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Unemployment Insurance and Training in an Equilibrium Matching Model with Heterogeneous Agents

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Abstract

This paper develops a joint evaluation of vocational training and unemployment insurance. This allows to analyze how these schemes complement each other from the viewpoints of labor market indicators and of welfare. For this purpose, a general equilibrium matching model is built where workers are heterogeneous and risk averse. Heterogeneity allows to look at the distribution of the effects. Job search effort and wages are endogenous in order to deal with the induced effects of these schemes. The net effect of these training programs appears to be gloomy. However, their impact on employment can be deeply affected by the design of passive policies. A declining time profile of benefit payments dominates a scheme with a constant replacement ratio. However, the optimal expected length of payment of 'high' benefits can vary a lot in the population. A reform that would relate this expected duration to search effort does not appear to produce substantial effects on any of the evaluation criteria. Performance indicators of the labor market and welfare criteria often vary in opposite directions after a reform. This questions the widespread focus on labor market indicators to guide the design of institutional reforms.

Keywords: training; unemployment insurance; sanctions; policy complementarities; wage bargaining; equilibrium unemployment; equilibrium search.

JEL classification : J63, J64, J65, J68.

1 Introduction

Most countries simultaneously use a mix of 'passive' and 'active' labor market policies to tackle often pervasive unemployment problems, especially for the low-skilled. Reforms are urgently called for by international institutions. Reforms can however have very different effects according to the criterion used to evaluate them. Moreover, since the labor force is heterogeneous, the distribution of the impact should be preferred to an average effect. This paper deals with four questions. First, to what extent should unemployment benefits decline with unemployment duration when the unemployment pool is made of groups with different intrinsic probabilities of exit to a job? Second, following the recent literature about sanctions, should there be a relationship between the time-profile of unemployment benefits and the level of search effort? Third, since training the unemployed simultaneously reduces firms' training cost, raises wages, affects participation decisions and is relatively costly, to what extent should training programs be developed in addition to unemployment insurance? Fourth, is it always true that "the more generous are passive unemployment policies, the less effective will be active unemployment policies" (Coe and Snower, 1997, p. 22)?

To deal with these questions, this paper develops a general equilibrium model based on a matching framework where individuals are risk averse, search effort and participation are endogenous and wages are bargained over (Pissarides, 2000). Workers are heterogeneous in two dimensions: Their skill and their utility when inactive. To the best of my knowledge, this model generalizes previous ones (see below). The paper develops both analytical results and simulation exercises for a country plagued with a pervasive unemployment problem for low-skilled individuals.

The related literature is quite large. Due to space limitation, it is only possible to present a selective and highly condensed review of some contributions.[1] After a lot of research in partial equilibrium (e.g. Shavell and Weiss, 1979, Wang and Williamson, 1996, and Hopenhayn and Nicolini, 1997), the design of unemployment insurance schemes has recently been studied in equilibrium search models with wage bargaining by Cahuc and Lehmann (2000), Fredriksson and Holmlund (2001) and Coles and Masters (2001). By carefully developing a strategic wage bargain between an individual job seeker and a firm, the latter focus on the relationship between the profile of duration dependent UI payment schemes and the level of wages. Assuming automatic renegotiation of individual wages and adopting a utilitarian criterion, Fredriksson and Holmlund (2001) show that a declining time profile of benefits dominates an insurance system characterized by indefinite payment of a constant replacement ratio if there is no discounting. With discounting, whether the sequencing of benefits should or not be declining remains an unsettled issue. A numerical analysis by Fredriksson and Holmlund (2001) concludes that the wage pressure effect of

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[1]A wide range of labor market policies and fiscal policies related to this market have recently been studied in general equilibrium settings (see e.g. Heckman, Lochner and Taber, 1999, Mortensen and Pissarides, 1999, Bassanini, Rasmussen and Scarpetta, 1999, Taber, 2000, Boone and Bovenberg, 2001, and Manning, 2001). This overview does not justice to this literature.

declining unemployment benefits (emphasized by Cahuc and Lehmann, 2000) is not strong enough to compensate the effect of a declining benefit sequence on search incentives. When the wage bargain is centralized, Kreiner and Whitta-Jacobsen (2002) have recently challenged the main result of Cahuc and Lehmann (2000) under the assumption that the incidence of long-term unemployment increases with aggregate unemployment. The present paper will revisit this literature in a generalized framework with active schemes.

In the recent past, a new branch of the literature has looked at the effects of sanctions. Sanctions are reductions or withdrawals of unemployment benefits due to some misconduct of the unemployed (see Grubb, 2001). Insufficient search for a job is the main example considered in the recent literature. Boadway and Cuff (1999), Boone and van Ours (2000) and Boone, Fredriksson, Holmlund and van Ours (2002) are major contributions to this literature. The broad message is that sanctions are effective to enhance search effort and that so doing they allow to insure the unemployment risk more efficiently. Sanctions can be seen as a particular mechanism by which the sequencing of benefits is declining. The specificity comes from the role played by the (imperfect) observation of search efforts on the rate at which benefits decline. Hence, it seems natural to merge the literature about the optimal sequencing of benefits and the one about sanctions by allowing for a dependency between the profile of unemployment benefits and the level of search.

Active programs have been studied by Holmlund and Lindén (1993) who highlight the wage-push effect of direct job creation schemes for the unemployed (also called 'relief jobs'). Calmfors and Lang (1995) added that these policies enhance or at least maintain effective labor-force participation in the presence of negative duration dependence. Masters (2000) introduces retraining programs for the unemployed in an equilibrium matching framework. These programs improve the matching effectiveness of the unemployed. Participation into a training program is decided by the unemployed. Assuming risk-neutral workers without credit constraints, Masters (2000) shows that the government should let the unemployed pay for the program under free entry of vacancies and in the absence of any 'hold-up' problem. Albrecht, van den Berg and Vroman (2002) evaluates a one-year program that is seen as enhancing the level of skills of the unemployed. In these two papers, search effort in unemployment is unaffected by the existence of the program. Moreover, the issue of complementarities between passive and active programs is not considered.

Another literature tries to assess possible complementarities between labor market policies and institutions. The empirical literature is quite large (see Nickell and Layard for a summary and Blanchard and Wolfers, 2000, Belot and van Ours, 2001, or Bertola, Blau and Kahn, 2001, for more recent contributions that focus on the interaction between institutions and shocks). These papers typically estimate a reduced-form equation linking an indicator of labor market performance to a set of explanatory factors inspired in particular by 'wage setting-price setting' models. Theoretical analyses of complementarities can be found in Coe and Snower (1997), L'Haridon (2001) and Burda and Weder (2002).Van der Linden and Dor (2002) is a previous effort to simultaneously study 'active' and 'passive' policies. Compared to that paper, the theoretical setting is here much more

general.[2] To the best of my knowledge, none of these articles has looked at the possible complementarities between the sequencing of unemployment benefits and training programs. Furthermore, except in L'Haridon (2001), the impact on welfare criteria has been neglected.

The rest of the paper is organized as follows. Section 2 develops the model and the analytical properties. A numerical analysis is conducted in Section 3. Section 4 concludes the paper.

2   The Model

2.1   Basic assumptions

This section develops a fairly general dynamic model that can simultaneously deal with active and passive labor market policies in a general equilibrium setting with heterogeneous agents. The model features a homogeneous good (the numeraire) and labor. The good market is perfectly competitive. Returns to scale are constant. Each firm uses one and only one type of skill. The labor market is therefore by assumption segmented in the skill dimension.[3] In the simulation exercise of Section 3, an aggregate budget constraint of the State will introduce a link between the labor markets.

Differences in skill and in utility levels while inactive are the two sources of heterogeneity. For simplicity, a representative firm will be modeled for each skill. Each firm is composed by filled and vacant occupations. Two levels of skill (l, h) will be introduced and the distribution of utilities in inactivity will be uniformly distributed. Both assumptions can be quite easily relaxed. To avoid non stationarity, several authors have assumed a stochastic two-tired framework (see e.g. Fredriksson and Holmlund, 2001). As in these papers, it will be assumed that (relatively) high benefits expire at a rate π. An insured unemployed whose high benefits has expired enters a state where (s)he indefinitely can benefit from a lower or equal unemployment benefit.[4] The latter could be an assistance benefit (Ortega and Rioux, 2002).

A markovian model is developed in a continuous-time setting and in steady state. It distinguishes six states for each skill n. For jobless individuals, three states are identified : Insured unemployment with high benefits (U_n), insured unemployment with low benefits (X_n) and training (T_n). 'Employment' should be understood as salaried employment. For reasons that will shortly be clear, a distinction has to be made according to the origin of those employed : E_n when coming directly from unemployment and $E_{T,n}$ when coming

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[2]First, search effort and participation decisions are endogeneized. Second, training programs do not only improve the intertemporal position of the unemployed, leading to a wage-push effect. These programs also reduce the fixed hiring cost for employers. Third, workers are risk averse and heterogeneous in skill. Finally, a more comprehensive set of evaluation criteria is used here.

[3]This implies that the so-called 'ladder effect' (or 'job competition') is by assumption ruled out (see Gautier, 2002).

[4]It is in principle possible to extend this setting to a larger number of states each characterized by a specific level of unemployment benefits (see Fredriksson and Holmlund, 2001).

from training programs. The sixth state, I_n, designates inactivity. These upper-case symbols will simultaneously designate the states and the number of individuals occupying them in steady state.

Training programs can be of different types. This paper does *not* deal with programs covering possibly several years and intending to be an alternative to formal education for the unemployed.[5] The present paper focuses instead on (relatively shorter) programs that facilitate the acquisition of vocational skills. Such short programs cannot lift a worker from the bottom of the skill distribution to the top. These training schemes are typically organized by the Public Employment Service ('PES') or by specialized agencies. Heckman, LaLonde and Smith (1999) summarize the conclusions of European evaluations of training programs. They conclude that their impact on participants' wages is negligible. Therefore, it is sound to assume that participation into such schemes does not modify the productivity of the worker. Heckman, LaLonde and Smith (1999) also conclude that the case for positive employment impacts is stronger. Hence, it is here assumed that participation into training can have a positive effect on matching effectiveness (see below). In addition, hiring former trainees lowers firms' training cost. Consequently, since these vocational training programs cannot be seen as an alternative to education, the skill composition of the population of working age is kept exogenous. $\mathcal{P}_n, n \in \{l, h\}$, denotes the constant size of this population.

The model being markovian, the benefit of training is lost as soon as the match ends. Masters (2000) adopt a similar assumption. In that sense, unemployment is associated with skill deterioration. Contrary to Acemoglu (1995) and Coles and Masters (2000), no further loss of skill is however generated as the unemployment spell lengthens. This assumption is motivated by the growing literature that shows that duration dependence is largely spurious in Continental Europe (see Machin and Manning, 1999).

Firms open skill-specific vacancies accessible either to trainees or to the other job-seekers. This assumption requires that participation to training programs is observable. Due to various imperfections that are not explicitly introduced in the model, the matching process is not instantaneous. So, the flows of hires, M_n and $M_{T,n}$, are a function of an indicator of the number of job-seekers, S_n and $S_{T,n}$, and of the number of vacancies, V_n and $V_{T,n}$. The matching functions are by assumption identical in all markets and they are written respectively $M_n = m(S_n, V_n)$ and $M_{T,n} = m(S_{T,n}, V_{T,n})$. The function $m(.,.)$ is assumed to be increasing, concave and homogeneous of degree 1.

At each moment, the timing of decisions is by assumption the following:

1. Firms post vacancies and this costs a fixed amount K_n per unit of time. Jobless workers search for a job or stay out of the labor force.

2. The firm incurs a fixed cost $H_{T,n}$ if the recruited worker has benefited from a training program and H_n otherwise ($H_{T,n} < H_n$). These match-specific fixed costs include training expenses.

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[5]See for instance Albrecht, van den Berg and Vroman (2002).

3. Having Continental Europe in mind where collective bargaining is widespread and following Cahuc and Lehmann (2000), it is assumed that the current wage is bargained over by incumbent employees on behalf of all workers. At this stage, $H_{T,n}, H_n$ are a sunk cost.[6] The fall-back level for theses 'insiders' is the intertemporal discounted utility of an unemployed entering state U_n.[7]

4. If an agreement is reached, production occurs and the total surplus is shared between the worker and the firm.

5. An exogenous fraction ϕ_n of the matches is destroyed. The workers who occupied these jobs enter insured unemployment[8] and these jobs become vacant. As will soon be clear, workers have no incentive to quit.

The behavior of jobless individuals could in principle be more complex. For, they could decide to postpone searching for a job or to reject job offers if entering a training program was more rewarding. However, it will soon be clear that equally productive workers are equally paid. For that reason, the behavior of jobless people consists in searching for a job. It will also turn out that the intertemporal value in training lies between the one in employment and the one in unemployment. So, entering a training program implies a gain for the unemployed. However, waiting for a job offer could be more advantageous. Nevertheless, if an unemployed receives an offer to enter a training scheme, the model assumes that it will be accepted. This simplifying assumption is motivated by the fact that in more and more countries refusing an active programs is a motive for being sanctioned.

As in Chapter 5 of Pissarides (2000), search intensity is *endogenous*. Here, it is specific to each group. It is assumed that participants to training program can search for a job. Let $s_{U,n}, s_{X,n}$ and $s_{T,n}$ denote these intensities. Job-seekers can have skill-specific attributes that influence their efficiency in the matching process and eventually their chosen level of search effort. A unique *exogenous* matching effectiveness parameter c_n will be associated to states U_n and X_n. For trainees, this parameter can be different and will be denoted $c_{T,n}$.[9] It is assumed that $c_{T,n} \geq c_n > 0$. The rationale for this hypothesis is the following. Perhaps because job-entry rates are unfortunately still used in the assessment of labor programs, the PES can for instance give priority to participants to these programs, in particular in the case of a closed treatment of job offers.[10] A signalling effect of training could also be invoked to motivate a specific matching effectiveness parameter for the

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[6]This creates a 'hold-up' problem. Since $H_{T,n} < H_n$, the latter is less acute when entrants exit from training programs.

[7]In a one-firm-one-job setting, an alternative setting would be to assume that the initial wage (individually) negotiated when workers enter the firm is automatically renegotiated (see Fredriksson and Holmlund, 2001, and Coles and Masters, 2001, for a critique).

[8]This assumption is a good approximation for several countries but clearly not for all of them.

[9]In Masters (2000), this captures *the* benefit of training for the unemployed.

[10]This refers to the case where the PES identifies those who are suitable for vacancies in their register. Cockx (2000) and Heckman, Heinrich and Smith (2002) emphasize the role of incentives on the behavior of programs' administrators.

trainees. So, $S_n \equiv c_n (s_{U,n} U_n + s_{X,n} X_n)$ and $S_{T,n} \equiv c_{T,n} s_{T,n} T_n$. The distinction between c_n and $c_{T,n}$ is also a way to relax the assumption of identical matching functions.

Due to the constant returns to scale in the matching process, the model can be developed in terms of tightness indicators measured in efficiency units, namely $\theta_n \equiv \frac{V_n}{S_n}$ and $\theta_{T,n} \equiv \frac{V_{T,n}}{S_{T,n}}$. The rate at which vacant jobs become filled is $q(\theta_n) \equiv M_n/V_n = m(\frac{1}{\theta_n}, 1), q'(\theta_n) < 0$ (respectively, $q(\theta_{T,n}) \equiv m(\frac{1}{\theta_{T,n}}, 1)$). An 'efficient job-seeker' moves into employment according to a Poisson process with rate $\alpha(\theta_n) \equiv \frac{h(S_n, V_n)}{S_n} = \theta_n q(\theta_n)$, with $\alpha'(\theta_n) > 0$ (respectively, $\alpha(\theta_{T,n}) \equiv \theta_{T,n} q(\theta_{T,n})$). Hence, an insured unemployed i endowed with skill n and searching with a search intensity $s_{U,n}^i$ flows from unemployment to employment at a rate to $c_n s_{U,n}^i \alpha(\theta_n)$. Similar expressions can easily be written for the two other groups.

In the literature, the rate at which unemployment benefits decrease is not a function of the individual level of search intensity. There is however a growing literature about sanctions that argues that allocative and risk-sharing efficiencies can be enhanced by closely monitoring the search activity of the unemployed and by imposing sanctions in case of a lack of search effort. Following this literature, it is here assumed that the rate (π) at which the first period of high benefits ends can be a function of $s_{U,n}$. For simplicity, let us take a linear relationship $\pi_n \equiv \pi_0 - \pi_1 s_{U,n}$ with $\pi_0 \geq \pi_1 \geq 0$.[11] Boone, Fredriksson, Holmlund and van Ours (2002) provide a rationale for such a specification. In their paper, unemployed people are monitored at a given rate (here, π_0). Conditional on an inspection, the rate of sanction is a linearly decreasing function of the observed level of search (here, this rate is equal to $1 - (\pi_1/\pi_0) s_{U,n}$). If $\pi_1 > 0$, it is assumed that $\pi_0/\pi_1 \geq \max[s_{U,l}, s_{U,h}]$.

The unemployed receive training offers at a rate γ_n. A training program ends at an exogenous rate λ_n. This parameter can be interpreted as the rate of failure of training schemes. As it is observed in several countries, participation to active programs is a sufficient condition to become eligible to high benefits again. In general, the rate of entry γ_n and the rate of exit λ_n could be different according to the state of origin. However, due to a lack of empirical evidence on such differences, it seems reasonable to assume common rates. With the type of training programs considered in this paper, it is quite natural to assume that $\lambda_n \geq \phi_n, \forall n$.

