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The revenue-effect of the buyer's option in multi-unit ascending auctions: the case of wine auctions at Drouot*

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Résumé

Cet article présente un modèle d'une vente aux enchères multi-unitaires avec option d'achat. L'option d'achat permet au vainqueur d'une enchère de choisir le nombre d'unités qu'il souhaite acquérir, en payant pour chaque unité le prix atteint à l'enchère. Nous déterminons la stratégie optimale des enchérisseurs. Le modèle est estimé à partir des résultats de plusieurs ventes de vin à Drouot. Des simulations montrent que le revenu (hypothétique) qu'aurait eu Drouot s'il avait vendu les biens séquentiellement (les unités sont alors vendues les unes après les autres) est identique à celui obtenu en utilisant le mécanisme avec option d'achat.

Abstract

This paper presents a model of bidding behavior in multi-unit ascending (English) auctions with a buyer's option. The buyer's option gives the winner of an auction the right to purchase any number of units at the winning price. We derive the optimal strategies for the bidders. The model is estimated using a unique data set on wine auctions held at the Paris-based auction house Drouot. A counterfactual comparison shows that the seller's revenue in a system where items are auctioned sequentially (i.e. one after the other) is the same as in a system based on the buyer's option.

1 Introduction

Whenever several *fully identical* goods have to be sold, many auction houses use the following auction mechanism. The first unit of the good is auctioned. Once this first auction is over, the winner of the auction is given the possibility to exercise a so-called buyer's option. The buyer's option gives him the right to buy any number of units of the good he desires, the price per unit being the price established at the first auction. In case the winner decides to purchase all units, the sale is over. If he decides to buy only part of the total quantity of goods on sale, a second auction is held for the remaining units. And this scheme repeats itself until all units are eventually sold. As an illustration, suppose there are initially 2 units of a particular good on sale, and let the first-auction price be p. The winning bidder then has the choice between buying just one unit (in which case he pays p to the seller) or both units (in which case he pays $2p$). If the winner does not exercise his option, another auction is organized for the second unit. It is important to note that the first-auction winner is allowed to participate and compete in this second auction.

The buyer's option is used in many auctions throughout the world. At the flower auction of Aalsmeer, the Netherlands, huge quantities of flowers and garden plants are auctioned each day. The products are sold via descending (or Dutch) auctions with a buyer's option (see van den Berg, van Ours, and Pradhan (2001)). Prestigious auction houses such as Christie's and Sotheby's in the UK and the USA (see Ashenfelter (1989) and Ginsburgh (1998)), and Drouot in France, systematically use the buyer's option in their ascending (or English) auctions of wine and champagne. Cassady (1967, pp. 154-156) gives historical examples of auctions where the buyer's option or variants of this mechanism have been practiced (fur auctions in Leningrad and London; fish auctions in English port markets).

A question that arises is why these auction houses sell their goods with the buyer's option arrangement, and not via some other auction mechanism? What explains that in the above examples the auction mechanism with a buyer's option is apparently preferred over a mechanism where all units are sold sequentially, i.e. one after the other, via independent auctions? Or, alternatively, what explains that the auction houses do not sell all available units simultaneously, in a single one-shot auction? According to Ashenfelter (1989), auctioneers use the buyer's option as a device to conceal the declining price anomaly. This phenomenon is typically observed when identical objects are auctioned sequentially. Auctioneers are afraid that bidders find something fishy about the fact that identical objects do not fetch identical prices. By introducing a buyer's option, auctioneers render any possible price decline less transparent, and they can thereby limit the extent to which bidders become aware of the declining price anomaly.

Cassady (1967) gives another explanation for the fact that the buyer's option arrangement is so popular among auctioneers. He argues that an auction mechanism based on the buyer's option is more flexible than a one-shot auction, and that it is faster than a pure sequential auction. Indeed, an auction wherein all units are sold simultaneously is not flexible in the sense that it may not fit the "small" buyers. Small buyers are not much interested in the total quantity of units on sale, and might not want to participate in a one-shot auction. An auction procedure that includes a buyer's option, however, accommodates even the small buyers as they are allowed to purchase any number of units, provided of course that they manage to win an auction. On the other hand, a sequential auction can be very time-consuming, especially when the total number of units is large, and according to Cassady a motive for auctioneers to introduce a buyer's option is to accelerate the auction operations. The auctioneers with whom we spoke at Drouot, the largest auction house in Paris, appear to share this point of view. They find that sequential auctions tend to take too much time, and mentioned higher speed as the main reason for making a buyer's option available.

A revenue argument is never invoked in this debate. That is, 'higher expected earnings' is never explicitly mentioned as a motive for adopting say the auction with a buyer's option, and not the pure sequential auction. Intuition suggests, however, that the expected revenue for the seller is not the same in these two auction mechanisms. This can be easily seen in our two-unit example. In a sequential auction, all agents know that if they lose the first auction, they still have a chance to win a unit in the subsequent second auction. However, when the seller provides a buyer's option, the losers do not have this second opportunity if the first-auction winner decides to exercise his buyer's option. This increased risk of ending up empty handed implies that bidding for the first unit should be more aggressive when a buyer's option is available. The revenue that the seller obtains from the first unit is therefore likely to be higher in an auction with the buyer's option. But the seller's revenue obtained from the second unit is expected to be lower in an auction with buyer's option. Indeed, if the first-auction winner does not exercise his buyer's option, the situation is analogous to the second round in a pure sequential auction mechanism, which implies that in this case there is no effect of the buyer's option on the revenue obtained from the second unit; if, on the contrary, the winner exercises his option there is no competition for the second unit, so that in this case the buyer's option has a negative effect. Nothing can be said, in general, on the total revenue-effect (effect on the first unit plus effect on the second unit). The example merely shows that the buyer's option can affect the seller's income, and that the decision to introduce a buyer's option or not ought to play a role in the sellers' choice of auction mechanism.

The purpose of this paper is to study, both theoretically and empirically, the revenue-effect of the buyer's option in ascending (or English) auctions. That is, we study the effect on the seller's income from switching from a system where units are sold sequentially via ascending auctions to a system whereby they are sold via ascending auctions with a buyer's option. We consider an Independent Private Value (IPV) model where two identical units are sold to n risk-neutral buyers. In modeling behavior at wine auctions, the IPV paradigm was also adopted by McAfee and Vincent (1993).

It is assumed that the valuations attributed to the two units are either the same (flat demand), or that a buyer's valuation for the second unit is less than his valuation for the first unit (decreasing demand). This is exactly the framework studied by Black and De Meza (1992). Our analysis differs from Black and De Meza since we assume that the units are sold via English auctions (while they considered second-price (or Vickrey) auctions). We derive the optimal bidding strategies, and, comparing our results with those given in Black and De Meza, show that English and second-price auctions with a buyer's option are not theoretically isomorphic (except when the number of bidders equals $n = 2$). This can be explained by the fact that in an "oral" auction, such as the English auction, more information is released than in a "sealed-bid" auction, such as the second-price auction. We also derive the equilibrium bidding strategies under the assumption that the two units are auctioned sequentially without buyer's option. Given these optimal strategies, we can theoretically determine what the revenue-effect is of the buyer's option. It turns out that, depending on the precise form of the distribution function of private values and the demand function, the impact of the buyer's option can be positive or negative.

The second objective of the paper is to estimate our model using data on wine auctions. Part of the data were collected by ourselves during an important sale of fine wine held at Drouot in April 2000. As explained in the data section of the paper, by being present in the auction room, we were able to register all sorts of information that is crucial for the identification and estimation of the econometric model (such as the numbers of potential bidders in the room at different points in time, the winning bid price for each unit of wine sold at auction, and especially details on the use of the buyer's option). These auction-specific data are matched with information that was available in wine catalogues. The catalogues could be consulted by potential buyers prior to the auction and describe, for each wine on sale, its name, colour, vintage, the production region (Bordeaux, Burgundy, Côtes du Rhône, etc.), the condition of the etiquette on the bottle (whether it was readable or not, whether it was numbered or not, etc.), the level of wine in the bottle, etc... We finally complemented this unique data set by adding to the data, for each wine, the highly influential Parker's grade, and also a grade taken from a wine web site.

