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Rhythm *versus* Nature of Technological Change

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Rhythm *versus* Nature of Technological Change*

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Abstract

An acceleration of the rate of technical change is often deemed responsible for the observed increase of job instability and wage inequality in some industrialised countries during the last 30 years. Numerous articles emphasize the role played by the rate of technical change. But, there has been relatively little attention paid to the question of how the nature of technological progress affects the instability of jobs and unemployment. In this paper, we argue that the introduction of technological improvements within firms is likely to modify the learning process on the job. Then, we examine the consequences of such changes in return to seniority on unemployment rate, job instability and wage dispersion. We conclude that when the rate of technical change increases, the unemployment rate and job instability should increase. But the alteration of the learning process can cancel this effect in a routine world, whereas this alteration amplifies it as jobs go out the routine. In terms of wages dispersion, compared to the existing literature, we show that the effects of the rate of technological change are non monotone, depending on the learning process. Finally, we study the question of diffusion of new technology. We suggest that the decrease of the rate of learning process is a factor which magnifies the effects of a collapse of price.

Keywords: Technological acceleration, learning process, job instability, wage inequality.
***JEL* Codes**: J23, J63, O33.

Résumé

Les effets du progrès technique sur le chômage et l'instabilité sont abordés dans la littérature essentiellement à travers les variations du taux de diffusion des innovations. Considérant que les innovations des trente dernières années ont eu pour effet de modifier la nature des emplois, nous montrons que dans un modèle standard d'appariement avec progrès technique, la prise en compte non pas du seul taux d'apparition des innovations mais également du rythme d'apprentissage permet d'élargir le champ des effets du progrès technique sur l'instabilité et sur le chômage dans le sens d'une reproduction plus fidèle de certaines observations. Nous montrons de plus que lorsque le rythme d'arrivée de nouvelles machines s'intensifie, les inégalités salariales tendent soit à se résorber, soit à s'aggraver selon le rythme d'apprentissage.

Mots clés: Progrès technique, apprentissage, durée des emplois, inégalités salariales.
Classification *JEL*: J23, J63, O33.

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1 Introduction

Economists have long been interested in studying the relation between technological progress and unemployment, at least since the beginning of the industrial revolution. In recent years, numerous theoretical contributions have been investigating this issue along several directions. A particular direction has consisted in examining the influence of the rate of technological change on the level of unemployment. Matching models of unemployment (Diamond [1982], Pissarides [1990]) have proved to be extremely helpful to tackle this question.

Let us briefly recall the main results of these models. The standard matching model with technological progress (Pissarides [1990], Postel-Vinay [1998]) predicts that a faster rate of technological progress is likely to reduce unemployment through a "capitalisation effect". In Schumpeterian models of growth (Aghion and Howitt [1992], Aghion and Howitt [1994], Mortensen and Pissarides [1998], Postel-Vinay [2002]), however, an other effect works in an opposite direction, namely a "creative destruction effect". The conventional view is that innovation stimulates job creation but induces simultaneously the destruction of other (older) jobs. Unemployment can then result from a lack of innovation and hence of job creation, or from a high level of innovation going along with a high level of job destruction. Job instability and unemployment emerge from this interaction between job creation and job destruction decisions. In a model with irreversible investment choice, Mortensen and Pissarides [1998] show for instance that higher productivity growth induces lower unemployment when capital equipment prices are low, but that the response of employment to growth switches from positive to negative as the cost of updating existing technology rises above a critical level. Common to all these models is the study of what happens when the rhythm of technological progress increases.

However, as far as we are aware, there has been relatively little attention paid to the question of how the nature of technological progress affects the instability of jobs and unemployment. This is the question that we would like to treat in this paper. More precisely, we argue that the role of technological acceleration in the increase of unemployment rate and instability should not hide the role played by the change of the nature of work induced by the technological change.

A variety of models in the literature have underlined the existence of this change of nature of technical progress. For instance, Galor and Moav [2000] show that technological progress may change the nature of occupations, jobs, and tasks. More closely related to our work, Givord and Maurin [2002] show in an empirical perspective that the increase of jobs instability results from the impact of technical progress on work organization, and more precisely from the increase of the substitutability between raw recruits and high seniority workers induced by the technological acceleration.

The aim of this paper is then to identify the theoretical consequences on jobs security and unemployment of changes in the rhythm and in the nature of technical progress. The nature of activities can be affected in several dimensions by the introduction of technological improvements within firms. In this paper, we focus our attention on one of them, namely the learning process. So the change of nature of activities induced by technological improvements is understood as a modification of the learning process on the job[1]. The contemporary technical change is likely to modify the labor-learning (see for instance Bahk and Gort [1993] for a distinction between labor learning, capital learning and organization learning). As Autor, Levy, and Murnane [2001] note, computerization is associated with declining relative industry demand for routine manual and cognitive tasks. Shifts are evident within industries, within detailed occupations, and within education groups within industries. After giving rise to taylorism and automatization, the technical progress is now associated with the end of routine for some activities. Jobs are on average more and more complex and consequently more difficult to perfectly fulfill. A worker no more performs a unique task throughout his career. He can instead face new problems for which ready-made solutions do not exist and he has to try to find original solutions. The hierarchical organization of firms becomes flatter and the learning process on the job is longer, which modifies the return to seniority in terms of specific skills gained in the course of one's career. In the same line, Maurin and Thesmar [2002] argue that "new technologies make it possible to allocate more human and material resources to non-routine activities". The strategy adopted by these authors is to test the assumption of an increasing decline of the most programmable activities in favor of those that are the least programmable. Nevertheless, at a more disaggregated level, we can not preclude that in some economic sectors, technological progress and innovations improve accessibility of technologies. Technologies become more user friendly (see for example Galor and Tsiddon [1997]), which means again a change in the learning process. We analyze the interplay between the change of the rhythm and the change of the nature of technological progress on jobs instability, unemployment and wage inequalities.

As seniority is the heart of the matter, the formalization of the lifespan of a job is of crucial importance. We follow Mortensen and Pissarides [1998] formalization and consider the length of a job as an endogenous variable. New jobs embody the most advanced known technology. The productivity of a new job grows at a constant rate, the rate of technical progress. Job creation commits the firm to the technology available at this date, unless the firm decides to update later by bearing fixed renovation costs. Once a job is created, the employer has three choices at any future date: continue to produce with the technology embodied in the job when it was created, pay a fixed renovation cost to update its technology and continue to produce with the same worker, or close down the job. In the last case, the worker becomes unemployed and the firm either leaves the market or reenters with a new job vacancy. Taking into account a learning process

[1] Some recent theoretical papers focus on learning gained on a job (see for instance Violante [2002], or Aghion, Howitt, and Violante [2000]). Nevertheless, both of these papers analyze the link between technological change and skill transferability, whereas we study the relation between technological change and specific skills acquisition. The importance of limited transferability of expertise is also studied by Jovanovic and Nyarko [1996] in a Bayesian model of learning-by-doing where the choice of which technology to implement is affected by the fraction of knowledge one can transfer from the current technology to the new one. Moreover, for Aghion, Howitt, and Violante [2000], it is the owner of machines, or firms that are learning rather than workers. By contrast, we will focus here on the learning process of the workers.

in this framework modifies of course the job productivity at each date, and consequently the optimal length of a job, the date of renovation and the correlation between growth and unemployment.