Figure 1 summarizes the flows in this economy.

In steady state, the flows have to verify the following equilibrium conditions $\forall n \in \{l, h\}$:

$$(c_{T,n} s_{T,n} \alpha(\theta_{T,n}) + \lambda_n) T_n = \gamma_n (U_n + X_n), \tag{1}$$

$$(c_n s_{X,n} \alpha(\theta_n) + \gamma_n) X_n = (\pi_0 - \pi_1 s_{U,n}) U_n, \tag{2}$$

$$(c_n s_{U,n} \alpha(\theta_n) + \pi_0 - \pi_1 s_{U,n} + \gamma_n) U_n = \lambda_n T_n + \phi_n (E_n + E_{T,n}), \tag{3}$$

$$\phi_n E_n = c_n (s_{U,n} U_n + s_{X,n} X_n) \alpha(\theta_n), \tag{4}$$

$$\phi_n E_{T,n} = c_{T,n} s_{T,n} \alpha(\theta_{T,n}) T_n, \tag{5}$$

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[11]These parameters could be skill-specific. As it is rarely so in observed unemployment legislation, π_0 and π_1 will not be indexed by n. I return to that issue in Section 3.

one of these equalities being redundant.

Let lower case letters e_n, u_n, x_n, t_n, v_n and $v_{T,n}$ be the rates obtained by dividing the absolute numbers by the corresponding size of the labor force L_n (e.g. $e_n \equiv \frac{E_n}{L_n}$, $v_n \equiv \frac{V_n}{L_n}$).

Equations (1), (2), (4), (5) and the identity $1 \equiv e_n + e_{T,n} + u_n + x_n + t_n$ determine $e_n, e_{T,n}, u_n, x_n$ and t_n as a function of the endogenous variables $s_{U,n}, s_{X,n}, s_{T,n}, \theta_n$ and $\theta_{T,n}$ and of the parameters $\tilde{Z} = (c_n, c_{T,n}, \phi_n, \gamma_n, \lambda_n, \pi_0, \pi_1)$. In general, available statistics do not make the distinction between e_n and $e_{T,n}$. So, let us consider the sum $e_n + e_{T,n}$, called 'the employment rate' (among active people endowed with skill n). If

$$
\begin{aligned}
\Delta_{0,n} &\equiv [c_{T,n} s_{T,n} \alpha(\theta_{T,n}) + \lambda_n] \left([c_n s_{U,n} \alpha(\theta_n) + \phi_n] [c_n s_{X,n} \alpha(\theta_n) + \gamma_n] \right. \\
&+ \pi_n [c_n s_{X,n} \alpha(\theta_n) + \phi_n]) + \gamma_n [c_{T,n} s_{T,n} \alpha(\theta_{T,n}) + \phi_n] [\pi_n + c_n s_{X,n} \alpha(\theta_n) + \gamma_n],
\end{aligned}
\tag{6}
$$

then:

$$
e_n + e_{T,n} = \mathcal{E}(\theta_n, \theta_{T,n}, s_{T,n}, s_{U,n}, s_{X,n} \mid \tilde{Z}_n) \equiv \left[[c_{T,n} s_{T,n} \alpha(\theta_{T,n}) + \lambda_n] (c_n s_{U,n} \alpha(\theta_n) \right. \tag{7}
$$

$$
\left. [c_n s_{X,n} \alpha(\theta_n) + \gamma_n] + \pi_n c_n s_{X,n} \alpha(\theta_n)) + \gamma_n c_{T,n} s_{T,n} \alpha(\theta_{T,n}) [\pi_n + c_n s_{X,n} \alpha(\theta_n) + \gamma_n] \right] \Delta_{0,n}^{-1}.
$$

Similar expressions can be defined for u_n, x_n and t_n.

2.2 Preferences and intertemporal utilities

All workers of type n have by assumption identical preferences. They are assumed to be risk averse and to have no access to capital markets. Equilibrium search model with risk averse workers are notoriously difficult to handle. So, a simple separable instantaneous utility function is adopted, namely $\ln(C) - \psi_n \frac{s^{\xi_n}}{\xi_n}$, with C denoting consumption and s effort while ψ_n and $\xi_n > 1$ are positive parameters. Appendix 4 summarizes the major changes when an isoelastic function of consumption is used instead of $\ln(C)$. Effort in employment is fixed and normalized to zero.[12]

Let W denote the net wage. The wage of a worker endowed with skill n is written $W = w_{T,n}$ if (s)he holds a job after a training spell and $W = w_n$ otherwise. Let $b_{\iota,n}$ be the level of benefit received ($\iota = U, X, T$). As such, levels of unemployment benefits are not a function of the skill. However, when they are (to some extent) indexed on wages, a dependency with n appears via the wage. The following very plausible ranking is assumed: $W_n > b_{T,n} > b_{U,n} > b_{X,n} > 0$.

Let r be the discount rate assumed to be common to the workers and the firms. Holding a job yields an intertemporal utility $V_{E,n}$ (respectively $V_{E,n|T}$ after a training programme) defined by

$$
r V_{E,n} = \ln(w_n) + \phi_n(V_{U,n} - V_{E,n}), \tag{8}
$$

$$
r V_{E,n|T} = \ln(w_{T,n}) + \phi_n(V_{U,n} - V_{E,n|T}). \tag{9}
$$

---

[12]It could equally well be normalized to any other value without changing the results.

The level of search of jobless individual i is optimized at any point in time. So, according to the position on the labor market and the level of skill, the intertemporal utility solves:

$$rV_{U,n}^i = \max_{s_{U,n}^i}\{\ln(b_{U,n}) - \psi_n \frac{(s_{U,n}^i)^{\xi_n}}{\xi_n} + c_n\, s_{U,n}^i\, \alpha(\theta_n)(V_{E,n} - V_{U,n}^i) + \gamma_n(V_{T,n}^i - V_{U,n}^i)$$

$$+ \quad (\pi_0 - \pi_1\, s_{U,n}^i)(V_{X,n}^i - V_{U,n}^i)\}, \tag{10}$$

$$rV_{T,n}^i = \max_{s_{T,n}^i}\{\ln(b_{T,n}) - \psi_n \frac{(s_{T,n}^i)^{\xi_n}}{\xi_n} + c_{T,n}\, s_{T,n}^i\, \alpha(\theta_{T,n})(V_{E,n|T} - V_{T,n}^i)$$

$$+ \quad \lambda_n(V_{U,n}^i - V_{T,n}^i)\}, \tag{11}$$

$$rV_{X,n}^i = \max_{s_{X,n}^i}\{\ln(b_{X,n}) - \psi_n \frac{(s_{X,n}^i)^{\xi_n}}{\xi_n} + c_n\, s_{X,n}^i\, \alpha(\theta_n)(V_{E,n} - V_{X,n}^i)$$

$$+ \quad \gamma_n(V_{T,n}^i - V_{X,n}^i). \tag{12}$$

Only symmetric equilibria are considered, where all insured unemployed and trainees have the same level of search effort. Henceforth, superscript i will be dropped.

The flows in Figure 1 require that jobless people have an incentive to accept job offers. Furthermore, even if active programs have more and more become compulsory, imposing that workers have an incentive to enter training schemes sounds plausible. Finally, unemployed people in state X_n have lower unemployment benefits. This will intuitively boost their search effort, leading to a lower level of instantaneous utility but also to an improved probability of exiting out of unemployment. So, the relationship between $V_{U,n}$ and $V_{X,n}$ could be ambiguous. The following proposition establishes the ranking of intertemporal utilities. Some of its assumptions introduce a hierarchy between endogenous variables that will have to be checked later.

Proposition 1. $\forall n$, if $w_{T,n} \geq w_n > b_{T,n} > b_{U,n} > b_{X,n} > 0$, $\theta_{T,n} \geq \theta_n$, $\pi_0 - \pi_1 s_{U,n} \geq 0$, $c_{T,n} \geq c_n$ and $\phi_n < \lambda_n$, then $V_{E,n} > V_{U,n} > V_{X,n}$ and $V_{E,n|T} > V_{T,n} > V_{U,n}$.

Proof. From Equations (8) to (12), $V_{E,n} - V_{U,n}$ can be written as:

$$V_{E,n} - V_{U,n} = \Bigg[(r + c_{T,n}s_{T,n}\alpha(\theta_{T,n}) + \lambda_n)\left[(r + c_n s_{X,n}\alpha(\theta_n) + \gamma_n)(v_{E,n} - v_{U,n})\right.$$

$$\left. + \pi_n(v_{E,n} - v_{X,n})\right] + \gamma_n(r + \pi_n + c_n s_{X,n}\alpha(\theta_n) + \gamma_n)(v_{E,n} - v_{T,n})\Bigg]\Big/\Delta_{1,n}, \tag{13}$$

where $v_{E,n} \equiv \ln(w_n)$, $v_{U,n} \equiv \ln(b_{U,n}) - \psi_n \frac{(s_{U,n})^{\xi_n}}{\xi_n}$, $v_{X,n} \equiv \ln(b_{X,n}) - \psi_n \frac{(s_{X,n})^{\xi_n}}{\xi_n}$, $v_{T,n} \equiv ln(b_{T,n}) - \psi_n \frac{(s_{T,n})^{\xi_n}}{\xi_n}$, and $\Delta_{1,n} \equiv (r + c_n s_{X,n}\alpha(\theta_n) + \gamma_n)\Bigg[[r + c_n s_{U,n}\alpha(\theta_n) + \phi_n][r +$

$c_{T,n}s_{T,n}\alpha(\theta_{T,n}) + \lambda_n] + \gamma_n[r + c_{T,n}s_{T,n}\alpha(\theta_{T,n}) + \phi_n] \Big] + \pi_n \Big[[r + c_{T,n}s_{T,n}\alpha(\theta_{T,n}) + \lambda_n][r +$

$c_n s_{X,n}\alpha(\theta_n) + \phi_n] + \gamma_n[r + c_{T,n}s_{T,n}\alpha(\theta_{T,n}) + \phi_n] \Big]$. The hypotheses $w_n > b_{T,n} > b_{U,n} > b_{X,n} > 0$ are sufficient to guarantee that $V_{E,n} - V_{U,n} > 0$.

$V_{U,n} - V_{X,n}$ can be written as:

$$V_{U,n} - V_{X,n} = [\delta_{UX,n} + c_n(s_{U,n} - s_{X,n})\alpha(\theta_n)(V_{E,n} - V_{U,n})] / \Delta_{2,n}, \tag{14}$$

where $\delta_{UX,n} \equiv v_{U,n} - v_{X,n}$ and $\Delta_{2,n} \equiv r + \pi_n + c_n s_{X,n}\alpha(\theta_n) + \gamma_n$. Appendix 1 shows that $V_{U,n} > V_{X,n}$.

The proof that $V_{T,n} > V_{U,n}$ is left to Appendix 1, too. If $w_{T,n} = w_n$, $V_{T,n} - V_{U,n}$ can be expressed in the following way:

$$V_{T,n} - V_{U,n} = \Big[(r + c_n s_{X,n}\alpha(\theta_n) + \gamma_n) \left(\delta_{TU,n} + (c_{T,n}s_{T,n}\alpha(\theta_{T,n}) - c_n s_{U,n}\alpha(\theta_n))(V_{E,n} - V_{U,n}) \right)$$

$$+ \pi_n \left(\delta_{TX,n} + (c_{T,n}s_{T,n}\alpha(\theta_{T,n}) - c_n s_{X,n}\alpha(\theta_n))(V_{E,n} - V_{U,n}) \right) \Big] / [\Delta_{2,n}\Delta_{3,n}], \tag{15}$$

where $\delta_{TU,n} \equiv v_{T,n} - v_{U,n}$, $\delta_{TX,n} \equiv v_{T,n} - v_{X,n}$ and $\Delta_{3,n} \equiv r + c_{T,n}s_{T,n}\alpha(\theta_{T,n}) + \lambda_n + \gamma_n$. $V_{E,n|T} - V_{T,n}$ can be expressed as:

$$V_{E,n|T} - V_{T,n} = [\delta_{ET,n} + (\lambda_n - \phi_n)(V_{T,n} - V_{U,n})] / \Delta_{4,n}, \tag{16}$$

where $\delta_{ET,n} \equiv \ln(w_{T,n}) - v_{T,n}$ and $\Delta_{4,n} \equiv r + c_{T,n}s_{T,n}\alpha(\theta_{T,n}) + \phi_n$. Imposing $\lambda_n > \phi_n$ is sufficient to guarantee that $V_{E,n|T} > V_{T,n}$ holds true. ∎

2.3 The optimal level of search

The optimal level of search effort $s_{U,n}, s_{X,n}$ and $s_{T,n}$ are respectively solution to the following (sufficient) first-order conditions:

$$\psi_n (s_{U,n})^{\xi_n - 1} = c_n \alpha(\theta_n)(V_{E,n} - V_{U,n}) - \pi_1(V_{X,n} - V_{U,n}), \tag{17}$$

$$\psi_n (s_{X,n})^{\xi_n - 1} = c_n \alpha(\theta_n)(V_{E,n} - V_{X,n}), \tag{18}$$

$$\psi_n (s_{T,n})^{\xi_n - 1} = c_{T,n}\alpha(\theta_{T,n})(V_{E,n|T} - V_{T,n}). \tag{19}$$

In a partial equilibrium perspective (i.e. when θ_n and the V's are given), from Proposition 1 and Equality (17), increasing π_1 raises $s_{U,n}$, while π_0 has no effect. In the same perspective, a higher hiring rate (through an increase in the c's or in the α's) stimulates search effort.

From (17) and (18),

$$s_{X,n} \gtreqless s_{U,n} \Leftrightarrow (c_n \alpha(\theta_n) - \pi_1)(V_{U,n} - V_{X,n}) \gtreqless 0. \tag{20}$$

9

From Proposition 1, we know that $V_{U,n} - V_{X,n} > 0$. Condition $c_n \alpha(\theta_n) - \pi_1 \geq 0$ says that the marginal effect of search effort on the exit rate out of unemployment should be at least as high as its marginal effect, π_1, on the rate π_n at which benefits decline. This condition is obviously fulfilled if $\pi_1 = 0$ and should still hold for moderate values of π_1. The exact meaning of the latter condition is clearly dependent of the skill-specific matching effectiveness (c_n). Even, for populations with high matching effectiveness, sufficiently high values of π_1 should imply that $s_{X,n}$ becomes lower than $s_{U,n}$ for it will later be shown that the equilibrium value of tightness decreases with π_1 (see Proposition 5). To sum up, for each skill n, one can expect that $s_{X,n}$ is higher or equal to $s_{U,n}$ as long as π_1 is sufficiently small. This can be called the *'ex-post effect'* of declining benefits.

Combining (18) and (19), $s_{T,n} \gtreqless s_{U,n}$ if and only if

$$
\begin{aligned}
& c_{T,n} \, \alpha(\theta_{T,n})(V_{E,n|T} - V_{E,n}) + (c_{T,n} \, \alpha(\theta_{T,n}) - c_n \, \alpha(\theta_n))(V_{E,n} - V_{U,n}) \\
& \gtreqless c_{T,n} \, \alpha(\theta_{T,n})(V_{T,n} - V_{U,n}) + \pi_1(V_{U,n} - V_{X,n})
\end{aligned}
\tag{21}
$$

Under the assumptions of Proposition 1, the left-hand side is clearly positive. Whatever the value of π_1, the right-hand side is positive too. If π_1 and the difference between $V_{T,n}$ and $V_{U,n}$ are sufficiently close to zero, one can expect that $s_{T,n}$ be higher than $s_{U,n}$.

2.4   The employment rates

This subsection looks at the marginal effects of $\theta_n, \theta_{T,n}, s_{U,n}, s_{X,n}, s_{T,n}, \phi_n, \gamma_n, \lambda_n, \pi_0$ and π_1 on the 'employment rate' $e_n + e_{T,n}$.[13]

Proposition 2.   *For each skill n,*

1. *The employment rate $e_n + e_{T,n}$ increases with $\theta_n, \theta_{T,n}, s_{U,n}, s_{X,n}, s_{T,n}$ and the parameters $c_n, c_{T,n}$ and it decreases with the rate of separation ϕ_n.*

2. $\dfrac{\partial e_n + e_{T,n}}{\partial \pi_0} \begin{array}{l} \geq 0 \\ < 0 \end{array}$ *and* $\dfrac{\partial e_n + e_{T,n}}{\partial \pi_1} \begin{array}{l} \leq 0 \\ > 0 \end{array}$ $\begin{array}{l} \text{if } s_{X,n} \geq s_{U,n} \\ \text{if } s_{X,n} < s_{U,n} \end{array}$

3. $\dfrac{\partial e_n + e_{T,n}}{\partial \gamma_n}$ *can be positive if $c_{T,n} s_{T,n} \alpha(\theta_{T,n})$ is sufficiently larger than $c_n s_{U,n} \alpha(\theta_n)$ and $c_n s_{X,n} \alpha(\theta_n)$,*

4. $\dfrac{\partial e_n + e_{T,n}}{\partial \lambda_n} \begin{array}{l} \leq 0 \\ > 0 \end{array}$ $\begin{array}{l} \text{if } c_{T,n} s_{T,n} \alpha(\theta_{T,n}) \geq c_n s_{U,n} \alpha(\theta_n) \text{ and } c_{T,n} s_{T,n} \alpha(\theta_{T,n}) \geq c_X s_X \alpha(\theta) \\ \text{if } c_{T,n} s_{T,n} \alpha(\theta_{T,n}) < c_n s_{U,n} \alpha(\theta_n) \text{ and } c_{T,n} s_{T,n} \alpha(\theta_{T,n}) < c_X s_X \alpha(\theta) \end{array}$

Proof.   See Appendix 2.   ∎

---

[13]The marginal effects on the participation rate should be added if these rates were defined with respect to the population of working age instead of the active population.