The fundamental parameter to be estimated in our model is the distribution function of the private values for the wines. There is evidence in our data that this distribution function can be well described by a log-normal distribution. We estimate the parameters of this distribution function (conditionally on the wine characteristics) using the maximum likelihood method. As a byproduct of this estimation, we can determine which characteristics influence the willingness to pay for wine. We thus determine how the different wine regions affect the willingness to pay for wine, and what are the effects of the condition of the label, the vintage, the Parker-rating, the level of wine in the bottle, etc... But the ultimate purpose of the estimation results is to evaluate the impact of the buyer's option. Using a counterfactual comparison based on simulations, we find that the seller's expected revenue in the pure sequential auction is the same as in the mechanism with buyer's option.

The paper proceeds as follows. The next section presents the theoretical model. Section 3 describes the wine auction data, section 4 derives the likelihood function, section 5 presents the estimation results and the revenue-comparison, and section 6 concludes.

2 A model for two-unit ascending auctions with and without the buyer's option

The two-unit auctions with and without buyer's option are modeled as non-cooperative games. First we consider the model with the buyer's option. Two units of a good are sold to $n \geq 2$ buyers. The first unit is auctioned using an English auction (ascending auction). At the end of the first auction, the winner has the possibility to use a buyer's option which allows him to purchase the second unit at the price of the first unit. If the winner uses this option, the game is ended. If the winner does not use the option, the second unit is auctioned, again via an English auction. The first-auction winner is allowed to participate in this second auction.

Adopting the IPV framework, let v_i represent the value that buyer i places on the first unit. The values v_i, $i = 1, ..., n$, are independently drawn from a distribution $F(\cdot)$ on the support $[0, +\infty[$. The associated density is denoted $f(\cdot)$, and it is assumed that this density is strictly positive on the whole support, and that beyond some large value of the valuation the density $f(\cdot)$ is a decreasing function. As usual in the IPV paradigm, only player i knows the valuation v_i. The opponents of i only know that this valuation is drawn from the commonly known distribution $F(\cdot)$.

It is assumed that the value that i places on the second unit is kv_i, with $0 < k \leq 1$. The value of k is the same for all bidders, and this is common knowledge. Note that $0 < k < 1$ implies that the second unit is valued less than the first unit (decreasing demand), and $k = 1$ implies that both units are valued the same (flat demand).

Let p_1 and p_2 be the winning prices in respectively the first and second auction. If the buyer's option is exercised we automatically have $p_2 = p_1$. The n players are supposed to be risk neutral. Thus, for a player with valuation v, the following outcomes are possible in the game: if he wins the first unit and uses the option, his utility is $(1 + k)v - 2p_1$; if he wins the first unit but does not exercise the option, he has utility $(1 + k)v - p_1 - p_2$ when he wins the second auction, and utility $v - p_1$ otherwise; if he loses the first auction but wins the second, his utility is $v - p_2$ (note that this outcome can only occur if the opponent who won the first auction does not use the buyer's option); finally, his utility equals 0 if he does not win any unit.

The above framework is identical to the one introduced by Black and De Meza (1992). There is one crucial difference however with their model and that is that we consider the case where each unit is sold via an English auction, and not, as in Black and De Meza, via a Vickrey auction (sealed-bid second price auction). For this reason, the Nash equilibrium strategy given in Proposition

1 below differs from their optimal bidding strategy (Black and De Meza, 1992, Proposition 5, page 613).

Proposition 1. *The following strategy forms a bayesian equilibrium of the game:*

1. *In the first auction:*

   - *As long as there are at least three active agents, each bidder i should bid up to his valuation v_i.*
   - *Once the $(n-2)$ "smallest" bidders have dropped out of the game, each of the two remaining players should bid according to the strategy $b(\cdot)$. The form of $b(v)$ depends on whether v is smaller or larger than $\frac{v_{(n-2)}}{k}$, with $v_{(n-2)}$ being the third largest valuation:*

   $$b(v) = \begin{cases} c(v) \text{ if } v_{(n-2)} \leq v \leq \frac{v_{(n-2)}}{k} \\ \\ d(v) \text{ if } v \geq \frac{v_{(n-2)}}{k}. \end{cases}$$

   The strategies $c(\cdot)$ and $d(\cdot)$ are the solutions of the following differential equations

   $$\begin{aligned} k(c(v) - v_{(n-2)})f(v) &= (v - c(v))f(c(v)/k)c'(v) \text{ if } v_{(n-2)} \leq v \leq \frac{v_{(n-2)}}{k} \\ k(d(v) - kv)f(v) &= (v - d(v))f(d(v)/k)d'(v) \text{ if } v \geq \frac{v_{(n-2)}}{k}. \end{aligned}$$

2. *The strategy $d(\cdot)$ is the unique solution of the second differential equation defined on $[\frac{v_{(n-2)}}{k}, +\infty[$, and $c(\cdot)$ is the unique solution of the first differential equation that verifies the conditions $c(\frac{v_{(n-2)}}{k}) = d(\frac{v_{(n-2)}}{k})$ and $c(v_{(n-2)}) = v_{(n-2)}$.*

3. *At equilibrium, the first-auction winner exercises the buyer's option iff $kv > p_1$, that is iff his second valuation is above the first-auction winning price.*

4. *In the second auction each bidder should bid up to his valuation. The first-auction winner (who has not exercised the buyer's option!) should thus participate until the price has reached the value he attributes to having a second unit, while all other bidders should continue until the price reaches their valuation for the first unit.*

Proof. The fourth claim in the proposition follows immediately from the standard dominated-strategies argument: in the second auction, players have a dominant strategy that consists in bidding until their valuation.

The proof of the third claim is also immediate. Because players bid more aggressively in the second auction than in the first one ($c(v) \leq v$; $d(v) \leq v$), the winner in the first auction must use his option whenever this decision implies a direct gain, i.e., iff his second valuation is higher than the first-auction equilibrium price.

Let us now turn to the proof of the first claim. Consider first the "final" stage of the first auction, that is the game once there are only 2 bidders left. We suppose, for the moment, that the players have not deviated from the optimal strategy *before* the final stage. We thus suppose that the $(n-2)$ smallest bidders have stopped bidding once the price reached their valuation, and that the 2 remaining bidders have valuations above $v_{(n-2)}$.

Suppose the ascending auction has reached the price p. Let $G(0,p)$ denote the expected total gain (gain in first plus second auction) for a player with valuation v who decides to withdraw at p. His opponent thus wins the first unit at the price p. The form of $G(0,p)$ depends on whether p is smaller or larger than $b(v_{(n-2)}/k)$:

$$\begin{aligned} G(0,p) &= \int_{b^{-1}(p)}^{p/k} (v - \max(v_{(n-2)}, kw)) \frac{f(w)dw}{1-F(b^{-1}(p))} \\ &= \int_{b^{-1}(p)}^{v_{(n-2)}/k} (v - v_{(n-2)}) \frac{f(w)dw}{1-F(b^{-1}(p))} \\ &\quad + \int_{v_{(n-2)}/k}^{p/k} (v - kw) \frac{f(w)dw}{1-F(b^{-1}(p))} \text{ if } b^{-1}(p) \le v_{(n-2)}/k \end{aligned}$$

and

$$G(0,p) = \int_{b^{-1}(p)}^{p/k} (v - kw) \frac{f(w)dw}{1-F(b^{-1}(p))} \text{ if } b^{-1}(p) \ge v_{(n-2)}/k.$$

The expression for $G(0,p)$ can be explained as follows. Clearly the first-auction loser (who decided to drop out at p) can only win something in the game if there is a second auction. A second auction takes place only if the first-auction winner does not exercise his option, that is if his valuation is smaller than p/k (note that the winner's valuation is necessarily above $b^{-1}(p)$). The first-auction loser then automatically wins the second auction at the price equal to the maximum of $v_{(n-2)}$ and k times the valuation of the winner. Note that the density in the above integrals is the conditional density of the valuation given that the valuation is larger than $b^{-1}(p)$.