The main result of this paper is that the impact of a change in the learning process on job insecurity can work in several directions depending on the initial conditions. For instance, it can cancel the negative impact of an increase of the rate of the technical change in a routine world, whereas it can amplify it as jobs go out the routine. This result is interesting as it can explain why job insecurity and wage inequality can remain relatively stable during a period in which the technical change evolves. Katz and Murphy [1992] reveal for instance that the wage differential between skilled and unskilled labor narrowed during the fourth, fifth, and sixth decades of the twentieth century in the United States, and has been widening in the seventies and eighties.

The intuition behind this result can be briefly explained. A decrease in the rhythm of skill acquisition induces a loss in productivity that may lead to a decrease of the length of a job. Note that our specification is a pure reallocation model, in which job destruction is motivated solely by the desire to break-up persistently unproductive employment relationship, thereby allowing the firm and worker to seek out more productive new relationships on the matching market. Nevertheless, the loss in productivity leads to fewer job creation, by the free entry condition. This tension on the job market depresses the outside opportunities for workers, in others words depresses wages, and may increase the expected length of a job. For an initial situation where jobs are very routine, this last effect can dominate the direct effect of a decrease in the rhythm of acquisition on productivity and length of jobs.

Moreover, an increase in the rhythm of technical change may induce a decrease in wage dispersion, depending on the rhythm of learning. This result can be explained by the fact that there are two sources of heterogeneity in this model : the different vintage of machines used by workers and the seniority of these workers. New machines are always better than old ones but as long as a firm continues to use its current technology, its employee accumulates expertise in that technology. We have then two effects: first, an increase in the rhythm of technical change means an increase of the obsolescence of installed technologies. The productivity gap between two vintage technologies increases. Therefore, as usual in this literature, the heterogeneity of wages increases among jobs. Secondly, any increase in the rhythm of technical change means a shorter learning process for all jobs, just because it is then more profitable to shorten the lifetime of jobs in order to take advantage of new technologies. Therefore, the impact of the second source of wage heterogeneity is reduced. We then observe a non-monotone relation. When the learning process is high or low, an increase in the rhythm of technical change increases the wage inequalities as it seems usual in the literature (see Violante [2002]). But, as the speed of learning is within two critical values, an increase in the rhythm of technical change decreases the wage inequalities.

The paper proceeds as follows. The setup of the model is described in the second section. Section 3 analyzes the case of the adoption of new technology through job destruction only. We focus there on the impact of a change of rhythm or nature of the technical progress on unemployment and the length of job relations. Section 4 presents a study of the evolution of wage inequality in this framework. Section 5 presents a more general case, where technologies can be retooled. A last section offers some concluding remarks.

2 The framework

The framework is borrowed from Mortensen and Pissarides [1998]. We consider an economy populated by a continuum of firms and workers. The total measure of workers is equal to one. We focus our attention on the case of a particular industry and we assume that the total measure of firms is larger than one. All workers have the same level of education (general human capital), denoted by x, but at each point in time, workers can differ in terms of productivity level, as will be clear below.

2.1 Production

Production takes place in one firm-one worker pairs. At the time of a match between a worker and a firm, the firm equips its employee with a machine embodying the technological frontier. The productivity of the technological frontier evolves at an exogenous constant rate, g. A crucial assumption is that the technological choice is irreversible. The output produced by a worker at date t on a job filled at time τ is denoted:

$$y(\tau,t) = p(\tau)\,\chi(\tau,t)$$

where $p(\tau) = e^{g\tau}$, and $\chi(\tau,t)$ represents the productivity of the worker in a job of tenure $t-\tau$. As long as a firm continues to use its current technology, its employee accumulates expertise in that technology, as a result of learning, so that the productivity of this worker rises when time elapses. We borrow the specification of learning from Parente [1994]. The increase of productivity due to learning is assumed to occur at a decreasing rate [2], γ. The law of motion governing the increase in productivity is

$$\dot{\chi}(\tau,t) = \gamma s - \gamma\chi(\tau,t)\,, \quad \gamma \geq 0 \quad \text{and } \chi(\tau,\tau) = x$$

Hence,

$$\chi(\tau,t) = s - (s-x)\,e^{-\gamma(t-\tau)} \tag{1}$$

Any worker starts at a first level x of productivity, and is able to reach at best a level s of productivity. In this section, a firm keeps the same technology until the job is destroyed. The parameter γ reflects the rhythm of learning within job. The higher γ is, the shorter the period of learning. A high value of γ is associated with the automation of tasks, and a routine job.

[2] See also Laing, Palivos, and Wang [1995]. In their framework, γ depends positively upon the level of schooling, which is homogenous for all workers in this paper.

Note that the knowledge accumulated on a job can not be transferred to another job, or machine. This knowledge is a specific skill gained by the memorization, routine and automation of tasks. We depart then from the debate on the impact of technical progress on the transferability of skill. We focus here on a change of the degree of automation of tasks.

2.2 The labor market

We use a matching process in the spirit of Diamond [1982] and Pissarides [1990]. At each moment in time a mass u of unemployed workers and a mass v of vacant jobs coexist on the labor market. Then jobs and workers meet pairwise at a Poisson rate $M(u,v)$, where $M(u,v)$ is the matching function. It represents the number of matches per unit time. The function $M : \Re_+^2 \rightarrow \Re_+$ is traditionally assumed to be strictly increasing and concave, exhibiting constant returns to scale. Furthermore it has the following properties : $M(0,v) = M(u,0) = 0$ and satisfies the Inada conditions. With these assumptions, we are able to define the Poisson probability for a firm posting a vacancy to interview a worker as follows: $M(u,v)/v = M(\theta,1)/\theta \equiv m(\theta)/\theta$, decreasing with respect to θ, where $\theta \equiv v/u$ represents the labor market tightness ratio. Symmetrically, the Poisson probability for any unemployed worker to meet a firm is given by: $M(u,v)/u = M(\theta,1) \equiv m(\theta)$, increasing with respect to θ.

We assume that there are two sources of job destruction, either in response to an exogenous shock that arrives with frequency δ, or in response to an endogenous motive : when the job ceases to be profitable, the firm chooses to close down.

2.3 Jobs value

In this section, we describe the asset price equations. We restrict attention to stationary equilibria. r is the interest rate. A job owned by a firm is either filled or vacant. The value of a job vacancy, denoted by $V(t)$, at date t is given by the following equation:

$$rV(t) = -c(t) + \frac{m(\theta)}{\theta}\left[J(t,t) - V(t)\right] + \frac{\partial V(t)}{\partial t} \tag{2}$$

where $J(\tau,t)$ is the value of an existing firm at date t with a job filled at time τ (i.e. a job of tenure $t-\tau$). If a firm decides to hire a worker, it has to pay a cost $c(t)$ per unit time until it meets a job searcher but it can obtain an expected capital gain, given by the two last terms of the above equality. To intuitively understand the above equation, we can think of an asset. For the asset to be held, it must provide an expected rate of return of r. In other words, its dividends per unit time plus the expected capital gain per unit time must be equal $rV(t)$. In any equilibrium, firms enter the market until all opportunities from new creation are exhausted. This free entry condition yields $V(t) = 0$ for all t.