Corollary 1. *If $\pi_n = 0$, $c_{T,n} s_{T,n} \alpha(\theta_{T,n}) > c_n s_{U,n} \alpha(\theta_n)$ is a sufficient condition for $\frac{\partial e_n + e_{T,n}}{\partial \gamma_n}$ to be positive.*

Proof. From Appendix 2, this result is immediate. ∎

The effect of the rate of entry into training programs (γ_n) strongly depends on the relative values of the hiring rates. If the exit rate of the trainees is the highest among the three groups of job seekers, the effect of an increase in γ_n can still be ambiguous for each additional unemployed who flows from state U_n into training would instead have increased his (her) search effort at the moment of entry in state X_n (if $\pi_n > 0$ and provided that the 'ex-post effect' is observed). The corollary implies that the conditions needed to get a positive effect of γ_n on the employment rate are less numerous when unemployment benefits are constant. One cannot say more since the level of the endogenous variables present in the above conditions vary whether unemployment benefits are constant or not.

2.5   Job creation

For simplicity, taxation (including social security contributions) is linear. Let $\tau_{0,n} + \tau_{1,n} w_n$ be the amount of taxes paid if the net wage is w_n ($\tau_{1,n} \geq 0$). Taxation is typically not skill-specific. This is however a convenient way of capturing the idea that marginal and average taxation can vary with the level of earnings. The firm's discounted expected return from an occupied job is denoted $\Pi_{E,n}$ if this firms operates in the skill segment n (respectively, $\Pi_{E,n|T}$ if a former trainee is occupied). The discounted expected return of vacant job is $\Pi_{V,n}$ (respectively, $\Pi_{V,n|T}$). The difference between hiring a trainee and hiring another job-seeker comes from the training cost. Since, conditional on their skill, workers are equally productive, let y_n be the constant marginal product of a filled vacancy. Remember that K_n captures the cost involved in posting a vacancy in segment n, searching for applicants and selecting them. Consider that K_n is proportional to y_n : $K_n \equiv k_n y_n$. Similarly, let us assume that the fixed hiring costs are proportional to y_n : $H_n \equiv \kappa_n y_n, H_{T,n} \equiv \kappa_{T,n} y_n$ ($\kappa_n > \kappa_{T,n}$). For each skill n, the discounted expected returns satisfy the following conditions:

$$r\Pi_{E,n} = y_n - [\tau_{0,n} + w_n(1 + \tau_{1,n})] + \phi_n \left(\max\left[\Pi_{V,n}, \Pi_{V,n|T}\right] - \Pi_{E,n}\right), \quad (22)$$

$$r\Pi_{E,n|T} = y_n - [\tau_{0,n} + w_{T,n}(1 + \tau_{1,n})] + \phi_n \left(\max\left[\Pi_{V,n}, \Pi_{V,n|T}\right] - \Pi_{E,n|T}\right), \quad (23)$$

$$r\Pi_{V,n} = -k_n y_n + q(\theta_n) \left(\Pi_{E,n} - \kappa_n y_n - \Pi_{V,n}\right), \quad (24)$$

$$r\Pi_{V,n|T} = -k_n y_n + q(\theta_{T,n}) \left(\Pi_{E,n|T} - \kappa_{T,n} y_n - \Pi_{V,n|T}\right). \quad (25)$$

In equilibrium, vacancies are opened as long as they yield a positive expected return. Therefore, in each segment n, the equilibrium conditions for the supply of vacancies are $\Pi_{V,n|T} = \Pi_{V,n} = 0$. These properties combined with (22), (23), (24), and (25) yield two

'vacancy-supply curves' for each n:

$$\left(\frac{k_n}{q(\theta_n)} + \kappa_n\right) y_n = \frac{y_n - \tau_{0,n} - w_n(1 + \tau_{1,n})}{r + \phi_n}, \tag{26}$$

$$\left(\frac{k_n}{q(\theta_{T,n})} + \kappa_{T,n}\right) y_n = \frac{y_n - \tau_{0,n} - w_{T,n}(1 + \tau_{1,n})}{r + \phi_n}. \tag{27}$$

It is easily checked that the 'vacancy-supply curves' establish a decreasing relationship between the net wage and the corresponding indicator of tightness. In particular, Equation (26) can be rewritten as:

$$w_n = VS(\theta_n \mid \kappa_n, k_n, y_n, r, \phi_n, \tau_{0,n}, \tau_{1,n}) \equiv \frac{y_n \left(1 - (r + \phi_n)\left(\frac{k_n}{q(\theta_n)} + \kappa_n\right)\right) - \tau_{0,n}}{1 + \tau_{1,n}}, \tag{28}$$

with $\frac{\partial VS}{\partial \theta_n} < 0, \frac{\partial VS}{\partial \phi_n} < 0, \frac{\partial VS}{\partial r} < 0, \frac{\partial VS}{\partial k_n} < 0, \frac{\partial VS}{\partial \kappa_n} < 0, \frac{\partial VS}{\partial \tau_{0,n}} < 0, \frac{\partial VS}{\partial \tau_{1,n}} < 0$ and $\frac{\partial VS}{\partial y_n} > 0$.

2.6 The wage bargain

It is assumed that wages are collectively bargained over in each segment (firm) n by insiders whose inter-temporal utility is $V_{U,n}$ in case of layoff.[14] Training expenditures are then sunk costs. In order to show that a single wage will appear in each firm, let us imagine that the insiders of type n bargain over two wages w_n and $w_{n,T}$. For each n, the Nash maximization program can be written as:

$$\max_{w_n} \left(V_{E,n} - V_{U,n}\right)^{\beta_n} \left(\Pi_{E,n} - \max\left[\Pi_{V,n}, \Pi_{V,n|T}\right]\right)^{1-\beta_n}, \tag{29}$$

$$\max_{w_{T,n}} \left(V_{E,n|T} - V_{U,n}\right)^{\beta_n} \left(\Pi_{E,n|T} - \max\left[\Pi_{V,n}, \Pi_{V,n|T}\right]\right)^{1-\beta_n}, \tag{30}$$

with $0 < \beta_n < 1$. The assumption of a single representative firm has been made for the sake of simplicity. It does not imply that the wage bargain is centralized in such a way that insiders and the representative firm take care of the equilibrium effect of wages on tightness. The first-order condition can be written as:

$$w_n = \frac{1}{1 + \tau_{1,n}} \frac{\beta_n}{1 - \beta_n} \frac{\Pi_{E,n} - \max\left[\Pi_{V,n}, \Pi_{V,n|T}\right]}{V_{E,n} - V_{U,n}}, \tag{31}$$

$$w_{T,n} = \frac{1}{1 + \tau_{1,n}} \frac{\beta_n}{1 - \beta_n} \frac{\Pi_{E,n|T} - \max\left[\Pi_{V,n}, \Pi_{V,n|T}\right]}{V_{E,n|T} - V_{U,n}}. \tag{32}$$

---

[14]The existence of a minimum wage will be taken into account in the numerical analysis below.

2.7 Wages and tightness in a symmetric equilibrium

Taking (8), (9), (22) and (23) into account, the first-order conditions (31) and (32) can be rewritten as under free entry (i.e. $\Pi_{V,n} = \Pi_{V,n|T} = 0$):

$$\ln(w_n) = rV_{U,n} + \frac{\beta_n}{1-\beta_n}\left(\frac{y_n - \tau_{0,n}}{w_n(1+\tau_{1,n})} - 1\right), \tag{33}$$

$$\ln(w_{T,n}) = rV_{U,n} + \frac{\beta_n}{1-\beta_n}\left(\frac{y_n - \tau_{0,n}}{w_{T,n}(1+\tau_{1,n})} - 1\right). \tag{34}$$

Therefore, $w_n = w_{T,n}$. These equations have an intuitive interpretation. If β_n was equal to zero, the instantaneous utility in employment would be equal to the minimum compensation that an unemployed worker requires to stop searching. As β_n increases, a growing share of the relative difference $y_n - \tau_{0,n} - w_n(1+\tau_{1,n})$ (scaled by $w_n(1+\tau_{1,n})$) accrues to the worker. The fact that $w_n = w_{T,n}$ in each segment n has two implications. First, $V_{E,n|T} = V_{E,n}$. Second, from (26) and (27), one has:

$$\kappa_n - \kappa_{T,n} = k_n\left(\frac{1}{q(\theta_{T,n})} - \frac{1}{q(\theta_n)}\right). \tag{35}$$

Therefore, $\theta_{T,n} > \theta_n$. This equality defines a relationship $\theta_{T,n} = \mathcal{T}(\theta_n \mid \kappa_{T,n}, \kappa_n, k_n)$ with $\frac{\partial \mathcal{T}}{\partial \theta_n} > 0, \frac{\partial \mathcal{T}}{\partial \kappa_{T,n}} < 0, \frac{\partial \mathcal{T}}{\partial \kappa_n} > 0$ and $\frac{\partial \mathcal{T}}{\partial k_n} < 0$. So, $\theta_{T,n}$ and θ_n move together.

The next step consists in replacing $rV_{U,n}$ in (33) by a function of $\theta_n, \theta_{T,n}, s_{U,n}, s_{X,n}, s_{T,n}$ and the parameters of the model. From (10), $rV_{U,n}$ is a function of $V_{E,n} - V_{U,n}, V_{U,n} - V_{X,n}$ and $V_{T,n} - V_{U,n}$. Substituting expressions (14) and (15) in (10) allows to write $rV_{U,n}$ as a function of $V_{E,n} - V_{U,n}$. This expression can then be substituted in (33). This yields the following relationship:

$$\ln(w_n) = \frac{[r + c_{T,n}s_{T,n}\alpha(\theta_{T,n}) + \lambda_n][r + \gamma_n + c_n s_{X,n}\alpha(\theta_n)]}{\Delta_{2,n}\Delta_{3,n}}\left[v_{U,n} + c_n s_{U,n}\alpha(\theta_n)(V_{E,n} - V_{U,n})\right]$$

$$+ \frac{\gamma_n}{\Delta_{3,n}}\left[v_{T,n} + c_{T,n}s_{T,n}\alpha(\theta_{T,n})(V_{E,n} - V_{U,n})\right] + \frac{\pi_n[r + c_{T,n}s_{T,n}\alpha(\theta_{T,n}) + \lambda_n]}{\Delta_{2,n}\Delta_{3,n}}$$

$$\left[v_{X,n} + c_n s_{X,n}\alpha(\theta_n)(V_{E,n} - V_{U,n})\right] + \frac{\beta_n}{1-\beta_n}\left(\frac{y_n - \tau_{0,n}}{w_n(1+\tau_{1,n})} - 1\right), \tag{36}$$

where $\Delta_{2,n}$ and $\Delta_{3,n}$ have been respectively defined after equations (14) and (15). Equation (31) combined with the free-entry conditions and with (22) and (26) implies that

$$V_{E,n} - V_{U,n} = \mathcal{V}(\theta_n \mid \beta_n, k_n, \kappa_n, r, \phi_n, y_n, \tau_{0,n}) \equiv \frac{\beta_n}{1-\beta_n} \cdot \frac{y_n(\frac{k_n}{q(\theta_n)} + \kappa_n)}{y_n\left(1 - (r+\phi_n)\left(\frac{k_n}{q(\theta_n)} + \kappa_n\right)\right) - \tau_{0,n}}. \tag{37}$$

with $\frac{\partial \mathcal{V}}{\partial \theta_n} > 0, \frac{\partial \mathcal{V}}{\partial \beta_n} > 0, \frac{\partial \mathcal{V}}{\partial k_n} > 0, \frac{\partial \mathcal{V}}{\partial \kappa_n} > 0, \frac{\partial \mathcal{V}}{\partial r} > 0, \frac{\partial \mathcal{V}}{\partial \phi_n} > 0, sign\left[\frac{\partial \mathcal{V}}{\partial y_n}\right] = -sign\left[\tau_{0,n}\right]$ and $\frac{\partial \mathcal{V}}{\partial \tau_{0,n}} > 0$. Henceforth, $\mathcal{V}(\cdot)$ will designate $\mathcal{V}(\theta_n \mid \beta_n, k_n, \kappa_n, r, \phi_n, y_n, \tau_{0,n})$.

Let $Z_n = (\tilde{Z}_n, , \kappa_n, \kappa_{T,n}, k_n, y_n, r, \beta_n) = (c_n, c_{T_n}, \phi_n, \gamma_n, \lambda_n, \pi_0, \pi_1, \kappa_n, \kappa_{T,n}, k_n, y_n, r, \beta_n)$ and $B_n = (b_{T,n}, b_{U,n}, b_{X,n})$. Substituting (37) into (36) and using (26) yield an explicit (net) 'wage-setting curve':

$$
\begin{aligned}
\ln(w_n) &= WS(\theta_n, \theta_{T,n}, s_{T,n}, s_{U,n}, s_{X,n} \mid Z_n, \tau_{0,n}, B_n) \equiv \\
&\frac{[r + c_{T,n}s_{T,n}\alpha(\theta_{T,n}) + \lambda_n][r + \gamma_n + c_n s_{X,n}\alpha(\theta_n)]}{\Delta_{2,n}\Delta_{3,n}}\left[v_{U,n} + c_n s_{U,n}\alpha(\theta_n)\mathcal{V}(\cdot)\right] \\
&+ \frac{\gamma_n}{\Delta_{3,n}}\left[v_{T,n} + c_{T,n}s_{T,n}\alpha(\theta_{T,n})\mathcal{V}(\cdot)\right] \\
&+ \frac{\pi_n[r + c_{T,n}s_{T,n}\alpha(\theta_{T,n}) + \lambda_n]}{\Delta_{2,n}\Delta_{3,n}}\left[v_{X,n} + c_n s_{X,n}\alpha(\theta_n)\mathcal{V}(\cdot)\right] + (r + \phi_n)\left[\mathcal{V}(\cdot)\right].
\end{aligned}
\tag{38}
$$

Proposition 3. *For each skill n, the 'wage-setting curve' $WS(\theta_n, \theta_{T,n}, s_{T,n}, s_{U,n}, s_{X,n} \mid Z_n, \tau_{0,n}, B_n)$ is upward-sloping in a (w_n, θ_n) space. It is not affected by marginal changes in search efforts. For any θ_n, the wage increases with $\theta_{T,n}$, the allowances $(b_{T,n}, b_{U,n}, b_{X,n}), \gamma_n, \pi_1, k_n, \kappa_n, \beta_n, \tau_{0,n}$ and ϕ_n. It decreases with π_0 and λ_n and does not vary with $\kappa_{T,n}$. Finally, still conditioning on θ_n, the sign of the marginal effect of y_n on the net wage is the opposite of the sign of $\tau_{0,n}$.*

Proof. The proof is left to Appendix 3. ■

The wage-push effect of active labor market policies was a major point made by Holmlund and Lindén (1993). The opposite effects of π_0 and π_1 on the wage-setting curve are worth to be mentioned.