Let us now again assume that the ascending auction has reached p, and let $G(\varepsilon,p)$ now be the expected total gain if the player with valuation v decides that he will stop participating once the auction has reached $p+\varepsilon$ (whereas $G(0,p)$ denotes the expected gain when he decides to stop *immediately*):

$$
\begin{aligned}
G(\varepsilon,p) &= \int_{b^{-1}(p)}^{b^{-1}(p+\varepsilon)} (v-b(w))\frac{f(w)dw}{1-F(b^{-1}(p))} + \int_{b^{-1}(p+\varepsilon)}^{v_{(n-2)}/k} (v-v_{(n-2)})\frac{f(w)dw}{1-F(b^{-1}(p))} \\
&+ \int_{v_{(n-2)}/k}^{(p+\varepsilon)/k} (v-kw)\frac{f(w)dw}{1-F(b^{-1}(p))} \text{ if } b^{-1}(p+\varepsilon) \le v_{(n-2)}/k
\end{aligned}
$$

and

$$
\begin{aligned}
G(\varepsilon,p) &= \int_{b^{-1}(p)}^{b^{-1}(p+\varepsilon)} (v-b(w))\frac{f(w)dw}{1-F(b^{-1}(p))} \\
&+ \int_{b^{-1}(p+\varepsilon)}^{(p+\varepsilon)/k} (v-kw)\frac{f(w)dw}{1-F(b^{-1}(p))} \text{ if } b^{-1}(p+\varepsilon) \ge v_{(n-2)}/k.
\end{aligned}
$$

The first part in the expression of $G(\varepsilon,p)$ corresponds to the total expected gain of the player if he wins the first auction. The player with valuation v wins the first auction if the valuation of his competitor is between $b^{-1}(p)$ and $b^{-1}(p+\varepsilon)$. It is easy to verify that, at the equilibrium, he will not exercise the buyer's option and he will not win the second auction. The remaining term(s) in $G(\varepsilon,p)$ correspond to the total expected gain if the valuation of the competitor exceeds $b^{-1}(p+\varepsilon)$. The agent with valuation v can then only hope to win the second auction and his total expected gain is determined similarly as above (indeed the expression of his gain in this case is very similar to $G(0,p+\varepsilon)$).

Derivation of $G(\varepsilon,p)$ with respect to ε gives

$$
\begin{aligned}
\frac{\partial G}{\partial \varepsilon}(\varepsilon,p) &= (b^{-1})'(p+\varepsilon)\frac{(v-(p+\varepsilon))f(b^{-1}(p+\varepsilon))}{1-F(b^{-1}(p))} \\
&-(b^{-1})'(p+\varepsilon)\frac{(v-v_{(n-2)})f(b^{-1}(p+\varepsilon))}{1-F(b^{-1}(p))} \\
&+\frac{\frac{1}{k}(v-(p+\varepsilon))f((p+\varepsilon)/k)}{1-F(b^{-1}(p))} \text{ if } b^{-1}(p+\varepsilon) \le v_{(n-2)}/k
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial G}{\partial \varepsilon}(\varepsilon,p) &= (b^{-1})'(p+\varepsilon)\frac{(v-(p+\varepsilon))f(b^{-1}(p+\varepsilon))}{1-F(b^{-1}(p))} \\
&-(b^{-1})'(p+\varepsilon)\frac{(v-kb^{-1}(p+\varepsilon))f(b^{-1}(p+\varepsilon))}{1-F(b^{-1}(p))} \\
&+\frac{\frac{1}{k}(v-(p+\varepsilon))f((p+\varepsilon)/k)}{1-F(b^{-1}(p))} \text{ if } b^{-1}(p+\varepsilon) \ge v_{(n-2)}/k.
\end{aligned}
$$

The equilibrium condition can be written as

$$\frac{\partial G}{\partial \varepsilon}(\varepsilon = 0, p = b(v)) = 0.$$

First we consider the equilibrium condition in the case $b^{-1}(p+\varepsilon) \leq v_{(n-2)}/k$. At $(\varepsilon = 0, p = b(v))$ this bound becomes $v \leq v_{(n-2)}/k$, so that in this case $b(v) = c(v)$. Note also that $v_{(n-2)} < v$. We have

$$\frac{\partial G}{\partial \varepsilon}(\varepsilon = 0, p = c(v)) = \frac{1}{c'(v)}(v_{(n-2)} - c(v))\frac{f(v)}{1-F(v)} + \frac{1}{k}(v - c(v))\frac{f(c(v)/k)}{1-F(v)} = 0$$

which can be rewritten as

$$k(c(v) - v_{(n-2)})f(v) = (v - c(v))f(c(v)/k)c'(v) \text{ if } v_{(n-2)} \leq v \leq \frac{v_{(n-2)}}{k}.$$

Next we consider the equilibrium condition in the case $b^{-1}(p+\varepsilon) \geq v_{(n-2)}/k$. At $(\varepsilon = 0, p = b(v))$ this bound becomes $v \geq v_{(n-2)}/k$, so that in this case $b(v) = d(v)$. We have

$$\frac{\partial G}{\partial \varepsilon}(\varepsilon = 0, p = d(v)) = \frac{1}{d'(v)}(kv - d(v))\frac{f(v)}{1-F(v)} + \frac{1}{k}(v - d(v))\frac{f(d(v)/k)}{1-F(v)} = 0$$

which can be rewritten as

$$k(d(v) - kv)f(v) = (v - d(v))f(d(v)/k)d'(v) \text{ if } v \geq \frac{v_{(n-2)}}{k}.$$

To end the proof of the first claim, we must now verify that it is not profitable for the players to deviate from the optimal strategy before the final stage of the first auction. We thus have to show that as long as there are at least three persons actively bidding, each bidder should bid up to his valuation. To show this, we first consider the deviation that consists in bidding *less* than one's valuation when two other players are still active. This strategy is clearly dominated by the following strategy (that we shall call "strategy s"): stop participating in the auction either when the price has reached the valuation or when one of the two remaining opponents has stopped bidding. Both strategies (the strategy that consists in bidding less than the valuation and "strategy s") lead to the same first-auction winner; similarly, under both strategies the identity of the second-auction winner is identical. However, "strategy s" leads to a higher first-auction equilibrium price, which reduces the probability that the first-auction winner uses the buyer's option, and the expected payoff is thus higher. The "strategy s" is in turn dominated by the strategy given in Proposition 1. We can therefore conclude that it is not profitable to quit the auction before the price reaches one's valuation if there are still at least two other players bidding. Next we have to consider the deviation that consists in bidding *above* one's valuation when at least two other players are active. This is clearly not a profitable deviation since at equilibrium the two remaining players have a greater valuation than the deviating agent, who is therefore sure to lose the second auction. The deviating agent can win the first auction but the profit he obtains is negative. We can therefore conclude that it is not in the interest of an agent to bid above the valuation when at least two other bidders still participate in the game.

Finally, the proof of the second claim is given in the appendix.

□

Unlike the standard IPV auction for a single unit, it turns out that in a sequential auction with buyer's option the English and second-price auctions are not theoretically isomorphic. This follows from the fact that the English auction mechanism reveals more information about competitors than the second-price auction.[1] Indeed, contrary to the sealed-bid auction, bidders in the oral auction can learn at which prices their competitors drop out. They learn in particular at which price the person with the third highest valuation abandons the first auction, and, given the equilibrium strategy, they thereby know the valuation $v_{(n-2)}$ of the third largest bidder. The two strongest competitors use this information in updating their optimal bidding strategy. The only case where the English and second-price auctions are strategically equivalent is when there are just 2 bidders. That the strategy obtained by Black and De Meza and our's coincide in this case is quite intuitive because $n = 2$ implies $v_{(n-2)} = 0$.

Next we study the pure sequential auction model. The framework described above remains exactly the same except that the 2 units are now always sold sequentially via two English auctions.