As long as the partners do not mutually decide to quit their relation, the unique source of destruction is exogenous with frequency δ per unit time. A firm gets a profit equal to $y(\tau,t) - w(\tau,t)$ per unit time until the exogenous or endogenous destruction of the relation with its worker. When the relation is hit by a negative shock, the capital loss for the firm is $J(\tau,t)$. Then, $J(\tau,t)$ is governed by the following asset equation:

$$rJ(\tau,t) = Max\left\{y(\tau,t) - w(\tau,t) - \delta J(\tau,t) + \frac{\partial J(\tau,t)}{\partial t}; 0\right\} \tag{3}$$

where $w(\tau,t)$ is the current wage at time t in a job of tenure $t-\tau$. Depending on the time t, either it is profitable for the partners to continue the relation and the value of a filled job is given by the instantaneous profit plus the expected capital loss, or it is not profitable because the expected value of this relation falls below zero, and the relation is destroyed. Thus, we need to include the operator Max because of the endogenous termination of job.

Concerning workers, they can be employed or unemployed at any date t. The asset price equations are written with a similar reasoning. In this model, only unemployed workers search for a job. Let $W(\tau,t)$ be the value of employment in a job at date t which was filled a time τ. If a worker is employed in a job paying a wage of $w(\tau,t)$ at date t, $W(\tau,t)$ is governed by the following asset equation:

$$rW(\tau,t) = Max\left\{w(\tau,t) + \delta\left[U(t) - W(\tau,t)\right] + \frac{\partial W(\tau,t)}{\partial t}; rU(t)\right\} \tag{4}$$

where $U(t)$ is the value of search for an unemployed agent. In other words, the return on being employed is an instantaneous wage of $w(\tau,t)$ per unit time plus the probability δ per unit time of a "capital loss" of $U(t) - W(\tau,t)$. Any unemployed worker receives an income flow $b(t)$ per unit time until it meets a firm which hires him. So, $U(t)$ is given by the following equation:

$$rU(t) = b(t) + m(\theta)\left[W(t,t) - U(t)\right] + \frac{\partial U(t)}{\partial t} \tag{5}$$

In order to ensure the existence of a balanced growth path, as usually we assume that all the exogenous parameters follow the pace of productivity growth. More formally, we define $c(t) = p(t)c$ and $b(t) = p(t)b$ where c and b are two positive exogenous parameters. Equation (2) and the free-entry condition imply:

$$\frac{\theta c}{m(\theta)} = \frac{J(t,t)}{p(t)} \tag{6}$$

2.4 Wages

As long as the total surplus $J(\tau,t) - V(t) + W(\tau,t) - U(t)$ at date t associated with a job created at time τ remains positive, the firm and its employee always find a mutually profitable contract to share that surplus in fixed proportions. Thus, $w(\tau,t)$ satisfies:

$$(1-\beta)\left[W(\tau,t) - U(t)\right] = \beta\left[J(\tau,t) - V(t)\right] \tag{7}$$

where β represents the relative weight of the employee in the bargaining[3].

From (4) and (5), we deduce

$$\begin{aligned}(r+\delta)\left[W(\tau,t)-U(t)\right] &= w(\tau,t)-b(t)-m(\theta)\left[W(t,t)-U(t)\right] \\ &\quad +\left[\frac{\partial W(\tau,t)}{\partial t}-\frac{\partial U(t)}{\partial t}\right]\end{aligned}$$

and from (3) and (2), we deduce

$$\begin{aligned}(r+\delta)\left[J(\tau,t)-V(t)\right] &= y(\tau,t)-w(\tau,t)+c(t)-\frac{m(\theta)}{\theta}\left[J(t,t)-V(t)\right] \\ &\quad +\left[\frac{\partial J(\tau,t)}{\partial t}-\frac{\partial V(t)}{\partial t}\right]\end{aligned}$$

Introducing these two equations in (7), with $V(t)=0$, yields:

$$w(\tau,t)=\beta p(\tau)\chi(\tau,t)+(1-\beta)p(t)\omega(\theta) \tag{8}$$

where we define:

$$\omega(\theta)\equiv b+\frac{\beta\theta c}{(1-\beta)} \tag{9}$$

$\omega(\theta)$ represents the outside option of any worker. Because of the growth of the productivity in new jobs due to productivity growth of the technological frontier, the outside option of workers grows. This increase of the outside option is a source of growth for wages in existing job. Another source is the improvement of the productivity of existing jobs induced by learning. Nevertheless, the increase of productivity occurs at a decreasing rate while the increase of the outside option takes place at a constant rate. Intuitively, the increase of wage induced by these two forces will eventually lead to job destruction.

3 Endogenous job destruction

3.1 The optimal age of job destruction

The partners choose to continue their relation as long as the surplus associated with the job, $S(\tau,t)=J(\tau,t)-V(t)+W(\tau,t)-U(t)$, remains positive. In any equilibrium, we have $V(t)=0$ and according to the rule of determination of wages, we deduce $S(\tau,t)=(1-\beta)^{-1}J(\tau,t)$. Thus, it is profitable for a firm to voluntarily cease its relation with its current worker at the date $\tau+T^*$, at which the surplus is negative or equal to zero, i.e. $J(\tau,\tau+T^*)\leq 0$, where T^* is the optimal duration of job. And it is also optimal for the worker to quit since $J(\tau,t)=\beta^{-1}(1-\beta)\left[W(\tau,t)-U(t)\right]$.

[3] This wage scheme implies a continuous bargaining of wages, which might seem hard to justify. More fundamentally, such a scheme leads to excessive wage volatility in response to an aggregate shock. It could be then interesting to combine all the other elements of the model with a wage as a fixed share of the product at each date. Nevertheless, this modification would lead to the same intuitions and to the same results in terms of wages inequality. We think that it is easier to understand the specific role played by the learning by adopting the same wage scheme as Mortensen and Pissarides [1998].

As the function J is continuous in t, T^* satisfies $J(\tau, \tau + T^*) = 0$. By virtue of (3) the maximal value of a filled job is:

$$J(\tau, t) = \underset{T}{Max} \left\{ \int_t^{\tau+T} e^{-(r+\delta)(\xi-t)} \left[p(\tau) \chi(\tau, \xi) - w(\tau, \xi) \right] d\xi \right\}$$

Using (8) in the previous equation gives

$$J(\tau, t) = \underset{T}{Max} \left\{ (1-\beta) p(t) \int_t^{\tau+T} e^{-(r+\delta)(\xi-t)} \left[e^{g(\tau-t)} \chi(\tau, \xi) - e^{g(\xi-t)} \omega(\theta) \right] d\xi \right\}$$

After a variable switch, $\nu = \xi - t$, when $\tau = t$, the above equation can be rewritten as:

$$J(t, t) = \underset{T}{Max} \left\{ (1-\beta) p(t) \int_0^T e^{-(r+\delta)\nu} \left[\chi(\nu) - e^{g\nu} \omega(\theta) \right] d\nu \right\}$$

where the productivity level, $\chi(.)$ depends only on the tenure of job. For simplicity, in the remainder of the article we will adopt the following notation: $\chi(0, \sigma) = \chi(\sigma)$ is the productivity level on a job of tenure σ.