From (35), $\theta_{T,n} = \mathcal{T}(\theta_n \mid \kappa_{T,n}, \kappa_n, k_n)$ can be substituted in (38) to yield another wage setting equation, namely:

$$
\mathcal{WS}(\theta_n, s_{T,n}, s_{U,n}, s_{X,n} \mid Z_n, \tau_{0,n}, B_n) \equiv WS(\theta_n, \mathcal{T}(\theta_n \mid \kappa_{T,n}, \kappa_n, k_n), s_{T,n}, s_{U,n}, s_{X,n} \mid Z_n, \tau_{0,n}, B_n)
\tag{39}
$$

Proposition 4. *The partial derivatives of $\mathcal{WS}(\theta_n, s_{T,n}, s_{U,n}, s_{X,n} \mid Z_n, \tau_{0,n}, B_n)$ have the same sign as those of $WS(\theta_n, \theta_{T,n}, s_{T,n}, s_{U,n}, s_{X,n} \mid Z_n, \tau_{0,n}, B_n)$ except that $\frac{\partial \mathcal{WS}}{\partial \kappa_{T,n}} < 0$ and $\frac{\partial \mathcal{WS}}{\partial k_n}$ becomes ambiguous.*

Proof. This is obvious from the properties of function $\mathcal{T}(.)$. ■

The downward-sloping 'vacancy-supply' curve (28) and the upward-sloping wage-setting equation $\ln(w_n) = \mathcal{WS}(\theta_n, s_{T,n}, s_{U,n}, s_{X,n} \mid Z_n, \tau_{0,n}, B_n)$ define the equilibrium value of w_n

and θ_n. Taking the ln of (28) yields an implicit equation for θ_n, namely $F(\theta_n, s_{T,n}, s_{U,n}, s_{X,n} \mid Z_n, \tau_{0,n}, \tau_{1,n}, B_n) = 0$ with :

$$
\begin{aligned}
F(\theta_n, s_{T,n}, s_{U,n}, s_{X,n} \mid Z_n, \tau_{0,n}, \tau_{1,n}, B_n) &\equiv \ln\left(VS(\theta_n \mid \kappa_n, k_n, y_n, r, \phi_n, \tau_{0,n}, \tau_{1,n})\right) \\
&- \mathcal{WS}(\theta_n, s_{T,n}, s_{U,n}, s_{X,n} \mid Z_n, \tau_{0,n}, B_n)
\end{aligned}
\tag{40}
$$

Marginal changes in search effort do not affect function F. For those parameters in Z_n that do not affect the vacancy-supply curve, any variation that has a wage-push effect leads to a higher equilibrium net wage and lower tightness θ_n and conversely. Combining the properties of (28) and of $\mathcal{WS}(\theta_n, s_{T,n}, s_{U,n}, s_{X,n} \mid Z_n, \tau_{0,n}, B_n)$ leads directly to the following proposition:

Proposition 5 *For each skill n, the equilibrium net wage w_n (respectively, the level of tightness θ_n) increases (respectively, decreases) with $b_{T,n}, b_{U,n}, b_{X,n}, \gamma_n, \pi_1, \tau_{0,n}$ and β_n. The equilibrium net wage w_n (respectively, the level of tightness θ_n) decreases (respectively, increases) with π_0, λ_n and $\kappa_{T,n}$. The marginal tax rate $\tau_{1,n}$ has a negative effect on the equilibrium wage and on tightness. ϕ_n and κ_n have a negative influence on θ_n but their effect on w_n cannot be signed. The impact of k_n is ambiguous on both θ_n and w_n. If $\tau_{0,n} = 0$, the marginal product y_n affects the equilibrium wage and tightness positively. If $\tau_{0,n} < 0$ (respectively, > 0), the effect of y_n on tightness (respectively, the wage) becomes ambiguous.*

So, in addition to its unclear effect on the employment rate (Proposition 2), an increase in the rate of entry into training will negatively affect tightness in equilibrium and hence the employment rate. On the contrary, the favorable effect of π_0 on the employment rate (if $s_{X,n} > s_{U,n}$; see Proposition 2) is here reinforced by a positive effect on the equilibrium level of tightness. The opposite holds for π_1. Therefore, the rationale for making π_n vary with search effort $s_{U,n}$ heavily depends on the effect of π_1 on search effort in equilibrium.

Before looking at this issue, it is useful to come back to the major result in Cahuc and Lehmann (2000). This result can be restated and generalized[15] as follows. Keep the tax parameters fixed and imagine that a marginal decrease in the lower level of benefits ($b_{X,n}$) is compensated by a marginal increase in the highest level of benefits ($db_{U,n} = -db_{X,n} > 0$). This is called "front-loading" the benefit system. A steeper profile will only affect the wage-setting curve through its effect on the inter-temporal utility of those entering unemployment, $V_{U,n}$. Since only marginal changes are considered here, the adjustment of search effort levels can be neglected as long as one only looks at the impact on wages and tightness. Therefore, differentiating (38) with respect to $b_{U,n}$ and $b_{X,n}$, it can be checked that the direction of change of the net wage w_n is given by the sign of the following

---

[15]The following result is more general for three reasons: Search effort is here endogenous, active programs are taken into account and the rate at which unemployed people enter the low-benefit state is a parameter (or a function of $s_{U,n}$ if $\pi_1 \neq 0$).

expression:

$$\frac{r + \gamma_n + c_{X,n}\, s_{X,n}\, \alpha(\theta_n)}{\pi_n} - \frac{b_{U,n}}{b_{X,n}}. \tag{41}$$

The equilibrium value of tightness varies in the opposite direction. The higher the rate π_n, the lower the intertemporal utility in the state of entry after a job loss. So, if π_n is high, front-loading has more chance of leading to wage moderation. The levels of the discount rate r and of the exit rate out of state X_n $(c_{X,n}\, s_{X,n}\, \alpha(\theta_n) + \gamma_n)$ have the opposite effect. If the first ratio in (41) is higher than one, then in the neighborhood of a situation where $b_{U,n} = b_{X,n}$, the equilibrium value of tightness will *decrease* (wages will increase). Front-loading the benefit system is then expected to raise tightness only when the ratio $b_{U,n}/b_{X,n}$ is already sufficiently high. *Consequently, a short expected period in the high benefit state and a low rate of entry into training schemes should be recommended if one intends to raise the number of vacancies per (efficient) job-seeker through a steeper profile of unemployment benefits.* Of course, front-loading the benefit system will also affect search effort. It is therefore time to look at the equilibrium levels of search.

2.8   The equilibrium levels of search

In a symmetric equilibrium, expressions (13), (14), (15) and (16) could be substituted in the optimality conditions (17), (18) and (19). However, this leads to very few clear-cut properties. Alternatively, the bargain over the rent created by each match combined with the free-entry condition for vacancies leads to (37). This expression can be substituted for $V_E - V_U$ in the first-order conditions (17), (18) and (19) in which $V_U - V_X$ has first been replaced by (14) and $V_T - V_U$ by (15). After some manipulation, this leads for each n to:

$$\Sigma_U(\theta_n, s_{U,n}, s_{X,n}, s_{T,n} \mid Z_n, \tau_{0,n}, B_n) = 0 \tag{42}$$

with $\Sigma_U \equiv \Delta_{2,n}\, \psi_n\, s_{U,n}^{\xi_n-1} - \pi_1 \delta_{UX,n} - c_n\, (\Delta_{2,n} + \pi_1[s_{U,n} - s_{X,n}])\, \alpha(\theta_n)\, \mathcal{V}(\cdot)$

$$\Sigma_X(\theta_n, s_{U,n}, s_{X,n}, s_{T,n} \mid Z_n, \tau_{0,n}, B_n) = 0 \tag{43}$$

with $\Sigma_X \equiv \Delta_{2,n}\, \psi_n\, s_{X,n}^{\xi_n-1} - c_n\, \alpha(\theta_n)\, [\delta_{UX,n} + (\Delta_{2,n} + c_n\, [s_{U,n} - s_{X,n}]\, \alpha(\theta_n))\, \mathcal{V}(\cdot)]$

$$\Sigma_T(\theta_n, \theta_{T,n}, s_{U,n}, s_{X,n}, s_{T,n} \mid Z_n, \tau_{0,n}, B_n) = 0 \tag{44}$$

with $\Sigma_T \equiv \Delta_{2,n}\, \Delta_{3,n}\, \psi_n\, s_{T,n}^{\xi_n-1} - c_{T,n}\, \alpha(\theta_{T,n}) \left[\left(\Delta_{2,n}\, \Delta_{3,n} - [r + c_n\, s_{X,n}\, \alpha(\theta_n) + \gamma_n] \right.\right.$

$$\left. [c_{T,n}\, s_{T,n}\, \alpha(\theta_{T,n}) - c_n\, s_{U,n}\, \alpha(\theta_n)] - \pi_n[c_{T,n}\, s_{T,n}\, \alpha(\theta_{T,n}) - c_n\, s_{X,n}\, \alpha(\theta_n)] \right) \mathcal{V}(\cdot)$$

$$\left. - (r + c_n s_{X,n}\alpha(\theta_n) + \gamma_n)\, \delta_{TU,n} - \pi_n\, \delta_{TX,n} \right].$$

This approach allows to derive a set of clear analytical properties. Their interpretation requires however a lot of care. For, exploiting (37) implies that wages are endogenous but

also that the behavior of firms is optimized. However, in (42), (43) and (44), the ratios θ and θ_T are taken as free variables. An interpretation of the following comparative statics would be that the number of vacancies is optimally chosen by the employers but S and S_T are adjusted to keep θ_n and $\theta_{T,n}$ unchanged. Obviously, the following properties should be considered as intermediate ones. In a next step, one needs to take the adjustment of θ_n and $\theta_{T,n}$ into account (according to (40) and $\theta_{T,n} = \mathcal{T}(\theta_n \mid \kappa_{T,n}, \kappa_n, k_n)$). The ambiguity reached then for many parameters (see Table 1) cannot easily be understood without the following intermediate result, however.

Totally differentiating equations (42), (43) and (44), it can be checked that $\frac{\partial \Sigma_\iota}{\partial s_{\iota'}} = 0 \; \forall \iota, \iota' \in \{\{T,n\}, \{X,n\}, \{U,n\}\}, \iota \neq \iota', n \in \{l,h\}$. Furthermore, if $\pi_1 = 0$, $s_{U,n}$ increases with θ_n. Otherwise, this relationship is ambiguous. Search effort $s_{U,n}$ increases with $\pi_1, \beta_n, k_n, \kappa_n$, and ϕ_n. If $\pi_1 > 0$, $s_{U,n}$ increases with $b_{U,n}$ and it decreases with $\gamma_n, b_{X,n}$ and π_0. Otherwise, $s_{U,n}$ is independent of $b_{U,n}, \; b_{X,n}, \pi_0$ and γ_n. The separability of the instantaneous utility function and the fact that $V_{E,n} - V_{U,n}$ is replaced by $\mathcal{V}(\theta_n, \beta_n, k_n, \kappa_n, r, \phi_n, y_n, \tau_{0,n})$ in (42) explain why the unemployment benefit $b_{U,n}$ can only influence s_U through the sanction rate π_1. Then, a higher $b_{U,n}$ *raises* the difference $V_{U,n} - V_{X,n}$ and therefore pushes $s_{U,n}$ *upwards* in order to reduce the risk of a reduction in unemployment benefits.

$s_{X,n}$ and $s_{T,n}$ increase with $b_{U,n}$ and π_1 and they decrease with π_0 (the so-called 'entitlement effect' due to Mortensen, 1977). By an 'entitlement effect', one here means that the prospect of higher or longer benefits in the first unemployment stage (U_n) stimulates search effort in the other states because entering a job is now more interesting taking the risk of a future layoff into account.

The effects of other parameters (conditional on θ_n and $\theta_{T,n}$) are also summarized in Table 1. As an example, raising the workers' bargaining power augments the share of the surplus they receive. Hence, search effort is stimulated by an increase in β_n (still keeping θ_n and $\theta_{T,n}$ fixed).

Table 1 also presents the comparative static properties *when the adjustment of θ_n and $\theta_{T,n}$ is taken into account.* The impact of the parameters on the equilibrium value of search in the first state (column '$s_{U,n}^*$' in Table 1) is only given when $\pi_1 = 0$ (otherwise, net effects become ambiguous). For the other states, the corresponding columns are $s_{X,n}^*$ and $s_{T,n}^*$. If $\pi_1 = 0$, because of the adjustment of tightness levels, $s_{U,n}^*$ is declining with the level of each of the three benefits and it is increasing with π_0. The latter effect can be called an *'ex-ante effect'* of declining benefits. Interestingly, π_1 has an ambiguous effect on search effort $s_{U,n}^*$: Conditional on tightness, as π_1 starts increasing search effort reacts positively but at the same time tightness is affected negatively (Proposition 5). As far as $s_{X,n}^*$ and $s_{T,n}^*$ are concerned, one cannot determine whether the 'entitlement effect' is more than offset by the impacts of $b_{U,n}, \pi_0, \pi_1$ on equilibrium tightness. Moreover, Table 1 indicates that in addition to its wage-push effect (Proposition 5), γ_n also reduces the equilibrium level of job-search effort in states U_n and X_n. The tax parameters affect equilibrium search efforts via their impacts on tightness. Finally, taking the negative influence of β_n

on tightness levels, the effect of the workers' bargaining power on the equilibrium levels of search effort is ambiguous.

2.9 Uniqueness of equilibrium conditional on a vector $(Z_n, \tau_{0,n}, \tau_{1,n}, B_n)$

Proposition 6 *Conditional on* $(Z_n, \tau_{0,n}, \tau_{1,n}, B_n)$, *the equilibrium, if any, is unique if* $\pi_1 = 0$.

Proof. From the properties of functions VS and WS, the solution θ_n to Equation (40), if any, is unique. Hence, along VS, w_n is uniquely determined, too. From (35), the same is then true for $\theta_{T,n}$. The properties of the functions Σ imply that search effort levels are increasing functions of tightness if $\pi_1 = 0$. So, for the equilibrium value of tightness, if there exists a solution to equations (42), (43) and (44), it is unique, too. From Equations (1) to (5), explicit expressions can then be derived for the rates $(e_n + e_{T,n}, u_n, x_n, t_n)$ (see e.g. Equation (7)). By an argument of continuity, the same property should obviously hold for sufficiently low values of π_1.

2.10 Summary of the analytical properties

Focusing on the effect of passive and active labor market policies on the employment rate, the analytical properties can be summarized as follows. With respect to the rate of entry into programs (γ_n), one knows that $\theta_{T,n} > \theta_n$, $\forall n$, in equilibrium. This and the plausible assumption that $c_{T,n} \geq c_n$ lead to the expectation that the hiring rate of trainees will be greater than the hiring rates of those in states U_n and X_n. From Proposition 2, it is then plausible (but not sure) that γ_n has a positive direct effect on the employment rate. The marginal effect of γ_n is however negative on tightness and on search effort levels in states U_n and X_n. Even if one ignores the financing of training schemes, their net effect on employment appears to be gloomy. The rate at which the high benefit stage ends is typically independent of the behavior of the unemployed. In that case (i.e. if $\pi_1 = 0$), increasing π_0 has a direct positive effect on the employment rate. Moreover, it induces higher tightness and more search effort among the unemployed benefiting from $b_{U,n}$. However, via a negative entitlement effect, increasing π_0 reduces the incentive the other types of job seekers have. So, the net effect of an increase in π_0 cannot be signed analytically. If π_1 becomes positive but sufficiently small,[16] increasing this rate would have two negative marginal effects, a direct one on the employment rate and an indirect one on tightness. Nevertheless, conditional on tightness, increasing π_1 gives an incentive to search more. In spite of the previous negative effects, using π_1 as an additional instrument could turn out to be interesting if the last effect is strong enough. Finally, it is interesting to emphasize a last analytical property, namely that a short expected period in the high

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[16]So that $s_{X,n}$ remains higher than $s_{U,n}$ and the relationship between search effort $s_{U,n}$ and θ_n remains positive.

benefit state and a low rate of entry into training schemes should be recommended if one intends to raise the number of vacancies per (efficient) job-seeker through a two-tired benefit system. Otherwise, increasing the difference between $b_{U,n}$ and $b_{X,n}$ could be detrimental to employment.

2.11 Extending the model

Up to now, the budget constraint of the State (including the Social Security system) has been ignored. This constraint can be written as follows:

$$Q + \sum_{n \in \{h,l\}} b_{U,n} U_n + b_{X,n} X_n + (b_{T,n} + C)T_n = \sum_{n \in \{h,l\}} (\tau_{0,n} + \tau_{1,n} w_n)(E_n + E_{T,n}), \quad (45)$$

where Q is an exogenous level of net expenses and C is the average cost of training programs.[17] Constraint (45) establishes the unique direct link between the two labor markets. To meet this constraint, one could either adjust the level of allowances (B_n) or the one of taxes. Adjusting taxes is the most standard approach. However, Rocheteau (1999) has shown that endogeneizing taxes in order to balance the budget of the State can lead to multiple equilibria. One should therefore care about this possibility. Appendix 5 shows that the uniqueness of equilibrium can be preserved if $\tau_{0,n} = \pi_1 = 0$, the replacement ratios are constant and $\tau_{1,n}$ is adjusted to clear the budget of the State.

The size of the labor force, L_n, matters when the budget of the State is introduced. Participation is modeled in a very simple way (see Pissarides, 2000). Inactive people have an arbitrage condition: Staying inactive or entering state X_n.[18] The lower the expected intertemporal utility in state X_n, $V_{X,n}$, the lower the participation rate $p_n \equiv L_n/\mathcal{P}_n$. Let $[V_{1,n}, V_{2,n}]$ be the finite support of the distribution of intertemporal utility levels in inactivity, $V_{I,n}$. With a uniform distribution, the participation rate is simply defined as

$$p_n = \frac{V_{X,n} - V_{1,n}}{V_{2,n} - V_{1,n}}. \quad (46)$$

2.12 Normative criteria

Entering into the debate about welfarist versus non-welfarist criteria would take us too far afield. As is standard in economics, let us assume that utility functions are an appropriate basis for a normative analysis. If the utility functions of the low-skilled and the skilled population are identical, adding the utility functions of the various individuals is justified. Utility levels will be represented by their certainty equivalents. This approach consists in

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[17]C is here taken as fixed and increasing π_1 does not add monitoring cost. These hypotheses could be relaxed.

[18]Alternatively, they could enter uninsured unemployment (i.e. start an unemployment spell without any benefit). However, in many OECD countries, people who are ready to take a job and have no income are eligible to a minimum income guarantee. The latter is typically related to the lowest level of unemployment benefits. So, the simplifying assumption made here is not a substantial limitation.

computing the level of income that would yield the intertemporal utility, say V, if this income was received forever. Hence, with the assumed utility function, $\exp[rV]$ will denote the certainty equivalent of V.