Proposition 2. *The following strategy forms a bayesian equilibrium of the game:*

1. *In the first auction:*
   - *As long as there are at least three active agents, each bidder i should bid up to his valuation v_i.*
   - *Once the $(n-2)$ "smallest" bidders have dropped out of the game, each of the two remaining players should bid according to the strategy:*

$$b(v) = \begin{cases} v_{(n-2)} \ \textit{if} \ v_{(n-2)} \leq v \leq \frac{v_{(n-2)}}{k} \\ \\ kv \ \textit{if} \ v \geq \frac{v_{(n-2)}}{k}. \end{cases}$$

2. *In the second auction each bidder should bid up to his valuation. The first-auction winner should thus participate until the price has reached the value he attributes to having a second unit, while all other bidders should continue until the price reaches their valuation for the first unit.*

[1] This fundamental difference between the two auction mechanisms also explains why in a common value model the optimal bidding strategies are no longer identical (see for instance Milgrom and Weber (1982)).

Proof. The proof is similar to the proof of Proposition 1. □

Let us now turn to the revenue-effect of the buyer's option. To do this one should calculate the revenue with and without buyer's option using the equilibrium strategies that we have just found, and compare the two outcomes. However, given the form of the equilibrium strategies with buyer's option, it is quite impossible to find a theoretical result and one has to use simulations to compare the revenue with and without buyer's option. Such simulations show that there is no general result about the revenue effect. That is, we can not say in general that one of the mechanisms dominates the other. The comparison between the two mechanisms depends on the distribution function considered and on the value of the parameter k. For example, if the distribution function is uniform, the revenue of the seller is greater with a buyer's option. To answer the question on the effect of the buyer's option at Drouot one cannot therefore rely on theory alone. The data should determine which is the appropriate distributing function and the relevant value of the demand parameter k. Using the estimates of the fundamental parameters then allows to determine which is the best mechanism for the seller.

3 The data

The data are based on auctions that were held at the Paris-based auction house Drouot. The wine auctions took place in the afternoon of 20 April (from 14:15 to 16:45), and in the morning of 21 April (from 11:15 to 12:30), and were headed by the same auctioneer. The auctioneers of Drouot organize wine auctions on almost every working day of the year. The two sale days were chosen arbitrarily by us, so that *a priori* there is no reason to think that the auction results are in some way specific or non-representative.

Auctions at Drouot are open to the public and anyone who wishes to attend them can do so. Two authors of this paper were present in the auction room of Drouot during the sales of April 20 and 21. They recorded all relevant information released during the auctions (number of bidders, winning prices, identity of the winners, use of the buyer's option, etc.). We added to these auction-specific data the information published in the wine catalogue of the sales. The catalogue could be consulted prior to the sales and record the precise characteristics of each wine on sale. We finally complemented the data set by adding for each wine two indicators for quality. One indicator is the grade assigned by the highly influential wine connoisseur Robert Parker, and the other is taken from a wine web site.

The next three subsections describe these three different sorts of information in our data set. The fourth subsection describes the link between the theoretical model and the real-life auctions as they were held at Drouot.

3.1 Auction-specific information

During the 2 sale days different sorts and kinds of wine were auctioned. The notion of "unit" that we have used so far in this paper can vary from wine to wine. In our data there is for instance a unit of wine consisting of a crate of 12 bottles of *Château Latour 1985*, a unit that is made up of 6 bottles of *Château Lafite Rothschild 1890*, a unit made up of a single bottle of *La Romanée Conti 1976*, etc... In the sequel, a "lot" of wine designates a group of strictly identical units of wine. In our data the lot of *Château Latour 1985* has 3 units (3 identical crates of 12 bottles), the lot of *Château Lafite Rothschild 1890* consists of 4 units (4 units of 6 bottles), and the lot of *La Romanée Conti 1976* only has 1 unit (of 1 bottle).

During the 2 sale days a total of 225 lots of wine were sold. The 225 lots were made up of 413 units, thus the average number of units per lot is 1.8. Table 1 gives the empirical distribution of the number of units per lot.

Table 1. Number of units per lot

Number of units	Number of lots
1	152
2	31
3	21
4	8
5	1
6	3
7	2
8	2
9	2
10	1
11	1
12	1

Thus 152 lots of wine were made up of just 1 unit, 31 lots had 2 units, 21 lots had 3 units, etc...

The lots were sold one after the other using ascending oral auctions. After announcing the number of units in a lot,[2] the auctioneer started the auction for the first unit at a low starting price. As in practically all ascending auctions, a bidder in the auction room could indicate that he wished to bid above the current price by raising his hand. The increments by which successive prices jumped were most of the time chosen by the auctioneer himself, but occasionally by a bidder (he could do this by simultaneously raising his hand and shouting out the new price). In any case, the winner was the bidder who was the last to remain active, and the winning price was the price established at the moment

[2] Each time a lot consisted of more than 1 unit, the auctioneer emphasized that the winner could exercise a buyer's option (at Drouot the buyer's option is called *faculté de réunion*).

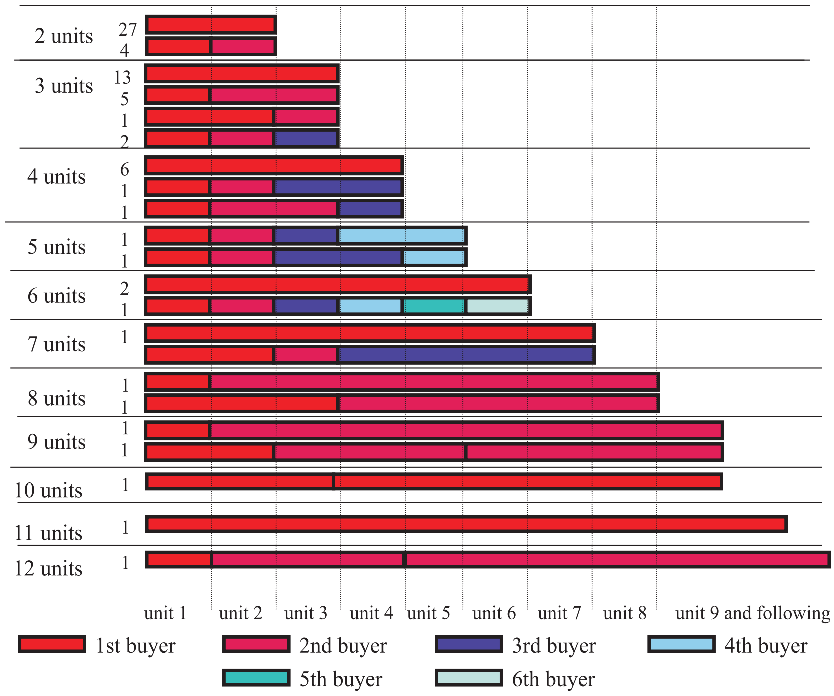

Figure 1: On the use of the buyer's option

he raised his hand for the last time. The auctioneer asked the winner how many units he desired to purchase (the price per unit being the winning price). If he bought all units in the lot, the auctioneer immediately went on with the sale of the next lot; if the winner purchased only part of the lot, a second ascending auction was organized for the remaining units, ..., and this sequence was repeated until all units in the lot were auctioned.

For each lot sold in this way we recorded the identity of the winner of each auction and the number of units purchased by the winner. This information is synthesized in Figure 1.

Thus of the 31 lots that were made up of 2 units, 27 were sold via a single auction (i.e. the first-auction winner exercised his buyer's option in these cases), and 4 were sold via 2 auctions (the winner did not use the option). Note that in all of these 4 cases the first -and second auction winners are different persons: exactly as predicted by our theoretical model, the first-auction winner who does not exercise his buyer's option never wins the second auction. Also perfectly in line with the theory is the fact that in these cases the winning price of the second auction is always lower than the winning price of the first auction. Note furthermore that there is a lot of variation in the outcomes of the sales of lots.

