken into account, the fact that a commodity has to be taxed at the social optimum depends not only on compensated price elasticities, but also on budget shares. Of course, it is always welfare improving to tax a commodity j with a low compensated price elasticity, as this makes higher the budget share of the group of taxed commodities and lower the price elasticity of this group. More precisely, if $\varepsilon_j < \varepsilon(G)$, then $V(R;G \cup \{j\}) > V(R;G)$. For any other commodity, the effect on social welfare is a priori ambiguous since introducing a commodity j with $\varepsilon_j > \varepsilon(G)$ increases simultaneously the budget share and the price elasticity of the group of taxed commodities. Our next result is precisely concerned with such commodities.

Corollary 2 Let R be arbitrarily close to 0. Consider a group G of commodities taxed at a positive rate, and a commodity $j \notin G$ such that $\varepsilon_j > \varepsilon(G)$. Then, introducing commodity j into G is welfare improving, i.e. $V(R;G \cup \{j\}) > V(R;G)$ if and only if $\varepsilon_j < \varepsilon(G) + \varepsilon(G \cup \{j\})$.

Proof. If directly follows from Lemma 1 that $V(R;G \cup \{j\}) > V(R;G)$ holds true if and only if $\pi(G)\varepsilon(G \cup \{j\}) < \pi(G \cup \{j\})\varepsilon(G)$, or equivalently,

$$\pi(G)\left(\frac{\pi(G)\varepsilon(G) + \pi_j\varepsilon_j}{\pi(G) + \pi_j}\right) < (\pi(G) + \pi_j)\,\varepsilon(G).$$

After straightforward manipulations, this condition rewrites

$$(\pi(G) + \pi_j)(\varepsilon_j - \varepsilon(G)) < \pi_j \varepsilon_j + \pi(G)\varepsilon(G).$$

Using (4), (5) and (6) leads to the result. ■

Hence, a commodity with high compensated price elasticity is taxed only if its budget share is large enough, and one could expect the optimal group G^* to be not connected if the effect of budget share is strong enough to overcome the effect of compensated price elasticity. Some commodities would be then exempted from taxation, while commodities with higher price elasticity would be taxed. This event does not arise, however, as Proposition 3 below shows.

Proposition 3 *Let R be arbitrarily close to 0. Consider a group G of commodities taxed at a positive rate, and a commodity $k \notin G$ with $\varepsilon_k > \varepsilon(G)$. Suppose that introducing commodity k into the group G is welfare improving, i.e. $V(R; G \cup \{k\}) > V(R; G)$. Then introducing any commodity $j \notin G$ with $\varepsilon_j < \varepsilon_k$ into G is also welfare improving, i.e. $V(R; G \cup \{j; k\}) > V(R; G \cup \{k\})$.*

Proof. Suppose that $V(R; G \cup \{k\}) > V(R; G)$. It follows from Corollary 2 that $\varepsilon_k < \varepsilon(G) + \varepsilon(G \cup \{k\})$. Since $\varepsilon_k > \varepsilon(G)$, we have also that $\varepsilon(G) < \varepsilon(G \cup \{k\})$, which implies in particular

$$\varepsilon_k < \varepsilon(G) + \varepsilon(G \cup \{k\}) < 2\varepsilon(G \cup \{k\}). \tag{7}$$

Observe now that $V(R; G \cup \{j; k\}) > V(R; G \cup \{k\})$ rewrites, using Lemma 1, $\pi(G \cup \{k\})\varepsilon(G \cup \{j; k\}) < \pi(G \cup \{j; k\})\varepsilon(G \cup \{k\})$, which is equivalent to

$$\left(2 + \frac{\pi_j}{\pi(G) + \pi_k}\right)\varepsilon(G \cup \{k\}) > \varepsilon_j. \tag{8}$$

We now proceed by contradiction to show that $V(R; G \cup \{k\}) > V(R; G)$ implies $V(R; G \cup \{j; k\}) > V(R; G \cup \{k\})$. Assuming $V(R; G \cup \{j; k\}) \leq V(R; G \cup \{k\})$, it follows from (8) that

$$\varepsilon(G \cup \{k\}) \leq \frac{1}{2}\varepsilon_j < \frac{1}{2}\varepsilon_k,$$

which is inconsistent with (7), and thus concludes the proof. ■

The optimal grouping structure does not violate Ramsey primary insights: it is connected in the sense that if some commodity is exempted from taxation, then any commodity with a higher compensated price elasticity is also exempted. Moreover, by Corollary 2, the optimal group involves taxing all the commodities with a low price elasticity, while the others are not subject to taxation.