Adopting the same marginal social weight for each type of agent, a utilitarian criterion, $r\Psi \equiv \sum_{n \in \{h,l\}} r\Psi_n \frac{L_n}{L}$, can be defined for the active population, with $L \equiv \sum_n L_n$ and:

$$r\Psi_n \equiv \left(\exp[rV_{E,n}](e_n + e_{T,n}) + \exp[rV_{U,n}]u_n + \exp[rV_{X,n}]x_n + \exp[rV_{T,n}]t_n \right). \qquad (47)$$

3 A numerical analysis

3.1 Calibration

The model is calibrated for Belgium with the month as unit of time. The discount rate is fixed to 0.004 (5% on an annual basis). As far as possible, the values of the parameters are based on observed data for the nineties. The period 1997-1998 has been used as a reference.[19] Belgium is a country plagued with long-term unemployment (LTU for short). For a very long time, more than 60% of the stock of unemployed is unoccupied for more than a year. Among these long-term unemployed, about 70% are jobless for more than two years. The median elapsed duration *in the stock* of 1997 was about 2 years. In Belgium, negative duration dependence is very strong but Dejemeppe and Cockx (2000) and Dejemeppe and Saks (2002) have shown that it is largely spurious. Furthermore, in Belgium, the relative unemployment rate of the low-skilled workers compared to the skilled ones is very large and increasing (see the previous references). Due to statistical availabilities, let us assume that holding at most a lower-secondary degree captures relatively well the notion of 'low skill'. Low-skilled workers then represent about 34% of the labor force. According to administrative data, the rate[20] of insured unemployment was equal to 0.105 in 1997. Within the group of skilled workers, the rate of short-term unemployment (STU for short) was equal to 0.031 and the rate of LTU amounted to 0.027. For the low-skilled, the same rates were respectively equal to 0.062 and 0.139. Adding these two rates for each skill group gives the skill-specific unemployment rate. With 64% of the unemployed being low skilled, one clearly sees that as in many other OECD countries the unemployment problem of the low-skilled is very acute.

Turning to the Belgian institutional setting, the level and the time-profile of unemployment benefits vary with the family composition. There is first a period of one year where unemployment benefits stay constant. For about two thirds of the insured unemployed, the level of benefits decreases afterwards. Unemployed people in charge of a family or living alone can receive insurance benefits for an indefinite length. Less than half of the unemployed (mostly women) can lose their entitlement after a period which differs a lot

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[19]The years close to 1993 were deeply affected by the major recession of the nineties. The last years of this decade were clearly a boom. During the period 1997-1998 the unemployment rate was fairly stable.

[20]Following the theoretical model, rates are obtained by dividing the stocks by the (skill-specific) size of the active population.

according to various criteria (the minimum length is 24 months and the maximum one is 99 months). The end of entitlement and sanctions are here ignored because the rate at which unemployed people move from the 'high' ($b_{U,n}$) to the 'low' ($b_{X,n}$) level of benefits is much more substantial. Within the stochastic framework presented in the previous section, the expected duration of the period during which $b_{U,n}$ is collected is fixed to a year. For the calibration, π_0 is therefore equal to 0.0833 and $\pi_1 = 0$. The level of unemployment benefits is in principle proportional to the previous wage, with upper- and lower-bounds. On the basis of administrative data about the level of benefit paid out to the unemployed (in EURO/month)[21], it is assumed that $b_{U,h} = 838$, $b_{X,h} = 608$, $b_{U,l} = 671$ and $b_{X,l} = 529$. On the basis of EUROSTAT statistics, the estimated average net wages are $w_h = 1650$ and $w_l = 1308$.[22] Hence, the net replacement ratios are $b_{U,h}/w_h = 0.51$, $b_{X,h}/w_h = 0.37$, $b_{U,l}/w_l = 0.51$ and $b_{X,l}/w_l = 0.40$. [23]

Vocational training for the unemployed is organized at the regional level in Belgium. In accordance with the model, programs that in a way or another put the unemployed back to school are ignored. In 1998, according to EUROSTAT data, the average stock of jobless people participating in training programs amounted to 0.67% of the active population and the average cost of training programs per worker amounted to 669 EURO/month (net of transfers to beneficiaries of these programs). Annual reports of the regional PES allow to split the stock of participants into low-skilled and skilled individuals. This leads to the skill-specific rates $t_h = 0.006$ and $t_l = 0.008$. These reports also allow to fix $\lambda_h = \lambda_l = 0.1, \gamma_h = 0.02$ and $\gamma_l = 0.006$. Belgian trainees receive a benefit that is marginally higher than the unemployment benefit. It is assumed that $b_{T,h} = 864$ and $b_{T,l} = 697$.

On the basis of administrative data and labour force surveys, the rate of salaried employment, $e_h + e_{T,h}$ (respectively, $e_l + e_{T,l}$), was equal to 0.75 (resp., 0.61) in 1997.[24] The participation rates p_h (resp. p_l) were equal to 0.72 (resp. 0.54). Administrative data on inflows into unemployment allow to fix ϕ_h to 0.006 and ϕ_l to 0.009.

On the basis of statistics collected by the Federal Planing Bureau and on the distribution of the tax burden, the tax wedge is huge: $\tau_{1,h} = 1.23$ and $\tau_{1,l} = 0.67$.[25]

The calibration uses information on the stock of vacancies. In 1997, the average stock of vacancies registered by the PES amounted to 24569. The market share of the PES is not well known. According to OECD (1997), it would be about 0.4. So, 60000 (respectively, 0.13) is an order of magnitude for the total stock of vacancies (resp. the aggregate V-U

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[21] All nominal quantities are in 1997 Belgian Francs, converted in EUROs.

[22] Adding social security contributions incident on workers and the income tax, this average wage for the low-skilled is above the average minimum gross wage (1074 EURO/month) and above the average minimum wage bargained over at the sectoral level (1188 EURO/month). These wage floors are taken into account during the simulations.

[23] These ratios are lower than reported values by the OECD on the basis of a range of earnings and family situations (see e.g. Table A.1 of OECD, 1999). However, for Belgium, these OECD statistics exclude some groups whose replacement ratio is quite low.

[24] The model is adapted to take the exogenous level of self-employment into account.

[25] The parameters $\tau_{0,n}$ are kept equal to zero throughout.

ratio). Various surveys[26] and published statistics are also used to fix the share of vacancies of each type. It is assumed that half of the vacancies are open in the low-skill segment.

As many other papers, let us assume a Cobb-Douglas matching function with two arguments: $m(S_n, V_n) \equiv m_0 S_n^{1-\mu} V_n^{\mu}$ and $m(S_{T,n}, V_{T,n}) \equiv m_0 S_{T,n}^{1-\mu} V_{T,n}^{\mu}$. Many authors have found that $\mu = 0.5$ is a good approximation (see e.g. Broersma and van Ours, 1999). So, μ is fixed to 0.5. Parameter m_0 is a scaling factor for the various c_ι. A range of values has been considered. It turns out that assuming that $m_0 = 0.5$ yields reasonable values. Since the model intends to represent the behavior of private firms, the calibration assumes that the endogenous numbers of vacancies, V_n and $V_{T,n}$, are multiplied by a coefficient that takes into account the existence of vacancies created by the public sector and by non-profit organizations. The latter are assumed to represent 18% of the total stock of vacancies.[27] The number of vacancies posted by the public sector and the non-profit organizations is kept fixed during the simulations.

The expected duration of a vacancy and the share of the low-skilled in the total number of recruitments is used to calibrate $\theta_n, n \in \{h, l\}$. Data on the former (2.5 month) and on the latter (0.38) is based on Denys et al (1998, 1999) and Delmotte et al (2001). This leads to $\theta_h = 0.83$ and $\theta_l = 2.22$.

The 'vacancy-supply curves' (26) are then used to calibrate k_n. At this stage, assumptions about y_n and κ_n are needed. In the absence of observed statistics on these parameters, the following procedure has been adopted. Starting from initial values, an iteration procedure has been implemented until the complete calibration yields reasonable values for the total number of vacancies and produces the observed share of vacancies open for the low-skilled unemployed. The following values are finally adopted : $y_h = 5800$ (again in EURO/month), $y_l = 3200$, $\kappa_h = 18$ and $\kappa_l = 7$. Hence, the assumed fixed cost paid once and for all when a new employee is recruited amount to 18 months of output for skilled workers and 7 months for low-skilled workers. Recall that this fixed cost include firm-specific training cost but also other match-specific setup costs. With these assumptions, the cost of opening vacancies can be calibrated. k_h amounts to 9.26 and k_l to 5.31. This hierarchy is plausible since recruitment costs are presumably much larger for skilled workers. However, the magnitude of these parameters looks large. Many authors have also found quite large values of k.[28] An interpretation is that in a model where investment is not modeled k_n (jointly with κ_n) implicitly captures these expenses and firing taxes.[29]

An assumption about $\kappa_{T,n}$ is needed in order to calibrate $\theta_{T,n}, n \in \{h, l\}$. No data are available about $\kappa_{T,n}$. For various values of the ratio $\kappa_{T,n}/\kappa_n$, the flow equilibrium conditions (1), (2), (4) and (5) are used to fix the products $c_\iota s_\iota$, $\iota = \{T, n\}, \{X, n\}, \{U, n\}, n \in$

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[26]Simoens, Denys and Denolf (1998), Denolf, Denys and Simoens (1999) and Delmotte, Van Hootegem and Dejonckheere (2001).

[27]This assumption is based on the survey made by Delmotte, Van Hootegem and Dejonckheere (2001).

[28]See Fredriksson and Holmlund (2001) and the references cited in their footnote 20. See also Lehmann and Van der Linden (2002).

[29]When they hire a worker, employers should take these firing taxes into account. Formally, it is easily checked that these taxes can be integrated in the parameters κ_n.

$\{l, h\}$. Notice that the ratio $\frac{c_n s_{U,n} \alpha(\theta_n)}{c_n s_{X,n} \alpha(\theta_n)}$ fixes $\frac{s_{U,n}}{s_{X,n}}$. Conditional on the values of the product $c_\iota s_\iota$'s, the calibration then fixes the c_ι's, the s_ι's, ξ_n, ψ_n and the bargaining power of the workers β_n. This part of the calibration is based on equations (38), (42), (43), (44) and on additional equations stipulating a value for the elasticity of unemployment duration with respect to the level of unemployment benefits.[30] As $\kappa_{T,n}/\kappa_n$ decreases, the ratio $V_{T,n}/V_n$ increases and the ratio $c_{T,n}/c_n$ shrinks. It turns out that $c_{T,h}/c_h$ becomes rapidly lower than 1 as $\kappa_{T,n}/\kappa_n$ decreases. Such an outcome being not plausible, one adopts the lowest ratio $\kappa_{T,n}/\kappa_n$ compatible with $c_{T,h}/c_h \geq 1$. So, $\kappa_{T,n}/\kappa_n = 0.85, \forall n$. Hence, from (35), $\theta_{T,h} = 1.09$ and $\theta_{T,l} = 2.49$. So, the labor market is more tight for trainees than for other job-seekers. The calibrated values are then $s_{U,h} = 0.21, s_{X,h} = 0.26, s_{T,h} = 0.28, c_h = 0.578, c_{T,h} = 0.580, \xi_h = 1.22, \psi_h = 15.03, s_{U,l} = 0.11, s_{X,l} = 0.14, s_{T,l} = 0.17, c_l = 0.24, c_{T,l} = 0.27, \xi_l = 1.20, \psi_l = 15.03, \beta_h = 0.39$ and $\beta_l = 0.58$. It turns out that skilled workers search more intensively. As expected, they have higher matching effectiveness parameters. Compared to the initial state (U_n), search effort is higher when occupying state X_n but the difference is not very large. Moreover, ξ_h is close to ξ_l, so that preferences are similar for both types of workers. The workers' bargaining power are such that $\beta_l > \mu > \beta_h$.[31] One could wonder why $\beta_l > \beta_h$. In Belgium, unionization is a widespread phenomenon, especially among blue-collar workers. This can explain why the bargaining power of low-skilled workers is higher. Combining the above information, the expected duration of a training spell is indeed quite short (3.7 months for the skilled and 5.8 months for the low-skilled workers). The $(V_{T,n} + V_n)/(U_n + X_n + T_n)$ ratios amount to 0.09 for the low-skilled and 0.15 for the skilled workers. The calibrated values also imply that the wage elasticity of salaried employment amounts to reasonable values, namely -0.72 for low-skilled workers and -0.25 for skilled ones.

Finally, as far as participation is concerned, the elasticity of p_n with respect to w_n allows to fix $V_{2,n} - V_{1,n}$. There is little consensus about this elasticity, especially for the large groups considered in this paper (see Saez, 2002). This author suggests that an elasticity equal or above 0.5 is plausible at the low end of the income distribution. On the basis of historical observations for Belgium, assuming such a value for this country leads to unrealistic adjustments in participation. A smaller value of 0.25 was assumed for both groups. Then, $V_{2,h} - V_{1,h} = 1215.5$ and $V_{2,l} - V_{1,l} = 1261.1$. This information and the

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[30]Since it is expressed for fixed values of tightness, this elasticity can be chosen by looking at micro-econometric evaluations. No such evaluation exists for Belgium. Therefore, one has to rely on evaluations made abroad, mostly in countries were unemployment duration is much lower than in Belgium. So, only the expected duration in state U_n is considered here. Meyer (2002) is the most recent published survey about the effect of unemployment benefits on unemployment duration. An elasticity of 0.5 would be a benchmark. However, no solution can be found to the system of equations when such a value is imposed. A *tâtonnement process* leads to the conclusion that the highest elasticity allowing a solution to the system of equations is 0.28 for the skilled workers. For the low-skilled, the constraint $\psi_l = \psi_h$ has to be imposed in order to find a solution. Then, the elasticity of unemployment duration in state U_l with respect to $b_{U,l}$ equals 0.16.

[31]Since workers are risk averse, the Hosios conditions $\beta_n = \mu$ does not necessarily guarantee that a *laissez-faire* economy is optimal. On this issue, see Lehmann and Van der Linden (2002).

participation rates in 1997 allow to fix the lower bound : $V_{1,h} = 879.6$ and $V_{1,l} = 954.7$.

3.2 Simulation results

3.2.1 Unemployment insurance

Three reforms will be envisaged in turn. They will respectively concern parameter π_0, the ratios $\frac{b_{U,n}}{b_{X,n}}$ and parameter π_1.

Let first π_0 increase when the replacement ratios and the tax rates are kept at their calibrated values and π_1 remains equal to 0. To ease comparisons with the existing literature, let us ignore training schemes ($\gamma_n = 0, \forall n \in \{h, l\}$). Theory predicts that the wage-setting curve shifts downwards, equilibrium tightness θ_n and search effort $s_{U,n}$ increase and the net wage rate w_n decreases. Due to space limitation, Figure 2 illustrates these results for $n = l$. These properties and the following ones hold however also for $n = h$. The net effect on search effort of unemployed people in stage X_n is theoretically unknown for the 'entitlement effect' and the improvement in tightness have opposite effects. Figure 2 illustrates that search effort $s_{X,l}$ is actually declining a little. Up to values of π_0 close to the current one (0.083), search effort $s_{U,l}$ and the employment rate of the low-skilled (resp., the aggregate unemployment rate $u + x$) are sharply increasing (resp., decreasing) with π_0. The improvement of these indicators becomes however negligible above $\pi_0 = 0.1$ (i.e. a first period of relatively high benefits that is expected to last 10 months). The participation rate of the low-skilled is somewhat declining with π_0. Interestingly, lower wages and more search effort in states U_n have a larger impact on intertemporal utilities than the higher hiring rates, so that the $rV_{.n}$ indicators are decreasing with π_0 *for all* groups. Therefore, the utilitarian criterion $r\Psi$ is decreasing, too. This first set of results highlights a phenomenon that often occurs in the simulations, namely that *performance indicators of the labor market and welfare criteria move in opposite directions. This questions the widespread focus on labor market indicators to guide the design of institutional reforms.*

For a range of values of the discount rate and in the case of a log utility function, simulations made by Fredriksson and Holmlund (2001) lead to the conclusion that employed workers and the (average) unemployed person would prefer a declining profile of benefits. These authors assume that the budget constraint of the State has to be balanced through an adjustment of the tax wedge. Under appropriate assumptions, Section 2.11 above has shown that this approach is sound in the sense that it does not lead to multiple equilibria. As π_0 increases, employment is higher and public spending for the unemployed is lower. So taxes can be cut, enhancing welfare. One can however wonder whether the conclusion of Fredriksson and Holmlund (2001) is verified when workers are heterogeneous. Two values of the discount rate have been considered, namely $r = 0.004$ and $r = 0.006$.[32] For these values, the conclusion is that $\pi_0 = 0$ is not desirable since the welfare of each type

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[32]Since the calibration is conditional on a given value of r, the model has been calibrated again for $r = 0.006$. Another exercise would consist in considering other values for the relative risk aversion of the workers.

of agent and labor market indicators are improving when π_0 is raised from zero. More interestingly, *the optimal value of π_0 strongly varies with the normative criterion used.* This property is clearer for $r = 0.006$. The utilitarian criterion is sharply increasing with π_0 for low values of this parameter and, as Figure 3 illustrates, it is a rather flat function around the optimum which is slightly above 0.2. On the contrary, from the viewpoint of a low-skilled entering unemployment, Figure 4 indicates that π_0 should be close to 0.03. Therefore, if such an approach is feasible in reality, this normative analysis points to the need of rates π_0 that are skill-specific.