Thus, $J(t, t) = p(t) J$ where

$$J = \underset{T}{Max} \ (1-\beta) \int_0^T e^{-(r+\delta)\nu} \left[\chi(\nu) - e^{g\nu} \omega(\theta) \right] d\nu \tag{10}$$

is the value of a job on the technological frontier. This value is independent of the current date. The optimal age of job destruction, denoted by T^*, is the solution to the problem defined by (10). The first-order condition gives the following condition for T^*:

$$\chi(T^*) = e^{gT^*} \omega(\theta)$$

or [4]

$$s - (s - x) e^{-\gamma T^*} = e^{gT^*} \omega(\theta), \quad \gamma > 0 \tag{11}$$

The left side of (11) is strictly increasing and concave in T while the right side is strictly increasing and convex. In the appendix, we show that if b, c and/or β are sufficiently small, there is a unique solution, denoted by T^*. These conditions are assumed to be satisfied throughout the remainder of the analysis. Moreover, (11) yields T^* as a decreasing function of θ. We denote this relation by $T^* = \Upsilon(\theta)$ with $\Upsilon'(\theta) < 0$. When we introduce the optimal value of T in the value of a job, we obtain:

$$J(\tau, t) = (1-\beta) p(\tau) \int_t^{\tau+T^*} e^{-(r+\delta)(\xi-t)} \left[\chi(\tau, \xi) - e^{g(\xi-\tau)} \omega(\theta) \right] d\xi \tag{12}$$

and

$$J^* = (1-\beta) \int_0^{T^*} e^{-(r+\delta)\xi} \left[\chi(\xi) - e^{g(\xi-T^*)} \chi(T^*) \right] d\xi \tag{13}$$

[4] The second order condition implies : $\dot{\chi}(T^*) / \chi(T^*) \leq g$ equivalent to $\chi(T^*) \geq \gamma s / (g + \gamma)$.

The endogenous variables are θ, J, T and u, the unemployment rate. u^* is determined once we know the optimal values of T^* et θ^*. Using (6) with $J(t,t) = p(t) J$, and (10) with $T^* = \Upsilon(\theta)$, we can finally represent J^* and θ^* in the plane (θ, J) thanks to the two following equations

$$\frac{\theta c}{m(\theta)} = J \tag{14}$$

with

$$J = (1-\beta)\int_0^{\Upsilon(\theta)} e^{-(r+\delta)\xi}\left[\chi(\xi) - e^{g\xi}\omega(\theta)\right] d\xi \tag{15}$$

(14) gives a positively sloped relation in the plane (θ, J) while (15) gives a negatively sloped relation. Consequently, there is at most a unique solution with θ and J positive provided that b is not too large (see appendix).

We finally turn to the long-run unemployment rate. In any steady state, the flows of workers in and out of the unemployment pool must be equal. As only unemployed workers search for a job, the flow of workers out of unemployment is given by $m(\theta^*) u^*$. At each date t, among filled jobs, a fraction $(1-u^*)\delta$ is hit by a negative exogenous shock, leading to their destruction and among the jobs occupied for T^* units of time, those not yet destroyed, i.e. a fraction $exp(-\delta T^*)$, become non-profitable and therefore are endogenously destroyed. Therefore, we deduce the following relation:

$$u^* = \frac{\delta}{\delta + [1 - e^{-\delta T^*}]\, m(\theta^*)} \tag{16}$$

3.2 Rhythm of technological progress, duration of jobs and unemployment

In this section, we focus our attention on the effects of the growth rate on the optimal destruction age. We notice that (14) is not affected by g while (15) is negatively affected. Therefore, when g increases, the labor market tightness declines in equilibrium. We substitute θ^* in the first-order condition to obtain the value of T^*. We define $\theta^* \equiv \theta(g)$ with $\theta'(g) < 0$, introduce in (11), and then differentiate. Finally, we obtain:

$$\frac{dT^*}{dg} g = \frac{\varepsilon_{\omega(\theta^*)/g} + gT^*}{\dot{\chi}(T^*)/\chi(T^*) - g} \tag{17}$$

where $\varepsilon_{\omega(\theta^*)/g}$ is the elasticity of $\omega(\theta^*)$ with respect to g, which is always negative. The denominator is always negative while the numerator has *a priori* an ambiguous sign. In fact, we show in appendix that it is possible to solve this indetermination. We obtain the following results:

$$\frac{dJ^*}{dg} < 0 \ , \ \frac{d\theta^*}{dg} < 0 \ , \ \frac{dT^*}{dg} < 0$$

The intuitive economic interpretation is as follows. Any increase of g has two direct effects which work in the direction of decreasing the life-time of production units : an increase of the growth rate of the outside option as well as a faster rhythm of obsolescence of existing technologies. The productivity of workers, $\chi(\tau, t)$, is however not affected by g. An indirect effect works through a decrease of labor market tightness, even though this effect seems to be of second-order. Thus, the destruction of jobs occurs faster when the growth rate is higher.

Therefore, from (16) and previous results, we deduce that the unemployment rate unambiguously increases with g. Two forces work in the same direction : the decline of the optimal destruction age and the faster rate of growth of wages. These effects are the direct and indirect creative destruction effects described in Aghion and Howitt [1994].

3.3 Nature of technological progress, duration of jobs and unemployment

The results are somewhat more difficult to establish regarding the effect of a variation of γ on T^*. We know that (14) is not affected by γ while (15) is positively affected so that the labor market tightness increases. Hence, we can deduce:

$$\frac{dJ^*}{d\gamma} > 0 \ , \ \frac{d\theta^*}{d\gamma} > 0$$

According to the previous results, we know that the equilibrium values of J^* and θ^* raise with γ because of the increase of firm's incentives to open new vacancies induced by the improvement of the productivity of workers. We introduce the value of θ^* in (11) and we differentiate this equation. This yields:

$$\frac{dT^*}{d\gamma}\gamma = \frac{\varepsilon_{\omega(\theta^*)/\gamma} - T^* \dot{\chi}(T^*) / \chi(T^*)}{\dot{\chi}(T^*) / \chi(T^*) - g} \tag{18}$$

where $\varepsilon_{\omega(\theta^*)/\gamma}$ is the elasticity of $\omega(\theta^*)$ with respect to γ, which is always positive. The denominator is always negative while the sign of the numerator remains ambiguous. Hence, the effect of γ on T^* seems to be ambiguous.

In fact, this effect is likely to be non-monotone (For a more formal discussion, see the appendix). We can see it by examining the first-order condition, (11). A firm decides to continue its relation with its employee as long as:

$$s - (s - x) e^{-\gamma T} \geq e^{gT} \omega(\theta), \quad \gamma > 0$$

Then, we distinguish between two effects induced by an increase in γ:

- First, the effects of learning are faster. The consequence is that we observe an instantaneous increase of production level, but the decreasing effects of learning are stronger. This effect works in the direction of increasing the optimal length of jobs.

- second, an increase in the value of γ increases the incentives of firms to open new vacancies because of the previous effect, namely the increased productivity. As a result, the outside option for workers increases, pushing towards an increase in wages. This effect works in the direction of decreasing the optimal duration of jobs.