This observation provides a simple method to determine the optimal grouping structure. Let us rank consumption goods in the order of increasing elasticity of demand, i.e. $\varepsilon_i < \varepsilon_j$ whenever $i < j$ $(i, j = 1, \ldots, n)$. Then Proposition 3 implies that there is a unique commodity s^* $(1 \le s^* \le n)$ such that $t_i^{G^*}(R) = t^{G^*}(R) > 0$ for $i \le s^*$ and $t_i^{G^*}(R) = 0$ for $i > s^*$. By Corollary 2, for the optimal grouping structure, the threshold s^* is the lowest index such that the inequality

$$\frac{\varepsilon_i}{\varepsilon_1} - \frac{\pi_i}{\pi_1} > 2. \tag{9}$$

is satisfied for any $i > s^*$.

Since commodity 1 is necessarily taxed at the social optimum, it is natural to find that ε_1 and π_1 play the role of an anchor in the tax system. Namely, from (9), if compensated price elasticities are high with respect to the elasticity of commodity 1, then the scope of taxation, measured by the optimal number of commodities taxed at a positive rate, tends to shrink. More generally, there is a narrower scope for taxation whenever the difference between high and low price elasticities is widening.[4] On the other hand, the optimal number of taxed commodities tends to decrease if commodities with low price elasticity have also large budget shares. Indeed, to collect fiscal liabilities, the government is forced to enlarge the basis for taxation if consumer expenditures are biased in favor of commodities with high price elasticity.

It is often argued that Ramsey taxation gives rise to a trade-off between efficiency and equity because the inverse elasticity rule implies that commodities with low price elasticity are taxed more heavily, and these commodities

[4] Of course, the constraint on the number of possible different tax rates is not relevant in case of identical price elasticities since then Ramsey tax rule recommends to tax all commodities at the same rate.

are consumed in a large proportion by poor individuals. This conflict is likely to be quite strengthened in our framework since condition (9) implies that efficiency in a poor society (with many poor and few rich individuals) involves to only tax necessities. In other words, a small government in a poor society should always exempt luxuries from taxation to enhance efficiency. Furthermore, the fact that the burden of taxation bears on low income individuals is reinforced if one considers the level of the optimal tax rate maximizing efficiency of the tax system, as our next result shows.

Proposition 4 Let R be arbitrarily close to 0. Let $\tau_i(R)$ be the Ramsey tax rate; by the inverse elasticity rule, $\tau_1(R) \geq \dots \geq \tau_n(R)$. If the number of taxed commodities s^* of the optimal group G^* is less than a threshold $\hat{s}$, then $t^{G^*}(R) > \tau_1(R)$. On the contrary, if $s^* > \hat{s}$, then $t^{G^*}(R) < \tau_1(R)$. Moreover we have $2 \leq \hat{s} \leq n - 1$.

Proof. We first prove that $t^{G_1}(R) > t^{G_2}(R)$ if $G_1 \subset G_2$ whatever G_1 and G_2 are. Since the government budget constraint is binding whatever G is, $G_1 \subset G_2$ implies

$$t^{G_1}(R) \sum_{i \in G_1} X_i(t^{G_1}(R)) > t^{G_2}(R) \sum_{i \in G_1} X_i(t^{G_2}(R)):$$

Observe now that fiscal revenue is an increasing function of tax rates whenever R is close enough to 0. Thus there exists $\hat{s}$, $1 \leq \hat{s} \leq n$, such that $t^{G^*}(R) < t_1(R)$ if and only if $s^* > \hat{s}$.