Keeping π_0 at its calibrated value (0.083), let us now look at the optimal degree of differentiation $b_{U,n}/b_{X,n}$. When simulating the effects of "front-loading" the benefit system, the level of unemployment benefits (not the replacement ratios) varies in the following way. Let ϵ be a positive parameter such that $b_{U,n} = \sqrt{\epsilon} b_{U,n}^c$ and $b_{X,n} = b_{X,n}^c/\sqrt{\epsilon}$, where superscript c denotes the calibrated values. Hence, $b_{U,n}/b_{X,n} = \epsilon(b_{U,n}^c/b_{X,n}^c), \forall n \in \{h, l\}$. Increasing ϵ means that both ratios $b_{U,n}/b_{X,n}$ increase in the same proportion. Since $b_{T,n} \geq b_{U,n}$, the following simulation also assumes that $b_{T,n} = \sqrt{\epsilon} b_{T,n}^c$. Henceforth, the discount rate is set at 0.004. Figure 5 displays how the level of benefits vary with ϵ for the low-skilled. Since $b_{U,n}$ ($b_{T,n}$) and $b_{X,n}$ move in opposite directions the net effects on wages and on tightness are theoretically ambiguous. Let us here ignore the budget constraint of the State. By generalizing Expression (41), it can be checked that the sign of $\frac{dw_n}{d\epsilon}$ is given by the following expression:

$$(r + c_{T,n}s_{T,n}\alpha(\theta_{T,n}) + \lambda_n)(r + \gamma_n + c_n s_{X,n}\alpha(\theta_n) - \pi_n) + \gamma_n(r + \pi_n + c_n s_{X,n}\alpha(\theta_n) + \gamma_n)$$

The sign of this expression critically depends on the value of the parameters. For the calibrated values, it turns out that it is positive for skilled workers and negative for the other group. So, the wage of the low-skilled decreases and the one of the skilled increases. In this example, the wage-push effect of "front-loading" (Cahuc and Lehmann, 2000) only emerges for skilled workers.[33] This effect is however not strong. Tightness varies in the opposite direction but again to a limited extent only. On the contrary, search effort $s_{X,l}$ is increasing a lot with ϵ (see Figure 6). Since θ_h is only decreasing a little with ϵ the same qualitative conclusion holds for $s_{X,h}$. The combination of all these effects is a substantial increase in the low-skill employment rate, $e_l + e_{T,l}$ (see Figure 7). Conversely, the net effect on skilled employment is negligible. The utilitarian criterion is slightly increasing with ϵ. However, the net effect of lower wages, more search effort and better chances of being hired is a decline in the intertemporal utility of the low-skilled in all positions (see Figure 8 in the case of jobless individuals). The opposite holds for the skilled except when they occupy state X_h. To sum up, "front loading" the benefit system can boost employment but, in this example at least, it also lowers welfare for the low-skilled.

Actual unemployment insurance systems do not establish a link between search effort and the length of time during which the highest level of benefit is paid. However, a growing

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[33]Unreported simulation results show that the same qualitative conclusions hold when $\gamma_h = \gamma_l = 0$. In accordance with the comment of Formula (41), wages are decreasing with ϵ for sufficiently high values of π_n.

literature tries to evaluate the pros and cons of conditioning the level of benefits on search effort. To ease comparisons, let again $\gamma_h = \gamma_l = 0$. Figure 9 summarizes a simulation exercise where π_1 increases while π_0 and the replacement ratios remain unchanged. Here, the tax rates $\tau_{1,n}$ are adjusted to keep the budget of the State balanced. From Table 1, one expects $s_{U,h}$ and $s_{U,l}$ to increase conditional on tightness. Proposition 5 states that wages will be pushed up and tightness down. This is actually what occurs but the effects are not large. Hence, the net effect on $s_{U,l}$ is positive. Therefore, for the largest value of π_1 (namely, 0.15), $s_{U,l}$ has nearly reached $s_{X,l}$ so that π_l now equals 0.062 (an expected duration of receipt of 'high' benefits of 16 months instead of 12). For the same value of π_1, this expected duration is now 22 months for the skilled unemployed. The effects on the aggregate unemployment rate $(u + x)$ and on its composition are rather small. The net effect on the intertemporal utility of the various groups is slightly positive but again quite low. Due to a lack of data, recall that the additional monitoring costs are here neglected.[34] So, this reform is apparently no big deal for the Belgian labor market.

3.2.2 Training programs

In the following simulations, the rates of entry γ_n are the same for both skill groups and vary from 0 to 0.1. Let us first keep all the other parameters at their calibrated values. Focusing on the low-skilled, Figure 10 summarizes a simulation where taxation and the level of unemployment benefits are fixed. Since $b_{T,n}$ is only slightly higher than $b_{U,n}$, the wage-push effect of training programs mainly comes through better employment prospects for trained individuals. Figure 10 highlights a moderate positive effect on w_l and a more substantial negative impact on tightness θ_l. Since $\theta_{T,l}$ is not a function of γ_l (see function $\mathcal{T}$), $\theta_{T,l}$ follows the decline in θ_l. Search effort levels in states U_l and X_l are decreasing with γ_l (directly and/or via tightness indicators; see Table 1). The total effect on search effort is strong. The net effect on the employment rates is negative, too. So, increasing the share of jobless people with more favorable exit rates thanks to training is here insufficient to compensate the previous negative effects. The aggregate unemployment rate $u + x$ is declining but 'open unemployment' $(u + x + t)$ is strongly increasing. Nevertheless, increasing γ improves the intertemporal utility levels of all groups in the active population (Figure 10 only displays the average).

Corollary 1 suggests that the direct effect of γ_n on $e_n + e_{T,n}$ depends on the level of π_n. To illustrate that point, let us keep the assumptions of the previous simulation except that π_n is now set equal to zero. As expected, the level of the endogenous variables is very different in Figures 11 and 10. More interestingly, the relationship between $e_l + e_{T,l}$ and γ_l is now (slightly) positive. Since a benefit system where $\pi_0 = 0$ is more generous than the one where $\pi_0 = 0.0833$, this example illustrates the lack of generality of the assertion of Coe and Snower (1997) according to which "the more generous are passive unemployment

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[34]Moreover, as soon as subjective evaluations of search effort enter the scene, a third party can be invoked (courts) entailing non negligible costs. In addition, one should take the imperfections of the monitoring system into account. So-called 'errors of type I and II' would lower welfare.

policies, the less effective will be active unemployment policies" (p. 22). However, this sentence holds true as far as the level of benefits is concerned. Unreported simulation results for $\pi_n = 0$ illustrate that the positive impact of γ_l on $e_l + e_{T,l}$ is more substantial for lower values of $(b_{U,n}, b_{X,n}, b_{T,n})$. The effect of γ_l on $e_l + e_{T,l}$ becomes even negative when the level of benefits increases above a certain threshold.

Training schemes entail a cost in addition to the transfer to the beneficiaries. This cost is now taken into account in a crude way (see (45)). To avoid multiple equilibria, the replacement ratios are now constant. π_n is also back to its calibrated value (0.0833). Figure 12 shows the main features in that situation. The wage-push effect of training schemes is now more than compensated by the depressing effect of higher tax rates $\tau_{1,n}$. By the assumption of constant replacement ratios, the level of benefits is declining, too. These effects would lead to the expectation that the profile of tightness will be more favorable compared to the case where taxes are fixed (Figure 10). This expectation is not verified for higher taxes are detrimental to the creation of vacancies. Eventually, the net effect on the employment rates is negative. In the inter-temporal utility functions, the decrease in search effort and the more probable entry in training schemes are more than compensated by the depressed employment perspectives for the unemployed and the lower levels of income. Hence, increasing γ_n harms each component of the workforce. Consequently, the type of training schemes considered in this paper should be abandoned.

Are these pessimistic conclusions robust? As expected from above, the same exercise with $\pi_n = 0$ will lead to a different picture for some indicators. Unreported simulation results show that $e_l + e_{T,l}$ is now increasing with γ_n. However, the welfare analysis leads exactly to the same qualitative conclusions. Another sensitivity analysis would consist in lowering the ratio $\frac{\kappa_{T,n}}{\kappa_n}$. Figure 13 features the consequences of an important improvement in the effectiveness of training schemes as this ratio amounts now to 0.5 (instead of 0.85). As far as labor market indicators are concerned, Figure 13 is qualitatively similar to but quantitatively different from Figure 12 (look e.g. at w_l) . Furthermore, even if it is still declining with γ, the level of tightness $\theta_{T,n}$ is now much higher for both skill groups. This really boosts search effort levels $s_{T,n}$. When γ is increasing, all tightness indicators are declining at a similar pace but search effort levels are more rapidly declining for the low skilled. This difference explains why intertemporal utility levels of the low and the high skilled people now vary in opposite directions. In sum, if training programs substantially reduced firms' training costs, labor market indicators would still be better without them. In addition, the favorable effects on the welfare of the low skilled would come from differences in the pace of decline of search effort levels when the rate of entry into training rises.

4   Conclusion

This paper has developed an equilibrium matching model with six labor market states (inactivity, insured unemployment with two distinct levels of benefits, participation to

vocational training, employment and employment after a training spell). This model is fairly general and could therefore be used to evaluate labor market policies in many countries. In this model, workers are risk averse and heterogeneous in skill, job-search is endogenous in each state, a state-specific parameter affects matching effectiveness and wages are endogenous via a bargaining framework. Training programs do not only improve the fall-back position of the workers. They also reduce the cost that firms incur when they recruit workers. These hiring costs are distinct from the cost of posting vacancies and screening applicants. This paper has shown that the steady-state equilibrium is unique. Under certain conditions, this property also holds when tax rates are endogenous.

Analytical properties are numerous. Under certain conditions that are more stringent in the presence of a two-tired benefit system, training programs have a positive direct effect on the employment rate. However, their indirect effects are detrimental to employment. The rate of entry into training programs not only increases equilibrium net wages and decreases tightness. It also reduces the equilibrium levels of job-search. A two-tired unemployment benefit structure has properties already studied by Cahuc and Lehmann (2000) and Fredriksson and Holmlund (2001) in general equilibrium matching frameworks. This paper has focused on the interaction between such a structure and training programs. A short expected period in the high benefit state and a low rate of entry into training schemes should be recommended if one intends to raise the employment rate through a more differentiated profile of unemployment benefits. Another contribution of this paper has been to merge the literature about the optimal sequencing of benefits and the one about sanctions. It has been assumed that the expected length of the period during which high benefits are received can be influenced by search effort. It has then been shown that the advantages of such a reform heavily depend on its effect on search effort because the other impacts turn out to be negative.

Many net effects cannot be signed and a normative analytical analysis is hard to conduct. Therefore, this paper has also developed a simulation exercise. The calibration has been based on an extensive and well-informed use of statistics and studies for Belgium. In this country, duration dependence is very strong but essentially spurious. The simulation exercises deal with the financing of the reforms. Simulation results strongly emphasize the importance of the choice of the evaluation criterion. Indeed, performance indicators of the labor market and welfare criteria often move in opposite directions after a reform, in particular because the latter takes care of the disutility of job-search effort. This questions the widespread focus on labor market indicators to guide the design of institutional reforms. The simulation exercise confirms the conclusion of Fredriksson and Holmlund (2001) according to which a declining time profile of benefit payments dominates a scheme with a constant replacement ratio. However, if this is feasible, the two-tired benefit structure should be skill-specific since the optimal expected length of payment of high benefits can vary a lot in the population. For low-skilled workers with gloomy employment perspectives, this optimal length amounts to 2.5 years in the simulations. A reform that would relate this expected length to search effort does not appear to produce substantial effects on any of the evaluation criteria. For the low-skilled, "front loading" the benefit system is

favorable to employment but is detrimental to their welfare. Microeconometric evaluation has shown that Belgian vocational training schemes improve the exit rate of the treated. In a setting where training programs reduce hiring costs, a moderate wage-push effect is confirmed when tax rates and the levels of unemployment benefits are fixed. Moreover, training programs affect negatively and strongly the level of search efforts of the unemployed. Eventually, training schemes reduce the employment-active population ratio in the presence of a two-tired benefit system. This property is however reversed when benefits are constant all along an unemployment spell. This indicates that the impact of active programs can be deeply affected by the design of passive policies and that this can happen in a way which is not the expected one (Coe and Snower, 1997). When the financing of training schemes is taken into account and replacement ratios are constant, the wage-push effect is more than compensated by the impact of higher tax rates. Simulations then show that nearly all the evaluation criteria worsen when more unemployed people enter training schemes. Despite their microeconomic favorable effects on the hiring rate, vocational training for the unemployment appears to have harmful net effects in Belgium. It should be emphasized that this paper has not dealt with programs covering possibly several years and intending to lift the productivity of the unemployed. Since microeconometric evaluation suggests that standard (and relatively shorter) programs do not have such effects, this paper has only dealt with training schemes that reduce firm-specific training costs and possibly improve the matching effectiveness of the participants.

Appendix 1. Proof of Proposition 1

Let us prove that $V_{U,n} > V_{X,n}$. If $V_{U,n}$ was lower or equal to $V_{X,n}$ and $s_{U,n}$ was optimally chosen by each unemployed, the following inequalities would hold:

$$
\begin{aligned}
rV_{U,n} =\,& \ln(b_{U,n}) - \psi_n \frac{(s_{U,n})^{\xi_n}}{\xi_n} + c_n s_{U,n} \alpha(\theta_n)(V_{E,n} - V_{U,n}) + \gamma_n(V_{T,n} - V_{U,n}) \\
& + (\pi_0 - \pi_1 s_{U,n})(V_{X,n} - V_{U,n}) \\
\geq\,& \ln(b_{U,n}) - \psi_n \frac{(s_{X,n})^{\xi_n}}{\xi_n} + c_n s_{X,n} \alpha(\theta_n)(V_{E,n} - V_{U,n}) + \gamma_n(V_{T,n} - V_{U,n}) \\
& + (\pi_0 - \pi_1 s_{X,n})(V_{X,n} - V_{U,n}) \\
>\,& \ln(b_{X,n}) - \psi_n \frac{(s_{X,n})^{\xi_n}}{\xi_n} + c_n s_{X,n} \alpha(\theta_n)(V_{E,n} - V_{X,n}) + \gamma_n(V_{T,n} - V_{X,n}) \\
=\,& rV_{X,n},
\end{aligned}
$$

which leads to a contradiction. Therefore, $V_{U,n} > V_{X,n}$.
Similarly, if $V_{U,n}$ was higher than $V_{T,n}$ and $s_{T,n}$ was optimally chosen by the trainee, the

following inequalities would be verified:

$$
\begin{aligned}
rV_{T,n} &= \ln(b_{T,n}) - \psi_n \frac{(s_{T,n})^{\xi_n}}{\xi_n} + c_{T,n}\, s_{T,n}\, \alpha(\theta_{T,n})(V_{E,n|T} - V_{T,n}) + \lambda_n(V_{U,n} - V_{T,n}) \\
&\geq \ln(b_{T,n}) - \psi_n \frac{(s_{U,n})^{\xi_n}}{\xi_n} + c_n\, s_{U,n}\, \alpha(\theta_{T,n})(V_{E,n|T} - V_{E,n} + V_{E,n} - V_{U,n} + V_{U,n} - V_{T,n}) \\
&+ (\lambda_n - \gamma_n + \gamma_n)(V_{U,n} - V_{T,n}) \\
&> \ln(b_{U,n}) - \psi_n \frac{(s_{U,n})^{\xi_n}}{\xi_n} + c_n\, s_{U,n}\, \alpha(\theta_{T,n})(V_{E,n|T} - V_{E,n} + V_{E,n} - V_{U,n}) \qquad (48)\\
&+ \gamma_n(V_{T,n} - V_{U,n}).
\end{aligned}
$$

From (8) and (9) and the assumption that $w_{T,n} \geq w_n$, one has $V_{E,n|T} - V_{E,n} = \frac{\ln(w_{T,n}) - \ln(w_n)}{r + \phi_n} \geq 0$. Therefore, assuming that $\theta_{T,n} \geq \theta_n$ and knowing that $V_{U,n} > V_{X,n}$, expression (48) leads to:

$$
\begin{aligned}
rV_{T,n} &> \ln(b_{U,n}) - \psi_n \frac{(s_{U,n})^{\xi_n}}{\xi_n} + c_n\, s_{U,n}\, \alpha(\theta_n)(V_{E,n} - V_{U,n}) + \gamma_n(V_{T,n} - V_{U,n}) \\
&+ (\pi_0 - \pi_1\, s_{U,n})(V_{X,n} - V_{U,n}) = rV_{U,n},
\end{aligned}
$$

which leads to a contradiction. So, $V_{T,n} > V_{U,n}$.