For instance, 13 lots of 3 units were sold in 1 shot, whereas 2 of such lots were each sold to 3 different buyers. Similarly, 2 lots of 6 units were sold in a single shot, and 1 lot was sold to 6 different buyers!

For each lot we also recorded the successive winning prices. To make the winning prices in different lots comparable, we normalized each price to the price of a single bottle containing 75 cl. For instance, if a unit of 12 bottles (containing 75 cl each) is sold at the price p, the normalized price of that unit is $p/12$. Table 2 gives statistics on the winning prices for all lots. If there is more than 1 winning price per lot, then we calculate an average of the winning prices for that lot (weighted by the number of units sold at each auction).[3] The statistics are therefore based on 225 observations. As the table indicates, the average winning price is quite high (more than FFr1300), and there is substantial heterogeneity in the winning prices, since they vary from slightly less than FFr30 to more than FFr30000 per bottle of 75 cl.

Table 2. Winning price per lot (in FFr)

Mean	1 322
Std. dev.	2 956
Max	30 833
75% quantile	1 200
50% quantile	500
25% quantile	217
Min	29

Finally we counted, every 15 minutes, the number of persons in the auction room. In counting the number of individuals we did not include the auctioneer, those who assisted him, or ourselves. Table 3 summarizes this information.

[3] For instance, if a two-unit lot is sold via 2 auctions with winning prices p_1 and p_2, the average winning price for that lot is simply $\frac{p_1+p_2}{2}$.

Table 3. Number of bidders in the auction room

Time	Lots auctioned	Number of bidders
	20 April	
14:15-14:30	1-23	65
14:30-14:45	24-45	65
14:45-15:00	46-53	70
15:00-15:15	54-65	80
15:15-15:30	66-75	80
15:30-15:45	75-86	75
15:45-16:00	87-105	60
16:00-16:15	106-118	70
16:15-16:30	119-123	60
16:30-16:45	124-136	50
	21 April	
11:15-11:30	137-145	45
11:30-11:45	146-176	50
11:45-12:00	177-185	65
12:00-12:15	186-207	65
12:15-12:30	208-225	55

Thus on April 20 at 14:15, that is when the sale started, 65 persons were present in the auction room; fifteen minutes later, when we counted again, there were still 65 person in the room, at 14:45 there were 70 participants, etc... Note that the number of participants over time follows the same pattern during both auction days: at the start of both sale days the number of bidders is relatively low, this number then increases to reach its peak at about halfway the sale day, and then decreases until the end of the sale. Table 3 also indicates which lots were sold during which time interval. Thus, on April 20 between 14:15 and 14:30 the lots 1 to 23 were sold, between 14:30 and 14:45 the lots 24 to 45, etc...

3.2 Data from the wine catalogue

The wine catalogue[4] could be consulted by potential buyers before and during the auctions. It records all kinds of information about the wines on sale. For each lot auctioned, the catalogue lists the number of units, the number of bottles of wine per unit (1 bottle, 6 bottles, 12 bottles, etc.), the type of bottle (a standard size bottle of 75 cl., or a magnum bottle of 150 cl.), and a number of other variables that characterize the wine in the lot. Table 4 gives these wine characteristics together with some descriptive statistics.

[4]Published by Millon & Associés, the auctioneer in charge of the April 20-21 sales. A copy of the catalogue can be obtained from the authors.

Table 4. Wine characteristics

Variable	Mean	Std. dev.
Age of wine (in years)	27.09	21.04
Authentic wooden crate	0.19	0.39
Label damaged	0.13	0.34
Level wine low	0.08	0.27
Wine reconditioned	0.13	0.34
Wine region:		
Bordeaux	0.65	0.48
Burgundy	0.05	0.22
Champagne	0.22	0.41
Côtes du Rhône	0.08	0.27

The variable "Age of wine" is missing for one observation;
descriptive statistics are therefore based on 224 observations.

All variables except "Age of wine" are 0-1 indicator variables. The variable "Authentic wooden crate" equals 1 if the wine is sold in an authentic wooden crate and 0 otherwise; the variable "Label damaged" equals 1 if the label on the bottle is in some way damaged, and 0 otherwise; "Level wine low" is equal to 1 if the level of wine in the bottle is low (this may indicate that the cave where the wine has been kept and stored was too dry and too hot), and 0 otherwise; the variable "Wine reconditioned" is 1 if the wine had at some point in time (while still at the château or domain) been uncorked and refilled with the original wine, and 0 otherwise; finally, the 4 region variables indicate from which wine producing region the wine originates (Bordeaux, Burgundy, Champagne or Côtes du Rhône).

3.3 Two quality indicators

Since the wine characteristics described in the previous subsection may perhaps not capture all the quality differences between the wines in our sample, we added two additional quality indicators to our data set. One is taken from Parker's guide (1995). Each wine in the guide is ranked between 1 star (lowest possible rank) and 5 stars (highest rank). This grading system reflects the overall and the long-term quality of each wine and does not depend on the vintage. Precisely because the Parker ranking does not take into account the specific vintage effects, we also added a grade reflecting the quality of the vintage. This Vintage grade is taken from the French wine web site http://www.vindelice.com. This site grades wines acording to their vintage, their production region and their colour of the wine (for instance it attributes a grade to red Burgundy 1989, or to white Bordeaux 1950, etc.). The site ranks wines as "To avoid", "Average", "Good", "Very good", or "Exceptional". Table 5 lists the frequencies of the values taken by the Parker grade and the Vintage grade.

Table 5. Parker grade and Vintage grade

		Parker grade			
	Unknown	**	***	****	*****
Number of lots	16	4	52	32	121
		Vintage grade			
	To avoid	Average	Good	Very good	Excellent
Number of lots	11	11	72	110	21

The majority of lots are made up of top quality wines: 121 lots consist of wines with 5 Parker stars; for 21 lots of wine the vintage grade is "Excellent" and for 110 lots it is "Very good".

3.4 The link between theory and practice

From the description of the sales at Drouot it is clear that our theoretical framework differs in several ways from practice. This subsections comments on these differences.

The first difference is that the model described in section 2 is a model for lots made up of 2 units. As Table 1 indicates, our data not only contain two-unit lots, but also single-unit lots and lots with more than 2 units. The optimal bidding strategy for the English auction of a single unit being well known, our method of statistical inference will also use the one-unit lots to estimate the parameters of interest of our model. However, we will not use the remaining observations in the sample, that is the observations corresponding to lots with more than 2 units. The reason is that we do not know the optimal bidding strategies in a model where more than 2 units are auctioned. Discarding these observations from the estimation sample does not, however, bias our estimates if the process that determines the number of units per lot is independent from bidders' valuations.

A second difference is that the model treats the sale of a lot as a completely isolated event. The model does not take into account that sales of other lots take place almost simultaneously and that a given bidder might be interested in buying several of these lots. However, at Drouot such inter-dependencies might well have existed. Most bidders stayed in the auction room not just for the sale of one particular lot, but for several different lots. It is plausible that some of such bidders were interested in more than 1 lot at the same time and that their bidding strategies reflected these synergies. It does not appear easy to extend our model to take into account these possible interactions between auctions of different lots. Furthermore, given our data, it would certainly be impossible to identify and estimate such an extended model. We will therefore ignore the (hopefully small) effects of inter-dependency. We thus assume that there are no interactions whatsoever between the auctions of the different lots, so that the sales of the different lots constitute independent observations.

The third difference is that in our model it has implicitly been assumed that bidders know at each moment the precise number of active competitors, and the prices at which each opponent drops out of the game. They are in particular assumed to know when and at which price the third strongest bidder quits the first auction. In contrast, the bidders in the auction room of Drouot had less information at their disposal. Basically, all they learned during the course of the auction was the sequence of successive prices proposed by the auctioneer and, for each of these prices, the identity of the bidder who raised his hand (see section 3.1). At first sight this suggests an incompatibility between theory and the real-life auctions as they were held at Drouot. Interpreting the model in a different way shows however that this is not true. It is in fact sufficient to assume that all the bidders always play according their "final" strategies ($c(.)$ and $d(.)$) and that they actualize the value of $v_{(n-2)}$ each time new information is revealed during the auction. In this spirit, the "two last bidders" assume that $v_{(n-2)}$ corresponds to the highest bid of all other bidders. As soon as the identity of the "two last bidders" changes, the value of $v_{(n-2)}$ is actualized.