Let us now show that $\hat{s} \leq n - 1$ by contradiction. To this aim, consider the group G formed by all the n commodities, and assume that $t^G(R) > t_1(R)$. This implies

$$R = t^G(R) \sum_{i=1}^{n} X_i(t^G(R)) > \sum_{i=1}^{n} \tau_i(R) X_i(\tau_i(R)).$$

Since the last term equals R, we have $\hat{s} \leq n - 1$, which proves the claim. Finally, in order to prove that $\hat{s} \geq 2$, consider the group G formed by commodity 1 only. Then we have

$$R = t^G(R) X_1(t^G(R)) = \sum_{i=1}^{n} \tau_i(R) X_i(\tau_i(R)).$$

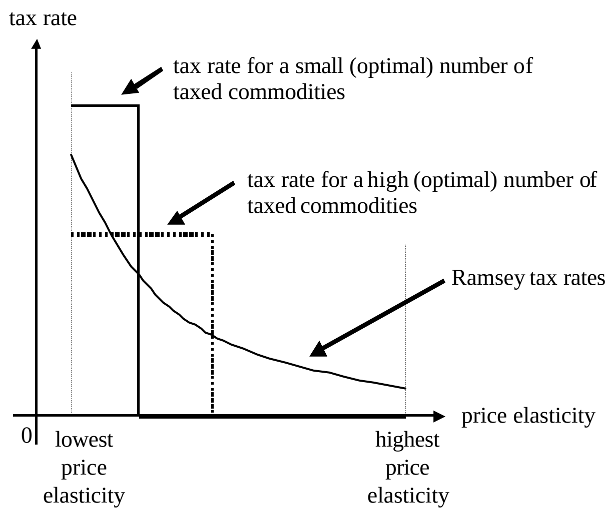

Figure 1: Optimal tax rate and Ramsey tax rates

$$\Leftrightarrow\; t^G(R)X_1(t^G(R)) - \tau_1(R)X_1(\tau_1(R)) = \sum_{i=2} \tau_i(R)X_i(\tau_i(R)) > 0.$$

Since, again, $tX_1(t)$ is increasing with t for R small enough, we have $t^G(R) > \tau_1(R)$, which completes the proof. It is worth noticing that the result actually holds independently of R if the non crazy case assumption (according to which a rise in the tax rate on one good increases fiscal revenue) is imposed. ∎

In a society where the optimal number of taxed commodities is large enough, the tax rate on necessities becomes lower than Ramsey tax rates. Given that the optimal group is connected, the social planner will compensate the fiscal loss on necessities by imposing a tax rate on goods with medium price elasticity higher than Ramsey tax rates, whereas luxuries avoid taxation in general (as depicted in Figure 1 above). The tax structure is quite different in a poor society, where necessities are the only taxed commodities and they are taxed at a higher rate than Ramsey's rule recommends (note that luxuries are still exempted from taxation). Intuitively, such properties should oppose to equity concern, and so it is reasonable to expect important changes in the characteristics of the tax policy when this kind of consideration is taken into account. In order to give a rule for the determination of the optimal group in this case, the next section introduces some heterogeneity among consumers.

4 A many-person analysis

We shall now assume that individuals differ according to their labor productivity. Let w^h stand for the wage of individual h ($h = 1, \ldots, H$). If the social planner maximizes a concave social welfare function $W(\cdot)$ of individual indirect utilities $V^h(R; G)$, then the following result provides a condition allowing us to compare social welfare for any two groups of taxed commodities. This condition relies on the social influence of individual h, measured by $\beta^h = W^h / w^h$ (where W^h stands for $\partial W / \partial V^h$), and on two new shares, $\pi^h(G)$ and $\pi(G)$. The first one is the share of expenditure devoted to a group G of taxed commodities by individual h in the aggregate consumers expenditure,

$$\pi^h(G) = \sum_{i \in G} X_i^h(0) \Big/ \sum_{h=1}^{H} \sum_{i=1}^{n} X_i^h(0).$$

The second one is the share of aggregate expenditure of commodities in G,

$$\pi(G) = \sum_{h=1}^{H} \sum_{i \in G} X_i^h(0) \Big/ \sum_{h=1}^{H} \sum_{i=1}^{n} X_i^h(0).$$