Appendix 2. Proof of Proposition 2

From (6) and (7), the marginal impact of $\phi_n, s_{T,n}, s_{U,n}, s_{X,n}, c_{T,n}, c_n, \theta_n$ and $\theta_{T,n}$ should intuitively be clear. Moreover,

$$
\frac{\partial e_n + e_{T,n}}{\partial \gamma_n} = \frac{\phi_n(c_{T,n}s_{T,n}\alpha(\theta_{T,n}) + \lambda_n)}{\Delta_{0,n}^2} \Bigg[\pi_n(c_{T,n}s_{T,n}\alpha(\theta_{T,n}) + \lambda_n + \gamma_n)\, c_n(s_{U,n} - s_{X,n})\alpha(\theta_n)
$$

$$
+ (\pi_n + c_n s_{X,n}\alpha(\theta_n) + \gamma_n)\Big((c_n s_{X,n}\alpha(\theta_n) + \gamma_n)(c_{T,n}s_{T,n}\alpha(\theta_{T,n}) - c_n s_{U,n}\alpha(\theta_n))
$$

$$
+ \pi_n(c_{T,n}s_{T,n}\alpha(\theta_{T,n}) - c_n s_{X,n}\alpha(\theta_n)) \Big) \Bigg]
$$

≥ 0 if $c_{T,n}s_{T,n}\alpha(\theta_{T,n})$ is sufficiently larger than $c_n s_{U,n}\alpha(\theta_n)$ and $c_n s_{X,n}\alpha(\theta_n)$
< 0 if $c_{T,n}s_{T,n}\alpha(\theta_{T,n}) < c_n s_{U,n}\alpha(\theta_n)$ and $c_{T,n}s_{T,n}\alpha(\theta_{T,n}) < c_n s_{X,n}\alpha(\theta_n)$

$$
\frac{\partial e_n + e_{T,n}}{\partial \lambda_n} = \frac{\phi_n \gamma_n(\pi_n + c_n s_{X,n}\alpha(\theta_n) + \gamma_n)}{\Delta_{0,n}^2} \Bigg[(c_n s_{X,n}\alpha(\theta_n) + \gamma_n)(c_n s_{U,n}\alpha(\theta_n) - c_{T,n}s_{T,n}\alpha(\theta_{T,n}))
$$

$$
+ \pi_n(c_n s_{X,n}\alpha(\theta_n) - c_{T,n}s_{T,n}\alpha(\theta_{T,n})) \Bigg]
$$

≤ 0 if $c_{T,n}s_{T,n}\alpha(\theta_{T,n}) \geq c_n s_{U,n}\alpha(\theta_n)$ and $c_{T,n}s_{T,n}\alpha(\theta_{T,n}) \geq c_n s_{X,n}\alpha(\theta_n)$
> 0 if $c_{T,n}s_{T,n}\alpha(\theta_{T,n}) < c_n s_{U,n}\alpha(\theta_n)$ and $c_{T,n}s_{T,n}\alpha(\theta_{T,n}) < c_n s_{X,n}\alpha(\theta_n)$

$$\frac{\partial e_n + e_{T,n}}{\partial \pi_0} = \frac{\phi_n(c_{T,n}s_{T,n}\alpha(\theta_n) + \lambda_n)}{\Delta_{0,n}^2}[c_{T,n}s_{T,n}\alpha(\theta_n) + \lambda_n + \gamma_n][c_n\,s_{X,n}\alpha(\theta_n) + \gamma_n]$$

$c_n\left[s_{X,n} - s_{U,n}\right]\alpha(\theta_n)$ whose sign is the one of $s_{X,n} - s_{U,n}$,

$$sign\left[\frac{\partial e_n + e_{T,n}}{\partial \pi_1}\right] = -sign\left[\frac{\partial e_n + e_{T,n}}{\partial \pi_0}\right]$$

Appendix 3. Proof of Proposition 3

$\frac{\partial WS}{\partial \theta_n} > 0$ because the derivative of the terms between large brackets are positive (since $\alpha'(\theta_n) > 0$ and $\frac{\partial \mathcal{V}}{\partial \theta_n} > 0$) and the derivatives of the other terms can be rewritten as $\frac{(r+c_{T,n}s_{T,n}\alpha(\theta_{T,n})+\lambda_n)\pi_n c_n s_{X,n}\alpha'(\theta_n)}{\Delta_{2,n}\Delta_{3,n}}(V_{U,n} - V_{X,n})$. Totally differentiating (38) yields $\frac{\partial WS}{\partial s_\iota} = 0$ for $\iota = \{T,n\}, \{X,n\}, \{U,n\}, n \in \{l,h\}$. This results has been established by Fredriksson and Holmlund (2001) and Lehmann and Van der Linden (2002) in similar contexts. Furthermore:

$$\frac{\partial WS}{\partial \theta_{T,n}} = \frac{\gamma_n c_{T,n}s_{T,n}\alpha'(\theta_{T,n})}{\Delta_{3,n}}(V_{E,n} - V_{T,n}) > 0 \text{ by proposition 1 in the case where} V_{E,n|T} = V_{E,n}.$$

The marginal impact of the b_ι's is obvious from the definition of the v_ι (for $\iota = \{T,n\}, \{X,n\}, \{U,n\}, n \in \{l,h\}$). In addition, remembering proposition 1, one has:

$$\frac{\partial WS}{\partial \gamma_n} = \frac{r + c_{T,n}s_{T,n}\alpha(\theta_{T,n}) + \lambda_n}{\Delta_{3,n}}(V_{T,n} - V_{X,n}) > 0,$$

$$\frac{\partial WS}{\partial \lambda_n} = -\frac{\gamma_n}{\Delta_{3,n}}(V_{T,n} - V_{U,n}) < 0, \tag{49}$$

$$\frac{\partial WS}{\partial \pi_0} = -\frac{[r + c_{T,n}s_{T,n}\alpha(\theta_{T,n}) + \lambda_n][r + \gamma_n + c_n s_{X,n}\alpha(\theta_n)]}{\Delta_{2,n}\Delta_{3,n}}(V_{U,n} - V_{X,n}) < 0, \tag{50}$$

$$\frac{\partial WS}{\partial \pi_1} = -s_{U,n}\frac{\partial WS}{\partial \pi_0} > 0. \tag{51}$$

The partial derivatives with respect to $k_n, \kappa_n, \beta_n, \phi_n, \tau_{0,n}$ and $\kappa_{T,n}$ are immediately derived from the properties of $\mathcal{V}$.

Appendix 4. The case of an isoelastic utility function of consumption

If $\ln(C)$ was replaced by $\frac{C^{\zeta_n}}{\zeta_n}, \zeta_n \leq 1, \zeta_n \neq 0$, Equation (33) would become

$$\frac{w_n^{\zeta_n}}{\zeta_n} = rV_{U,n} + \frac{\beta_n}{1 - \beta_n}w_n^{\zeta_n}\left(\frac{y_n - \tau_{0,n}}{w_n(1 + \tau_{1,n})} - 1\right). \tag{52}$$

Substituting an explicit expression for $rV_{U,n}$ yield an implicit (net) 'wage-setting curve':

$$\frac{w_n^{\zeta_n}}{\zeta_n} = \tilde{W}S(w_n, \theta_n, \theta_{T,n}, s_{T,n}, s_{U,n}, s_{X,n} \mid Z_n, \tau_{0,n}, B_n) \equiv$$

$$\frac{[r + c_{T,n}s_{T,n}\alpha(\theta_{T,n}) + \lambda_n][r + \gamma_n + c_n s_{X,n}\alpha(\theta_n)]}{\Delta_{2,n}\Delta_{3,n}}\left[v_{U,n} + c_n s_{U,n}\alpha(\theta_n)w_n^{\zeta_n}\mathcal{V}(\cdot)\right]$$

$$+ \frac{\gamma_n}{\Delta_{3,n}}\left[v_{T,n} + c_{T,n}s_{T,n}\alpha(\theta_{T,n})w_n^{\zeta_n}\mathcal{V}(\cdot)\right]$$ (53)

$$+ \frac{\pi_n[r + c_{T,n}s_{T,n}\alpha(\theta_{T,n}) + \lambda_n]}{\Delta_{2,n}\Delta_{3,n}}\left[v_{X,n} + c_n s_{X,n}\alpha(\theta_n)w_n^{\zeta_n}\mathcal{V}(\cdot)\right] + (r + \phi_n)w_n^{\zeta_n}\left[\mathcal{V}(\cdot)\right].$$

After substitution of function $\mathcal{T}$, Equation (40) becomes then

$$\tilde{F}(w_n, \theta_n, s_{T,n}, s_{U,n}, s_{X,n} \mid Z_n, \tau_{0,n}, \tau_{1,n}, B_n) \equiv \ln\left(y_n\left(1 - (r + \phi_n)\left[\frac{k_n}{q(\theta_n)} + \kappa_n\right]\right) - \tau_{0,n}\right)$$

$$- \ln(1 + \tau_{1,n}) - \ln(w_n) - \tilde{\mathcal{W}S}(\theta_n, s_{T,n}, s_{U,n}, s_{X,n} \mid Z_n, \tau_{0,n}, B_n) + \frac{w_n^{\zeta_n}}{\zeta_n}.$$ (54)

One modification should be introduced in Equations (42),(43) and (44), namely function $\mathcal{V}(\cdot)$ is replaced by $w_n^{\zeta_n}\mathcal{V}(\cdot)$.

Appendix 5. The budget constraint of the State and the uniqueness of equilibrium

Assume that $\tau_{0,n} = 0$ and also that the net replacement ratios are constant (e.g. $\rho_{U,n} = b_{U,n}/w_n$). Let R_n designate the vector of net replacement ratios with $\mathcal{R} = (R_h, R_l)$. Let also $\mathcal{L} = (L_h, L_l)$. Equation (45) implies that at least one of the marginal tax rates has to become endogenous. In Section 3, both tax rates are assumed to vary proportionately. Rearranging (45), both marginal tax rates can then be written as functions of the rate of individuals in the various states, say

$$\tau_{1,n} = g_n(e_n + e_{T,n}, u_n, x_n, t_n \mid \mathcal{R}, \mathcal{L}, Q, C), \ n \in \{h, l\}.$$ (55)

Obviously, $\tau_{1,n}$ increases with the rates u_n, x_n, t_n and decreases with the employment rate. Adapting (40), equilibrium tightness is now potentially a function of these rates:

$$F(\theta_n, s_{T,n}, s_{U,n}, s_{X,n} \mid Z_n, 0, g_n(e_n + e_{T,n}, u_n, x_n, t_n \mid \mathcal{R}, \mathcal{L}, Q, C), R_n \cdot w_n) = 0,$$ (56)

with $w_n = VS(\theta_n \mid \kappa_n, k_n, y_n, r, \phi_n, 0, g_n(e_n + e_{T,n}, u_n, x_n, t_n \mid \mathcal{R}, \mathcal{L}, Q, C))$. Totally differentiating (56) and making use of (38) and (40) lead to $\frac{dF}{d\theta_n} = -\frac{\partial \mathcal{WS}}{\partial \theta_n}$ and $\frac{dF}{d(e_n + e_{T,n})} = \frac{dF}{du_n} = \frac{dF}{dx_n} = \frac{dF}{dt_n} = 0$. Hence, equilibrium tightness is not a function of $e_n + e_{T,n}, u_n, x_n, t_n$. Furthermore, the functions Σ are independent of $\tau_{1,n}$. Therefore, the levels of equilibrium search effort are not influenced by (55). These properties and the proof of Proposition 6 allow to conclude that the equilibrium is unique if $\tau_{0,n} = \pi_1 = 0$, the replacement ratios are constant and $\tau_{1,n}$ is adjusted to clear the budget of the State.

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parameter	$s_{U,n} \mid \theta_n$ °	$s_{U,n}{}^*$ if $\pi_1 = 0$	$s_{X,n} \mid \theta_n$‡	$s_{X,n}{}^*$	$s_{T,n} \mid \theta_n, \theta_{T,n}$†	$s_{T,n}{}^*$
$b_{U,n}$	0 if $\pi_1 = 0$ + if $\pi_1 > 0$	-	+	?	+	?
$b_{X,n}$	0 if $\pi_1 = 0$ - if $\pi_1 > 0$	-	-	-	+	?
$b_{T,n}$	0	-	0	-	-	-
π_0	0 if $\pi_1 = 0$ - if $\pi_1 > 0$	+	-	?	-	?
π_1	+	?	+	?	+	?
γ_n	0 if $\pi_1 = 0$ - if $\pi_1 > 0$	-	-	-	+	?
λ_n	0	+	0	+	+	+
ϕ_n	+	?	+	?	+	?
κ_n	+	?	+	?	+	?
$\kappa_{T,n}$	0	+	0	+	0	+
k_n	+	?	+	?	+	?
y_n	0 0	+ if $\tau_{0,n} \geq 0$? if $\tau_{0,n} < 0$	0 0	+ if $\tau_{0,n} \geq 0$? if $\tau_{0,n} < 0$	0 0	+ if $\tau_{0,n} \geq 0$? if $\tau_{0,n} < 0$
$\tau_{0,n}, \tau_{1,n}$	0	-	0	-	0	-
β_n	+	?	+	?	+	?

° i.e. $s_{U,n}$ solving $\Sigma_U(\theta_n, s_{U,n}, s_{X,n}, s_{T,n} \mid Z_n) = 0$.

‡ i.e. $s_{X,n}$ solving $\Sigma_X(\theta_n, s_{U,n}, s_{X,n}, s_{T,n} \mid Z_n) = 0$.

† i.e. $s_{T,n}$ solving $\Sigma_T(\theta_n, \theta_{T,n}, s_{U,n}, s_{X,n}, s_{T,n} \mid Z_n) = 0$.

* means that the adjustment of θ_n (and $\theta_{T,n}$) in equilibrium is taken into account.

Table 1. Search effort levels : Comparative statics, $n \in \{l, h\}$.
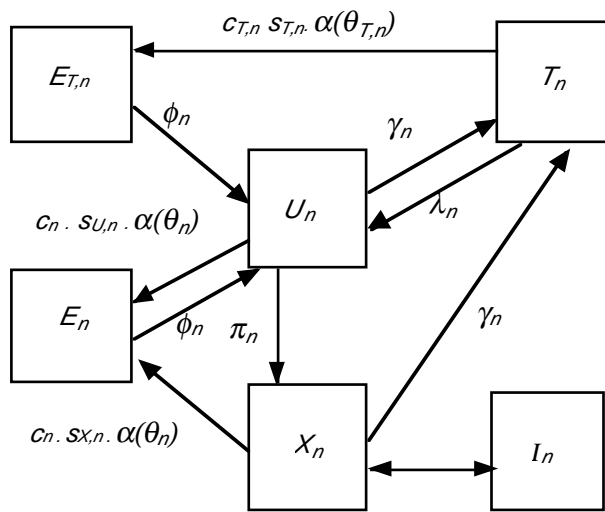
Figure 1: Labor market flows.
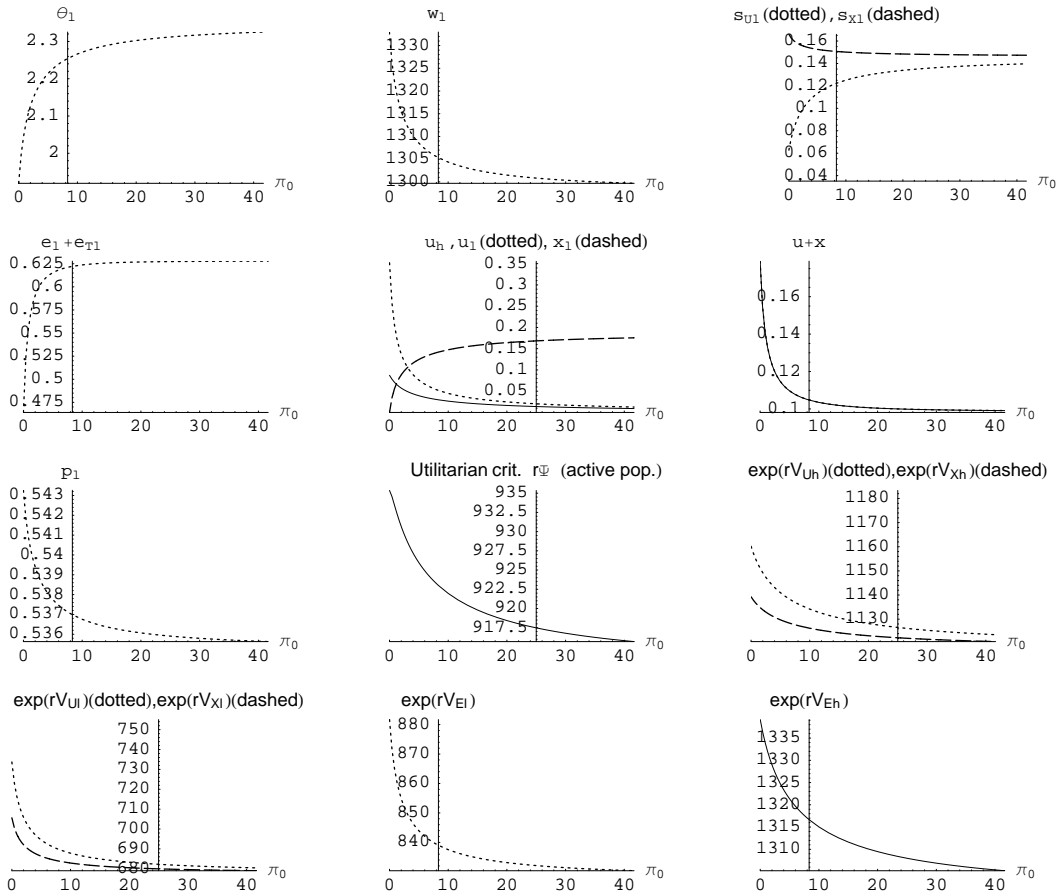
Figure 2: Simulations of an increase in π_0 when the budget constraint of the State is ignored; $\gamma_n = 0$. Scale on the horizontal axis: $100 * \pi_0$.