4 The likelihood function

As mentioned in the previous section, the different lots are treated as "independent" entities, and as such each lot contributes independently to the likelihood function. We also mentioned that we only consider the lots made up of 1 or 2 units.

4.1 One-unit lots

First we consider the contribution to the likelihood of a one-unit lot. To capture the between-lot heterogeneity of wines, we introduce the vector of variables z characterizing the wine sold in the lot. Thus z contains all variables that appear in Tables 3 and 4. Let us denote the parametric density and distribution functions of the valuation v given z by respectively $f(v|z;\theta)$ and $F(v|z;\theta)$, with θ an unknown parameter that has to be estimated. Let n be the number of bidders present in the bidding room at the moment the lot is auctioned (see Table 2), and let p be the (normalized) price at which the single unit in the lot is sold. Let $f_{(n-1),(n)}(v_{(n-1)}, v_{(n)}|z;\theta)$ be the joint density of the second largest and largest evaluation conditional on z in a sample of size n (see Mood, Graybill, and Boes (1982)):

$$\begin{aligned} & f_{(n-1),(n)}(v_{(n-1)}, v_{(n)}|z;\theta) \\ = \;& n(n-1)F(v_{(n-1)}|z;\theta)^{(n-2)} f(v_{(n-1)}|z;\theta) f(v_{(n)}|z;\theta)\mathbf{1}\{v_{(n-1)} \leq v_{(n)}\}. \end{aligned}$$

It is well known (see for example the survey of Klemperer (1999)) that in an English auction for a single good the optimal strategy is for all agents to bid up to their private value. The contribution to the likelihood function of a single-unit lot sold at the price p (when n bidders are in the room and the characteristics of the wine are z) is therefore the probability of the event that the second largest evaluation equals p and the largest evaluation exceeds p:

$$
\begin{aligned}
& l_1(p,n,z;k,\theta) \\
= & \int_p^{\infty} f_{(n-1),(n)}(p, v_{(n)}|z;\theta) dv_{(n)} \\
= & n(n-1) f(p|z;\theta) F^{n-2}(p|z;\theta)(1-F(p|z;\theta)). \qquad (1)
\end{aligned}
$$

This corresponds of course to the contribution to the likelihood function of a standard IPV single-good-English-auction observation (see for example Paarsch (1997)). Note that in writing the contribution to the likelihood (1), it is implicitly assumed that the n evaluations are independent drawings from the density $f(\cdot|z;\theta)$, and not from some truncated density of v. It is thus assumed that potentially all types of bidders could have been present in the bidding room, and not just the bidders with an evaluation above some truncation point.[5]

4.2 Two-unit lots

Let us now consider the contribution to the likelihood function of a two-unit lot. Let p_1 and p_2 now represent the (normalized) winning prices in the first and second auction. If the buyer's option is exercised then there is no second auction and we automatically have $p_2 = p_1$. Let $f_{(n-2),(n-1),(n)}(v_{(n-2)}, v_{(n-1)}, v_{(n)}|z;\theta)$ be the joint density of the third largest evaluation, the second largest evaluation, and the largest evaluation, conditional on z, in a sample of size n (see Mood, Graybill, and Boes (1982)):

$$
\begin{aligned}
& f_{(n-2),(n-1),(n)}(v_{(n-2)}, v_{(n-1)}, v_{(n)}|z;\theta) \\
= & n(n-1)(n-2) F(v_{(n-2)}|z;\theta)^{(n-3)} f(v_{(n-2)}|z;\theta) \\
& \times f(v_{(n-1)}|z;\theta) f(v_{(n)}|z;\theta) \mathbf{1}\{v_{(n-2)} \leq v_{(n-1)} \leq v_{(n)}\}.
\end{aligned}
$$

The form of the likelihood contribution depends on whether the buyer's option is exercised or not. First consider the case where the first auction winner does not use the buyer's option. For simplicity, we denote $p \equiv p_1 = p_2$. From section 2 we know that in the first auction the $n-3$ "smallest" bidders should in equilibrium bid up to their evaluation, and that the two bidders with the two highest evaluations should play according to the bid function $b(\cdot)$, solution of the differential equation given in Proposition 1. We therefore know that the winning price in the first auction exceeds the third largest evaluation, i.e., $v_{(n-2)} \leq p$. We also know that the winning bid equals the bid of the agent with the second highest evaluation, i.e., $p = b(v_{(n-1)}, v_{(n-2)})$, so that the second highest

[5]For instance, had the auctioneer published, ex-ante, a binding reservation price for a unit of wine, it would have been appropriate to assume that only bidders with an evaluation above the reservation price are in the bidding room, in which case the evaluations are drawings from the truncated distribution of v (truncated at the reservation price).

evaluation equals $v_{(n-1)} = b^{-1}(p, v_{(n-2)})$.[6] Finally, since the first-auction winner exercises the option we also know that $kv_{(n)} > p$, so that $v_{(n)} > \frac{p}{k}$. The contribution to the likelihood of this type of observation is thus

$$\begin{aligned}
& l_2^{\textbf{option used}}(p, n, z; k, \theta) \\
= & \int_0^p \left[\int_{\frac{p}{k}}^{\infty} f_{(n-2),(n-1),(n)}(v_{(n-2)}, b^{-1}(p, v_{(n-2)}), v_{(n)}|z;\theta) dv_{(n)} \right] dv_{(n-2)} \\
= & n(n-1)(n-2)(1 - F(\frac{p}{k})|z;\theta) \\
& \times \int_0^p F^{n-3}(v_{(n-2)}|z;\theta) f(v_{(n-2)}|z;\theta) f(b^{-1}(p, v_{(n-2)})|z;\theta) dv_{(n-2)}. \qquad (2)
\end{aligned}$$

We next turn to the contribution of the likelihood of lots where the first-auction winner does exercise his buyer's option. For the same reason as in the previous case, we still have that $v_{(n-1)} = b^{-1}(p_1, v_{(n-2)})$. Unlike the previous case, since the winner does not use his option it must now be that the value of a second unit is worth less to him than the first-auction winning price, i.e., $kv_{(n)} < p_1$. Also unlike the previous case, there is now a second auction. In the second auction, all bidders should bid up to their value for the unit they attempt to obtain: the first-auction winner should thus participate in the auction until the price has reached $kv_{(n)}$, and the bidders with the second and third largest evaluations should bid up to respectively $v_{(n-1)}$ and $v_{(n-2)}$. Since $kv_{(n)} < p_1 = b(v_{(n-1)}, v_{(n-2)}) < v_{(n-1)}$, it is the agent with the second highest evaluation who wins the second auction at the winning price $p_2 = \max(kv_{(n)}, v_{(n-2)})$. We must now again distinguish two cases.

- Case A: $p_2 = kv_{(n)} > v_{(n-2)}$

  In this case we thus have $v_{(n)} = \frac{p_2}{k}$ and $v_{(n-2)} < p_2$. The probability of this type of observation is therefore

$$\begin{aligned}
& l_{2A}^{\textbf{option not used}}(p_1, p_2, n, z; k, \theta) \\
= & \int_0^{p_2} f_{(n-2),(n-1),(n)}(v_{(n-2)}, b^{-1}(p_1, v_{(n-2)}), \frac{p_2}{k}) dv_{(n-2)}.
\end{aligned}$$

  It should be verified under which conditions the above probability is strictly positive. We automatically have for all values of $v_{(n-2)}$ between 0 and p_2 that $v_{(n-2)} < b^{-1}(p_1, v_{(n-2)})$ because $v_{(n-2)} < p_1$. So for the probability to be strictly positive, it needs to be verified that there is at least one value for $v_{(n-2)}$ such that $b^{-1}(p_1, v_{(n-2)}) < p_2/k$. Since $b^{-1}(p_1, v_{(n-2)})$ is a decreasing function of $v_{(n-2)}$, the condition that must be imposed is

$$b^{-1}(p_1, p_2) < p_2/k. \qquad (3)$$

[6] From now on we explicitly indicate the dependence of $b(\cdot)$ on the third highest evaluation $v_{(n-2)}$. In the expression $b^{-1}(p, v_{(n-2)})$, the inverse is taken with respect to the first variable.