So, the effects of an increase in the value of γ are as follows. When γ is relatively high, the output level produced by a worker comes quickly near its ceiling-level. Hence, the second effect is likely to dominate in order to entail $dT^*/d\gamma < 0$. By contrast, when γ is sufficiently low, the first effect is likely to dominate.

Observe that some cross-effects between g and γ are at stake. For instance, when g is low, the effects working through the outside option might be less relevant.

Finally, we study the effects of a variation of the value of γ on unemployment. From (16) with the preceding results, two cases have to be distinguished. First, when the initial value of γ is high, any decrease of this parameter yields ambiguous effects on the unemployment rate : on the one hand, the incentives for firms to create vacancies decrease, but on the other hand, the lifetime of filled jobs raises. Hence, the unemployment rate may decrease. In this case, the effects of γ might compensate the effects of a rise of g in order to keep the unemployment rate unchanged. Second, when γ is initially sufficiently low, any decrease of the value of γ induces an increase of the unemployment rate. So, the effects of a decrease in γ reinforces the effects of an increase in g. We can then summarize the results within a tabular:

	After an increase of g	After a decrease of γ	
		Initial high level of γ	Initial low level of γ
Impact on the expect. wait of the unemployed	+	+	+
Impact on the unemployment rate	+	-	+
Impact on the expected job lifespan	-	+	-

4 Wages inequality

4.1 Productivity of workers and wages

The wage of a worker at date t, with a seniority equal to T ($T = t - \tau \geq 0$), is given by (8). In order to analyze the effects of tenure on wages, we rewrite wages as adjusted to productivity level. Thus, in stationary equilibrium:

$$\tilde{w}(T) \equiv \frac{w(\tau, t)}{p(t)} = \beta e^{-gT}\left[s - (s-x)e^{-\gamma T}\right] + (1-\beta)\,\omega(\theta^*)$$

According to this equation, we have to distinguish between two components in the wages : a component linked to the own productivity of a worker on the job and a component related to labor market tightness. The higher the labor market tightness, the higher the outside option for each worker, pushing towards a higher individual wage. The expression in brackets in the above equation is the productivity level reached by any worker after T units of time of expertise on a technology. The higher this level, the higher the wage. A faster growth rate makes the obsolescence of existing technologies faster. We recall that the initial investment of any firm is irreversible. Thus, the factor e^{-gT} in the above equation measures the gap between the technology currently operated by a worker on a job created for T units of time and the current technological frontier. This is the "*obsolescence effect*".

So, the adjusted wage raises with the level of the outside option as well as with the productivity level reached by a worker. But, two forces may entail a collapse of the adjusted wage : first, the learning technology has decreasing returns and second, the assumption of irreversibility of the initial investment implies an increasing technological gap as just mentioned.

More formally, the growth rate of the adjusted wage can be expressed as:

$$\frac{\dot{\tilde{w}}\left(T\right)}{\tilde{w}\left(T\right)}=\beta e^{-gT}\frac{\left[\dot{\chi}\left(T\right)-g\chi\left(T\right)\right]}{\tilde{w}\left(T\right)}$$

This growth rate depends positively on the difference between two terms : $\dot{\chi}\left(T\right)$ and $g\chi\left(T\right)$. The former measures the ability of a worker to improve his own productivity, which is decreasing in T, and the latter is simply the level of productivity reached by a worker after T units of time weighted by the growth rate of the technological frontier. In order to analyse the evolution of wages with respect to T, we distinguish between, on the one hand, two extreme values of γ, namely $\gamma\rightarrow 0$ and $\gamma\rightarrow+\infty$, and on the other hand, what we will call intermediate values of γ.

Let us first examine what happens when γ is low and $\gamma\rightarrow+\infty$. Intuitively, these cases may yield some similar results because the growth rate of the individual productivity is similar, and to be more precise, it is close to zero. More formally, γ low enough, and more precisely[5] $\gamma\leq gx/\left(s-x\right)$, or $\gamma\rightarrow+\infty$, is likely to lead to $\dot{\chi}\left(T\right)/\chi\left(T\right)<g\ \forall T>0$. Then, the growth rate of adjusted wage is negative throughout the career of the worker in a production unit, as in Mortensen and Pissarides [1998] framework. Similarly to their work, the highest wage in the economy is associated with the more recent technology.

By contrast, for some intermediate values of γ, what we will simply denote by $\gamma>gx/\left(s-x\right)$, the growth rate of adjusted wage is positive up to a certain time, denoted by $\tilde{T}$, and afterwards it becomes negative. This U-inverted shape is due to the fact that at the outset of the career, the "*learning effect*" dominates the "*obsolescence effect*", while it is the contrary when the seniority raises. The highest wage, in cross-section, is reached by the worker for who the growth rate of his own productivity is just equal to the growth rate of technical change ($\tilde{T}$ is such that $\dot{\chi}\left(\tilde{T}\right)/\chi\left(\tilde{T}\right)=g$).

From the learning technology, we deduce:

$$\chi\left(\tilde{T}\right)=\frac{\gamma s}{\left(\gamma+g\right)}$$

Hence,

$$\tilde{T}=\frac{\ln\left(\gamma+g\right)\left(s-x\right)-\ln gs}{\gamma}$$

which is decreasing in g. Any increase of g leads to a decrease of $\tilde{T}$, leading to a decrease of the period of increasing of the adjusted-wage.

[5] From the equation of individual productivity, we show that the growth rate of productivity, $\dot{\chi}\left(T\right)/\chi\left(T\right)$, is lower than g provided that the productivity level is higher than the threshold $\gamma s/\left(\gamma+g\right)$. As $\chi\left(T\right)$ is an increasing function, if this property is true for the initial date for which we have $\chi\left(0\right)=x$, then it is also true for all following dates.

For $\tilde{T}$ to be positive, γ must satisfy $\gamma > gx/(s-x) \equiv \hat{\gamma}_2$. We also have[6]

$$\frac{d\tilde{T}}{d\gamma}\begin{cases} \geq 0 & \text{if } \gamma \in [\hat{\gamma}_2, \bar{\gamma}] \\ \leq 0 & \text{if } \gamma \geq \bar{\gamma} \end{cases} \quad \text{and} \quad \lim_{\gamma \to +\infty} \tilde{T} = 0$$

These results indicate that the evolution of $\tilde{T}$ as a function of the parameter γ describes a U-inverted curve. We deduce that there are two ranges of values of γ for which $\tilde{T}$ gets nearer to zero, namely for all values in the interval $[0; \hat{\gamma}_2]$ or when γ approaches the infinity. In the next section, we will show that these analytical results are of great value for the study of inequality.

Finally, the growth rate of (non-adjusted) wage, $w(T)$, is

$$\frac{\dot{w}(T)}{w(T)} = g + \beta \frac{\left[\dot{\chi}(T) - g\chi(T)\right]}{e^{gT} w(T)} = g + \frac{\dot{\tilde{w}}(T)}{\tilde{w}(T)}$$

which is always positive. Thus, within a particular relation, the growth rate of wage is higher than the growth rate of technological progress as long as the duration of implementation of a technology is below $\tilde{T}$ units of time. After a higher duration, the growth rate of wage becomes lower than that of technological progress.