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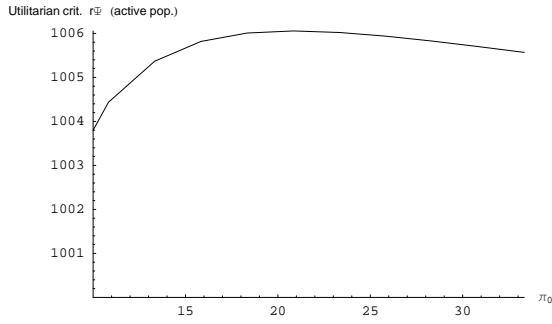
Figure 3: The impact of π_0 from a utilitarian point of view when $r = 0.006$ and tax rates are endogenous. Scale:$100 * \pi_0$.
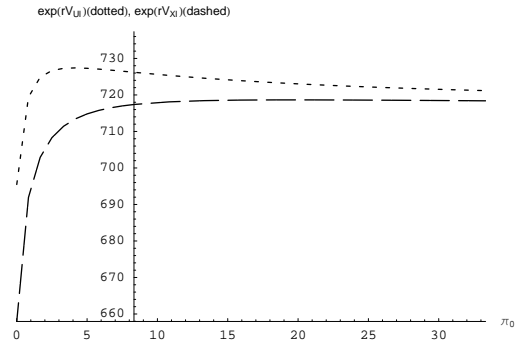

Figure 4: The impact of π_0 on the low-skilled unemployed when $r = 0.006$ and tax rates are endogenous. Scale:$100 * \pi_0$.
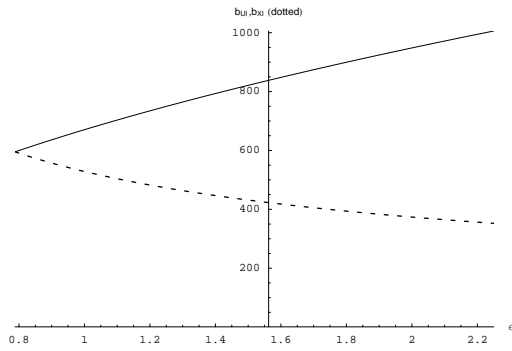

Figure 5: The assumed evolution of unemployed benefits $(n = l)$.
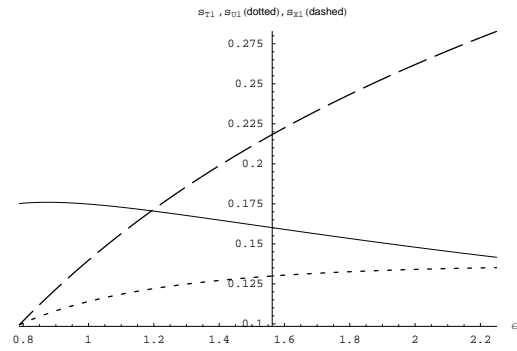
Figure 6: The evolution of search effort among the low-skilled.
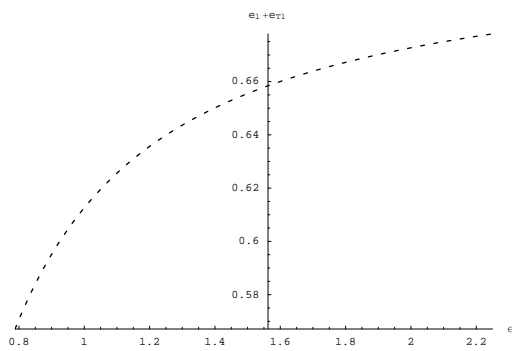
Figure 7: The evolution of the low-skilled employment rate.
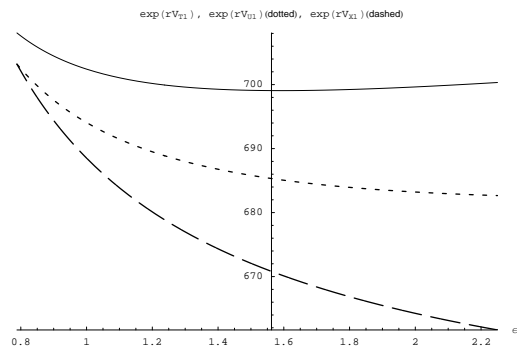
Figure 8: The evolution the low-skilled intertemporal utility levels measured in certainty equivalents (jobless individuals).
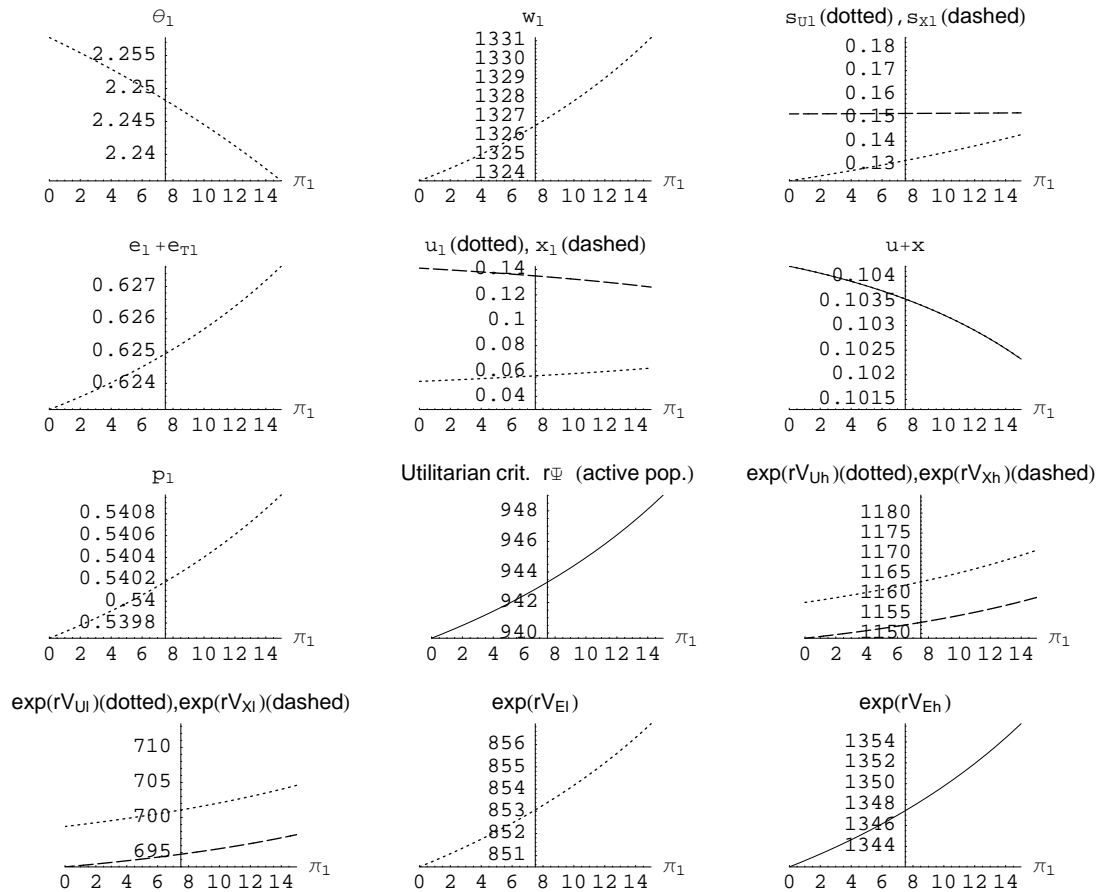
Figure 9: Simulations of an increase in π_1 when the tax rates $\tau_{1,n}$ are adjusted to keep the budget of the State balanced; $\gamma_n = 0$. Scale on the horizontal axis : $100 * \pi_1$.
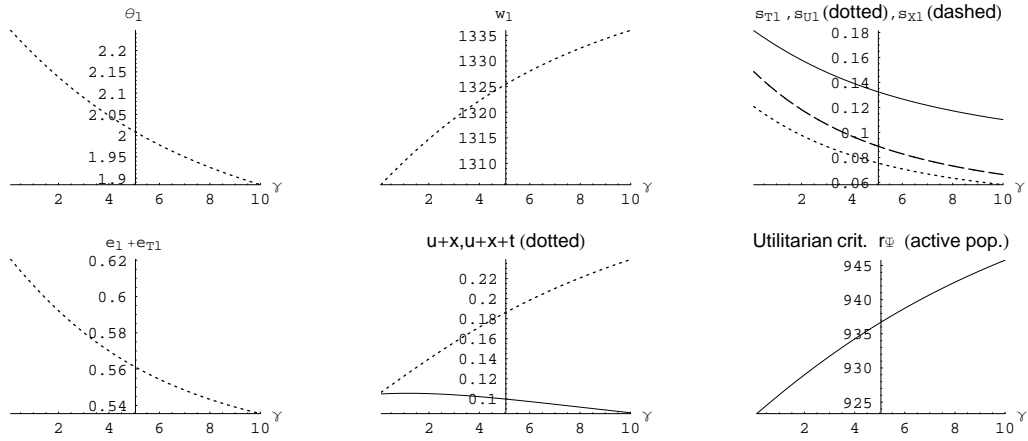
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Figure 10: Simulations of changes in γ keeping benefits and tax rates unchanged; $\pi_n = 0.0833$. Scale on the horizontal axis : $100 * \gamma$
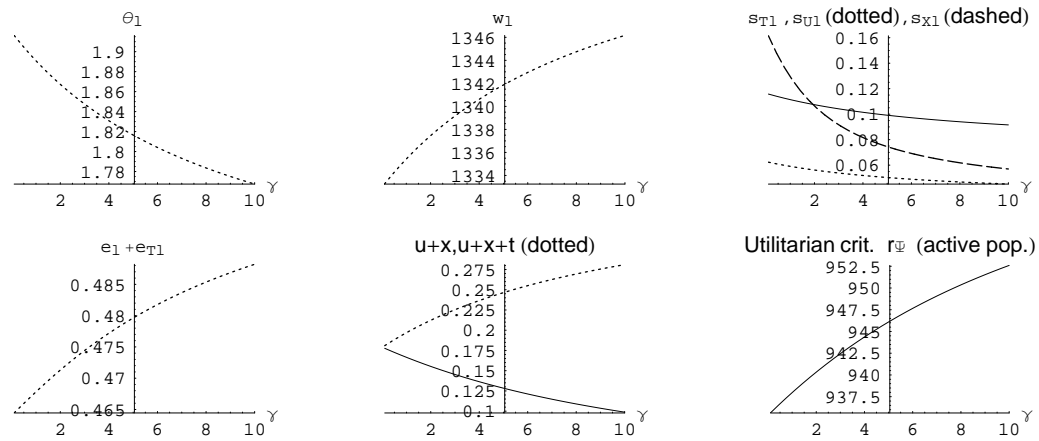

Figure 11: Simulations of changes in γ keeping benefits and tax rates unchanged; $\pi_n = 0$. Scale on the horizontal axis : $100 * \gamma$
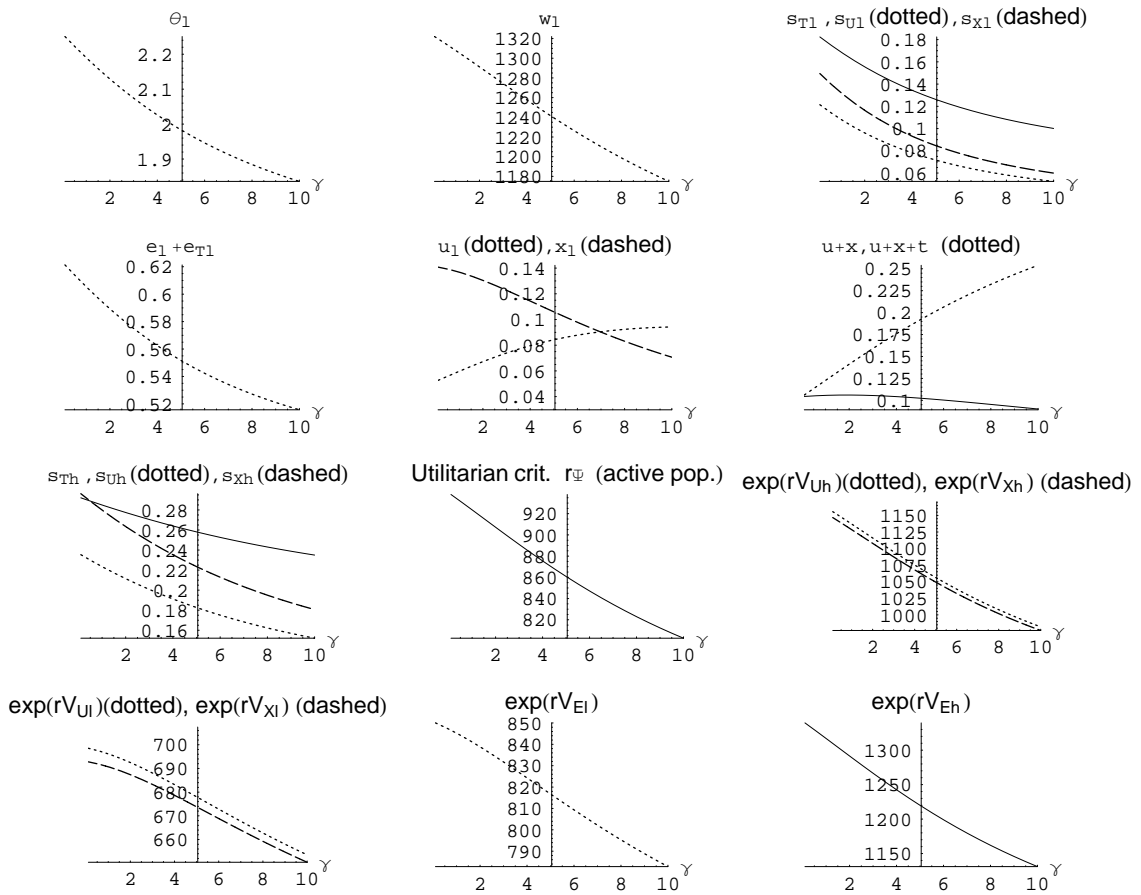
Figure 12: Simulations of changes in γ when the tax rates $\tau_{1,n}$ are adjusted to keep the budget of the State balanced; $\pi_n = 0.0833$. Scale on the horizontal axis : $100 * \gamma$
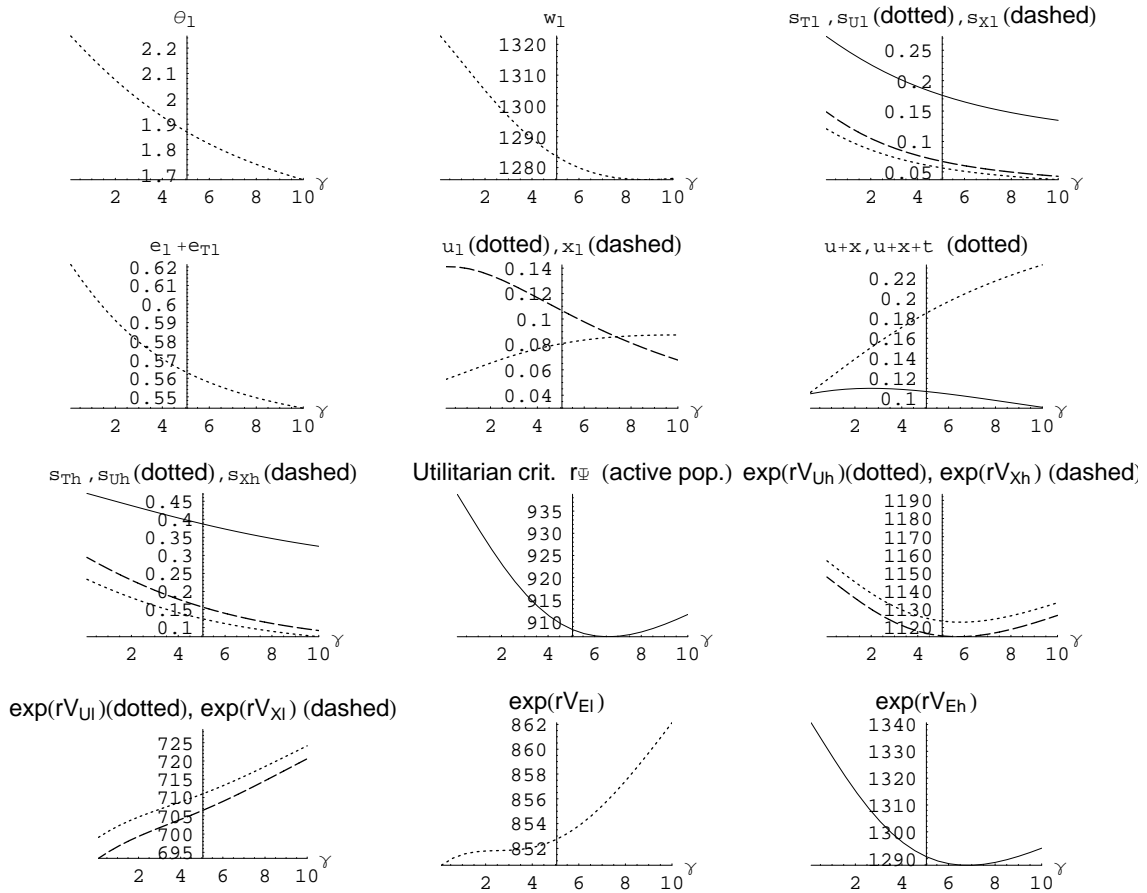
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Figure 13: Sensitivity analysis: Simulations of changes in γ if $\frac{\kappa_{T,n}}{\kappa_n}$ drops from 0.85 to 0.5. The tax rates $\tau_{1,n}$ are adjusted to keep the budget of the State balanced and $\pi_n = 0.0833$. Scale on the horizontal axis : $100 * \gamma$