Proposition 5 Let R be arbitrarily close to 0. Consider two groups G_1 and G_2 of taxed commodities. Then we have

$$W(V^1(R; G_1), \ldots, V^H(R; G_1)) > W(V^1(R; G_2), \ldots, V^H(R; G_2))$$

if and only if

$$\sum_{h=1}^{H} \beta^h \frac{\pi^h(G_1)}{\pi(G_1)} < \sum_{h=1}^{H} \beta^h \frac{\pi^h(G_2)}{\pi(G_2)},$$

or equivalently,

$$\operatorname{cov}\left(\beta^h, \frac{H\pi^h(G_1)}{\pi(G_1)}\right) < \operatorname{cov}\left(\beta^h, \frac{H\pi^h(G_2)}{\pi(G_2)}\right).$$

Proof. The problem of consumer h is to determine a consumption bundle $(X_1^h, \ldots, X_n^h)$ and a labor supply L^h maximizing (1) under the budget constraint

$$\sum_{i=1}^{n} (1 + t_i) X_i \leq w^h L,$$

where $t_i = t > 0$ if $i \in G$ and $t_i = 0$ otherwise. The tax rate for some arbitrary grouping structure G is given by the government budget constraint

$$\sum_{i \in G} t X_i(t) \equiv R.$$

Let $t^G(R)$ be the solution of this equation. It follows that the optimal choice of consumer h only depends on R and G, so that he obtains the indirect utility level $V^h(R; G) \equiv V^h(t^G(R))$. The problem of the government is to determine the grouping structure G^* maximizing social welfare function $W(V^h(t^G(R)); \ldots; V^H(t^G(R)))$. At first order, for R small enough, social welfare equals

$$W(V^1(0); \ldots; V^H(0)) + R \sum_{h=1}^{H} \frac{\partial W}{\partial V^h}(0) \frac{dV^h}{dt}(0) \frac{dt^G}{dR}(0).$$

Thus it is welfare improving to tax commodities of a group G_1 instead of commodities of another group G_2 if and only if

$$\sum_{h=1}^{H} \frac{\partial W}{\partial V^h}(0) \frac{dV^h}{dt^{G_1}}(0) \frac{dt^{G_1}}{dR}(0) > \sum_{h=1}^{H} \frac{\partial W}{\partial V^h}(0) \frac{dV^h}{dt^{G_2}}(0) \frac{dt^{G_2}}{dR}(0).$$

Using Roy's identity, it is then readily verified that

$$\frac{dV^h}{dt}(0) = -\sum_{i \in G} \frac{X_i^h(0)}{w^h}$$

and

$$\frac{dt^G}{dR}(0) = \left(\sum_{h=1}^{H} \sum_{i \in G} X_i^h(0) \right)^{-1}.$$

This leads directly to the result. ■

In order to interpret conditions given in Proposition 5, observe first that β^h measures the marginal increase in social welfare which results from transferring one additional unit of income to individual h. If all these social weights are equal, i.e. in the case of a representative agent, then Proposition 5 does not allow us to determine the optimal grouping structure; compensated price elasticity then matter, as underlined in Lemma 1. Nevertheless, as soon as both equity and efficiency enter the social objective, so that different individuals have different social weights, Proposition 5 shows that budget share becomes the only relevant characteristic; thus, the optimal group will be not connected according to compensated price elasticities.

In fact, the departure from this type of commodity grouping is potentially large, as introducing equity into analysis tends to favor taxation of luxuries and to exonerate goods with low price elasticity, at least whenever poor individuals mainly consume necessities. To see this point most clearly, let us notice that, from Proposition 5, the social planner prefers to tax groups of commodities that are consumed in a relatively low proportion by individuals with a high social weight. Indeed, at the optimum, the share of taxed commodities in the aggregate consumption expenditure will be typically lower for individuals with high social value than for the whole society; that is, with the notations introduced in Proposition 5, $\pi^h(G)$ should be smaller than $\pi(G)$ for every consumer h endowed with a high β^h by the planner. The equity concern leads us to set social weights inversely related to labor productivity, which amounts, for the class of utility functions (1), to assign a high social weight to poor individuals. Since, for any h,

$$\pi^h(G) = \frac{\sum_{i=1}^{n} X_i^h(0)}{\sum_{h=1}^{H}\sum_{i=1}^{n} X_i^h(0)} \frac{\sum_{i \in G} X_i^h(0)}{\sum_{i=1}^{n} X_i^h(0)},$$

and given that spending of a poor individual h represents a small part of aggregate expenditure, the optimal group is such that taxed commodities are consumed in a low proportion of their income by poor individuals, and so at least a part of this group typically consists of luxuries. Actually the following corollary of Proposition 5 highlights that the optimal rule recommends to tax luxuries only.