It can be shown that condition (3) implies that $b^{-1}(p_1, v_{(n-2)}) < p_2/k$ for all relevant values of $v_{(n-2)}$. Thus under condition (3), the 3 elements in the density function $f_{(n-2),(n-1),(n)}$ are all the time ranked as they should, and the above probability can then be rewritten as

$$\begin{aligned} & l_{2A}^{\textbf{option not used}}(p_1, p_2, n, z; k, \theta) \qquad (4) \\ = \quad & n(n-1)(n-2) f(\frac{p_2}{k}|z;\theta) \\ & \times \int_0^{p_2} F^{n-3}(v_{(n-2)}|z;\theta) f(v_{(n-2)}|z;\theta) f(b^{-1}(p_1, v_{(n-2)})|z;\theta) dv_{(n-2)}. \end{aligned}$$

- Case B: $p_2 = v_{(n-2)} > kv_{(n)}$

  We know in this case that $v_{(n-2)} = p_2$ and $v_{(n-1)} = b^{-1}(p_1, v_{(n-2)}) = b^{-1}(p_1, p_2)$. Concerning the largest valuation $v_{(n)}$, we should have that $v_{(n-1)} = b^{-1}(p_1, p_2) < v_{(n)}$ and $v_{(n)} < v_{(n-2)}/k = p_2/k$. The 2 bounds on $v_{(n)}$ are coherent if $b^{-1}(p_1, p_2) < p_2/k$. There is thus coherency if condition 3 holds. The probability for this type of observation can then be written as

$$\begin{aligned} & l_{2B}^{\textbf{option not used}}(p_1, p_2, n, z; k, \theta) \qquad (5) \\ = \quad & \int_{b^{-1}(p_1,p_2)}^{\frac{p_2}{k}} f_{(n-2),(n-1),(n)}(p_2, b^{-1}(p_1, p_2), v_{(n)}|z;\theta) dv_{(n)} \\ = \quad & n(n-1)(n-2) F^{n-3}(p_2|z;\theta) f(p_2|z;\theta) f(b^{-1}(p_1, p_2)|z;\theta) \\ & \times[(F(\frac{p_2}{k}|z;\theta) - F(b^{-1}(p_1, p_2)|z;\theta)]. \end{aligned}$$

Since we do not know whether case A or case B is the relevant case, the likelihood of observing that the first-auction winner does not exercise his option (while the winning prices are p_1 and p_2, there are n bidders in the room, etc.) is the sum of the probabilities (4) and (5):

$$\begin{aligned} & l_{2}^{\textbf{option not used}}(p_1, p_2, n, z; k, \theta) \qquad (6) \\ = \quad & l_{2A}^{\textbf{option not used}}(p_1, p_2, n, z; k, \theta) + l_{2B}^{\textbf{option not used}}(p_1, p_2, n, z; k, \theta). \end{aligned}$$

This contribution to the likelihood is strictly positive if condition (3) holds, and is equal to 0 otherwise. There are discontinuity problems at the boundary, that is the contribution (6) is not continuous at the values of θ and k such that $b^{-1}(p_1, p_2) = p_2/k$.[7] This is due to the fact that $l_{2A}^{\textbf{option not used}}$ is discontinuous at these values of θ and k ($l_{2B}^{\textbf{option not used}}$ however is continuous at these points).

[7]Note that the optimal bid function $b(\cdot)$ implicitly depends on θ and k.

The total likelihood function consists of the product of 3 terms. The first term is the product of all likelihood contributions corresponding to the one-unit lots, that is the contributions of the form (1); the second term is the product of all contributions of the two-unit lots where the buyer's option has been exercised, i.e., the contributions of the form (2); and the third term is the product of all contributions of two-unit lots where the buyer's option has not been exercised, i.e., the contributions of the form (6). To obtain the maximum likelihood estimates of k and θ, the total likelihood must be maximized with respect with to these parameters. Maximization must take place under the constraint $0 < k \leq 1$, and the constraints of the form (3). There are as many constraints (3) as there are contributions of the type (6), that is 4 in our sample (see Figure 1).

5 Results

The distribution function of private values is assumed to be the log-normal distribution. The log-normal distribution passes the standard Kolmogorov test of fit, so the parametric assumption that we have made seems justified.[8] We assume that the conditional mean $E(v|z) = (1, z')\beta$, and the conditional variance $Var(v|z) = \sigma$, where β and σ are unknown parameters. The complete vector of parameters describing the distribution function is thus $\theta = (\beta', \sigma)'$. The parameters to be estimated are θ and k.

We did not manage to maximize the total likelihood with respect to these parameters. This is due to the discontinuity problem raised in the previous section. To solve this problem we estimated the parameters of interest using the following two-step method. In the first step we maximized the first term of the likelihood (the contribution of the one-unit lots) with respect to θ. The resulting estimate of θ is given in Table 6, together with the asymptotic T-statistic (based on the standard asymptotic variance of the estimate). In the second step we maximized the second and third term with respect to k, with θ fixed at its first-step estimate. In this second step each contribution of the form (6) was replaced by $l_2^{\textbf{option not used}}(p_1, p_2, n, z; k, \theta) S_h(b^{-1}(p_1, p_2) - p_2/k)$ where

$$S_h(x) = 1 - \int_{-\infty}^{x} K_h(t)dt$$

and $K_h(\cdot)$ is the Epanechnikov kernel with bandwidth h. We thus smoothed the likelihood at its points of discontinuity. We took $h = 0.01$. At each iteration the the strategy $b(\cdot)$ appearing in the likelihood is approximated numerically. The resulting estimate of k is given in the last line of Table 6. In calculating the asymptotic variance (and thus the T-statistic) we did not correct for the fact that the likelihood is smoothed.

[8]We used the Kolmogorov test corrected for the fact that the parameter in the log-normal distribution is estimated by maximum likelihood (see D'Agostino and Stephens (1984)).

Most parameters are significant and take the expected sign. The value bidders attribute to wine increases with its age and if the bottles are sold in an authentic wooden crate. When the label on the bottle is damaged the willingness to pay for the wine is reduced. Wine experts tend to say however that a damaged label is an indicator of good quality. Our result suggest that consumers neglect this and prefer the label to be in good shape. As expected, the value of wine decreases when the level in the bottle is low and increases when the wine has been reconditioned. Burgundy wines are valued the most, followed by Côtes du Rhône, Bordeaux and Champagne. The Parker and vintage parameters are ranked as expected. The parameter k equals 0.83 and is significantly smaller than 1. This suggest that the bidders in our sample had decreasing demand for wine.

Table 6. Estimation of the parameters θ and k

Variable	Est.	T-stat.
Mean of valuation (β):		
Constant	3.40	13.56^{++}
Age of wine	0.01	4.67^{++}
Authentic wooden crate	0.36	2.08^{++}
Label damaged	-0.23	-1.65^{+}
Level wine low	-0.24	-1.70^{+}
Wine reconditioned	0.88	4.63^{++}
Wine region:		
Bordeaux	0.28	2.08^{++}
Burgundy	1.15	5.94^{++}
Côtes du Rhône	0.99	4.93^{++}
Parker grade:		
Unknown	-2.08	-12.24^{++}
** or ***	-1.09	-8.72^{++}
****	-0.89	-6.29^{++}
Vintage grade:		
Average or good	-0.38	-3.36^{++}
Very good	-0.19	-1.78^{+}
Variance of valuation (σ)	1.40	16.29^{++}
k	0.83	280.96^{++}

++ = significant at 5%; + = significant at 10%. Reference variables are Champagne, Parker grade=*****, Vintage grade=Excellent.