4.2 Dispersion of wages

We are also interested in studying inequality through the comparison between the highest and the lowest wage. This indicator is usual in literature. We know that the lowest wage is paid by the oldest firm. Hence, this wage is $\tilde{w}(T^*)$. According to the previous results, the highest wage depends on the value of γ. It is given by $\max\left\{\tilde{w}(0); \tilde{w}\left(\tilde{T}\right)\right\}$. In traditional matching model, the highest wage is always associated with the newest technology. It is not necessarily the case in our model. More precisely, when the learning process is fast, a non-monotone evolution of the adjusted-wage level throughout the career of a worker emerges. We denote by I_w the dispersion index:

$$I_w = \frac{\max\left\{\tilde{w}(0); \tilde{w}\left(\tilde{T}\right)\right\}}{\tilde{w}(T^*)}$$

So, we have to distinguish between three cases. Intuitively, according to the previous section, since $\tilde{T}$ gets near zero for two ranges of values of γ, we expect to obtain similar results for these values of the learning parameter. But, for some intermediate values, the results should be different. We indeed establish the following results:

- If $\gamma < gx/(s-x) \equiv \hat{\gamma}_2$, then,

$$I_{w0} = \frac{\tilde{w}(0)}{\tilde{w}(T_0^*)} = \frac{\beta x + (1-\beta)\,\omega(\theta_0^*)}{\omega(\theta_0^*)}$$

[6] When γ is sufficiently high in comparison with g, we are able to approximate the value of $\bar{\gamma}$: we find $\bar{\gamma} = \hat{\gamma}_2 + \left[gs\left(e^{1/2} - 1\right)\right]/(s-x)$.

which is always above one provided that $x > \omega(\theta^*)$. This ratio is decreasing with θ^*. Since θ^* is a decreasing function of g, the higher the growth rate of technological frontier is, the higher wages inequality between a beginner and a worker on a mature job is. And as θ^* is an increasing function of γ, the lower γ is, the higher wages inequality is.

- If $\gamma > gx/(s-x) \equiv \hat{\gamma}_2$, then,

$$I_w = \frac{\tilde{w}\left(\tilde{T}\right)}{\tilde{w}\left(T^*\right)} = \frac{\beta e^{-g\tilde{T}}\chi\left(\tilde{T}\right) + (1-\beta)\,\omega\left(\theta^*\right)}{\omega\left(\theta^*\right)}$$

It is straightforward to show that the term $e^{-g\tilde{T}}\chi\left(\tilde{T}\right)$ is increasing (respect. decreasing) with respect to γ (respect. g). Moreover, we know that the tightness parameter is increasing (respect. decreasing) with respect to γ (respect. g). Nevertheless, the final impact of γ and g on the inequality indicator when $\gamma > \hat{\gamma}_2$ is more difficult to determine. The two next subsections discuss this question.

- If $\gamma \to +\infty$,

$$I_{w\infty} = \frac{\tilde{w}(0)}{\tilde{w}\left(T^*_\infty\right)} = \frac{\beta s + (1-\beta)\,\omega\left(\theta^*_\infty\right)}{\omega\left(\theta^*_\infty\right)}$$

which is very similar to the first indicator I_{w0} expect that x is replaced with s.. Obviously, this result comes from the fact that the individual productivity is equal to s when $\gamma \to +\infty$. So, we have the same conclusions about the evolution of inequality with respect to g and γ. However, we have to notice that we cannot compare the index $I_{w\infty}$ with the index I_{w0}, since on the one hand, $s > x$, and on the other hand, $\omega\left(\theta^*_\infty\right) > \omega\left(\theta^*_0\right)$.

Impact of the rate of technical change

In this section, we argue that the effects of the rate of technological change on wages inequality are not monotone, depending on the learning process. Compared to the existing literature, this is a new result.

According to the previous section, we know that when $\gamma \to 0$ or $\gamma \to +\infty$, the index of wages inequality is an increasing function of g. But, for some intermediate values of the parameter γ, the conclusion becomes less clear. Formally, we have:

$$\frac{dI_w}{dg}\begin{cases} > 0 & \text{if } \dot{\chi}\left(\tilde{T}\right)/\chi\left(\tilde{T}\right)\tilde{T} = g\tilde{T} < \left|\varepsilon_{\omega(\theta^*)/g}\right| \\ < 0 & \text{if } \dot{\chi}\left(\tilde{T}\right)/\chi\left(\tilde{T}\right)\tilde{T} = g\tilde{T} > \left|\varepsilon_{\omega(\theta^*)/g}\right| \end{cases}$$

where $\varepsilon_{\omega(\theta^*)/g}$ is the elasticity of $\omega\left(\theta^*\right)$ with respect to g, which is always negative. According to the section 4.1, for extreme values of γ, $\tilde{T} \to 0$, and then, the sign of dI_w/dg is likely to be positive. But, for intermediate values of γ, we know that $\tilde{T}$ can reach some high values and the sign of dI_w/dg may be negative. This result indicates that if we want to say something about on how the rate of technological change affects wages inequality, we have to take into account another dimension of technological change, namely its impact on the learning process on the job.

What are the intuitions? Recall that there are two sources of heterogeneity in this model : the different vintage of machines used by workers and the seniority of these workers. New machines are always better than old ones but as long as a firm continues to use its current technology, its employee accumulates expertise in that technology. To understand the effects on wage inequalities induced by an increase of g, we must take two effects into account.

- Firstly, any increase of g means an increase of the obsolescence of installed technologies. The productivity gap between two vintage technologies increases. Therefore, the heterogeneity of wages increases among jobs.

- Secondly, any increase of g means a shorter learning process for the highest wage (associated to $\tilde{T}$), and the lowest wage (associated with T^*) which correspond to the outside option, namely $\omega(\theta^*)$. Both wages decrease but, as the return of learning is decreasing, the highest wage is more reduced than the other. Therefore, the heterogeneity of wages decreases among jobs.

5 Renovation of technology

A firm might find profitable to update its technology rather than destroy its job. This will be the case if there is a date of updating such that $T^o \leq T^*$. First, we calculate this date. We assume, like Mortensen and Pissarides [1998] that there is a fixed cost, I, paid by a firm when it updates its technology. Hence, the value of a filled job can be written as follows:

$$
\begin{aligned}
J(\tau,t) &= \underset{T}{Max} \int_t^{\tau+T} e^{-(r+\delta)(\nu-t)} \left[p(\tau)\chi(\tau,\nu) - w(\tau,\nu)\right] d\nu \\
&\quad + e^{-(r+\delta)(\tau+T-t)} \left[J(\tau+T;\tau+T) - p(\tau+T)I\right] \\
&= \underset{T}{Max}\,(1-\beta) \int_t^{\tau+T} e^{-(r+\delta)(\nu-t)} \left[p(\tau)\chi(\tau,\nu) - p(\nu)\omega(\theta)\right] d\nu \\
&\quad + e^{-(r+\delta)(\tau+T-t)} \left[J(\tau+T;\tau+T) - p(\tau+T)I\right]
\end{aligned}
$$