Corollary 6 Let R be arbitrarily close to 0. If there is a social concern for equity, that is, $\beta^h \neq \beta^{h'}$ for two individuals h and h', then the optimal group G^* typically consists of only one commodity i^* which satisfies

$$\mathrm{cov}\left(\beta^h, \frac{H\pi^h(\{i^*\})}{\pi(\{i^*\})}\right) < \mathrm{cov}\left(\beta^h, \frac{H\pi^h(\{i\})}{\pi(\{i\})}\right)$$

for any $i \neq i^*$.

Proof. It directly follows from Proposition 5 that the optimal group G^* is such that, for any G,

$$\sum_{h=1}^{H} \beta_h \frac{\pi^h(G^*)}{\pi(G^*)} \leq \sum_{h=1}^{H} \beta_h \frac{\pi^h(G)}{\pi(G)}.$$

Since, for any G, we have

$$\sum_{h=1}^{H} \beta_h \frac{\pi^h(G)}{\pi(G)} = \sum_{i \in G} \frac{\pi(\{i\})}{\pi(G)} \left[\sum_{h=1}^{H} \beta_h \frac{\pi^h(\{i\})}{\pi(\{i\})} \right] \geq \inf_{i \in G} \left[\sum_{h=1}^{H} \beta_h \frac{\pi^h(\{i\})}{\pi(\{i\})} \right],$$

the optimal group consists of commodity i^* such that, for any i,

$$\sum_{h=1}^{H} \beta_h \frac{\pi^h(\{i^*\})}{\pi(\{i^*\})} \leq \sum_{h=1}^{H} \beta_h \frac{\pi^h(\{i\})}{\pi(\{i\})}.$$

Observe now that

$$\sum_{h=1}^{H} \beta_h \frac{\pi^h(\{i^*\})}{\pi(\{i^*\})} = \operatorname{cov}\left(\beta_h; \frac{H\pi^h(\{i^*\})}{\pi(\{i^*\})} \right) + \frac{1}{H} \sum_{h=1}^{H} \beta_h,$$

where β^h is independent of G for R close enough to 0. This leads to the result. ■

Therefore, if there is only one available tax rate, the social planner should tax only one commodity;[5] this commodity is mainly consumed by individuals endowed with low social weights, in the sense that it is associated with the least Feldstein's (1972) distributional characteristic (see, e.g. Atkinson and Stiglitz, 1980, Section 12.5). It directly follows that the optimal group is connected according to distributional characteristics of commodities.

5 Conclusion

This paper characterizes the optimal tax rule in presence of a constraint on the number of possible different tax rates. This rule depends on a complex

[5] Note that in the particular event where several goods satisfy the optimality criterion given in Corollary 6, any of these goods should be taxed.

interaction between compensated price elasticities and budget shares. In the case of a representative agent, the optimal group of taxed commodities is shown to be connected according to compensated price elasticities; namely, goods with low price elasticities should always be taxed whereas luxuries will eventually be exempted. The more the individual consumes necessities, the smaller the optimal number of taxed commodities is; taxation is then restricted to necessities. This kind of tax structure does not accord with equity considerations, which recommends in general to tax some luxuries. Indeed, in the many-person framework, a single commodity should be taxed; it is the one with the lowest distributional characteristic. As a result, the optimal group of taxed commodities is connected according to distributional characteristics of commodities, and not according to their price elasticity.

The attention of the paper is focused on the particular case of a small government, where collected fiscal liabilities are low enough. So far we did not succeed to relax this assumption in order to compare social welfare for any two different groups of taxed commodities. Of course, it would be worth to analyze how tax structure is changed whenever fiscal liability is set arbitrarily; the framework could also be extended to the case where demand schedules are no longer independent and income effects matter. Still, to our view, the optimal commodity grouping characterized in the paper could already be used to get preliminary insights into actual preferences of governments in the design of tax rules. In fact, given the large number of commodities that usually enter VAT systems in developed countries, one may conjecture that indirect taxation weights heavily efficiency and tends, to a large extent, disregards equity considerations.

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