To evaluate the effect of the buyer's option on the revenue of the seller we simulated, for each of the 31 two-unit lots in our sample, the revenue under both the pure sequential auction (using the equilibrium strategy in Proposition 2) and the auction with buyer's option. We simulated 5000 times and calculated for each lot the simulated revenues over the 5000 observations. The average revenue obtained from the first unit is (standard error) FFr17429 (35.2) in the sequential auction and FFr18602 (36.6) in the auction with buyer's option. The average revenue obtained from the second unit is FFr19813 (41.4) in the sequential auction and FFr18490 (36.3) in the auction with buyer's option. The total simulated revenue is FFr37243 (76.4) in the sequential auction and FFr37092 (73) in the auction with buyer's option. Revenue is thus slightly higher in a pure sequential auction, but the difference is not significant.

6 Conclusion

This paper has presented a theoretical model for two-unit English auctions. The two units are sold sequentially. Equilibrium strategies have been derived for the case where the buyer's option is available and also when it is not. Using structural econometric techniques, the model has been estimated using a rich data set on wine auctions. The analysis in this paper suggests that the buyer's option has no impact on the revenue of the seller. Given that the mechanism with buyer's option is quicker and thus saves time, this result can be seen as an ex-post justification for the fact that so many auction houses use the buyer's option.

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Appendix

A Study of the differential equations

Let us first consider the following differential equation:

$$k(d(v) - kv)f(v) = (v - d(v))f(d(v)/k)d'(v)$$

We look for a solution defined on $[0, +\infty[$ such that $\forall v$, $d(v) \in [kv, v]$. On this interval, the function is increasing as $d' > 0$.

Let us introduce the solution d_A of this differential equation that verifies the condition $d_A(A) = kA$, $A \in [0, +\infty[$. The Cauchy-Liptchitz conditions are verified which proves the existence and the uniqueness of the solution. Furthermore, as $d'_A(A) = 0$, d verifies $d(v) \in [kv, v]$ at least locally for $v \leq A$. We will prove that d_A is defined on $[0, A]$ and verifies the condition $c_A(0) = 0$.

- Suppose that there exists B, $B < A$, such that $d_A(B) = kB$. If there is more than one B satisfying this condition, then we consider the greatest real value. If $B \neq 0$, we then obtain that $d'_A(B) = 0$. So locally $d_A(v) < kv$ for $v > B$. This implies that B can not be the greatest real that verifies the previous condition.

- Suppose then that there exists $B \neq 0$ such that $d_A(B) = B$. Suppose again that B is the greatest real that verifies this condition. Because $d'_A(B) = +\infty$, locally $d_A(v) > v$ for $v > B$, again a contradiction with the definition of B.

We then conclude that d_A is defined on $[0, A]$ and verifies $d_A(0) = 0$.

We prove in the same way that the solution d^A of the differential equation that verifies the condition $d^A(A) = A$, $A \in [0, +\infty[$ is defined on $[0, A]$ and verifies $d^A(0) = 0$.

Let then $x \neq 0$ be a real fixed number. We introduce $I = \{d_A(x),\ A \in [x, +\infty[\}$ and $J = \{d^A(x),\ A \in [x, +\infty[\}$.

- We first prove that I is an interval of the form $[kx, \underline{x}[$. As $d_x(x) = kx$, we have $kx \in I$. Moreover, let us verify that if $y \in I$, then $z \in [kx, y[$ also belongs to I. Indeed, the Cauchy-Liptchitz properties imply that the solutions $d_z(.)$ and $d_y(.)$ are functions that do not cross each other. We then deduce that $d_z(.) < d_y(.)$ and consequently, d_z crosses the straight line $f(v) = kv$ before $d_y(.)$ which implies that z belongs to I. This proves that I is an interval. One must now prove that it is an open interval. Let $\underline{x}$ be the superior bound of the interval. If $\underline{x} \in I$, then there exists $A_{\underline{x}}$ such that $d_{A_{\underline{x}}}(A_{\underline{x}}) = kA_{\underline{x}}$. The same reasoning as previously allows to prove that a solution d_A with $A > A_{\underline{x}}$ verifies $d_A(x) > d_{A_{\underline{x}}}(x)$, a contradiction with the definition of $\underline{x}$.

  We have then proved that I is of the form $[kx, \underline{x}[$.

- Using the same reasoning, one can prove that J is of the form $]\overline{x}, x]$.
- Finally, I and J are two disjointed sets as the solutions that correspond to the interval I can not cross the line $g(v) = v$ and the solutions that correspond to the interval J can not cross the line $f(v) = kv$. We then conclude that $\underline{x} \leq \overline{x}$.
- To prove that there exists a unique solution of the differential equation that is defined on $[0, +\infty[$, one has to show that $\underline{x} = \overline{x}$.

  Suppose then that $\underline{x} \neq \overline{x}$, and consider the two solutions $d_{\underline{x}}(.)$ and $d_{\overline{x}}(.)$ that are defined on $[0, +\infty[$. If there exists v such that $d_{\underline{x}}(v) = d_{\overline{x}}(v)$, then the Cauchy-Liptschitz conditions imply that $d_{\underline{x}}(.) = d_{\overline{x}}(.)$. We can therefore conclude that $d_{\underline{x}}(v) < d_{\overline{x}}(v)$ for every v. Using the differential equation, we obtain

$$d'_{\overline{x}}(v) f(\frac{d_{\overline{x}}(v)}{k}) - d'_{\underline{x}}(v) f(\frac{d_{\underline{x}}(v)}{k}) = \frac{k(1-k)v[d_{\overline{x}}(v) - d_{\underline{x}}(v)]f(v)}{(v - d_{\overline{x}}(v))(v - d_{\underline{x}}(v))}$$

  The function $f(v)$ being decreasing for v large enough, we obtain for v large enough

$$\begin{aligned}\frac{d'_{\overline{x}}(v) - d'_{\underline{x}}(v)}{d_{\overline{x}}(v) - d_{\underline{x}}(v)} &\geq \frac{k(1-k)vf(v)}{(v - d_{\overline{x}}(v))(v - d_{\underline{x}}(v))f(\frac{d_{\underline{x}}(v)}{k})} \\ &\geq \frac{kf(v)}{(1-k)vf(\frac{d_{\underline{x}}(v)}{k})} \\ &\geq \frac{1}{(1-k)v}\end{aligned}$$

  where the second inequality follows from the fact that $v - d_{\underline{x}}(v) \leq (1-k)v$, and $v - d_{\overline{x}}(v) \leq (1-k)v$. The first and third inequalities follow from the fact that f is decreasing.

  Integrating the previous inequality allows us to obtain that

$$d_{\overline{x}}(v) - d_{\underline{x}}(v) \geq Cv^{1/(1-k)}$$

  which is a contradiction for v large enough with the fact that $d_{\overline{x}}(v) - d_{\underline{x}}(v) \leq (1-k)v$

  This allows us to conclude that $\underline{x} = \overline{x}$ and that there exists a unique solution of the differential equation: $d(\cdot) = d_{\overline{x}}(\cdot) = d_{\underline{x}}(\cdot)$ that is defined on $[0, +\infty[$. This solution verifies in particular that $\forall v$, $d(v) \in]kv, v[$.

In fact, this unique solution interests us only on the interval $[\frac{v_{(n-2)}}{k}, +\infty[$ and we now have to study the second differential equation:

$$k(c(v) - v_{(n-2)})f(v) = (v - c(v))f(c(v)/k)c'(v)$$

with the two conditions $c(\frac{v_{(n-2)}}{k}) = d(\frac{v_{(n-2)}}{k})$ and $c(v_{(n-2)}) = v_{(n-2)}$.

The same kind of demonstration as above can be applied. The Cauchy-Liptchitz conditions show that there exists a unique solution of this differential equation that verifies $c(\frac{v_{(n-2)}}{k}) = d(\frac{v_{(n-2)}}{k})$ and one can prove that this solution also verifies $c(v_{(n-2)}) = v_{(n-2)}$.