At a date $\tau = t$, we obtain

$$
\begin{aligned}
J &= \underset{T}{Max}\,(1-\beta) \int_0^{T} e^{-(r+\delta)\nu} \left[\chi(\nu) - e^{g\nu}\omega(\theta)\right] d\nu \\
&\quad + e^{-(r+\delta-g)T}\left[J - I\right]
\end{aligned} \tag{19}
$$

The first-order condition is then

$$
(1-\beta)\left[\chi(T^o) - e^{-gT^o}\omega(\theta)\right] = (r+\delta-g)\,e^{gT^o}\left[J - I\right]
$$

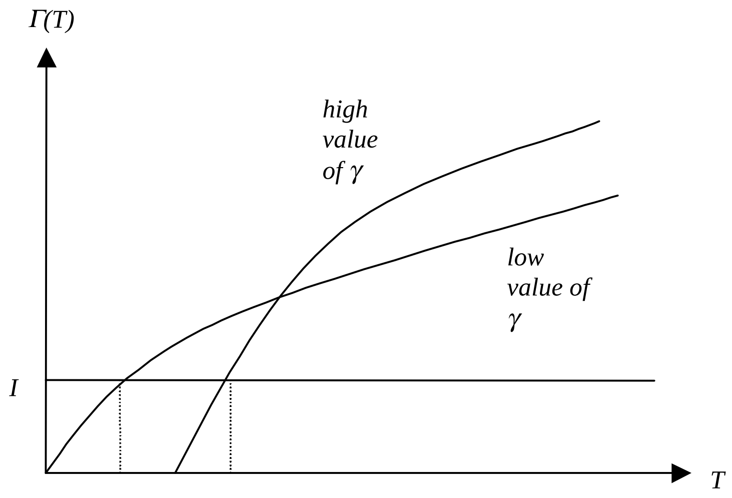

Figure 1: T^0 according to the value of γ

We introduce this expression in (19) in order to eliminate the wage:

$$I = (1-\beta) \int_0^{T^o} e^{-(r+\delta)\nu} \left[\chi(\nu) - e^{g(\nu - T^o)} \chi(T^o) \right] d\nu \tag{20}$$

Two reasons can explain the diffusion of new technologies : on the one hand, the collapse of the price of new equipment leads to a lower renovation cost and on the other hand, the decrease of the learning process is a factor which has magnified the effects of the collapse of prices. We can graphically illustrate this result: on the figure 1, we represent the function $\Gamma(T)$. We define (the study of this function is reported in the appendix):

$$\Gamma(T) \equiv (1-\beta) \int_0^{T} e^{-(r+\delta)\nu} \left[\chi(\nu) - e^{g(\nu - T)} \chi(T) \right] d\nu$$

On the figure 1, we see that for a relatively low renovation cost, a slow learning process leads to a shorter renovation date. The intuition is straightforward. When γ is high, a firm wish to take advantage of the possibility for a worker to accumulate expertise on its current technology rather than paying the renovation cost. When γ is low, the accumulation of expertise takes place too slowly such that a firm prefers to increase its productivity by changing its technology. In this case, firms update more frequently. By consequence, if the cost of renovation is low, contrary to the results from Mortensen and Pissarides [1998], the lower the value of a filled job is, the more often firms update.

6 Conclusion

In the model presented, the alteration of the learning process can be for instance represented by a decrease in the value of the parameter γ. Recall that this parameter describes the increase of productivity of a worker due to learning. In this model, we distinguish between two situations : a high and a low value of γ.

So long as the learning rate is high enough, firms expect to take advantage of a relatively high productivity level within a relatively short period. So, when this rate decreases, as the job creation and therefore the outside option of workers decreases, it might be profitable for firms to increase the lifetime of jobs. In other words, on the one hand the learning process is slower, but fast enough such that firms think it does pay to allow workers to reach a high level of productivity. That is why the optimal destruction date increases. Under these circumstances, jobs become less unstable, wage inequality between workers as well as the unemployment rate might decrease.

Without taking the learning process into account, the traditional matching model cannot explain why inequality can remain relatively stable when technical change evolves. In our model, such a phenomenon can emerge. More accurately, when the rate of technical change increases, the unemployment rate and job instability should increase. But this effect might be canceled by the alteration of the learning process. We indeed showed that from high initial value of the learning rate, any decrease of this rate might induce for instance a decrease of the unemployment rate.

By contrast, from a lower learning rate, the effects of a decrease of this rate might entail an increase of wage inequality and unemployment rate. The intuition for this result is as follows. When the learning rate is low, it takes a long time for a worker to reach a "good" level of productivity. So, it does not pay from the point of view of the firms to keep a worker for a long time because his own productivity increases too slowly. It is then more profitable to shorten the lifetime of jobs in order to take advantage of new technologies. That is why, under these circumstances the lifetime of jobs decreases, and wage inequality as well as the unemployment rate might increase. Note that these effects are reinforced by any increase in the rate of technical change.

An interesting extension of this model could be the study of different sources of learning. More particularly, as Laing, Palivos, and Wang [1995] do, we could assume that schooling and formal education enhance ability to acquire additional skills once employed. The level of schooling also acts as a key determinant of job instability and wage inequality. Reciprocally, job instability acts on the returns to education, and on the investment in schooling when young. Such an interaction between schooling and job instability is left for further research.

Appendix

Existence of a unique equilibrium

We want to establish the conditions to have a unique equilibrium from the system $(14)-(15)$. In the plane (θ, J), the first equation is positively sloped, concave and for $\theta = 0$, then $J = 0$. The second equation is negatively sloped. To have a solution with $\theta > 0$ and $J > 0$, from (15), we must have $J > 0$ for $\theta = 0$ and this is true if b is not too large.

Once we have a solution for θ, we can check that the solution for T, i.e. the endogenous length of job, is unique. To do so, we introduce the solution of θ in the equation (11) and we obtain the equilibrium value of T, denoted by T^* and given by:

$$se^{-gT^*} - (s-x)\, e^{-(g+\gamma)T^*} = \omega(\theta^*) > 0$$

First, we can notice that we have $se^{-gT} > (s-x)\, e^{-(g+\gamma)T}\ \forall T \geq 0$. Second, we have to distinguish two separate cases depending on the values of the parameters. Let us define the function $G(T) \equiv se^{-gT} - (s-x)\, e^{-(g+\gamma)T}$. This function is concave with $G(0) = x$ and $\lim_{T\to+\infty} G(T) = 0$. The highest value is obtained for a particular value of T, denoted by $\hat{T}$ with $\hat{T} = \gamma^{-1} \ln\left[\frac{gs}{(g+\gamma)(s-x)}\right]$. We deduce that we have $\hat{T} < 0$ if and only if $x > \gamma s/(g+\gamma)$ and $\hat{T} > 0$ if and only if $x < \gamma s/(g+\gamma)$.

In the first case, i.e. when $x > \gamma s/(g+\gamma)$, the function G is strictly decreasing in T for all $T > 0$ and there is a unique solution T^* such that $se^{-gT^*} - (s-x)\, e^{-(g+\gamma)T^*} = \omega(\theta^*)$ provided that $\omega(\theta^*) < x$, i.e. provided that b and/or c and/or β are sufficiently small.

In the second case, i.e. $x > \gamma s/(g+\gamma)$, the function G is increasing in T and then decreasing for all $T > \hat{T}$. So, if $\omega(\theta^*) < x$, there is a unique solution like in the previous case. But if $\omega(\theta^*) > x$, there are two potential solutions. In fact, it is straightforward to show that the second order condition $(\chi(T^*) \geq \gamma s/(g+\gamma)$, see the footnote (4)) is satisfied only in the decreasing part of the curve representative of G. However, to have a solution $\omega(\theta^*)$ doesn't have to be too large. More precisely, the highest value of G is given by $G\left(\hat{T}\right)$:

$$G\left(\hat{T}\right) = \left[\frac{(g+\gamma)(s-x)}{gs}\right]^{\frac{g}{\gamma}} \left[x - \frac{(s-x)}{gs}\left[s\gamma - (g+\gamma)x\right]\right]$$

So, we have a solution if $\omega(\theta^*) \leq G\left(\hat{T}\right)$, i.e. if b and/or c and/or β are sufficiently small.

Impact of g on T*

First, we differentiate the system formed by (14) and (15). We obtain

$$\begin{bmatrix} 1 & -\eta(\theta^*)\, c \\ 1 & (1-\beta)\int_0^{T^*} e^{-(r+\delta-g)v}\omega'(\theta^*)\, dv \end{bmatrix} \begin{bmatrix} \frac{dJ^*}{dg} \\ \frac{d\theta^*}{dg} \end{bmatrix}$$

$$
= \left[\begin{array}{c} 0 \\ -(1-\beta)\int_0^{T^*} e^{-(r+\delta-g)v}\omega(\theta^*)\, v dv \end{array} \right]
$$

where we denote by A, the Jacobian matrix, (2×2), of this system and $\eta(\theta^*) \equiv \left(1 - \varepsilon_{m(\theta)/\theta}\right)/m(\theta)$ with $\varepsilon_{m(\theta)/\theta}$, the elasticity of $m(\theta)$ with respect to θ. This elasticity is smaller than one. It is straightforward to see that we have $\det A > 0$ for all positive values of T^*. Then, by Cramer's rule, we deduce the sign of $d\theta^*/dg$:

$$
\frac{d\theta^*}{dg} = \frac{-(1-\beta)\int_0^{T^*} e^{-(r+\delta-g)v}\omega(\theta^*)\, v dv}{\det A} < 0
$$

Second, we introduce the expression in (17), and after some calculus, we finally obtain:

$$
\begin{aligned}
& e^{-gT^*}\frac{dT^*}{dg}\left[\gamma(s-x)e^{-\gamma T^*} - g e^{gT^*}\omega(\theta)\right] \\
& = \omega(\theta^*)\, T^* \left\{ \frac{\frac{\beta c}{(r+\delta-g)}\left[1 - \frac{1-e^{-(r+\delta-g)T^*}}{(r+\delta-g)T^*}\right] + \eta(\theta^*)\, c}{\frac{\beta c}{(r+\delta-g)}\left[1 - e^{-(r+\delta-g)T^*}\right] + \eta(\theta^*)\, c} \right\}
\end{aligned}
$$

The term in brackets to the left of just above equality is negative. Hence, dT^*/dg has the same sign as the opposite of the term to the right of this equality. A sufficient condition for the term in brackets to be positive is

$$
1 - e^{-(r+\delta-g)T^*} < (r+\delta-g)\, T^*
$$

It turns out that this inequality is always true provided that T^* has any positive value. We conclude that we have

$$
\frac{dT^*}{dg} < 0
$$

Impact of γ on $\mathbf{T}^*$

We study the equilibrium effects of γ on the following equality:

$$
\omega(\theta^*) = e^{-gT^*}\chi(T^*)
$$

We know that the left-hand side is increasing with respect to γ. Hence, any decrease of γ induces a decrease of the right-hand side. We have to take two effects into account. First, a direct effect and second, an indirect effect of γ. The direct effect on the right-hand side is always positive. The indirect effect works through T^* and the right-hand side is decreasing with respect of T^* (we can check that we have $\partial e^{-gT^*}\chi(T^*)/\partial T^* < 0$). So, the overall effect of a decrease of γ on T^* depends on the extent of the direct effect of γ.

Then, we have to evaluate the extent of the direct effect of γ on $e^{-gT^*}\chi(T^*)$ according to the initial value of γ. We find that, for any given value of T^*, the first derivative is positive and the second derivative is negative. This implies that when γ decreases, the right-hand side of the above equality decreases, but the lower γ is, the higher this decrease is. In other words, when γ has an high initial value, the direct effect seems to be negligible but becomes more significant for lower initial values of γ.

These results allow to conjecture the following general result: T^* is likely to be decreasing (respect. increasing) with respect to γ for high (respect. low) initial value of γ.

Study of $\mathbf{\Gamma}(T)$

$$\Gamma(T) \equiv (1-\beta)\int_0^T e^{-(r+\delta)(\nu)}\left[\chi(\nu) - e^{g(\nu-T)}\chi(T)\right]d\nu$$

First, notice that we have $\Gamma(0) = 0$. Second, the derivative gives the following result

$$\begin{aligned}\frac{\partial\Gamma(T)}{\partial T} &= \frac{(1-\beta)\left[e^{-gT} - e^{-(r+\delta)T}\right]}{(r+\delta-g)}\left[gs - (g+\gamma)(s-x)e^{-\gamma T}\right] \\ &= \frac{(1-\beta)\left[e^{-gT} - e^{-(r+\delta)T}\right]}{(r+\delta-g)}\left[g\chi(T) - \gamma(s-x)e^{-\gamma T}\right]\end{aligned}$$

For $r > g$, sign$\partial\Gamma(T)/\partial T =$sign$\psi(T)$ where we define $\psi(T) \equiv \left[gs - (g+\gamma)(s-x)e^{-\gamma T}\right]$. Moreover, for some values of parameters, we can have $\psi(0) < 0$, and in this case, the slope of $\Gamma(T)$ is negative for $T \in \left[0, \widetilde{T}\right]$. The value for $\widetilde{T}$ is given by

$$\widetilde{T} = \frac{-1}{\gamma}\log\left(\frac{gs}{(g+\gamma)(s-x)}\right)$$

For $T \in \left]\widetilde{T}, \infty\right[$, the slope of $\Gamma(T)$ is strictly positive with an horizontal asymptote:

$$\lim_{T\to+\infty}\Gamma(T) = \frac{(1-\beta)}{(r+\delta)(r+\delta+\gamma)}\left[\gamma s + (r+\delta)x\right] > 0$$

The value of this asymptote is strictly increasing in γ.

We have $\psi(0) < 0$ if $(g+\gamma)x - \gamma s < 0$, i.e. if γ is sufficiently high. We also can calculate the slope in a neighbourhood of zero. We obtain:

$$\lim_{T\to 0^+} e^{-gT} - e^{-(r+\delta)T} = 0^+$$

and

$$\lim_{T\to 0^+} g\chi(T) - \gamma(s-x)e^{-\gamma T} = (g+\gamma)x - \gamma s$$

Hence,

$$\begin{aligned}\tfrac{\partial\Gamma}{\partial T}(T=0) &> 0 \quad \text{for any sufficiently low value of } \gamma \\ &< 0 \quad \text{otherwise.}\end{aligned}$$

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